Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.4-e-x-^m-a+b-x^n-^p-c+d-x^n-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1081]. This is test number [27].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1081)	% 0. (0)
Mathematica	% 100. (1081)	% 0. (0)
Maple	% 66.6 (720)	% 33.4 (361)
Maxima	% 19.89 (215)	% 80.11 (866)
Fricas	% 61.24 (662)	% 38.76 (419)
Sympy	% 33.77 (365)	% 66.23 (716)
Giac	% 45.33 (490)	% 54.67 (591)

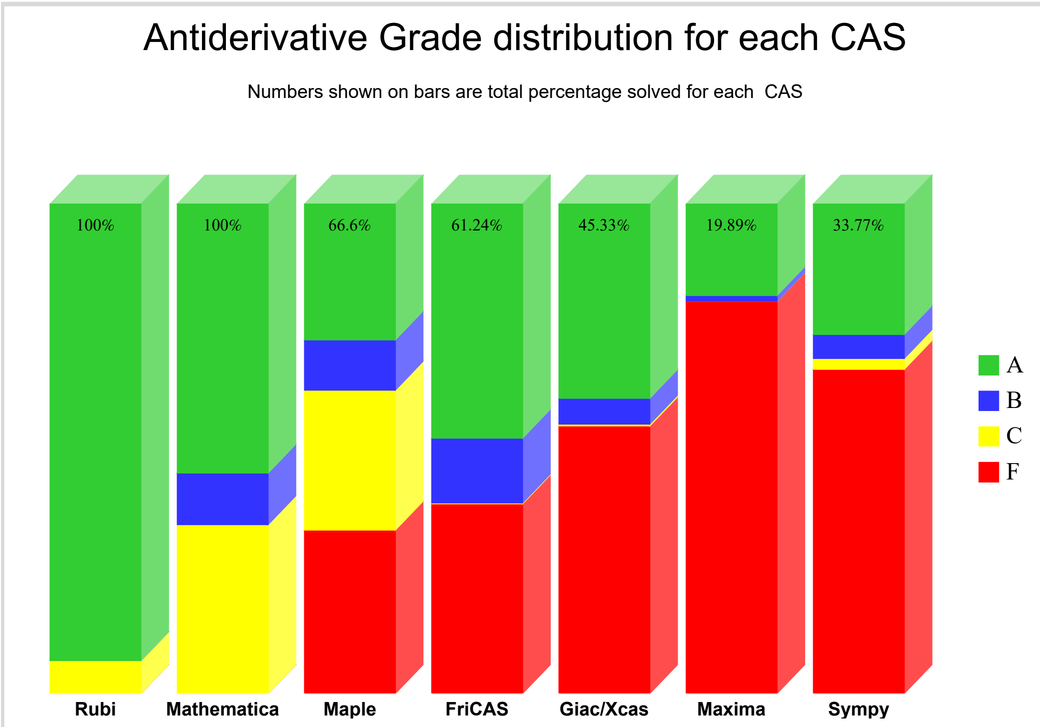
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

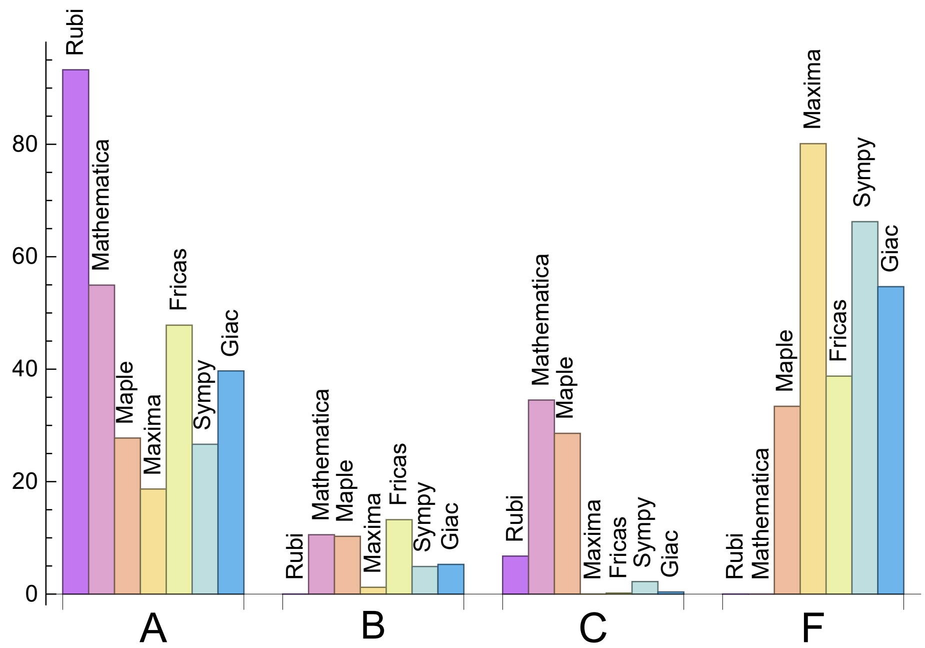
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	93.25	0.	6.75	0.
Mathematica	54.95	10.55	34.51	0.
Maple	27.75	10.27	28.58	33.4
Maxima	18.69	1.2	0.	80.11
Fricas	47.83	13.23	0.19	38.76
Sympy	26.64	4.9	2.22	66.23
Giac	39.69	5.27	0.37	54.67

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.31	203.5	0.99	106.	1.
Mathematica	0.22	136.98	1.16	104.	0.89
Maple	0.02	883.22	5.54	425.	1.69
Maxima	1.08	112.54	1.85	103.	1.4
Fricas	2.86	893.42	6.01	497.	4.58
Sympy	20.22	187.81	1.83	107.	1.05
Giac	1.33	220.3	1.85	159.	1.42

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {455, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 622, 634, 635, 636, 637, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 754, 755, 756, 758, 759, 760, 795, 796, 797, 798, 799, 800, 816, 817, 818, 819, 820, 821}

Mathematica {264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 513, 514, 515, 574, 575, 576, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 724, 725, 726, 727, 730, 731, 738, 739, 740, 741, 742, 743, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 813, 814, 815, 816, 817, 818, 819, 820, 821, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 845, 850, 861, 862, 863, 864, 865, 866, 867, 868, 869, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1011, 1012}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

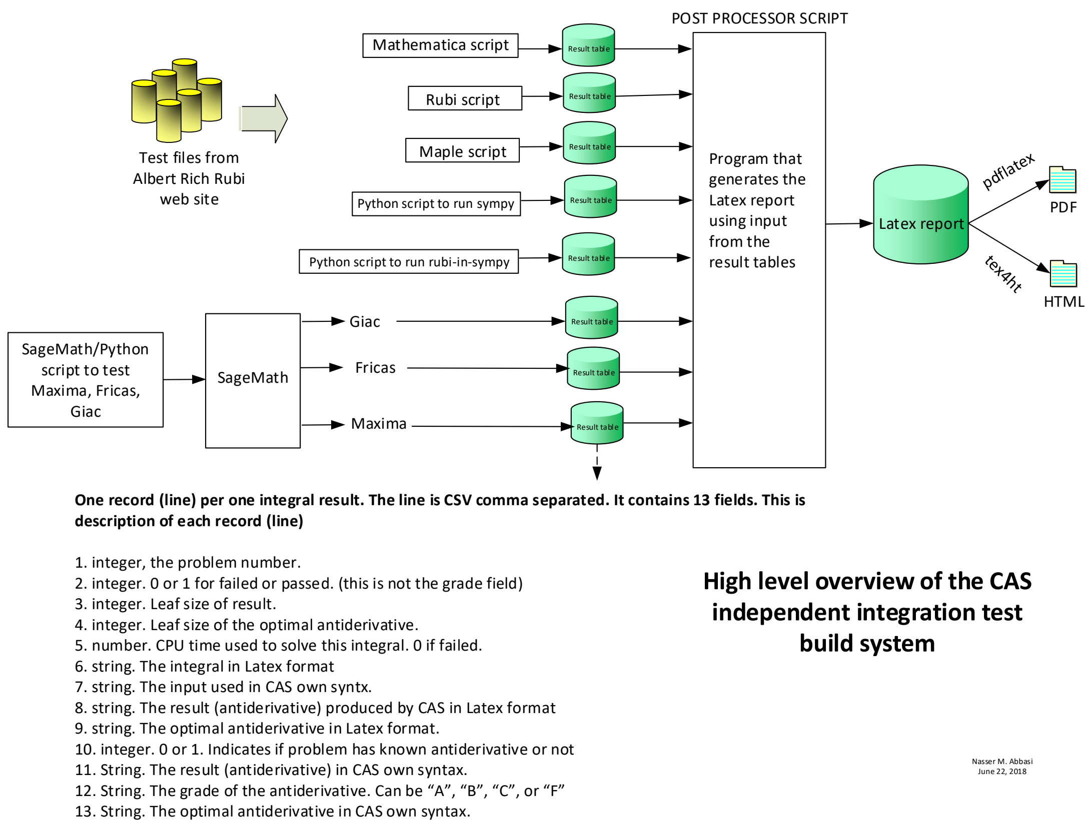
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 757, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081 }

B grade: { }

C grade: { 455, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 622, 634, 635, 636, 637, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 754, 755, 756, 758, 759, 760 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 161, 163, 166, 169, 171, 174, 177, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 244, 245, 246, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 278, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 321, 327, 358, 359, 360, 361, 362, 364, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 389, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 443, 460, 461, 462, 463, 464, 470, 471, 473, 474, 480, 481, 482, 483, 484, 490, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 598, 599, 605, 606, 607, 608, 609, 610, 611, 614, 616, 617, 623, 624, 625, 626, 627, 628, 641, 651, 658, 659, 660, 661, 662, 663, 664, 693, 696, 697, 698, 699, 700, 701, 723, 728, 729, 732, 733, 734, 735, 736, 737, 744, 763, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 793, 801, 802, 803, 805, 806, 807, 808, 809, 810, 811, 812, 822, 823, 824, 825, 826, 827, 828, 829, 830, 833, 841, 842, 843, 844, 846, 847, 848, 849, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 863, 864, 865, 870, 871, 872, 873, 874, 875, 876, 887, 888, 889, 890, 891, 892, 893, 894, 903, 904, 908, 909, 910, 911, 912, 913, 914, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 945, 946, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078 }

B grade: { 30, 54, 267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 366, 367, 373, 374, 375, 376, 377, 385, 386, 387, 393, 394, 395, 396, 397, 437, 438, 439, 440, 441, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 513, 514, 515, 672, 673, 674, 675, 676, 691, 692, 694, 695, 709, 710, 711, 712, 713, 727, 730, 731, 743, 745, 746, 761, 762, 764, 765, 766, 804, 845, 850, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 905, 906, 907, 924, 925, 926, 927, 928, 1014, 1051, 1052, 1053 }

C grade: { 156, 157, 159, 160, 162, 164, 165, 167, 168, 170, 172, 173, 175, 176, 178, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 276, 277, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 388, 390, 391, 392, 405, 406, 407, 408, 409, 410, 414, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 472, 491, 492, 493, 494, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 600, 601, 602, 603, 604, 612, 613, 615, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 738, 739, 740, 741, 742, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 792, 794, 795, 796, 797, 798, 799, 800, 813, 814, 815, 816, 817, 818, 819, 820, 821, 831, 832, 834, 835, 836, 837, 838, 839, 840, 861, 862, 877, 878, 879, 895, 896, 897, 898, 899, 900, 901, 902, 915, 916, 917, 918, 919, 920, 921, 922, 923, 944, 947, 948, 959, 961, 984, 985, 1024, 1055, 1079, 1080, 1081 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 557, 562, 565, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 929, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 945, 946, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 962, 963, 964, 965, 966, 967, 968, 969, 970, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1029, 1036, 1040, 1041, 1042, 1043, 1044, 1047, 1048, 1049, 1050, 1056, 1057, 1058, 1059, 1078, 1079, 1080 }

B grade: { 27, 30, 54, 124, 125, 126, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 932, 943, 944, 947, 948, 959, 960, 961, 971, 1005, 1014, 1023, 1039, 1045, 1046, 1051, 1052, 1053, 1054, 1060, 1061, 1062, 1063, 1065 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 563, 564, 566, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840 }

F grade: { 127, 128, 129, 130, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 801, 802, 803, 804, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1024, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1055, 1064, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1081 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 88, 89, 90, 91, 92, 93, 94, 107, 110, 113, 116, 119, 122, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 179, 180, 181, 196, 197, 198, 213, 214, 215, 228, 229, 230, 244, 245, 246, 605, 606, 607, 608, 609, 623, 624, 625, 626, 638, 639, 640, 641, 642, 767, 768, 769, 770, 771, 772, 933, 934, 935, 936, 937, 938, 939, 940, 949, 950, 951, 952, 953, 954, 955, 956, 965, 966, 967, 968, 969, 976, 977, 978, 979, 980, 981, 982, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1055, 1056, 1057, 1058, 1064, 1078, 1079, 1080, 1081

B grade: { 30, 54, 1014, 1044, 1050, 1051, 1052, 1053, 1059, 1060, 1061, 1062, 1063 }

C grade: { }

F grade: { 56, 58, 59, 61, 62, 64, 65, 67, 68, 70, 71, 73, 74, 76, 77, 79, 80, 82, 83, 85, 86, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 111, 112, 114, 115, 117, 118, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 941, 942, 943, 944, 945, 946, 947, 948, 957, 958, 959, 960, 961, 962, 963, 964, 970, 971, 972, 973, 974, 975, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1054, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 161, 163, 166, 169, 171, 174, 177, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 390, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 464, 470, 471, 472, 473, 474, 481, 483, 484, 506, 509, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 605, 606, 607, 608, 609, 611, 612, 623, 624, 625, 626, 627, 628, 629, 630, 634, 637, 638, 639, 640, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 665, 667, 682, 683, 696, 697, 698, 699, 700, 701, 702, 703, 714, 715, 719, 720, 721, 722, 751, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 814, 822, 824, 826, 827, 828, 829, 833, 853, 854, 855, 856, 857, 858, 859, 862, 871, 873, 874, 875, 879, 887, 888, 889, 890, 891, 892, 893, 896, 909, 911, 912, 913, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059, 1064, 1066, 1067, 1068, 1069, 1071, 1072, 1073, 1074, 1075, 1078, 1079, 1080, 1081 }

B grade: { 27, 30, 54, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 124, 125, 126, 156, 157, 159, 160, 162, 164, 165, 167, 168, 170, 172, 173, 175, 176, 178, 276, 281, 317, 356, 357, 388, 389, 391, 392, 463, 480, 482, 490, 491, 492, 493, 494, 507, 508, 576, 614, 615, 616, 617, 620, 631, 632, 633, 641, 645, 646, 647, 648, 649, 650, 651, 666, 677, 678, 679, 680, 681, 684, 685, 686, 716, 717, 718, 732, 733, 734, 735, 736, 737, 738, 739, 747, 748, 749, 750, 752, 753, 754, 755, 779, 780, 781, 782, 783, 784, 785, 786, 812, 813, 823, 825, 830, 831, 832, 860, 861, 870, 872, 876, 877, 878, 894, 895, 908, 910, 914, 915, 916, 949, 975, 1014, 1036, 1044, 1050, 1051, 1052, 1053, 1054, 1055, 1060, 1061, 1062, 1063, 1065, 1070, 1076, 1077 }

C grade: { 610, 613 }

F grade: { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 577, 578, 579, 580, 581, 582, 583, 584, 585, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 621, 622, 635, 636, 652, 653, 654, 655, 656, 657, 668, 669, 670, 671, 672, 673, 674, 675, 676, 687, 688, 689, 690, 691, 692, 693, 694, 695, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 723, 724, 725, 726, 727, 728, 729, 730, 731, 740, 741, 742, 743, 744, 745, 746, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 863, 864, 865, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 112, 114, 115, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 159, 160, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 250, 251, 255, 256, 259, 260, 261, 262, 271, 272, 273, 282, 283, 284, 285, 286, 297, 298, 299, 310, 311, 312, 327, 328, 329, 358, 359, 360, 361, 369, 370, 371, 380, 381, 390, 391, 519, 522, 546, 549, 554, 787, 789, 791, 807, 808, 855, 856, 889, 890, 930, 931, 932, 933, 934, 935, 936, 941, 942, 943, 947, 948, 949, 958, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 973, 975, 976, 977, 978, 979, 982, 985, 1023, 1029, 1036, 1041, 1056, 1057, 1058, 1059, 1078 }

B grade: { 27, 30, 107, 108, 110, 111, 113, 117, 184, 201, 218, 232, 530, 533, 538, 767, 768, 769, 770, 774, 775, 776, 777, 929, 937, 938, 939, 940, 944, 945, 946, 950, 951, 952, 953, 954, 955, 956, 957, 959, 960, 961, 968, 974, 980, 981, 983, 984, 1024, 1051, 1052, 1053, 1060 }

C grade: { 127, 501, 502, 503, 518, 520, 521, 524, 531, 532, 539, 540, 544, 545, 547, 548, 550, 555, 841, 842, 843, 848, 849, 1013 }

F grade: { 52, 53, 54, 55, 116, 118, 119, 120, 121, 122, 123, 128, 129, 130, 155, 156, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 233, 243, 248, 249, 252, 253, 254, 257, 258, 263, 264, 265, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 379, 382, 383, 384, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 523, 525, 526, 527, 528, 529, 534, 535, 536, 537, 541, 542, 543, 551, 552, 553, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 771, 772, 773, 778, 779, 780, 781, 782, 783, 784, 785, 786, 788, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 845, 846, 847, 850, 851, 852, 853, 854, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1039, 1040, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1055, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1079, 1080, 1081 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 111, 112, 114, 115, 117, 118, 120, 121, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 525, 549, 562, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 681, 682, 683, 696, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 752, 753, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 853, 854, 855, 856, 857, 860, 870, 871, 872, 873, 874, 879, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 908, 909, 910, 911, 912, 913, 914, 916, 917, 929, 930, 931, 937, 938, 939, 940, 942, 943, 944, 945, 946, 957, 958, 959, 960, 961, 963, 973, 974, 1006, 1009, 1010, 1015, 1016, 1017, 1023, 1024, 1056, 1057, 1058, 1060, 1061, 1062, 1064, 1078 }

B grade: { 27, 30, 54, 124, 125, 126, 196, 197, 198, 508, 509, 680, 697, 750, 751, 773, 774, 775, 776, 777, 778, 831, 859, 861, 862, 875, 876, 877, 878, 915, 932, 933, 934, 935, 936, 941, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 962, 1007, 1008, 1051, 1052, 1053, 1054, 1055, 1079, 1080, 1081 }

C grade: { 779, 784, 785, 786 }

F grade: { 107, 110, 113, 116, 119, 122, 127, 128, 129, 130, 157, 165, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 780, 781, 782, 783, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 858, 863, 864, 865, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 964, 965, 966, 967, 968, 969, 970, 971, 972, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1011, 1012, 1013, 1014, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1059, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	74	29	39
normalized size	1	1.	1.	0.85	1.09	2.24	0.88	1.18
time (sec)	N/A	0.033	0.008	0.004	1.212	1.225	0.06	1.189

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	36	74	29	39
normalized size	1	1.	1.	0.85	1.09	2.24	0.88	1.18
time (sec)	N/A	0.017	0.005	0.	1.443	1.289	0.061	1.154

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	66	26	35
normalized size	1	1.	1.	0.89	1.14	2.36	0.93	1.25
time (sec)	N/A	0.013	0.005	0.	1.27	1.261	0.057	1.156

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	38	65	27	38
normalized size	1	1.	1.	0.97	1.31	2.24	0.93	1.31
time (sec)	N/A	0.021	0.008	0.02	1.367	1.428	0.254	1.197

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	36	66	26	39
normalized size	1	1.	1.	0.97	1.16	2.13	0.84	1.26
time (sec)	N/A	0.015	0.01	0.038	1.152	1.385	0.251	1.202

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	32	63	24	31
normalized size	1	1.	1.	0.86	1.14	2.25	0.86	1.11
time (sec)	N/A	0.016	0.009	0.003	1.266	1.381	0.258	1.155

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	38	70	26	54
normalized size	1	1.	1.	0.9	1.31	2.41	0.9	1.86
time (sec)	N/A	0.022	0.011	0.007	1.045	1.441	0.336	1.172

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	28	39	63	29	42
normalized size	1	1.	1.03	0.9	1.26	2.03	0.94	1.35
time (sec)	N/A	0.017	0.012	0.006	1.306	1.374	0.378	1.209

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	30	25	36	69	27	39
normalized size	1	1.	1.07	0.89	1.29	2.46	0.96	1.39
time (sec)	N/A	0.016	0.011	0.006	1.331	1.405	0.409	1.132

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	28	41	73	27	50
normalized size	1	1.	1.07	0.97	1.41	2.52	0.93	1.72
time (sec)	N/A	0.021	0.016	0.006	1.336	1.474	0.65	1.197

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	69	131	54	72
normalized size	1	1.	1.21	1.24	1.64	3.12	1.29	1.71
time (sec)	N/A	0.072	0.015	0.001	1.127	1.266	0.068	1.189

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	69	131	54	72
normalized size	1	1.	1.	0.95	1.25	2.38	0.98	1.31
time (sec)	N/A	0.034	0.008	0.	1.683	1.266	0.069	1.187

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	123	51	68
normalized size	1	1.	1.	0.98	1.3	2.46	1.02	1.36
time (sec)	N/A	0.025	0.007	0.002	1.282	1.222	0.067	1.168

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	52	70	116	53	70
normalized size	1	1.	1.11	1.13	1.52	2.52	1.15	1.52
time (sec)	N/A	0.033	0.014	0.001	1.217	1.449	0.282	1.115

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	69	116	49	70
normalized size	1	1.	1.	1.	1.3	2.19	0.92	1.32
time (sec)	N/A	0.028	0.016	0.004	1.204	1.422	0.28	1.107

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	119	49	65
normalized size	1	1.	1.	0.98	1.3	2.38	0.98	1.3
time (sec)	N/A	0.027	0.014	0.003	1.153	1.387	0.285	1.215

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	51	70	122	51	93
normalized size	1	1.	0.96	1.	1.37	2.39	1.	1.82
time (sec)	N/A	0.05	0.023	0.007	1.113	1.611	0.401	1.269

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	50	72	119	51	73
normalized size	1	1.	0.96	0.94	1.36	2.25	0.96	1.38
time (sec)	N/A	0.031	0.016	0.006	1.171	1.706	0.442	1.199

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	46	69	119	51	69
normalized size	1	1.	1.	0.92	1.38	2.38	1.02	1.38
time (sec)	N/A	0.028	0.018	0.004	1.281	1.69	0.471	1.122

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	51	73	122	49	95
normalized size	1	1.	1.	1.	1.43	2.39	0.96	1.86
time (sec)	N/A	0.038	0.019	0.006	1.194	1.568	0.863	1.203

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	48	73	119	54	76
normalized size	1	1.	1.02	0.91	1.38	2.25	1.02	1.43
time (sec)	N/A	0.029	0.015	0.006	1.031	1.446	1.021	1.171

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	69	119	51	72
normalized size	1	1.	1.	0.9	1.38	2.38	1.02	1.44
time (sec)	N/A	0.029	0.021	0.006	1.175	1.339	1.102	1.2

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	323	136	169
normalized size	1	1.	1.	1.06	1.38	2.76	1.16	1.44
time (sec)	N/A	0.098	0.02	0.004	1.127	1.241	0.084	1.167

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	124	161	316	136	169
normalized size	1	1.	1.13	1.31	1.69	3.33	1.43	1.78
time (sec)	N/A	0.237	0.025	0.002	1.409	1.264	0.085	1.137

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	319	134	169
normalized size	1	1.	1.	1.06	1.38	2.73	1.15	1.44
time (sec)	N/A	0.075	0.018	0.002	1.198	1.279	0.086	1.208

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	319	136	169
normalized size	1	1.	1.	1.06	1.38	2.73	1.16	1.44
time (sec)	N/A	0.068	0.018	0.002	1.089	1.227	0.085	1.441

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	107	124	161	309	138	169
normalized size	1	1.	1.6	1.85	2.4	4.61	2.06	2.52
time (sec)	N/A	0.148	0.024	0.002	1.266	1.257	0.087	1.181

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	313	136	169
normalized size	1	1.	1.	1.06	1.38	2.68	1.16	1.44
time (sec)	N/A	0.07	0.016	0.002	1.175	1.257	0.086	1.245

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	308	133	167
normalized size	1	1.	1.	1.06	1.38	2.63	1.14	1.43
time (sec)	N/A	0.065	0.016	0.001	1.285	1.228	0.085	1.163

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	107	124	161	305	136	169
normalized size	1	1.	2.55	2.95	3.83	7.26	3.24	4.02
time (sec)	N/A	0.069	0.025	0.001	1.322	1.286	0.084	1.17

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	161	305	134	167
normalized size	1	1.	1.	1.06	1.38	2.61	1.15	1.43
time (sec)	N/A	0.063	0.016	0.001	1.395	1.224	0.087	1.162

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	121	155	290	128	162
normalized size	1	1.	1.	1.11	1.42	2.66	1.17	1.49
time (sec)	N/A	0.055	0.015	0.001	1.31	1.29	0.083	1.194

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	113	124	162	269	134	167
normalized size	1	1.	1.28	1.41	1.84	3.06	1.52	1.9
time (sec)	N/A	0.067	0.027	0.002	1.717	1.43	0.376	1.134

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	125	159	281	129	167
normalized size	1	1.	1.	1.12	1.42	2.51	1.15	1.49
time (sec)	N/A	0.061	0.033	0.004	1.252	1.436	0.376	1.121

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	120	157	279	128	161
normalized size	1	1.	1.	1.07	1.4	2.49	1.14	1.44
time (sec)	N/A	0.061	0.03	0.004	1.114	1.369	0.377	1.229

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	162	281	133	193
normalized size	1	1.	1.02	1.09	1.43	2.49	1.18	1.71
time (sec)	N/A	0.117	0.039	0.007	1.485	1.476	0.506	1.228

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	163	275	131	171
normalized size	1	1.	1.02	1.09	1.44	2.43	1.16	1.51
time (sec)	N/A	0.067	0.034	0.006	1.143	1.412	0.545	1.193

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	119	162	275	131	167
normalized size	1	1.	1.	1.05	1.43	2.43	1.16	1.48
time (sec)	N/A	0.063	0.036	0.006	1.001	1.368	0.596	1.207

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	106	124	165	273	129	200
normalized size	1	1.	0.93	1.09	1.45	2.39	1.13	1.75
time (sec)	N/A	0.106	0.046	0.009	1.334	1.474	1.044	1.18

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	117	163	274	126	171
normalized size	1	1.	1.	1.06	1.48	2.49	1.15	1.55
time (sec)	N/A	0.067	0.033	0.007	1.052	1.436	1.224	1.158

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	114	162	271	129	167
normalized size	1	1.	1.	1.01	1.43	2.4	1.14	1.48
time (sec)	N/A	0.062	0.033	0.004	1.223	1.43	1.284	1.16

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	106	124	166	270	126	203
normalized size	1	1.	0.93	1.09	1.46	2.37	1.11	1.78
time (sec)	N/A	0.104	0.051	0.008	1.331	1.491	2.51	1.16

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	118	111	165	273	126	171
normalized size	1	1.	1.03	0.97	1.43	2.37	1.1	1.49
time (sec)	N/A	0.063	0.023	0.007	1.143	1.411	3.318	1.207

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	108	162	275	126	167
normalized size	1	1.	1.	0.99	1.49	2.52	1.16	1.53
time (sec)	N/A	0.064	0.036	0.007	1.184	1.417	3.84	1.19

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	118	124	166	274	124	201
normalized size	1	1.	1.04	1.09	1.46	2.4	1.09	1.76
time (sec)	N/A	0.099	0.038	0.007	1.284	1.45	7.531	1.167

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	117	107	165	277	128	173
normalized size	1	1.	1.02	0.93	1.43	2.41	1.11	1.5
time (sec)	N/A	0.062	0.034	0.007	1.037	1.387	18.399	1.226

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	102	161	277	122	166
normalized size	1	1.	1.	0.93	1.46	2.52	1.11	1.51
time (sec)	N/A	0.068	0.041	0.009	1.392	1.413	27.684	1.229

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	116	123	166	282	122	196
normalized size	1	1.	1.03	1.09	1.47	2.5	1.08	1.73
time (sec)	N/A	0.091	0.055	0.009	1.11	1.449	36.786	1.195

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	118	104	165	281	126	173
normalized size	1	1.	1.03	0.9	1.43	2.44	1.1	1.5
time (sec)	N/A	0.062	0.031	0.006	1.037	1.434	52.724	1.181

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	101	161	285	122	169
normalized size	1	1.	1.	0.92	1.46	2.59	1.11	1.54
time (sec)	N/A	0.061	0.043	0.007	1.313	1.407	125.843	1.199

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	121	124	166	281	124	184
normalized size	1	1.	1.33	1.36	1.82	3.09	1.36	2.02
time (sec)	N/A	0.054	0.038	0.007	1.103	1.453	153.16	1.186

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	163	296	0	171
normalized size	1	1.	1.05	0.92	1.44	2.62	0.	1.51
time (sec)	N/A	0.064	0.03	0.008	1.128	1.413	0.	1.213

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	163	297	0	171
normalized size	1	1.	1.03	0.89	1.39	2.54	0.	1.46
time (sec)	N/A	0.059	0.031	0.007	1.117	1.406	0.	1.176

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	118	104	163	267	0	171
normalized size	1	1.	2.46	2.17	3.4	5.56	0.	3.56
time (sec)	N/A	0.032	0.029	0.009	1.294	1.327	0.	1.117

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	163	304	0	171
normalized size	1	1.	1.	0.89	1.39	2.6	0.	1.46
time (sec)	N/A	0.065	0.042	0.007	1.225	1.417	0.	1.116

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	171	249	0	397	110	293
normalized size	1	1.	0.93	1.36	0.	2.17	0.6	1.6
time (sec)	N/A	0.149	0.106	0.007	0.	1.527	1.221	1.123

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	68	108	44	70
normalized size	1	1.	0.87	1.15	1.26	2.	0.81	1.3
time (sec)	N/A	0.057	0.02	0.004	1.141	1.449	1.124	1.174

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	154	226	0	390	112	279
normalized size	1	1.	0.92	1.35	0.	2.34	0.67	1.67
time (sec)	N/A	0.12	0.082	0.004	0.	1.474	0.782	1.167

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	152	221	0	343	87	251
normalized size	1	1.	0.94	1.36	0.	2.12	0.54	1.55
time (sec)	N/A	0.116	0.078	0.003	0.	1.524	0.825	1.139

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	42	65	27	43
normalized size	1	1.	0.89	1.14	1.2	1.86	0.77	1.23
time (sec)	N/A	0.033	0.012	0.001	1.142	1.383	0.691	1.116

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	152	198	0	895	92	247
normalized size	1	1.	1.01	1.32	0.	5.97	0.61	1.65
time (sec)	N/A	0.09	0.048	0.001	0.	1.524	1.042	1.123

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	0	900	71	217
normalized size	1	1.	0.89	1.34	0.	6.21	0.49	1.5
time (sec)	N/A	0.077	0.061	0.002	0.	1.514	1.158	1.12

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	47	74	26	46
normalized size	1	1.	1.	1.09	1.38	2.18	0.76	1.35
time (sec)	N/A	0.033	0.013	0.006	1.206	1.48	1.937	1.132

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	134	195	0	891	90	239
normalized size	1	1.	0.91	1.33	0.	6.06	0.61	1.63
time (sec)	N/A	0.086	0.081	0.004	0.	1.503	0.743	1.19

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	135	195	0	960	73	217
normalized size	1	1.	0.91	1.31	0.	6.44	0.49	1.46
time (sec)	N/A	0.093	0.099	0.003	0.	1.574	0.857	1.122

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	56	65	111	41	93
normalized size	1	1.	0.98	1.12	1.3	2.22	0.82	1.86
time (sec)	N/A	0.049	0.021	0.006	1.296	1.451	1.36	1.101

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	154	216	0	371	112	266
normalized size	1	1.	0.93	1.31	0.	2.25	0.68	1.61
time (sec)	N/A	0.115	0.116	0.008	0.	1.482	1.409	1.144

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	154	217	0	419	99	238
normalized size	1	1.	0.92	1.29	0.	2.49	0.59	1.42
time (sec)	N/A	0.119	0.126	0.007	0.	1.509	1.694	1.132

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	70	81	95	158	61	134
normalized size	1	1.	1.01	1.17	1.38	2.29	0.88	1.94
time (sec)	N/A	0.064	0.028	0.007	1.368	1.585	2.771	1.127

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	173	247	0	424	139	292
normalized size	1	1.	0.94	1.34	0.	2.3	0.76	1.59
time (sec)	N/A	0.134	0.135	0.007	0.	1.702	1.302	1.139

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	203	288	0	636	153	329
normalized size	1	1.	0.87	1.24	0.	2.73	0.66	1.41
time (sec)	N/A	0.138	0.134	0.017	0.	1.806	1.59	1.129

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	72	97	111	251	78	143
normalized size	1	1.	0.88	1.18	1.35	3.06	0.95	1.74
time (sec)	N/A	0.091	0.065	0.016	0.962	1.727	1.535	1.11

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	185	266	0	599	151	319
normalized size	1	1.	0.86	1.24	0.	2.79	0.7	1.48
time (sec)	N/A	0.14	0.125	0.009	0.	1.657	2.52	1.104

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	181	257	0	551	124	285
normalized size	1	1.	0.85	1.21	0.	2.59	0.58	1.34
time (sec)	N/A	0.13	0.124	0.009	0.	1.777	1.44	1.127

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	74	81	165	56	123
normalized size	1	1.	0.83	1.23	1.35	2.75	0.93	2.05
time (sec)	N/A	0.058	0.035	0.01	0.97	1.662	1.208	1.121

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	165	235	0	1314	126	285
normalized size	1	1.	0.84	1.2	0.	6.7	0.64	1.45
time (sec)	N/A	0.11	0.119	0.01	0.	1.798	2.244	1.131

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	160	228	0	1289	102	254
normalized size	1	1.	0.84	1.2	0.	6.78	0.54	1.34
time (sec)	N/A	0.106	0.121	0.01	0.	1.837	1.916	1.137

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	54	92	36	88
normalized size	1	1.	1.	1.15	1.32	2.24	0.88	2.15
time (sec)	N/A	0.038	0.014	0.01	0.975	1.615	1.42	1.128

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	146	223	0	1235	117	273
normalized size	1	1.	0.85	1.3	0.	7.22	0.68	1.6
time (sec)	N/A	0.088	0.09	0.008	0.	1.792	1.048	1.121

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	0	1233	97	246
normalized size	1	1.	0.86	1.31	0.	7.3	0.57	1.46
time (sec)	N/A	0.086	0.09	0.007	0.	1.898	1.251	1.142

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	69	151	46	82
normalized size	1	1.	0.9	1.04	1.35	2.96	0.9	1.61
time (sec)	N/A	0.046	0.03	0.014	1.035	1.712	1.157	1.119

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	164	241	0	1296	122	273
normalized size	1	1.	0.84	1.23	0.	6.61	0.62	1.39
time (sec)	N/A	0.106	0.123	0.01	0.	1.841	1.366	1.142

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	163	237	0	1396	109	254
normalized size	1	1.	0.83	1.21	0.	7.12	0.56	1.3
time (sec)	N/A	0.103	0.125	0.012	0.	1.774	0.945	1.147

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	87	103	247	70	108
normalized size	1	1.	0.84	1.14	1.36	3.25	0.92	1.42
time (sec)	N/A	0.076	0.047	0.014	0.949	1.639	1.563	1.103

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	185	257	0	585	153	312
normalized size	1	1.	0.86	1.2	0.	2.72	0.71	1.45
time (sec)	N/A	0.127	0.136	0.013	0.	1.815	1.245	1.653

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	183	252	0	633	138	278
normalized size	1	1.	0.85	1.17	0.	2.94	0.64	1.29
time (sec)	N/A	0.127	0.143	0.013	0.	1.653	1.606	1.149

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	85	116	143	327	100	201
normalized size	1	1.	0.88	1.2	1.47	3.37	1.03	2.07
time (sec)	N/A	0.099	0.095	0.016	0.967	1.485	2.597	1.136

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	94	134	155	362	112	177
normalized size	1	1.	0.88	1.25	1.45	3.38	1.05	1.65
time (sec)	N/A	0.14	0.065	0.012	0.957	1.434	3.092	1.115

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	110	127	289	94	126
normalized size	1	1.	1.05	1.25	1.44	3.28	1.07	1.43
time (sec)	N/A	0.088	0.037	0.013	0.985	1.406	2.605	1.087

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	81	97	184	70	82
normalized size	1	1.	0.97	1.23	1.47	2.79	1.06	1.24
time (sec)	N/A	0.068	0.025	0.009	0.945	1.438	1.99	1.14

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	57	86	42	38
normalized size	1	1.	0.94	1.22	1.78	2.69	1.31	1.19
time (sec)	N/A	0.022	0.013	0.008	0.964	1.363	1.113	1.101

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	68	104	250	75	100
normalized size	1	1.	0.87	1.	1.53	3.68	1.1	1.47
time (sec)	N/A	0.058	0.045	0.013	0.961	1.5	1.11	1.104

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	87	117	147	412	107	184
normalized size	1	1.	0.86	1.16	1.46	4.08	1.06	1.82
time (sec)	N/A	0.1	0.055	0.015	1.002	1.468	3.182	1.137

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	108	147	184	478	133	177
normalized size	1	1.	0.89	1.2	1.51	3.92	1.09	1.45
time (sec)	N/A	0.13	0.077	0.014	0.958	1.543	9.366	1.127

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	216	308	0	838	189	350
normalized size	1	1.	0.88	1.25	0.	3.41	0.77	1.42
time (sec)	N/A	0.156	0.166	0.012	0.	1.496	3.722	1.127

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	210	299	0	779	162	316
normalized size	1	1.	0.86	1.23	0.	3.19	0.66	1.3
time (sec)	N/A	0.165	0.153	0.013	0.	1.513	2.61	1.14

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	194	275	0	1705	162	313
normalized size	1	1.	0.87	1.24	0.	7.68	0.73	1.41
time (sec)	N/A	0.14	0.155	0.011	0.	1.571	3.236	1.144

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	188	268	0	1712	141	282
normalized size	1	1.	0.85	1.22	0.	7.78	0.64	1.28
time (sec)	N/A	0.126	0.155	0.011	0.	1.603	2.223	1.137

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	181	241	0	1651	153	300
normalized size	1	1.	0.9	1.2	0.	8.21	0.76	1.49
time (sec)	N/A	0.107	0.168	0.01	0.	1.772	2.272	1.167

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	178	239	0	1651	134	274
normalized size	1	1.	0.89	1.2	0.	8.3	0.67	1.38
time (sec)	N/A	0.113	0.158	0.012	0.	1.97	1.444	1.121

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	178	251	0	1648	153	301
normalized size	1	1.	0.89	1.25	0.	8.2	0.76	1.5
time (sec)	N/A	0.115	0.129	0.01	0.	1.773	1.192	1.128

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	175	249	0	1648	133	273
normalized size	1	1.	0.89	1.26	0.	8.37	0.68	1.39
time (sec)	N/A	0.103	0.123	0.01	0.	1.58	0.987	1.147

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	193	281	0	1716	162	305
normalized size	1	1.	0.85	1.24	0.	7.56	0.71	1.34
time (sec)	N/A	0.123	0.166	0.012	0.	1.668	1.551	1.137

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	189	277	0	1785	143	282
normalized size	1	1.	0.83	1.22	0.	7.86	0.63	1.24
time (sec)	N/A	0.131	0.156	0.013	0.	1.648	2.012	1.14

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	214	299	0	817	189	343
normalized size	1	1.	0.87	1.22	0.	3.32	0.77	1.39
time (sec)	N/A	0.158	0.167	0.015	0.	1.568	3.876	1.159

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	210	295	0	876	173	309
normalized size	1	1.	0.85	1.2	0.	3.56	0.7	1.26
time (sec)	N/A	0.142	0.17	0.016	0.	1.512	10.684	1.126

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	92	142	201	0
normalized size	1	1.	0.94	0.93	1.31	2.03	2.87	0.
time (sec)	N/A	0.066	0.033	0.009	0.931	2.696	7.527	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	242	269	0	659	663	436
normalized size	1	1.	0.8	0.89	0.	2.19	2.2	1.45
time (sec)	N/A	0.31	0.143	0.008	0.	2.934	167.128	1.18

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	238	266	0	555	452	416
normalized size	1	1.	0.8	0.9	0.	1.88	1.53	1.41
time (sec)	N/A	0.267	0.122	0.009	0.	1.685	90.217	1.152

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	66	92	144	0
normalized size	1	1.	0.81	0.94	1.25	1.74	2.72	0.
time (sec)	N/A	0.05	0.02	0.007	0.931	1.793	2.897	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	246	0	590	573	386
normalized size	1	1.	0.78	0.85	0.	2.05	1.99	1.34
time (sec)	N/A	0.154	0.09	0.008	0.	1.67	11.714	1.137

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	246	0	487	342	375
normalized size	1	1.	0.78	0.85	0.	1.69	1.19	1.3
time (sec)	N/A	0.148	0.085	0.009	0.	1.625	18.328	1.161

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	55	69	138	0
normalized size	1	1.	0.69	0.93	1.22	1.53	3.07	0.
time (sec)	N/A	0.031	0.018	0.006	0.925	1.528	2.51	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	0	497	515	392
normalized size	1	1.	0.78	0.77	0.	1.73	1.79	1.36
time (sec)	N/A	0.143	0.099	0.007	0.	1.628	11.114	1.158

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	0	606	447	375
normalized size	1	1.	0.78	0.77	0.	2.1	1.55	1.3
time (sec)	N/A	0.142	0.1	0.007	0.	1.936	46.495	1.158

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	82	123	0	0
normalized size	1	1.	0.87	0.95	1.32	1.98	0.	0.
time (sec)	N/A	0.06	0.029	0.01	0.949	3.514	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	244	257	0	585	661	412
normalized size	1	1.	0.82	0.86	0.	1.96	2.21	1.38
time (sec)	N/A	0.272	0.137	0.01	0.	1.897	39.33	1.159

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	259	257	0	711	0	417
normalized size	1	1.	0.86	0.85	0.	2.36	0.	1.39
time (sec)	N/A	0.255	0.152	0.008	0.	7.551	0.	1.139

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	87	117	200	0	0
normalized size	1	1.	1.01	1.	1.34	2.3	0.	0.
time (sec)	N/A	0.092	0.04	0.01	0.959	11.267	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	282	291	0	697	0	443
normalized size	1	1.	0.89	0.92	0.	2.19	0.	1.39
time (sec)	N/A	0.38	0.182	0.012	0.	6.761	0.	1.124

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	282	293	0	811	0	454
normalized size	1	1.	0.88	0.91	0.	2.53	0.	1.41
time (sec)	N/A	0.456	0.179	0.011	0.	2.213	0.	1.166

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	124	158	255	0	0
normalized size	1	1.	1.	1.04	1.33	2.14	0.	0.
time (sec)	N/A	0.129	0.054	0.013	0.948	35.014	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	304	334	0	759	0	509
normalized size	1	1.	0.86	0.95	0.	2.16	0.	1.45
time (sec)	N/A	0.501	0.195	0.014	0.	2.639	0.	1.139

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	137	1078	0	2048	5418	1797
normalized size	1	1.	0.93	7.28	0.	13.84	36.61	12.14
time (sec)	N/A	0.099	0.173	0.026	0.	1.673	35.189	1.221

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	0	497	1057	448
normalized size	1	1.	0.93	3.69	0.	7.	14.89	6.31
time (sec)	N/A	0.04	0.051	0.008	0.	1.568	3.788	1.129

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	110	0	213	410	193
normalized size	1	1.	0.93	2.44	0.	4.73	9.11	4.29
time (sec)	N/A	0.021	0.029	0.004	0.	1.525	1.462	1.126

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0
normalized size	1	1.	0.83	0.	0.	0.	2.88	0.
time (sec)	N/A	0.035	0.059	0.104	0.	0.	26.943	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.058	0.044	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.056	0.033	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.063	0.061	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	85	46	39
normalized size	1	1.	0.85	0.82	0.92	2.18	1.18	1.
time (sec)	N/A	0.015	0.015	0.004	0.956	1.502	35.84	1.117

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	88	46	39
normalized size	1	1.	0.85	0.82	0.92	2.26	1.18	1.
time (sec)	N/A	0.016	0.014	0.004	0.935	1.441	17.561	1.105

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	85	46	39
normalized size	1	1.	0.85	0.82	0.92	2.18	1.18	1.
time (sec)	N/A	0.015	0.013	0.006	0.935	1.647	5.734	1.111

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	36	77	46	39
normalized size	1	1.	0.85	0.82	0.92	1.97	1.18	1.
time (sec)	N/A	0.016	0.013	0.003	0.961	1.695	2.524	1.091

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	36	76	44	39
normalized size	1	1.	0.89	0.86	0.97	2.05	1.19	1.05
time (sec)	N/A	0.015	0.013	0.005	0.947	1.71	2.07	1.104

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	32	36	76	44	39
normalized size	1	1.	0.95	0.86	0.97	2.05	1.19	1.05
time (sec)	N/A	0.015	0.01	0.004	0.948	1.702	2.521	1.099

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	34	32	36	69	46	39
normalized size	1	1.	0.87	0.82	0.92	1.77	1.18	1.
time (sec)	N/A	0.015	0.011	0.004	0.937	1.712	3.106	1.129

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	32	36	74	42	39
normalized size	1	1.	0.97	0.86	0.97	2.	1.14	1.05
time (sec)	N/A	0.016	0.011	0.004	0.983	1.746	4.236	1.107

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	138	80	72
normalized size	1	1.	0.84	0.89	1.1	2.19	1.27	1.14
time (sec)	N/A	0.031	0.03	0.006	0.946	1.672	51.847	1.08

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	69	149	80	72
normalized size	1	1.	1.	0.89	1.1	2.37	1.27	1.14
time (sec)	N/A	0.031	0.028	0.009	0.959	1.683	46.928	1.13

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	147	80	72
normalized size	1	1.	0.84	0.89	1.1	2.33	1.27	1.14
time (sec)	N/A	0.03	0.028	0.009	0.937	1.637	18.718	1.13

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	69	134	80	72
normalized size	1	1.	0.84	0.89	1.1	2.13	1.27	1.14
time (sec)	N/A	0.029	0.027	0.006	0.936	1.706	4.467	1.131

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	69	135	78	72
normalized size	1	1.	0.87	0.92	1.13	2.21	1.28	1.18
time (sec)	N/A	0.029	0.029	0.007	0.948	1.644	7.225	1.108

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	60	56	69	131	78	72
normalized size	1	1.	0.98	0.92	1.13	2.15	1.28	1.18
time (sec)	N/A	0.03	0.019	0.006	0.941	1.689	7.952	1.101

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	56	69	124	80	72
normalized size	1	1.	0.9	0.89	1.1	1.97	1.27	1.14
time (sec)	N/A	0.03	0.016	0.008	0.948	1.744	9.449	1.107

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	69	130	76	72
normalized size	1	1.	0.93	0.92	1.13	2.13	1.25	1.18
time (sec)	N/A	0.031	0.016	0.005	0.943	1.726	12.01	1.16

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	71	80	99	194	114	104
normalized size	1	1.	0.84	0.94	1.16	2.28	1.34	1.22
time (sec)	N/A	0.044	0.041	0.004	0.946	1.626	114.892	1.135

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	211	114	104
normalized size	1	1.	1.	0.94	1.16	2.48	1.34	1.22
time (sec)	N/A	0.041	0.037	0.006	0.95	1.662	69.107	1.112

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	99	208	114	104
normalized size	1	1.	1.	0.94	1.16	2.45	1.34	1.22
time (sec)	N/A	0.042	0.041	0.009	0.937	1.733	45.046	1.123

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	71	80	99	182	114	104
normalized size	1	1.	0.84	0.94	1.16	2.14	1.34	1.22
time (sec)	N/A	0.041	0.038	0.007	0.992	1.725	7.637	1.126

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	80	99	192	112	104
normalized size	1	1.	1.	0.96	1.19	2.31	1.35	1.25
time (sec)	N/A	0.043	0.039	0.009	0.941	1.672	18.321	1.136

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	81	80	99	190	112	104
normalized size	1	1.	0.98	0.96	1.19	2.29	1.35	1.25
time (sec)	N/A	0.041	0.024	0.006	0.942	1.725	26.727	1.121

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	80	99	171	112	104
normalized size	1	1.	0.91	0.94	1.16	2.01	1.32	1.22
time (sec)	N/A	0.045	0.023	0.008	0.967	1.843	35.207	1.119

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	80	99	186	110	104
normalized size	1	1.	0.94	0.96	1.19	2.24	1.33	1.25
time (sec)	N/A	0.041	0.024	0.006	0.936	1.676	39.641	1.112

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	67	78	0	321	0	86
normalized size	1	1.	0.92	1.07	0.	4.4	0.	1.18
time (sec)	N/A	0.048	0.08	0.013	0.	1.773	0.	1.12

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	54	371	0	5104	0	390
normalized size	1	1.	0.19	1.29	0.	17.72	0.	1.35
time (sec)	N/A	0.519	0.049	0.115	0.	2.343	0.	1.161

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	45	350	0	7606	867	0
normalized size	1	1.	0.17	1.3	0.	28.17	3.21	0.
time (sec)	N/A	0.551	0.041	0.043	0.	2.752	79.262	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	53	0	261	0	53
normalized size	1	1.	0.98	1.	0.	4.92	0.	1.
time (sec)	N/A	0.036	0.045	0.007	0.	1.769	0.	1.106

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	43	347	0	5096	864	378
normalized size	1	1.	0.16	1.29	0.	19.01	3.22	1.41
time (sec)	N/A	0.473	0.026	0.03	0.	2.305	39.569	1.142

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	46	349	0	7648	867	378
normalized size	1	1.	0.17	1.3	0.	28.54	3.24	1.41
time (sec)	N/A	0.552	0.013	0.034	0.	3.068	54.164	1.375

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	0	282	0	53
normalized size	1	1.	1.	1.	0.	5.32	0.	1.
time (sec)	N/A	0.037	0.028	0.01	0.	2.089	0.	1.096

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	47	352	0	5083	0	378
normalized size	1	1.	0.17	1.3	0.	18.83	0.	1.4
time (sec)	N/A	0.485	0.015	0.035	0.	2.381	0.	1.126

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	93	0	478	0	92
normalized size	1	1.	0.81	0.98	0.	5.03	0.	0.97
time (sec)	N/A	0.055	0.104	0.016	0.	1.764	0.	1.112

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	76	399	0	5975	0	423
normalized size	1	1.	0.24	1.28	0.	19.15	0.	1.36
time (sec)	N/A	0.502	0.076	0.039	0.	2.523	0.	1.202

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	62	381	0	8817	0	0
normalized size	1	1.	0.21	1.32	0.	30.51	0.	0.
time (sec)	N/A	0.549	0.051	0.037	0.	2.694	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	74	0	414	0	85
normalized size	1	1.	1.	1.04	0.	5.83	0.	1.2
time (sec)	N/A	0.042	0.052	0.015	0.	1.823	0.	1.631

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	68	381	0	5873	0	408
normalized size	1	1.	0.24	1.32	0.	20.32	0.	1.41
time (sec)	N/A	0.468	0.03	0.037	0.	2.336	0.	1.181

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	70	395	0	9007	0	414
normalized size	1	1.	0.22	1.24	0.	28.32	0.	1.3
time (sec)	N/A	0.686	0.074	0.042	0.	2.745	0.	1.347

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	79	93	0	498	0	89
normalized size	1	1.	0.81	0.96	0.	5.13	0.	0.92
time (sec)	N/A	0.057	0.116	0.019	0.	1.93	0.	1.103

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	74	389	0	6660	0	423
normalized size	1	1.	0.23	1.22	0.	20.94	0.	1.33
time (sec)	N/A	0.504	0.086	0.039	0.	2.324	0.	1.179

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	96	0	664	0	113
normalized size	1	1.	0.89	0.92	0.	6.38	0.	1.09
time (sec)	N/A	0.057	0.142	0.016	0.	1.745	0.	1.118

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	92	416	0	6793	0	443
normalized size	1	1.	0.28	1.27	0.	20.77	0.	1.35
time (sec)	N/A	0.493	0.096	0.043	0.	2.56	0.	1.159

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	62	411	0	10242	0	443
normalized size	1	1.	0.19	1.26	0.	31.32	0.	1.35
time (sec)	N/A	0.597	0.054	0.042	0.	3.248	0.	1.295

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	97	0	664	0	113
normalized size	1	1.	0.9	0.93	0.	6.38	0.	1.09
time (sec)	N/A	0.06	0.073	0.014	0.	2.113	0.	1.135

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	91	401	0	6384	0	435
normalized size	1	1.	0.28	1.25	0.	19.89	0.	1.36
time (sec)	N/A	0.522	0.044	0.041	0.	2.618	0.	1.204

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	113	435	0	9719	0	444
normalized size	1	1.	0.32	1.24	0.	27.69	0.	1.26
time (sec)	N/A	0.616	0.09	0.045	0.	2.899	0.	1.403

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	102	133	0	738	0	119
normalized size	1	1.	0.78	1.02	0.	5.68	0.	0.92
time (sec)	N/A	0.072	0.153	0.02	0.	1.93	0.	1.119

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	96	429	0	7052	0	451
normalized size	1	1.	0.27	1.22	0.	20.09	0.	1.28
time (sec)	N/A	0.541	0.11	0.043	0.	2.401	0.	1.175

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	75	77	159	220	219	144
normalized size	1	1.	0.73	0.75	1.54	2.14	2.13	1.4
time (sec)	N/A	0.084	0.055	0.007	0.929	1.715	3.671	1.114

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	113	166	168	107
normalized size	1	1.	0.78	0.73	1.55	2.27	2.3	1.47
time (sec)	N/A	0.06	0.039	0.005	0.937	1.692	1.612	1.104

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	66	113	117	63
normalized size	1	1.	0.74	0.67	1.43	2.46	2.54	1.37
time (sec)	N/A	0.04	0.023	0.007	0.922	1.64	0.629	1.126

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	60	50	0	301	76	82
normalized size	1	1.	0.94	0.78	0.	4.7	1.19	1.28
time (sec)	N/A	0.043	0.047	0.184	0.	1.773	18.829	1.097

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	63	72	0	333	134	92
normalized size	1	1.	0.75	0.86	0.	3.96	1.6	1.1
time (sec)	N/A	0.063	0.04	0.021	0.	1.825	23.239	1.14

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	93	96	0	397	160	162
normalized size	1	1.	1.06	1.09	0.	4.51	1.82	1.84
time (sec)	N/A	0.07	0.068	0.022	0.	1.624	50.891	1.112

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	89	658	0	0	83	0
normalized size	1	1.	0.29	2.17	0.	0.	0.27	0.
time (sec)	N/A	0.151	0.133	0.021	0.	0.	2.269	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	75	618	0	0	82	0
normalized size	1	1.	0.28	2.31	0.	0.	0.31	0.
time (sec)	N/A	0.091	0.063	0.014	0.	0.	1.901	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	81	596	0	0	85	0
normalized size	1	1.	0.3	2.22	0.	0.	0.32	0.
time (sec)	N/A	0.099	0.062	0.02	0.	0.	2.223	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	80	616	0	0	94	0
normalized size	1	1.	0.29	2.26	0.	0.	0.35	0.
time (sec)	N/A	0.103	0.083	0.021	0.	0.	2.873	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	80	660	0	0	97	0
normalized size	1	1.	0.26	2.16	0.	0.	0.32	0.
time (sec)	N/A	0.138	0.09	0.023	0.	0.	4.832	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	91	966	0	0	83	0
normalized size	1	1.	0.16	1.66	0.	0.	0.14	0.
time (sec)	N/A	0.355	0.132	0.021	0.	0.	2.826	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	75	926	0	0	83	0
normalized size	1	1.	0.14	1.69	0.	0.	0.15	0.
time (sec)	N/A	0.241	0.09	0.016	0.	0.	2.048	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	81	902	0	0	85	0
normalized size	1	1.	0.15	1.66	0.	0.	0.16	0.
time (sec)	N/A	0.249	0.068	0.017	0.	0.	2.221	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	80	920	0	0	92	0
normalized size	1	1.	0.15	1.68	0.	0.	0.17	0.
time (sec)	N/A	0.251	0.08	0.02	0.	0.	2.403	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	80	964	0	0	97	0
normalized size	1	1.	0.14	1.66	0.	0.	0.17	0.
time (sec)	N/A	0.32	0.083	0.023	0.	0.	3.037	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	80	1006	0	0	97	0
normalized size	1	1.	0.13	1.64	0.	0.	0.16	0.
time (sec)	N/A	0.379	0.097	0.024	0.	0.	4.232	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	159	286	267	323
normalized size	1	1.	0.76	0.75	1.54	2.78	2.59	3.14
time (sec)	N/A	0.079	0.06	0.007	0.95	1.695	8.342	1.16

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	113	219	216	247
normalized size	1	1.	0.78	0.73	1.55	3.	2.96	3.38
time (sec)	N/A	0.058	0.038	0.006	0.927	1.713	5.316	1.207

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	66	162	165	162
normalized size	1	1.	0.74	0.67	1.43	3.52	3.59	3.52
time (sec)	N/A	0.04	0.025	0.005	0.924	1.586	2.781	1.19

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	66	0	409	82	108
normalized size	1	1.	0.99	0.81	0.	5.05	1.01	1.33
time (sec)	N/A	0.057	0.067	0.017	0.	1.742	39.024	1.268

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	80	101	0	405	223	139
normalized size	1	1.	0.73	0.92	0.	3.68	2.03	1.26
time (sec)	N/A	0.084	0.047	0.02	0.	1.811	28.635	1.22

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	59	107	0	439	243	177
normalized size	1	1.	0.51	0.93	0.	3.82	2.11	1.54
time (sec)	N/A	0.087	0.026	0.023	0.	1.819	62.971	1.156

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	93	694	0	0	172	0
normalized size	1	1.	0.28	2.07	0.	0.	0.51	0.
time (sec)	N/A	0.175	0.132	0.023	0.	0.	6.061	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	77	654	0	0	170	0
normalized size	1	1.	0.26	2.19	0.	0.	0.57	0.
time (sec)	N/A	0.124	0.057	0.014	0.	0.	4.053	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	83	629	0	0	172	0
normalized size	1	1.	0.28	2.13	0.	0.	0.58	0.
time (sec)	N/A	0.127	0.049	0.02	0.	0.	3.934	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	82	626	0	0	184	0
normalized size	1	1.	0.28	2.11	0.	0.	0.62	0.
time (sec)	N/A	0.121	0.071	0.022	0.	0.	4.301	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	82	653	0	0	196	0
normalized size	1	1.	0.27	2.16	0.	0.	0.65	0.
time (sec)	N/A	0.126	0.078	0.024	0.	0.	5.502	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	96	1002	0	0	172	0
normalized size	1	1.	0.16	1.63	0.	0.	0.28	0.
time (sec)	N/A	0.37	0.135	0.023	0.	0.	4.833	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	78	962	0	0	172	0
normalized size	1	1.	0.13	1.66	0.	0.	0.3	0.
time (sec)	N/A	0.292	0.099	0.014	0.	0.	3.599	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	573	573	83	937	0	0	173	0
normalized size	1	1.	0.14	1.64	0.	0.	0.3	0.
time (sec)	N/A	0.294	0.036	0.017	0.	0.	3.974	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	578	578	85	932	0	0	182	0
normalized size	1	1.	0.15	1.61	0.	0.	0.31	0.
time (sec)	N/A	0.292	0.074	0.021	0.	0.	6.096	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	576	576	82	957	0	0	194	0
normalized size	1	1.	0.14	1.66	0.	0.	0.34	0.
time (sec)	N/A	0.297	0.075	0.025	0.	0.	4.951	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	608	608	82	1002	0	0	199	0
normalized size	1	1.	0.13	1.65	0.	0.	0.33	0.
time (sec)	N/A	0.355	0.075	0.023	0.	0.	6.483	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	159	173	175	140
normalized size	1	1.	0.76	0.75	1.54	1.68	1.7	1.36
time (sec)	N/A	0.074	0.055	0.006	0.938	1.722	3.368	1.139

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	53	112	117	124	99
normalized size	1	1.	0.77	0.73	1.53	1.6	1.7	1.36
time (sec)	N/A	0.055	0.04	0.008	0.929	1.685	1.592	1.119

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	65	69	75	58
normalized size	1	1.	0.72	0.65	1.41	1.5	1.63	1.26
time (sec)	N/A	0.038	0.022	0.006	0.938	1.747	0.768	1.111

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	0	252	65	54
normalized size	1	1.	1.	0.77	0.	5.25	1.35	1.12
time (sec)	N/A	0.032	0.026	0.016	0.	1.865	6.973	1.143

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	62	0	302	80	84
normalized size	1	1.	0.98	1.07	0.	5.21	1.38	1.45
time (sec)	N/A	0.046	0.039	0.02	0.	1.78	16.302	1.15

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	81	102	0	409	163	163
normalized size	1	1.	0.9	1.13	0.	4.54	1.81	1.81
time (sec)	N/A	0.07	0.206	0.022	0.	1.822	31.319	1.122

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	89	624	0	0	80	0
normalized size	1	1.	0.33	2.31	0.	0.	0.3	0.
time (sec)	N/A	0.101	0.077	0.023	0.	0.	3.575	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	74	586	0	0	78	0
normalized size	1	1.	0.31	2.45	0.	0.	0.33	0.
time (sec)	N/A	0.063	0.037	0.016	0.	0.	2.245	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	78	587	0	0	82	0
normalized size	1	1.	0.32	2.42	0.	0.	0.34	0.
time (sec)	N/A	0.069	0.031	0.019	0.	0.	1.9	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	78	625	0	0	90	0
normalized size	1	1.	0.28	2.28	0.	0.	0.33	0.
time (sec)	N/A	0.102	0.033	0.022	0.	0.	2.447	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	91	932	0	0	80	0
normalized size	1	1.	0.17	1.7	0.	0.	0.15	0.
time (sec)	N/A	0.245	0.077	0.022	0.	0.	2.456	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	75	892	0	0	80	0
normalized size	1	1.	0.15	1.73	0.	0.	0.15	0.
time (sec)	N/A	0.186	0.054	0.013	0.	0.	2.094	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	77	891	0	0	82	0
normalized size	1	1.	0.15	1.75	0.	0.	0.16	0.
time (sec)	N/A	0.195	0.031	0.021	0.	0.	1.757	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	78	929	0	0	88	0
normalized size	1	1.	0.14	1.69	0.	0.	0.16	0.
time (sec)	N/A	0.252	0.035	0.02	0.	0.	2.175	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	78	970	0	0	94	0
normalized size	1	1.	0.13	1.67	0.	0.	0.16	0.
time (sec)	N/A	0.308	0.034	0.023	0.	0.	2.95	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	77	157	186	175	131
normalized size	1	1.	0.75	0.75	1.52	1.81	1.7	1.27
time (sec)	N/A	0.077	0.052	0.007	0.943	1.756	3.708	1.142

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	52	109	131	124	88
normalized size	1	1.	0.75	0.71	1.49	1.79	1.7	1.21
time (sec)	N/A	0.055	0.034	0.007	0.936	1.74	1.808	1.125

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	63	85	75	47
normalized size	1	1.	0.72	0.65	1.37	1.85	1.63	1.02
time (sec)	N/A	0.037	0.022	0.006	0.942	1.753	0.885	1.139

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	57	0	371	56	72
normalized size	1	1.	1.	0.98	0.	6.4	0.97	1.24
time (sec)	N/A	0.04	0.052	0.026	0.	1.816	9.944	1.097

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	57	100	0	508	264	134
normalized size	1	1.	0.66	1.16	0.	5.91	3.07	1.56
time (sec)	N/A	0.065	0.018	0.022	0.	1.728	78.477	1.123

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	120	60	141	0	630	0	185
normalized size	1	1.02	0.51	1.19	0.	5.34	0.	1.57
time (sec)	N/A	0.09	0.02	0.026	0.	1.795	0.	1.126

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	103	666	0	0	80	0
normalized size	1	1.	0.34	2.22	0.	0.	0.27	0.
time (sec)	N/A	0.139	0.099	0.027	0.	0.	51.451	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	78	627	0	0	80	0
normalized size	1	1.	0.29	2.33	0.	0.	0.3	0.
time (sec)	N/A	0.1	0.076	0.018	0.	0.	16.395	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	73	613	0	0	78	0
normalized size	1	1.	0.29	2.44	0.	0.	0.31	0.
time (sec)	N/A	0.063	0.033	0.018	0.	0.	9.667	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	86	631	0	0	82	0
normalized size	1	1.	0.32	2.32	0.	0.	0.3	0.
time (sec)	N/A	0.101	0.035	0.021	0.	0.	39.287	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	72	667	0	0	90	0
normalized size	1	1.	0.24	2.19	0.	0.	0.3	0.
time (sec)	N/A	0.136	0.034	0.026	0.	0.	139.688	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	547	79	937	0	0	80	0
normalized size	1	1.	0.14	1.71	0.	0.	0.15	0.
time (sec)	N/A	0.255	0.069	0.026	0.	0.	22.51	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	524	71	921	0	0	80	0
normalized size	1	1.	0.14	1.76	0.	0.	0.15	0.
time (sec)	N/A	0.195	0.059	0.017	0.	0.	10.191	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	72	939	0	0	82	0
normalized size	1	1.	0.13	1.71	0.	0.	0.15	0.
time (sec)	N/A	0.244	0.031	0.023	0.	0.	28.518	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	580	72	975	0	0	88	0
normalized size	1	1.	0.12	1.68	0.	0.	0.15	0.
time (sec)	N/A	0.304	0.033	0.024	0.	0.	107.752	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	611	72	1018	0	0	0	0
normalized size	1	1.	0.12	1.67	0.	0.	0.	0.
time (sec)	N/A	0.362	0.034	0.025	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	73	76	157	201	338	124
normalized size	1	1.	0.71	0.74	1.52	1.95	3.28	1.2
time (sec)	N/A	0.078	0.059	0.008	0.929	1.73	6.096	1.128

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	53	113	155	240	82
normalized size	1	1.	0.74	0.73	1.55	2.12	3.29	1.12
time (sec)	N/A	0.056	0.037	0.006	0.925	1.743	2.419	1.138

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	66	111	144	43
normalized size	1	1.	0.72	0.65	1.43	2.41	3.13	0.93
time (sec)	N/A	0.036	0.023	0.006	0.939	1.683	1.166	1.143

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	85	0	522	76	90
normalized size	1	1.	0.81	1.1	0.	6.78	0.99	1.17
time (sec)	N/A	0.05	0.022	0.027	0.	1.826	17.142	1.129

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	57	157	0	757	0	136
normalized size	1	1.	0.5	1.39	0.	6.7	0.	1.2
time (sec)	N/A	0.087	0.02	0.028	0.	1.779	0.	1.174

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	108	683	0	0	0	0
normalized size	1	1.	0.36	2.28	0.	0.	0.	0.
time (sec)	N/A	0.132	0.128	0.035	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	99	669	0	0	80	0
normalized size	1	1.	0.35	2.36	0.	0.	0.28	0.
time (sec)	N/A	0.103	0.112	0.022	0.	0.	171.476	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	103	674	0	0	78	0
normalized size	1	1.	0.36	2.38	0.	0.	0.28	0.
time (sec)	N/A	0.099	0.055	0.018	0.	0.	170.077	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	116	689	0	0	0	0
normalized size	1	1.	0.39	2.3	0.	0.	0.	0.
time (sec)	N/A	0.13	0.058	0.026	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	83	722	0	0	0	0
normalized size	1	1.	0.25	2.16	0.	0.	0.	0.
time (sec)	N/A	0.166	0.041	0.029	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	109	997	0	0	0	0
normalized size	1	1.	0.19	1.73	0.	0.	0.	0.
time (sec)	N/A	0.318	0.104	0.036	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	559	559	92	981	0	0	80	0
normalized size	1	1.	0.16	1.75	0.	0.	0.14	0.
time (sec)	N/A	0.265	0.094	0.022	0.	0.	173.245	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	81	986	0	0	80	0
normalized size	1	1.	0.14	1.75	0.	0.	0.14	0.
time (sec)	N/A	0.248	0.071	0.019	0.	0.	170.762	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	578	578	86	1001	0	0	0	0
normalized size	1	1.	0.15	1.73	0.	0.	0.	0.
time (sec)	N/A	0.3	0.038	0.026	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	83	1034	0	0	0	0
normalized size	1	1.	0.14	1.7	0.	0.	0.	0.
time (sec)	N/A	0.356	0.04	0.028	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	77	506	0	397	85	111
normalized size	1	1.	0.79	5.22	0.	4.09	0.88	1.14
time (sec)	N/A	0.094	0.062	0.187	0.	2.102	55.309	1.118

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	65	446	0	332	68	92
normalized size	1	1.	0.86	5.87	0.	4.37	0.89	1.21
time (sec)	N/A	0.062	0.028	0.01	0.	2.095	19.229	1.137

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	425	0	281	51	59
normalized size	1	1.	0.95	7.46	0.	4.93	0.89	1.04
time (sec)	N/A	0.046	0.014	0.006	0.	2.005	4.405	1.123

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	468	0	382	66	68
normalized size	1	1.	0.91	7.2	0.	5.88	1.02	1.05
time (sec)	N/A	0.056	0.016	0.019	0.	1.87	8.85	1.132

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	511	0	497	0	101
normalized size	1	1.	1.	5.81	0.	5.65	0.	1.15
time (sec)	N/A	0.077	0.04	0.022	0.	1.892	0.	1.105

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	689	689	133	1309	0	0	0	0
normalized size	1	1.	0.19	1.9	0.	0.	0.	0.
time (sec)	N/A	0.477	0.076	0.034	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	659	659	63	848	0	0	0	0
normalized size	1	1.	0.1	1.29	0.	0.	0.	0.
time (sec)	N/A	0.167	0.03	0.007	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	697	697	136	1306	0	0	0	0
normalized size	1	1.	0.2	1.87	0.	0.	0.	0.
time (sec)	N/A	0.384	0.071	0.02	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	236	1003	0	0	0	0
normalized size	1	1.	3.58	15.2	0.	0.	0.	0.
time (sec)	N/A	0.054	0.275	0.033	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	165	696	0	0	0	0
normalized size	1	1.	2.58	10.88	0.	0.	0.	0.
time (sec)	N/A	0.028	0.142	0.006	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	244	1002	0	0	0	0
normalized size	1	1.	3.7	15.18	0.	0.	0.	0.
time (sec)	N/A	0.056	0.127	0.02	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	467	0	332	0	86
normalized size	1	1.	0.83	5.99	0.	4.26	0.	1.1
time (sec)	N/A	0.074	0.054	0.035	0.	1.53	0.	1.106

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	425	0	294	65	66
normalized size	1	1.	0.95	7.2	0.	4.98	1.1	1.12
time (sec)	N/A	0.048	0.02	0.011	0.	1.573	16.486	1.132

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	413	0	232	37	39
normalized size	1	1.	1.	10.32	0.	5.8	0.92	0.98
time (sec)	N/A	0.036	0.009	0.098	0.	1.561	7.887	1.113

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	433	0	392	63	72
normalized size	1	1.	0.91	6.66	0.	6.03	0.97	1.11
time (sec)	N/A	0.055	0.018	0.018	0.	1.533	12.594	1.114

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	477	0	505	0	101
normalized size	1	1.	1.	5.42	0.	5.74	0.	1.15
time (sec)	N/A	0.077	0.032	0.02	0.	1.604	0.	1.122

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	667	667	67	848	0	0	0	0
normalized size	1	1.	0.1	1.27	0.	0.	0.	0.
time (sec)	N/A	0.235	0.032	0.033	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	67	416	0	5685	0	0
normalized size	1	1.	0.33	2.02	0.	27.6	0.	0.
time (sec)	N/A	0.032	0.026	0.007	0.	7.159	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	697	697	136	874	0	0	0	0
normalized size	1	1.	0.2	1.25	0.	0.	0.	0.
time (sec)	N/A	0.388	0.075	0.019	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	0	0	0
normalized size	1	1.	1.02	10.55	0.	0.	0.	0.
time (sec)	N/A	0.053	0.032	0.032	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	165	416	0	0	0	0
normalized size	1	1.	2.58	6.5	0.	0.	0.	0.
time (sec)	N/A	0.028	0.047	0.006	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	243	722	0	0	0	0
normalized size	1	1.	3.68	10.94	0.	0.	0.	0.
time (sec)	N/A	0.056	0.158	0.02	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	3299	0	0
normalized size	1	1.	0.22	1.29	0.	25.98	0.	0.
time (sec)	N/A	0.019	0.017	0.247	0.	3.352	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	81	582	0	420	99	135
normalized size	1	1.	0.73	5.24	0.	3.78	0.89	1.22
time (sec)	N/A	0.097	0.075	0.056	0.	1.8	155.865	1.091

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	70	507	0	360	82	112
normalized size	1	1.	0.78	5.63	0.	4.	0.91	1.24
time (sec)	N/A	0.085	0.058	0.011	0.	1.774	47.644	1.121

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	446	0	296	65	93
normalized size	1	1.	0.84	6.46	0.	4.29	0.94	1.35
time (sec)	N/A	0.057	0.032	0.007	0.	1.81	14.721	1.115

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	425	0	247	46	58
normalized size	1	1.	0.94	8.5	0.	4.94	0.92	1.16
time (sec)	N/A	0.043	0.017	0.007	0.	1.761	4.456	1.137

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	468	0	342	60	65
normalized size	1	1.	0.91	8.07	0.	5.9	1.03	1.12
time (sec)	N/A	0.054	0.017	0.011	0.	1.568	8.177	1.118

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	511	0	462	0	104
normalized size	1	1.	1.	6.31	0.	5.7	0.	1.28
time (sec)	N/A	0.072	0.029	0.012	0.	1.638	0.	1.142

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	96	574	0	439	0	126
normalized size	1	1.	0.9	5.36	0.	4.1	0.	1.18
time (sec)	N/A	0.094	0.052	0.024	0.	1.518	0.	1.115

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	648	648	150	1788	0	0	0	0
normalized size	1	1.	0.23	2.76	0.	0.	0.	0.
time (sec)	N/A	0.888	0.088	0.037	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	624	624	130	1310	0	0	0	0
normalized size	1	1.	0.21	2.1	0.	0.	0.	0.
time (sec)	N/A	0.747	0.077	0.01	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	601	601	63	848	0	0	0	0
normalized size	1	1.	0.1	1.41	0.	0.	0.	0.
time (sec)	N/A	0.532	0.033	0.007	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	632	632	137	1306	0	0	0	0
normalized size	1	1.	0.22	2.07	0.	0.	0.	0.
time (sec)	N/A	0.717	0.082	0.014	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	654	654	153	1782	0	0	0	0
normalized size	1	1.	0.23	2.72	0.	0.	0.	0.
time (sec)	N/A	0.838	0.094	0.022	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	678	678	164	2280	0	0	0	0
normalized size	1	1.	0.24	3.36	0.	0.	0.	0.
time (sec)	N/A	0.951	0.093	0.026	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	93	634	0	495	0	158
normalized size	1	1.	0.72	4.88	0.	3.81	0.	1.22
time (sec)	N/A	0.113	0.096	0.043	0.	1.38	0.	1.131

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	81	541	0	429	0	135
normalized size	1	1.	0.74	4.96	0.	3.94	0.	1.24
time (sec)	N/A	0.102	0.07	0.012	0.	1.384	0.	1.125

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	70	462	0	363	90	112
normalized size	1	1.	0.8	5.25	0.	4.12	1.02	1.27
time (sec)	N/A	0.072	0.036	0.01	0.	1.357	78.069	1.138

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	441	0	293	65	88
normalized size	1	1.	0.87	6.58	0.	4.37	0.97	1.31
time (sec)	N/A	0.057	0.035	0.01	0.	1.389	28.126	1.161

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	500	0	396	73	82
normalized size	1	1.	1.	6.85	0.	5.42	1.	1.12
time (sec)	N/A	0.07	0.026	0.021	0.	1.431	21.31	1.134

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	556	0	462	0	92
normalized size	1	1.	1.	7.13	0.	5.92	0.	1.18
time (sec)	N/A	0.072	0.042	0.023	0.	1.445	0.	1.101

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	96	617	0	532	0	127
normalized size	1	1.	0.92	5.93	0.	5.12	0.	1.22
time (sec)	N/A	0.095	0.052	0.026	0.	1.438	0.	1.103

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	669	669	163	1840	0	0	0	0
normalized size	1	1.	0.24	2.75	0.	0.	0.	0.
time (sec)	N/A	0.966	0.108	0.042	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	645	645	150	1344	0	0	0	0
normalized size	1	1.	0.23	2.08	0.	0.	0.	0.
time (sec)	N/A	0.827	0.08	0.012	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	627	627	127	864	0	0	0	0
normalized size	1	1.	0.2	1.38	0.	0.	0.	0.
time (sec)	N/A	0.723	0.079	0.009	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	626	626	137	1339	0	0	0	0
normalized size	1	1.	0.22	2.14	0.	0.	0.	0.
time (sec)	N/A	0.718	0.076	0.022	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	651	651	154	1810	0	0	0	0
normalized size	1	1.	0.24	2.78	0.	0.	0.	0.
time (sec)	N/A	0.844	0.091	0.025	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	675	675	167	2306	0	0	0	0
normalized size	1	1.	0.25	3.42	0.	0.	0.	0.
time (sec)	N/A	0.936	0.1	0.024	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	69	528	0	360	0	111
normalized size	1	1.	0.77	5.87	0.	4.	0.	1.23
time (sec)	N/A	0.081	0.07	0.036	0.	1.327	0.	1.112

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	58	468	0	296	0	88
normalized size	1	1.	0.82	6.59	0.	4.17	0.	1.24
time (sec)	N/A	0.069	0.049	0.013	0.	1.313	0.	1.107

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	425	0	255	61	65
normalized size	1	1.	0.94	8.17	0.	4.9	1.17	1.25
time (sec)	N/A	0.044	0.018	0.012	0.	1.323	17.751	1.109

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	413	0	193	32	36
normalized size	1	1.	1.	12.52	0.	5.85	0.97	1.09
time (sec)	N/A	0.033	0.01	0.008	0.	1.347	7.572	1.134

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	433	0	347	58	73
normalized size	1	1.	0.88	7.47	0.	5.98	1.	1.26
time (sec)	N/A	0.052	0.016	0.01	0.	1.305	10.774	1.119

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	477	0	462	0	103
normalized size	1	1.	1.	5.89	0.	5.7	0.	1.27
time (sec)	N/A	0.073	0.03	0.01	0.	1.372	0.	1.124

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	95	540	0	525	0	127
normalized size	1	1.	0.89	5.05	0.	4.91	0.	1.19
time (sec)	N/A	0.098	0.061	0.024	0.	1.342	0.	1.118

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	630	630	130	1311	0	0	0	0
normalized size	1	1.	0.21	2.08	0.	0.	0.	0.
time (sec)	N/A	0.723	0.082	0.033	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	601	601	67	848	0	0	0	0
normalized size	1	1.	0.11	1.41	0.	0.	0.	0.
time (sec)	N/A	0.581	0.032	0.01	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	67	416	0	5659	0	0
normalized size	1	1.	0.48	2.95	0.	40.13	0.	0.
time (sec)	N/A	0.413	0.026	0.007	0.	9.376	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	632	632	137	874	0	0	0	0
normalized size	1	1.	0.22	1.38	0.	0.	0.	0.
time (sec)	N/A	0.723	0.078	0.01	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	654	654	152	1351	0	0	0	0
normalized size	1	1.	0.23	2.07	0.	0.	0.	0.
time (sec)	N/A	0.83	0.091	0.025	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	678	678	167	1849	0	0	0	0
normalized size	1	1.	0.25	2.73	0.	0.	0.	0.
time (sec)	N/A	0.929	0.108	0.024	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	0	0	0
normalized size	1	1.	1.02	10.55	0.	0.	0.	0.
time (sec)	N/A	0.054	0.036	0.03	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	166	416	0	0	0	0
normalized size	1	1.	2.59	6.5	0.	0.	0.	0.
time (sec)	N/A	0.028	0.146	0.006	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	242	722	0	0	0	0
normalized size	1	1.	3.67	10.94	0.	0.	0.	0.
time (sec)	N/A	0.057	0.155	0.01	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	261	1047	0	0	0	0
normalized size	1	1.	3.95	15.86	0.	0.	0.	0.
time (sec)	N/A	0.059	0.19	0.025	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	66	560	0	436	0	111
normalized size	1	1.	0.73	6.22	0.	4.84	0.	1.23
time (sec)	N/A	0.104	0.045	0.053	0.	1.584	0.	1.539

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	53	501	0	377	0	78
normalized size	1	1.	0.75	7.06	0.	5.31	0.	1.1
time (sec)	N/A	0.073	0.033	0.01	0.	1.545	0.	1.121

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	456	0	344	58	63
normalized size	1	1.	0.94	8.77	0.	6.62	1.12	1.21
time (sec)	N/A	0.046	0.03	0.01	0.	1.548	19.006	1.13

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	43	435	0	339	51	65
normalized size	1	1.	0.78	7.91	0.	6.16	0.93	1.18
time (sec)	N/A	0.049	0.012	0.008	0.	1.558	18.408	1.123

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	63	485	0	517	78	92
normalized size	1	1.	0.83	6.38	0.	6.8	1.03	1.21
time (sec)	N/A	0.07	0.021	0.027	0.	1.531	19.061	1.108

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	77	549	0	629	0	128
normalized size	1	1.	0.77	5.49	0.	6.29	0.	1.28
time (sec)	N/A	0.095	0.029	0.024	0.	1.371	0.	1.141

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	91	636	0	695	0	146
normalized size	1	1.	0.71	4.97	0.	5.43	0.	1.14
time (sec)	N/A	0.124	0.036	0.029	0.	1.413	0.	1.131

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	629	629	127	1810	0	0	0	0
normalized size	1	1.	0.2	2.88	0.	0.	0.	0.
time (sec)	N/A	0.724	0.077	0.048	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	635	635	126	1346	0	0	0	0
normalized size	1	1.	0.2	2.12	0.	0.	0.	0.
time (sec)	N/A	0.73	0.079	0.01	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	632	632	124	875	0	0	0	0
normalized size	1	1.	0.2	1.38	0.	0.	0.	0.
time (sec)	N/A	0.709	0.07	0.009	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	653	653	140	1361	0	0	0	0
normalized size	1	1.	0.21	2.08	0.	0.	0.	0.
time (sec)	N/A	0.826	0.081	0.023	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	675	675	153	1864	0	0	0	0
normalized size	1	1.	0.23	2.76	0.	0.	0.	0.
time (sec)	N/A	0.921	0.087	0.026	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	699	699	167	2389	0	0	0	0
normalized size	1	1.	0.24	3.42	0.	0.	0.	0.
time (sec)	N/A	1.098	0.105	0.026	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	233	1038	0	0	0	0
normalized size	1	1.	3.53	15.73	0.	0.	0.	0.
time (sec)	N/A	0.061	0.168	0.036	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	230	721	0	0	0	0
normalized size	1	1.	3.59	11.27	0.	0.	0.	0.
time (sec)	N/A	0.03	0.128	0.005	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	248	1053	0	0	0	0
normalized size	1	1.	3.76	15.95	0.	0.	0.	0.
time (sec)	N/A	0.064	0.148	0.024	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	261	1402	0	0	0	0
normalized size	1	1.	3.95	21.24	0.	0.	0.	0.
time (sec)	N/A	0.064	0.213	0.028	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	737	737	80	977	0	0	0	0
normalized size	1	1.	0.11	1.33	0.	0.	0.	0.
time (sec)	N/A	0.294	0.085	0.166	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	757	757	80	924	0	0	0	0
normalized size	1	1.	0.11	1.22	0.	0.	0.	0.
time (sec)	N/A	0.288	0.086	0.261	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	774	774	87	926	0	0	0	0
normalized size	1	1.	0.11	1.2	0.	0.	0.	0.
time (sec)	N/A	0.281	0.064	0.068	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	768	768	90	983	0	0	0	0
normalized size	1	1.	0.12	1.28	0.	0.	0.	0.
time (sec)	N/A	0.283	0.054	0.066	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	738	738	80	977	0	0	0	0
normalized size	1	1.	0.11	1.32	0.	0.	0.	0.
time (sec)	N/A	0.251	0.09	0.145	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	758	758	80	924	0	0	0	0
normalized size	1	1.	0.11	1.22	0.	0.	0.	0.
time (sec)	N/A	0.237	0.078	0.134	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	774	774	89	926	0	0	0	0
normalized size	1	1.	0.11	1.2	0.	0.	0.	0.
time (sec)	N/A	0.237	0.049	0.067	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	768	768	89	983	0	0	0	0
normalized size	1	1.	0.12	1.28	0.	0.	0.	0.
time (sec)	N/A	0.231	0.071	0.065	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	318	318	83	538	0	0	0	0
normalized size	1	1.	0.26	1.69	0.	0.	0.	0.
time (sec)	N/A	0.064	0.067	0.065	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	83	509	0	0	0	0
normalized size	1	1.	0.26	1.57	0.	0.	0.	0.
time (sec)	N/A	0.07	0.073	0.063	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	85	510	0	0	0	0
normalized size	1	1.	0.26	1.55	0.	0.	0.	0.
time (sec)	N/A	0.067	0.071	0.06	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	87	541	0	0	0	0
normalized size	1	1.	0.26	1.64	0.	0.	0.	0.
time (sec)	N/A	0.067	0.067	0.059	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	83	538	0	0	0	0
normalized size	1	1.	0.27	1.74	0.	0.	0.	0.
time (sec)	N/A	0.051	0.09	0.063	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	316	316	83	509	0	0	0	0
normalized size	1	1.	0.26	1.61	0.	0.	0.	0.
time (sec)	N/A	0.054	0.073	0.063	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	84	510	0	12766	0	0
normalized size	1	1.	0.26	1.59	0.	39.89	0.	0.
time (sec)	N/A	0.053	0.066	0.06	0.	90.7	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	86	541	0	12785	0	0
normalized size	1	1.	0.27	1.68	0.	39.7	0.	0.
time (sec)	N/A	0.054	0.071	0.063	0.	79.134	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	121	514	0	603	128	188
normalized size	1	1.	0.97	4.11	0.	4.82	1.02	1.5
time (sec)	N/A	0.129	0.265	0.054	0.	1.37	50.936	1.137

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	458	0	427	95	130
normalized size	1	1.	0.95	4.92	0.	4.59	1.02	1.4
time (sec)	N/A	0.078	0.073	0.008	0.	1.331	15.584	1.102

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	434	0	331	68	89
normalized size	1	1.	1.	6.2	0.	4.73	0.97	1.27
time (sec)	N/A	0.059	0.03	0.006	0.	1.295	5.532	1.103

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	476	0	851	85	117
normalized size	1	1.	0.95	5.6	0.	10.01	1.	1.38
time (sec)	N/A	0.074	0.043	0.01	0.	1.463	10.453	1.126

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	107	518	0	1160	0	163
normalized size	1	1.	0.93	4.5	0.	10.09	0.	1.42
time (sec)	N/A	0.125	0.121	0.01	0.	1.545	0.	1.11

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	241	1012	0	0	0	0
normalized size	1	1.	3.77	15.81	0.	0.	0.	0.
time (sec)	N/A	0.053	0.414	0.029	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	857	0	0	0	0
normalized size	1	1.	1.02	13.39	0.	0.	0.	0.
time (sec)	N/A	0.037	0.027	0.03	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	161	705	0	0	0	0
normalized size	1	1.	2.73	11.95	0.	0.	0.	0.
time (sec)	N/A	0.026	0.152	0.006	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	139	1314	0	0	0	0
normalized size	1	1.	2.24	21.19	0.	0.	0.	0.
time (sec)	N/A	0.052	0.101	0.01	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	335	1010	0	0	0	0
normalized size	1	1.	5.23	15.78	0.	0.	0.	0.
time (sec)	N/A	0.052	0.247	0.008	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	145	605	0	883	0	261
normalized size	1	1.	0.94	3.93	0.	5.73	0.	1.69
time (sec)	N/A	0.155	0.152	0.033	0.	2.711	0.	1.13

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	111	531	0	639	116	204
normalized size	1	1.	0.92	4.42	0.	5.32	0.97	1.7
time (sec)	N/A	0.104	0.11	0.009	0.	1.975	94.566	1.111

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	85	507	0	446	90	153
normalized size	1	1.	0.89	5.28	0.	4.65	0.94	1.59
time (sec)	N/A	0.081	0.071	0.006	0.	1.881	33.845	1.105

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	105	565	0	1080	102	159
normalized size	1	1.	1.01	5.43	0.	10.38	0.98	1.53
time (sec)	N/A	0.112	0.073	0.009	0.	2.108	30.248	1.134

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	108	620	0	1220	0	182
normalized size	1	1.	0.93	5.34	0.	10.52	0.	1.57
time (sec)	N/A	0.145	0.107	0.01	0.	2.147	0.	1.12

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	280	1101	0	0	0	0
normalized size	1	1.	4.31	16.94	0.	0.	0.	0.
time (sec)	N/A	0.056	0.456	0.036	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	149	930	0	0	0	0
normalized size	1	1.	2.29	14.31	0.	0.	0.	0.
time (sec)	N/A	0.04	0.152	0.027	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	351	776	0	0	0	0
normalized size	1	1.	5.85	12.93	0.	0.	0.	0.
time (sec)	N/A	0.028	0.327	0.004	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	148	1404	0	0	0	0
normalized size	1	1.	2.35	22.29	0.	0.	0.	0.
time (sec)	N/A	0.058	0.121	0.01	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	343	1096	0	0	0	0
normalized size	1	1.	5.28	16.86	0.	0.	0.	0.
time (sec)	N/A	0.058	0.335	0.02	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	488	0	593	0	143
normalized size	1	1.	0.88	4.69	0.	5.7	0.	1.38
time (sec)	N/A	0.105	0.145	0.035	0.	1.849	0.	1.115

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	448	0	437	0	86
normalized size	1	1.	1.	6.05	0.	5.91	0.	1.16
time (sec)	N/A	0.07	0.07	0.01	0.	1.709	0.	1.094

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	426	0	288	39	54
normalized size	1	1.	1.	8.35	0.	5.65	0.76	1.06
time (sec)	N/A	0.05	0.015	0.006	0.	1.545	7.797	1.112

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	453	0	948	70	107
normalized size	1	1.	0.95	5.33	0.	11.15	0.82	1.26
time (sec)	N/A	0.076	0.078	0.009	0.	1.687	14.122	1.123

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	151	498	0	1261	0	159
normalized size	1	1.	1.29	4.26	0.	10.78	0.	1.36
time (sec)	N/A	0.118	0.113	0.01	0.	1.808	0.	1.117

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	719	0	0	0	0
normalized size	1	1.	1.02	11.23	0.	0.	0.	0.
time (sec)	N/A	0.051	0.029	0.03	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	429	0	0	0	0
normalized size	1	1.	1.02	6.7	0.	0.	0.	0.
time (sec)	N/A	0.037	0.027	0.026	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	161	429	0	0	0	0
normalized size	1	1.	2.73	7.27	0.	0.	0.	0.
time (sec)	N/A	0.026	0.028	0.004	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	141	890	0	0	0	0
normalized size	1	1.	2.27	14.35	0.	0.	0.	0.
time (sec)	N/A	0.053	0.1	0.009	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	339	738	0	0	0	0
normalized size	1	1.	5.3	11.53	0.	0.	0.	0.
time (sec)	N/A	0.053	0.262	0.009	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	100	527	0	896	0	139
normalized size	1	1.	0.93	4.93	0.	8.37	0.	1.3
time (sec)	N/A	0.121	0.069	0.037	0.	2.465	0.	1.138

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	88	487	0	682	0	105
normalized size	1	1.	1.07	5.94	0.	8.32	0.	1.28
time (sec)	N/A	0.074	0.086	0.007	0.	2.092	0.	1.124

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	52	463	0	494	66	99
normalized size	1	1.	0.68	6.01	0.	6.42	0.86	1.29
time (sec)	N/A	0.066	0.013	0.005	0.	1.824	16.419	1.112

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	89	512	0	1663	104	158
normalized size	1	1.	0.78	4.49	0.	14.59	0.91	1.39
time (sec)	N/A	0.113	0.029	0.022	0.	2.103	23.871	1.122

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	117	575	0	2290	0	248
normalized size	1	1.	0.74	3.64	0.	14.49	0.	1.57
time (sec)	N/A	0.222	0.045	0.01	0.	6.033	0.	1.163

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	231	1069	0	0	0	0
normalized size	1	1.	3.45	15.96	0.	0.	0.	0.
time (sec)	N/A	0.057	0.169	0.03	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	142	907	0	0	0	0
normalized size	1	1.	2.12	13.54	0.	0.	0.	0.
time (sec)	N/A	0.041	0.116	0.03	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	338	753	0	0	0	0
normalized size	1	1.	5.45	12.15	0.	0.	0.	0.
time (sec)	N/A	0.029	0.319	0.005	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	193	1392	0	0	0	0
normalized size	1	1.	2.97	21.42	0.	0.	0.	0.
time (sec)	N/A	0.059	0.197	0.008	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	425	1084	0	0	0	0
normalized size	1	1.	6.34	16.18	0.	0.	0.	0.
time (sec)	N/A	0.057	0.579	0.008	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	101	952	0	502	0	149
normalized size	1	1.	0.86	8.14	0.	4.29	0.	1.27
time (sec)	N/A	0.094	0.067	0.039	0.	2.085	0.	1.121

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	90	892	0	433	0	126
normalized size	1	1.	0.88	8.75	0.	4.25	0.	1.24
time (sec)	N/A	0.076	0.05	0.014	0.	2.1	0.	1.121

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	79	874	0	378	0	100
normalized size	1	1.	0.96	10.66	0.	4.61	0.	1.22
time (sec)	N/A	0.059	0.04	0.01	0.	1.83	0.	1.119

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	61	439	0	342	0	72
normalized size	1	1.	0.95	6.86	0.	5.34	0.	1.12
time (sec)	N/A	0.049	0.055	0.007	0.	1.809	0.	1.092

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	102	912	0	536	0	107
normalized size	1	1.	1.16	10.36	0.	6.09	0.	1.22
time (sec)	N/A	0.072	0.048	0.015	0.	1.889	0.	1.104

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	97	957	0	636	0	150
normalized size	1	1.	0.78	7.72	0.	5.13	0.	1.21
time (sec)	N/A	0.102	0.136	0.013	0.	1.957	0.	1.141

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	112	1020	0	706	0	140
normalized size	1	1.	0.68	6.22	0.	4.3	0.	0.85
time (sec)	N/A	0.131	0.172	0.015	0.	2.039	0.	1.114

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	663	663	176	2198	0	0	0	0
normalized size	1	1.	0.27	3.32	0.	0.	0.	0.
time (sec)	N/A	0.851	0.153	0.034	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	641	641	167	1740	0	0	0	0
normalized size	1	1.	0.26	2.71	0.	0.	0.	0.
time (sec)	N/A	0.731	0.139	0.01	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	644	644	164	882	0	0	0	0
normalized size	1	1.	0.25	1.37	0.	0.	0.	0.
time (sec)	N/A	0.723	0.086	0.008	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	665	665	179	2193	0	0	0	0
normalized size	1	1.	0.27	3.3	0.	0.	0.	0.
time (sec)	N/A	0.824	0.108	0.013	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	687	687	199	2671	0	0	0	0
normalized size	1	1.	0.29	3.89	0.	0.	0.	0.
time (sec)	N/A	0.944	0.134	0.013	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	711	711	209	3169	0	0	0	0
normalized size	1	1.	0.29	4.46	0.	0.	0.	0.
time (sec)	N/A	1.071	0.143	0.013	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	111	998	0	562	0	171
normalized size	1	1.	0.83	7.45	0.	4.19	0.	1.28
time (sec)	N/A	0.11	0.069	0.039	0.	1.891	0.	1.122

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	102	920	0	509	0	150
normalized size	1	1.	0.86	7.73	0.	4.28	0.	1.26
time (sec)	N/A	0.094	0.075	0.014	0.	1.885	0.	1.129

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	90	902	0	437	0	126
normalized size	1	1.	0.93	9.3	0.	4.51	0.	1.3
time (sec)	N/A	0.075	0.052	0.012	0.	1.9	0.	1.125

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	43	451	0	373	0	93
normalized size	1	1.	0.56	5.86	0.	4.84	0.	1.21
time (sec)	N/A	0.06	0.015	0.007	0.	1.729	0.	1.092

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	100	956	0	524	0	95
normalized size	1	1.	1.18	11.25	0.	6.16	0.	1.12
time (sec)	N/A	0.079	0.045	0.013	0.	1.639	0.	1.121

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	97	1014	0	636	0	153
normalized size	1	1.	0.8	8.38	0.	5.26	0.	1.26
time (sec)	N/A	0.103	0.133	0.015	0.	1.644	0.	1.162

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	112	1075	0	707	0	161
normalized size	1	1.	0.7	6.68	0.	4.39	0.	1.
time (sec)	N/A	0.133	0.157	0.016	0.	1.639	0.	1.127

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	681	681	191	2223	0	0	0	0
normalized size	1	1.	0.28	3.26	0.	0.	0.	0.
time (sec)	N/A	0.939	0.158	0.035	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	657	657	176	1747	0	0	0	0
normalized size	1	1.	0.27	2.66	0.	0.	0.	0.
time (sec)	N/A	0.805	0.16	0.01	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	141	873	0	0	0	0
normalized size	1	1.	0.22	1.37	0.	0.	0.	0.
time (sec)	N/A	0.718	0.183	0.007	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	522	522	242	2217	0	0	0	0
normalized size	1	1.	0.46	4.25	0.	0.	0.	0.
time (sec)	N/A	0.256	0.618	0.013	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	684	684	199	2690	0	0	0	0
normalized size	1	1.	0.29	3.93	0.	0.	0.	0.
time (sec)	N/A	0.94	0.134	0.015	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	708	708	212	3186	0	0	0	0
normalized size	1	1.	0.3	4.5	0.	0.	0.	0.
time (sec)	N/A	1.076	0.151	0.014	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	916	0	455	0	126
normalized size	1	1.	0.96	9.64	0.	4.79	0.	1.33
time (sec)	N/A	0.073	0.049	0.036	0.	1.824	0.	1.155

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	874	0	390	0	93
normalized size	1	1.	0.99	10.53	0.	4.7	0.	1.12
time (sec)	N/A	0.064	0.042	0.013	0.	1.781	0.	1.105

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	861	0	356	0	78
normalized size	1	1.	0.98	13.45	0.	5.56	0.	1.22
time (sec)	N/A	0.051	0.047	0.012	0.	1.795	0.	1.11

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	442	0	351	0	80
normalized size	1	1.	0.96	6.6	0.	5.24	0.	1.19
time (sec)	N/A	0.049	0.029	0.007	0.	1.829	0.	1.107

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	83	880	0	544	0	107
normalized size	1	1.	0.94	10.	0.	6.18	0.	1.22
time (sec)	N/A	0.075	0.073	0.014	0.	1.857	0.	1.122

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	97	926	0	653	0	153
normalized size	1	1.	0.78	7.47	0.	5.27	0.	1.23
time (sec)	N/A	0.101	0.133	0.014	0.	1.784	0.	1.134

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	112	989	0	722	0	158
normalized size	1	1.	0.68	6.03	0.	4.4	0.	0.96
time (sec)	N/A	0.129	0.18	0.013	0.	1.736	0.	1.097

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	641	641	167	1737	0	0	0	0
normalized size	1	1.	0.26	2.71	0.	0.	0.	0.
time (sec)	N/A	0.711	0.147	0.035	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	647	647	166	1304	0	0	0	0
normalized size	1	1.	0.26	2.02	0.	0.	0.	0.
time (sec)	N/A	0.715	0.083	0.011	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	644	644	164	882	0	0	0	0
normalized size	1	1.	0.25	1.37	0.	0.	0.	0.
time (sec)	N/A	0.699	0.073	0.009	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	665	665	180	1761	0	0	0	0
normalized size	1	1.	0.27	2.65	0.	0.	0.	0.
time (sec)	N/A	0.828	0.122	0.014	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	687	687	196	2240	0	0	0	0
normalized size	1	1.	0.29	3.26	0.	0.	0.	0.
time (sec)	N/A	0.927	0.138	0.013	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	711	711	212	2738	0	0	0	0
normalized size	1	1.	0.3	3.85	0.	0.	0.	0.
time (sec)	N/A	1.044	0.153	0.011	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	239	1431	0	0	0	0
normalized size	1	1.	3.62	21.68	0.	0.	0.	0.
time (sec)	N/A	0.054	0.299	0.035	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	237	1150	0	0	0	0
normalized size	1	1.	3.59	17.42	0.	0.	0.	0.
time (sec)	N/A	0.055	0.168	0.008	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	237	728	0	0	0	0
normalized size	1	1.	3.7	11.38	0.	0.	0.	0.
time (sec)	N/A	0.028	0.16	0.006	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	266	1455	0	0	0	0
normalized size	1	1.	4.03	22.05	0.	0.	0.	0.
time (sec)	N/A	0.055	0.172	0.013	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	279	1782	0	0	0	0
normalized size	1	1.	4.23	27.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.201	0.013	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	970	0	532	0	119
normalized size	1	1.	0.93	10.21	0.	5.6	0.	1.25
time (sec)	N/A	0.076	0.037	0.046	0.	1.863	0.	1.156

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	71	926	0	495	0	90
normalized size	1	1.	0.86	11.16	0.	5.96	0.	1.08
time (sec)	N/A	0.065	0.088	0.016	0.	2.13	0.	1.173

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	68	908	0	487	0	103
normalized size	1	1.	0.8	10.68	0.	5.73	0.	1.21
time (sec)	N/A	0.062	0.024	0.01	0.	2.074	0.	1.126

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	43	463	0	482	0	97
normalized size	1	1.	0.49	5.26	0.	5.48	0.	1.1
time (sec)	N/A	0.064	0.012	0.007	0.	1.931	0.	1.129

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	97	953	0	724	0	126
normalized size	1	1.	0.92	8.99	0.	6.83	0.	1.19
time (sec)	N/A	0.095	0.037	0.013	0.	1.689	0.	1.106

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	117	1019	0	837	0	171
normalized size	1	1.	0.82	7.13	0.	5.85	0.	1.2
time (sec)	N/A	0.125	0.046	0.01	0.	1.671	0.	1.104

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	135	1106	0	919	0	180
normalized size	1	1.	0.73	5.98	0.	4.97	0.	0.97
time (sec)	N/A	0.163	0.054	0.015	0.	1.661	0.	1.143

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	668	668	168	2255	0	0	0	0
normalized size	1	1.	0.25	3.38	0.	0.	0.	0.
time (sec)	N/A	0.801	0.105	0.04	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	671	671	169	1788	0	0	0	0
normalized size	1	1.	0.25	2.66	0.	0.	0.	0.
time (sec)	N/A	0.8	0.1	0.012	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	665	665	167	903	0	0	0	0
normalized size	1	1.	0.25	1.36	0.	0.	0.	0.
time (sec)	N/A	0.799	0.112	0.01	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	686	686	180	2269	0	0	0	0
normalized size	1	1.	0.26	3.31	0.	0.	0.	0.
time (sec)	N/A	0.923	0.128	0.014	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	708	708	198	2774	0	0	0	0
normalized size	1	1.	0.28	3.92	0.	0.	0.	0.
time (sec)	N/A	1.032	0.15	0.014	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	732	732	210	3299	0	0	0	0
normalized size	1	1.	0.29	4.51	0.	0.	0.	0.
time (sec)	N/A	1.178	0.165	0.014	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	C	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	256	66	189	1791	0	0	0	0
normalized size	1	0.26	0.74	7.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.519	0.039	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	242	1478	0	0	0	0
normalized size	1	1.	3.67	22.39	0.	0.	0.	0.
time (sec)	N/A	0.058	0.259	0.011	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	253	747	0	0	0	0
normalized size	1	1.	3.95	11.67	0.	0.	0.	0.
time (sec)	N/A	0.029	0.228	0.004	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	259	1805	0	0	0	0
normalized size	1	1.	3.92	27.35	0.	0.	0.	0.
time (sec)	N/A	0.057	0.207	0.013	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	283	2156	0	0	0	0
normalized size	1	1.	4.29	32.67	0.	0.	0.	0.
time (sec)	N/A	0.061	0.196	0.013	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	147	917	0	975	0	184
normalized size	1	1.	0.91	5.7	0.	6.06	0.	1.14
time (sec)	N/A	0.192	0.247	0.037	0.	1.908	0.	1.118

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	117	897	0	707	0	150
normalized size	1	1.	0.86	6.6	0.	5.2	0.	1.1
time (sec)	N/A	0.109	0.139	0.01	0.	1.842	0.	1.108

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	453	0	533	0	107
normalized size	1	1.	1.	5.66	0.	6.66	0.	1.34
time (sec)	N/A	0.065	0.083	0.007	0.	1.653	0.	1.108

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	112	934	0	1800	0	170
normalized size	1	1.	0.93	7.72	0.	14.88	0.	1.4
time (sec)	N/A	0.118	0.213	0.012	0.	1.761	0.	1.121

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	190	978	0	1858	0	258
normalized size	1	1.	1.18	6.07	0.	11.54	0.	1.6
time (sec)	N/A	0.218	0.229	0.013	0.	1.808	0.	1.164

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	235	1468	0	0	0	0
normalized size	1	1.	3.67	22.94	0.	0.	0.	0.
time (sec)	N/A	0.051	0.215	0.037	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	153	908	0	0	0	0
normalized size	1	1.	2.39	14.19	0.	0.	0.	0.
time (sec)	N/A	0.041	0.103	0.034	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	232	753	0	0	0	0
normalized size	1	1.	3.93	12.76	0.	0.	0.	0.
time (sec)	N/A	0.027	0.201	0.005	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	172	2227	0	0	0	0
normalized size	1	1.	2.77	35.92	0.	0.	0.	0.
time (sec)	N/A	0.054	0.136	0.013	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	338	1768	0	0	0	0
normalized size	1	1.	5.28	27.62	0.	0.	0.	0.
time (sec)	N/A	0.051	0.293	0.01	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	162	1003	0	944	0	285
normalized size	1	1.	0.86	5.31	0.	4.99	0.	1.51
time (sec)	N/A	0.237	0.162	0.039	0.	2.183	0.	1.159

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	125	983	0	679	0	234
normalized size	1	1.	0.77	6.03	0.	4.17	0.	1.44
time (sec)	N/A	0.137	0.102	0.009	0.	1.632	0.	1.117

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	54	466	0	493	0	161
normalized size	1	1.	0.57	4.96	0.	5.24	0.	1.71
time (sec)	N/A	0.081	0.023	0.005	0.	1.573	0.	1.146

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	122	1036	0	1485	0	223
normalized size	1	1.	0.93	7.91	0.	11.34	0.	1.7
time (sec)	N/A	0.14	0.163	0.011	0.	1.819	0.	1.151

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	142	1093	0	1782	0	300
normalized size	1	1.	0.84	6.43	0.	10.48	0.	1.76
time (sec)	N/A	0.258	0.219	0.011	0.	1.845	0.	1.151

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	338	1587	0	0	0	0
normalized size	1	1.	5.2	24.42	0.	0.	0.	0.
time (sec)	N/A	0.056	0.423	0.039	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	177	955	0	0	0	0
normalized size	1	1.	2.72	14.69	0.	0.	0.	0.
time (sec)	N/A	0.041	0.181	0.035	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	339	801	0	0	0	0
normalized size	1	1.	5.65	13.35	0.	0.	0.	0.
time (sec)	N/A	0.027	0.317	0.005	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	190	2364	0	0	0	0
normalized size	1	1.	3.02	37.52	0.	0.	0.	0.
time (sec)	N/A	0.057	0.177	0.014	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	370	1902	0	0	0	0
normalized size	1	1.	5.69	29.26	0.	0.	0.	0.
time (sec)	N/A	0.055	0.377	0.011	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	911	0	983	0	181
normalized size	1	1.	0.87	7.41	0.	7.99	0.	1.47
time (sec)	N/A	0.144	0.252	0.037	0.	1.73	0.	1.113

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	98	892	0	722	0	157
normalized size	1	1.	0.99	9.01	0.	7.29	0.	1.59
time (sec)	N/A	0.084	0.092	0.01	0.	1.635	0.	1.104

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	457	0	639	0	124
normalized size	1	1.	0.98	5.25	0.	7.34	0.	1.43
time (sec)	N/A	0.073	0.085	0.007	0.	1.599	0.	1.12

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	915	0	1831	0	207
normalized size	1	1.	0.93	6.93	0.	13.87	0.	1.57
time (sec)	N/A	0.139	0.233	0.01	0.	1.962	0.	1.693

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	961	0	2504	0	362
normalized size	1	1.	0.88	5.19	0.	13.54	0.	1.96
time (sec)	N/A	0.243	0.546	0.013	0.	2.325	0.	1.132

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	238	1207	0	0	0	0
normalized size	1	1.	3.72	18.86	0.	0.	0.	0.
time (sec)	N/A	0.052	0.212	0.036	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	172	923	0	0	0	0
normalized size	1	1.	2.69	14.42	0.	0.	0.	0.
time (sec)	N/A	0.039	0.154	0.034	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	392	769	0	0	0	0
normalized size	1	1.	6.64	13.03	0.	0.	0.	0.
time (sec)	N/A	0.028	0.235	0.004	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	226	1818	0	0	0	0
normalized size	1	1.	3.65	29.32	0.	0.	0.	0.
time (sec)	N/A	0.054	0.232	0.011	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	411	1512	0	0	0	0
normalized size	1	1.	6.42	23.62	0.	0.	0.	0.
time (sec)	N/A	0.052	0.548	0.011	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	134	978	0	1470	0	263
normalized size	1	1.	0.9	6.56	0.	9.87	0.	1.77
time (sec)	N/A	0.195	0.278	0.053	0.	1.9	0.	1.138

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	91	958	0	1278	0	244
normalized size	1	1.	0.68	7.15	0.	9.54	0.	1.82
time (sec)	N/A	0.114	0.029	0.01	0.	1.793	0.	1.146

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	54	485	0	923	0	203
normalized size	1	1.	0.5	4.49	0.	8.55	0.	1.88
time (sec)	N/A	0.091	0.017	0.007	0.	1.849	0.	1.105

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	123	1002	0	3657	0	319
normalized size	1	1.	0.72	5.83	0.	21.26	0.	1.85
time (sec)	N/A	0.223	0.104	0.01	0.	3.27	0.	1.123

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	189	1067	0	4741	0	510
normalized size	1	1.	0.78	4.43	0.	19.67	0.	2.12
time (sec)	N/A	0.357	0.113	0.013	0.	5.043	0.	1.145

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	381	1593	0	0	0	0
normalized size	1	1.	5.69	23.78	0.	0.	0.	0.
time (sec)	N/A	0.057	0.288	0.045	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	216	986	0	0	0	0
normalized size	1	1.	3.22	14.72	0.	0.	0.	0.
time (sec)	N/A	0.043	0.257	0.039	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	381	830	0	0	0	0
normalized size	1	1.	6.15	13.39	0.	0.	0.	0.
time (sec)	N/A	0.028	0.525	0.004	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	308	2383	0	0	0	0
normalized size	1	1.	4.74	36.66	0.	0.	0.	0.
time (sec)	N/A	0.058	0.376	0.011	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	515	1919	0	0	0	0
normalized size	1	1.	7.69	28.64	0.	0.	0.	0.
time (sec)	N/A	0.056	0.915	0.011	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	113	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.111	0.026	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	111	0	0	0	252	0
normalized size	1	1.	0.84	0.	0.	0.	1.91	0.
time (sec)	N/A	0.074	0.079	0.022	0.	0.	56.089	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	122	0
normalized size	1	1.	0.84	0.	0.	0.	0.93	0.
time (sec)	N/A	0.074	0.071	0.023	0.	0.	8.684	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	119	0
normalized size	1	1.	0.84	0.	0.	0.	0.91	0.
time (sec)	N/A	0.071	0.077	0.024	0.	0.	5.499	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.085	0.021	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.089	0.02	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	123	0	0	597	0	140
normalized size	1	1.	1.4	0.	0.	6.78	0.	1.59
time (sec)	N/A	0.093	0.18	0.059	0.	1.407	0.	1.177

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	85	0	0	440	0	73
normalized size	1	1.	1.77	0.	0.	9.17	0.	1.52
time (sec)	N/A	0.059	0.065	0.04	0.	1.296	0.	1.148

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	455	0	120
normalized size	1	1.	1.	0.	0.	9.48	0.	2.5
time (sec)	N/A	0.048	0.016	0.046	0.	1.495	0.	1.124

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	636	0	558
normalized size	1	1.	1.	0.	0.	6.99	0.	6.13
time (sec)	N/A	0.08	0.05	0.038	0.	1.873	0.	1.314

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.047	0.046	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.045	0.046	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.043	0.04	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	170	0	0	0	0	0
normalized size	1	1.	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.23	0.039	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	189	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.157	0.047	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	365	0	0	0	0	0
normalized size	1	1.	4.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.254	0.046	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	145	7293	0	656	0	0
normalized size	1	1.	0.9	45.3	0.	4.07	0.	0.
time (sec)	N/A	0.113	0.207	0.388	0.	4.425	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	112	4175	0	0	0	0
normalized size	1	1.	0.35	12.89	0.	0.	0.	0.
time (sec)	N/A	0.32	0.135	0.068	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	94	5358	0	0	97	0
normalized size	1	1.	0.16	9.22	0.	0.	0.17	0.
time (sec)	N/A	0.621	0.097	0.066	0.	0.	70.217	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	119	6858	0	501	201	0
normalized size	1	1.	0.98	56.68	0.	4.14	1.66	0.
time (sec)	N/A	0.089	0.125	0.056	0.	4.4	13.338	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	93	3721	0	0	97	0
normalized size	1	1.	0.33	13.01	0.	0.	0.34	0.
time (sec)	N/A	0.229	0.06	0.053	0.	0.	6.831	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	580	98	5736	0	0	100	0
normalized size	1	1.	0.17	9.89	0.	0.	0.17	0.
time (sec)	N/A	0.592	0.056	0.062	0.	0.	6.571	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	87	6668	0	486	160	0
normalized size	1	1.	0.74	56.51	0.	4.12	1.36	0.
time (sec)	N/A	0.084	0.187	0.059	0.	4.329	22.141	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	97	3512	0	0	0	0
normalized size	1	1.	0.34	12.41	0.	0.	0.	0.
time (sec)	N/A	0.227	0.056	0.056	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	564	564	81	5911	0	0	97	0
normalized size	1	1.	0.14	10.48	0.	0.	0.17	0.
time (sec)	N/A	0.53	0.093	0.106	0.	0.	112.877	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	87	3759	0	437	0	147
normalized size	1	1.	1.1	47.58	0.	5.53	0.	1.86
time (sec)	N/A	0.046	0.163	0.066	0.	2.183	0.	1.19

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	80	3690	0	0	0	0
normalized size	1	1.	0.3	13.72	0.	0.	0.	0.
time (sec)	N/A	0.199	0.087	0.068	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	167	7705	0	791	0	0
normalized size	1	1.	0.83	38.33	0.	3.94	0.	0.
time (sec)	N/A	0.139	0.257	0.065	0.	4.379	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	116	4619	0	0	0	0
normalized size	1	1.	0.32	12.69	0.	0.	0.	0.
time (sec)	N/A	0.309	0.156	0.059	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	96	5790	0	0	0	0
normalized size	1	1.	0.15	9.32	0.	0.	0.	0.
time (sec)	N/A	0.661	0.116	0.062	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	143	7290	0	614	335	0
normalized size	1	1.	0.89	45.28	0.	3.81	2.08	0.
time (sec)	N/A	0.113	0.171	0.047	0.	4.286	37.908	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	96	4173	0	0	199	0
normalized size	1	1.	0.3	12.88	0.	0.	0.61	0.
time (sec)	N/A	0.256	0.071	0.042	0.	0.	25.848	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	84	6142	0	0	202	0
normalized size	1	1.	0.14	10.	0.	0.	0.33	0.
time (sec)	N/A	0.644	0.074	0.052	0.	0.	31.626	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	126	7108	0	590	289	0
normalized size	1	1.	0.83	46.76	0.	3.88	1.9	0.
time (sec)	N/A	0.104	0.146	0.049	0.	4.197	62.041	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	85	3966	0	0	0	0
normalized size	1	1.	0.27	12.63	0.	0.	0.	0.
time (sec)	N/A	0.247	0.071	0.046	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	188	8117	0	940	0	0
normalized size	1	1.	0.78	33.68	0.	3.9	0.	0.
time (sec)	N/A	0.159	0.293	0.063	0.	4.583	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	116	5063	0	0	0	0
normalized size	1	1.	0.29	12.53	0.	0.	0.	0.
time (sec)	N/A	0.344	0.178	0.059	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	661	99	6202	0	0	0	0
normalized size	1	1.	0.15	9.38	0.	0.	0.	0.
time (sec)	N/A	0.708	0.133	0.061	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	146	7702	0	738	413	0
normalized size	1	1.	0.73	38.32	0.	3.67	2.05	0.
time (sec)	N/A	0.127	0.377	0.049	0.	4.312	111.188	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	84	4617	0	0	308	0
normalized size	1	1.	0.23	12.68	0.	0.	0.85	0.
time (sec)	N/A	0.284	0.064	0.045	0.	0.	133.694	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	87	6530	0	0	311	0
normalized size	1	1.	0.13	10.05	0.	0.	0.48	0.
time (sec)	N/A	0.698	0.044	0.056	0.	0.	159.126	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	150	7544	0	714	0	0
normalized size	1	1.	0.8	40.13	0.	3.8	0.	0.
time (sec)	N/A	0.128	0.193	0.053	0.	4.339	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	88	4422	0	0	0	0
normalized size	1	1.	0.25	12.56	0.	0.	0.	0.
time (sec)	N/A	0.287	0.047	0.049	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	97	6861	0	551	0	0
normalized size	1	1.	0.8	56.7	0.	4.55	0.	0.
time (sec)	N/A	0.086	0.121	0.059	0.	4.16	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	98	3723	0	0	94	0
normalized size	1	1.	0.34	13.02	0.	0.	0.33	0.
time (sec)	N/A	0.222	0.1	0.053	0.	0.	169.649	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	80	4914	0	0	94	0
normalized size	1	1.	0.15	9.05	0.	0.	0.17	0.
time (sec)	N/A	0.501	0.089	0.056	0.	0.	30.201	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	6424	0	417	107	0
normalized size	1	1.	0.94	77.4	0.	5.02	1.29	0.
time (sec)	N/A	0.067	0.042	0.044	0.	4.229	6.054	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	80	3275	0	0	94	0
normalized size	1	1.	0.32	13.15	0.	0.	0.38	0.
time (sec)	N/A	0.183	0.056	0.042	0.	0.	3.388	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	83	5385	0	0	97	0
normalized size	1	1.	0.15	9.94	0.	0.	0.18	0.
time (sec)	N/A	0.505	0.042	0.051	0.	0.	5.235	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	65	3397	0	425	60	146
normalized size	1	1.	0.87	45.29	0.	5.67	0.8	1.95
time (sec)	N/A	0.057	0.038	0.042	0.	2.406	29.51	1.25

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	82	3303	0	0	97	0
normalized size	1	1.	0.33	13.43	0.	0.	0.39	0.
time (sec)	N/A	0.183	0.046	0.043	0.	0.	176.107	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	109	7016	0	667	0	0
normalized size	1	1.	0.91	58.47	0.	5.56	0.	0.
time (sec)	N/A	0.086	0.163	0.065	0.	4.339	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	87	3760	0	0	0	0
normalized size	1	1.	0.3	13.15	0.	0.	0.	0.
time (sec)	N/A	0.221	0.111	0.059	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	77	5392	0	0	0	0
normalized size	1	1.	0.14	9.75	0.	0.	0.	0.
time (sec)	N/A	0.503	0.106	0.065	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	93	3654	0	510	95	0
normalized size	1	1.	1.09	42.99	0.	6.	1.12	0.
time (sec)	N/A	0.062	0.087	0.04	0.	2.53	53.142	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	79	3565	0	0	94	0
normalized size	1	1.	0.31	13.82	0.	0.	0.36	0.
time (sec)	N/A	0.188	0.056	0.044	0.	0.	142.286	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	77	5563	0	0	0	0
normalized size	1	1.	0.13	9.51	0.	0.	0.	0.
time (sec)	N/A	0.571	0.042	0.052	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	39	0	117	0	0
normalized size	1	1.	0.67	0.58	0.	1.75	0.	0.
time (sec)	N/A	0.029	0.021	0.006	0.	1.311	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	95	3783	0	0	0	0
normalized size	1	1.	0.34	13.37	0.	0.	0.	0.
time (sec)	N/A	0.223	0.05	0.046	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	119	7081	0	726	0	0
normalized size	1	1.	1.04	62.11	0.	6.37	0.	0.
time (sec)	N/A	0.075	0.273	0.076	0.	2.524	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	108	7083	0	0	0	0
normalized size	1	1.	0.36	23.69	0.	0.	0.	0.
time (sec)	N/A	0.229	0.148	0.078	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	86	10786	0	0	0	0
normalized size	1	1.	0.14	18.1	0.	0.	0.	0.
time (sec)	N/A	0.573	0.117	0.096	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	128	0	86
normalized size	1	1.	0.56	0.49	0.	1.62	0.	1.09
time (sec)	N/A	0.031	0.037	0.006	0.	1.367	0.	1.479

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	107	7077	0	0	0	0
normalized size	1	1.	0.36	23.83	0.	0.	0.	0.
time (sec)	N/A	0.222	0.093	0.059	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	624	85	10961	0	0	0	0
normalized size	1	1.	0.14	17.57	0.	0.	0.	0.
time (sec)	N/A	0.639	0.042	0.085	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	65	62	0	193	0	0
normalized size	1	1.	0.62	0.6	0.	1.86	0.	0.
time (sec)	N/A	0.046	0.037	0.006	0.	1.322	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	121	7299	0	0	0	0
normalized size	1	1.	0.38	22.81	0.	0.	0.	0.
time (sec)	N/A	0.258	0.075	0.066	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	230	0	0	520	0	0
normalized size	1	1.	1.05	0.	0.	2.36	0.	0.
time (sec)	N/A	0.236	0.197	0.063	0.	1.348	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	207	0	0	483	0	0
normalized size	1	1.	1.19	0.	0.	2.78	0.	0.
time (sec)	N/A	0.178	0.124	0.043	0.	1.338	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	186	0	0	451	0	0
normalized size	1	1.	1.08	0.	0.	2.62	0.	0.
time (sec)	N/A	0.149	0.097	0.045	0.	1.27	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	167	0	0	417	0	0
normalized size	1	1.	1.11	0.	0.	2.78	0.	0.
time (sec)	N/A	0.119	0.048	0.039	0.	1.263	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	233	0	0	809	0	0
normalized size	1	1.	1.09	0.	0.	3.78	0.	0.
time (sec)	N/A	0.174	0.098	0.04	0.	1.391	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	280	0	0	905	0	0
normalized size	1	1.	1.04	0.	0.	3.38	0.	0.
time (sec)	N/A	0.247	0.109	0.055	0.	1.391	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	314	0	0	952	0	0
normalized size	1	1.	1.11	0.	0.	3.36	0.	0.
time (sec)	N/A	0.264	0.128	0.065	0.	1.381	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	268	66	177	0	0	980	0	0
normalized size	1	0.25	0.66	0.	0.	3.66	0.	0.
time (sec)	N/A	0.06	0.191	0.05	0.	1.423	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	233	66	160	0	0	933	0	0
normalized size	1	0.28	0.69	0.	0.	4.	0.	0.
time (sec)	N/A	0.063	0.149	0.049	0.	1.455	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	201	66	63	0	0	869	0	0
normalized size	1	0.33	0.31	0.	0.	4.32	0.	0.
time (sec)	N/A	0.043	0.038	0.033	0.	1.394	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	156	77	45	0	0	0	0	0
normalized size	1	0.49	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.017	0.046	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	183	117	112	0	0	0	0	0
normalized size	1	0.64	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.422	4.857	0.046	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	210	244	135	0	0	0	0	0
normalized size	1	1.16	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	19.78	5.106	0.052	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	237	423	130	0	0	0	0	0
normalized size	1	1.78	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	29.209	5.154	0.056	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	521	66	234	0	0	0	0	0
normalized size	1	0.13	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.332	0.052	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	494	66	225	0	0	0	0	0
normalized size	1	0.13	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.194	0.049	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	416	61	154	0	0	0	0	0
normalized size	1	0.15	0.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.145	0.037	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	496	66	231	0	0	0	0	0
normalized size	1	0.13	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.138	0.049	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	523	66	243	0	0	0	0	0
normalized size	1	0.13	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.174	0.054	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	163	0	0	559	0	0
normalized size	1	1.	0.73	0.	0.	2.51	0.	0.
time (sec)	N/A	0.25	0.253	0.052	0.	2.608	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	153	0	0	517	0	0
normalized size	1	1.	0.86	0.	0.	2.92	0.	0.
time (sec)	N/A	0.2	0.164	0.042	0.	2.654	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	143	0	0	493	0	0
normalized size	1	1.	0.82	0.	0.	2.82	0.	0.
time (sec)	N/A	0.156	0.129	0.04	0.	1.841	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	130	0	0	454	0	0
normalized size	1	1.	0.85	0.	0.	2.97	0.	0.
time (sec)	N/A	0.126	0.071	0.04	0.	1.563	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	164	0	0	1530	0	0
normalized size	1	1.	0.77	0.	0.	7.15	0.	0.
time (sec)	N/A	0.182	0.07	0.041	0.	1.697	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	213	0	0	1727	0	0
normalized size	1	1.	0.79	0.	0.	6.42	0.	0.
time (sec)	N/A	0.263	0.084	0.058	0.	1.694	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	247	0	0	1816	0	0
normalized size	1	1.	0.87	0.	0.	6.39	0.	0.
time (sec)	N/A	0.279	0.141	0.063	0.	1.721	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	264	66	244	0	0	1875	0	0
normalized size	1	0.25	0.92	0.	0.	7.1	0.	0.
time (sec)	N/A	0.063	0.264	0.052	0.	1.694	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	229	66	216	0	0	1786	0	0
normalized size	1	0.29	0.94	0.	0.	7.8	0.	0.
time (sec)	N/A	0.063	0.227	0.049	0.	1.697	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	200	61	156	0	0	1659	0	0
normalized size	1	0.3	0.78	0.	0.	8.3	0.	0.
time (sec)	N/A	0.033	0.154	0.033	0.	1.643	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	157	79	47	0	0	0	0	0
normalized size	1	0.5	0.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.016	0.043	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	182	121	124	0	0	0	0	0
normalized size	1	0.66	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.429	4.67	0.046	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	209	244	179	0	0	0	0	0
normalized size	1	1.17	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	10.676	5.213	0.05	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	236	391	196	0	0	0	0	0
normalized size	1	1.66	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	17.601	5.223	0.055	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	512	66	147	0	0	0	0	0
normalized size	1	0.13	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.105	0.051	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	485	66	127	0	0	0	0	0
normalized size	1	0.14	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.082	0.046	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	457	66	63	0	0	0	0	0
normalized size	1	0.14	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.039	0.033	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	483	64	136	0	0	0	0	0
normalized size	1	0.13	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.096	0.052	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	512	66	148	0	0	0	0	0
normalized size	1	0.13	0.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.081	0.053	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	113	0	161	359	0	0
normalized size	1	1.	0.89	0.	1.27	2.83	0.	0.
time (sec)	N/A	0.093	0.101	0.059	1.461	2.232	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	128	0	161	448	0	0
normalized size	1	1.	1.	0.	1.26	3.5	0.	0.
time (sec)	N/A	0.088	0.051	0.039	1.434	2.393	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	96	0	131	328	0	0
normalized size	1	1.	0.99	0.	1.35	3.38	0.	0.
time (sec)	N/A	0.077	0.055	0.017	1.419	2.13	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	0	131	419	0	0
normalized size	1	1.	0.97	0.	1.34	4.28	0.	0.
time (sec)	N/A	0.064	0.031	0.018	1.448	1.783	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	116	282	0	0
normalized size	1	1.	0.89	0.	1.41	3.44	0.	0.
time (sec)	N/A	0.059	0.018	0.	1.444	1.716	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	133	0	0	1335	0	0
normalized size	1	1.	0.97	0.	0.	9.74	0.	0.
time (sec)	N/A	0.091	0.039	0.038	0.	11.553	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	153	0	0	541	0	0
normalized size	1	1.	0.97	0.	0.	3.45	0.	0.
time (sec)	N/A	0.103	0.149	0.05	0.	1.706	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	226	144	0	0	572	0	0
normalized size	1	1.47	0.94	0.	0.	3.71	0.	0.
time (sec)	N/A	0.149	0.111	0.047	0.	1.808	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	207	26	0	0	1392	0	0
normalized size	1	1.53	0.19	0.	0.	10.31	0.	0.
time (sec)	N/A	0.105	0.018	0.037	0.	11.712	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	122	112	0	0	660	0	0
normalized size	1	1.39	1.27	0.	0.	7.5	0.	0.
time (sec)	N/A	0.057	0.066	0.034	0.	21.97	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	139	82	0	0	841	0	0
normalized size	1	1.32	0.78	0.	0.	8.01	0.	0.
time (sec)	N/A	0.072	0.36	0.041	0.	22.064	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	140	123	0	0	726	0	0
normalized size	1	1.13	0.99	0.	0.	5.85	0.	0.
time (sec)	N/A	0.088	5.137	0.023	0.	22.035	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	175	133	0	0	876	0	0
normalized size	1	1.24	0.94	0.	0.	6.21	0.	0.
time (sec)	N/A	0.092	5.095	0.025	0.	21.635	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	271	26	40	0	0	0	0	0
normalized size	1	0.1	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.027	0.052	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	254	26	26	0	0	0	0	0
normalized size	1	0.1	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.018	0.033	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	233	26	26	0	0	1049	0	0
normalized size	1	0.11	0.11	0.	0.	4.5	0.	0.
time (sec)	N/A	0.013	0.013	0.033	0.	16.705	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	270	24	67	0	0	0	0	0
normalized size	1	0.09	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.048	0.048	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	289	26	76	0	0	0	0	0
normalized size	1	0.09	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.04	0.053	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	151	0	161	463	0	0
normalized size	1	1.	1.21	0.	1.29	3.7	0.	0.
time (sec)	N/A	0.095	0.067	0.066	1.438	1.521	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	135	0	131	346	0	0
normalized size	1	1.	1.38	0.	1.34	3.53	0.	0.
time (sec)	N/A	0.075	0.036	0.046	1.417	1.534	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	118	0	131	431	0	0
normalized size	1	1.	1.24	0.	1.38	4.54	0.	0.
time (sec)	N/A	0.062	0.028	0.017	1.431	1.541	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	94	0	116	301	0	0
normalized size	1	1.	1.13	0.	1.4	3.63	0.	0.
time (sec)	N/A	0.056	0.023	0.031	1.424	1.513	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	179	0	0	605	0	0
normalized size	1	1.	1.31	0.	0.	4.42	0.	0.
time (sec)	N/A	0.09	0.038	0.037	0.	1.51	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	196	0	0	560	0	0
normalized size	1	1.	1.24	0.	0.	3.54	0.	0.
time (sec)	N/A	0.103	0.066	0.059	0.	1.593	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	228	78	0	0	698	0	0
normalized size	1	1.42	0.49	0.	0.	4.36	0.	0.
time (sec)	N/A	0.163	0.055	0.058	0.	1.59	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	207	26	0	0	555	0	0
normalized size	1	1.49	0.19	0.	0.	3.99	0.	0.
time (sec)	N/A	0.103	0.034	0.037	0.	1.544	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	122	38	0	0	797	0	0
normalized size	1	1.39	0.43	0.	0.	9.06	0.	0.
time (sec)	N/A	0.061	0.007	0.036	0.	13.814	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	137	81	0	0	694	0	0
normalized size	1	1.33	0.79	0.	0.	6.74	0.	0.
time (sec)	N/A	0.072	0.371	0.058	0.	13.694	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	140	145	0	0	856	0	0
normalized size	1	1.13	1.17	0.	0.	6.9	0.	0.
time (sec)	N/A	0.084	8.518	0.053	0.	13.023	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	291	26	115	0	0	949	0	0
normalized size	1	0.09	0.4	0.	0.	3.26	0.	0.
time (sec)	N/A	0.019	0.119	0.056	0.	17.8	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	294	26	26	0	0	0	0	0
normalized size	1	0.09	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.033	0.036	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	293	21	111	0	0	0	0	0
normalized size	1	0.07	0.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.118	0.032	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	294	26	120	0	0	1095	0	0
normalized size	1	0.09	0.41	0.	0.	3.72	0.	0.
time (sec)	N/A	0.019	0.087	0.049	0.	19.692	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	53	0	173	406	0	0
normalized size	1	1.	0.38	0.	1.23	2.88	0.	0.
time (sec)	N/A	0.114	0.023	0.049	1.422	1.783	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	48	0	161	500	0	0
normalized size	1	1.	0.37	0.	1.24	3.85	0.	0.
time (sec)	N/A	0.101	0.017	0.025	1.432	1.708	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	41	0	146	377	0	0
normalized size	1	1.	0.36	0.	1.27	3.28	0.	0.
time (sec)	N/A	0.09	0.013	0.021	1.474	1.814	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	95	0	131	468	0	0
normalized size	1	1.	0.95	0.	1.31	4.68	0.	0.
time (sec)	N/A	0.068	0.072	0.023	1.441	1.818	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	34	0	131	363	0	0
normalized size	1	1.	0.34	0.	1.31	3.63	0.	0.
time (sec)	N/A	0.068	0.007	0.02	1.439	1.758	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	54	0	0	701	0	0
normalized size	1	1.	0.35	0.	0.	4.55	0.	0.
time (sec)	N/A	0.111	0.015	0.056	0.	1.81	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	64	0	0	633	0	0
normalized size	1	1.	0.37	0.	0.	3.62	0.	0.
time (sec)	N/A	0.116	0.021	0.056	0.	1.837	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	174	26	152	0	0	776	0	0
normalized size	1	0.15	0.87	0.	0.	4.46	0.	0.
time (sec)	N/A	0.018	0.173	0.052	0.	1.934	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	153	26	142	0	0	655	0	0
normalized size	1	0.17	0.93	0.	0.	4.28	0.	0.
time (sec)	N/A	0.018	0.138	0.047	0.	1.859	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	106	38	38	0	0	876	0	0
normalized size	1	0.36	0.36	0.	0.	8.26	0.	0.
time (sec)	N/A	0.018	0.009	0.052	0.	21.769	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	140	100	0	0	741	0	0
normalized size	1	1.32	0.94	0.	0.	6.99	0.	0.
time (sec)	N/A	0.076	0.042	0.02	0.	21.742	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	124	204	192	0	0	900	0	0
normalized size	1	1.65	1.55	0.	0.	7.26	0.	0.
time (sec)	N/A	8.121	2.118	0.024	0.	23.254	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	144	397	373	0	0	780	0	0
normalized size	1	2.76	2.59	0.	0.	5.42	0.	0.
time (sec)	N/A	8.318	5.015	0.026	0.	22.106	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	162	643	136	0	0	933	0	0
normalized size	1	3.97	0.84	0.	0.	5.76	0.	0.
time (sec)	N/A	10.595	5.166	0.027	0.	22.223	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	292	26	71	0	0	0	0	0
normalized size	1	0.09	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.073	0.054	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	274	26	66	0	0	0	0	0
normalized size	1	0.09	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.064	0.049	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	274	26	66	0	0	0	0	0
normalized size	1	0.09	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.056	0.042	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	274	26	45	0	0	0	0	0
normalized size	1	0.09	0.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	0.032	0.042	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	292	24	76	0	0	0	0	0
normalized size	1	0.08	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.059	0.053	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	308	26	79	0	0	0	0	0
normalized size	1	0.08	0.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.081	0.053	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	270	0	0	749	0	512
normalized size	1	1.	1.02	0.	0.	2.84	0.	1.94
time (sec)	N/A	0.386	0.535	0.058	0.	1.777	0.	1.267

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	230	0	0	644	0	432
normalized size	1	1.	1.05	0.	0.	2.93	0.	1.96
time (sec)	N/A	0.259	0.331	0.045	0.	1.746	0.	1.226

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	204	0	0	535	0	362
normalized size	1	1.	1.1	0.	0.	2.88	0.	1.95
time (sec)	N/A	0.202	0.338	0.042	0.	2.13	0.	1.209

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	205	0	0	483	0	301
normalized size	1	1.	1.29	0.	0.	3.04	0.	1.89
time (sec)	N/A	0.155	0.148	0.037	0.	2.043	0.	1.221

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	268	0	0	714	0	447
normalized size	1	1.	1.09	0.	0.	2.9	0.	1.82
time (sec)	N/A	0.234	0.309	0.037	0.	2.233	0.	2.527

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	366	0	0	1044	0	525
normalized size	1	1.	1.08	0.	0.	3.07	0.	1.54
time (sec)	N/A	0.384	0.93	0.059	0.	2.695	0.	2.608

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	411	0	0	1142	0	653
normalized size	1	1.	1.11	0.	0.	3.09	0.	1.76
time (sec)	N/A	0.489	1.356	0.066	0.	11.634	0.	2.737

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	336	64	226	0	0	1175	0	0
normalized size	1	0.19	0.67	0.	0.	3.5	0.	0.
time (sec)	N/A	0.054	0.285	0.063	0.	11.684	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	276	64	185	0	0	1077	0	0
normalized size	1	0.23	0.67	0.	0.	3.9	0.	0.
time (sec)	N/A	0.054	0.179	0.048	0.	2.591	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	234	64	65	0	0	807	0	0
normalized size	1	0.27	0.28	0.	0.	3.45	0.	0.
time (sec)	N/A	0.038	0.031	0.036	0.	1.897	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	168	87	81	0	0	0	0	0
normalized size	1	0.52	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.028	0.046	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	204	145	146	0	0	0	0	0
normalized size	1	0.71	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.399	0.044	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	258	451	451	0	0	0	0	0
normalized size	1	1.75	1.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.94	1.595	0.051	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	318	905	905	0	0	0	0	0
normalized size	1	2.85	2.85	0.	0.	0.	0.	0.
time (sec)	N/A	2.642	3.79	0.056	0.	0.	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	281	0	0	0	0	0
normalized size	1	1.	4.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.542	0.056	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	240	0	0	0	0	0
normalized size	1	1.	3.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.281	0.046	0.	0.	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0
normalized size	1	1.	2.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.04	0.03	0.	0.	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	327	0	0	0	0	0
normalized size	1	1.	5.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.243	0.052	0.	0.	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	289	0	0	0	0	0
normalized size	1	1.	4.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.354	0.063	0.	0.	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	148	0	0	1033	0	552
normalized size	1	1.	0.56	0.	0.	3.88	0.	2.08
time (sec)	N/A	0.322	0.106	0.047	0.	1.887	0.	1.211

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	104	0	0	900	0	473
normalized size	1	1.	0.47	0.	0.	4.04	0.	2.12
time (sec)	N/A	0.259	0.068	0.043	0.	1.62	0.	1.202

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	68	0	0	815	0	402
normalized size	1	1.	0.36	0.	0.	4.34	0.	2.14
time (sec)	N/A	0.199	0.025	0.041	0.	1.598	0.	1.194

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	47	0	0	736	0	350
normalized size	1	1.	0.29	0.	0.	4.54	0.	2.16
time (sec)	N/A	0.17	0.016	0.039	0.	1.536	0.	1.235

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	115	0	0	1033	0	487
normalized size	1	1.	0.47	0.	0.	4.22	0.	1.99
time (sec)	N/A	0.229	0.069	0.038	0.	1.764	0.	2.909

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	202	0	0	2473	0	585
normalized size	1	1.	0.58	0.	0.	7.13	0.	1.69
time (sec)	N/A	0.389	0.099	0.057	0.	2.379	0.	2.679

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	240	0	0	2691	0	698
normalized size	1	1.	0.65	0.	0.	7.27	0.	1.89
time (sec)	N/A	0.496	0.273	0.066	0.	9.316	0.	2.644

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	334	64	525	0	0	2734	0	0
normalized size	1	0.19	1.57	0.	0.	8.19	0.	0.
time (sec)	N/A	0.056	0.614	0.059	0.	9.232	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	272	64	286	0	0	2546	0	0
normalized size	1	0.24	1.05	0.	0.	9.36	0.	0.
time (sec)	N/A	0.059	0.497	0.049	0.	2.364	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	233	59	161	0	0	1102	0	0
normalized size	1	0.25	0.69	0.	0.	4.73	0.	0.
time (sec)	N/A	0.027	0.041	0.031	0.	1.726	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	169	89	83	0	0	0	0	0
normalized size	1	0.53	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.034	0.044	0.	0.	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	206	148	148	0	0	0	0	0
normalized size	1	0.72	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.389	0.047	0.	0.	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	257	451	451	0	0	0	0	0
normalized size	1	1.75	1.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.971	1.628	0.056	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	320	819	819	0	0	0	0	0
normalized size	1	2.56	2.56	0.	0.	0.	0.	0.
time (sec)	N/A	2.626	3.92	0.056	0.	0.	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0
normalized size	1	1.	2.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.201	0.063	0.	0.	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.095	0.051	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.032	0.035	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	138	0	0	0	0	0
normalized size	1	1.	2.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.091	0.055	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0
normalized size	1	1.	2.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.157	0.06	0.	0.	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	258	0	0	829	0	532
normalized size	1	1.	1.03	0.	0.	3.3	0.	2.12
time (sec)	N/A	0.363	0.433	0.046	0.	2.482	0.	1.192

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	255	0	0	683	0	470
normalized size	1	1.	1.21	0.	0.	3.24	0.	2.23
time (sec)	N/A	0.245	0.256	0.043	0.	2.54	0.	1.213

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	232	0	0	572	0	401
normalized size	1	1.	1.24	0.	0.	3.06	0.	2.14
time (sec)	N/A	0.208	0.295	0.042	0.	2.577	0.	1.246

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	331	0	0	811	0	510
normalized size	1	1.	1.27	0.	0.	3.11	0.	1.95
time (sec)	N/A	0.304	0.539	0.039	0.	2.629	0.	2.807

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	389	0	0	946	0	568
normalized size	1	1.	0.97	0.	0.	2.37	0.	1.42
time (sec)	N/A	0.484	1.21	0.056	0.	3.541	0.	2.863

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	429	0	0	1204	0	689
normalized size	1	1.	0.98	0.	0.	2.74	0.	1.57
time (sec)	N/A	0.623	0.982	0.063	0.	12.008	0.	2.218

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	334	65	225	0	0	1281	0	0
normalized size	1	0.19	0.67	0.	0.	3.84	0.	0.
time (sec)	N/A	0.059	0.267	0.061	0.	11.954	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	A	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	277	65	198	0	0	968	0	0
normalized size	1	0.23	0.71	0.	0.	3.49	0.	0.
time (sec)	N/A	0.044	0.216	0.056	0.	3.503	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	254	63	161	0	0	0	0	0
normalized size	1	0.25	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.323	0.055	0.	0.	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	201	90	84	0	0	0	0	0
normalized size	1	0.45	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.033	0.049	0.	0.	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	250	169	179	0	0	0	0	0
normalized size	1	0.68	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.507	0.423	0.051	0.	0.	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	318	260	270	0	0	0	0	0
normalized size	1	0.82	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	1.801	1.651	0.056	0.	0.	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	392	1446	1446	0	0	0	0	0
normalized size	1	3.69	3.69	0.	0.	0.	0.	0.
time (sec)	N/A	4.391	3.917	0.063	0.	0.	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	343	0	0	0	0	0
normalized size	1	1.	5.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.603	0.063	0.	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	280	0	0	0	0	0
normalized size	1	1.	4.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.445	0.053	0.	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0
normalized size	1	1.	5.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.303	0.047	0.	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	341	0	0	0	0	0
normalized size	1	1.	5.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.342	0.058	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	286	0	0	0	0	0
normalized size	1	1.	4.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.387	0.062	0.	0.	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	157	0	0	2228	0	613
normalized size	1	1.	0.54	0.	0.	7.68	0.	2.11
time (sec)	N/A	0.317	0.204	0.047	0.	1.962	0.	1.269

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	145	0	0	1904	0	501
normalized size	1	1.	0.59	0.	0.	7.8	0.	2.05
time (sec)	N/A	0.243	0.093	0.043	0.	2.032	0.	1.257

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	103	0	0	1716	0	423
normalized size	1	1.	0.51	0.	0.	8.45	0.	2.08
time (sec)	N/A	0.214	0.061	0.043	0.	1.969	0.	1.222

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	69	0	0	1497	0	347
normalized size	1	1.	0.41	0.	0.	8.91	0.	2.07
time (sec)	N/A	0.161	0.02	0.04	0.	1.702	0.	1.239

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	50	0	0	1357	0	305
normalized size	1	1.	0.34	0.	0.	9.36	0.	2.1
time (sec)	N/A	0.125	0.015	0.032	0.	1.359	0.	1.28

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	140	0	0	1623	0	470
normalized size	1	1.	0.57	0.	0.	6.65	0.	1.93
time (sec)	N/A	0.212	0.09	0.04	0.	1.482	0.	2.953

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	156	0	0	2056	0	567
normalized size	1	1.	0.53	0.	0.	6.95	0.	1.92
time (sec)	N/A	0.314	0.365	0.057	0.	2.028	0.	3.433

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	394	288	0	0	2026	0	0
normalized size	1	1.44	1.05	0.	0.	7.42	0.	0.
time (sec)	N/A	0.509	0.521	0.051	0.	2.028	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	346	65	0	0	1848	0	0
normalized size	1	1.48	0.28	0.	0.	7.93	0.	0.
time (sec)	N/A	0.291	0.034	0.038	0.	1.477	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	207	168	0	0	0	0	0
normalized size	1	1.4	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.142	0.034	0.	0.	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	235	124	0	0	0	0	0
normalized size	1	1.34	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.057	0.042	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	271	207	0	0	0	0	0
normalized size	1	1.27	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.286	0.778	0.047	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	317	821	0	0	0	0	0
normalized size	1	1.21	3.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.325	2.29	0.051	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	144	0	0	0	0	0
normalized size	1	1.	2.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.113	0.055	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.031	0.035	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.03	0.033	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0
normalized size	1	1.	2.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.097	0.049	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	183	0	0	0	0	0
normalized size	1	1.	2.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.171	0.062	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	251	0	0	2795	0	502
normalized size	1	1.	1.04	0.	0.	11.6	0.	2.08
time (sec)	N/A	0.263	0.491	0.049	0.	2.081	0.	1.243

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	211	0	0	2496	0	421
normalized size	1	1.	1.05	0.	0.	12.42	0.	2.09
time (sec)	N/A	0.212	0.341	0.043	0.	1.636	0.	1.181

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	202	0	0	2276	0	342
normalized size	1	1.	1.22	0.	0.	13.79	0.	2.07
time (sec)	N/A	0.155	0.103	0.04	0.	1.529	0.	1.181

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	164	0	0	2037	0	298
normalized size	1	1.	1.13	0.	0.	14.05	0.	2.06
time (sec)	N/A	0.124	0.046	0.033	0.	1.499	0.	1.17

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	308	0	0	1115	0	462
normalized size	1	1.	1.26	0.	0.	4.55	0.	1.89
time (sec)	N/A	0.211	0.278	0.037	0.	1.502	0.	2.616

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	303	0	0	1296	0	567
normalized size	1	1.	1.01	0.	0.	4.33	0.	1.9
time (sec)	N/A	0.317	0.417	0.059	0.	4.663	0.	2.369

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	400	190	0	0	1305	0	0
normalized size	1	1.43	0.68	0.	0.	4.68	0.	0.
time (sec)	N/A	0.502	0.191	0.055	0.	4.631	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	346	65	0	0	1206	0	0
normalized size	1	1.48	0.28	0.	0.	5.15	0.	0.
time (sec)	N/A	0.308	0.035	0.036	0.	1.529	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	208	83	0	0	0	0	0
normalized size	1	1.4	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.025	0.028	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	232	128	0	0	0	0	0
normalized size	1	1.34	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	0.056	0.043	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	269	267	0	0	0	0	0
normalized size	1	1.25	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.297	1.532	0.046	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	249	0	0	0	0	0
normalized size	1	1.	3.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.334	0.047	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.034	0.035	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0
normalized size	1	1.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.035	0.031	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	338	0	0	0	0	0
normalized size	1	1.	5.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.264	0.049	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	157	0	0	2768	0	582
normalized size	1	1.	0.45	0.	0.	7.98	0.	1.68
time (sec)	N/A	0.441	0.213	0.065	0.	2.243	0.	1.226

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	147	0	0	2417	0	502
normalized size	1	1.	0.58	0.	0.	9.55	0.	1.98
time (sec)	N/A	0.34	0.088	0.053	0.	1.606	0.	1.237

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	101	0	0	2159	0	439
normalized size	1	1.	0.5	0.	0.	10.64	0.	2.16
time (sec)	N/A	0.242	0.06	0.046	0.	1.549	0.	1.2

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	77	0	0	1890	0	406
normalized size	1	1.	0.44	0.	0.	10.86	0.	2.33
time (sec)	N/A	0.171	0.031	0.036	0.	1.484	0.	1.198

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	50	0	0	599	0	385
normalized size	1	1.	0.3	0.	0.	3.59	0.	2.31
time (sec)	N/A	0.165	0.015	0.034	0.	1.335	0.	1.179

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	86	0	0	2310	0	552
normalized size	1	1.	0.32	0.	0.	8.52	0.	2.04
time (sec)	N/A	0.316	0.03	0.04	0.	1.661	0.	2.104

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	117	0	0	3062	0	699
normalized size	1	1.	0.33	0.	0.	8.58	0.	1.96
time (sec)	N/A	0.394	0.048	0.065	0.	6.199	0.	2.9

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	322	67	504	0	0	2989	0	0
normalized size	1	0.21	1.57	0.	0.	9.28	0.	0.
time (sec)	N/A	0.06	0.828	0.053	0.	6.059	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	260	67	309	0	0	2562	0	0
normalized size	1	0.26	1.19	0.	0.	9.85	0.	0.
time (sec)	N/A	0.059	0.322	0.034	0.	1.994	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	172	92	86	0	0	0	0	0
normalized size	1	0.53	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.034	0.034	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	238	256	0	0	0	0	0
normalized size	1	1.33	1.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	0.083	0.03	0.	0.	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	229	542	542	0	0	0	0	0
normalized size	1	2.37	2.37	0.	0.	0.	0.	0.
time (sec)	N/A	1.287	0.686	0.048	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	287	950	950	0	0	0	0	0
normalized size	1	3.31	3.31	0.	0.	0.	0.	0.
time (sec)	N/A	3.857	1.785	0.054	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	351	1486	1486	0	0	0	0	0
normalized size	1	4.23	4.23	0.	0.	0.	0.	0.
time (sec)	N/A	8.295	3.743	0.066	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	194	0	0	0	0	0
normalized size	1	1.	2.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.218	0.06	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	144	0	0	0	0	0
normalized size	1	1.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.119	0.039	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	129	0	0	0	0	0
normalized size	1	1.	1.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.084	0.036	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0
normalized size	1	1.	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.107	0.033	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	193	0	0	0	0	0
normalized size	1	1.	2.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.196	0.053	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	264	0	0	0	0	0
normalized size	1	1.	3.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.285	0.066	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	92	89	113	196	230	0
normalized size	1	1.	1.02	0.99	1.26	2.18	2.56	0.
time (sec)	N/A	0.096	0.047	0.007	0.935	28.199	9.562	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	92	142	201	0
normalized size	1	1.	0.94	0.93	1.31	2.03	2.87	0.
time (sec)	N/A	0.064	0.033	0.009	0.929	7.862	7.355	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	66	92	144	0
normalized size	1	1.	0.81	0.94	1.25	1.74	2.72	0.
time (sec)	N/A	0.05	0.023	0.009	0.933	2.842	4.546	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	55	69	138	0
normalized size	1	1.	0.69	0.93	1.22	1.53	3.07	0.
time (sec)	N/A	0.031	0.019	0.007	0.927	1.295	1.718	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	82	123	0	0
normalized size	1	1.	0.87	0.95	1.32	1.98	0.	0.
time (sec)	N/A	0.064	0.03	0.007	0.927	9.125	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	87	117	200	0	0
normalized size	1	1.	1.01	1.	1.34	2.3	0.	0.
time (sec)	N/A	0.09	0.041	0.011	0.938	38.272	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	104	105	0	1125	0	722
normalized size	1	1.	0.93	0.94	0.	10.04	0.	6.45
time (sec)	N/A	0.273	0.158	0.011	0.	10.322	0.	1.242

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	81	0	848	932	482
normalized size	1	1.	0.89	0.88	0.	9.22	10.13	5.24
time (sec)	N/A	0.118	0.134	0.009	0.	2.542	10.827	1.198

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	0	657	576	271
normalized size	1	1.	0.84	0.76	0.	8.32	7.29	3.43
time (sec)	N/A	0.062	0.038	0.005	0.	1.347	38.052	1.148

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	0	668	719	261
normalized size	1	1.	0.84	0.76	0.	8.46	9.1	3.3
time (sec)	N/A	0.048	0.046	0.008	0.	1.352	21.493	1.175

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	169	81	0	910	1103	508
normalized size	1	1.	1.84	0.88	0.	9.89	11.99	5.52
time (sec)	N/A	0.122	0.203	0.008	0.	2.9	11.934	1.202

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	193	105	0	1191	0	736
normalized size	1	1.	1.72	0.94	0.	10.63	0.	6.57
time (sec)	N/A	0.218	0.213	0.012	0.	13.461	0.	1.695

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	377	328	0	2708	0	1670
normalized size	1	1.	0.82	0.72	0.	5.93	0.	3.65
time (sec)	N/A	0.443	0.21	0.016	0.	2.319	0.	2.264

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	0	2657	0	0
normalized size	1	1.	0.76	0.71	0.	5.92	0.	0.
time (sec)	N/A	0.285	0.092	0.006	0.	1.711	0.	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	296	0	2491	0	0
normalized size	1	1.	0.76	0.66	0.	5.55	0.	0.
time (sec)	N/A	0.274	0.093	0.006	0.	1.378	0.	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	296	0	2515	0	0
normalized size	1	1.	0.76	0.66	0.	5.6	0.	0.
time (sec)	N/A	0.27	0.118	0.009	0.	1.467	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	0	2668	0	0
normalized size	1	1.	0.76	0.71	0.	5.94	0.	0.
time (sec)	N/A	0.259	0.113	0.007	0.	2.036	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	385	331	0	2730	0	2279
normalized size	1	1.	0.84	0.72	0.	5.93	0.	4.95
time (sec)	N/A	0.445	0.209	0.012	0.	2.628	0.	4.774

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	406	343	0	2822	0	2225
normalized size	1	1.	0.88	0.74	0.	6.11	0.	4.82
time (sec)	N/A	0.432	0.226	0.01	0.	20.476	0.	5.521

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	428	365	0	2942	0	1737
normalized size	1	1.	0.89	0.76	0.	6.14	0.	3.63
time (sec)	N/A	0.597	0.254	0.013	0.	36.764	0.	3.53

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	1015	0	428	90	130
normalized size	1	1.	0.95	10.91	0.	4.6	0.97	1.4
time (sec)	N/A	0.082	0.065	0.054	0.	1.47	20.688	1.107

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	114	1066	0	1577	0	136
normalized size	1	1.	0.95	8.88	0.	13.14	0.	1.13
time (sec)	N/A	0.154	0.15	0.025	0.	2.353	0.	1.586

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	69	988	0	331	65	89
normalized size	1	1.	0.99	14.11	0.	4.73	0.93	1.27
time (sec)	N/A	0.061	0.032	0.006	0.	1.559	7.516	1.175

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	89	1000	0	1310	0	157
normalized size	1	1.	0.98	10.99	0.	14.4	0.	1.73
time (sec)	N/A	0.076	0.042	0.006	0.	1.808	0.	1.192

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	1037	0	851	82	117
normalized size	1	1.	0.95	12.2	0.	10.01	0.96	1.38
time (sec)	N/A	0.075	0.035	0.016	0.	1.657	11.206	1.144

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	53	1075	0	586	0	89
normalized size	1	1.	0.7	14.14	0.	7.71	0.	1.17
time (sec)	N/A	0.086	0.018	0.016	0.	1.659	0.	1.135

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	107	1107	0	1160	0	163
normalized size	1	1.	0.93	9.63	0.	10.09	0.	1.42
time (sec)	N/A	0.124	0.121	0.016	0.	1.682	0.	1.137

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	221	1116	0	684	0	131
normalized size	1	1.	2.01	10.15	0.	6.22	0.	1.19
time (sec)	N/A	0.16	1.07	0.014	0.	2.009	0.	1.214

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	857	1067	141	421	0	0	0	0
normalized size	1	1.25	0.16	0.49	0.	0.	0.	0.
time (sec)	N/A	1.855	0.104	0.061	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	700	904	241	368	0	0	0	0
normalized size	1	1.29	0.34	0.53	0.	0.	0.	0.
time (sec)	N/A	1.248	0.411	0.019	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	786	1012	65	299	0	0	0	0
normalized size	1	1.29	0.08	0.38	0.	0.	0.	0.
time (sec)	N/A	1.038	0.031	0.006	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	679	881	161	273	0	0	0	0
normalized size	1	1.3	0.24	0.4	0.	0.	0.	0.
time (sec)	N/A	0.978	0.149	0.005	0.	0.	0.	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	809	1031	138	421	0	0	0	0
normalized size	1	1.27	0.17	0.52	0.	0.	0.	0.
time (sec)	N/A	1.506	0.092	0.014	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	703	893	333	370	0	0	0	0
normalized size	1	1.27	0.47	0.53	0.	0.	0.	0.
time (sec)	N/A	1.19	0.266	0.016	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.035	0.05	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.033	0.043	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.031	0.049	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	143	0	0	0	0	0
normalized size	1	1.	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.114	0.079	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	378	0	594	0	143
normalized size	1	1.	0.88	3.63	0.	5.71	0.	1.38
time (sec)	N/A	0.112	0.181	0.033	0.	1.729	0.	1.099

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	335	0	437	0	86
normalized size	1	1.	0.97	4.53	0.	5.91	0.	1.16
time (sec)	N/A	0.065	0.063	0.009	0.	1.571	0.	1.098

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	316	0	288	37	54
normalized size	1	1.	1.	6.2	0.	5.65	0.73	1.06
time (sec)	N/A	0.049	0.016	0.006	0.	1.503	11.823	1.115

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	347	0	948	66	107
normalized size	1	1.	0.95	4.08	0.	11.15	0.78	1.26
time (sec)	N/A	0.073	0.077	0.013	0.	1.659	16.389	1.068

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	151	402	0	1261	0	159
normalized size	1	1.	1.29	3.44	0.	10.78	0.	1.36
time (sec)	N/A	0.12	0.115	0.016	0.	1.701	0.	1.099

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	118	408	0	1623	0	140
normalized size	1	1.	0.96	3.32	0.	13.2	0.	1.14
time (sec)	N/A	0.149	0.13	0.027	0.	2.485	0.	1.332

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	356	0	1358	0	107
normalized size	1	1.	0.99	3.91	0.	14.92	0.	1.18
time (sec)	N/A	0.087	0.057	0.01	0.	2.227	0.	1.132

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	95	322	0	518	0	97
normalized size	1	1.	1.76	5.96	0.	9.59	0.	1.8
time (sec)	N/A	0.044	0.056	0.006	0.	2.054	0.	1.141

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	179	350	0	698	0	86
normalized size	1	1.	2.24	4.38	0.	8.72	0.	1.08
time (sec)	N/A	0.088	0.685	0.015	0.	2.251	0.	1.099

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	253	383	0	856	0	144
normalized size	1	1.	2.2	3.33	0.	7.44	0.	1.25
time (sec)	N/A	0.164	1.584	0.015	0.	2.307	0.	1.11

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	872	872	249	363	0	0	0	0
normalized size	1	1.	0.29	0.42	0.	0.	0.	0.
time (sec)	N/A	1.019	0.33	0.023	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	638	837	65	265	0	0	0	0
normalized size	1	1.31	0.1	0.42	0.	0.	0.	0.
time (sec)	N/A	0.682	0.033	0.007	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	638	742	161	191	0	0	0	0
normalized size	1	1.16	0.25	0.3	0.	0.	0.	0.
time (sec)	N/A	0.602	0.045	0.004	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	677	864	337	288	0	0	0	0
normalized size	1	1.28	0.5	0.43	0.	0.	0.	0.
time (sec)	N/A	0.991	0.256	0.015	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	804	982	65	292	0	0	0	0
normalized size	1	1.22	0.08	0.36	0.	0.	0.	0.
time (sec)	N/A	0.963	0.035	0.018	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	656	756	65	191	0	0	0	0
normalized size	1	1.15	0.1	0.29	0.	0.	0.	0.
time (sec)	N/A	0.71	0.033	0.007	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	833	1007	141	310	0	0	0	0
normalized size	1	1.21	0.17	0.37	0.	0.	0.	0.
time (sec)	N/A	1.309	0.101	0.015	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	175	923	0	1270	0	243
normalized size	1	1.	1.	5.27	0.	7.26	0.	1.39
time (sec)	N/A	0.173	0.19	0.043	0.	1.637	0.	2.028

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	876	0	983	0	181
normalized size	1	1.	0.87	7.12	0.	7.99	0.	1.47
time (sec)	N/A	0.146	0.229	0.014	0.	1.586	0.	1.556

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	98	851	0	722	0	157
normalized size	1	1.	0.99	8.6	0.	7.29	0.	1.59
time (sec)	N/A	0.087	0.089	0.012	0.	1.568	0.	1.329

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	541	0	639	0	124
normalized size	1	1.	0.98	6.22	0.	7.34	0.	1.43
time (sec)	N/A	0.074	0.087	0.006	0.	1.576	0.	1.519

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	880	0	1831	0	207
normalized size	1	1.	0.93	6.67	0.	13.87	0.	1.57
time (sec)	N/A	0.142	0.237	0.013	0.	1.704	0.	1.373

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	938	0	2504	0	362
normalized size	1	1.	0.88	5.07	0.	13.54	0.	1.96
time (sec)	N/A	0.251	0.547	0.012	0.	1.6	0.	1.669

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	150	953	0	2853	0	238
normalized size	1	1.	0.79	4.99	0.	14.94	0.	1.25
time (sec)	N/A	0.365	0.365	0.027	0.	4.949	0.	2.084

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	135	893	0	2271	0	205
normalized size	1	1.	0.96	6.33	0.	16.11	0.	1.45
time (sec)	N/A	0.162	0.198	0.011	0.	2.743	0.	1.166

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	124	861	0	883	0	124
normalized size	1	1.	1.33	9.26	0.	9.49	0.	1.33
time (sec)	N/A	0.092	0.446	0.01	0.	1.582	0.	1.253

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	407	867	0	967	0	320
normalized size	1	1.	3.91	8.34	0.	9.3	0.	3.08
time (sec)	N/A	0.094	0.772	0.006	0.	1.741	0.	1.563

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	869	885	0	1256	0	181
normalized size	1	1.	5.83	5.94	0.	8.43	0.	1.21
time (sec)	N/A	0.199	1.566	0.015	0.	1.848	0.	1.604

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	175	923	0	1547	0	244
normalized size	1	1.	0.84	4.44	0.	7.44	0.	1.17
time (sec)	N/A	0.328	5.639	0.014	0.	2.65	0.	1.575

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	996	996	253	604	0	0	0	0
normalized size	1	1.	0.25	0.61	0.	0.	0.	0.
time (sec)	N/A	1.308	0.279	0.03	0.	0.	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	908	908	238	530	0	0	0	0
normalized size	1	1.	0.26	0.58	0.	0.	0.	0.
time (sec)	N/A	0.906	0.195	0.011	0.	0.	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	983	983	392	333	0	0	0	0
normalized size	1	1.	0.4	0.34	0.	0.	0.	0.
time (sec)	N/A	1.033	0.241	0.007	0.	0.	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1046	1046	408	626	0	0	0	0
normalized size	1	1.	0.39	0.6	0.	0.	0.	0.
time (sec)	N/A	1.557	0.563	0.013	0.	0.	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1146	1146	162	556	0	0	0	0
normalized size	1	1.	0.14	0.49	0.	0.	0.	0.
time (sec)	N/A	1.563	0.123	0.026	0.	0.	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1144	1144	172	359	0	0	0	0
normalized size	1	1.	0.15	0.31	0.	0.	0.	0.
time (sec)	N/A	1.441	0.146	0.007	0.	0.	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1225	1225	226	674	0	0	0	0
normalized size	1	1.	0.18	0.55	0.	0.	0.	0.
time (sec)	N/A	1.898	0.254	0.013	0.	0.	0.	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	194	164	0	0	0	185	0
normalized size	1	0.97	0.82	0.	0.	0.	0.92	0.
time (sec)	N/A	0.212	0.137	0.029	0.	0.	55.62	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	115	110	0	0	0	119	0
normalized size	1	0.93	0.89	0.	0.	0.	0.97	0.
time (sec)	N/A	0.059	0.085	0.022	0.	0.	6.723	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	56	0
normalized size	1	1.	0.97	0.	0.	0.	0.82	0.
time (sec)	N/A	0.019	0.016	0.023	0.	0.	0.953	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	125	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.095	0.032	0.	0.	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.138	0.031	0.	0.	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.06	0.032	0.	0.	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	167	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	0.141	0.028	0.	0.	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	113	0	0	0	119	0
normalized size	1	1.	0.86	0.	0.	0.	0.9	0.
time (sec)	N/A	0.06	0.091	0.02	0.	0.	163.386	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0
normalized size	1	1.	0.97	0.	0.	0.	0.79	0.
time (sec)	N/A	0.023	0.018	0.018	0.	0.	1.657	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	169	0	0	0	0	0
normalized size	1	1.	2.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.136	0.03	0.	0.	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.069	0.03	0.	0.	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.079	0.03	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	594	0	143
normalized size	1	1.	0.88	0.	0.	5.71	0.	1.38
time (sec)	N/A	0.107	0.177	0.059	0.	1.095	0.	1.148

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	0	0	437	0	86
normalized size	1	1.	0.97	0.	0.	5.91	0.	1.16
time (sec)	N/A	0.063	0.063	0.045	0.	1.051	0.	1.124

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	288	37	54
normalized size	1	1.	1.	0.	0.	5.65	0.73	1.06
time (sec)	N/A	0.046	0.015	0.031	0.	1.049	18.685	1.163

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	0	0	948	66	107
normalized size	1	1.	0.95	0.	0.	11.15	0.78	1.26
time (sec)	N/A	0.074	0.076	0.043	0.	1.104	24.803	1.091

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	151	0	0	1264	0	159
normalized size	1	1.	1.29	0.	0.	10.8	0.	1.36
time (sec)	N/A	0.119	0.117	0.065	0.	1.33	0.	1.153

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	118	0	0	1632	0	0
normalized size	1	1.	0.96	0.	0.	13.27	0.	0.
time (sec)	N/A	0.153	0.135	0.068	0.	1.785	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	1366	0	223
normalized size	1	1.	0.99	0.	0.	15.01	0.	2.45
time (sec)	N/A	0.087	0.058	0.037	0.	1.481	0.	1.2

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	95	0	0	522	0	97
normalized size	1	1.	1.76	0.	0.	9.67	0.	1.8
time (sec)	N/A	0.049	0.062	0.033	0.	1.378	0.	1.17

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	179	0	0	702	0	185
normalized size	1	1.	2.24	0.	0.	8.78	0.	2.31
time (sec)	N/A	0.089	0.693	0.053	0.	1.412	0.	1.222

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	253	0	0	859	0	277
normalized size	1	1.	2.2	0.	0.	7.47	0.	2.41
time (sec)	N/A	0.163	1.73	0.054	0.	1.54	0.	1.324

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.04	0.037	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.036	0.038	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.034	0.035	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0
normalized size	1	1.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.166	0.036	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0
normalized size	1	1.	2.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.114	0.063	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.11	0.059	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.116	0.062	0.	0.	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	0	0	984	0	181
normalized size	1	1.	0.87	0.	0.	8.	0.	1.47
time (sec)	N/A	0.14	0.237	0.056	0.	2.152	0.	1.143

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	98	0	0	724	0	157
normalized size	1	1.	0.99	0.	0.	7.31	0.	1.59
time (sec)	N/A	0.08	0.088	0.042	0.	1.888	0.	1.117

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	0	640	0	124
normalized size	1	1.	0.98	0.	0.	7.36	0.	1.43
time (sec)	N/A	0.069	0.088	0.034	0.	1.534	0.	1.127

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	0	0	1833	0	207
normalized size	1	1.	0.93	0.	0.	13.89	0.	1.57
time (sec)	N/A	0.127	0.233	0.045	0.	1.903	0.	1.141

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	0	0	2525	0	362
normalized size	1	1.	0.88	0.	0.	13.65	0.	1.96
time (sec)	N/A	0.228	0.537	0.072	0.	2.362	0.	1.195

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	135	0	0	2279	0	482
normalized size	1	1.	0.96	0.	0.	16.16	0.	3.42
time (sec)	N/A	0.158	0.21	0.046	0.	4.168	0.	1.249

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	124	0	0	887	0	248
normalized size	1	1.	1.33	0.	0.	9.54	0.	2.67
time (sec)	N/A	0.09	0.444	0.042	0.	2.441	0.	1.277

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	407	0	0	971	0	320
normalized size	1	1.	3.91	0.	0.	9.34	0.	3.08
time (sec)	N/A	0.098	0.772	0.036	0.	2.775	0.	1.183

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	869	0	0	1260	0	386
normalized size	1	1.	5.83	0.	0.	8.46	0.	2.59
time (sec)	N/A	0.194	1.591	0.054	0.	2.862	0.	1.384

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	1535	0	0	1553	0	494
normalized size	1	1.	7.38	0.	0.	7.47	0.	2.38
time (sec)	N/A	0.308	2.739	0.06	0.	3.537	0.	1.439

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	169	0	0	0	0	0
normalized size	1	1.	2.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.172	0.041	0.	0.	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	168	0	0	0	0	0
normalized size	1	1.	2.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.169	0.04	0.	0.	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	172	0	0	0	0	0
normalized size	1	1.	2.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.155	0.04	0.	0.	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	329	0	0	0	0	0
normalized size	1	1.	5.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.282	0.039	0.	0.	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0
normalized size	1	1.	3.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.261	0.063	0.	0.	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0
normalized size	1	1.	3.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.248	0.057	0.	0.	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	225	0	0	0	0	0
normalized size	1	1.	3.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.26	0.064	0.	0.	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	595	0	143
normalized size	1	1.	0.88	0.	0.	5.72	0.	1.38
time (sec)	N/A	0.099	0.171	0.059	0.	1.366	0.	1.176

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	0	0	437	0	86
normalized size	1	1.	0.97	0.	0.	5.91	0.	1.16
time (sec)	N/A	0.061	0.063	0.043	0.	1.31	0.	1.123

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	288	37	54
normalized size	1	1.	1.	0.	0.	5.65	0.73	1.06
time (sec)	N/A	0.047	0.015	0.03	0.	1.305	37.556	1.113

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	0	0	948	66	107
normalized size	1	1.	0.95	0.	0.	11.15	0.78	1.26
time (sec)	N/A	0.079	0.077	0.043	0.	1.387	44.69	1.143

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	151	0	0	1264	0	159
normalized size	1	1.	1.29	0.	0.	10.8	0.	1.36
time (sec)	N/A	0.113	0.117	0.066	0.	1.448	0.	1.162

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	118	0	0	1638	0	140
normalized size	1	1.	0.96	0.	0.	13.32	0.	1.14
time (sec)	N/A	0.131	0.132	0.067	0.	1.752	0.	1.392

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	1372	0	107
normalized size	1	1.	0.99	0.	0.	15.08	0.	1.18
time (sec)	N/A	0.11	0.056	0.039	0.	1.437	0.	1.246

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	95	0	0	525	0	97
normalized size	1	1.	1.76	0.	0.	9.72	0.	1.8
time (sec)	N/A	0.055	0.066	0.036	0.	1.289	0.	1.205

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	179	0	0	705	0	86
normalized size	1	1.	2.24	0.	0.	8.81	0.	1.08
time (sec)	N/A	0.087	0.699	0.05	0.	1.429	0.	1.21

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	253	0	0	867	0	144
normalized size	1	1.	2.2	0.	0.	7.54	0.	1.25
time (sec)	N/A	0.161	1.751	0.058	0.	1.542	0.	1.173

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	851	851	65	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	1.101	0.04	0.045	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	754	754	65	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.766	0.034	0.033	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	878	878	141	0	0	0	0	0
normalized size	1	1.	0.16	0.	0.	0.	0.	0.
time (sec)	N/A	1.222	0.114	0.065	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1005	1005	65	0	0	0	0	0
normalized size	1	1.	0.06	0.	0.	0.	0.	0.
time (sec)	N/A	1.363	0.043	0.043	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	768	768	65	0	0	0	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.935	0.037	0.033	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1032	1032	141	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	1.612	0.105	0.063	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.038	0.037	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.037	0.036	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0
normalized size	1	1.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.166	0.036	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0
normalized size	1	1.	2.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.117	0.065	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.114	0.059	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	0	0	984	0	181
normalized size	1	1.	0.87	0.	0.	8.	0.	1.47
time (sec)	N/A	0.142	0.237	0.059	0.	1.702	0.	1.475

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	98	0	0	724	0	157
normalized size	1	1.	0.99	0.	0.	7.31	0.	1.59
time (sec)	N/A	0.083	0.088	0.04	0.	1.649	0.	1.491

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	0	640	0	124
normalized size	1	1.	0.98	0.	0.	7.36	0.	1.43
time (sec)	N/A	0.073	0.088	0.036	0.	1.628	0.	1.468

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	123	0	0	1833	0	207
normalized size	1	1.	0.93	0.	0.	13.89	0.	1.57
time (sec)	N/A	0.137	0.242	0.044	0.	1.655	0.	1.619

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	0	0	2525	0	362
normalized size	1	1.	0.88	0.	0.	13.65	0.	1.96
time (sec)	N/A	0.229	0.569	0.074	0.	1.894	0.	1.412

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	135	0	0	2284	0	205
normalized size	1	1.	0.96	0.	0.	16.2	0.	1.45
time (sec)	N/A	0.16	0.21	0.046	0.	3.758	0.	1.74

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	124	0	0	890	0	124
normalized size	1	1.	1.33	0.	0.	9.57	0.	1.33
time (sec)	N/A	0.086	0.472	0.042	0.	2.005	0.	1.326

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	407	0	0	973	0	320
normalized size	1	1.	3.91	0.	0.	9.36	0.	3.08
time (sec)	N/A	0.09	0.779	0.036	0.	2.267	0.	1.21

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	869	0	0	1268	0	181
normalized size	1	1.	5.83	0.	0.	8.51	0.	1.21
time (sec)	N/A	0.185	1.571	0.056	0.	2.494	0.	1.216

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	1535	0	0	1561	0	244
normalized size	1	1.	7.38	0.	0.	7.5	0.	1.17
time (sec)	N/A	0.297	2.754	0.063	0.	3.537	0.	1.188

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	924	924	159	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	1.288	0.139	0.044	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	999	999	169	0	0	0	0	0
normalized size	1	1.	0.17	0.	0.	0.	0.	0.
time (sec)	N/A	1.406	0.157	0.035	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1060	1060	225	0	0	0	0	0
normalized size	1	1.	0.21	0.	0.	0.	0.	0.
time (sec)	N/A	1.983	0.273	0.066	0.	0.	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1164	1164	159	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	1.936	0.134	0.044	0.	0.	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1162	1162	169	0	0	0	0	0
normalized size	1	1.	0.15	0.	0.	0.	0.	0.
time (sec)	N/A	1.91	0.162	0.033	0.	0.	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1243	1243	226	0	0	0	0	0
normalized size	1	1.	0.18	0.	0.	0.	0.	0.
time (sec)	N/A	2.384	0.263	0.065	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.178	0.04	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.173	0.042	0.	0.	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	328	0	0	0	0	0
normalized size	1	1.	5.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.301	0.04	0.	0.	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0
normalized size	1	1.	3.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.263	0.068	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0
normalized size	1	1.	3.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.271	0.059	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	121	162	0	533	226	193
normalized size	1	1.	0.98	1.32	0.	4.33	1.84	1.57
time (sec)	N/A	0.094	0.199	0.017	0.	1.664	37.839	1.149

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	100	122	0	431	144	142
normalized size	1	1.	1.11	1.36	0.	4.79	1.6	1.58
time (sec)	N/A	0.068	0.105	0.007	0.	1.371	28.825	1.146

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	127	0	358	107	124
normalized size	1	1.	0.85	1.51	0.	4.26	1.27	1.48
time (sec)	N/A	0.058	0.16	0.011	0.	1.32	32.215	1.255

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	82	110	0	385	75	220
normalized size	1	1.	1.39	1.86	0.	6.53	1.27	3.73
time (sec)	N/A	0.043	0.116	0.011	0.	1.359	17.041	1.398

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	48	66	132	58	338
normalized size	1	1.	1.02	1.04	1.43	2.87	1.26	7.35
time (sec)	N/A	0.037	0.016	0.006	0.941	1.323	3.3	2.196

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	69	70	113	188	78	419
normalized size	1	1.	0.93	0.95	1.53	2.54	1.05	5.66
time (sec)	N/A	0.059	0.022	0.005	0.944	1.347	3.835	3.891

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	79	94	159	238	112	500
normalized size	1	1.	0.76	0.9	1.53	2.29	1.08	4.81
time (sec)	N/A	0.079	0.049	0.007	0.932	1.551	4.626	3.468

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	90	118	205	302	146	581
normalized size	1	1.	0.67	0.88	1.53	2.25	1.09	4.34
time (sec)	N/A	0.102	0.063	0.007	0.958	1.8	5.689	7.097

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	108	113	213	294	1386	217
normalized size	1	1.	0.72	0.75	1.42	1.96	9.24	1.45
time (sec)	N/A	0.073	0.071	0.006	0.95	1.352	9.082	1.134

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	86	89	167	232	910	180
normalized size	1	1.	0.74	0.76	1.43	1.98	7.78	1.54
time (sec)	N/A	0.062	0.054	0.006	0.97	1.337	6.334	1.133

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	64	65	122	181	422	142
normalized size	1	1.	0.76	0.77	1.45	2.15	5.02	1.69
time (sec)	N/A	0.04	0.04	0.007	0.944	1.261	4.361	1.189

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	43	74	127	119	99
normalized size	1	1.	0.79	0.81	1.4	2.4	2.25	1.87
time (sec)	N/A	0.025	0.023	0.004	0.944	1.299	3.081	1.103

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	84	83	0	365	107	157
normalized size	1	1.	1.27	1.26	0.	5.53	1.62	2.38
time (sec)	N/A	0.037	0.047	0.009	0.	1.357	3.048	1.166

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	75	135	0	375	107	103
normalized size	1	1.	0.88	1.59	0.	4.41	1.26	1.21
time (sec)	N/A	0.046	0.078	0.012	0.	1.439	4.179	1.187

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	100	175	0	450	144	176
normalized size	1	1.	1.1	1.92	0.	4.95	1.58	1.93
time (sec)	N/A	0.051	0.115	0.012	0.	1.409	6.512	1.238

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	68	220	0	552	226	207
normalized size	1	1.	0.55	1.79	0.	4.49	1.84	1.68
time (sec)	N/A	0.081	0.026	0.014	0.	1.388	10.858	1.247

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	123	162	0	543	253	194
normalized size	1	1.	1.	1.32	0.	4.41	2.06	1.58
time (sec)	N/A	0.09	0.118	0.007	0.	1.478	52.032	1.16

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	89	174	0	462	216	170
normalized size	1	1.	0.77	1.51	0.	4.02	1.88	1.48
time (sec)	N/A	0.084	0.177	0.013	0.	1.366	89.858	1.298

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	78	216	0	456	187	304
normalized size	1	1.	0.71	1.96	0.	4.15	1.7	2.76
time (sec)	N/A	0.073	0.083	0.013	0.	1.368	33.146	1.71

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	90	153	0	493	73	343
normalized size	1	1.	1.18	2.01	0.	6.49	0.96	4.51
time (sec)	N/A	0.054	0.047	0.014	0.	1.347	37.223	1.806

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	48	66	180	138	500
normalized size	1	1.	1.07	1.04	1.43	3.91	3.	10.87
time (sec)	N/A	0.036	0.018	0.006	0.947	1.379	10.193	5.439

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	70	113	239	194	581
normalized size	1	1.	0.96	0.95	1.53	3.23	2.62	7.85
time (sec)	N/A	0.056	0.025	0.007	0.938	1.598	11.826	4.182

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	94	159	305	262	662
normalized size	1	1.	0.9	0.9	1.53	2.93	2.52	6.37
time (sec)	N/A	0.072	0.03	0.008	0.933	1.738	13.995	7.535

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	115	118	205	360	326	743
normalized size	1	1.	0.86	0.88	1.53	2.69	2.43	5.54
time (sec)	N/A	0.096	0.034	0.009	0.944	2.156	15.966	12.433

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	110	115	213	352	3351	440
normalized size	1	1.	0.73	0.77	1.42	2.35	22.34	2.93
time (sec)	N/A	0.072	0.074	0.009	0.959	1.348	19.192	1.105

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	89	91	167	298	2304	363
normalized size	1	1.	0.76	0.78	1.43	2.55	19.69	3.1
time (sec)	N/A	0.057	0.055	0.007	0.941	1.367	14.434	1.127

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	66	67	122	232	1340	288
normalized size	1	1.	0.79	0.8	1.45	2.76	15.95	3.43
time (sec)	N/A	0.042	0.041	0.006	0.96	1.33	10.172	1.104

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	45	74	174	498	203
normalized size	1	1.	0.83	0.85	1.4	3.28	9.4	3.83
time (sec)	N/A	0.025	0.027	0.004	0.937	1.314	7.043	1.099

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	81	99	0	473	184	189
normalized size	1	1.	0.94	1.15	0.	5.5	2.14	2.2
time (sec)	N/A	0.055	0.1	0.01	0.	1.372	5.723	1.091

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	105	170	0	447	202	155
normalized size	1	1.	0.87	1.4	0.	3.69	1.67	1.28
time (sec)	N/A	0.058	0.063	0.01	0.	1.349	7.117	1.177

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	68	213	0	490	216	196
normalized size	1	1.	0.61	1.9	0.	4.38	1.93	1.75
time (sec)	N/A	0.062	0.032	0.012	0.	1.403	11.128	1.166

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	126	259	0	562	253	234
normalized size	1	1.	1.02	2.11	0.	4.57	2.06	1.9
time (sec)	N/A	0.064	0.099	0.013	0.	1.451	17.672	1.198

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	71	302	0	674	287	289
normalized size	1	1.	0.45	1.9	0.	4.24	1.81	1.82
time (sec)	N/A	0.089	0.029	0.02	0.	1.524	27.737	1.22

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	95	129	0	440	150	220
normalized size	1	1.	1.06	1.43	0.	4.89	1.67	2.44
time (sec)	N/A	0.067	0.079	0.01	0.	1.324	32.297	1.18

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	79	90	0	335	66	107
normalized size	1	1.	1.34	1.53	0.	5.68	1.12	1.81
time (sec)	N/A	0.04	0.037	0.007	0.	1.425	38.144	1.172

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	73	69	0	298	63	0
normalized size	1	1.	1.7	1.6	0.	6.93	1.47	0.
time (sec)	N/A	0.033	0.029	0.009	0.	1.37	14.126	0.

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	47	65	88	139	0
normalized size	1	1.	0.91	1.09	1.51	2.05	3.23	0.
time (sec)	N/A	0.036	0.025	0.008	0.93	1.326	5.261	0.

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	70	112	139	206	0
normalized size	1	1.	0.83	0.97	1.56	1.93	2.86	0.
time (sec)	N/A	0.052	0.03	0.005	0.936	1.29	9.57	0.

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	94	159	192	270	0
normalized size	1	1.	0.9	0.93	1.57	1.9	2.67	0.
time (sec)	N/A	0.069	0.027	0.007	0.945	1.342	13.642	0.

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	56	67	115	132	338	0
normalized size	1	1.	0.68	0.82	1.4	1.61	4.12	0.
time (sec)	N/A	0.033	0.044	0.007	0.95	1.356	3.243	0.

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	44	66	82	70	0
normalized size	1	1.	0.67	0.86	1.29	1.61	1.37	0.
time (sec)	N/A	0.02	0.028	0.003	0.938	1.284	2.375	0.

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	73	0	305	39	0
normalized size	1	1.	1.51	1.55	0.	6.49	0.83	0.
time (sec)	N/A	0.029	0.031	0.01	0.	1.373	2.611	0.

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	80	105	0	346	66	0
normalized size	1	1.	1.31	1.72	0.	5.67	1.08	0.
time (sec)	N/A	0.038	0.053	0.012	0.	1.37	4.689	0.

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	107	146	0	459	150	0
normalized size	1	1.	1.15	1.57	0.	4.94	1.61	0.
time (sec)	N/A	0.05	0.121	0.01	0.	1.721	8.334	0.

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	120	111	140	0	662	177	261
normalized size	1	1.02	0.94	1.19	0.	5.61	1.5	2.21
time (sec)	N/A	0.085	0.145	0.012	0.	1.881	52.476	1.17

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	89	114	0	543	264	182
normalized size	1	1.	1.03	1.33	0.	6.31	3.07	2.12
time (sec)	N/A	0.058	0.116	0.009	0.	1.628	27.298	1.177

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	73	75	0	427	49	0
normalized size	1	1.	1.4	1.44	0.	8.21	0.94	0.
time (sec)	N/A	0.04	0.062	0.008	0.	1.605	10.215	0.

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	46	62	92	68	0
normalized size	1	1.	0.86	1.1	1.48	2.19	1.62	0.
time (sec)	N/A	0.035	0.018	0.006	0.927	1.479	3.39	0.

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	69	109	150	61	0
normalized size	1	1.	0.88	1.01	1.6	2.21	0.9	0.
time (sec)	N/A	0.052	0.023	0.006	0.956	1.587	7.431	0.

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	81	94	157	204	90	0
normalized size	1	1.	0.81	0.94	1.57	2.04	0.9	0.
time (sec)	N/A	0.071	0.026	0.007	0.949	1.613	10.017	0.

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	104	118	204	252	122	0
normalized size	1	1.	0.83	0.94	1.62	2.	0.97	0.
time (sec)	N/A	0.089	0.03	0.007	0.956	1.487	12.66	0.

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	80	91	173	200	561	0
normalized size	1	1.	0.72	0.82	1.56	1.8	5.05	0.
time (sec)	N/A	0.049	0.052	0.006	0.947	1.322	11.38	0.

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	66	122	147	267	0
normalized size	1	1.	0.72	0.84	1.54	1.86	3.38	0.
time (sec)	N/A	0.03	0.038	0.008	0.943	1.318	7.561	0.

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	43	72	93	65	0
normalized size	1	1.	0.73	0.96	1.6	2.07	1.44	0.
time (sec)	N/A	0.03	0.023	0.005	0.943	1.28	6.424	0.

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	71	79	0	424	206	0
normalized size	1	1.	1.2	1.34	0.	7.19	3.49	0.
time (sec)	N/A	0.034	0.032	0.01	0.	1.086	10.899	0.

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	57	132	0	551	262	0
normalized size	1	1.	0.62	1.43	0.	5.99	2.85	0.
time (sec)	N/A	0.05	0.024	0.01	0.	1.087	19.553	0.

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	60	157	0	680	180	0
normalized size	1	1.	0.49	1.28	0.	5.53	1.46	0.
time (sec)	N/A	0.069	0.027	0.011	0.	1.13	34.553	0.

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	101	115	0	0	0	0	0
normalized size	1	0.96	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.129	0.227	0.	0.	0.	0.

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.096	0.065	0.	0.	0.	0.

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.088	0.063	0.	0.	0.	0.

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.092	0.064	0.	0.	0.	0.

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.085	0.063	0.	0.	0.	0.

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.083	0.073	0.	0.	0.	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.085	0.059	0.	0.	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	106	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.109	0.063	0.	0.	0.	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	110	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.043	0.064	0.	0.	0.	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.188	0.072	0.	0.	0.	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.112	0.024	0.	0.	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.113	0.025	0.	0.	0.	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.103	0.026	0.	0.	0.	0.

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.109	0.026	0.	0.	0.	0.

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.117	0.025	0.	0.	0.	0.

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.121	0.025	0.	0.	0.	0.

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	111	75	77	201	0	0
normalized size	1	1.	0.82	0.56	0.57	1.49	0.	0.
time (sec)	N/A	0.061	0.058	0.014	0.96	0.925	0.	0.

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	99	65	63	184	0	0
normalized size	1	1.	0.95	0.62	0.61	1.77	0.	0.
time (sec)	N/A	0.049	0.037	0.007	0.933	1.052	0.	0.

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	87	52	50	170	0	0
normalized size	1	1.	1.19	0.71	0.68	2.33	0.	0.
time (sec)	N/A	0.032	0.03	0.007	0.92	0.983	0.	0.

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	72	72	35	151	0	0
normalized size	1	1.	1.95	1.95	0.95	4.08	0.	0.
time (sec)	N/A	0.021	0.021	0.007	0.937	0.972	0.	0.

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	74	47	36	166	0	65
normalized size	1	1.	1.1	0.7	0.54	2.48	0.	0.97
time (sec)	N/A	0.031	0.039	0.007	1.398	1.013	0.	3.152

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	14	93	0	65
normalized size	1	1.	1.	0.74	0.45	3.	0.	2.1
time (sec)	N/A	0.009	0.011	0.006	1.421	0.995	0.	1.074

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	36	28	28	108	0	122
normalized size	1	1.	0.57	0.44	0.44	1.71	0.	1.94
time (sec)	N/A	0.02	0.014	0.006	1.432	0.994	0.	1.117

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	41	33	42	124	0	150
normalized size	1	1.	0.44	0.35	0.45	1.32	0.	1.6
time (sec)	N/A	0.031	0.016	0.007	1.405	0.971	0.	1.125

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	46	38	55	138	0	178
normalized size	1	1.	0.37	0.3	0.44	1.1	0.	1.42
time (sec)	N/A	0.042	0.018	0.013	1.41	0.919	0.	1.157

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	67	65	63	186	0	0
normalized size	1	1.	0.64	0.62	0.61	1.79	0.	0.
time (sec)	N/A	0.045	0.035	0.013	0.936	0.947	0.	0.

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	62	55	50	170	0	0
normalized size	1	1.	0.85	0.75	0.68	2.33	0.	0.
time (sec)	N/A	0.032	0.025	0.01	0.924	0.951	0.	0.

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	55	41	32	151	83	0
normalized size	1	1.	1.57	1.17	0.91	4.31	2.37	0.
time (sec)	N/A	0.021	0.01	0.01	0.924	0.966	6.587	0.

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	26	40	22	85	0	0
normalized size	1	1.	3.25	5.	2.75	10.62	0.	0.
time (sec)	N/A	0.011	0.006	0.002	0.918	0.969	0.	0.

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	14	74	0	34
normalized size	1	1.	1.	0.69	0.48	2.55	0.	1.17
time (sec)	N/A	0.01	0.009	0.01	1.429	1.018	0.	1.094

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	36	25	28	99	0	65
normalized size	1	1.	0.57	0.4	0.44	1.57	0.	1.03
time (sec)	N/A	0.02	0.012	0.01	1.407	1.002	0.	1.069

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	41	30	42	111	0	93
normalized size	1	1.	0.44	0.32	0.45	1.18	0.	0.99
time (sec)	N/A	0.033	0.014	0.01	1.431	0.988	0.	1.136

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.03	0.174	0.	0.	0.	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.014	0.142	0.	0.	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.013	0.143	0.	0.	0.	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.024	0.238	0.	0.	0.	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	78	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.015	0.144	0.	0.	0.	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	20	38	10	22
normalized size	1	1.	1.	2.4	1.33	2.53	0.67	1.47
time (sec)	N/A	0.014	0.005	0.01	0.916	0.944	0.117	1.064

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	110	0	49	107	39	51
normalized size	1	1.	5.	0.	2.23	4.86	1.77	2.32
time (sec)	N/A	0.015	0.13	0.07	1.197	1.031	9.477	1.099

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	88	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.069	0.07	0.	0.	0.	0.

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.054	0.076	0.	0.	0.	0.

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.052	0.07	0.	0.	0.	0.

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.032	0.066	0.	0.	0.	0.

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	69	93	123	335	0
normalized size	1	1.	0.89	1.1	1.48	1.95	5.32	0.
time (sec)	N/A	0.068	0.048	0.013	0.948	1.085	86.273	0.

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	74	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.053	0.073	0.	0.	0.	0.

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.052	0.079	0.	0.	0.	0.

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	141	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.288	0.185	0.081	0.	0.	0.	0.

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	135	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.159	0.148	0.	0.	0.	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.155	0.123	0.	0.	0.	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.14	0.108	0.	0.	0.	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	131	204	458	2227	0
normalized size	1	1.	0.96	1.3	2.02	4.53	22.05	0.
time (sec)	N/A	0.105	0.174	0.012	0.947	1.149	12.612	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	133	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.154	0.144	0.	0.	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	136	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.152	0.135	0.	0.	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	115	284	312	363	0	0
normalized size	1	1.	0.88	2.18	2.4	2.79	0.	0.
time (sec)	N/A	0.138	0.196	0.028	0.968	1.046	0.	0.

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	173	203	227	0	0
normalized size	1	1.	0.91	1.92	2.26	2.52	0.	0.
time (sec)	N/A	0.089	0.113	0.023	0.95	1.111	0.	0.

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	87	112	119	105	0
normalized size	1	1.	0.83	1.45	1.87	1.98	1.75	0.
time (sec)	N/A	0.054	0.047	0.02	0.964	1.041	36.535	0.

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	44	59	81	92	0	0
normalized size	1	1.	0.81	1.09	1.5	1.7	0.	0.
time (sec)	N/A	0.054	0.035	0.02	0.945	1.071	0.	0.

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	58	109	163	244	0	0
normalized size	1	1.	0.77	1.45	2.17	3.25	0.	0.
time (sec)	N/A	0.062	0.097	0.03	0.952	1.073	0.	0.

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	97	203	328	540	0	0
normalized size	1	1.	0.92	1.93	3.12	5.14	0.	0.
time (sec)	N/A	0.09	0.115	0.056	0.97	1.106	0.	0.

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	138	342	386	471	0	0
normalized size	1	1.	0.87	2.16	2.44	2.98	0.	0.
time (sec)	N/A	0.151	0.216	0.036	0.971	1.084	0.	0.

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	103	236	259	306	0	0
normalized size	1	1.	0.87	2.	2.19	2.59	0.	0.
time (sec)	N/A	0.115	0.146	0.026	0.961	1.062	0.	0.

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	125	151	170	0	0
normalized size	1	1.	0.91	1.45	1.76	1.98	0.	0.
time (sec)	N/A	0.076	0.077	0.023	0.961	1.08	0.	0.

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	78	109	142	0	0
normalized size	1	1.	0.93	1.1	1.54	2.	0.	0.
time (sec)	N/A	0.069	0.067	0.026	0.938	1.093	0.	0.

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	90	163	198	324	0	0
normalized size	1	1.	0.95	1.72	2.08	3.41	0.	0.
time (sec)	N/A	0.09	0.097	0.03	0.949	1.108	0.	0.

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	112	214	354	598	0	0
normalized size	1	1.	0.93	1.78	2.95	4.98	0.	0.
time (sec)	N/A	0.105	0.14	0.054	0.973	1.106	0.	0.

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	172	155	208	387	175	208
normalized size	1	1.	12.29	11.07	14.86	27.64	12.5	14.86
time (sec)	N/A	0.002	0.005	0.003	0.922	0.844	0.105	1.086

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	211	402	182	211
normalized size	1	1.	11.38	9.81	13.19	25.12	11.38	13.19
time (sec)	N/A	0.041	0.006	0.002	0.958	0.784	0.109	1.076

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	211	408	185	211
normalized size	1	1.	11.62	9.81	13.19	25.5	11.56	13.19
time (sec)	N/A	0.036	0.006	0.003	0.929	0.839	0.109	1.085

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	0	467	0	255
normalized size	1	1.	1.	10.95	0.	22.24	0.	12.14
time (sec)	N/A	0.016	0.125	0.033	0.	1.044	0.	1.361

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	107	0	22	77	0	95
normalized size	1	1.	8.23	0.	1.69	5.92	0.	7.31
time (sec)	N/A	0.014	0.156	0.075	1.154	1.017	0.	1.6

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	9	9	9	12	24	8	15
normalized size	1	1.12	1.12	1.12	1.5	3.	1.	1.88
time (sec)	N/A	0.006	0.005	0.003	0.924	0.94	0.27	1.07

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	39	12	24
normalized size	1	1.	1.	0.93	1.53	2.6	0.8	1.6
time (sec)	N/A	0.015	0.007	0.005	0.935	0.909	0.314	1.067

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	39	12	20
normalized size	1	1.	1.	0.93	1.53	2.6	0.8	1.33
time (sec)	N/A	0.014	0.007	0.005	0.926	0.939	0.324	1.058

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	63	42	29	0
normalized size	1	1.	1.	1.13	4.2	2.8	1.93	0.
time (sec)	N/A	0.015	0.014	0.001	0.931	1.093	0.576	0.

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	177	109	171	87	18
normalized size	1	1.	1.	12.64	7.79	12.21	6.21	1.29
time (sec)	N/A	0.002	0.02	0.02	0.971	0.951	3.773	1.068

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	109	177	0	20
normalized size	1	1.	1.	12.31	6.81	11.06	0.	1.25
time (sec)	N/A	0.014	0.028	0.019	0.984	0.953	0.	1.074

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	109	177	0	20
normalized size	1	1.	1.	12.31	6.81	11.06	0.	1.25
time (sec)	N/A	0.013	0.037	0.013	0.974	0.996	0.	1.099

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	826	236	0	0
normalized size	1	1.	1.	9.67	39.33	11.24	0.	0.
time (sec)	N/A	0.013	0.182	0.05	1.154	1.197	0.	0.

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	44	0	72	138	0	76
normalized size	1	1.	0.85	0.	1.38	2.65	0.	1.46
time (sec)	N/A	0.028	0.018	0.103	1.422	0.998	0.	1.073

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	142	143	0	1628	0	0
normalized size	1	1.	1.53	1.54	0.	17.51	0.	0.
time (sec)	N/A	0.086	0.46	0.043	0.	2.106	0.	0.

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	223	0	0	1342	0	0
normalized size	1	1.	0.88	0.	0.	5.33	0.	0.
time (sec)	N/A	0.234	0.808	0.072	0.	1.373	0.	0.

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	178	0	0	1031	0	0
normalized size	1	1.	0.89	0.	0.	5.18	0.	0.
time (sec)	N/A	0.164	0.557	0.066	0.	1.223	0.	0.

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	141	0	0	801	0	0
normalized size	1	1.	0.97	0.	0.	5.49	0.	0.
time (sec)	N/A	0.121	0.432	0.064	0.	1.166	0.	0.

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	123	0	0	645	0	0
normalized size	1	1.	1.38	0.	0.	7.25	0.	0.
time (sec)	N/A	0.088	0.199	0.065	0.	1.087	0.	0.

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	122	0	0	879	0	0
normalized size	1	1.	1.34	0.	0.	9.66	0.	0.
time (sec)	N/A	0.089	0.716	0.067	0.	1.755	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	0	281	0	0
normalized size	1	1.	0.6	0.	0.	2.96	0.	0.
time (sec)	N/A	0.065	0.05	0.067	0.	2.196	0.	0.

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	274	0	0	1719	0	0
normalized size	1	1.	0.77	0.	0.	4.8	0.	0.
time (sec)	N/A	0.405	1.592	0.065	0.	1.647	0.	0.

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	241	0	0	1335	0	0
normalized size	1	1.	0.83	0.	0.	4.59	0.	0.
time (sec)	N/A	0.316	0.741	0.068	0.	1.383	0.	0.

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	191	0	0	1019	0	0
normalized size	1	1.	0.86	0.	0.	4.61	0.	0.
time (sec)	N/A	0.245	0.496	0.065	0.	1.209	0.	0.

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	157	0	0	810	0	0
normalized size	1	1.	1.05	0.	0.	5.4	0.	0.
time (sec)	N/A	0.152	0.304	0.062	0.	1.17	0.	0.

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	185	0	0	1143	0	0
normalized size	1	1.	1.39	0.	0.	8.59	0.	0.
time (sec)	N/A	0.161	0.434	0.067	0.	1.877	0.	0.

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	217	0	0	1620	0	0
normalized size	1	1.	1.48	0.	0.	11.02	0.	0.
time (sec)	N/A	0.142	0.681	0.066	0.	3.049	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	39	53	46	47
normalized size	1	1.	1.	1.05	1.95	2.65	2.3	2.35
time (sec)	N/A	0.004	0.011	0.003	1.084	1.011	2.894	1.577

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	47	72	0	70
normalized size	1	1.	3.59	0.96	1.74	2.67	0.	2.59
time (sec)	N/A	0.009	0.079	0.004	1.093	1.065	0.	1.102

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	47	72	0	76
normalized size	1	1.	3.59	0.96	1.74	2.67	0.	2.81
time (sec)	N/A	0.009	0.077	0.004	1.097	1.075	0.	1.1

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	101	0	53	77	0	89
normalized size	1	1.	3.74	0.	1.96	2.85	0.	3.3
time (sec)	N/A	0.012	0.125	0.08	1.164	1.072	0.	1.098

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [451] had the largest ratio of [0.52]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	18	0.111
2	A	2	1	1.	16	0.062
3	A	2	1	1.	15	0.067
4	A	3	2	1.	18	0.111
5	A	2	1	1.	18	0.056
6	A	2	1	1.	18	0.056
7	A	3	2	1.	18	0.111
8	A	2	1	1.	18	0.056
9	A	2	1	1.	18	0.056
10	A	3	2	1.	18	0.111
11	A	3	2	1.	20	0.1
12	A	2	1	1.	18	0.056
13	A	2	1	1.	17	0.059
14	A	4	3	1.	20	0.15
15	A	2	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	2	1	1.	20	0.05
17	A	3	2	1.	20	0.1
18	A	2	1	1.	20	0.05
19	A	2	1	1.	20	0.05
20	A	3	2	1.	20	0.1
21	A	2	1	1.	20	0.05
22	A	2	1	1.	20	0.05
23	A	2	1	1.	20	0.05
24	A	3	2	1.	20	0.1
25	A	2	1	1.	20	0.05
26	A	2	1	1.	20	0.05
27	A	3	2	1.	20	0.1
28	A	2	1	1.	20	0.05
29	A	2	1	1.	20	0.05
30	A	3	2	1.	20	0.1
31	A	2	1	1.	18	0.056
32	A	2	1	1.	17	0.059
33	A	4	3	1.	20	0.15
34	A	2	1	1.	20	0.05
35	A	2	1	1.	20	0.05
36	A	3	2	1.	20	0.1
37	A	2	1	1.	20	0.05
38	A	2	1	1.	20	0.05
39	A	3	2	1.	20	0.1
40	A	2	1	1.	20	0.05
41	A	2	1	1.	20	0.05
42	A	3	2	1.	20	0.1
43	A	2	1	1.	20	0.05
44	A	2	1	1.	20	0.05
45	A	3	2	1.	20	0.1
46	A	2	1	1.	20	0.05
47	A	2	1	1.	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
48	A	3	2	1.	20	0.1
49	A	2	1	1.	20	0.05
50	A	2	1	1.	20	0.05
51	A	4	3	1.	20	0.15
52	A	2	1	1.	20	0.05
53	A	2	1	1.	20	0.05
54	A	3	3	1.	20	0.15
55	A	2	1	1.	20	0.05
56	A	9	8	1.	20	0.4
57	A	3	2	1.	20	0.1
58	A	8	8	1.	20	0.4
59	A	8	8	1.	20	0.4
60	A	3	2	1.	20	0.1
61	A	7	7	1.	18	0.389
62	A	7	7	1.	17	0.412
63	A	3	2	1.	20	0.1
64	A	7	7	1.	20	0.35
65	A	7	7	1.	20	0.35
66	A	3	2	1.	20	0.1
67	A	8	8	1.	20	0.4
68	A	8	8	1.	20	0.4
69	A	3	2	1.	20	0.1
70	A	9	8	1.	20	0.4
71	A	9	8	1.	20	0.4
72	A	3	2	1.	20	0.1
73	A	9	8	1.	20	0.4
74	A	9	8	1.	20	0.4
75	A	3	2	1.	20	0.1
76	A	8	8	1.	20	0.4
77	A	8	8	1.	20	0.4
78	A	3	2	1.	20	0.1
79	A	7	7	1.	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
80	A	7	7	1.	17	0.412
81	A	3	2	1.	20	0.1
82	A	8	8	1.	20	0.4
83	A	8	8	1.	20	0.4
84	A	3	2	1.	20	0.1
85	A	9	8	1.	20	0.4
86	A	9	8	1.	20	0.4
87	A	3	2	1.	20	0.1
88	A	3	2	1.	20	0.1
89	A	3	2	1.	20	0.1
90	A	3	2	1.	20	0.1
91	A	2	2	1.	20	0.1
92	A	3	2	1.	20	0.1
93	A	3	2	1.	20	0.1
94	A	3	2	1.	20	0.1
95	A	10	9	1.	20	0.45
96	A	10	9	1.	20	0.45
97	A	9	9	1.	20	0.45
98	A	9	9	1.	20	0.45
99	A	8	8	1.	20	0.4
100	A	8	8	1.	20	0.4
101	A	8	8	1.	18	0.444
102	A	8	8	1.	17	0.471
103	A	9	9	1.	20	0.45
104	A	9	9	1.	20	0.45
105	A	10	9	1.	20	0.45
106	A	10	9	1.	20	0.45
107	A	3	2	1.	22	0.091
108	A	15	8	1.	22	0.364
109	A	14	8	1.	22	0.364
110	A	3	2	1.	22	0.091
111	A	13	7	1.	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	13	7	1.	22	0.318
113	A	4	3	1.	22	0.136
114	A	13	7	1.	20	0.35
115	A	13	7	1.	19	0.368
116	A	3	2	1.	22	0.091
117	A	15	8	1.	22	0.364
118	A	14	8	1.	22	0.364
119	A	3	2	1.	22	0.091
120	A	16	9	1.	22	0.409
121	A	15	9	1.	22	0.409
122	A	3	2	1.	22	0.091
123	A	17	9	1.	22	0.409
124	A	2	1	1.	20	0.05
125	A	2	1	1.	20	0.05
126	A	2	1	1.	18	0.056
127	A	2	2	1.	20	0.1
128	A	2	2	1.	20	0.1
129	A	2	2	1.	20	0.1
130	A	3	2	1.	24	0.083
131	A	2	1	1.	20	0.05
132	A	2	1	1.	20	0.05
133	A	2	1	1.	20	0.05
134	A	2	1	1.	20	0.05
135	A	2	1	1.	20	0.05
136	A	2	1	1.	20	0.05
137	A	2	1	1.	20	0.05
138	A	2	1	1.	20	0.05
139	A	2	1	1.	22	0.045
140	A	2	1	1.	22	0.045
141	A	2	1	1.	22	0.045
142	A	2	1	1.	22	0.045
143	A	2	1	1.	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
144	A	2	1	1.	22	0.045
145	A	2	1	1.	22	0.045
146	A	2	1	1.	22	0.045
147	A	2	1	1.	22	0.045
148	A	2	1	1.	22	0.045
149	A	2	1	1.	22	0.045
150	A	2	1	1.	22	0.045
151	A	2	1	1.	22	0.045
152	A	2	1	1.	22	0.045
153	A	2	1	1.	22	0.045
154	A	2	1	1.	22	0.045
155	A	5	5	1.	22	0.227
156	A	13	9	1.	22	0.409
157	A	12	8	1.	22	0.364
158	A	4	4	1.	22	0.182
159	A	12	8	1.	22	0.364
160	A	12	8	1.	22	0.364
161	A	4	4	1.	22	0.182
162	A	12	8	1.	22	0.364
163	A	5	5	1.	22	0.227
164	A	13	9	1.	22	0.409
165	A	12	8	1.	22	0.364
166	A	4	4	1.	22	0.182
167	A	12	8	1.	22	0.364
168	A	13	9	1.	22	0.409
169	A	5	5	1.	22	0.227
170	A	13	9	1.	22	0.409
171	A	5	5	1.	22	0.227
172	A	13	9	1.	22	0.409
173	A	13	9	1.	22	0.409
174	A	5	5	1.	22	0.227
175	A	13	9	1.	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	14	10	1.	22	0.454
177	A	6	6	1.	22	0.273
178	A	14	10	1.	22	0.454
179	A	3	2	1.	22	0.091
180	A	3	2	1.	22	0.091
181	A	3	2	1.	22	0.091
182	A	5	5	1.	22	0.227
183	A	5	5	1.	22	0.227
184	A	5	5	1.	22	0.227
185	A	4	4	1.	22	0.182
186	A	3	3	1.	19	0.158
187	A	3	3	1.	22	0.136
188	A	3	3	1.	22	0.136
189	A	4	4	1.	22	0.182
190	A	6	6	1.	22	0.273
191	A	5	5	1.	20	0.25
192	A	5	5	1.	22	0.227
193	A	5	5	1.	22	0.227
194	A	6	6	1.	22	0.273
195	A	7	6	1.	22	0.273
196	A	3	2	1.	22	0.091
197	A	3	2	1.	22	0.091
198	A	3	2	1.	22	0.091
199	A	6	5	1.	22	0.227
200	A	6	5	1.	22	0.227
201	A	6	6	1.	22	0.273
202	A	5	4	1.	22	0.182
203	A	4	3	1.	19	0.158
204	A	4	3	1.	22	0.136
205	A	4	4	1.	22	0.182
206	A	4	3	1.	22	0.136
207	A	7	6	1.	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
208	A	6	5	1.	20	0.25
209	A	6	5	1.	22	0.227
210	A	6	6	1.	22	0.273
211	A	6	5	1.	22	0.227
212	A	7	6	1.	22	0.273
213	A	3	2	1.	22	0.091
214	A	3	2	1.	22	0.091
215	A	3	2	1.	22	0.091
216	A	4	4	1.	22	0.182
217	A	4	4	1.	22	0.182
218	A	5	5	1.	22	0.227
219	A	3	3	1.	22	0.136
220	A	2	2	1.	19	0.105
221	A	2	2	1.	22	0.091
222	A	3	3	1.	22	0.136
223	A	5	5	1.	22	0.227
224	A	4	4	1.	20	0.2
225	A	4	4	1.	22	0.182
226	A	5	5	1.	22	0.227
227	A	6	5	1.	22	0.227
228	A	3	2	1.	22	0.091
229	A	3	2	1.	22	0.091
230	A	3	2	1.	22	0.091
231	A	4	4	1.	22	0.182
232	A	5	5	1.	22	0.227
233	A	6	5	1.02	22	0.227
234	A	4	4	1.	22	0.182
235	A	3	3	1.	22	0.136
236	A	2	2	1.	19	0.105
237	A	3	3	1.	22	0.136
238	A	4	4	1.	22	0.182
239	A	5	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	4	4	1.	20	0.2
241	A	5	5	1.	22	0.227
242	A	6	6	1.	22	0.273
243	A	7	6	1.	22	0.273
244	A	3	2	1.	22	0.091
245	A	3	2	1.	22	0.091
246	A	3	2	1.	22	0.091
247	A	5	5	1.	22	0.227
248	A	6	5	1.	22	0.227
249	A	4	3	1.	22	0.136
250	A	3	3	1.	22	0.136
251	A	3	3	1.	19	0.158
252	A	4	3	1.	22	0.136
253	A	5	4	1.	22	0.182
254	A	6	5	1.	22	0.227
255	A	5	5	1.	22	0.227
256	A	5	5	1.	20	0.25
257	A	6	5	1.	22	0.227
258	A	7	6	1.	22	0.273
259	A	6	5	1.	26	0.192
260	A	5	5	1.	26	0.192
261	A	4	4	1.	26	0.154
262	A	6	5	1.	26	0.192
263	A	7	6	1.	26	0.231
264	A	7	6	1.	26	0.231
265	A	5	5	1.	24	0.208
266	A	7	6	1.	26	0.231
267	A	2	2	1.	26	0.077
268	A	2	2	1.	23	0.087
269	A	2	2	1.	26	0.077
270	A	5	4	1.	26	0.154
271	A	4	4	1.	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	3	3	1.	26	0.115
273	A	6	5	1.	26	0.192
274	A	7	6	1.	26	0.231
275	A	5	5	1.	26	0.192
276	A	1	1	1.	24	0.042
277	A	7	6	1.	26	0.231
278	A	2	2	1.	26	0.077
279	A	2	2	1.	23	0.087
280	A	2	2	1.	26	0.077
281	A	1	1	1.	22	0.045
282	A	6	5	1.	27	0.185
283	A	6	5	1.	27	0.185
284	A	5	5	1.	27	0.185
285	A	4	4	1.	27	0.148
286	A	6	5	1.	27	0.185
287	A	7	6	1.	27	0.222
288	A	8	7	1.	27	0.259
289	A	15	13	1.	27	0.482
290	A	14	12	1.	27	0.444
291	A	12	11	1.	25	0.44
292	A	14	12	1.	27	0.444
293	A	15	13	1.	27	0.482
294	A	16	13	1.	27	0.482
295	A	7	5	1.	27	0.185
296	A	7	5	1.	27	0.185
297	A	6	5	1.	27	0.185
298	A	5	4	1.	27	0.148
299	A	7	6	1.	27	0.222
300	A	7	6	1.	27	0.222
301	A	8	7	1.	27	0.259
302	A	16	13	1.	27	0.482
303	A	15	13	1.	27	0.482

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
304	A	14	12	1.	25	0.48
305	A	14	12	1.	27	0.444
306	A	15	13	1.	27	0.482
307	A	16	13	1.	27	0.482
308	A	5	4	1.	27	0.148
309	A	5	4	1.	27	0.148
310	A	4	4	1.	27	0.148
311	A	3	3	1.	27	0.111
312	A	6	5	1.	27	0.185
313	A	7	6	1.	27	0.222
314	A	8	7	1.	27	0.259
315	A	14	12	1.	27	0.444
316	A	12	11	1.	27	0.407
317	A	8	7	1.	25	0.28
318	A	14	12	1.	27	0.444
319	A	15	13	1.	27	0.482
320	A	16	13	1.	27	0.482
321	A	2	2	1.	27	0.074
322	A	2	2	1.	24	0.083
323	A	2	2	1.	27	0.074
324	A	2	2	1.	27	0.074
325	A	7	5	1.	27	0.185
326	A	5	4	1.	27	0.148
327	A	4	4	1.	27	0.148
328	A	4	4	1.	27	0.148
329	A	7	6	1.	27	0.222
330	A	8	7	1.	27	0.259
331	A	9	8	1.	27	0.296
332	A	14	12	1.	27	0.444
333	A	14	12	1.	27	0.444
334	A	14	12	1.	25	0.48
335	A	15	13	1.	27	0.482

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	16	13	1.	27	0.482
337	A	17	13	1.	27	0.482
338	A	2	2	1.	27	0.074
339	A	2	2	1.	24	0.083
340	A	2	2	1.	27	0.074
341	A	2	2	1.	27	0.074
342	A	5	5	1.	33	0.152
343	A	5	5	1.	35	0.143
344	A	5	5	1.	35	0.143
345	A	5	5	1.	37	0.135
346	A	5	5	1.	33	0.152
347	A	5	5	1.	35	0.143
348	A	5	5	1.	36	0.139
349	A	5	5	1.	36	0.139
350	A	1	1	1.	33	0.03
351	A	1	1	1.	35	0.029
352	A	1	1	1.	35	0.029
353	A	1	1	1.	37	0.027
354	A	1	1	1.	33	0.03
355	A	1	1	1.	35	0.029
356	A	1	1	1.	36	0.028
357	A	1	1	1.	36	0.028
358	A	6	5	1.	24	0.208
359	A	5	5	1.	24	0.208
360	A	4	4	1.	24	0.167
361	A	6	4	1.	24	0.167
362	A	7	5	1.	24	0.208
363	A	2	2	1.	24	0.083
364	A	2	2	1.	22	0.091
365	A	2	2	1.	21	0.095
366	A	2	2	1.	24	0.083
367	A	2	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	7	5	1.	24	0.208
369	A	6	5	1.	24	0.208
370	A	5	4	1.	24	0.167
371	A	7	5	1.	24	0.208
372	A	7	5	1.	24	0.208
373	A	2	2	1.	24	0.083
374	A	2	2	1.	22	0.091
375	A	2	2	1.	21	0.095
376	A	2	2	1.	24	0.083
377	A	2	2	1.	24	0.083
378	A	5	4	1.	24	0.167
379	A	4	4	1.	24	0.167
380	A	3	3	1.	24	0.125
381	A	6	4	1.	24	0.167
382	A	7	5	1.	24	0.208
383	A	2	2	1.	24	0.083
384	A	2	2	1.	22	0.091
385	A	2	2	1.	21	0.095
386	A	2	2	1.	24	0.083
387	A	2	2	1.	24	0.083
388	A	5	4	1.	24	0.167
389	A	4	4	1.	24	0.167
390	A	4	4	1.	24	0.167
391	A	7	5	1.	24	0.208
392	A	8	6	1.	24	0.25
393	A	2	2	1.	24	0.083
394	A	2	2	1.	22	0.091
395	A	2	2	1.	21	0.095
396	A	2	2	1.	24	0.083
397	A	2	2	1.	24	0.083
398	A	6	6	1.	27	0.222
399	A	6	6	1.	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
400	A	5	5	1.	27	0.185
401	A	4	4	1.	27	0.148
402	A	7	6	1.	27	0.222
403	A	8	7	1.	27	0.259
404	A	9	7	1.	27	0.259
405	A	15	13	1.	27	0.482
406	A	14	12	1.	27	0.444
407	A	14	12	1.	25	0.48
408	A	15	13	1.	27	0.482
409	A	16	13	1.	27	0.482
410	A	17	13	1.	27	0.482
411	A	7	7	1.	27	0.259
412	A	7	6	1.	27	0.222
413	A	6	5	1.	27	0.185
414	A	5	5	1.	27	0.185
415	A	7	6	1.	27	0.222
416	A	8	7	1.	27	0.259
417	A	9	7	1.	27	0.259
418	A	16	14	1.	27	0.518
419	A	15	13	1.	27	0.482
420	A	14	12	1.	25	0.48
421	A	6	6	1.	27	0.222
422	A	16	13	1.	27	0.482
423	A	17	13	1.	27	0.482
424	A	5	5	1.	27	0.185
425	A	5	5	1.	27	0.185
426	A	4	4	1.	27	0.148
427	A	4	4	1.	27	0.148
428	A	7	6	1.	27	0.222
429	A	8	7	1.	27	0.259
430	A	9	7	1.	27	0.259
431	A	14	12	1.	27	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
432	A	14	12	1.	27	0.444
433	A	14	12	1.	25	0.48
434	A	15	13	1.	27	0.482
435	A	16	13	1.	27	0.482
436	A	17	13	1.	27	0.482
437	A	2	2	1.	27	0.074
438	A	2	2	1.	27	0.074
439	A	2	2	1.	24	0.083
440	A	2	2	1.	27	0.074
441	A	2	2	1.	27	0.074
442	A	5	5	1.	27	0.185
443	A	5	5	1.	27	0.185
444	A	5	5	1.	27	0.185
445	A	5	4	1.	27	0.148
446	A	8	7	1.	27	0.259
447	A	9	8	1.	27	0.296
448	A	10	8	1.	27	0.296
449	A	15	13	1.	27	0.482
450	A	15	13	1.	27	0.482
451	A	15	13	1.	25	0.52
452	A	16	14	1.	27	0.518
453	A	17	14	1.	27	0.518
454	A	18	14	1.	27	0.518
455	C	2	2	0.26	27	0.074
456	A	2	2	1.	27	0.074
457	A	2	2	1.	24	0.083
458	A	2	2	1.	27	0.074
459	A	2	2	1.	27	0.074
460	A	6	6	1.	24	0.25
461	A	5	5	1.	24	0.208
462	A	4	4	1.	24	0.167
463	A	7	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
464	A	8	6	1.	24	0.25
465	A	2	2	1.	24	0.083
466	A	2	2	1.	22	0.091
467	A	2	2	1.	21	0.095
468	A	2	2	1.	24	0.083
469	A	2	2	1.	24	0.083
470	A	7	6	1.	24	0.25
471	A	6	5	1.	24	0.208
472	A	5	5	1.	24	0.208
473	A	7	5	1.	24	0.208
474	A	8	6	1.	24	0.25
475	A	2	2	1.	24	0.083
476	A	2	2	1.	22	0.091
477	A	2	2	1.	21	0.095
478	A	2	2	1.	24	0.083
479	A	2	2	1.	24	0.083
480	A	5	5	1.	24	0.208
481	A	4	4	1.	24	0.167
482	A	4	4	1.	24	0.167
483	A	7	5	1.	24	0.208
484	A	8	6	1.	24	0.25
485	A	2	2	1.	24	0.083
486	A	2	2	1.	22	0.091
487	A	2	2	1.	21	0.095
488	A	2	2	1.	24	0.083
489	A	2	2	1.	24	0.083
490	A	5	5	1.	24	0.208
491	A	5	5	1.	24	0.208
492	A	5	4	1.	24	0.167
493	A	8	6	1.	24	0.25
494	A	9	7	1.	24	0.292
495	A	2	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
496	A	2	2	1.	22	0.091
497	A	2	2	1.	21	0.095
498	A	2	2	1.	24	0.083
499	A	2	2	1.	24	0.083
500	A	3	3	1.	24	0.125
501	A	3	3	1.	24	0.125
502	A	3	3	1.	24	0.125
503	A	3	3	1.	24	0.125
504	A	3	3	1.	24	0.125
505	A	3	3	1.	24	0.125
506	A	5	5	1.	26	0.192
507	A	4	4	1.	26	0.154
508	A	3	3	1.	26	0.115
509	A	4	4	1.	26	0.154
510	A	3	2	1.	26	0.077
511	A	3	2	1.	26	0.077
512	A	3	2	1.	24	0.083
513	A	3	2	1.	23	0.087
514	A	3	2	1.	26	0.077
515	A	3	2	1.	26	0.077
516	A	7	7	1.	26	0.269
517	A	5	5	1.	26	0.192
518	A	6	6	1.	26	0.231
519	A	6	6	1.	26	0.231
520	A	4	4	1.	26	0.154
521	A	6	6	1.	26	0.231
522	A	6	6	1.	26	0.231
523	A	4	4	1.	26	0.154
524	A	6	6	1.	24	0.25
525	A	6	6	1.	24	0.25
526	A	4	4	1.	24	0.167
527	A	8	7	1.	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
528	A	6	5	1.	26	0.192
529	A	7	6	1.	26	0.231
530	A	7	6	1.	26	0.231
531	A	5	4	1.	26	0.154
532	A	7	6	1.	26	0.231
533	A	7	6	1.	26	0.231
534	A	5	4	1.	26	0.154
535	A	9	7	1.	26	0.269
536	A	7	5	1.	26	0.192
537	A	8	6	1.	26	0.231
538	A	8	6	1.	26	0.231
539	A	6	4	1.	26	0.154
540	A	8	6	1.	26	0.231
541	A	8	6	1.	26	0.231
542	A	6	4	1.	26	0.154
543	A	6	6	1.	26	0.231
544	A	4	4	1.	26	0.154
545	A	5	5	1.	26	0.192
546	A	5	5	1.	26	0.192
547	A	3	3	1.	26	0.115
548	A	5	5	1.	26	0.192
549	A	5	5	1.	26	0.192
550	A	3	3	1.	26	0.115
551	A	6	6	1.	26	0.231
552	A	4	4	1.	26	0.154
553	A	5	5	1.	26	0.192
554	A	5	5	1.	26	0.192
555	A	3	3	1.	26	0.115
556	A	6	6	1.	26	0.231
557	A	2	2	1.	26	0.077
558	A	4	4	1.	26	0.154
559	A	6	6	1.	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
560	A	4	4	1.	26	0.154
561	A	6	6	1.	26	0.231
562	A	2	2	1.	26	0.077
563	A	4	4	1.	26	0.154
564	A	7	6	1.	26	0.231
565	A	3	3	1.	26	0.115
566	A	5	4	1.	26	0.154
567	A	8	7	1.	28	0.25
568	A	8	7	1.	28	0.25
569	A	7	7	1.	28	0.25
570	A	6	6	1.	28	0.214
571	A	10	6	1.	28	0.214
572	A	13	8	1.	28	0.286
573	A	12	8	1.	28	0.286
574	C	2	2	0.25	28	0.071
575	C	2	2	0.28	28	0.071
576	C	2	2	0.33	26	0.077
577	C	2	2	0.49	28	0.071
578	C	2	2	0.64	28	0.071
579	C	2	2	1.16	28	0.071
580	C	2	2	1.78	28	0.071
581	C	2	2	0.13	28	0.071
582	C	2	2	0.13	28	0.071
583	C	2	2	0.15	25	0.08
584	C	2	2	0.13	28	0.071
585	C	2	2	0.13	28	0.071
586	A	8	7	1.	28	0.25
587	A	8	7	1.	28	0.25
588	A	7	7	1.	28	0.25
589	A	6	6	1.	28	0.214
590	A	10	6	1.	28	0.214
591	A	13	8	1.	28	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
592	A	12	8	1.	28	0.286
593	C	2	2	0.25	28	0.071
594	C	2	2	0.29	28	0.071
595	C	2	2	0.3	25	0.08
596	C	2	2	0.5	28	0.071
597	C	2	2	0.66	28	0.071
598	C	2	2	1.17	28	0.071
599	C	2	2	1.66	28	0.071
600	C	2	2	0.13	28	0.071
601	C	2	2	0.14	28	0.071
602	C	2	2	0.14	26	0.077
603	C	2	2	0.13	28	0.071
604	C	2	2	0.13	28	0.071
605	A	7	6	1.	22	0.273
606	A	7	6	1.	22	0.273
607	A	7	6	1.	22	0.273
608	A	6	6	1.	22	0.273
609	A	5	5	1.	22	0.227
610	A	10	7	1.	22	0.318
611	A	11	8	1.	22	0.364
612	A	15	10	1.47	22	0.454
613	A	14	9	1.53	22	0.409
614	A	7	7	1.39	19	0.368
615	A	8	8	1.32	22	0.364
616	A	9	8	1.13	22	0.364
617	A	9	8	1.24	22	0.364
618	C	1	1	0.1	22	0.045
619	C	1	1	0.1	22	0.045
620	C	1	1	0.11	20	0.05
621	C	1	1	0.09	22	0.045
622	C	1	1	0.09	22	0.045
623	A	7	6	1.	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
624	A	7	6	1.	22	0.273
625	A	6	6	1.	22	0.273
626	A	5	5	1.	22	0.227
627	A	10	7	1.	22	0.318
628	A	11	8	1.	22	0.364
629	A	14	10	1.42	22	0.454
630	A	14	9	1.49	22	0.409
631	A	7	7	1.39	20	0.35
632	A	8	8	1.33	22	0.364
633	A	9	8	1.13	22	0.364
634	C	1	1	0.09	22	0.045
635	C	1	1	0.09	22	0.045
636	C	1	1	0.07	19	0.053
637	C	1	1	0.09	22	0.045
638	A	11	9	1.	22	0.409
639	A	11	9	1.	22	0.409
640	A	9	8	1.	22	0.364
641	A	6	6	1.	22	0.273
642	A	6	6	1.	22	0.273
643	A	11	8	1.	22	0.364
644	A	13	9	1.	22	0.409
645	C	1	1	0.15	22	0.045
646	C	1	1	0.17	22	0.045
647	C	1	1	0.36	22	0.045
648	A	8	8	1.32	19	0.421
649	C	1	1	1.65	22	0.045
650	C	1	1	2.76	22	0.045
651	C	1	1	3.97	22	0.045
652	C	1	1	0.09	22	0.045
653	C	1	1	0.09	22	0.045
654	C	1	1	0.09	22	0.045
655	C	1	1	0.09	20	0.05

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	C	1	1	0.08	22	0.045
657	C	1	1	0.08	22	0.045
658	A	8	7	1.	24	0.292
659	A	8	7	1.	24	0.292
660	A	7	7	1.	24	0.292
661	A	6	6	1.	24	0.25
662	A	10	7	1.	24	0.292
663	A	13	9	1.	24	0.375
664	A	12	9	1.	24	0.375
665	C	2	2	0.19	24	0.083
666	C	2	2	0.23	24	0.083
667	C	2	2	0.27	22	0.091
668	C	2	2	0.52	24	0.083
669	C	2	2	0.71	24	0.083
670	C	2	2	1.75	24	0.083
671	C	2	2	2.85	24	0.083
672	A	2	2	1.	24	0.083
673	A	2	2	1.	24	0.083
674	A	2	2	1.	21	0.095
675	A	2	2	1.	24	0.083
676	A	2	2	1.	24	0.083
677	A	8	7	1.	24	0.292
678	A	8	7	1.	24	0.292
679	A	7	7	1.	24	0.292
680	A	6	6	1.	24	0.25
681	A	10	7	1.	24	0.292
682	A	13	9	1.	24	0.375
683	A	12	9	1.	24	0.375
684	C	2	2	0.19	24	0.083
685	C	2	2	0.24	24	0.083
686	C	2	2	0.25	21	0.095
687	C	2	2	0.53	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
688	C	2	2	0.72	24	0.083
689	C	2	2	1.75	24	0.083
690	C	2	2	2.56	24	0.083
691	A	2	2	1.	24	0.083
692	A	2	2	1.	24	0.083
693	A	2	2	1.	22	0.091
694	A	2	2	1.	24	0.083
695	A	2	2	1.	24	0.083
696	A	9	7	1.	24	0.292
697	A	8	7	1.	24	0.292
698	A	7	6	1.	24	0.25
699	A	11	8	1.	24	0.333
700	A	15	9	1.	24	0.375
701	A	14	10	1.	24	0.417
702	C	2	2	0.19	24	0.083
703	C	2	2	0.23	22	0.091
704	C	2	2	0.25	24	0.083
705	C	2	2	0.45	24	0.083
706	C	2	2	0.68	24	0.083
707	C	2	2	0.82	24	0.083
708	C	2	2	3.69	24	0.083
709	A	2	2	1.	24	0.083
710	A	2	2	1.	24	0.083
711	A	2	2	1.	21	0.095
712	A	2	2	1.	24	0.083
713	A	2	2	1.	24	0.083
714	A	7	6	1.	24	0.25
715	A	7	6	1.	24	0.25
716	A	7	6	1.	24	0.25
717	A	6	6	1.	24	0.25
718	A	5	5	1.	24	0.208
719	A	10	7	1.	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
720	A	11	8	1.	24	0.333
721	A	15	9	1.44	24	0.375
722	A	14	8	1.48	24	0.333
723	A	7	7	1.4	21	0.333
724	A	8	8	1.34	24	0.333
725	A	9	8	1.27	24	0.333
726	A	9	8	1.21	24	0.333
727	A	2	2	1.	24	0.083
728	A	2	2	1.	24	0.083
729	A	2	2	1.	22	0.091
730	A	2	2	1.	24	0.083
731	A	2	2	1.	24	0.083
732	A	7	6	1.	24	0.25
733	A	7	6	1.	24	0.25
734	A	6	6	1.	24	0.25
735	A	5	5	1.	24	0.208
736	A	10	7	1.	24	0.292
737	A	11	8	1.	24	0.333
738	A	16	9	1.43	24	0.375
739	A	14	8	1.48	24	0.333
740	A	7	7	1.4	22	0.318
741	A	8	8	1.34	24	0.333
742	A	9	8	1.25	24	0.333
743	A	2	2	1.	24	0.083
744	A	2	2	1.	24	0.083
745	A	2	2	1.	21	0.095
746	A	2	2	1.	24	0.083
747	A	11	7	1.	24	0.292
748	A	9	7	1.	24	0.292
749	A	7	6	1.	24	0.25
750	A	6	6	1.	24	0.25
751	A	6	6	1.	24	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
752	A	11	8	1.	24	0.333
753	A	13	9	1.	24	0.375
754	C	2	2	0.21	24	0.083
755	C	2	2	0.26	24	0.083
756	C	2	2	0.53	24	0.083
757	A	8	8	1.33	21	0.381
758	C	2	2	2.37	24	0.083
759	C	2	2	3.31	24	0.083
760	C	2	2	4.23	24	0.083
761	A	2	2	1.	24	0.083
762	A	2	2	1.	24	0.083
763	A	2	2	1.	24	0.083
764	A	2	2	1.	22	0.091
765	A	2	2	1.	24	0.083
766	A	2	2	1.	24	0.083
767	A	3	2	1.	22	0.091
768	A	3	2	1.	22	0.091
769	A	3	2	1.	22	0.091
770	A	4	3	1.	22	0.136
771	A	3	2	1.	22	0.091
772	A	3	2	1.	22	0.091
773	A	6	5	1.	22	0.227
774	A	5	4	1.	22	0.182
775	A	4	3	1.	22	0.136
776	A	4	3	1.	20	0.15
777	A	5	4	1.	22	0.182
778	A	6	5	1.	22	0.227
779	A	20	8	1.	22	0.364
780	A	19	7	1.	22	0.318
781	A	19	7	1.	22	0.318
782	A	19	7	1.	22	0.318
783	A	19	7	1.	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
784	A	21	8	1.	22	0.364
785	A	20	8	1.	22	0.364
786	A	22	9	1.	22	0.409
787	A	5	5	1.	24	0.208
788	A	7	7	1.	24	0.292
789	A	4	4	1.	24	0.167
790	A	6	6	1.	22	0.273
791	A	6	4	1.	24	0.167
792	A	5	5	1.	24	0.208
793	A	7	5	1.	24	0.208
794	A	6	6	1.	24	0.25
795	A	13	8	1.25	24	0.333
796	A	10	6	1.29	24	0.25
797	A	11	7	1.29	24	0.292
798	A	9	5	1.3	21	0.238
799	A	13	8	1.27	24	0.333
800	A	10	6	1.27	24	0.25
801	A	3	3	1.	28	0.107
802	A	3	3	1.	28	0.107
803	A	3	3	1.	28	0.107
804	A	3	3	1.	28	0.107
805	A	5	4	1.	24	0.167
806	A	4	4	1.	24	0.167
807	A	3	3	1.	24	0.125
808	A	6	4	1.	24	0.167
809	A	7	5	1.	24	0.208
810	A	7	7	1.	24	0.292
811	A	6	6	1.	24	0.25
812	A	3	3	1.	22	0.136
813	A	5	5	1.	24	0.208
814	A	6	6	1.	24	0.25
815	A	10	6	1.	24	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
816	A	9	5	1.31	24	0.208
817	A	7	4	1.16	21	0.19
818	A	10	6	1.28	24	0.25
819	A	11	7	1.22	24	0.292
820	A	7	4	1.15	24	0.167
821	A	13	8	1.21	24	0.333
822	A	5	5	1.	24	0.208
823	A	5	5	1.	24	0.208
824	A	4	4	1.	24	0.167
825	A	4	4	1.	24	0.167
826	A	7	5	1.	24	0.208
827	A	8	6	1.	24	0.25
828	A	8	8	1.	24	0.333
829	A	7	7	1.	24	0.292
830	A	5	5	1.	24	0.208
831	A	4	4	1.	22	0.182
832	A	6	6	1.	24	0.25
833	A	7	6	1.	24	0.25
834	A	10	6	1.	24	0.25
835	A	10	6	1.	24	0.25
836	A	10	6	1.	21	0.286
837	A	11	7	1.	24	0.292
838	A	13	8	1.	24	0.333
839	A	13	8	1.	24	0.333
840	A	14	9	1.	24	0.375
841	A	4	4	0.97	26	0.154
842	A	3	3	0.93	24	0.125
843	A	2	2	1.	17	0.118
844	A	2	2	1.	26	0.077
845	A	2	2	1.	26	0.077
846	A	2	2	1.	26	0.077
847	A	4	4	1.	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
848	A	3	3	1.	24	0.125
849	A	2	2	1.	17	0.118
850	A	2	2	1.	26	0.077
851	A	2	2	1.	26	0.077
852	A	2	2	1.	26	0.077
853	A	5	4	1.	24	0.167
854	A	4	4	1.	24	0.167
855	A	3	3	1.	24	0.125
856	A	6	4	1.	24	0.167
857	A	7	5	1.	24	0.208
858	A	7	7	1.	24	0.292
859	A	6	6	1.	24	0.25
860	A	3	3	1.	24	0.125
861	A	5	5	1.	24	0.208
862	A	6	6	1.	24	0.25
863	A	2	2	1.	24	0.083
864	A	3	3	1.	24	0.125
865	A	3	3	1.	22	0.136
866	A	2	2	1.	21	0.095
867	A	2	2	1.	24	0.083
868	A	3	3	1.	24	0.125
869	A	3	3	1.	24	0.125
870	A	5	5	1.	24	0.208
871	A	4	4	1.	24	0.167
872	A	4	4	1.	24	0.167
873	A	7	5	1.	24	0.208
874	A	8	6	1.	24	0.25
875	A	7	7	1.	24	0.292
876	A	5	5	1.	24	0.208
877	A	4	4	1.	24	0.167
878	A	6	6	1.	24	0.25
879	A	7	6	1.	24	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
880	A	2	2	1.	24	0.083
881	A	3	3	1.	24	0.125
882	A	3	3	1.	22	0.136
883	A	2	2	1.	21	0.095
884	A	2	2	1.	24	0.083
885	A	3	3	1.	24	0.125
886	A	3	3	1.	24	0.125
887	A	5	4	1.	24	0.167
888	A	4	4	1.	24	0.167
889	A	3	3	1.	24	0.125
890	A	6	4	1.	24	0.167
891	A	7	5	1.	24	0.208
892	A	7	7	1.	24	0.292
893	A	6	6	1.	24	0.25
894	A	3	3	1.	24	0.125
895	A	5	5	1.	24	0.208
896	A	6	6	1.	24	0.25
897	A	10	6	1.	24	0.25
898	A	8	5	1.	22	0.227
899	A	11	7	1.	24	0.292
900	A	12	8	1.	24	0.333
901	A	8	5	1.	24	0.208
902	A	14	9	1.	24	0.375
903	A	2	2	1.	24	0.083
904	A	2	2	1.	24	0.083
905	A	2	2	1.	21	0.095
906	A	2	2	1.	24	0.083
907	A	2	2	1.	24	0.083
908	A	5	5	1.	24	0.208
909	A	4	4	1.	24	0.167
910	A	4	4	1.	24	0.167
911	A	7	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
912	A	8	6	1.	24	0.25
913	A	7	7	1.	24	0.292
914	A	5	5	1.	24	0.208
915	A	4	4	1.	24	0.167
916	A	6	6	1.	24	0.25
917	A	7	6	1.	24	0.25
918	A	11	7	1.	24	0.292
919	A	11	7	1.	22	0.318
920	A	12	8	1.	24	0.333
921	A	14	9	1.	24	0.375
922	A	14	9	1.	24	0.375
923	A	15	10	1.	24	0.417
924	A	2	2	1.	24	0.083
925	A	2	2	1.	24	0.083
926	A	2	2	1.	21	0.095
927	A	2	2	1.	24	0.083
928	A	2	2	1.	24	0.083
929	A	6	6	1.	22	0.273
930	A	5	5	1.	22	0.227
931	A	5	5	1.	20	0.25
932	A	5	5	1.	22	0.227
933	A	3	2	1.	22	0.091
934	A	3	2	1.	22	0.091
935	A	3	2	1.	22	0.091
936	A	3	2	1.	22	0.091
937	A	5	3	1.	22	0.136
938	A	4	3	1.	22	0.136
939	A	3	3	1.	22	0.136
940	A	2	2	1.	22	0.091
941	A	5	5	1.	22	0.227
942	A	5	5	1.	19	0.263
943	A	5	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
944	A	6	6	1.	22	0.273
945	A	6	5	1.	22	0.227
946	A	6	6	1.	22	0.273
947	A	6	5	1.	20	0.25
948	A	6	5	1.	22	0.227
949	A	3	2	1.	22	0.091
950	A	3	2	1.	22	0.091
951	A	3	2	1.	22	0.091
952	A	3	2	1.	22	0.091
953	A	5	3	1.	22	0.136
954	A	4	3	1.	22	0.136
955	A	3	3	1.	22	0.136
956	A	2	2	1.	22	0.091
957	A	6	5	1.	22	0.227
958	A	6	6	1.	22	0.273
959	A	6	5	1.	19	0.263
960	A	6	5	1.	22	0.227
961	A	7	6	1.	22	0.273
962	A	5	5	1.	22	0.227
963	A	4	4	1.	20	0.2
964	A	4	4	1.	22	0.182
965	A	3	2	1.	22	0.091
966	A	3	2	1.	22	0.091
967	A	3	2	1.	22	0.091
968	A	3	3	1.	22	0.136
969	A	2	2	1.	22	0.091
970	A	4	4	1.	19	0.21
971	A	4	4	1.	22	0.182
972	A	5	5	1.	22	0.227
973	A	6	5	1.02	22	0.227
974	A	5	5	1.	20	0.25
975	A	4	4	1.	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
976	A	3	2	1.	22	0.091
977	A	3	2	1.	22	0.091
978	A	3	2	1.	22	0.091
979	A	3	2	1.	22	0.091
980	A	4	4	1.	22	0.182
981	A	3	3	1.	22	0.136
982	A	3	3	1.	19	0.158
983	A	4	4	1.	22	0.182
984	A	5	5	1.	22	0.227
985	A	6	6	1.	22	0.273
986	A	4	3	0.96	24	0.125
987	A	4	3	1.	22	0.136
988	A	3	3	1.	22	0.136
989	A	4	3	1.	22	0.136
990	A	3	3	1.	20	0.15
991	A	4	3	1.	19	0.158
992	A	3	3	1.	22	0.136
993	A	4	3	1.	22	0.136
994	A	3	3	1.	22	0.136
995	A	4	3	1.	22	0.136
996	A	4	3	1.	26	0.115
997	A	4	3	1.	26	0.115
998	A	4	3	1.	26	0.115
999	A	4	3	1.	26	0.115
1000	A	4	3	1.	26	0.115
1001	A	4	3	1.	26	0.115
1002	A	6	4	1.	28	0.143
1003	A	5	4	1.	28	0.143
1004	A	4	4	1.	28	0.143
1005	A	3	3	1.	28	0.107
1006	A	4	4	1.	28	0.143
1007	A	1	1	1.	28	0.036

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1008	A	2	2	1.	28	0.071
1009	A	3	2	1.	28	0.071
1010	A	4	2	1.	28	0.071
1011	A	5	3	1.	28	0.107
1012	A	4	3	1.	28	0.107
1013	A	3	3	1.	28	0.107
1014	A	2	2	1.	28	0.071
1015	A	1	1	1.	28	0.036
1016	A	2	2	1.	28	0.071
1017	A	3	2	1.	28	0.071
1018	A	3	3	1.	24	0.125
1019	A	3	3	1.	22	0.136
1020	A	3	3	1.	21	0.143
1021	A	4	4	1.	24	0.167
1022	A	3	3	1.	24	0.125
1023	A	3	2	1.	18	0.111
1024	A	1	1	1.	33	0.03
1025	A	3	2	1.	24	0.083
1026	A	3	2	1.	22	0.091
1027	A	3	2	1.	20	0.1
1028	A	3	2	1.	19	0.105
1029	A	3	2	1.	22	0.091
1030	A	3	2	1.	22	0.091
1031	A	3	2	1.	22	0.091
1032	A	5	3	1.	24	0.125
1033	A	5	3	1.	22	0.136
1034	A	5	3	1.	20	0.15
1035	A	4	3	1.	19	0.158
1036	A	3	2	1.	22	0.091
1037	A	5	3	1.	22	0.136
1038	A	5	3	1.	22	0.136
1039	A	3	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1040	A	3	2	1.	26	0.077
1041	A	3	2	1.	24	0.083
1042	A	3	2	1.	26	0.077
1043	A	3	2	1.	26	0.077
1044	A	3	2	1.	26	0.077
1045	A	3	2	1.	26	0.077
1046	A	3	2	1.	26	0.077
1047	A	3	2	1.	24	0.083
1048	A	3	2	1.	26	0.077
1049	A	3	2	1.	26	0.077
1050	A	3	2	1.	26	0.077
1051	A	1	1	1.	17	0.059
1052	A	2	2	1.	21	0.095
1053	A	2	2	1.	21	0.095
1054	A	2	2	1.	25	0.08
1055	A	1	1	1.	31	0.032
1056	A	2	1	1.12	17	0.059
1057	A	3	2	1.	21	0.095
1058	A	3	2	1.	21	0.095
1059	A	3	2	1.	21	0.095
1060	A	1	1	1.	17	0.059
1061	A	2	2	1.	21	0.095
1062	A	2	2	1.	21	0.095
1063	A	2	2	1.	25	0.08
1064	A	5	5	1.	22	0.227
1065	A	7	7	1.	26	0.269
1066	A	8	6	1.	30	0.2
1067	A	7	6	1.	30	0.2
1068	A	6	6	1.	30	0.2
1069	A	5	5	1.	30	0.167
1070	A	5	5	1.	30	0.167
1071	A	3	3	1.	30	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1072	A	9	7	1.	30	0.233
1073	A	8	7	1.	30	0.233
1074	A	7	7	1.	30	0.233
1075	A	6	6	1.	30	0.2
1076	A	6	6	1.	30	0.2
1077	A	6	6	1.	30	0.2
1078	A	1	1	1.	17	0.059
1079	A	1	1	1.	27	0.037
1080	A	1	1	1.	27	0.037
1081	A	1	1	1.	27	0.037

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(a+bx^3\right) \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=33

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Rubi [A] time = 0.0325437, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)dx &= \frac{1}{3}\operatorname{Subst}\left(\int (a+bx)(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(aA+(Ab+aB)x+bBx^2\right)dx,x,x^3\right)\\ &= \frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9\end{aligned}$$

Mathematica [A] time = 0.0084485, size = 33, normalized size = 1.

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Maple [A] time = 0.004, size = 28, normalized size = 0.9

$$\frac{aAx^3}{3}+\frac{(Ab+Ba)\,x^6}{6}+\frac{bBx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Maxima [A] time = 1.21202, size = 36, normalized size = 1.09

$$\frac{1}{9}Bbx^9 + \frac{1}{6}(Ba + Ab)x^6 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

Fricas [A] time = 1.22531, size = 74, normalized size = 2.24

$$\frac{1}{9}x^9bB + \frac{1}{6}x^6aB + \frac{1}{6}x^6bA + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b*B + 1/6*x^6*a*B + 1/6*x^6*b*A + 1/3*x^3*a*A
```

Sympy [A] time = 0.059812, size = 29, normalized size = 0.88

$$\frac{Aax^3}{3} + \frac{Bbx^9}{9} + x^6\left(\frac{Ab}{6} + \frac{Ba}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)
```

Giac [A] time = 1.18944, size = 39, normalized size = 1.18

$$\frac{1}{9}Bbx^9 + \frac{1}{6}Bax^6 + \frac{1}{6}Abx^6 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3
```

3.2 $\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Rubi [A] time = 0.0169825, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {448}

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aAx+(Ab+aB)x^4+bBx^7\right)\,dx\\ &= \frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8\end{aligned}$$

Mathematica [A] time = 0.0050731, size = 33, normalized size = 1.

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Maple [A] time = 0., size = 28, normalized size = 0.9

$$\frac{aAx^2}{2}+\frac{(Ab+Ba)\,x^5}{5}+\frac{bBx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Maxima [A] time = 1.44324, size = 36, normalized size = 1.09

$$\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

Fricas [A] time = 1.28918, size = 74, normalized size = 2.24

$$\frac{1}{8}x^8bB+\frac{1}{5}x^5aB+\frac{1}{5}x^5bA+\frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b*B + 1/5*x^5*a*B + 1/5*x^5*b*A + 1/2*x^2*a*A
```

Sympy [A] time = 0.060681, size = 29, normalized size = 0.88

$$\frac{Aax^2}{2} + \frac{Bbx^8}{8} + x^5\left(\frac{Ab}{5} + \frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)
```

Giac [A] time = 1.1539, size = 39, normalized size = 1.18

$$\frac{1}{8}\,Bbx^8 + \frac{1}{5}\,Bax^5 + \frac{1}{5}\,Abx^5 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2
```

3.3 $\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

```
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Rubi [A] time = 0.0129807, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aA+(Ab+aB)x^3+bBx^6\right)\,dx\\ &= aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7\end{aligned}$$

Mathematica [A] time = 0.0054895, size = 28, normalized size = 1.

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Maple [A] time = 0., size = 25, normalized size = 0.9

$$aAx + \frac{(Ab + Ba)\,x^4}{4} + \frac{bBx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A),x)
```

```
[Out] a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Maxima [A] time = 1.26996, size = 32, normalized size = 1.14

$$\frac{1}{7}\,Bbx^7 + \frac{1}{4}\,(Ba + Ab)x^4 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

Fricas [A] time = 1.26144, size = 66, normalized size = 2.36

$$\frac{1}{7}x^7bB + \frac{1}{4}x^4aB + \frac{1}{4}x^4bA + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*B + 1/4*x^4*a*B + 1/4*x^4*b*A + x*a*A
```

Sympy [A] time = 0.057488, size = 26, normalized size = 0.93

$$Aax + \frac{Bbx^7}{7} + x^4\left(\frac{Ab}{4} + \frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)
```

Giac [A] time = 1.15625, size = 35, normalized size = 1.25

$$\frac{1}{7}Bbx^7 + \frac{1}{4}Bax^4 + \frac{1}{4}Abx^4 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x
```

3.4 $\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^3(aB + Ab) + aA\log(x) + \frac{1}{6}bBx^6$$

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Rubi [A] time = 0.0213559, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\frac{1}{3}x^3(aB + Ab) + aA\log(x) + \frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)\left(A+Bx^3\right)}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(Ab+aB+\frac{aA}{x}+bBx\right)dx, x, x^3\right)$$
$$= \frac{1}{3}(Ab+aB)x^3+\frac{1}{6}bBx^6+aA\log(x)$$

Mathematica [A] time = 0.0084466, size = 29, normalized size = 1.

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Maple [A] time = 0.02, size = 28, normalized size = 1.

$$\frac{bBx^6}{6}+\frac{Ax^3b}{3}+\frac{Bx^3a}{3}+aA\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x,x)
```

```
[Out] 1/6*b*B*x^6+1/3*A*x^3*b+1/3*B*x^3*a+a*A*ln(x)
```

Maxima [A] time = 1.36668, size = 38, normalized size = 1.31

$$\frac{1}{6}\,Bbx^6+\frac{1}{3}\,(Ba+Ab)x^3+\frac{1}{3}\,Aa\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)
```

Fricas [A] time = 1.42804, size = 65, normalized size = 2.24

$$\frac{1}{6}Bbx^6 + \frac{1}{3}(Ba + Ab)x^3 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)
```

Sympy [A] time = 0.253742, size = 27, normalized size = 0.93

$$Aa\log(x) + \frac{Bbx^6}{6} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x,x)
```

```
[Out] A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)
```

Giac [A] time = 1.19681, size = 38, normalized size = 1.31

$$\frac{1}{6}Bbx^6 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aa\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))
```

3.5 $\int \frac{(a+bx^3)(A+Bx^3)}{x^2} dx$

Optimal. Leaf size=31

$$\frac{1}{2}x^2(aB + Ab) - \frac{aA}{x} + \frac{1}{5}bBx^5$$

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Rubi [A] time = 0.0153374, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{2}x^2(aB + Ab) - \frac{aA}{x} + \frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a + bx^3)(A + Bx^3)}{x^2} dx &= \int \left(\frac{aA}{x^2} + (Ab + aB)x + bBx^4\right) dx \\ &= -\frac{aA}{x} + \frac{1}{2}(Ab + aB)x^2 + \frac{1}{5}bBx^5\end{aligned}$$

Mathematica [A] time = 0.0099321, size = 31, normalized size = 1.

$$\frac{1}{2}x^2(aB + Ab) - \frac{aA}{x} + \frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Maple [A] time = 0.038, size = 30, normalized size = 1.

$$\frac{bBx^5}{5}+\frac{Ax^2b}{2}+\frac{Bx^2a}{2}-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^2,x)
```

```
[Out] 1/5*b*B*x^5+1/2*A*x^2*b+1/2*B*x^2*a-a*A/x
```

Maxima [A] time = 1.15168, size = 36, normalized size = 1.16

$$\frac{1}{5}\,Bbx^5+\frac{1}{2}\,(Ba+Ab)x^2-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x
```

Fricas [A] time = 1.38473, size = 66, normalized size = 2.13

$$\frac{2\,Bbx^6+5\,(Ba+Ab)x^3-10\,Aa}{10\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x
```

Sympy [A] time = 0.250824, size = 26, normalized size = 0.84

$$-\frac{Aa}{x}+\frac{Bbx^5}{5}+x^2\left(\frac{Ab}{2}+\frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)
```

Giac [A] time = 1.20218, size = 39, normalized size = 1.26

$$\frac{1}{5}\,Bbx^5+\frac{1}{2}\,Bax^2+\frac{1}{2}\,Abx^2-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x
```

3.6 $\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx$

Optimal. Leaf size=28

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

```
[Out] -(a*A)/(2*x^2) + (A*b + a*B)*x + (b*B*x^4)/4
```

Rubi [A] time = 0.0159127, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) + (A*b + a*B)*x + (b*B*x^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)\left(A+Bx^3\right)}{x^3}\,dx &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^3}+bBx^3\right)dx\\ &= -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4\end{aligned}$$

Mathematica [A] time = 0.0086834, size = 28, normalized size = 1.

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) + (A*b + a*B)*x + (b*B*x^4)/4
```

Maple [A] time = 0.003, size = 24, normalized size = 0.9

$$\frac{bBx^4}{4} + Abx + Bax - \frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^3,x)
```

```
[Out] 1/4*b*B*x^4+A*b*x+B*a*x-1/2*a*A/x^2
```

Maxima [A] time = 1.26629, size = 32, normalized size = 1.14

$$\frac{1}{4}\,Bbx^4 + (Ba + Ab)x - \frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2
```

Fricas [A] time = 1.38094, size = 63, normalized size = 2.25

$$\frac{Bbx^6 + 4\,(Ba + Ab)x^3 - 2\,Aa}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2
```

Sympy [A] time = 0.258407, size = 24, normalized size = 0.86

$$-\frac{Aa}{2x^2} + \frac{Bbx^4}{4} + x\left(Ab + Ba\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)
```

Giac [A] time = 1.15505, size = 31, normalized size = 1.11

$$\frac{1}{4}\,Bbx^4 + Bax + Abx - \frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2
```

3.7 $\int \frac{(a+bx^3)(A+Bx^3)}{x^4} \, dx$

Optimal. Leaf size=29

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

```
[Out] -(a*A)/(3*x^3) + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Rubi [A] time = 0.0215009, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(bB+\frac{aA}{x^2}+\frac{Ab+aB}{x}\right)dx, x, x^3\right)\\ &= -\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)\end{aligned}$$

Mathematica [A] time = 0.0105077, size = 29, normalized size = 1.

$$\log(x)(aB+Ab)-\frac{aA}{3x^3}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Maple [A] time = 0.007, size = 26, normalized size = 0.9

$$\frac{bBx^3}{3}-\frac{Aa}{3\,x^3}+A\ln(x)\,b+B\ln(x)\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^4,x)
```

```
[Out] 1/3*b*B*x^3-1/3*a*A/x^3+A*ln(x)*b+B*ln(x)*a
```

Maxima [A] time = 1.04531, size = 38, normalized size = 1.31

$$\frac{1}{3}\,Bbx^3+\frac{1}{3}\,(Ba+Ab)\log\left(x^3\right)-\frac{Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*B*b*x^3 + 1/3*(B*a + A*b)*log(x^3) - 1/3*A*a/x^3
```

Fricas [A] time = 1.44147, size = 70, normalized size = 2.41

$$\frac{Bbx^6 + 3\,(Ba + Ab)x^3\log\left(x\right) - Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3
```

Sympy [A] time = 0.336231, size = 26, normalized size = 0.9

$$-\frac{Aa}{3x^3} + \frac{Bbx^3}{3} + (Ab + Ba)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*log(x)
```

Giac [A] time = 1.17242, size = 54, normalized size = 1.86

$$\frac{1}{3}\,Bbx^3 + (Ba + Ab)\log\left(|x|\right) - \frac{Bax^3 + Abx^3 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3
```

3.8 $\int \frac{(a+bx^3)(A+Bx^3)}{x^5} dx$

Optimal. Leaf size=31

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/x + (b*B*x^2)/2
```

Rubi [A] time = 0.0173916, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/x + (b*B*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^5} dx &= \int \left(\frac{aA}{x^5}+\frac{Ab+aB}{x^2}+bBx\right) dx\\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2\end{aligned}$$

Mathematica [A] time = 0.0118031, size = 32, normalized size = 1.03

$$\frac{-aB-Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) + (-(A*b) - a*B)/x + (b*B*x^2)/2
```

Maple [A] time = 0.006, size = 28, normalized size = 0.9

$$\frac{bBx^2}{2} - \frac{Aa}{4x^4} - \frac{Ab + Ba}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^5,x)
```

```
[Out] 1/2*b*B*x^2-1/4*a*A/x^4-(A*b+B*a)/x
```

Maxima [A] time = 1.30636, size = 39, normalized size = 1.26

$$\frac{1}{2}Bbx^2 - \frac{4(Ba + Ab)x^3 + Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4
```

Fricas [A] time = 1.37449, size = 63, normalized size = 2.03

$$\frac{2Bbx^6 - 4(Ba + Ab)x^3 - Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4
```

Sympy [A] time = 0.378447, size = 29, normalized size = 0.94

$$\frac{Bbx^2}{2} - \frac{Aa + x^3\left(4Ab + 4Ba\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**5,x)
```

```
[Out] B*b*x**2/2 - (A*a + x**3*(4*A*b + 4*B*a))/(4*x**4)
```

Giac [A] time = 1.20865, size = 42, normalized size = 1.35

$$\frac{1}{2}\,Bbx^2 - \frac{4\,Bax^3 + 4\,Abx^3 + Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4
```

3.9 $\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx$

Optimal. Leaf size=28

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(2*x^2) + b*B*x
```

Rubi [A] time = 0.0164566, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(2*x^2) + b*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)\left(A+Bx^3\right)}{x^6}\,dx &= \int \left(bB+\frac{aA}{x^6}+\frac{Ab+aB}{x^3}\right)dx\\ &= -\frac{aA}{5x^5}-\frac{Ab+aB}{2x^2}+bBx\end{aligned}$$

Mathematica [A] time = 0.0111974, size = 30, normalized size = 1.07

$$\frac{-aB-Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) + (-(A*b) - a*B)/(2*x^2) + b*B*x
```

Maple [A] time = 0.006, size = 25, normalized size = 0.9

$$bBx-\frac{Ab+Ba}{2\,x^{2}}-\frac{Aa}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^6,x)
```

```
[Out] b*B*x-1/2*(A*b+B*a)/x^2-1/5*a*A/x^5
```

Maxima [A] time = 1.33122, size = 36, normalized size = 1.29

$$Bbx-\frac{5\,(Ba+Ab)x^{3}+2\,Aa}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5
```

Fricas [A] time = 1.40485, size = 69, normalized size = 2.46

$$\frac{10\,Bbx^{6}-5\,(Ba+Ab)x^{3}-2\,Aa}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5
```

Sympy [A] time = 0.408755, size = 27, normalized size = 0.96

$$Bbx - \frac{2Aa + x^3 (5Ab + 5Ba)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**6,x)
```

```
[Out] B*b*x - (2*A*a + x**3*(5*A*b + 5*B*a))/(10*x**5)
```

Giac [A] time = 1.13208, size = 39, normalized size = 1.39

$$Bbx - \frac{5\,Bax^3 + 5\,Abx^3 + 2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5
```

3.10 $\int \frac{(a+bx^3)(A+Bx^3)}{x^7} dx$

Optimal. Leaf size=29

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(3*x^3) + b*B*Log[x]
```

Rubi [A] time = 0.0211706, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(3*x^3) + b*B*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{aA}{x^3} + \frac{Ab+aB}{x^2} + \frac{bB}{x}\right) dx, x, x^3\right)$$
$$= -\frac{aA}{6x^6} - \frac{Ab+aB}{3x^3} + bB\log(x)$$

Mathematica [A] time = 0.016098, size = 31, normalized size = 1.07

$$\frac{-aB-Ab}{3x^3} - \frac{aA}{6x^6} + bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]
```

Maple [A] time = 0.006, size = 28, normalized size = 1.

$$-\frac{Ab}{3\,x^3} - \frac{Ba}{3\,x^3} - \frac{Aa}{6\,x^6} + bB\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^7,x)
```

```
[Out] -1/3/x^3*A*b-1/3/x^3*B*a-1/6*a*A/x^6+b*B*ln(x)
```

Maxima [A] time = 1.33586, size = 41, normalized size = 1.41

$$\frac{1}{3}\,Bb\log\left(x^3\right) - \frac{2\,(Ba+Ab)x^3 + Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6
```

Fricas [A] time = 1.47408, size = 73, normalized size = 2.52

$$\frac{6\,Bbx^6\log(x) - 2\,(Ba + Ab)x^3 - Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6
```

Sympy [A] time = 0.65047, size = 27, normalized size = 0.93

$$Bb\log(x) - \frac{Aa + x^3\,(2Ab + 2Ba)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**7,x)
```

```
[Out] B*b*log(x) - (A*a + x**3*(2*A*b + 2*B*a))/(6*x**6)
```

Giac [A] time = 1.19674, size = 50, normalized size = 1.72

$$Bb\log(|x|) - \frac{3\,Bbx^6 + 2\,Bax^3 + 2\,Abx^3 + Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6
```

3.11 $\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^3\right)^3 (Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^3)^3)/(9*b^2) + (B*(a + b*x^3)^4)/(12*b^2)
```

Rubi [A] time = 0.0721012, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{\left(a+bx^3\right)^3 (Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]

[Out] ((A*b - a*B)*(a + b*x^3)^3)/(9*b^2) + (B*(a + b*x^3)^4)/(12*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)dx=\frac{1}{3}\text{Subst}\left(\int(a+bx)^2(A+Bx)\,dx,x,x^3\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right)dx,x,x^3\right)$$
$$=\frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

Mathematica [A] time = 0.0145113, size = 51, normalized size = 1.21

$$\frac{1}{36}x^3\left(12a^2A+4bx^6(2aB+Ab)+6ax^3(aB+2Ab)+3b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x^
9))/36
```

Maple [A] time = 0.001, size = 52, normalized size = 1.2

$$\frac{b^2Bx^{12}}{12}+\frac{\left(b^2A+2\,abB\right)x^9}{9}+\frac{\left(2\,abA+a^2B\right)x^6}{6}+\frac{a^2Ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/12*b^2*B*x^12+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*a^2*A*x
^3
```

Maxima [A] time = 1.12715, size = 69, normalized size = 1.64

$$\frac{1}{12}\,Bb^2x^{12}+\frac{1}{9}\left(2\,Bab+Ab^2\right)x^9+\frac{1}{6}\left(Ba^2+2\,Aab\right)x^6+\frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1
/3*A*a^2*x^3
```

Fricas [A] time = 1.26638, size = 131, normalized size = 3.12

$$\frac{1}{12}x^{12}b^2B + \frac{2}{9}x^9baB + \frac{1}{9}x^9b^2A + \frac{1}{6}x^6a^2B + \frac{1}{3}x^6baA + \frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^2*B + 2/9*x^9*b*a*B + 1/9*x^9*b^2*A + 1/6*x^6*a^2*B + 1/3*x^6*b
*a*A + 1/3*x^3*a^2*A
```

Sympy [A] time = 0.067793, size = 54, normalized size = 1.29

$$\frac{Aa^2x^3}{3} + \frac{Bb^2x^{12}}{12} + x^9\left(\frac{Ab^2}{9} + \frac{2Bab}{9}\right) + x^6\left(\frac{Aab}{3} + \frac{Ba^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*b
/3 + B*a**2/6)
```

Giac [A] time = 1.18901, size = 72, normalized size = 1.71

$$\frac{1}{12}Bb^2x^{12} + \frac{2}{9}Babx^9 + \frac{1}{9}Ab^2x^9 + \frac{1}{6}Ba^2x^6 + \frac{1}{3}Aabx^6 + \frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*b
*x^6 + 1/3*A*a^2*x^3
```

3.12 $\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Rubi [A] time = 0.0337621, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx&=\int\left(a^2Ax+a(2Ab+aB)x^4+b(Ab+2aB)x^7+b^2Bx^{10}\right)\,dx\\&=\frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}\end{aligned}$$

Mathematica [A] time = 0.0075756, size = 55, normalized size = 1.

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Maple [A] time = 0., size = 52, normalized size = 1.

$$\frac{b^2Bx^{11}}{11}+\frac{\left(b^2A+2\,abB\right)x^8}{8}+\frac{\left(2\,abA+a^2B\right)x^5}{5}+\frac{a^2Ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/11*b^2*B*x^11+1/8*(A*b^2+2*B*a*b)*x^8+1/5*(2*A*a*b+B*a^2)*x^5+1/2*a^2*A*x
^2
```

Maxima [A] time = 1.68294, size = 69, normalized size = 1.25

$$\frac{1}{11}\,Bb^2x^{11}+\frac{1}{8}\left(2\,Bab+Ab^2\right)x^8+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5+\frac{1}{2}\,Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1
/2*A*a^2*x^2
```

Fricas [A] time = 1.26594, size = 131, normalized size = 2.38

$$\frac{1}{11}x^{11}b^2B+\frac{1}{4}x^8baB+\frac{1}{8}x^8b^2A+\frac{1}{5}x^5a^2B+\frac{2}{5}x^5baA+\frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^2*B + 1/4*x^8*b*a*B + 1/8*x^8*b^2*A + 1/5*x^5*a^2*B + 2/5*x^5*b
*a*A + 1/2*x^2*a^2*A
```

Sympy [A] time = 0.068951, size = 54, normalized size = 0.98

$$\frac{Aa^2x^2}{2} + \frac{Bb^2x^{11}}{11} + x^8\left(\frac{Ab^2}{8} + \frac{Bab}{4}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ba^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

Giac [A] time = 1.18727, size = 72, normalized size = 1.31

$$\frac{1}{11}Bb^2x^{11} + \frac{1}{4}Babx^8 + \frac{1}{8}Ab^2x^8 + \frac{1}{5}Ba^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/11*B*b^2*x^11 + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*b
*x^5 + 1/2*A*a^2*x^2
```

3.13 $\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

Optimal. Leaf size=50

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Rubi [A] time = 0.0254353, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx &= \int \left(a^2A+a(2Ab+aB)x^3+b(Ab+2aB)x^6+b^2Bx^9\right) dx\\ &= a^2Ax+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{10}b^2Bx^{10}\end{aligned}$$

Mathematica [A] time = 0.0071112, size = 50, normalized size = 1.

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Maple [A] time = 0.002, size = 49, normalized size = 1.

$$\frac{b^2Bx^{10}}{10}+\frac{\left(b^2A+2\,abB\right)x^7}{7}+\frac{\left(2\,abA+a^2B\right)x^4}{4}+a^2Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/10*b^2*B*x^10+1/7*(A*b^2+2*B*a*b)*x^7+1/4*(2*A*a*b+B*a^2)*x^4+a^2*A*x
```

Maxima [A] time = 1.28197, size = 65, normalized size = 1.3

$$\frac{1}{10}\,Bb^2x^{10}+\frac{1}{7}\left(2\,Bab+Ab^2\right)x^7+\frac{1}{4}\left(Ba^2+2\,Aab\right)x^4+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A
*a^2*x
```

Fricas [A] time = 1.22204, size = 123, normalized size = 2.46

$$\frac{1}{10}x^{10}b^2B+\frac{2}{7}x^7baB+\frac{1}{7}x^7b^2A+\frac{1}{4}x^4a^2B+\frac{1}{2}x^4baA+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2*B + 2/7*x^7*b*a*B + 1/7*x^7*b^2*A + 1/4*x^4*a^2*B + 1/2*x^4*b
*a*A + x*a^2*A
```

Sympy [A] time = 0.066957, size = 51, normalized size = 1.02

$$Aa^2x + \frac{Bb^2x^{10}}{10} + x^7\left(\frac{Ab^2}{7} + \frac{2Bab}{7}\right) + x^4\left(\frac{Aab}{2} + \frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 +
B*a**2/4)
```

Giac [A] time = 1.16846, size = 68, normalized size = 1.36

$$\frac{1}{10}\,Bb^2x^{10} + \frac{2}{7}\,Babx^7 + \frac{1}{7}\,Ab^2x^7 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{2}\,Aabx^4 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/10*B*b^2*x^10 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*b
*x^4 + A*a^2*x
```

3.14 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x} \, dx$

Optimal. Leaf size=46

$$a^2 A \log(x) + \frac{2}{3} a A b x^3 + \frac{B (a + b x^3)^3}{9b} + \frac{1}{6} A b^2 x^6$$

```
[Out] (2*a*A*b*x^3)/3 + (A*b^2*x^6)/6 + (B*(a + b*x^3)^3)/(9*b) + a^2*A*Log[x]
```

Rubi [A] time = 0.0326931, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 80, 43}

$$a^2 A \log(x) + \frac{2}{3} a A b x^3 + \frac{B (a + b x^3)^3}{9b} + \frac{1}{6} A b^2 x^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*b*x^3)/3 + (A*b^2*x^6)/6 + (B*(a + b*x^3)^3)/(9*b) + a^2*A*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x}\,dx, x, x^3\right)\\ &= \frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{3}A\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^3\right)\\ &= \frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{3}A\,\mathrm{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx, x, x^3\right)\\ &= \frac{2}{3}aAbx^3+\frac{1}{6}Ab^2x^6+\frac{B\left(a+bx^3\right)^3}{9b}+a^2A\log(x)\end{aligned}$$

Mathematica [A] time = 0.0142697, size = 51, normalized size = 1.11

$$a^2A\log(x)+\frac{1}{6}bx^6(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*L
og[x]
```

Maple [A] time = 0.001, size = 52, normalized size = 1.1

$$\frac{Bb^2x^9}{9}+\frac{Ab^2x^6}{6}+\frac{Bx^6ab}{3}+\frac{2\,aAbx^3}{3}+\frac{Bx^3a^2}{3}+a^2A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x,x)
```

```
[Out] 1/9*B*b^2*x^9+1/6*A*b^2*x^6+1/3*B*x^6*a*b+2/3*a*A*b*x^3+1/3*B*x^3*a^2+a^2*A
*ln(x)
```

Maxima [A] time = 1.21736, size = 70, normalized size = 1.52

$$\frac{1}{9}\,Bb^2x^9 + \frac{1}{6}\left(2\,Bab + Ab^2\right)x^6 + \frac{1}{3}\left(Ba^2 + 2\,Aab\right)x^3 + \frac{1}{3}\,Aa^2\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + 1/3
*A*a^2*log(x^3)
```

Fricas [A] time = 1.44867, size = 116, normalized size = 2.52

$$\frac{1}{9}\,Bb^2x^9 + \frac{1}{6}\left(2\,Bab + Ab^2\right)x^6 + \frac{1}{3}\left(Ba^2 + 2\,Aab\right)x^3 + Aa^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + A*a
^2*log(x)
```

Sympy [A] time = 0.282266, size = 53, normalized size = 1.15

$$Aa^2\log\left(x\right) + \frac{Bb^2x^9}{9} + x^6\left(\frac{Ab^2}{6} + \frac{Bab}{3}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**9/9 + x**6*(A*b**2/6 + B*a*b/3) + x**3*(2*A*a*b/3
 + B*a**2/3)
```

Giac [A] time = 1.11482, size = 70, normalized size = 1.52

$$\frac{1}{9}\,Bb^{2}x^{9} + \frac{1}{3}\,Babx^{6} + \frac{1}{6}\,Ab^{2}x^{6} + \frac{1}{3}\,Ba^{2}x^{3} + \frac{2}{3}\,Aabx^{3} + Aa^{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*log(abs(x))
```

3.15 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^2}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Rubi [A] time = 0.0282968, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^2}\,dx &= \int \left(\frac{a^2A}{x^2}+a(2Ab+aB)x+b(Ab+2aB)x^4+b^2Bx^7\right)dx\\ &= -\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8\end{aligned}$$

Mathematica [A] time = 0.0164872, size = 53, normalized size = 1.

$$-\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Maple [A] time = 0.004, size = 53, normalized size = 1.

$$\frac{b^2Bx^8}{8}+\frac{Ax^5b^2}{5}+\frac{2\,Bx^5ab}{5}+Ax^2ab+\frac{Bx^2a^2}{2}-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^2,x)
```

```
[Out] 1/8*b^2*B*x^8+1/5*A*x^5*b^2+2/5*B*x^5*a*b+A*x^2*a*b+1/2*B*x^2*a^2-a^2*A/x
```

Maxima [A] time = 1.20415, size = 69, normalized size = 1.3

$$\frac{1}{8}\,Bb^2x^8+\frac{1}{5}\left(2\,Bab+Ab^2\right)x^5+\frac{1}{2}\left(Ba^2+2\,Aab\right)x^2-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/2*(B*a^2 + 2*A*a*b)*x^2 - A*a
^2/x
```

Fricas [A] time = 1.42236, size = 116, normalized size = 2.19

$$\frac{5\,Bb^2x^9+8\left(2\,Bab+Ab^2\right)x^6+20\left(Ba^2+2\,Aab\right)x^3-40\,Aa^2}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 40
*A*a^2)/x
```

Sympy [A] time = 0.280175, size = 49, normalized size = 0.92

$$-\frac{Aa^2}{x} + \frac{Bb^2x^8}{8} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*a
**2/2)
```

Giac [A] time = 1.10672, size = 70, normalized size = 1.32

$$\frac{1}{8}Bb^2x^8 + \frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{2}Ba^2x^2 + Aabx^2 - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2 -
 A*a^2/x
```

3.16 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$

Optimal. Leaf size=50

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

```
[Out] -(a^2*A)/(2*x^2) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Rubi [A] time = 0.0266763, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3}\,dx &= \int \left(a(2Ab+aB)+\frac{a^2A}{x^3}+b(Ab+2aB)x^3+b^2Bx^6\right)dx\\ &= -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7\end{aligned}$$

Mathematica [A] time = 0.0142723, size = 50, normalized size = 1.

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Maple [A] time = 0.003, size = 49, normalized size = 1.

$$\frac{b^2Bx^7}{7}+\frac{Ax^4b^2}{4}+\frac{Bx^4ab}{2}+2\,abAx+a^2Bx-\frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^3,x)
```

```
[Out] 1/7*b^2*B*x^7+1/4*A*x^4*b^2+1/2*B*x^4*a*b+2*a*b*A*x+a^2*B*x-1/2*a^2*A/x^2
```

Maxima [A] time = 1.15342, size = 65, normalized size = 1.3

$$\frac{1}{7}\,Bb^2x^7+\frac{1}{4}\left(2\,Bab+Ab^2\right)x^4+\left(Ba^2+2\,Aab\right)x-\frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^2
/x^2
```

Fricas [A] time = 1.38677, size = 119, normalized size = 2.38

$$\frac{4\,Bb^2x^9+7\left(2\,Bab+Ab^2\right)x^6+28\left(Ba^2+2\,Aab\right)x^3-14\,Aa^2}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 14
*A*a^2)/x^2
```

Sympy [A] time = 0.284578, size = 49, normalized size = 0.98

$$-\frac{Aa^2}{2x^2} + \frac{Bb^2x^7}{7} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x\left(2Aab + Ba^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b +
 B*a**2)
```

Giac [A] time = 1.2148, size = 65, normalized size = 1.3

$$\frac{1}{7}\,Bb^2x^7 + \frac{1}{2}\,Babx^4 + \frac{1}{4}\,Ab^2x^4 + Ba^2x + 2\,Aabx - \frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*A
*a^2/x^2
```

3.17 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

```
[Out] -(a^2*A)/(3*x^3) + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^6)/6 + a*(2*A*b + a*B
)*Log[x]
```

Rubi [A] time = 0.0500513, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^6)/6 + a*(2*A*b + a*B
)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^4}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(b(Ab+2aB)+\frac{a^2A}{x^2}+\frac{a(2Ab+aB)}{x}+b^2Bx\right)dx, x, x^3\right)$$
$$= -\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)$$

Mathematica [A] time = 0.023097, size = 49, normalized size = 0.96

$$\frac{1}{6}\left(-\frac{2a^2A}{x^3}+2bx^3(2aB+Ab)+6a\log(x)(aB+2Ab)+b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] ((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Log
[x])/6
```

Maple [A] time = 0.007, size = 51, normalized size = 1.

$$\frac{b^2Bx^6}{6}+\frac{Ax^3b^2}{3}+\frac{2\,Bx^3ab}{3}-\frac{Aa^2}{3\,x^3}+2\,A\ln(x)\,ab+B\ln(x)\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^4,x)
```

```
[Out] 1/6*b^2*B*x^6+1/3*A*x^3*b^2+2/3*B*x^3*a*b-1/3*a^2*A/x^3+2*A*ln(x)*a*b+B*ln(
x)*a^2
```

Maxima [A] time = 1.11259, size = 70, normalized size = 1.37

$$\frac{1}{6}\,Bb^2x^6+\frac{1}{3}\left(2\,Bab+Ab^2\right)x^3+\frac{1}{3}\left(Ba^2+2\,Aab\right)\log\left(x^3\right)-\frac{Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3)
- 1/3*A*a^2/x^3
```

Fricas [A] time = 1.61107, size = 122, normalized size = 2.39

$$\frac{Bb^2x^9 + 2\left(2\,Bab + Ab^2\right)x^6 + 6\left(Ba^2 + 2\,Aab\right)x^3\log\left(x\right) - 2\,Aa^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x) -
 2*A*a^2)/x^3
```

Sympy [A] time = 0.400952, size = 51, normalized size = 1.

$$-\frac{Aa^2}{3x^3} + \frac{Bb^2x^6}{6} + a\left(2Ab + Ba\right)\log\left(x\right) + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3
+ 2*B*a*b/3)
```

Giac [A] time = 1.26942, size = 93, normalized size = 1.82

$$\frac{1}{6}\,Bb^2x^6 + \frac{2}{3}\,Babx^3 + \frac{1}{3}\,Ab^2x^3 + \left(Ba^2 + 2\,Aab\right)\log\left(|x|\right) - \frac{Ba^2x^3 + 2\,Aabx^3 + Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(x
)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3
```

3.18 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^5}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x
^5)/5
```

Rubi [A] time = 0.030959, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x
^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^5}\,dx &= \int \left(\frac{a^2A}{x^5}+\frac{a(2Ab+aB)}{x^2}+b(Ab+2aB)x+b^2Bx^4\right)dx\\ &= -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{x}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{5}b^2Bx^5\end{aligned}$$

Mathematica [A] time = 0.0156971, size = 51, normalized size = 0.96

$$\frac{-5a^2A+10bx^6(2aB+Ab)-20ax^3(aB+2Ab)+4b^2Bx^9}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] (-5*a^2*A - 20*a*(2*A*b + a*B)*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)/
(20*x^4)
```

Maple [A] time = 0.006, size = 50, normalized size = 0.9

$$\frac{b^2Bx^5}{5}+\frac{Ax^2b^2}{2}+Bx^2ab-\frac{Aa^2}{4\,x^4}-\frac{a\,(2\,Ab+Ba)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^5,x)
```

```
[Out] 1/5*b^2*B*x^5+1/2*A*x^2*b^2+B*x^2*a*b-1/4*a^2*A/x^4-a*(2*A*b+B*a)/x
```

Maxima [A] time = 1.17113, size = 72, normalized size = 1.36

$$\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\left(2\,Bab+Ab^2\right)x^2-\frac{4\left(Ba^2+2\,Aab\right)x^3+Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/2*(2*B*a*b + A*b^2)*x^2 - 1/4*(4*(B*a^2 + 2*A*a*b)*x^3 +
A*a^2)/x^4
```

Fricas [A] time = 1.70558, size = 119, normalized size = 2.25

$$\frac{4\,Bb^2x^9+10\left(2\,Bab+Ab^2\right)x^6-20\left(Ba^2+2\,Aab\right)x^3-5\,Aa^2}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^4
```

Sympy [A] time = 0.442399, size = 51, normalized size = 0.96

$$\frac{Bb^2x^5}{5} + x^2\left(\frac{Ab^2}{2} + Bab\right) - \frac{Aa^2 + x^3\left(8Aab + 4Ba^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**2*x**5/5 + x**2*(A*b**2/2 + B*a*b) - (A*a**2 + x**3*(8*A*a*b + 4*B*a**
2))/(4*x**4)
```

Giac [A] time = 1.1991, size = 73, normalized size = 1.38

$$\frac{1}{5}Bb^2x^5 + Babx^2 + \frac{1}{2}Ab^2x^2 - \frac{4\,Ba^2x^3 + 8\,Aabx^3 + Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3
+ A*a^2)/x^4
```

3.19 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Rubi [A] time = 0.0283431, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^6}\,dx &= \int \left(b(Ab+2aB)+\frac{a^2A}{x^6}+\frac{a(2Ab+aB)}{x^3}+b^2Bx^3\right)dx\\ &= -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{2x^2}+b(Ab+2aB)x+\frac{1}{4}b^2Bx^4\end{aligned}$$

Mathematica [A] time = 0.0182721, size = 50, normalized size = 1.

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Maple [A] time = 0.004, size = 46, normalized size = 0.9

$$\frac{b^2Bx^4}{4}+b^2Ax+2\,abBx-\frac{a\,(2\,Ab+Ba)}{2\,x^2}-\frac{Aa^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^6,x)
```

```
[Out] 1/4*b^2*B*x^4+b^2*A*x+2*a*b*B*x-1/2*a*(2*A*b+B*a)/x^2-1/5*a^2*A/x^5
```

Maxima [A] time = 1.28127, size = 69, normalized size = 1.38

$$\frac{1}{4}\,Bb^2x^4+\left(2\,Bab+Ab^2\right)x-\frac{5\left(Ba^2+2\,Aab\right)x^3+2\,Aa^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*a
^2)/x^5
```

Fricas [A] time = 1.69, size = 119, normalized size = 2.38

$$\frac{5\,Bb^2x^9+20\left(2\,Bab+Ab^2\right)x^6-10\left(Ba^2+2\,Aab\right)x^3-4\,Aa^2}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^5
```

Sympy [A] time = 0.470905, size = 51, normalized size = 1.02

$$\frac{Bb^2x^4}{4} + x\left(Ab^2 + 2Bab\right) - \frac{2Aa^2 + x^3\left(10Aab + 5Ba^2\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) - (2*A*a**2 + x**3*(10*A*a*b + 5*B*a**
2))/(10*x**5)
```

Giac [A] time = 1.12228, size = 69, normalized size = 1.38

$$\frac{1}{4}\,Bb^2x^4 + 2\,Babx + Ab^2x - \frac{5\,Ba^2x^3 + 10\,Aabx^3 + 2\,Aa^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2*
A*a^2)/x^5
```

3.20 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{3x^3}+b\log(x)(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(3*x^3) + (b^2*B*x^3)/3 + b*(A*b + 2*a
*B)*Log[x]
```

Rubi [A] time = 0.0375813, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{3x^3}+b\log(x)(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(3*x^3) + (b^2*B*x^3)/3 + b*(A*b + 2*a
*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \frac{1}{3} \text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3} \, dx, x, x^3\right)$$
$$= \frac{1}{3} \text{Subst}\left(\int \left(b^2B + \frac{a^2A}{x^3} + \frac{a(2Ab+aB)}{x^2} + \frac{b(Ab+2aB)}{x}\right) dx, x, x^3\right)$$
$$= -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{3x^3} + \frac{1}{3}b^2Bx^3 + b(Ab+2aB)\log(x)$$

Mathematica [A] time = 0.0193914, size = 51, normalized size = 1.

$$\frac{1}{6}\left(-\frac{a^2(A+2Bx^3)}{x^6} + 6b\log(x)(2aB+Ab) - \frac{4aAb}{x^3} + 2b^2Bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] ((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B)
*Log[x])/6
```

Maple [A] time = 0.006, size = 51, normalized size = 1.

$$\frac{b^2Bx^3}{3} - \frac{Aa^2}{6x^6} - \frac{2abA}{3x^3} - \frac{a^2B}{3x^3} + A\ln(x)b^2 + 2B\ln(x)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^7,x)
```

```
[Out] 1/3*b^2*B*x^3-1/6*a^2*A/x^6-2/3*a/x^3*A*b-1/3*a^2/x^3*B+A*ln(x)*b^2+2*B*ln(
x)*a*b
```

Maxima [A] time = 1.1939, size = 73, normalized size = 1.43

$$\frac{1}{3}Bb^2x^3 + \frac{1}{3}(2Bab + Ab^2)\log(x^3) - \frac{2(Ba^2 + 2Aab)x^3 + Aa^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*x
^3 + A*a^2)/x^6
```

Fricas [A] time = 1.56835, size = 122, normalized size = 2.39

$$\frac{2\,Bb^2x^9 + 6\left(2\,Bab + Ab^2\right)x^6\log\left(x\right) - 2\left(Ba^2 + 2\,Aab\right)x^3 - Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^3
 - A*a^2)/x^6
```

Sympy [A] time = 0.863031, size = 49, normalized size = 0.96

$$\frac{Bb^2x^3}{3} + b\left(Ab + 2Ba\right)\log\left(x\right) - \frac{Aa^2 + x^3\left(4Aab + 2Ba^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) - (A*a**2 + x**3*(4*A*a*b + 2*B*a**2
))/(6*x**6)
```

Giac [A] time = 1.20273, size = 95, normalized size = 1.86

$$\frac{1}{3}\,Bb^2x^3 + \left(2\,Bab + Ab^2\right)\log\left(|x|\right) - \frac{6\,Babx^6 + 3\,Ab^2x^6 + 2\,Ba^2x^3 + 4\,Aabx^3 + Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2*
x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6
```

3.21 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8} \, dx$

Optimal. Leaf size=53

$$-\frac{a^2 A}{7x^7} - \frac{a(aB+2Ab)}{4x^4} - \frac{b(2aB+Ab)}{x} + \frac{1}{2}b^2 Bx^2$$

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*B
*x^2)/2
```

Rubi [A] time = 0.0287355, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2 A}{7x^7} - \frac{a(aB+2Ab)}{4x^4} - \frac{b(2aB+Ab)}{x} + \frac{1}{2}b^2 Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*B
*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8} \, dx &= \int \left(\frac{a^2 A}{x^8} + \frac{a(2Ab+aB)}{x^5} + \frac{b(Ab+2aB)}{x^2} + b^2 Bx\right) dx \\ &= -\frac{a^2 A}{7x^7} - \frac{a(2Ab+aB)}{4x^4} - \frac{b(Ab+2aB)}{x} + \frac{1}{2}b^2 Bx^2\end{aligned}$$

Mathematica [A] time = 0.0151625, size = 54, normalized size = 1.02

$$-\frac{a^2 \left(4A+7Bx^3\right)+14abx^3 \left(A+4Bx^3\right)-14b^2x^6 \left(Bx^3-2A\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A + 7*B*x^
3))/(28*x^7)
```

Maple [A] time = 0.006, size = 48, normalized size = 0.9

$$-\frac{Aa^2}{7\,x^7}-\frac{a\,(2\,Ab+Ba)}{4\,x^4}-\frac{b\,(Ab+2\,Ba)}{x}+\frac{b^2Bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^8,x)
```

```
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Maxima [A] time = 1.031, size = 73, normalized size = 1.38

$$\frac{1}{2}\,Bb^2x^2-\frac{28\left(2\,Bab+Ab^2\right)x^6+7\left(Ba^2+2\,Aab\right)x^3+4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 +
4*A*a^2)/x^7
```

Fricas [A] time = 1.44573, size = 119, normalized size = 2.25

$$\frac{14\,Bb^2x^9-28\left(2\,Bab+Ab^2\right)x^6-7\left(Ba^2+2\,Aab\right)x^3-4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^7
```

Sympy [A] time = 1.0215, size = 54, normalized size = 1.02

$$\frac{Bb^2x^2}{2} - \frac{4Aa^2 + x^6\left(28Ab^2 + 56Bab\right) + x^3\left(14Aab + 7Ba^2\right)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**2*x**2/2 - (4*A*a**2 + x**6*(28*A*b**2 + 56*B*a*b) + x**3*(14*A*a*b +
7*B*a**2))/(28*x**7)
```

Giac [A] time = 1.17072, size = 76, normalized size = 1.43

$$\frac{1}{2}\,Bb^2x^2 - \frac{56\,Babx^6 + 28\,Ab^2x^6 + 7\,Ba^2x^3 + 14\,Aabx^3 + 4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b*
x^3 + 4*A*a^2)/x^7
```

3.22 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9} \, dx$

Optimal. Leaf size=50

$$-\frac{a^2 A}{8x^8} - \frac{a(aB+2Ab)}{5x^5} - \frac{b(2aB+Ab)}{2x^2} + b^2 Bx$$

```
[Out] -(a^2*A)/(8*x^8) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Rubi [A] time = 0.0287994, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2 A}{8x^8} - \frac{a(aB+2Ab)}{5x^5} - \frac{b(2aB+Ab)}{2x^2} + b^2 Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^2*A)/(8*x^8) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9} \, dx &= \int \left(b^2 B + \frac{a^2 A}{x^9} + \frac{a(2Ab+aB)}{x^6} + \frac{b(Ab+2aB)}{x^3}\right) dx \\ &= -\frac{a^2 A}{8x^8} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{2x^2} + b^2 Bx\end{aligned}$$

Mathematica [A] time = 0.021259, size = 50, normalized size = 1.

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^2*A)/(8*x^8) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Maple [A] time = 0.006, size = 45, normalized size = 0.9

$$-\frac{Aa^2}{8\,x^8}-\frac{a\,(2\,Ab+Ba)}{5\,x^5}-\frac{b\,(Ab+2\,Ba)}{2\,x^2}+b^2Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^9,x)
```

```
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Maxima [A] time = 1.17527, size = 69, normalized size = 1.38

$$Bb^2x-\frac{20\left(2\,Bab+Ab^2\right)x^6+8\left(Ba^2+2\,Aab\right)x^3+5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a^
2)/x^8
```

Fricas [A] time = 1.33918, size = 119, normalized size = 2.38

$$\frac{40\,Bb^2x^9-20\left(2\,Bab+Ab^2\right)x^6-8\left(Ba^2+2\,Aab\right)x^3-5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^8
```

Sympy [A] time = 1.10184, size = 51, normalized size = 1.02

$$Bb^2x - \frac{5Aa^2 + x^6\left(20Ab^2 + 40Bab\right) + x^3\left(16Aab + 8Ba^2\right)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**2*x - (5*A*a**2 + x**6*(20*A*b**2 + 40*B*a*b) + x**3*(16*A*a*b + 8*B*a
**2))/(40*x**8)
```

Giac [A] time = 1.19952, size = 72, normalized size = 1.44

$$Bb^2x - \frac{40\,Babx^6 + 20\,Ab^2x^6 + 8\,Ba^2x^3 + 16\,Aabx^3 + 5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 +
5*A*a^2)/x^8
```

3.23 $\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx$

Optimal. Leaf size=117

$$\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+$$

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Rubi [A] time = 0.0975507, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \int \left(a^5Ax^9+a^4(5Ab+aB)x^{12}+5a^3b(2Ab+aB)x^{15}+10a^2b^2(Ab+aB)x^{18}+5ab^3(Ab+\right.\\ &= \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab\end{aligned}$$

Mathematica [A] time = 0.0204761, size = 117, normalized size = 1.

$$\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Maple [A] time = 0.004, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{28}}{28}+\frac{\left(b^5A+5\,ab^4B\right)x^{25}}{25}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{22}}{22}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{19}}{19}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{16}}{16}+\frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/28*b^5*B*x^28+1/25*(A*b^5+5*B*a*b^4)*x^25+1/22*(5*A*a*b^4+10*B*a^2*b^3)*x
^22+1/19*(10*A*a^2*b^3+10*B*a^3*b^2)*x^19+1/16*(10*A*a^3*b^2+5*B*a^4*b)*x^1
6+1/13*(5*A*a^4*b+B*a^5)*x^13+1/10*a^5*A*x^10
```

Maxima [A] time = 1.12747, size = 161, normalized size = 1.38

$$\frac{1}{28}Bb^5x^{28}+\frac{1}{25}\left(5\,Bab^4+Ab^5\right)x^{25}+\frac{5}{22}\left(2\,Ba^2b^3+Aab^4\right)x^{22}+\frac{10}{19}\left(Ba^3b^2+Aa^2b^3\right)x^{19}+\frac{5}{16}\left(Ba^4b+2\,Aa^3b^2\right)x^{16}+\frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*b
^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b^2
)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

Fricas [A] time = 1.24066, size = 323, normalized size = 2.76

$$\frac{1}{28}x^{28}b^5B + \frac{1}{5}x^{25}b^4aB + \frac{1}{25}x^{25}b^5A + \frac{5}{11}x^{22}b^3a^2B + \frac{5}{22}x^{22}b^4aA + \frac{10}{19}x^{19}b^2a^3B + \frac{10}{19}x^{19}b^3a^2A + \frac{5}{16}x^{16}ba^4B + \frac{5}{8}x^{16}b^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/28*x^28*b^5*B + 1/5*x^25*b^4*a*B + 1/25*x^25*b^5*A + 5/11*x^22*b^3*a^2*B
+ 5/22*x^22*b^4*a*A + 10/19*x^19*b^2*a^3*B + 10/19*x^19*b^3*a^2*A + 5/16*x^
16*b*a^4*B + 5/8*x^16*b^2*a^3*A + 1/13*x^13*a^5*B + 5/13*x^13*b*a^4*A + 1/1
0*x^10*a^5*A
```

Sympy [A] time = 0.084399, size = 136, normalized size = 1.16

$$\frac{Aa^5x^{10}}{10} + \frac{Bb^5x^{28}}{28} + x^{25}\left(\frac{Ab^5}{25} + \frac{Bab^4}{5}\right) + x^{22}\left(\frac{5Aab^4}{22} + \frac{5Ba^2b^3}{11}\right) + x^{19}\left(\frac{10Aa^2b^3}{19} + \frac{10Ba^3b^2}{19}\right) + x^{16}\left(\frac{5Aa^3b^2}{8} + \frac{5Ba^4b}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22*
(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3*b
**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13 +
B*a**5/13)
```

Giac [A] time = 1.16651, size = 169, normalized size = 1.44

$$\frac{1}{28}Bb^5x^{28} + \frac{1}{5}Bab^4x^{25} + \frac{1}{25}Ab^5x^{25} + \frac{5}{11}Ba^2b^3x^{22} + \frac{5}{22}Aab^4x^{22} + \frac{10}{19}Ba^3b^2x^{19} + \frac{10}{19}Aa^2b^3x^{19} + \frac{5}{16}Ba^4bx^{16} + \frac{5}{8}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/28*B*b^5*x^28 + 1/5*B*a*b^4*x^25 + 1/25*A*b^5*x^25 + 5/11*B*a^2*b^3*x^22
+ 5/22*A*a*b^4*x^22 + 10/19*B*a^3*b^2*x^19 + 10/19*A*a^2*b^3*x^19 + 5/16*B*
```

```
a^4*b*x^16 + 5/8*A*a^3*b^2*x^16 + 1/13*B*a^5*x^13 + 5/13*A*a^4*b*x^13 + 1/1
0*A*a^5*x^10
```

3.24 $\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=95

$$\frac{a^2 \left(a+bx^3\right)^6 (Ab-aB)}{18b^4}+\frac{\left(a+bx^3\right)^8 (Ab-3aB)}{24b^4}-\frac{a \left(a+bx^3\right)^7 (2Ab-3aB)}{21b^4}+\frac{B \left(a+bx^3\right)^9}{27b^4}$$

```
[Out] (a^2*(A*b - a*B)*(a + b*x^3)^6)/(18*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^7
)/(21*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(24*b^4) + (B*(a + b*x^3)^9)/(27
*b^4)
```

Rubi [A] time = 0.236616, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{a^2 \left(a+bx^3\right)^6 (Ab-aB)}{18b^4}+\frac{\left(a+bx^3\right)^8 (Ab-3aB)}{24b^4}-\frac{a \left(a+bx^3\right)^7 (2Ab-3aB)}{21b^4}+\frac{B \left(a+bx^3\right)^9}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^3)^6)/(18*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^7
)/(21*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(24*b^4) + (B*(a + b*x^3)^9)/(27
*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx = \frac{1}{3}\operatorname{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3} + \frac{a(-2Ab+3aB)(a+bx)^6}{b^3} + \frac{(Ab-3aB)(a+}{b^3}\right.\right.$$
$$= \frac{a^2(Ab-aB)\left(a+bx^3\right)^6}{18b^4} - \frac{a(2Ab-3aB)\left(a+bx^3\right)^7}{21b^4} + \frac{(Ab-3aB)\left(a+bx^3\right)^8}{24b^4} + \frac{B\left(a+\right.}{27}$$

Mathematica [A] time = 0.0250756, size = 107, normalized size = 1.13

$$\frac{x^9 \left(840a^2b^2x^9(aB+Ab)+504a^3bx^6(aB+2Ab)+126a^4x^3(aB+5Ab)+168a^5A+63b^4x^{15}(5aB+Ab)+360ab^3x^{12}(2a\right.}{1512}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6 +
 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b +
 5*a*B)*x^15 + 56*b^5*B*x^18))/1512
```

Maple [A] time = 0.002, size = 124, normalized size = 1.3

$$\frac{b^5Bx^{27}}{27} + \frac{\left(b^5A+5\,ab^4B\right)x^{24}}{24} + \frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{21}}{21} + \frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{18}}{18} + \frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{15}}{15} + \frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/27*b^5*B*x^27+1/24*(A*b^5+5*B*a*b^4)*x^24+1/21*(5*A*a*b^4+10*B*a^2*b^3)*x
^21+1/18*(10*A*a^2*b^3+10*B*a^3*b^2)*x^18+1/15*(10*A*a^3*b^2+5*B*a^4*b)*x^1
5+1/12*(5*A*a^4*b+B*a^5)*x^12+1/9*a^5*A*x^9
```

Maxima [A] time = 1.40945, size = 161, normalized size = 1.69

$$\frac{1}{27}\,Bb^{5}x^{27} + \frac{1}{24}\left(5\,Bab^{4} + Ab^{5}\right)x^{24} + \frac{5}{21}\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{21} + \frac{5}{9}\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{18} + \frac{1}{3}\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{15} + \frac{1}{9}\,Aa^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*b
^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)*x
^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Fricas [A] time = 1.26366, size = 316, normalized size = 3.33

$$\frac{1}{27}x^{27}b^{5}B + \frac{5}{24}x^{24}b^{4}aB + \frac{1}{24}x^{24}b^{5}A + \frac{10}{21}x^{21}b^{3}a^{2}B + \frac{5}{21}x^{21}b^{4}aA + \frac{5}{9}x^{18}b^{2}a^{3}B + \frac{5}{9}x^{18}b^{3}a^{2}A + \frac{1}{3}x^{15}ba^{4}B + \frac{2}{3}x^{15}b^{2}a^{3}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/27*x^27*b^5*B + 5/24*x^24*b^4*a*B + 1/24*x^24*b^5*A + 10/21*x^21*b^3*a^2*
B + 5/21*x^21*b^4*a*A + 5/9*x^18*b^2*a^3*B + 5/9*x^18*b^3*a^2*A + 1/3*x^15*
b*a^4*B + 2/3*x^15*b^2*a^3*A + 1/12*x^12*a^5*B + 5/12*x^12*b*a^4*A + 1/9*x^
9*a^5*A
```

Sympy [A] time = 0.084674, size = 136, normalized size = 1.43

$$\frac{Aa^{5}x^{9}}{9} + \frac{Bb^{5}x^{27}}{27} + x^{24}\left(\frac{Ab^{5}}{24} + \frac{5Bab^{4}}{24}\right) + x^{21}\left(\frac{5Aab^{4}}{21} + \frac{10Ba^{2}b^{3}}{21}\right) + x^{18}\left(\frac{5Aa^{2}b^{3}}{9} + \frac{5Ba^{3}b^{2}}{9}\right) + x^{15}\left(\frac{2Aa^{3}b^{2}}{3} + \frac{Ba^{4}b}{3}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**21
*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*b*
*2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*a**
```

```
5/12)
```

Giac [A] time = 1.13727, size = 169, normalized size = 1.78

$$\frac{1}{27}Bb^5x^{27}+\frac{5}{24}Bab^4x^{24}+\frac{1}{24}Ab^5x^{24}+\frac{10}{21}Ba^2b^3x^{21}+\frac{5}{21}Aab^4x^{21}+\frac{5}{9}Ba^3b^2x^{18}+\frac{5}{9}Aa^2b^3x^{18}+\frac{1}{3}Ba^4bx^{15}+\frac{2}{3}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/27*B*b^5*x^27 + 5/24*B*a*b^4*x^24 + 1/24*A*b^5*x^24 + 10/21*B*a^2*b^3*x^2
1 + 5/21*A*a*b^4*x^21 + 5/9*B*a^3*b^2*x^18 + 5/9*A*a^2*b^3*x^18 + 1/3*B*a^4
*b*x^15 + 2/3*A*a^3*b^2*x^15 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/9*A*
a^5*x^9
```

3.25 $\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{8}a^5Ax^8+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab$$

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Rubi [A] time = 0.0749274, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{8}a^5Ax^8+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)dx=\int\left(a^5Ax^7+a^4(5Ab+aB)x^{10}+5a^3b(2Ab+aB)x^{13}+10a^2b^2(Ab+aB)x^{16}+5ab^3(Ab+\right.$$

$$=\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(A$$

Mathematica [A] time = 0.0180576, size = 117, normalized size = 1.

$$\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{8}a^5Ax^8+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Maple [A] time = 0.002, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{26}}{26}+\frac{\left(b^5A+5\,ab^4B\right)x^{23}}{23}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{20}}{20}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{17}}{17}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{14}}{14}+\frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/26*b^5*B*x^26+1/23*(A*b^5+5*B*a*b^4)*x^23+1/20*(5*A*a*b^4+10*B*a^2*b^3)*x
^20+1/17*(10*A*a^2*b^3+10*B*a^3*b^2)*x^17+1/14*(10*A*a^3*b^2+5*B*a^4*b)*x^1
4+1/11*(5*A*a^4*b+B*a^5)*x^11+1/8*a^5*A*x^8
```

Maxima [A] time = 1.19773, size = 161, normalized size = 1.38

$$\frac{1}{26}Bb^5x^{26}+\frac{1}{23}\left(5\,Bab^4+Ab^5\right)x^{23}+\frac{1}{4}\left(2\,Ba^2b^3+Aab^4\right)x^{20}+\frac{10}{17}\left(Ba^3b^2+Aa^2b^3\right)x^{17}+\frac{5}{14}\left(Ba^4b+2\,Aa^3b^2\right)x^{14}+\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b^
4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^2)
*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Fricas [A] time = 1.27853, size = 319, normalized size = 2.73

$$\frac{1}{26}x^{26}b^5B + \frac{5}{23}x^{23}b^4aB + \frac{1}{23}x^{23}b^5A + \frac{1}{2}x^{20}b^3a^2B + \frac{1}{4}x^{20}b^4aA + \frac{10}{17}x^{17}b^2a^3B + \frac{10}{17}x^{17}b^3a^2A + \frac{5}{14}x^{14}ba^4B + \frac{5}{7}x^{14}b^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/26*x^26*b^5*B + 5/23*x^23*b^4*a*B + 1/23*x^23*b^5*A + 1/2*x^20*b^3*a^2*B
+ 1/4*x^20*b^4*a*A + 10/17*x^17*b^2*a^3*B + 10/17*x^17*b^3*a^2*A + 5/14*x^1
4*b*a^4*B + 5/7*x^14*b^2*a^3*A + 1/11*x^11*a^5*B + 5/11*x^11*b*a^4*A + 1/8*
x^8*a^5*A
```

Sympy [A] time = 0.085915, size = 134, normalized size = 1.15

$$\frac{Aa^5x^8}{8} + \frac{Bb^5x^{26}}{26} + x^{23}\left(\frac{Ab^5}{23} + \frac{5Bab^4}{23}\right) + x^{20}\left(\frac{Aab^4}{4} + \frac{Ba^2b^3}{2}\right) + x^{17}\left(\frac{10Aa^2b^3}{17} + \frac{10Ba^3b^2}{17}\right) + x^{14}\left(\frac{5Aa^3b^2}{7} + \frac{5Ba^4b}{14}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**26/26 + x**23*(A*b**5/23 + 5*B*a*b**4/23) + x**20
*(A*a*b**4/4 + B*a**2*b**3/2) + x**17*(10*A*a**2*b**3/17 + 10*B*a**3*b**2/1
7) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**11*(5*A*a**4*b/11 + B*a**
5/11)
```

Giac [A] time = 1.20776, size = 169, normalized size = 1.44

$$\frac{1}{26}Bb^5x^{26} + \frac{5}{23}Bab^4x^{23} + \frac{1}{23}Ab^5x^{23} + \frac{1}{2}Ba^2b^3x^{20} + \frac{1}{4}Aab^4x^{20} + \frac{10}{17}Ba^3b^2x^{17} + \frac{10}{17}Aa^2b^3x^{17} + \frac{5}{14}Ba^4bx^{14} + \frac{5}{7}Aa^3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/26*B*b^5*x^26 + 5/23*B*a*b^4*x^23 + 1/23*A*b^5*x^23 + 1/2*B*a^2*b^3*x^20
+ 1/4*A*a*b^4*x^20 + 10/17*B*a^3*b^2*x^17 + 10/17*A*a^2*b^3*x^17 + 5/14*B*a
```

```
^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/8*
A*a^5*x^8
```

3.26 $\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+A$$

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Rubi [A] time = 0.0681405, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+A$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5Ax^6+a^4(5Ab+aB)x^9+5a^3b(2Ab+aB)x^{12}+10a^2b^2(Ab+aB)x^{15}+5ab^3(Ab+\right.\\ &= \frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{19}ab^3(A\end{aligned}$$

Mathematica [A] time = 0.0182178, size = 117, normalized size = 1.

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Maple [A] time = 0.002, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{25}}{25}+\frac{\left(b^5A+5\,ab^4B\right)x^{22}}{22}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{19}}{19}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{16}}{16}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{13}}{13}+\frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/25*b^5*B*x^25+1/22*(A*b^5+5*B*a*b^4)*x^22+1/19*(5*A*a*b^4+10*B*a^2*b^3)*x
^19+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/13*(10*A*a^3*b^2+5*B*a^4*b)*x^1
3+1/10*(5*A*a^4*b+B*a^5)*x^10+1/7*a^5*A*x^7
```

Maxima [A] time = 1.08949, size = 161, normalized size = 1.38

$$\frac{1}{25}\,Bb^5x^{25}+\frac{1}{22}\left(5\,Bab^4+Ab^5\right)x^{22}+\frac{5}{19}\left(2\,Ba^2b^3+Aab^4\right)x^{19}+\frac{5}{8}\left(Ba^3b^2+Aa^2b^3\right)x^{16}+\frac{5}{13}\left(Ba^4b+2\,Aa^3b^2\right)x^{13}+\frac{1}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*b
^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*
x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Fricas [A] time = 1.22717, size = 319, normalized size = 2.73

$$\frac{1}{25}x^{25}b^5B + \frac{5}{22}x^{22}b^4aB + \frac{1}{22}x^{22}b^5A + \frac{10}{19}x^{19}b^3a^2B + \frac{5}{19}x^{19}b^4aA + \frac{5}{8}x^{16}b^2a^3B + \frac{5}{8}x^{16}b^3a^2A + \frac{5}{13}x^{13}ba^4B + \frac{10}{13}x^{13}b^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/25*x^25*b^5*B + 5/22*x^22*b^4*a*B + 1/22*x^22*b^5*A + 10/19*x^19*b^3*a^2*
B + 5/19*x^19*b^4*a*A + 5/8*x^16*b^2*a^3*B + 5/8*x^16*b^3*a^2*A + 5/13*x^13
*b*a^4*B + 10/13*x^13*b^2*a^3*A + 1/10*x^10*a^5*B + 1/2*x^10*b*a^4*A + 1/7*
x^7*a^5*A
```

Sympy [A] time = 0.084589, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7} + \frac{Bb^5x^{25}}{25} + x^{22}\left(\frac{Ab^5}{22} + \frac{5Bab^4}{22}\right) + x^{19}\left(\frac{5Aab^4}{19} + \frac{10Ba^2b^3}{19}\right) + x^{16}\left(\frac{5Aa^2b^3}{8} + \frac{5Ba^3b^2}{8}\right) + x^{13}\left(\frac{10Aa^3b^2}{13} + \frac{5Ba^4b}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**25/25 + x**22*(A*b**5/22 + 5*B*a*b**4/22) + x**19
*(5*A*a*b**4/19 + 10*B*a**2*b**3/19) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b*
*2/8) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**10*(A*a**4*b/2 + B*a
**5/10)
```

Giac [A] time = 1.44056, size = 169, normalized size = 1.44

$$\frac{1}{25}Bb^5x^{25} + \frac{5}{22}Bab^4x^{22} + \frac{1}{22}Ab^5x^{22} + \frac{10}{19}Ba^2b^3x^{19} + \frac{5}{19}Aab^4x^{19} + \frac{5}{8}Ba^3b^2x^{16} + \frac{5}{8}Aa^2b^3x^{16} + \frac{5}{13}Ba^4bx^{13} + \frac{10}{13}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/25*B*b^5*x^25 + 5/22*B*a*b^4*x^22 + 1/22*A*b^5*x^22 + 10/19*B*a^2*b^3*x^1
9 + 5/19*A*a*b^4*x^19 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/13*B*a^
```

```
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/7*
A*a^5*x^7
```

3.27 $\int x^5 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=67

$$\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{21b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{18b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}$$

```
[Out] -(a*(A*b - a*B)*(a + b*x^3)^6)/(18*b^3) + ((A*b - 2*a*B)*(a + b*x^3)^7)/(21
*b^3) + (B*(a + b*x^3)^8)/(24*b^3)
```

Rubi [A] time = 0.148173, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{21b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{18b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^3)^6)/(18*b^3) + ((A*b - 2*a*B)*(a + b*x^3)^7)/(21
*b^3) + (B*(a + b*x^3)^8)/(24*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)(a+bx)^5}{b^2}+\frac{(Ab-2aB)(a+bx)^6}{b^2}+\frac{B(a+bx)^7}{b^2}\right)dx,x,x^3\right)\\ &= -\frac{a(Ab-aB)\left(a+bx^3\right)^6}{18b^3}+\frac{(Ab-2aB)\left(a+bx^3\right)^7}{21b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}\end{aligned}$$

Mathematica [A] time = 0.0240713, size = 107, normalized size = 1.6

$$\frac{1}{504}x^6\left(336a^2b^2x^9(aB+Ab)+210a^3bx^6(aB+2Ab)+56a^4x^3(aB+5Ab)+84a^5A+24b^4x^{15}(5aB+Ab)+140ab^3x^{12}(2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B)*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 + 3
36*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b + 5
*a*B)*x^15 + 21*b^5*B*x^18))/504
```

Maple [B] time = 0.002, size = 124, normalized size = 1.9

$$\frac{b^5Bx^{24}}{24}+\frac{\left(b^5A+5\,ab^4B\right)x^{21}}{21}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{18}}{18}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{15}}{15}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{12}}{12}+\frac{(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/24*b^5*B*x^24+1/21*(A*b^5+5*B*a*b^4)*x^21+1/18*(5*A*a*b^4+10*B*a^2*b^3)*x
^18+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/12*(10*A*a^3*b^2+5*B*a^4*b)*x^1
2+1/9*(5*A*a^4*b+B*a^5)*x^9+1/6*a^5*A*x^6
```

Maxima [A] time = 1.26554, size = 161, normalized size = 2.4

$$\frac{1}{24}Bb^5x^{24}+\frac{1}{21}\left(5\,Bab^4+Ab^5\right)x^{21}+\frac{5}{18}\left(2\,Ba^2b^3+Aab^4\right)x^{18}+\frac{2}{3}\left(Ba^3b^2+Aa^2b^3\right)x^{15}+\frac{5}{12}\left(Ba^4b+2\,Aa^3b^2\right)x^{12}+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*
x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Fricas [B] time = 1.25691, size = 309, normalized size = 4.61

$$\frac{1}{24}x^{24}b^5B + \frac{5}{21}x^{21}b^4aB + \frac{1}{21}x^{21}b^5A + \frac{5}{9}x^{18}b^3a^2B + \frac{5}{18}x^{18}b^4aA + \frac{2}{3}x^{15}b^2a^3B + \frac{2}{3}x^{15}b^3a^2A + \frac{5}{12}x^{12}ba^4B + \frac{5}{6}x^{12}b^2a^3A +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^5*B + 5/21*x^21*b^4*a*B + 1/21*x^21*b^5*A + 5/9*x^18*b^3*a^2*B
+ 5/18*x^18*b^4*a*A + 2/3*x^15*b^2*a^3*B + 2/3*x^15*b^3*a^2*A + 5/12*x^12*b
*a^4*B + 5/6*x^12*b^2*a^3*A + 1/9*x^9*a^5*B + 5/9*x^9*b*a^4*A + 1/6*x^6*a^5
*A
```

Sympy [B] time = 0.086819, size = 138, normalized size = 2.06

$$\frac{Aa^5x^6}{6} + \frac{Bb^5x^{24}}{24} + x^{21}\left(\frac{Ab^5}{21} + \frac{5Bab^4}{21}\right) + x^{18}\left(\frac{5Aab^4}{18} + \frac{5Ba^2b^3}{9}\right) + x^{15}\left(\frac{2Aa^2b^3}{3} + \frac{2Ba^3b^2}{3}\right) + x^{12}\left(\frac{5Aa^3b^2}{6} + \frac{5Ba^4b}{12}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**18
*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b**2
/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a**5
/9)
```

Giac [B] time = 1.18118, size = 169, normalized size = 2.52

$$\frac{1}{24}Bb^5x^{24} + \frac{5}{21}Bab^4x^{21} + \frac{1}{21}Ab^5x^{21} + \frac{5}{9}Ba^2b^3x^{18} + \frac{5}{18}Aab^4x^{18} + \frac{2}{3}Ba^3b^2x^{15} + \frac{2}{3}Aa^2b^3x^{15} + \frac{5}{12}Ba^4bx^{12} + \frac{5}{6}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/24*B*b^5*x^24 + 5/21*B*a*b^4*x^21 + 1/21*A*b^5*x^21 + 5/9*B*a^2*b^3*x^18
+ 5/18*A*a*b^4*x^18 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/6*A*a^5*x
^6
```

3.28 $\int x^4 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)$$

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Rubi [A] time = 0.0696903, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)dx=\int\left(a^5Ax^4+a^4(5Ab+aB)x^7+5a^3b(2Ab+aB)x^{10}+10a^2b^2(Ab+aB)x^{13}+5ab^3(Ab+\right.$$
$$=\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab$$

Mathematica [A] time = 0.0155362, size = 117, normalized size = 1.

$$\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Maple [A] time = 0.002, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{23}}{23}+\frac{\left(b^5A+5\,ab^4B\right)x^{20}}{20}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{17}}{17}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{14}}{14}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{11}}{11}+\frac{(5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/23*b^5*B*x^23+1/20*(A*b^5+5*B*a*b^4)*x^20+1/17*(5*A*a*b^4+10*B*a^2*b^3)*x
^17+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/11*(10*A*a^3*b^2+5*B*a^4*b)*x^1
1+1/8*(5*A*a^4*b+B*a^5)*x^8+1/5*a^5*A*x^5
```

Maxima [A] time = 1.1747, size = 161, normalized size = 1.38

$$\frac{1}{23}Bb^5x^{23}+\frac{1}{20}\left(5\,Bab^4+Ab^5\right)x^{20}+\frac{5}{17}\left(2\,Ba^2b^3+Aab^4\right)x^{17}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{5}{11}\left(Ba^4b+2\,Aa^3b^2\right)x^{11}+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*
x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Fricas [A] time = 1.25708, size = 313, normalized size = 2.68

$$\frac{1}{23}x^{23}b^5B + \frac{1}{4}x^{20}b^4aB + \frac{1}{20}x^{20}b^5A + \frac{10}{17}x^{17}b^3a^2B + \frac{5}{17}x^{17}b^4aA + \frac{5}{7}x^{14}b^2a^3B + \frac{5}{7}x^{14}b^3a^2A + \frac{5}{11}x^{11}ba^4B + \frac{10}{11}x^{11}b^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/23*x^23*b^5*B + 1/4*x^20*b^4*a*B + 1/20*x^20*b^5*A + 10/17*x^17*b^3*a^2*B
 + 5/17*x^17*b^4*a*A + 5/7*x^14*b^2*a^3*B + 5/7*x^14*b^3*a^2*A + 5/11*x^11*
b*a^4*B + 10/11*x^11*b^2*a^3*A + 1/8*x^8*a^5*B + 5/8*x^8*b*a^4*A + 1/5*x^5*
a^5*A
```

Sympy [A] time = 0.085864, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5} + \frac{Bb^5x^{23}}{23} + x^{20}\left(\frac{Ab^5}{20} + \frac{Bab^4}{4}\right) + x^{17}\left(\frac{5Aab^4}{17} + \frac{10Ba^2b^3}{17}\right) + x^{14}\left(\frac{5Aa^2b^3}{7} + \frac{5Ba^3b^2}{7}\right) + x^{11}\left(\frac{10Aa^3b^2}{11} + \frac{5Ba^4b}{11}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**23/23 + x**20*(A*b**5/20 + B*a*b**4/4) + x**17*(5
*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2/
7) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**8*(5*A*a**4*b/8 + B*a**
5/8)
```

Giac [A] time = 1.24538, size = 169, normalized size = 1.44

$$\frac{1}{23}Bb^5x^{23} + \frac{1}{4}Bab^4x^{20} + \frac{1}{20}Ab^5x^{20} + \frac{10}{17}Ba^2b^3x^{17} + \frac{5}{17}Aab^4x^{17} + \frac{5}{7}Ba^3b^2x^{14} + \frac{5}{7}Aa^2b^3x^{14} + \frac{5}{11}Ba^4bx^{11} + \frac{10}{11}Aa^3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/23*B*b^5*x^23 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 10/17*B*a^2*b^3*x^17
 + 5/17*A*a*b^4*x^17 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/11*B*a^4
```

```
*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/5*A*a^
5*x^5
```

3.29 $\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{4}a^5Ax^4+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)$$

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Rubi [A] time = 0.065377, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{4}a^5Ax^4+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)dx=\int\left(a^5Ax^3+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^9+10a^2b^2(Ab+aB)x^{12}+5ab^3(Ab+2\right.$$
$$=\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab$$

Mathematica [A] time = 0.0164245, size = 117, normalized size = 1.

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{4}a^5Ax^4+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Maple [A] time = 0.001, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{22}}{22}+\frac{\left(b^5A+5\,ab^4B\right)x^{19}}{19}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{16}}{16}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{13}}{13}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^{10}}{10}+\frac{(5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/22*b^5*B*x^22+1/19*(A*b^5+5*B*a*b^4)*x^19+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x
^16+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^1
0+1/7*(5*A*a^4*b+B*a^5)*x^7+1/4*a^5*A*x^4
```

Maxima [A] time = 1.28491, size = 161, normalized size = 1.38

$$\frac{1}{22}\,Bb^5x^{22}+\frac{1}{19}\left(5\,Bab^4+Ab^5\right)x^{19}+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^{10}+\frac{1}{4}\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^2)
*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Fricas [A] time = 1.22838, size = 308, normalized size = 2.63

$$\frac{1}{22}x^{22}b^5B + \frac{5}{19}x^{19}b^4aB + \frac{1}{19}x^{19}b^5A + \frac{5}{8}x^{16}b^3a^2B + \frac{5}{16}x^{16}b^4aA + \frac{10}{13}x^{13}b^2a^3B + \frac{10}{13}x^{13}b^3a^2A + \frac{1}{2}x^{10}ba^4B + x^{10}b^2a^3A +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5*B + 5/19*x^19*b^4*a*B + 1/19*x^19*b^5*A + 5/8*x^16*b^3*a^2*B
+ 5/16*x^16*b^4*a*A + 10/13*x^13*b^2*a^3*B + 10/13*x^13*b^3*a^2*A + 1/2*x^1
0*b*a^4*B + x^10*b^2*a^3*A + 1/7*x^7*a^5*B + 5/7*x^7*b*a^4*A + 1/4*x^4*a^5*
A
```

Sympy [A] time = 0.085181, size = 133, normalized size = 1.14

$$\frac{Aa^5x^4}{4} + \frac{Bb^5x^{22}}{22} + x^{19}\left(\frac{Ab^5}{19} + \frac{5Bab^4}{19}\right) + x^{16}\left(\frac{5Aab^4}{16} + \frac{5Ba^2b^3}{8}\right) + x^{13}\left(\frac{10Aa^2b^3}{13} + \frac{10Ba^3b^2}{13}\right) + x^{10}\left(Aa^3b^2 + \frac{Ba^4b}{2}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b
**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5/7)
```

Giac [A] time = 1.16333, size = 167, normalized size = 1.43

$$\frac{1}{22}Bb^5x^{22} + \frac{5}{19}Bab^4x^{19} + \frac{1}{19}Ab^5x^{19} + \frac{5}{8}Ba^2b^3x^{16} + \frac{5}{16}Aab^4x^{16} + \frac{10}{13}Ba^3b^2x^{13} + \frac{10}{13}Aa^2b^3x^{13} + \frac{1}{2}Ba^4bx^{10} + Aa^3b^2x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 1/2*B*a
^4*b*x^10 + A*a^3*b^2*x^10 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/4*A*a^5*x^
4
```

3.30 $\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^3\right)^6 (Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

```
[Out] ((A*b - a*B)*(a + b*x^3)^6)/(18*b^2) + (B*(a + b*x^3)^7)/(21*b^2)
```

Rubi [A] time = 0.068837, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{\left(a+bx^3\right)^6 (Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^6)/(18*b^2) + (B*(a + b*x^3)^7)/(21*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx = \frac{1}{3}\,\text{Subst}\left(\int (a+bx)^5(A+Bx)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right) dx, x, x^3\right)$$
$$= \frac{(Ab-aB)\left(a+bx^3\right)^6}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

Mathematica [B] time = 0.0250203, size = 107, normalized size = 2.55

$$\frac{1}{126}x^3\left(105a^2b^2x^9(aB+Ab)+70a^3bx^6(aB+2Ab)+21a^4x^3(aB+5Ab)+42a^5A+7b^4x^{15}(5aB+Ab)+42ab^3x^{12}(2aB+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 10
5*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*a*
B)*x^15 + 6*b^5*B*x^18))/126
```

Maple [B] time = 0.001, size = 124, normalized size = 3.

$$\frac{b^5Bx^{21}}{21}+\frac{\left(b^5A+5\,ab^4B\right)x^{18}}{18}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{15}}{15}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{12}}{12}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^9}{9}+\frac{\left(5\,a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/21*b^5*B*x^21+1/18*(A*b^5+5*B*a*b^4)*x^18+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x
^15+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+
1/6*(5*A*a^4*b+B*a^5)*x^6+1/3*a^5*A*x^3
```

Maxima [B] time = 1.32201, size = 161, normalized size = 3.83

$$\frac{1}{21}\,Bb^5x^{21}+\frac{1}{18}\left(5\,Bab^4+Ab^5\right)x^{18}+\frac{1}{3}\left(2\,Ba^2b^3+Aab^4\right)x^{15}+\frac{5}{6}\left(Ba^3b^2+Aa^2b^3\right)x^{12}+\frac{5}{9}\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{3}\,Aa^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^
9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Fricas [B] time = 1.2861, size = 305, normalized size = 7.26

$$\frac{1}{21}x^{21}b^5B + \frac{5}{18}x^{18}b^4aB + \frac{1}{18}x^{18}b^5A + \frac{2}{3}x^{15}b^3a^2B + \frac{1}{3}x^{15}b^4aA + \frac{5}{6}x^{12}b^2a^3B + \frac{5}{6}x^{12}b^3a^2A + \frac{5}{9}x^9ba^4B + \frac{10}{9}x^9b^2a^3A +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^5*B + 5/18*x^18*b^4*a*B + 1/18*x^18*b^5*A + 2/3*x^15*b^3*a^2*B
+ 1/3*x^15*b^4*a*A + 5/6*x^12*b^2*a^3*B + 5/6*x^12*b^3*a^2*A + 5/9*x^9*b*a^
4*B + 10/9*x^9*b^2*a^3*A + 1/6*x^6*a^5*B + 5/6*x^6*b*a^4*A + 1/3*x^3*a^5*A
```

Sympy [B] time = 0.084238, size = 136, normalized size = 3.24

$$\frac{Aa^5x^3}{3} + \frac{Bb^5x^{21}}{21} + x^{18}\left(\frac{Ab^5}{18} + \frac{5Bab^4}{18}\right) + x^{15}\left(\frac{Aab^4}{3} + \frac{2Ba^2b^3}{3}\right) + x^{12}\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^9\left(\frac{10Aa^3b^2}{9} + \frac{5Ba^4b}{9}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/6)
 + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

Giac [B] time = 1.17044, size = 169, normalized size = 4.02

$$\frac{1}{21}Bb^5x^{21} + \frac{5}{18}Bab^4x^{18} + \frac{1}{18}Ab^5x^{18} + \frac{2}{3}Ba^2b^3x^{15} + \frac{1}{3}Aab^4x^{15} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*b*
x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x^3
```

3.31 $\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=117

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{2}a^5Ax^2+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)$$

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Rubi [A] time = 0.0632876, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{2}a^5Ax^2+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^5*(A + B*x^3),x]
```


```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```


Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \int \left(a^5Ax+a^4(5Ab+aB)x^4+5a^3b(2Ab+aB)x^7+10a^2b^2(Ab+aB)x^{10}+5ab^3(Ab+2\right.\\ &= \frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{14}ab^3(Ab\end{aligned}$$

Mathematica [A] time = 0.0156807, size = 117, normalized size = 1.

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{2}a^5Ax^2+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Maple [A] time = 0.001, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{20}}{20}+\frac{\left(b^5A+5\,ab^4B\right)x^{17}}{17}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{14}}{14}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{11}}{11}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^8}{8}+\frac{(5\,a^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/20*b^5*B*x^20+1/17*(A*b^5+5*B*a*b^4)*x^17+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x
^14+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+
1/5*(5*A*a^4*b+B*a^5)*x^5+1/2*a^5*A*x^2
```

Maxima [A] time = 1.39543, size = 161, normalized size = 1.38

$$\frac{1}{20}Bb^5x^{20}+\frac{1}{17}\left(5\,Bab^4+Ab^5\right)x^{17}+\frac{5}{14}\left(2\,Ba^2b^3+Aab^4\right)x^{14}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{5}{8}\left(Ba^4b+2\,Aa^3b^2\right)x^8+\frac{1}{2}Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^2)
*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Fricas [A] time = 1.22414, size = 305, normalized size = 2.61

$$\frac{1}{20}x^{20}b^5B + \frac{5}{17}x^{17}b^4aB + \frac{1}{17}x^{17}b^5A + \frac{5}{7}x^{14}b^3a^2B + \frac{5}{14}x^{14}b^4aA + \frac{10}{11}x^{11}b^2a^3B + \frac{10}{11}x^{11}b^3a^2A + \frac{5}{8}x^8ba^4B + \frac{5}{4}x^8b^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5*B + 5/17*x^17*b^4*a*B + 1/17*x^17*b^5*A + 5/7*x^14*b^3*a^2*B
+ 5/14*x^14*b^4*a*A + 10/11*x^11*b^2*a^3*B + 10/11*x^11*b^3*a^2*A + 5/8*x^8
*b*a^4*B + 5/4*x^8*b^2*a^3*A + 1/5*x^5*a^5*B + x^5*b*a^4*A + 1/2*x^2*a^5*A
```

Sympy [A] time = 0.086919, size = 134, normalized size = 1.15

$$\frac{Aa^5x^2}{2} + \frac{Bb^5x^{20}}{20} + x^{17}\left(\frac{Ab^5}{17} + \frac{5Bab^4}{17}\right) + x^{14}\left(\frac{5Aab^4}{14} + \frac{5Ba^2b^3}{7}\right) + x^{11}\left(\frac{10Aa^2b^3}{11} + \frac{10Ba^3b^2}{11}\right) + x^8\left(\frac{5Aa^3b^2}{4} + \frac{5Ba^4b}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b
**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**5/5
)
```

Giac [A] time = 1.1619, size = 167, normalized size = 1.43

$$\frac{1}{20}Bb^5x^{20} + \frac{5}{17}Bab^4x^{17} + \frac{1}{17}Ab^5x^{17} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/8*B*a
^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x^2
```

3.32 $\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=109

$$a^2b^2x^{10}(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{4}a^4x^4(aB+5Ab)+a^5Ax+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^{5}$$

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Rubi [A] time = 0.0554363, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2b^2x^{10}(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{4}a^4x^4(aB+5Ab)+a^5Ax+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5A+a^4(5Ab+aB)x^3+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^9+5ab^3(Ab+2aB)x^{1}\right.\\ &= a^5Ax+\frac{1}{4}a^4(5Ab+aB)x^4+\frac{5}{7}a^3b(2Ab+aB)x^7+a^2b^2(Ab+aB)x^{10}+\frac{5}{13}ab^3(Ab+2aB)x^{1}\end{aligned}$$

Mathematica [A] time = 0.0148256, size = 109, normalized size = 1.

$$a^2b^2x^{10}(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{4}a^4x^4(aB+5Ab)+a^5Ax+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Maple [A] time = 0.001, size = 121, normalized size = 1.1

$$\frac{b^5Bx^{19}}{19}+\frac{\left(b^5A+5\,ab^4B\right)x^{16}}{16}+\frac{\left(5\,ab^4A+10\,a^2b^3B\right)x^{13}}{13}+\frac{\left(10\,a^2b^3A+10\,a^3b^2B\right)x^{10}}{10}+\frac{\left(10\,a^3b^2A+5\,a^4bB\right)x^7}{7}+\frac{(5\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/19*b^5*B*x^19+1/16*(A*b^5+5*B*a*b^4)*x^16+1/13*(5*A*a*b^4+10*B*a^2*b^3)*x
^13+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/7*(10*A*a^3*b^2+5*B*a^4*b)*x^7+
1/4*(5*A*a^4*b+B*a^5)*x^4+a^5*A*x
```

Maxima [A] time = 1.31041, size = 155, normalized size = 1.42

$$\frac{1}{19}\,Bb^5x^{19}+\frac{1}{16}\left(5\,Bab^4+Ab^5\right)x^{16}+\frac{5}{13}\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\left(Ba^3b^2+Aa^2b^3\right)x^{10}+\frac{5}{7}\left(Ba^4b+2\,Aa^3b^2\right)x^7+Aa^5x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 +
 A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Fricas [A] time = 1.28977, size = 290, normalized size = 2.66

$$\frac{1}{19}x^{19}b^5B + \frac{5}{16}x^{16}b^4aB + \frac{1}{16}x^{16}b^5A + \frac{10}{13}x^{13}b^3a^2B + \frac{5}{13}x^{13}b^4aA + x^{10}b^2a^3B + x^{10}b^3a^2A + \frac{5}{7}x^7ba^4B + \frac{10}{7}x^7b^2a^3A + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5*B + 5/16*x^16*b^4*a*B + 1/16*x^16*b^5*A + 10/13*x^13*b^3*a^2*
B + 5/13*x^13*b^4*a*A + x^10*b^2*a^3*B + x^10*b^3*a^2*A + 5/7*x^7*b*a^4*B +
 10/7*x^7*b^2*a^3*A + 1/4*x^4*a^5*B + 5/4*x^4*b*a^4*A + x*a^5*A
```

Sympy [A] time = 0.082786, size = 128, normalized size = 1.17

$$Aa^5x + \frac{Bb^5x^{19}}{19} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{13}\left(\frac{5Aab^4}{13} + \frac{10Ba^2b^3}{13}\right) + x^{10}\left(Aa^2b^3 + Ba^3b^2\right) + x^7\left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^4\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**19/19 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**13*(5*A
*a*b**4/13 + 10*B*a**2*b**3/13) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x**7*
(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

Giac [A] time = 1.19373, size = 162, normalized size = 1.49

$$\frac{1}{19}Bb^5x^{19} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{7}Ba^4bx^7 + \frac{10}{7}Aa^3b^2x^7 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 10/13*B*a^2*b^3*x^1
3 + 5/13*A*a*b^4*x^13 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/7*B*a^4*b*x^7 +
 10/7*A*a^3*b^2*x^7 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + A*a^5*x
```

3.33 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx$

Optimal. Leaf size=88

$$\frac{10}{9}a^2Ab^3x^9 + \frac{5}{3}a^3Ab^2x^6 + \frac{5}{3}a^4Abx^3 + a^5A\log(x) + \frac{5}{12}aAb^4x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15}Ab^5x^{15}$$

```
[Out] (5*a^4*A*b*x^3)/3 + (5*a^3*A*b^2*x^6)/3 + (10*a^2*A*b^3*x^9)/9 + (5*a*A*b^4
*x^12)/12 + (A*b^5*x^15)/15 + (B*(a + b*x^3)^6)/(18*b) + a^5*A*Log[x]
```

Rubi [A] time = 0.0672439, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 80, 43}

$$\frac{10}{9}a^2Ab^3x^9 + \frac{5}{3}a^3Ab^2x^6 + \frac{5}{3}a^4Abx^3 + a^5A\log(x) + \frac{5}{12}aAb^4x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15}Ab^5x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (5*a^4*A*b*x^3)/3 + (5*a^3*A*b^2*x^6)/3 + (10*a^2*A*b^3*x^9)/9 + (5*a*A*b^4
*x^12)/12 + (A*b^5*x^15)/15 + (B*(a + b*x^3)^6)/(18*b) + a^5*A*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x}\,dx,x,x^3\right)$$

$$= \frac{B\left(a+bx^3\right)^6}{18b} + \frac{1}{3}A\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx,x,x^3\right)$$

$$= \frac{B\left(a+bx^3\right)^6}{18b} + \frac{1}{3}A\operatorname{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx,x,x^3\right)$$

$$= \frac{5}{3}a^4Abx^3+\frac{5}{3}a^3Ab^2x^6+\frac{10}{9}a^2Ab^3x^9+\frac{5}{12}aAb^4x^{12}+\frac{1}{15}Ab^5x^{15}+\frac{B\left(a+bx^3\right)^6}{18b}+a^5A\log($$

Mathematica [A] time = 0.0271966, size = 113, normalized size = 1.28

$$\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{5}{6}a^3bx^6(aB+2Ab)+\frac{1}{3}a^4x^3(aB+5Ab)+a^5A\log(x)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{12}ab^3x^{12}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A*
b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^15
)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]
```

Maple [A] time = 0.002, size = 124, normalized size = 1.4

$$\frac{Bb^5x^{18}}{18}+\frac{Ab^5x^{15}}{15}+\frac{Bx^{15}ab^4}{3}+\frac{5\,aAb^4x^{12}}{12}+\frac{5\,Bx^{12}a^2b^3}{6}+\frac{10\,a^2Ab^3x^9}{9}+\frac{10\,Bx^9a^3b^2}{9}+\frac{5\,a^3Ab^2x^6}{3}+\frac{5\,Bx^6a^4b}{6}+\frac{5\,a^4Ab}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x,x)
```

```
[Out] 1/18*B*b^5*x^18+1/15*A*b^5*x^15+1/3*B*x^15*a*b^4+5/12*a*A*b^4*x^12+5/6*B*x^
12*a^2*b^3+10/9*a^2*A*b^3*x^9+10/9*B*x^9*a^3*b^2+5/3*a^3*A*b^2*x^6+5/6*B*x^
6*a^4*b+5/3*a^4*A*b*x^3+1/3*B*x^3*a^5+a^5*A*ln(x)
```

Maxima [A] time = 1.71705, size = 162, normalized size = 1.84

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{15}\left(5\,Bab^4+Ab^5\right)x^{15}+\frac{5}{12}\left(2\,Ba^2b^3+Aab^4\right)x^{12}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{6}\left(Ba^4b+2\,Aa^3b^2\right)x^6+\frac{1}{3}\,Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 1/3*A*a^5*log(x^3) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Fricas [A] time = 1.43029, size = 269, normalized size = 3.06

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{15}\left(5\,Bab^4+Ab^5\right)x^{15}+\frac{5}{12}\left(2\,Ba^2b^3+Aab^4\right)x^{12}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{6}\left(Ba^4b+2\,Aa^3b^2\right)x^6+Aa^5\,l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + A*a^5*log(x) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Sympy [A] time = 0.376198, size = 134, normalized size = 1.52

$$Aa^5\log{(x)}+\frac{Bb^5x^{18}}{18}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{12}\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^9\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^6\left(\frac{5Aa^3b^2}{3}+\frac{5Ba^4}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**18/18 + x**15*(A*b**5/15 + B*a*b**4/3) + x**12*(5
*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2/9
) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

Giac [A] time = 1.13411, size = 167, normalized size = 1.9

$$\frac{1}{18}Bb^5x^{18} + \frac{1}{3}Bab^4x^{15} + \frac{1}{15}Ab^5x^{15} + \frac{5}{6}Ba^2b^3x^{12} + \frac{5}{12}Aab^4x^{12} + \frac{10}{9}Ba^3b^2x^9 + \frac{10}{9}Aa^2b^3x^9 + \frac{5}{6}Ba^4bx^6 + \frac{5}{3}Aa^3b^2x^6 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 5/6*B*a^2*b^3*x^12 +
 5/12*A*a*b^4*x^12 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/6*B*a^4*b*
x^6 + 5/3*A*a^3*b^2*x^6 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*log(abs(x
))
```

3.34 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} \, dx$

Optimal. Leaf size=112

$$\frac{5}{4}a^2b^2x^8(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{2}a^4x^2(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5$$

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Rubi [A] time = 0.060787, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{5}{4}a^2b^2x^8(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{2}a^4x^2(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} \, dx = \int \left(\frac{a^5A}{x^2}+a^4(5Ab+aB)x+5a^3b(2Ab+aB)x^4+10a^2b^2(Ab+aB)x^7+5ab^3(Ab+2aB)x\right.$$

$$= -\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5+\frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+2aB)x$$

Mathematica [A] time = 0.0332409, size = 112, normalized size = 1.

$$\frac{5}{4}a^2b^2x^8(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{2}a^4x^2(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5B$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Maple [A] time = 0.004, size = 125, normalized size = 1.1

$$\frac{b^5Bx^{17}}{17}+\frac{Ax^{14}b^5}{14}+\frac{5\,Bx^{14}ab^4}{14}+\frac{5\,Ax^{11}ab^4}{11}+\frac{10\,Bx^{11}a^2b^3}{11}+\frac{5\,Ax^8a^2b^3}{4}+\frac{5\,Bx^8a^3b^2}{4}+2\,Ax^5a^3b^2+Bx^5a^4b+\frac{5\,Ax^2a^4b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^2,x)
```

```
[Out] 1/17*b^5*B*x^17+1/14*A*x^14*b^5+5/14*B*x^14*a*b^4+5/11*A*x^11*a*b^4+10/11*B
*x^11*a^2*b^3+5/4*A*x^8*a^2*b^3+5/4*B*x^8*a^3*b^2+2*A*x^5*a^3*b^2+B*x^5*a^4
*b+5/2*A*x^2*a^4*b+1/2*B*x^2*a^5-a^5*A/x
```

Maxima [A] time = 1.25233, size = 159, normalized size = 1.42

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{14}\left(5\,Bab^4+Ab^5\right)x^{14}+\frac{5}{11}\left(2\,Ba^2b^3+Aab^4\right)x^{11}+\frac{5}{4}\left(Ba^3b^2+Aa^2b^3\right)x^8+\left(Ba^4b+2\,Aa^3b^2\right)x^5-\frac{Aa^5}{x}+\frac{1}{2}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5 -
A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Fricas [A] time = 1.43568, size = 281, normalized size = 2.51

$$\frac{308\,Bb^5x^{18} + 374\left(5\,Bab^4 + Ab^5\right)x^{15} + 2380\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 6545\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 5236\left(Ba^4b + 2\,Aa^3b^2\right)x}{5236\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5236*(308*B*b^5*x^18 + 374*(5*B*a*b^4 + A*b^5)*x^15 + 2380*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5236*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 5236*A*a^5 + 2618*(B*a^5 + 5*A*a^4*b)*x^3)/x
```

Sympy [A] time = 0.37606, size = 129, normalized size = 1.15

$$-\frac{Aa^5}{x} + \frac{Bb^5x^{17}}{17} + x^{14}\left(\frac{Ab^5}{14} + \frac{5Bab^4}{14}\right) + x^{11}\left(\frac{5Aab^4}{11} + \frac{10Ba^2b^3}{11}\right) + x^8\left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^5\left(2Aa^3b^2 + Ba^4b\right) + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5*
A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4)
 + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

Giac [A] time = 1.12063, size = 167, normalized size = 1.49

$$\frac{1}{17}\,Bb^5x^{17} + \frac{5}{14}\,Bab^4x^{14} + \frac{1}{14}\,Ab^5x^{14} + \frac{10}{11}\,Ba^2b^3x^{11} + \frac{5}{11}\,Aab^4x^{11} + \frac{5}{4}\,Ba^3b^2x^8 + \frac{5}{4}\,Aa^2b^3x^8 + Ba^4bx^5 + 2\,Aa^3b^2x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + B*a^4*b*x^5
 + 2*A*a^3*b^2*x^5 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 - A*a^5/x
```

3.35 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^3}\,dx$

Optimal. Leaf size=112

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5A}{2x^2}+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5B$$

```
[Out] -(a^5*A)/(2*x^2) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Rubi [A] time = 0.0606458, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5A}{2x^2}+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5B$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^3}\,dx=\int \left(a^4(5Ab+aB)+\frac{a^5A}{x^3}+5a^3b(2Ab+aB)x^3+10a^2b^2(Ab+aB)x^6+5ab^3(Ab+2aB)x^9\right.$$

$$=-\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab+2aB)x^{10}$$

Mathematica [A] time = 0.0301264, size = 112, normalized size = 1.

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5A}{2x^2}+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Maple [A] time = 0.004, size = 120, normalized size = 1.1

$$\frac{b^5Bx^{16}}{16}+\frac{Ax^{13}b^5}{13}+\frac{5\,Bx^{13}ab^4}{13}+\frac{Ax^{10}ab^4}{2}+Bx^{10}a^2b^3+\frac{10\,Ax^7a^2b^3}{7}+\frac{10\,Bx^7a^3b^2}{7}+\frac{5\,Ax^4a^3b^2}{2}+\frac{5\,Bx^4a^4b}{4}+5\,a^4bAx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^3,x)
```

```
[Out] 1/16*b^5*B*x^16+1/13*A*x^13*b^5+5/13*B*x^13*a*b^4+1/2*A*x^10*a*b^4+B*x^10*a
^2*b^3+10/7*A*x^7*a^2*b^3+10/7*B*x^7*a^3*b^2+5/2*A*x^4*a^3*b^2+5/4*B*x^4*a^
4*b+5*a^4*b*A*x+a^5*B*x-1/2*a^5*A/x^2
```

Maxima [A] time = 1.11409, size = 157, normalized size = 1.4

$$\frac{1}{16}Bb^5x^{16}+\frac{1}{13}\left(5\,Bab^4+Ab^5\right)x^{13}+\frac{1}{2}\left(2\,Ba^2b^3+Aab^4\right)x^{10}+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4-\frac{Aa^5}{2\,x^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*x^
4 - 1/2*A*a^5/x^2 + (B*a^5 + 5*A*a^4*b)*x
```

Fricas [A] time = 1.3694, size = 279, normalized size = 2.49

$$\frac{91\,Bb^5x^{18} + 112\left(5\,Bab^4 + Ab^5\right)x^{15} + 728\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 2080\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 1820\left(Ba^4b + 2\,Aa^3b^2\right)x^6 - 7}{1456\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/1456*(91*B*b^5*x^18 + 112*(5*B*a*b^4 + A*b^5)*x^15 + 728*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1820*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 728*A*a^5 + 1456*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
```

Sympy [A] time = 0.377499, size = 128, normalized size = 1.14

$$-\frac{Aa^5}{2x^2} + \frac{Bb^5x^{16}}{16} + x^{13}\left(\frac{Ab^5}{13} + \frac{5Bab^4}{13}\right) + x^{10}\left(\frac{Aab^4}{2} + Ba^2b^3\right) + x^7\left(\frac{10Aa^2b^3}{7} + \frac{10Ba^3b^2}{7}\right) + x^4\left(\frac{5Aa^3b^2}{2} + \frac{5Ba^4b}{4}\right) + x\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**16/16 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x*
*10*(A*a*b**4/2 + B*a**2*b**3) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7)
 + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x*(5*A*a**4*b + B*a**5)
```

Giac [A] time = 1.22865, size = 161, normalized size = 1.44

$$\frac{1}{16}\,Bb^5x^{16} + \frac{5}{13}\,Bab^4x^{13} + \frac{1}{13}\,Ab^5x^{13} + Ba^2b^3x^{10} + \frac{1}{2}\,Aab^4x^{10} + \frac{10}{7}\,Ba^3b^2x^7 + \frac{10}{7}\,Aa^2b^3x^7 + \frac{5}{4}\,Ba^4bx^4 + \frac{5}{2}\,Aa^3b^2x^4 + B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/4*B*a^4*b*x^4
+ 5/2*A*a^3*b^2*x^4 + B*a^5*x + 5*A*a^4*b*x - 1/2*A*a^5/x^2
```

3.36 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} \, dx$

Optimal. Leaf size=113

$$\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4\log(x)(aB+5Ab)-\frac{a^5A}{3x^3}+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}$$

```
[Out] -(a^5*A)/(3*x^3) + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + a^4*(5*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.117054, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4\log(x)(aB+5Ab)-\frac{a^5A}{3x^3}+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^4}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^2}+\frac{a^4(5Ab+aB)}{x}+10a^2b^2(Ab+aB)x+5ab^3(Ab+2\right.\right.$$
$$= -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab+5aB)$$

Mathematica [A] time = 0.0394044, size = 115, normalized size = 1.02

$$\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+\log(x)\left(5a^4Ab+a^5B\right)-\frac{a^5A}{3x^3}+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]
```

Maple [A] time = 0.007, size = 123, normalized size = 1.1

$$\frac{b^5Bx^{15}}{15}+\frac{Ax^{12}b^5}{12}+\frac{5\,Bx^{12}ab^4}{12}+\frac{5\,Ax^9ab^4}{9}+\frac{10\,Bx^9a^2b^3}{9}+\frac{5\,Ax^6a^2b^3}{3}+\frac{5\,Bx^6a^3b^2}{3}+\frac{10\,Ax^3a^3b^2}{3}+\frac{5\,Bx^3a^4b}{3}-\frac{Aa^5}{3\,x^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^4,x)
```

```
[Out] 1/15*b^5*B*x^15+1/12*A*x^12*b^5+5/12*B*x^12*a*b^4+5/9*A*x^9*a*b^4+10/9*B*x^
9*a^2*b^3+5/3*A*x^6*a^2*b^3+5/3*B*x^6*a^3*b^2+10/3*A*x^3*a^3*b^2+5/3*B*x^3*
a^4*b-1/3*a^5*A/x^3+5*A*ln(x)*a^4*b+B*ln(x)*a^5
```

Maxima [A] time = 1.48549, size = 162, normalized size = 1.43

$$\frac{1}{15}Bb^5x^{15} + \frac{1}{12}\left(5Bab^4 + Ab^5\right)x^{12} + \frac{5}{9}\left(2Ba^2b^3 + Aab^4\right)x^9 + \frac{5}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^6 + \frac{5}{3}\left(Ba^4b + 2Aa^3b^2\right)x^3 - \frac{Aa^5}{3x^3} + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*x^3
- 1/3*A*a^5/x^3 + 1/3*(B*a^5 + 5*A*a^4*b)*log(x^3)
```

Fricas [A] time = 1.47553, size = 281, normalized size = 2.49

$$\frac{12Bb^5x^{18} + 15\left(5Bab^4 + Ab^5\right)x^{15} + 100\left(2Ba^2b^3 + Aab^4\right)x^{12} + 300\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 300\left(Ba^4b + 2Aa^3b^2\right)x^6 - 60}{180x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2)*
x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3
```

Sympy [A] time = 0.506031, size = 133, normalized size = 1.18

$$-\frac{Aa^5}{3x^3} + \frac{Bb^5x^{15}}{15} + a^4(5Ab + Ba)\log(x) + x^{12}\left(\frac{Ab^5}{12} + \frac{5Bab^4}{12}\right) + x^9\left(\frac{5Aab^4}{9} + \frac{10Ba^2b^3}{9}\right) + x^6\left(\frac{5Aa^2b^3}{3} + \frac{5Ba^3b^2}{3}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*log(x) + x**12*(A*b
**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(5*
A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3)
```

Giac [A] time = 1.22758, size = 193, normalized size = 1.71

$$\frac{1}{15}Bb^5x^{15} + \frac{5}{12}Bab^4x^{12} + \frac{1}{12}Ab^5x^{12} + \frac{10}{9}Ba^2b^3x^9 + \frac{5}{9}Aab^4x^9 + \frac{5}{3}Ba^3b^2x^6 + \frac{5}{3}Aa^2b^3x^6 + \frac{5}{3}Ba^4bx^3 + \frac{10}{3}Aa^3b^2x^3 + ($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/3*B*a^4*b*x^3
 + 10/3*A*a^3*b^2*x^3 + (B*a^5 + 5*A*a^4*b)*log(abs(x)) - 1/3*(B*a^5*x^3 +
5*A*a^4*b*x^3 + A*a^5)/x^3
```

3.37 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} dx$

Optimal. Leaf size=113

$$2a^2b^2x^5(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)-\frac{a^4(aB+5Ab)}{x}-\frac{a^5A}{4x^4}+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a
*B)*x^11)/11 + (b^5*B*x^14)/14
```

Rubi [A] time = 0.0667424, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$2a^2b^2x^5(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)-\frac{a^4(aB+5Ab)}{x}-\frac{a^5A}{4x^4}+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a
*B)*x^11)/11 + (b^5*B*x^14)/14
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} dx &= \int \left(\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^2}+5a^3b(2Ab+aB)x+10a^2b^2(Ab+aB)x^4+5ab^3(Ab+2aB)x^7\right.\\ &= -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2+2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+2aB)x^8+\end{aligned}$$

Mathematica [A] time = 0.0344469, size = 115, normalized size = 1.02

$$2a^2b^2x^5(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)+\frac{a^5(-B)-5a^4Ab}{x}-\frac{a^5A}{4x^4}+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*
a*B)*x^11)/11 + (b^5*B*x^14)/14
```

Maple [A] time = 0.006, size = 123, normalized size = 1.1

$$\frac{b^5Bx^{14}}{14}+\frac{Ax^{11}b^5}{11}+\frac{5\,Bx^{11}ab^4}{11}+\frac{5\,Ax^8ab^4}{8}+\frac{5\,Bx^8a^2b^3}{4}+2\,Ax^5a^2b^3+2\,Bx^5a^3b^2+5\,Ax^2a^3b^2+\frac{5\,Bx^2a^4b}{2}-\frac{Aa^5}{4\,x^4}-\frac{a^4\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^5,x)
```

```
[Out] 1/14*b^5*B*x^14+1/11*A*x^11*b^5+5/11*B*x^11*a*b^4+5/8*A*x^8*a*b^4+5/4*B*x^8
*a^2*b^3+2*A*x^5*a^2*b^3+2*B*x^5*a^3*b^2+5*A*x^2*a^3*b^2+5/2*B*x^2*a^4*b-1/
4*a^5*A/x^4-a^4*(5*A*b+B*a)/x
```

Maxima [A] time = 1.14339, size = 163, normalized size = 1.44

$$\frac{1}{14}\,Bb^5x^{14}+\frac{1}{11}\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{5}{8}\left(2\,Ba^2b^3+Aab^4\right)x^8+2\left(Ba^3b^2+Aa^2b^3\right)x^5+\frac{5}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^2-\frac{Aa^5+4\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2 -
1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

Fricas [A] time = 1.41169, size = 275, normalized size = 2.43

$$\frac{44\,Bb^5x^{18} + 56\left(5\,Bab^4 + Ab^5\right)x^{15} + 385\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 1232\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 1540\left(Ba^4b + 2\,Aa^3b^2\right)x^6 -}{616\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/616*(44*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 385*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

Sympy [A] time = 0.545273, size = 131, normalized size = 1.16

$$\frac{Bb^5x^{14}}{14} + x^{11}\left(\frac{Ab^5}{11} + \frac{5Bab^4}{11}\right) + x^8\left(\frac{5Aab^4}{8} + \frac{5Ba^2b^3}{4}\right) + x^5\left(2Aa^2b^3 + 2Ba^3b^2\right) + x^2\left(5Aa^3b^2 + \frac{5Ba^4b}{2}\right) - \frac{Aa^5 + x^3\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 +
5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*b*
*2 + 5*B*a**4*b/2) - (A*a**5 + x**3*(20*A*a**4*b + 4*B*a**5))/(4*x**4)
```

Giac [A] time = 1.19314, size = 171, normalized size = 1.51

$$\frac{1}{14}\,Bb^5x^{14} + \frac{5}{11}\,Bab^4x^{11} + \frac{1}{11}\,Ab^5x^{11} + \frac{5}{4}\,Ba^2b^3x^8 + \frac{5}{8}\,Aab^4x^8 + 2\,Ba^3b^2x^5 + 2\,Aa^2b^3x^5 + \frac{5}{2}\,Ba^4bx^2 + 5\,Aa^3b^2x^2 - \frac{4\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 5/4*B*a^2*b^3*x^8 +
 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4
```

3.38 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^6}\,dx$

Optimal. Leaf size=113

$$\frac{5}{2}a^2b^2x^4(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)-\frac{a^5A}{5x^5}+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Rubi [A] time = 0.062649, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{5}{2}a^2b^2x^4(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)-\frac{a^5A}{5x^5}+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^6}\,dx=\int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^3}+10a^2b^2(Ab+aB)x^3+5ab^3(Ab+2aB)x^6+\right.$$
$$=-\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{1}$$

Mathematica [A] time = 0.0355137, size = 113, normalized size = 1.

$$\frac{5}{2}a^2b^2x^4(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)-\frac{a^5A}{5x^5}+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Maple [A] time = 0.006, size = 119, normalized size = 1.1

$$\frac{b^5Bx^{13}}{13}+\frac{Ax^{10}b^5}{10}+\frac{Bx^{10}ab^4}{2}+\frac{5\,Ax^7ab^4}{7}+\frac{10\,Bx^7a^2b^3}{7}+\frac{5\,Ax^4a^2b^3}{2}+\frac{5\,Bx^4a^3b^2}{2}+10\,a^3b^2Ax+5\,a^4bBx-\frac{Aa^5}{5\,x^5}-\frac{a^4(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^6,x)
```

```
[Out] 1/13*b^5*B*x^13+1/10*A*x^10*b^5+1/2*B*x^10*a*b^4+5/7*A*x^7*a*b^4+10/7*B*x^7
*a^2*b^3+5/2*A*x^4*a^2*b^3+5/2*B*x^4*a^3*b^2+10*a^3*b^2*A*x+5*a^4*b*B*x-1/5
*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2
```

Maxima [A] time = 1.00114, size = 162, normalized size = 1.43

$$\frac{1}{13}\,Bb^5x^{13}+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{7}\left(2\,Ba^2b^3+Aab^4\right)x^7+\frac{5}{2}\left(Ba^3b^2+Aa^2b^3\right)x^4+5\left(Ba^4b+2\,Aa^3b^2\right)x-\frac{2\,Aa^5+5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b^
4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/
10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Fricas [A] time = 1.36795, size = 275, normalized size = 2.43

$$\frac{70\,Bb^5x^{18} + 91\left(5\,Bab^4 + Ab^5\right)x^{15} + 650\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 2275\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 4550\left(Ba^4b + 2\,Aa^3b^2\right)x^6 - 1}{910\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/910*(70*B*b^5*x^18 + 91*(5*B*a*b^4 + A*b^5)*x^15 + 650*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 4550*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 182*A*a^5 - 455*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Sympy [A] time = 0.596036, size = 131, normalized size = 1.16

$$\frac{Bb^5x^{13}}{13} + x^{10}\left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^7\left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^4\left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x\left(10Aa^3b^2 + 5Ba^4b\right) - \frac{2Aa^5 + x^3\left(2\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10*
B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*b*
*2 + 5*B*a**4*b) - (2*A*a**5 + x**3*(25*A*a**4*b + 5*B*a**5))/(10*x**5)
```

Giac [A] time = 1.20654, size = 167, normalized size = 1.48

$$\frac{1}{13}\,Bb^5x^{13} + \frac{1}{2}\,Bab^4x^{10} + \frac{1}{10}\,Ab^5x^{10} + \frac{10}{7}\,Ba^2b^3x^7 + \frac{5}{7}\,Aab^4x^7 + \frac{5}{2}\,Ba^3b^2x^4 + \frac{5}{2}\,Aa^2b^3x^4 + 5\,Ba^4bx + 10\,Aa^3b^2x - \frac{5\,Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/7*B*a^2*b^3*x^7 +
 5/7*A*a*b^4*x^7 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5*B*a^4*b*x + 10
*A*a^3*b^2*x - 1/10*(5*B*a^5*x^3 + 25*A*a^4*b*x^3 + 2*A*a^5)/x^5
```

3.39 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7} \, dx$

Optimal. Leaf size=114

$$\frac{10}{3}a^2b^2x^3(aB+Ab) - \frac{a^4(aB+5Ab)}{3x^3} + 5a^3b\log(x)(aB+2Ab) - \frac{a^5A}{6x^6} + \frac{1}{9}b^4x^9(5aB+Ab) + \frac{5}{6}ab^3x^6(2aB+Ab) + \frac{1}{12}b^5$$

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(3*x^3) + (10*a^2*b^2*(A*b + a*B)*x^
3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x
^12)/12 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.1058, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {446, 76}

$$\frac{10}{3}a^2b^2x^3(aB+Ab) - \frac{a^4(aB+5Ab)}{3x^3} + 5a^3b\log(x)(aB+2Ab) - \frac{a^5A}{6x^6} + \frac{1}{9}b^4x^9(5aB+Ab) + \frac{5}{6}ab^3x^6(2aB+Ab) + \frac{1}{12}b^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(3*x^3) + (10*a^2*b^2*(A*b + a*B)*x^
3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x
^12)/12 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^7}\,dx=\frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\operatorname{Subst}\left(\int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^3}+\frac{a^4(5Ab+aB)}{x^2}+\frac{5a^3b(2Ab+aB)}{x}+5ab^3(Ab+2a\right.\right.$$
$$=-\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{9}b^4(Ab+5aB)x^9+$$

Mathematica [A] time = 0.0462325, size = 106, normalized size = 0.93

$$\frac{1}{36}\left(120a^2b^2x^3(aB+Ab)-\frac{12a^4(aB+5Ab)}{x^3}+180a^3b\log(x)(aB+2Ab)-\frac{6a^5A}{x^6}+4b^4x^9(5aB+Ab)+30ab^3x^6(2aB+Ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] ((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 180
*a^3*b*(2*A*b + a*B)*Log[x])/36
```

Maple [A] time = 0.009, size = 124, normalized size = 1.1

$$\frac{b^5Bx^{12}}{12}+\frac{Ax^9b^5}{9}+\frac{5\,Bx^9ab^4}{9}+\frac{5\,Ax^6ab^4}{6}+\frac{5\,Bx^6a^2b^3}{3}+\frac{10\,Ax^3a^2b^3}{3}+\frac{10\,Bx^3a^3b^2}{3}-\frac{Aa^5}{6\,x^6}-\frac{5\,a^4bA}{3\,x^3}-\frac{a^5B}{3\,x^3}+10\,A\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^7,x)
```

```
[Out] 1/12*b^5*B*x^12+1/9*A*x^9*b^5+5/9*B*x^9*a*b^4+5/6*A*x^6*a*b^4+5/3*B*x^6*a^2
*b^3+10/3*A*x^3*a^2*b^3+10/3*B*x^3*a^3*b^2-1/6*a^5*A/x^6-5/3*a^4/x^3*A*b-1/
3*a^5/x^3*B+10*A*ln(x)*a^3*b^2+5*B*ln(x)*a^4*b
```

Maxima [A] time = 1.33352, size = 165, normalized size = 1.45

$$\frac{1}{12}Bb^5x^{12}+\frac{1}{9}\left(5\,Bab^4+Ab^5\right)x^9+\frac{5}{6}\left(2\,Ba^2b^3+Aab^4\right)x^6+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3+\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)\log\left(x^3\right)-\frac{Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*log(x
^3) - 1/6*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Fricas [A] time = 1.47394, size = 273, normalized size = 2.39

$$\frac{3\,Bb^5x^{18}+4\left(5\,Bab^4+Ab^5\right)x^{15}+30\left(2\,Ba^2b^3+Aab^4\right)x^{12}+120\left(Ba^3b^2+Aa^2b^3\right)x^9+180\left(Ba^4b+2\,Aa^3b^2\right)x^6\log\left(x\right)-}{36\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^6*
log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Sympy [A] time = 1.04359, size = 129, normalized size = 1.13

$$\frac{Bb^5x^{12}}{12}+5a^3b\left(2Ab+Ba\right)\log\left(x\right)+x^9\left(\frac{Ab^5}{9}+\frac{5Bab^4}{9}\right)+x^6\left(\frac{5Aab^4}{6}+\frac{5Ba^2b^3}{3}\right)+x^3\left(\frac{10Aa^2b^3}{3}+\frac{10Ba^3b^2}{3}\right)-\frac{Aa^5+x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**5*x**12/12 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**9*(A*b**5/9 + 5*B*a*b*
*4/9) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**3*(10*A*a**2*b**3/3 + 10
*B*a**3*b**2/3) - (A*a**5 + x**3*(10*A*a**4*b + 2*B*a**5))/(6*x**6)
```

Giac [A] time = 1.1803, size = 200, normalized size = 1.75

$$\frac{1}{12}Bb^5x^{12} + \frac{5}{9}Bab^4x^9 + \frac{1}{9}Ab^5x^9 + \frac{5}{3}Ba^2b^3x^6 + \frac{5}{6}Aab^4x^6 + \frac{10}{3}Ba^3b^2x^3 + \frac{10}{3}Aa^2b^3x^3 + 5\left(Ba^4b + 2Aa^3b^2\right)\log\left(|x|\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*(B*a^4*b + 2*A*a
^3*b^2)*log(abs(x)) - 1/6*(15*B*a^4*b*x^6 + 30*A*a^3*b^2*x^6 + 2*B*a^5*x^3
+ 10*A*a^4*b*x^3 + A*a^5)/x^6
```

3.40 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx$

Optimal. Leaf size=110

$$5a^2b^2x^2(aB+Ab) - \frac{a^4(aB+5Ab)}{4x^4} - \frac{5a^3b(aB+2Ab)}{x} - \frac{a^5A}{7x^7} + \frac{1}{8}b^4x^8(5aB+Ab) + ab^3x^5(2aB+Ab) + \frac{1}{11}b^5Bx^{11}$$

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Rubi [A] time = 0.0671996, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$5a^2b^2x^2(aB+Ab) - \frac{a^4(aB+5Ab)}{4x^4} - \frac{5a^3b(aB+2Ab)}{x} - \frac{a^5A}{7x^7} + \frac{1}{8}b^4x^8(5aB+Ab) + ab^3x^5(2aB+Ab) + \frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = \int \left(\frac{a^5A}{x^8} + \frac{a^4(5Ab+aB)}{x^5} + \frac{5a^3b(2Ab+aB)}{x^2} + 10a^2b^2(Ab+aB)x + 5ab^3(Ab+2aB)x^4 +\right.$$

$$= -\frac{a^5A}{7x^7} - \frac{a^4(5Ab+aB)}{4x^4} - \frac{5a^3b(2Ab+aB)}{x} + 5a^2b^2(Ab+aB)x^2 + ab^3(Ab+2aB)x^5 + \frac{1}{8}b$$

Mathematica [A] time = 0.0333246, size = 110, normalized size = 1.

$$5a^2b^2x^2(aB+Ab)-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}-\frac{a^5A}{7x^7}+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Maple [A] time = 0.007, size = 117, normalized size = 1.1

$$\frac{b^5Bx^{11}}{11}+\frac{Ax^8b^5}{8}+\frac{5\,Bx^8ab^4}{8}+Ax^5ab^4+2\,Bx^5a^2b^3+5\,Ax^2a^2b^3+5\,Bx^2a^3b^2-\frac{a^4\,(5\,Ab+Ba)}{4\,x^4}-5\,\frac{a^3b\,(2\,Ab+Ba)}{x}-\frac{Aa}{7\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^8,x)
```

```
[Out] 1/11*b^5*B*x^11+1/8*A*x^8*b^5+5/8*B*x^8*a*b^4+A*x^5*a*b^4+2*B*x^5*a^2*b^3+5
*A*x^2*a^2*b^3+5*B*x^2*a^3*b^2-1/4*a^4*(5*A*b+B*a)/x^4-5*a^3*b*(2*A*b+B*a)/
x-1/7*a^5*A/x^7
```

Maxima [A] time = 1.05217, size = 163, normalized size = 1.48

$$\frac{1}{11}\,Bb^5x^{11}+\frac{1}{8}\left(5\,Bab^4+Ab^5\right)x^8+\left(2\,Ba^2b^3+Aab^4\right)x^5+5\left(Ba^3b^2+Aa^2b^3\right)x^2-\frac{140\left(Ba^4b+2\,Aa^3b^2\right)x^6+4\,Aa^5+7\left(B\right.}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^5
 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6 +
4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

Fricas [A] time = 1.43642, size = 274, normalized size = 2.49

$$\frac{56\,Bb^5x^{18} + 77\left(5\,Bab^4 + Ab^5\right)x^{15} + 616\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 3080\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 3080\left(Ba^4b + 2\,Aa^3b^2\right)x^6 -}{616\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/616*(56*B*b^5*x^18 + 77*(5*B*a*b^4 + A*b^5)*x^15 + 616*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

Sympy [A] time = 1.22447, size = 126, normalized size = 1.15

$$\frac{Bb^5x^{11}}{11} + x^8\left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^5\left(Aab^4 + 2Ba^2b^3\right) + x^2\left(5Aa^2b^3 + 5Ba^3b^2\right) - \frac{4Aa^5 + x^6\left(280Aa^3b^2 + 140Ba^4b\right) + x^3\left(\right.}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a**
2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) - (4*A*a**5 + x**6*(280*A*a*
*3*b**2 + 140*B*a**4*b) + x**3*(35*A*a**4*b + 7*B*a**5))/(28*x**7)
```

Giac [A] time = 1.15847, size = 171, normalized size = 1.55

$$\frac{1}{11}\,Bb^5x^{11} + \frac{5}{8}\,Bab^4x^8 + \frac{1}{8}\,Ab^5x^8 + 2\,Ba^2b^3x^5 + Aab^4x^5 + 5\,Ba^3b^2x^2 + 5\,Aa^2b^3x^2 - \frac{140\,Ba^4bx^6 + 280\,Aa^3b^2x^6 + 7\,Ba^5}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*b
^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*A*
a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7
```

3.41 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^9} \, dx$

Optimal. Leaf size=113

$$10a^2b^2x(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{a^5A}{8x^8}+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Rubi [A] time = 0.0623747, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$10a^2b^2x(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{a^5A}{8x^8}+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^9}\,dx &= \int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^9}+\frac{a^4(5Ab+aB)}{x^6}+\frac{5a^3b(2Ab+aB)}{x^3}+5ab^3(Ab+2aB)x^3+b^4\right.\\ &= -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b\end{aligned}$$

Mathematica [A] time = 0.0327947, size = 113, normalized size = 1.

$$10a^2b^2x(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{a^5A}{8x^8}+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Maple [A] time = 0.004, size = 114, normalized size = 1.

$$\frac{b^5Bx^{10}}{10}+\frac{Ax^7b^5}{7}+\frac{5\,Bx^7ab^4}{7}+\frac{5\,Ax^4ab^4}{4}+\frac{5\,Bx^4a^2b^3}{2}+10\,a^2b^3Ax+10\,a^3b^2Bx-\frac{a^4\,(5\,Ab+Ba)}{5\,x^5}-\frac{Aa^5}{8\,x^8}-\frac{5\,a^3b\,(2\,Ab}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^9,x)
```

```
[Out] 1/10*b^5*B*x^10+1/7*A*x^7*b^5+5/7*B*x^7*a*b^4+5/4*A*x^4*a*b^4+5/2*B*x^4*a^2
*b^3+10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/5*a^4*(5*A*b+B*a)/x^5-1/8*a^5*A/x^8-5/
2*a^3*b*(2*A*b+B*a)/x^2
```

Maxima [A] time = 1.22319, size = 162, normalized size = 1.43

$$\frac{1}{10}\,Bb^5x^{10}+\frac{1}{7}\left(5\,Bab^4+Ab^5\right)x^7+\frac{5}{4}\left(2\,Ba^2b^3+Aab^4\right)x^4+10\left(Ba^3b^2+Aa^2b^3\right)x-\frac{100\left(Ba^4b+2\,Aa^3b^2\right)x^6+5\,Aa^5+}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4)
*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x^6
 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Fricas [A] time = 1.42995, size = 271, normalized size = 2.4

$$\frac{28\,Bb^5x^{18} + 40\left(5\,Bab^4 + Ab^5\right)x^{15} + 350\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 2800\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 700\left(Ba^4b + 2\,Aa^3b^2\right)x^6 - 35}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/280*(28*B*b^5*x^18 + 40*(5*B*a*b^4 + A*b^5)*x^15 + 350*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 700*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 35*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Sympy [A] time = 1.28377, size = 129, normalized size = 1.14

$$\frac{Bb^5x^{10}}{10} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^4\left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2}\right) + x\left(10Aa^2b^3 + 10Ba^3b^2\right) - \frac{5Aa^5 + x^6\left(200Aa^3b^2 + 100Ba^4b\right) + x}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**5*x**10/10 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**4*(5*A*a*b**4/4 + 5*B
*a**2*b**3/2) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) - (5*A*a**5 + x**6*(200
*A*a**3*b**2 + 100*B*a**4*b) + x**3*(40*A*a**4*b + 8*B*a**5))/(40*x**8)
```

Giac [A] time = 1.16013, size = 167, normalized size = 1.48

$$\frac{1}{10}\,Bb^5x^{10} + \frac{5}{7}\,Bab^4x^7 + \frac{1}{7}\,Ab^5x^7 + \frac{5}{2}\,Ba^2b^3x^4 + \frac{5}{4}\,Aab^4x^4 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{100\,Ba^4bx^6 + 200\,Aa^3b^2x^6 + 8\,Ba}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/2*B*a^2*b^3*x^4 + 5/4
*A*a*b^4*x^4 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/40*(100*B*a^4*b*x^6 + 20
0*A*a^3*b^2*x^6 + 8*B*a^5*x^3 + 40*A*a^4*b*x^3 + 5*A*a^5)/x^8
```

3.42 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} dx$

Optimal. Leaf size=114

$$10a^2b^2\log(x)(aB+Ab)-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{a^5A}{9x^9}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*
x^9)/9 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rubi [A] time = 0.104487, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$10a^2b^2\log(x)(aB+Ab)-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{a^5A}{9x^9}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*
x^9)/9 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

qQ[p, 1])

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{10}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^2}+\frac{10a^2b^2(Ab+\right.\right.$$
$$= -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{9}b^5$$

Mathematica [A] time = 0.0509723, size = 106, normalized size = 0.93

$$\frac{1}{18}\left(180a^2b^2\log(x)(aB+Ab)-\frac{30a^3b(aB+2Ab)}{x^3}-\frac{3a^4(aB+5Ab)}{x^6}-\frac{2a^5A}{x^9}+3b^4x^6(5aB+Ab)+30ab^3x^3(2aB+Ab)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] ((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 180*
a^2*b^2*(A*b + a*B)*Log[x])/18
```

Maple [A] time = 0.008, size = 124, normalized size = 1.1

$$\frac{b^5Bx^9}{9}+\frac{Ax^6b^5}{6}+\frac{5\,Bx^6ab^4}{6}+\frac{5\,Ax^3ab^4}{3}+\frac{10\,Bx^3a^2b^3}{3}-\frac{Aa^5}{9\,x^9}-\frac{10\,a^3b^2A}{3\,x^3}-\frac{5\,a^4bB}{3\,x^3}-\frac{5\,a^4bA}{6\,x^6}-\frac{a^5B}{6\,x^6}+10\,A\ln(x)\,a^2b^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^10,x)
```

```
[Out] 1/9*b^5*B*x^9+1/6*A*x^6*b^5+5/6*B*x^6*a*b^4+5/3*A*x^3*a*b^4+10/3*B*x^3*a^2*
b^3-1/9*a^5*A/x^9-10/3*a^3*b^2/x^3*A-5/3*a^4*b/x^3*B-5/6*a^4/x^6*A*b-1/6*a^
5/x^6*B+10*A*ln(x)*a^2*b^3+10*B*ln(x)*a^3*b^2
```

Maxima [A] time = 1.33105, size = 166, normalized size = 1.46

$$\frac{1}{9}Bb^5x^9 + \frac{1}{6}\left(5\,Bab^4 + Ab^5\right)x^6 + \frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)x^3 + \frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)\log\left(x^3\right) - \frac{30\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + 2\,Aa}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x
^3 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*log(x^3) - 1/18*(30*(B*a^4*b + 2*A*a^3*b^
2)*x^6 + 2*A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Fricas [A] time = 1.49055, size = 270, normalized size = 2.37

$$\frac{2\,Bb^5x^{18} + 3\left(5\,Bab^4 + Ab^5\right)x^{15} + 30\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 180\left(Ba^3b^2 + Aa^2b^3\right)x^9\log\left(x\right) - 30\left(Ba^4b + 2\,Aa^3b^2\right)x^6 -}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Sympy [A] time = 2.50999, size = 126, normalized size = 1.11

$$\frac{Bb^5x^9}{9} + 10a^2b^2\left(Ab + Ba\right)\log\left(x\right) + x^6\left(\frac{Ab^5}{6} + \frac{5Bab^4}{6}\right) + x^3\left(\frac{5Aab^4}{3} + \frac{10Ba^2b^3}{3}\right) - \frac{2Aa^5 + x^6\left(60Aa^3b^2 + 30Ba^4b\right) +}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)
```

```
[Out] B*b**5*x**9/9 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b*
*4/6) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) - (2*A*a**5 + x**6*(60*A*a**
3*b**2 + 30*B*a**4*b) + x**3*(15*A*a**4*b + 3*B*a**5))/(18*x**9)
```

Giac [A] time = 1.16011, size = 203, normalized size = 1.78

$$\frac{1}{9}Bb^5x^9 + \frac{5}{6}Bab^4x^6 + \frac{1}{6}Ab^5x^6 + \frac{10}{3}Ba^2b^3x^3 + \frac{5}{3}Aab^4x^3 + 10\left(Ba^3b^2 + Aa^2b^3\right)\log\left(|x|\right) - \frac{110\,Ba^3b^2x^9 + 110\,Aa^2b^3x^9 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 10/3*B*a^2*b^3*x^3 + 5/3*
A*a*b^4*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/18*(110*B*a^3*b^2*
x^9 + 110*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 3*B*a^5*x^3 +
 15*A*a^4*b*x^3 + 2*A*a^5)/x^9
```

3.43 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}} dx$

Optimal. Leaf size=115

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^5A}{10x^{10}}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))/
(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4
*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

Rubi [A] time = 0.0631634, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^5A}{10x^{10}}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))/
(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4
*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}} dx = \int \left(\frac{a^5A}{x^{11}}+\frac{a^4(5Ab+aB)}{x^8}+\frac{5a^3b(2Ab+aB)}{x^5}+\frac{10a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+2aB)x+b\right.$$

$$=-\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{10a^2b^2(Ab+aB)}{x}+\frac{5}{2}ab^3(Ab+2aB)x^2+$$

Mathematica [A] time = 0.0231462, size = 118, normalized size = 1.03

$$\frac{1400a^2b^3x^9\left(Bx^3-2A\right)-700a^3b^2x^6\left(A+4Bx^3\right)-50a^4bx^3\left(4A+7Bx^3\right)-4a^5\left(7A+10Bx^3\right)+140ab^4x^{12}\left(5A+2Bx^3\right)+}{280x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] (1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^3
*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A + 7
*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)
```

Maple [A] time = 0.007, size = 111, normalized size = 1.

$$\frac{b^5Bx^8}{8}+\frac{Ax^5b^5}{5}+Bx^5ab^4+\frac{5\,Ax^2ab^4}{2}+5\,Bx^2a^2b^3-\frac{5\,a^3b\,(2\,Ab+Ba)}{4\,x^4}-\frac{Aa^5}{10\,x^{10}}-10\,\frac{a^2b^2\,(Ab+Ba)}{x}-\frac{a^4\,(5\,Ab+Ba)}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^11,x)
```

```
[Out] 1/8*b^5*B*x^8+1/5*A*x^5*b^5+B*x^5*a*b^4+5/2*A*x^2*a*b^4+5*B*x^2*a^2*b^3-5/4
*a^3*b*(2*A*b+B*a)/x^4-1/10*a^5*A/x^10-10*a^2*b^2*(A*b+B*a)/x-1/7*a^4*(5*A*
b+B*a)/x^7
```

Maxima [A] time = 1.14326, size = 165, normalized size = 1.43

$$\frac{1}{8}\,Bb^5x^8+\frac{1}{5}\left(5\,Bab^4+Ab^5\right)x^5+\frac{5}{2}\left(2\,Ba^2b^3+Aab^4\right)x^2-\frac{1400\left(Ba^3b^2+Aa^2b^3\right)x^9+175\left(Ba^4b+2\,Aa^3b^2\right)x^6+14\,Aa^5+}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x
^2 - 1/140*(1400*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 175*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 14*A*a^5 + 20*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

Fricas [A] time = 1.41073, size = 273, normalized size = 2.37

$$\frac{35\,Bb^{5}x^{18}+56\left(5\,Bab^{4}+Ab^{5}\right)x^{15}+700\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{12}-2800\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{9}-350\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{6}-2}{280\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] 1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*a
*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

Sympy [A] time = 3.31842, size = 126, normalized size = 1.1

$$\frac{Bb^{5}x^{8}}{8}+x^{5}\left(\frac{Ab^{5}}{5}+Bab^{4}\right)+x^{2}\left(\frac{5Aab^{4}}{2}+5Ba^{2}b^{3}\right)-\frac{14Aa^{5}+x^{9}\left(1400Aa^{2}b^{3}+1400Ba^{3}b^{2}\right)+x^{6}\left(350Aa^{3}b^{2}+175Ba^{4}b\right.}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)
```

```
[Out] B*b**5*x**8/8 + x**5*(A*b**5/5 + B*a*b**4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*
b**3) - (14*A*a**5 + x**9*(1400*A*a**2*b**3 + 1400*B*a**3*b**2) + x**6*(350
*A*a**3*b**2 + 175*B*a**4*b) + x**3*(100*A*a**4*b + 20*B*a**5))/(140*x**10)
```

Giac [A] time = 1.2071, size = 171, normalized size = 1.49

$$\frac{1}{8}\,Bb^{5}x^{8}+Bab^{4}x^{5}+\frac{1}{5}\,Ab^{5}x^{5}+5\,Ba^{2}b^{3}x^{2}+\frac{5}{2}\,Aab^{4}x^{2}-\frac{1400\,Ba^{3}b^{2}x^{9}+1400\,Aa^{2}b^{3}x^{9}+175\,Ba^{4}bx^{6}+350\,Aa^{3}b^{2}x^{6}+}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^4
*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 + 3
50*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10
```

3.44 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{12}}\,dx$

Optimal. Leaf size=109

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^5A}{11x^{11}}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Rubi [A] time = 0.064277, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^5A}{11x^{11}}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{12}}\,dx=\int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^9}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{10a^2b^2(Ab+aB)}{x^3}+b^4(\right.$$

$$=-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+2aB)x+\frac{1}{4}b^4(A$$

Mathematica [A] time = 0.0360838, size = 109, normalized size = 1.

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^5A}{11x^{11}}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Maple [A] time = 0.007, size = 108, normalized size = 1.

$$\frac{b^5Bx^7}{7}+\frac{Ax^4b^5}{4}+\frac{5\,Bx^4ab^4}{4}+5\,ab^4Ax+10\,a^2b^3Bx-\frac{a^4\,(5\,Ab+Ba)}{8\,x^8}-\frac{a^3b\,(2\,Ab+Ba)}{x^5}-5\,\frac{a^2b^2\,(Ab+Ba)}{x^2}-\frac{Aa^5}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^12,x)
```

```
[Out] 1/7*b^5*B*x^7+1/4*A*x^4*b^5+5/4*B*x^4*a*b^4+5*a*b^4*A*x+10*a^2*b^3*B*x-1/8*
a^4*(5*A*b+B*a)/x^8-a^3*b*(2*A*b+B*a)/x^5-5*a^2*b^2*(A*b+B*a)/x^2-1/11*a^5*
A/x^11
```

Maxima [A] time = 1.18392, size = 162, normalized size = 1.49

$$\frac{1}{7}\,Bb^5x^7+\frac{1}{4}\left(5\,Bab^4+Ab^5\right)x^4+5\left(2\,Ba^2b^3+Aab^4\right)x-\frac{440\left(Ba^3b^2+Aa^2b^3\right)x^9+88\left(Ba^4b+2\,Aa^3b^2\right)x^6+8\,Aa^5+11}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 8
*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Fricas [A] time = 1.41697, size = 275, normalized size = 2.52

$$\frac{88\,Bb^5x^{18}+154\left(5\,Bab^4+Ab^5\right)x^{15}+3080\left(2\,Ba^2b^3+Aab^4\right)x^{12}-3080\left(Ba^3b^2+Aa^2b^3\right)x^9-616\left(Ba^4b+2\,Aa^3b^2\right)x^6-5}{616\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")
```

```
[Out] 1/616*(88*B*b^5*x^18 + 154*(5*B*a*b^4 + A*b^5)*x^15 + 3080*(2*B*a^2*b^3 + A
*a*b^4)*x^12 - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Sympy [A] time = 3.83957, size = 126, normalized size = 1.16

$$\frac{Bb^5x^7}{7}+x^4\left(\frac{Ab^5}{4}+\frac{5Bab^4}{4}\right)+x\left(5Aab^4+10Ba^2b^3\right)-\frac{8Aa^5+x^9\left(440Aa^2b^3+440Ba^3b^2\right)+x^6\left(176Aa^3b^2+88Ba^4b\right)+x}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)
```

```
[Out] B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) - (8*A*a**5 + x**9*(440*A*a**2*b**3 + 440*B*a**3*b**2) + x**6*(176*A*
a**3*b**2 + 88*B*a**4*b) + x**3*(55*A*a**4*b + 11*B*a**5))/(88*x**11)
```

Giac [A] time = 1.1897, size = 167, normalized size = 1.53

$$\frac{1}{7}\,Bb^5x^7+\frac{5}{4}\,Bab^4x^4+\frac{1}{4}\,Ab^5x^4+10\,Ba^2b^3x+5\,Aab^4x-\frac{440\,Ba^3b^2x^9+440\,Aa^2b^3x^9+88\,Ba^4bx^6+176\,Aa^3b^2x^6+11\,B}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*A*
a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^11
```

3.45 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{13}}\,dx$

Optimal. Leaf size=114

$$-\frac{10a^2b^2(aB+Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{a^5A}{12x^{12}}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

```
[Out] -(a^5*A)/(12*x^12) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (10*a^2*b^2*(A*b + a*B))/(3*x^3) + (b^4*(A*b + 5*a*B)*x^3)/3 + (b
^5*B*x^6)/6 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rubi [A] time = 0.0987126, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{10a^2b^2(aB+Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{a^5A}{12x^{12}}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -(a^5*A)/(12*x^12) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (10*a^2*b^2*(A*b + a*B))/(3*x^3) + (b^4*(A*b + 5*a*B)*x^3)/3 + (b
^5*B*x^6)/6 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{13}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(b^4(Ab+5aB)+\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^3}+\frac{10a^2b^2(Ab+aB}{x^2}\right.\right.$$
$$= -\frac{a^5A}{12x^{12}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{10a^2b^2(Ab+aB)}{3x^3}+\frac{1}{3}b^4(Ab+5aB)x^3+\frac{1}{6}b^5$$

Mathematica [A] time = 0.0382892, size = 118, normalized size = 1.04

$$-\frac{60a^3b^2x^6\left(A+2Bx^3\right)+120a^2Ab^3x^9+10a^4bx^3\left(2A+3Bx^3\right)+a^5\left(3A+4Bx^3\right)-180ab^3x^{12}\log(x)(2aB+Ab)-60ab^4B}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60*a^3*b
^2*x^6*(A + 2*B*x^3) + 10*a^4*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B*x^3) -
 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/(36*x^12)
```

Maple [A] time = 0.007, size = 124, normalized size = 1.1

$$\frac{b^5Bx^6}{6}+\frac{Ax^3b^5}{3}+\frac{5\,Bx^3ab^4}{3}-\frac{Aa^5}{12\,x^{12}}-\frac{5\,a^4bA}{9\,x^9}-\frac{a^5B}{9\,x^9}-\frac{5\,a^3b^2A}{3\,x^6}-\frac{5\,a^4bB}{6\,x^6}-\frac{10\,a^2b^3A}{3\,x^3}-\frac{10\,a^3b^2B}{3\,x^3}+5\,A\ln(x)\,ab^4+10$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^13,x)
```

```
[Out] 1/6*b^5*B*x^6+1/3*A*x^3*b^5+5/3*B*x^3*a*b^4-1/12*a^5*A/x^12-5/9*a^4/x^9*A*b
-1/9*a^5/x^9*B-5/3*a^3*b^2/x^6*A-5/6*a^4*b/x^6*B-10/3*a^2*b^3/x^3*A-10/3*a^
3*b^2/x^3*B+5*A*ln(x)*a*b^4+10*B*ln(x)*a^2*b^3
```

Maxima [A] time = 1.28447, size = 166, normalized size = 1.46

$$\frac{1}{6}Bb^5x^6 + \frac{1}{3}\left(5\,Bab^4 + Ab^5\right)x^3 + \frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)\log\left(x^3\right) - \frac{120\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 30\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + 3\,A}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^3) - 1/36*(120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 3*A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Fricas [A] time = 1.44972, size = 274, normalized size = 2.4

$$\frac{6\,Bb^5x^{18} + 12\left(5\,Bab^4 + Ab^5\right)x^{15} + 180\left(2\,Ba^2b^3 + Aab^4\right)x^{12}\log\left(x\right) - 120\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 30\left(Ba^4b + 2\,Aa^3b^2\right)x^6}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*b
^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Sympy [A] time = 7.53096, size = 124, normalized size = 1.09

$$\frac{Bb^5x^6}{6} + 5ab^3\left(Ab + 2Ba\right)\log\left(x\right) + x^3\left(\frac{Ab^5}{3} + \frac{5Bab^4}{3}\right) - \frac{3Aa^5 + x^9\left(120Aa^2b^3 + 120Ba^3b^2\right) + x^6\left(60Aa^3b^2 + 30Ba^4b\right)}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)
```

```
[Out] B*b**5*x**6/6 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**4
/3) - (3*A*a**5 + x**9*(120*A*a**2*b**3 + 120*B*a**3*b**2) + x**6*(60*A*a**
3*b**2 + 30*B*a**4*b) + x**3*(20*A*a**4*b + 4*B*a**5))/(36*x**12)
```

Giac [A] time = 1.16675, size = 201, normalized size = 1.76

$$\frac{1}{6}Bb^5x^6+\frac{5}{3}Bab^4x^3+\frac{1}{3}Ab^5x^3+5\left(2Ba^2b^3+Aab^4\right)\log\left(|x|\right)-\frac{250Ba^2b^3x^{12}+125Aab^4x^{12}+120Ba^3b^2x^9+120Aa^2b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)
*log(abs(x)) - 1/36*(250*B*a^2*b^3*x^12 + 125*A*a*b^4*x^12 + 120*B*a^3*b^2*
x^9 + 120*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 4*B*a^5*x^3 +
 20*A*a^4*b*x^3 + 3*A*a^5)/x^12
```

3.46 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}} dx$

Optimal. Leaf size=115

$$-\frac{5a^2b^2(aB+Ab)}{2x^4}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{a^5A}{13x^{13}}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x + (
b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

Rubi [A] time = 0.0621777, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{2x^4}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{a^5A}{13x^{13}}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x + (
b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}dx = \int \left(\frac{a^5A}{x^{14}}+\frac{a^4(5Ab+aB)}{x^{11}}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^5}+\frac{5ab^3(Ab+2aB)}{x^2}+b^4\right.$$
$$= -\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{10x^{10}}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{x}+\frac{1}{2}b^4$$

Mathematica [A] time = 0.0342063, size = 117, normalized size = 1.02

$$-\frac{2275a^2b^3x^9\left(A+4Bx^3\right)+325a^3b^2x^6\left(4A+7Bx^3\right)+65a^4bx^3\left(7A+10Bx^3\right)+a^5\left(70A+91Bx^3\right)-2275ab^4x^{12}\left(Bx^3-2\right.}{910x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -(-2275*a*b^4*x^12*(-2*A + B*x^3) - 91*b^5*x^15*(5*A + 2*B*x^3) + 2275*a^2*
b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*x^3*(7*A
 + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/(910*x^13)
```

Maple [A] time = 0.007, size = 107, normalized size = 0.9

$$\frac{b^5Bx^5}{5}+\frac{Ax^2b^5}{2}+\frac{5\,Bx^2ab^4}{2}-\frac{Aa^5}{13\,x^{13}}-\frac{a^4\,(5\,Ab+Ba)}{10\,x^{10}}-5\,\frac{ab^3\,(Ab+2\,Ba)}{x}-\frac{5\,a^2b^2\,(Ab+Ba)}{2\,x^4}-\frac{5\,a^3b\,(2\,Ab+Ba)}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^14,x)
```

```
[Out] 1/5*b^5*B*x^5+1/2*A*x^2*b^5+5/2*B*x^2*a*b^4-1/13*a^5*A/x^13-1/10*a^4*(5*A*b
+B*a)/x^10-5*a*b^3*(A*b+2*B*a)/x-5/2*a^2*b^2*(A*b+B*a)/x^4-5/7*a^3*b*(2*A*b
+B*a)/x^7
```

Maxima [A] time = 1.03738, size = 165, normalized size = 1.43

$$\frac{1}{5}\,Bb^5x^5+\frac{1}{2}\left(5\,Bab^4+Ab^5\right)x^2-\frac{4550\left(2\,Ba^2b^3+Aab^4\right)x^{12}+2275\left(Ba^3b^2+Aa^2b^3\right)x^9+650\left(Ba^4b+2\,Aa^3b^2\right)x^6+70\,A}{910\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 - 1/910*(4550*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 650*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 70*A*a^5 + 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

Fricas [A] time = 1.38707, size = 277, normalized size = 2.41

$$\frac{182\,Bb^5x^{18} + 455\left(5\,Bab^4 + Ab^5\right)x^{15} - 4550\left(2\,Ba^2b^3 + Aab^4\right)x^{12} - 2275\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 650\left(Ba^4b + 2\,Aa^3b^2\right)x^6}{910\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")
```

```
[Out] 1/910*(182*B*b^5*x^18 + 455*(5*B*a*b^4 + A*b^5)*x^15 - 4550*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

Sympy [A] time = 18.3993, size = 128, normalized size = 1.11

$$\frac{Bb^5x^5}{5} + x^2\left(\frac{Ab^5}{2} + \frac{5Bab^4}{2}\right) - \frac{70Aa^5 + x^{12}\left(4550Aab^4 + 9100Ba^2b^3\right) + x^9\left(2275Aa^2b^3 + 2275Ba^3b^2\right) + x^6\left(1300Aa^3b^2\right.}{910x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)
```

```
[Out] B*b**5*x**5/5 + x**2*(A*b**5/2 + 5*B*a*b**4/2) - (70*A*a**5 + x**12*(4550*A
*a*b**4 + 9100*B*a**2*b**3) + x**9*(2275*A*a**2*b**3 + 2275*B*a**3*b**2) +
x**6*(1300*A*a**3*b**2 + 650*B*a**4*b) + x**3*(455*A*a**4*b + 91*B*a**5))/(
910*x**13)
```

Giac [A] time = 1.2257, size = 173, normalized size = 1.5

$$\frac{1}{5}\,Bb^5x^5 + \frac{5}{2}\,Bab^4x^2 + \frac{1}{2}\,Ab^5x^2 - \frac{9100\,Ba^2b^3x^{12} + 4550\,Aab^4x^{12} + 2275\,Ba^3b^2x^9 + 2275\,Aa^2b^3x^9 + 650\,Ba^4bx^6 + 130}{910\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^1
2 + 4550*A*a*b^4*x^12 + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a^4
```

```
*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)/x^
13
```

3.47 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} \, dx$

Optimal. Leaf size=110

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{a^5A}{14x^{14}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4x(5aB+Ab)+\frac{1}{4}b^5Bx^4$$

```
[Out] -(a^5*A)/(14*x^14) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Rubi [A] time = 0.0677994, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{a^5A}{14x^{14}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4x(5aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -(a^5*A)/(14*x^14) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} \, dx = \int \left(b^4(Ab+5aB)+\frac{a^5A}{x^{15}}+\frac{a^4(5Ab+aB)}{x^{12}}+\frac{5a^3b(2Ab+aB)}{x^9}+\frac{10a^2b^2(Ab+aB)}{x^6}+\frac{5ab^3}{}\right.$$

$$= -\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(A$$

Mathematica [A] time = 0.0412247, size = 110, normalized size = 1.

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{a^5A}{14x^{14}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4x(5aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -(a^5*A)/(14*x^14) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Maple [A] time = 0.009, size = 102, normalized size = 0.9

$$\frac{b^5Bx^4}{4}+b^5Ax+5\,ab^4Bx-\frac{a^4\,(5\,Ab+Ba)}{11\,x^{11}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{2\,x^2}-\frac{Aa^5}{14\,x^{14}}-2\,\frac{a^2b^2\,(Ab+Ba)}{x^5}-\frac{5\,a^3b\,(2\,Ab+Ba)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^15,x)
```

```
[Out] 1/4*b^5*B*x^4+b^5*A*x+5*a*b^4*B*x-1/11*a^4*(5*A*b+B*a)/x^11-5/2*a*b^3*(A*b+
2*B*a)/x^2-1/14*a^5*A/x^14-2*a^2*b^2*(A*b+B*a)/x^5-5/8*a^3*b*(2*A*b+B*a)/x^
8
```

Maxima [A] time = 1.39243, size = 161, normalized size = 1.46

$$\frac{1}{4}\,Bb^5x^4+\left(5\,Bab^4+Ab^5\right)x-\frac{1540\left(2\,Ba^2b^3+Aab^4\right)x^{12}+1232\left(Ba^3b^2+Aa^2b^3\right)x^9+385\left(Ba^4b+2\,Aa^3b^2\right)x^6+44\,Aa^5}{616\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + (5*B*a*b^4 + A*b^5)*x - 1/616*(1540*(2*B*a^2*b^3 + A*a*b^4)
*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 385*(B*a^4*b + 2*A*a^3*b^2)*x^6
+ 44*A*a^5 + 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Fricas [A] time = 1.4132, size = 277, normalized size = 2.52

$$\frac{154\,Bb^5x^{18} + 616\left(5\,Bab^4 + Ab^5\right)x^{15} - 1540\left(2\,Ba^2b^3 + Aab^4\right)x^{12} - 1232\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 385\left(Ba^4b + 2\,Aa^3b^2\right)x^6}{616\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")
```

```
[Out] 1/616*(154*B*b^5*x^18 + 616*(5*B*a*b^4 + A*b^5)*x^15 - 1540*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 385*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 44*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Sympy [A] time = 27.6844, size = 122, normalized size = 1.11

$$\frac{Bb^5x^4}{4} + x\left(Ab^5 + 5Bab^4\right) - \frac{44Aa^5 + x^{12}\left(1540Aab^4 + 3080Ba^2b^3\right) + x^9\left(1232Aa^2b^3 + 1232Ba^3b^2\right) + x^6\left(770Aa^3b^2 + 3\right.}{616x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

```
[Out] B*b**5*x**4/4 + x*(A*b**5 + 5*B*a*b**4) - (44*A*a**5 + x**12*(1540*A*a*b**4
 + 3080*B*a**2*b**3) + x**9*(1232*A*a**2*b**3 + 1232*B*a**3*b**2) + x**6*(7
70*A*a**3*b**2 + 385*B*a**4*b) + x**3*(280*A*a**4*b + 56*B*a**5))/(616*x**1
4)
```

Giac [A] time = 1.22931, size = 166, normalized size = 1.51

$$\frac{1}{4}\,Bb^5x^4 + 5\,Bab^4x + Ab^5x - \frac{3080\,Ba^2b^3x^{12} + 1540\,Aab^4x^{12} + 1232\,Ba^3b^2x^9 + 1232\,Aa^2b^3x^9 + 385\,Ba^4bx^6 + 770\,Aa^3b}{616\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5*B*a*b^4*x + A*b^5*x - 1/616*(3080*B*a^2*b^3*x^12 + 1540*A
*a*b^4*x^12 + 1232*B*a^3*b^2*x^9 + 1232*A*a^2*b^3*x^9 + 385*B*a^4*b*x^6 + 7
70*A*a^3*b^2*x^6 + 56*B*a^5*x^3 + 280*A*a^4*b*x^3 + 44*A*a^5)/x^14
```

3.48 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}}\,dx$

Optimal. Leaf size=113

$$-\frac{5a^2b^2(aB+Ab)}{3x^6}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{a^5A}{15x^{15}}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^4\log(x)(5aB+Ab)+\frac{1}{3}b^5Bx^3$$

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (5*a*b^3*(A*b + 2*a*B))/(3*x^
3) + (b^5*B*x^3)/3 + b^4*(A*b + 5*a*B)*Log[x]
```

Rubi [A] time = 0.0911369, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{3x^6}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{a^5A}{15x^{15}}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^4\log(x)(5aB+Ab)+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (5*a*b^3*(A*b + 2*a*B))/(3*x^
3) + (b^5*B*x^3)/3 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
```

```
qQ[p, 1])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{16}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(b^5B+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+aB)}{x^3}+\frac{5ab^3}{}\right.\right.$$
$$= -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab+2aB)}{3x^3}+\frac{1}{3}b^5$$

Mathematica [A] time = 0.0549728, size = 116, normalized size = 1.03

$$b^4\log(x)(5aB+Ab)-\frac{300a^2b^3x^9\left(A+2Bx^3\right)+100a^3b^2x^6\left(2A+3Bx^3\right)+25a^4bx^3\left(3A+4Bx^3\right)+3a^5\left(4A+5Bx^3\right)+3}{180x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) + 100*a^
3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4*A + 5*B
*x^3))/(180*x^15) + b^4*(A*b + 5*a*B)*Log[x]
```

Maple [A] time = 0.009, size = 123, normalized size = 1.1

$$\frac{b^5Bx^3}{3}-\frac{5a^2b^3A}{3x^6}-\frac{5a^3b^2B}{3x^6}-\frac{5ab^4A}{3x^3}-\frac{10a^2b^3B}{3x^3}-\frac{Aa^5}{15x^{15}}-\frac{10a^3b^2A}{9x^9}-\frac{5a^4bB}{9x^9}-\frac{5a^4bA}{12x^{12}}-\frac{a^5B}{12x^{12}}+A\ln(x)b^5+5B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^16,x)
```

```
[Out] 1/3*b^5*B*x^3-5/3*a^2*b^3/x^6*A-5/3*a^3*b^2/x^6*B-5/3*a*b^4/x^3*A-10/3*a^2*
b^3/x^3*B-1/15*a^5*A/x^15-10/9*a^3*b^2/x^9*A-5/9*a^4*b/x^9*B-5/12*a^4/x^12*
A*b-1/12*a^5/x^12*B+A*ln(x)*b^5+5*B*ln(x)*a*b^4
```

Maxima [A] time = 1.10972, size = 166, normalized size = 1.47

$$\frac{1}{3}\,Bb^5x^3+\frac{1}{3}\left(5\,Bab^4+Ab^5\right)\log\left(x^3\right)-\frac{300\left(2\,Ba^2b^3+Aab^4\right)x^{12}+300\left(Ba^3b^2+Aa^2b^3\right)x^9+100\left(Ba^4b+2\,Aa^3b^2\right)x^6+1}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + 1/3*(5*B*a*b^4 + A*b^5)*log(x^3) - 1/180*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 100*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 12*A*a^5 + 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Fricas [A] time = 1.44939, size = 282, normalized size = 2.5

$$\frac{60\,Bb^5x^{18}+180\left(5\,Bab^4+Ab^5\right)x^{15}\log\left(x\right)-300\left(2\,Ba^2b^3+Aab^4\right)x^{12}-300\left(Ba^3b^2+Aa^2b^3\right)x^9-100\left(Ba^4b+2\,Aa^3b^2\right)x}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Sympy [A] time = 36.7863, size = 122, normalized size = 1.08

$$\frac{Bb^5x^3}{3}+b^4\left(Ab+5Ba\right)\log\left(x\right)-\frac{12Aa^5+x^{12}\left(300Aab^4+600Ba^2b^3\right)+x^9\left(300Aa^2b^3+300Ba^3b^2\right)+x^6\left(200Aa^3b^2+10\right.}{180x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

```
[Out] B*b**5*x**3/3 + b**4*(A*b + 5*B*a)*log(x) - (12*A*a**5 + x**12*(300*A*a*b**
4 + 600*B*a**2*b**3) + x**9*(300*A*a**2*b**3 + 300*B*a**3*b**2) + x**6*(200
*A*a**3*b**2 + 100*B*a**4*b) + x**3*(75*A*a**4*b + 15*B*a**5))/(180*x**15)
```

Giac [A] time = 1.19474, size = 196, normalized size = 1.73

$$\frac{1}{3}Bb^5x^3 + \left(5\,Bab^4 + Ab^5\right)\log\left(|x|\right) - \frac{685\,Bab^4x^{15} + 137\,Ab^5x^{15} + 600\,Ba^2b^3x^{12} + 300\,Aab^4x^{12} + 300\,Ba^3b^2x^9 + 300\,Aa}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/180*(685*B*a*b^4*x^15 +
 137*A*b^5*x^15 + 600*B*a^2*b^3*x^12 + 300*A*a*b^4*x^12 + 300*B*a^3*b^2*x^9
 + 300*A*a^2*b^3*x^9 + 100*B*a^4*b*x^6 + 200*A*a^3*b^2*x^6 + 15*B*a^5*x^3 +
 75*A*a^4*b*x^3 + 12*A*a^5)/x^15
```

3.49 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{17}}\,dx$

Optimal. Leaf size=115

$$-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{a^5A}{16x^{16}}-\frac{5ab^3(2aB+Ab)}{4x^4}-\frac{b^4(5aB+Ab)}{x}+\frac{1}{2}b^5Bx^2$$

```
[Out] -(a^5*A)/(16*x^16) - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))/
(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*x^
4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

Rubi [A] time = 0.0615403, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{a^5A}{16x^{16}}-\frac{5ab^3(2aB+Ab)}{4x^4}-\frac{b^4(5aB+Ab)}{x}+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -(a^5*A)/(16*x^16) - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))/
(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*x^
4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{17}}\,dx=\int \left(\frac{a^5A}{x^{17}}+\frac{a^4(5Ab+aB)}{x^{14}}+\frac{5a^3b(2Ab+aB)}{x^{11}}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(Ab+2aB)}{x^5}+\frac{b^4(}{}\right.$$

$$=-\frac{a^5A}{16x^{16}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{a^3b(2Ab+aB)}{2x^{10}}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{5ab^3(Ab+2aB)}{4x^4}-\frac{b^4(Ab}{}$$

Mathematica [A] time = 0.0310625, size = 118, normalized size = 1.03

$$-\frac{520a^2b^3x^9\left(4A+7Bx^3\right)+208a^3b^2x^6\left(7A+10Bx^3\right)+56a^4bx^3\left(10A+13Bx^3\right)+7a^5\left(13A+16Bx^3\right)+1820ab^4x^{12}\left(A\right.}{1456x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -(-728*b^5*x^15*(-2*A + B*x^3) + 1820*a*b^4*x^12*(A + 4*B*x^3) + 520*a^2*b^
3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*b*x^3*(10
*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/(1456*x^16)
```

Maple [A] time = 0.006, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{16\,x^{16}}-\frac{a^4\,(5\,Ab+Ba)}{13\,x^{13}}-\frac{a^3b\,(2\,Ab+Ba)}{2\,x^{10}}-\frac{10\,a^2b^2\,(Ab+Ba)}{7\,x^7}-\frac{5\,ab^3\,(Ab+2\,Ba)}{4\,x^4}-\frac{b^4\,(Ab+5\,Ba)}{x}+\frac{b^5Bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^17,x)
```

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

Maxima [A] time = 1.03681, size = 165, normalized size = 1.43

$$\frac{1}{2}\,Bb^5x^2-\frac{1456\left(5\,Bab^4+Ab^5\right)x^{15}+1820\left(2\,Ba^2b^3+Aab^4\right)x^{12}+2080\left(Ba^3b^2+Aa^2b^3\right)x^9+728\left(Ba^4b+2\,Aa^3b^2\right)x^6+}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^15 + 1820*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

Fricas [A] time = 1.43416, size = 281, normalized size = 2.44

$$\frac{728\,Bb^5x^{18} - 1456\left(5\,Bab^4 + Ab^5\right)x^{15} - 1820\left(2\,Ba^2b^3 + Aab^4\right)x^{12} - 2080\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 728\left(Ba^4b + 2\,Aa^3b^2\right)x^6}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="fricas")
```

```
[Out] 1/1456*(728*B*b^5*x^18 - 1456*(5*B*a*b^4 + A*b^5)*x^15 - 1820*(2*B*a^2*b^3
+ A*a*b^4)*x^12 - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

Sympy [A] time = 52.7244, size = 126, normalized size = 1.1

$$\frac{Bb^5x^2}{2} - \frac{91Aa^5 + x^{15}\left(1456Ab^5 + 7280Bab^4\right) + x^{12}\left(1820Aab^4 + 3640Ba^2b^3\right) + x^9\left(2080Aa^2b^3 + 2080Ba^3b^2\right) + x^6\left(1456\right.}{1456x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)
```

```
[Out] B*b**5*x**2/2 - (91*A*a**5 + x**15*(1456*A*b**5 + 7280*B*a*b**4) + x**12*(1
820*A*a*b**4 + 3640*B*a**2*b**3) + x**9*(2080*A*a**2*b**3 + 2080*B*a**3*b**
2) + x**6*(1456*A*a**3*b**2 + 728*B*a**4*b) + x**3*(560*A*a**4*b + 112*B*a*
*5))/(1456*x**16)
```

Giac [A] time = 1.18053, size = 173, normalized size = 1.5

$$\frac{1}{2}\,Bb^5x^2 - \frac{7280\,Bab^4x^{15} + 1456\,Ab^5x^{15} + 3640\,Ba^2b^3x^{12} + 1820\,Aab^4x^{12} + 2080\,Ba^3b^2x^9 + 2080\,Aa^2b^3x^9 + 728\,Ba^4bx^6}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^15 + 1456*A*b^5*x^15 + 3640*B*a^2*b^
3*x^12 + 1820*A*a*b^4*x^12 + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 728*
```

```
B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*A*a
^5)/x^16
```

3.50 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx$

Optimal. Leaf size=110

$$-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5A}{17x^{17}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{2x^2}+b^5Bx$$

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Rubi [A] time = 0.0613283, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5A}{17x^{17}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{2x^2}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx = \int \left(b^5B+\frac{a^5A}{x^{18}}+\frac{a^4(5Ab+aB)}{x^{15}}+\frac{5a^3b(2Ab+aB)}{x^{12}}+\frac{10a^2b^2(Ab+aB)}{x^9}+\frac{5ab^3(Ab+2aB)}{x^6}\right.$$

$$= -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\frac{b^4(Ab}{2x}$$

Mathematica [A] time = 0.0432395, size = 110, normalized size = 1.

$$-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5A}{17x^{17}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{2x^2}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Maple [A] time = 0.007, size = 101, normalized size = 0.9

$$-\frac{Aa^5}{17\,x^{17}}-\frac{a^4\,(5\,Ab+Ba)}{14\,x^{14}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{11\,x^{11}}-\frac{5\,a^2b^2\,(Ab+Ba)}{4\,x^8}-\frac{ab^3\,(Ab+2\,Ba)}{x^5}-\frac{b^4\,(Ab+5\,Ba)}{2\,x^2}+b^5Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^18,x)
```

```
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

Maxima [A] time = 1.31328, size = 161, normalized size = 1.46

$$Bb^5x-\frac{2618\left(5\,Bab^4+Ab^5\right)x^{15}+5236\left(2\,Ba^2b^3+Aab^4\right)x^{12}+6545\left(Ba^3b^2+Aa^2b^3\right)x^9+2380\left(Ba^4b+2\,Aa^3b^2\right)x^6+3}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^15 + 5236*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Fricas [A] time = 1.40694, size = 285, normalized size = 2.59

$$\frac{5236\,Bb^5x^{18} - 2618\left(5\,Bab^4 + Ab^5\right)x^{15} - 5236\left(2\,Ba^2b^3 + Aab^4\right)x^{12} - 6545\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 2380\left(Ba^4b + 2\,Aa^3b^2\right)x}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")
```

```
[Out] 1/5236*(5236*B*b^5*x^18 - 2618*(5*B*a*b^4 + A*b^5)*x^15 - 5236*(2*B*a^2*b^3
 + A*a*b^4)*x^12 - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Sympy [A] time = 125.843, size = 122, normalized size = 1.11

$$Bb^5x - \frac{308Aa^5 + x^{15}\left(2618Ab^5 + 13090Bab^4\right) + x^{12}\left(5236Aab^4 + 10472Ba^2b^3\right) + x^9\left(6545Aa^2b^3 + 6545Ba^3b^2\right) + x^6\left(47\right.}{5236x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)
```

```
[Out] B*b**5*x - (308*A*a**5 + x**15*(2618*A*b**5 + 13090*B*a*b**4) + x**12*(5236
*A*a*b**4 + 10472*B*a**2*b**3) + x**9*(6545*A*a**2*b**3 + 6545*B*a**3*b**2)
 + x**6*(4760*A*a**3*b**2 + 2380*B*a**4*b) + x**3*(1870*A*a**4*b + 374*B*a*
*5))/(5236*x**17)
```

Giac [A] time = 1.19891, size = 169, normalized size = 1.54

$$Bb^5x - \frac{13090\,Bab^4x^{15} + 2618\,Ab^5x^{15} + 10472\,Ba^2b^3x^{12} + 5236\,Aab^4x^{12} + 6545\,Ba^3b^2x^9 + 6545\,Aa^2b^3x^9 + 2380\,Ba^4bx^6}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/5236*(13090*B*a*b^4*x^15 + 2618*A*b^5*x^15 + 10472*B*a^2*b^3*x^
12 + 5236*A*a*b^4*x^12 + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B*a
^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A*a^
5)/x^17
```

3.51 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} dx$

Optimal. Leaf size=91

$$-\frac{5a^2b^3B}{3x^6}-\frac{10a^3b^2B}{9x^9}-\frac{5a^4bB}{12x^{12}}-\frac{a^5B}{15x^{15}}-\frac{A(a+bx^3)^6}{18ax^{18}}-\frac{5ab^4B}{3x^3}+b^5B\log(x)$$

```
[Out] -(a^5*B)/(15*x^15) - (5*a^4*b*B)/(12*x^12) - (10*a^3*b^2*B)/(9*x^9) - (5*a^
2*b^3*B)/(3*x^6) - (5*a*b^4*B)/(3*x^3) - (A*(a + b*x^3)^6)/(18*a*x^18) + b^
5*B*Log[x]
```

Rubi [A] time = 0.0538979, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 78, 43}

$$-\frac{5a^2b^3B}{3x^6}-\frac{10a^3b^2B}{9x^9}-\frac{5a^4bB}{12x^{12}}-\frac{a^5B}{15x^{15}}-\frac{A(a+bx^3)^6}{18ax^{18}}-\frac{5ab^4B}{3x^3}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -(a^5*B)/(15*x^15) - (5*a^4*b*B)/(12*x^12) - (10*a^3*b^2*B)/(9*x^9) - (5*a^
2*b^3*B)/(3*x^6) - (5*a*b^4*B)/(3*x^3) - (A*(a + b*x^3)^6)/(18*a*x^18) + b^
5*B*Log[x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{19}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx, x, x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^6}{18ax^{18}}+\frac{1}{3}B\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx, x, x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^6}{18ax^{18}}+\frac{1}{3}B\operatorname{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx, x, x^3\right)\\
&= -\frac{a^5B}{15x^{15}}-\frac{5a^4bB}{12x^{12}}-\frac{10a^3b^2B}{9x^9}-\frac{5a^2b^3B}{3x^6}-\frac{5ab^4B}{3x^3}-\frac{A\left(a+bx^3\right)^6}{18ax^{18}}+b^5B\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0376973, size = 121, normalized size = 1.33

$$
-\frac{100a^2b^3x^9\left(2A+3Bx^3\right)+50a^3b^2x^6\left(3A+4Bx^3\right)+15a^4bx^3\left(4A+5Bx^3\right)+2a^5\left(5A+6Bx^3\right)+150ab^4x^{12}\left(A+2Bx^3\right)+}{180x^{18}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*A + 3*B
*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^3) + 2*a
^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/(180*x^18)
```

Maple [A] time = 0.007, size = 124, normalized size = 1.4

$$
-\frac{5\,ab^4A}{6\,x^6}-\frac{5\,a^2b^3B}{3\,x^6}-\frac{b^5A}{3\,x^3}-\frac{5\,ab^4B}{3\,x^3}-\frac{a^4bA}{3\,x^{15}}-\frac{a^5B}{15\,x^{15}}-\frac{Aa^5}{18\,x^{18}}-\frac{10\,Ab^3a^2}{9\,x^9}-\frac{10\,a^3b^2B}{9\,x^9}-\frac{5\,a^3b^2A}{6\,x^{12}}-\frac{5\,a^4bB}{12\,x^{12}}+b^5B\ln(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^19,x)
```

```
[Out] -5/6*a*b^4/x^6*A-5/3*a^2*b^3*B/x^6-1/3*b^5/x^3*A-5/3*a*b^4*B/x^3-1/3*a^4/x^
15*A*b-1/15*a^5*B/x^15-1/18*A*a^5/x^18-10/9*a^2*b^3/x^9*A-10/9*a^3*b^2*B/x^
9-5/6*a^3*b^2/x^12*A-5/12*a^4*b*B/x^12+b^5*B*ln(x)
```

Maxima [A] time = 1.10264, size = 166, normalized size = 1.82

$$\frac{1}{3}\,Bb^5\log\left(x^3\right)-\frac{60\left(5\,Bab^4+Ab^5\right)x^{15}+150\left(2\,Ba^2b^3+Aab^4\right)x^{12}+200\left(Ba^3b^2+Aa^2b^3\right)x^9+75\left(Ba^4b+2\,Aa^3b^2\right)x^6+}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

Fricas [A] time = 1.45278, size = 281, normalized size = 3.09

$$\frac{180\,Bb^5x^{18}\log\left(x\right)-60\left(5\,Bab^4+Ab^5\right)x^{15}-150\left(2\,Ba^2b^3+Aab^4\right)x^{12}-200\left(Ba^3b^2+Aa^2b^3\right)x^9-75\left(Ba^4b+2\,Aa^3b^2\right)x}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")
```

```
[Out] 1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*a^
3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

Sympy [A] time = 153.16, size = 124, normalized size = 1.36

$$Bb^5\log\left(x\right)-\frac{10Aa^5+x^{15}\left(60Ab^5+300Bab^4\right)+x^{12}\left(150Aab^4+300Ba^2b^3\right)+x^9\left(200Aa^2b^3+200Ba^3b^2\right)+x^6\left(150Aa\right.}{180x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

```
[Out] B*b**5*log(x) - (10*A*a**5 + x**15*(60*A*b**5 + 300*B*a*b**4) + x**12*(150*
A*a*b**4 + 300*B*a**2*b**3) + x**9*(200*A*a**2*b**3 + 200*B*a**3*b**2) + x*
*6*(150*A*a**3*b**2 + 75*B*a**4*b) + x**3*(60*A*a**4*b + 12*B*a**5))/(180*x
**18)
```

Giac [A] time = 1.18607, size = 184, normalized size = 2.02

$$Bb^5 \log(|x|) - \frac{147\,Bb^5x^{18} + 300\,Bab^4x^{15} + 60\,Ab^5x^{15} + 300\,Ba^2b^3x^{12} + 150\,Aab^4x^{12} + 200\,Ba^3b^2x^9 + 200\,Aa^2b^3x^9 + 75}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")
```

```
[Out] B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^1
5 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2*b
^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*x^3
 + 10*A*a^5)/x^18
```

3.52 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} dx$

Optimal. Leaf size=113

$$-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{b^5B}{x}$$

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7)
- (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

Rubi [A] time = 0.0643632, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7)
- (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} dx = \int \left(\frac{a^5A}{x^{20}}+\frac{a^4(5Ab+aB)}{x^{17}}+\frac{5a^3b(2Ab+aB)}{x^{14}}+\frac{10a^2b^2(Ab+aB)}{x^{11}}+\frac{5ab^3(Ab+2aB)}{x^8}+\frac{b}{}\right.$$

$$= -\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{a^2b^2(Ab+aB)}{x^{10}}-\frac{5ab^3(Ab+2aB)}{7x^7}-\frac{b^4(Ab}{}$$

Mathematica [A] time = 0.0295196, size = 119, normalized size = 1.05

$$-\frac{3952a^2b^3x^9\left(7A+10Bx^3\right)+2128a^3b^2x^6\left(10A+13Bx^3\right)+665a^4bx^3\left(13A+16Bx^3\right)+91a^5\left(16A+19Bx^3\right)+4940ab^4x}{27664x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -(6916*b^5*x^15*(A + 4*B*x^3) + 4940*a*b^4*x^12*(4*A + 7*B*x^3) + 3952*a^2*
b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 665*a^4*b*x
^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/(27664*x^19)
```

Maple [A] time = 0.008, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{19\,x^{19}}-\frac{a^4\,(5\,Ab+Ba)}{16\,x^{16}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{13\,x^{13}}-\frac{a^2b^2\,(Ab+Ba)}{x^{10}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{7\,x^7}-\frac{b^4\,(Ab+5\,Ba)}{4\,x^4}-\frac{Bb^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^20,x)
```

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

Maxima [A] time = 1.12837, size = 163, normalized size = 1.44

$$-\frac{27664\,Bb^5x^{18}+6916\left(5\,Bab^4+Ab^5\right)x^{15}+19760\left(2\,Ba^2b^3+Aab^4\right)x^{12}+27664\left(Ba^3b^2+Aa^2b^3\right)x^9+10640\left(Ba^4b+2\,A\right.}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

Fricas [A] time = 1.41254, size = 296, normalized size = 2.62

$$-\frac{27664\,Bb^5x^{18}+6916\left(5\,Bab^4+Ab^5\right)x^{15}+19760\left(2\,Ba^2b^3+Aab^4\right)x^{12}+27664\left(Ba^3b^2+Aa^2b^3\right)x^9+10640\left(Ba^4b+2\right.}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2132, size = 171, normalized size = 1.51

$$-\frac{27664\,Bb^5x^{18}+34580\,Bab^4x^{15}+6916\,Ab^5x^{15}+39520\,Ba^2b^3x^{12}+19760\,Aab^4x^{12}+27664\,Ba^3b^2x^9+27664\,Aa^2b^3x^9}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 34580*B*a*b^4*x^15 + 6916*A*b^5*x^15 + 39520*B
*a^2*b^3*x^12 + 19760*A*a*b^4*x^12 + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^3*
x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*a^4
*b*x^3 + 1456*A*a^5)/x^19
```

3.53 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{21}}\,dx$

Optimal. Leaf size=117

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{a^5A}{20x^{20}}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{b^5B}{2x^2}$$

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

Rubi [A] time = 0.0588298, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{a^5A}{20x^{20}}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{b^5B}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{21}}\,dx &= \int \left(\frac{a^5A}{x^{21}}+\frac{a^4(5Ab+aB)}{x^{18}}+\frac{5a^3b(2Ab+aB)}{x^{15}}+\frac{10a^2b^2(Ab+aB)}{x^{12}}+\frac{5ab^3(Ab+2aB)}{x^9}+\frac{b^4(\right.\\ &= -\frac{a^5A}{20x^{20}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2aB)}{8x^8}-\frac{b^4(A\end{aligned}$$

Mathematica [A] time = 0.0305845, size = 121, normalized size = 1.03

$$-\frac{5950a^2b^3x^9\left(8A+11Bx^3\right)+3400a^3b^2x^6\left(11A+14Bx^3\right)+1100a^4bx^3\left(14A+17Bx^3\right)+154a^5\left(17A+20Bx^3\right)+6545a}{52360x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -(5236*b^5*x^15*(2*A + 5*B*x^3) + 6545*a*b^4*x^12*(5*A + 8*B*x^3) + 5950*a^
2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) + 1100*a^4*
b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/(52360*x^20)
```

Maple [A] time = 0.007, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{20\,x^{20}}-\frac{a^4\,(5\,Ab+Ba)}{17\,x^{17}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{14\,x^{14}}-\frac{10\,a^2b^2\,(Ab+Ba)}{11\,x^{11}}-\frac{5\,ab^3\,(Ab+2\,Ba)}{8\,x^8}-\frac{b^4\,(Ab+5\,Ba)}{5\,x^5}-\frac{Bb^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^21,x)
```

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

Maxima [A] time = 1.1166, size = 163, normalized size = 1.39

$$-\frac{26180\,Bb^5x^{18}+10472\left(5\,Bab^4+Ab^5\right)x^{15}+32725\left(2\,Ba^2b^3+Aab^4\right)x^{12}+47600\left(Ba^3b^2+Aa^2b^3\right)x^9+18700\left(Ba^4b+\right.}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Fricas [A] time = 1.40634, size = 297, normalized size = 2.54

$$-\frac{26180\,Bb^5x^{18}+10472\left(5\,Bab^4+Ab^5\right)x^{15}+32725\left(2\,Ba^2b^3+Aab^4\right)x^{12}+47600\left(Ba^3b^2+Aa^2b^3\right)x^9+18700\left(Ba^4b+2\right.}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**21,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17573, size = 171, normalized size = 1.46

$$-\frac{26180\,Bb^5x^{18}+52360\,Bab^4x^{15}+10472\,Ab^5x^{15}+65450\,Ba^2b^3x^{12}+32725\,Aab^4x^{12}+47600\,Ba^3b^2x^9+47600\,Aa^2b^3x^9}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 52360*B*a*b^4*x^15 + 10472*A*b^5*x^15 + 65450*
B*a^2*b^3*x^12 + 32725*A*a*b^4*x^12 + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b^3
*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*A*a
^4*b*x^3 + 2618*A*a^5)/x^20
```

3.54 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^3\right)^6(Ab-7aB)}{126a^2x^{18}}-\frac{A\left(a+bx^3\right)^6}{21ax^{21}}$$

```
[Out] -(A*(a + b*x^3)^6)/(21*a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(126*a^2*x^1
8)
```

Rubi [A] time = 0.031515, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 78, 37}

$$\frac{\left(a+bx^3\right)^6(Ab-7aB)}{126a^2x^{18}}-\frac{A\left(a+bx^3\right)^6}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -(A*(a + b*x^3)^6)/(21*a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(126*a^2*x^1
8)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{22}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx, x, x^3\right)\\ &= -\frac{A\left(a+bx^3\right)^6}{21ax^{21}}+\frac{(-Ab+7aB)\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^3\right)}{21a}\\ &= -\frac{A\left(a+bx^3\right)^6}{21ax^{21}}+\frac{(Ab-7aB)\left(a+bx^3\right)^6}{126a^2x^{18}}\end{aligned}$$

Mathematica [B] time = 0.0293528, size = 118, normalized size = 2.46

$$-\frac{35a^2b^3x^9\left(3A+4Bx^3\right)+21a^3b^2x^6\left(4A+5Bx^3\right)+7a^4bx^3\left(5A+6Bx^3\right)+a^5\left(6A+7Bx^3\right)+35ab^4x^{12}\left(2A+3Bx^3\right)+21}{126x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2*b^3*x^
9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5*A + 6*B
*x^3) + a^5*(6*A + 7*B*x^3))/(126*x^21)
```

Maple [B] time = 0.009, size = 104, normalized size = 2.2

$$-\frac{a^4(5Ab+Ba)}{18x^{18}}-\frac{Aa^5}{21x^{21}}-\frac{Bb^5}{3x^3}-\frac{5a^2b^2(Ab+Ba)}{6x^{12}}-\frac{b^4(Ab+5Ba)}{6x^6}-\frac{a^3b(2Ab+Ba)}{3x^{15}}-\frac{5ab^3(Ab+2Ba)}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^22,x)
```

```
[Out] -1/18*a^4*(5*A*b+B*a)/x^18-1/21*A*a^5/x^21-1/3*B*b^5/x^3-5/6*a^2*b^2*(A*b+B
*a)/x^12-1/6*b^4*(A*b+5*B*a)/x^6-1/3*a^3*b*(2*A*b+B*a)/x^15-5/9*a*b^3*(A*b+
2*B*a)/x^9
```

Maxima [B] time = 1.29429, size = 163, normalized size = 3.4

$$-\frac{42\,Bb^5x^{18}+21\left(5\,Bab^4+Ab^5\right)x^{15}+70\left(2\,Ba^2b^3+Aab^4\right)x^{12}+105\left(Ba^3b^2+Aa^2b^3\right)x^9+42\left(Ba^4b+2\,Aa^3b^2\right)x^6+6\,A}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Fricas [B] time = 1.32716, size = 267, normalized size = 5.56

$$-\frac{42\,Bb^5x^{18}+21\left(5\,Bab^4+Ab^5\right)x^{15}+70\left(2\,Ba^2b^3+Aab^4\right)x^{12}+105\left(Ba^3b^2+Aa^2b^3\right)x^9+42\left(Ba^4b+2\,Aa^3b^2\right)x^6+6\,A}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)
```

[Out] Timed out

Giac [B] time = 1.1171, size = 171, normalized size = 3.56

$$-\frac{42\,Bb^5x^{18}+105\,Bab^4x^{15}+21\,Ab^5x^{15}+140\,Ba^2b^3x^{12}+70\,Aab^4x^{12}+105\,Ba^3b^2x^9+105\,Aa^2b^3x^9+42\,Ba^4bx^6+84\,Aa}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 105*B*a*b^4*x^15 + 21*A*b^5*x^15 + 140*B*a^2*b^3*x^
12 + 70*A*a*b^4*x^12 + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b*x
^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^21
```

3.55 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} \, dx$

Optimal. Leaf size=117

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{22x^{22}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{4x^4}$$

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Rubi [A] time = 0.0649493, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{22x^{22}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} \, dx = \int \left(\frac{a^5A}{x^{23}}+\frac{a^4(5Ab+aB)}{x^{20}}+\frac{5a^3b(2Ab+aB)}{x^{17}}+\frac{10a^2b^2(Ab+aB)}{x^{14}}+\frac{5ab^3(Ab+2aB)}{x^{11}}+\frac{b}{}\right.$$

$$= -\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(A}{}$$

Mathematica [A] time = 0.0415233, size = 117, normalized size = 1.

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{22x^{22}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Maple [A] time = 0.007, size = 104, normalized size = 0.9

$$-\frac{Aa^5}{22\,x^{22}}-\frac{a^4\,(5\,Ab+Ba)}{19\,x^{19}}-\frac{5\,a^3b\,(2\,Ab+Ba)}{16\,x^{16}}-\frac{10\,a^2b^2\,(Ab+Ba)}{13\,x^{13}}-\frac{ab^3\,(Ab+2\,Ba)}{2\,x^{10}}-\frac{b^4\,(Ab+5\,Ba)}{7\,x^7}-\frac{Bb^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^23,x)
```

```
[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/1
3*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7
-1/4*b^5*B/x^4
```

Maple [A] time = 1.22549, size = 163, normalized size = 1.39

$$-\frac{76076\,Bb^5x^{18}+43472\left(5\,Bab^4+Ab^5\right)x^{15}+152152\left(2\,Ba^2b^3+Aab^4\right)x^{12}+234080\left(Ba^3b^2+Aa^2b^3\right)x^9+95095\left(Ba^4b+\right.}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

Fricas [A] time = 1.41678, size = 304, normalized size = 2.6

$$-\frac{76076\,Bb^5x^{18}+43472\left(5\,Bab^4+Ab^5\right)x^{15}+152152\left(2\,Ba^2b^3+Aab^4\right)x^{12}+234080\left(Ba^3b^2+Aa^2b^3\right)x^9+95095\left(Ba^4b\right.}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11635, size = 171, normalized size = 1.46

$$-\frac{76076\,Bb^5x^{18}+217360\,Bab^4x^{15}+43472\,Ab^5x^{15}+304304\,Ba^2b^3x^{12}+152152\,Aab^4x^{12}+234080\,Ba^3b^2x^9+234080\,A}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 217360*B*a*b^4*x^15 + 43472*A*b^5*x^15 + 3043
04*B*a^2*b^3*x^12 + 152152*A*a*b^4*x^12 + 234080*B*a^3*b^2*x^9 + 234080*A*a
^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3 + 8
0080*A*a^4*b*x^3 + 13832*A*a^5)/x^22
```

3.56 $\int \frac{x^6 (A+Bx^3)}{a+bx^3} \, dx$

Optimal. Leaf size=183

$$-\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}-\frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}}+\frac{x^4(Ab}{4b}$$

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^7)/(7*b) - (a^(
4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*b^(10/3)) + (a^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) -
 (a^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^
(10/3))
```

Rubi [A] time = 0.149132, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {459, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}-\frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}}+\frac{x^4(Ab}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^7)/(7*b) - (a^(
4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*b^(10/3)) + (a^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) -
 (a^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^
(10/3))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{7b}$$

$$= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{7b}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^2(Ab-aB)\right)\int \frac{1}{a+bx^3}\,dx}{b^3}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^3} + \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{2\sqrt[3]{a}\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{\cdots}}}{3b^3}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{\cdots}}{a^{2/3}-\sqrt[3]{a}\cdots}}{6b^{10/3}}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\cdots\right.}{6b^{10/3}}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} - \frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\cdots\right.}{3b^{10/3}}$$

Mathematica [A] time = 0.106341, size = 171, normalized size = 0.93

$$\frac{14a^{4/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-28a^{4/3}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}a^{4/3}(aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\cdots}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B*
x^7 + 28*Sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*
(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*b^(10/3)
)
```

Maple [A] time = 0.007, size = 249, normalized size = 1.4

$$\frac{Bx^7}{7b}+\frac{Ax^4}{4b}-\frac{Bx^4a}{4b^2}-\frac{aAx}{b^2}+\frac{a^2Bx}{b^3}+\frac{a^2A}{3b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{Ba^3}{3b^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{a^2A}{6b^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/7*B*x^7/b+1/4/b*A*x^4-1/4/b^2*B*x^4*a-1/b^2*a*A*x+1/b^3*a^2*B*x+1/3*a^2/b
^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)*B-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*a^3/b^4
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3*a^2/b^3/(a/b)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3*a^3/b^4/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52742, size = 397, normalized size = 2.17

$$\frac{12\,Bb^2x^7-21\left(Bab-Ab^2\right)x^4-28\,\sqrt{3}\left(Ba^2-Aab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+14\left(Ba^2-Aab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{84\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(a
/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a^2
```

```
 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2 -
A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x)/b^3
```

Sympy [A] time = 1.22068, size = 110, normalized size = 0.6

$$\frac{Bx^7}{7b} + \operatorname{RootSum}\left(27t^3b^{10} - A^3a^4b^3 + 3A^2Ba^5b^2 - 3AB^2a^6b + B^3a^7, \left(t \mapsto t\log\left(-\frac{3tb^3}{-Aab + Ba^2} + x\right)\right)\right) - \frac{x^4(-Ab + Ba)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**7/(7*b) + RootSum(27*_t**3*b**10 - A**3*a**4*b**3 + 3*A**2*B*a**5*b**2
 - 3*A*B**2*a**6*b + B**3*a**7, Lambda(_t, _t*log(-3*_t*b**3/(-A*a*b + B*a*
*2) + x))) - x**4*(-A*b + B*a)/(4*b**2) + x*(-A*a*b + B*a**2)/b**3
```

Giac [A] time = 1.1233, size = 293, normalized size = 1.6

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - \left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - \left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (-a
*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B*a^
3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(
4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^5*x)/
b^7
```

3.57 $\int \frac{x^5(A+Bx^3)}{a+bx^3} dx$

Optimal. Leaf size=54

$$\frac{x^3(Ab-aB)}{3b^2} - \frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3} + \frac{Bx^6}{6b}$$

```
[Out] ((A*b - a*B)*x^3)/(3*b^2) + (B*x^6)/(6*b) - (a*(A*b - a*B)*Log[a + b*x^3])/
(3*b^3)
```

Rubi [A] time = 0.0572361, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{x^3(Ab-aB)}{3b^2} - \frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3} + \frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*b^2) + (B*x^6)/(6*b) - (a*(A*b - a*B)*Log[a + b*x^3])/
(3*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^5 \left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(-Ab+aB)}{b^2(a+bx)}\right)dx,x,x^3\right)$$
$$= \frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}$$

Mathematica [A] time = 0.0204856, size = 47, normalized size = 0.87

$$\frac{bx^3\left(-2aB+2Ab+bBx^3\right)+2a(aB-Ab)\log\left(a+bx^3\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b^
3)
```

Maple [A] time = 0.004, size = 62, normalized size = 1.2

$$\frac{Bx^6}{6\,b}+\frac{Ax^3}{3\,b}-\frac{Bx^3a}{3\,b^2}-\frac{a\ln\left(bx^3+a\right)A}{3\,b^2}+\frac{a^2\ln\left(bx^3+a\right)B}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/6*B*x^6/b+1/3/b*A*x^3-1/3/b^2*B*x^3*a-1/3*a/b^2*ln(b*x^3+a)*A+1/3*a^2/b^3
*ln(b*x^3+a)*B
```

Maxima [A] time = 1.14137, size = 68, normalized size = 1.26

$$\frac{Bbx^6-2\,(Ba-Ab)x^3}{6\,b^2}+\frac{\left(Ba^2-Aab\right)\log\left(bx^3+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)/
b^3
```

Fricas [A] time = 1.44918, size = 108, normalized size = 2.

$$\frac{Bb^2x^6 - 2\left(Bab - Ab^2\right)x^3 + 2\left(Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))/
b^3
```

Sympy [A] time = 1.12372, size = 44, normalized size = 0.81

$$\frac{Bx^6}{6b} + \frac{a\left(-Ab + Ba\right)\log\left(a + bx^3\right)}{3b^3} - \frac{x^3\left(-Ab + Ba\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) - x**3*(-A*b + B*a)/
(3*b**2)
```

Giac [A] time = 1.17429, size = 70, normalized size = 1.3

$$\frac{Bbx^6 - 2\,Bax^3 + 2\,Abx^3}{6\,b^2} + \frac{\left(Ba^2 - Aab\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*x
^3 + a))/b^3
```

3.58 $\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=167

$$-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}+\frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{x^2(A$$

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^5)/(5*b) + (a^(2/3)*(A*b - a*B)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (a^(2/3)*(A*b
- a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) - (a^(2/3)*(A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))
```

Rubi [A] time = 0.120236, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {459, 321, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}+\frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{x^2(A$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3),x]
```


```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^5)/(5*b) + (a^(2/3)*(A*b - a*B)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (a^(2/3)*(A*b
- a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) - (a^(2/3)*(A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4 \left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{5b}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} - \frac{(a(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{b^2}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{7/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{7/3}}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{8/3}} - $$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(A}{}$$

Mathematica [A] time = 0.0816788, size = 154, normalized size = 0.92

$$\frac{5a^{2/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-10a^{2/3}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}a^{2/3}(aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+1}{30b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b)
+ a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) + a
*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(30*b^(8/3))
```

Maple [A] time = 0.004, size = 226, normalized size = 1.4

$$\frac{Bx^5}{5b} + \frac{Ax^2}{2b} - \frac{Bx^2a}{2b^2} + \frac{aA}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{a^2B}{3b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{aA}{6b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{a^2B}{6b^3}\ln\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/5*B*x^5/b+1/2/b*A*x^2-1/2/b^2*B*x^2*a+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))*A-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B-1/6*a/b^2/(a/b)^(1/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))*B-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*A+1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47417, size = 390, normalized size = 2.34

$$\frac{6\,Bbx^5 - 15\,(Ba - Ab)x^2 + 10\,\sqrt{3}(Ba - Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}a}{3\,a}\right) + 5\,(Ba - Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2 - bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + a\left(\right.\right.}{30\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1/
3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A*b
)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10
*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b^2
```

Sympy [A] time = 0.781595, size = 112, normalized size = 0.67

$$\frac{Bx^5}{5b} + \operatorname{RootSum}\left(27t^3b^8 - A^3a^2b^3 + 3A^2Ba^3b^2 - 3AB^2a^4b + B^3a^5, \left(t \mapsto t\log\left(\frac{9t^2b^5}{A^2ab^2 - 2ABa^2b + B^2a^3} + x\right)\right)\right) - \frac{x^2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**5/(5*b) + RootSum(27*_t**3*b**8 - A**3*a**2*b**3 + 3*A**2*B*a**3*b**2
- 3*A*B**2*a**4*b + B**3*a**5, Lambda(_t, _t*log(9*_t**2*b**5/(A**2*a*b**2
- 2*A*B*a**2*b + B**2*a**3) + x))) - x**2*(-A*b + B*a)/(2*b**2)
```

Giac [A] time = 1.16652, size = 279, normalized size = 1.67

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2)^
(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3*(-
a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5
```

3.59 $\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{x(Ab-aB)}{b^2}-\frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}b^{7/3}}$$

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^4)/(4*b) + (a^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (a^(1/3)*(A*b - a*B)
*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) + (a^(1/3)*(A*b - a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))
```

Rubi [A] time = 0.115521, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {459, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{x(Ab-aB)}{b^2}-\frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3),x]
```


```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^4)/(4*b) + (a^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (a^(1/3)*(A*b - a*B)
*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) + (a^(1/3)*(A*b - a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3 \left(A + Bx^3\right)}{a + bx^3} \, dx = \frac{Bx^4}{4b} - \frac{(-4Ab + 4aB) \int \frac{x^3}{a+bx^3} \, dx}{4b}$$
$$= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{(a(Ab - aB)) \int \frac{1}{a+bx^3} \, dx}{b^2}$$
$$= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\left(\sqrt[3]{a}(Ab - aB)\right) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} \, dx}{3b^2} - \frac{\left(\sqrt[3]{a}(Ab - aB)\right) \int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} \, dx}{3b^2}$$
$$= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\left(\sqrt[3]{a}(Ab - aB)\right) \int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} \, dx}{6b^{7/3}} - \frac{\left(a^{2/\cdots\right.}{\cdots}$$
$$= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab - aB) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{6b^{7/3}} - \cdots$$
$$= \frac{(Ab - aB)x}{b^2} + \frac{Bx^4}{4b} + \frac{\sqrt[3]{a}(Ab - aB) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{\sqrt[3]{a}(Ab - aB) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab - \cdots}{\cdots}$$

Mathematica [A] time = 0.0784405, size = 152, normalized size = 0.94

$$\frac{-2\sqrt[3]{a}(aB - Ab) \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right) + 12\sqrt[3]{b}x(Ab - aB) + 4\sqrt[3]{a}(aB - Ab) \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right) - 4\sqrt{3}\sqrt[3]{a}(aB - Ab) \text{ta}\cdots}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) + a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B)*
Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))
```

Maple [A] time = 0.003, size = 221, normalized size = 1.4

$$\frac{Bx^4}{4\,b} + \frac{Ax}{b} - \frac{Bax}{b^2} - \frac{aA}{3\,b^2} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{a^2B}{3\,b^3} \ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{aA}{6\,b^2} \ln\left(x^2 - \sqrt[3]{\frac{a}{b}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{a^2B}{6\,b^3} \ln\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/4*B*x^4/b+1/b*A*x-1/b^2*B*a*x-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1
/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*A-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))*B-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*A+1/3*a^2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52449, size = 343, normalized size = 2.12

$$\frac{3\,Bbx^4 - 4\,\sqrt{3}(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right) + 2\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4\,(Ba-Ab}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*B*b*x^4 - 4*sqrt(3)*(B*a - A*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*
b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 2*(B*a - A*b)*(-a/b)^(1/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3)) - 4*(B*a - A*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/
3)) - 12*(B*a - A*b)*x)/b^2
```

Sympy [A] time = 0.825358, size = 87, normalized size = 0.54

$$\frac{Bx^4}{4b} + \text{RootSum}\left(27t^3b^7 + A^3ab^3 - 3A^2Ba^2b^2 + 3AB^2a^3b - B^3a^4, \left(t \mapsto t\log\left(\frac{3tb^2}{-Ab+Ba} + x\right)\right)\right) - \frac{x(-Ab+Ba)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**4/(4*b) + RootSum(27*_t**3*b**7 + A**3*a*b**3 - 3*A**2*B*a**2*b**2 + 3
*A*B**2*a**3*b - B**3*a**4, Lambda(_t, _t*log(3*_t*b**2/(-A*b + B*a) + x)))
 - x*(-A*b + B*a)/b**2
```

Giac [A] time = 1.13934, size = 251, normalized size = 1.55

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3} - \frac{(Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(
1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 - 1/3*(B*a^2*b^2 - A
*a*b^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(B*b^3*x^4 -
4*B*a*b^2*x + 4*A*b^3*x)/b^4
```

3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3} \, dx$

Optimal. Leaf size=35

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}+\frac{Bx^3}{3b}$$

```
[Out] (B*x^3)/(3*b) + ((A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.0327442, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^3)/(3*b) + ((A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^3\right)}{a+bx^3} dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{a+bx} dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right) dx, x, x^3\right)$$
$$= \frac{Bx^3}{3b}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}$$

Mathematica [A] time = 0.0122157, size = 31, normalized size = 0.89

$$\frac{(Ab-aB)\log\left(a+bx^3\right)+bBx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Maple [A] time = 0.001, size = 40, normalized size = 1.1

$$\frac{Bx^3}{3\,b}+\frac{\ln\left(bx^3+a\right)A}{3\,b}-\frac{\ln\left(bx^3+a\right)Ba}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/3*B*x^3/b+1/3/b*ln(b*x^3+a)*A-1/3/b^2*ln(b*x^3+a)*B*a
```

Maxima [A] time = 1.14153, size = 42, normalized size = 1.2

$$\frac{Bx^3}{3\,b}-\frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(b*x^3 + a)/b^2
```

Fricas [A] time = 1.38283, size = 65, normalized size = 1.86

$$\frac{Bbx^3 - (Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2
```

Sympy [A] time = 0.691102, size = 27, normalized size = 0.77

$$\frac{Bx^3}{3b} - \frac{(-Ab + Ba)\log\left(a + bx^3\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**3/(3*b) - (-A*b + B*a)*log(a + b*x**3)/(3*b**2)
```

Giac [A] time = 1.11574, size = 43, normalized size = 1.23

$$\frac{Bx^3}{3\,b} - \frac{(Ba - Ab)\log\left(\left|bx^3 + a\right|\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2
```

3.61 $\int \frac{x\left(A+Bx^3\right)}{a+bx^3} dx$

Optimal. Leaf size=150

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}+\frac{Bx^2}{2b}$$

```
[Out] (B*x^2)/(2*b) - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*b^(5/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(1/3)*b^(5/3))
```

Rubi [A] time = 0.0903092, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {459, 292, 31, 634, 617, 204, 628}

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3),x]
```


```
[Out] (B*x^2)/(2*b) - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*b^(5/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(1/3)*b^(5/3))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(-2Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{2b}$$
$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}b^{4/3}} + \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}b^{4/3}}$$
$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x}}{2b^{4/3}}$$
$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\,\text{Subst}\left(\int\right.}{\sqrt[3]{a}}$$
$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{6\sqrt[3]{a}b^{5/3}}$$

Mathematica [A] time = 0.0482049, size = 152, normalized size = 1.01

$$-\frac{(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}} + \frac{(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} - \frac{(aB-Ab)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} + \frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*
x]])/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]])/(6*a^(1/3)*b^(5/3))
```

Maple [A] time = 0.001, size = 198, normalized size = 1.3

$$\frac{Bx^2}{2\,b} - \frac{A}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{Ba}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{A}{6\,b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{Ba}{6\,b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/2*B*x^2/b-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A+1/3/b^2/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))*B*a+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/
b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B*a+1/3/b*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/b^2*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52438, size = 895, normalized size = 5.97

$$\left[\frac{3\,Bab^2x^2 - 3\,\sqrt{\frac{1}{3}}\left(Ba^2b - Aab^2\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right) - \left(-ab^2\right)^{\frac{2}{3}}(Ba -}{6\,ab^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^
2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (-a
*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
+ 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6*(3*B
```

```
*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(
2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*(-a
*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]
```

Sympy [A] time = 1.04161, size = 92, normalized size = 0.61

$$\frac{Bx^2}{2b} + \operatorname{RootSum}\left(27t^3ab^5 + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2ab^3}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**2/(2*b) + RootSum(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A*
B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(9*_t**2*a*b**3/(A**2*b**2 - 2*A*
B*a*b + B**2*a**2) + x)))
```

Giac [A] time = 1.12312, size = 247, normalized size = 1.65

$$\frac{Bx^2}{2\,b} + \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3} - \frac{((}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^
2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3)
 - 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a*b^3)
```

3.62 $\int \frac{A+Bx^3}{a+bx^3} dx$

Optimal. Leaf size=145

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{Bx}{b}$$

```
[Out] (B*x)/b - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rubi [A] time = 0.0768879, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (B*x)/b - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+bx^3}\,dx}{b}$$
$$= \frac{Bx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b} + \frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b}$$
$$= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} + \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,d}{2\sqrt[3]{a}b}$$
$$= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{-3-}\right.}{a^{2/3}b^{4/}}$$
$$= \frac{Bx}{b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.}{6a^{2/3}b^{4/3}}$$

Mathematica [A] time = 0.0608954, size = 129, normalized size = 0.89

$$\frac{-(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}Bx+2(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Maple [A] time = 0.002, size = 195, normalized size = 1.3

$$\frac{Bx}{b} + \frac{A}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{Ba}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{A}{6\,b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{Ba}{6\,b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a),x)
```

```
[Out] B*x/b+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))*B*a-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/b^2/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B*a+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51415, size = 900, normalized size = 6.21

$$\left[\frac{6\,Ba^2bx-3\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+\left(a^2b\right)^{\frac{2}{3}}\left(Ba-Ab\right)\mathrm{lo}}{6\,a^2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*B*a^2*b*x - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(a^2*b)^(1/3)/b)*
log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (a^2*b
)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(a
^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*B*a^2
```

```
*b*x - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/
3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*
b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(
a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2)]
```

Sympy [A] time = 1.15771, size = 71, normalized size = 0.49

$$\frac{Bx}{b} + \operatorname{RootSum}\left(27t^3a^2b^4 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(-\frac{3tab}{-Ab+Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x/b + RootSum(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2
*a**2*b + B**3*a**3, Lambda(_t, _t*log(-3*_t*a*b/(-A*b + B*a) + x)))
```

Giac [A] time = 1.11964, size = 217, normalized size = 1.5

$$\frac{Bx}{b} + \frac{(Ba - Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] B*x/b + 1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3
*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x
+ (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^
(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

3.63 $\int \frac{A+Bx^3}{x(a+bx^3)}\,dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^3])/(3*a*b)
```

Rubi [A] time = 0.0333877, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 72}

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^3])/(3*a*b)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{A}{ax} + \frac{-Ab+aB}{a(a+bx)}\right)dx, x, x^3\right)$$
$$= \frac{A\log(x)}{a} - \frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

Mathematica [A] time = 0.0126488, size = 34, normalized size = 1.

$$\frac{(aB-Ab)\log\left(a+bx^3\right)}{3ab} + \frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)
```

Maple [A] time = 0.006, size = 37, normalized size = 1.1

$$-\frac{\ln\left(bx^3+a\right)A}{3\,a} + \frac{\ln\left(bx^3+a\right)B}{3\,b} + \frac{A\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a),x)
```

```
[Out] -1/3/a*ln(b*x^3+a)*A+1/3/b*ln(b*x^3+a)*B+A*ln(x)/a
```

Maxima [A] time = 1.20634, size = 47, normalized size = 1.38

$$\frac{A\log\left(x^3\right)}{3\,a} + \frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*A*log(x^3)/a + 1/3*(B*a - A*b)*log(b*x^3 + a)/(a*b)
```

Fricas [A] time = 1.48043, size = 74, normalized size = 2.18

$$\frac{3\,Ab\log\left(x\right)+\left(Ba-Ab\right)\log\left(bx^{3}+a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)
```

Sympy [A] time = 1.93668, size = 26, normalized size = 0.76

$$\frac{A\log\left(x\right)}{a}+\frac{\left(-Ab+Ba\right)\log\left(\frac{a}{b}+x^{3}\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a),x)
```

```
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**3)/(3*a*b)
```

Giac [A] time = 1.13221, size = 46, normalized size = 1.35

$$\frac{A\log\left(|x|\right)}{a}+\frac{\left(Ba-Ab\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)
```

3.64 $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=147

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}-\frac{A}{ax}$$

```
[Out] -(A/(a*x)) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(4/3)*b^(2/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
4/3)*b^(2/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(4/3)*b^(2/3))
```

Rubi [A] time = 0.0856454, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {453, 292, 31, 634, 617, 204, 628}

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```


```
[Out] -(A/(a*x)) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(4/3)*b^(2/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
4/3)*b^(2/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(4/3)*b^(2/3))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = -\frac{A}{ax} - \frac{(Ab-aB)\int \frac{x}{a+bx^3}\,dx}{a}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}\sqrt[3]{b}} - \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{2/3}} - \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/}}}{2a\sqrt[3]{b}}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}} - \frac{(Ab-aB)\,\text{Subst}\Big(}{a^4}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{6a^{4/3}b^{2/3}}$$

Mathematica [A] time = 0.0807601, size = 134, normalized size = 0.91

$$\frac{-x(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-6\sqrt[3]{a}Ab^{2/3}+2x(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}x(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{4/3}b^{2/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*x
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)
```

Maple [A] time = 0.004, size = 195, normalized size = 1.3

$$-\frac{A}{ax}+\frac{A}{3a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{B}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{A}{6a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{B}{6b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a),x)
```

```
[Out] -A/a/x+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))*B-1/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/b/(a/b)^(
1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50286, size = 891, normalized size = 6.06

$$\left[-\frac{6\,Aab^2+3\,\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)x\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^3-ab-3\,\sqrt{\frac{1}{3}}\left(abx+2\left(ab^2\right)^{\frac{2}{3}}x^2-\left(ab^2\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}-3\left(ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)-\left(ab^2\right)^{\frac{2}{3}}\left(Ba-Ab\right)x}{6\,a^2b^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*A*a*b^2 + 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt(-(a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^
(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - (a*b^2)
^(2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(
a*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x), -1/6*(6*A
```

```
*a*b^2 + 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt((a*b^2)^(1/3)/a)*arctan(-sq
rt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (a*b^2)^(2/3)*(B
*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(a*b^2)^(2
/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x)]
```

Sympy [A] time = 0.742758, size = 90, normalized size = 0.61

$$-\frac{A}{ax}+\operatorname{RootSum}\left(27t^3a^4b^2-A^3b^3+3A^2Bab^2-3AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{9t^2a^3b}{A^2b^2-2ABab+B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a),x)
```

```
[Out] -A/(a*x) + RootSum(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B
**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(9*_t**2*a**3*b/(A**2*b**2 - 2*A*B
*a*b + B**2*a**2) + x)))
```

Giac [A] time = 1.18995, size = 239, normalized size = 1.63

$$-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}-\frac{A}{ax}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2}+\frac{((-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/a^2 - A/(a*x) - 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*
b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/6*((
-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^2*b^2)
```

3.65 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=149

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}-\frac{A}{2ax^2}$$

```
[Out] -A/(2*a*x^2) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(5/3)*b^(1/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a
^(5/3)*b^(1/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(6*a^(5/3)*b^(1/3))
```

Rubi [A] time = 0.0932772, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {453, 200, 31, 634, 617, 204, 628}

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] -A/(2*a*x^2) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(5/3)*b^(1/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a
^(5/3)*b^(1/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(6*a^(5/3)*b^(1/3))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{A}{2ax^2}-\frac{(2Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{2a}$$
$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}}-\frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}}$$
$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}}+\frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b}}{6a^{5/3}\sqrt[3]{b}}$$
$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\,\text{Subst}\left(\right.}{a}$$
$$= -\frac{A}{2ax^2}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{6a^{5/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.0991995, size = 135, normalized size = 0.91

$$\frac{\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{3a^{2/3}A}{x^2}+\frac{2(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3
) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(
6*a^(5/3))
```

Maple [A] time = 0.003, size = 195, normalized size = 1.3

$$-\frac{A}{2\,ax^2}-\frac{A}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{A}{6\,a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{6\,b}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a),x)
```

```
[Out] -1/2*A/a/x^2-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/3/b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*B+1/6/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/b/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57357, size = 960, normalized size = 6.44

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(Ba^2b - Aab^2\right)x^2\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+\left(-a^2b\right)^{\frac{2}{3}}(Ba-Ab)x^2}{6\,a^3bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt((-a^2*b)^(1/3)/b)*log((2*a*
b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3
)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) + (-a^2*b)^(2/
```

```
3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(
-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^(2/3)) + 3*A*a^2*b)/(a^3
*b*x^2), 1/6*(6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt(-(-a^2*b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3
)/b)/a^2) - (-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
 (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^
(2/3)) - 3*A*a^2*b)/(a^3*b*x^2)]
```

Sympy [A] time = 0.856832, size = 73, normalized size = 0.49

$$-\frac{A}{2ax^2} + \operatorname{RootSum}\left(27t^3a^5b + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{3ta^2}{-Ab+Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a),x)
```

```
[Out] -A/(2*a*x**2) + RootSum(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A
*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(3*_t*a**2/(-A*b + B*a) + x)))
```

Giac [A] time = 1.12158, size = 217, normalized size = 1.46

$$-\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*
((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a^2*b) + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2*A/(a*x^2)
```

3.66 $\int \frac{A+Bx^3}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=50

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

```
[Out] -A/(3*a*x^3) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3*a
^2)
```

Rubi [A] time = 0.0490322, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -A/(3*a*x^3) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3*a
^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}-\frac{b(-Ab+aB)}{a^2(a+bx)}\right)dx, x, x^3\right)$$
$$= -\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}$$

Mathematica [A] time = 0.020785, size = 49, normalized size = 0.98

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}+\frac{\log(x)(aB-Ab)}{a^2}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -A/(3*a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(
3*a^2)
```

Maple [A] time = 0.006, size = 56, normalized size = 1.1

$$\frac{\ln\left(bx^3+a\right)Ab}{3\,a^2}-\frac{\ln\left(bx^3+a\right)B}{3\,a}-\frac{A}{3\,ax^3}-\frac{A\ln(x)\,b}{a^2}+\frac{\ln(x)\,B}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3/a^2*ln(b*x^3+a)*A*b-1/3/a*ln(b*x^3+a)*B-1/3*A/a/x^3-1/a^2*ln(x)*A*b+1/a
*ln(x)*B
```

Maxima [A] time = 1.29595, size = 65, normalized size = 1.3

$$-\frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,a^2}+\frac{(Ba-Ab)\log\left(x^3\right)}{3\,a^2}-\frac{A}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A/
(a*x^3)
```

Fricas [A] time = 1.45138, size = 111, normalized size = 2.22

$$-\frac{(Ba - Ab)x^3 \log\left(bx^3 + a\right) - 3\,(Ba - Ab)x^3 \log(x) + Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^2
*x^3)
```

Sympy [A] time = 1.35958, size = 41, normalized size = 0.82

$$-\frac{A}{3ax^3} + \frac{(-Ab + Ba)\log(x)}{a^2} - \frac{(-Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a),x)
```

```
[Out] -A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3*
a**2)
```

Giac [A] time = 1.1008, size = 93, normalized size = 1.86

$$\frac{(Ba - Ab)\log(|x|)}{a^2} - \frac{\left(Bab - Ab^2\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^2b} - \frac{Bax^3 - Abx^3 + Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2*
b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)
```

3.67 $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}}-\frac{\sqrt[3]{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}$$

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(a^2*x) - (b^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (b^(1/3)*(A*b - a*B)*
Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)) + (b^(1/3)*(A*b - a*B)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3))
```

Rubi [A] time = 0.114788, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}}-\frac{\sqrt[3]{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```


```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(a^2*x) - (b^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (b^(1/3)*(A*b - a*B)*
Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)) + (b^(1/3)*(A*b - a*B)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = -\frac{A}{4ax^4} - \frac{(4Ab-4aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{4a}$$
$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} + \frac{(b(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{a^2}$$
$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}} + \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{7/3}}$$
$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\left(\sqrt[3]{b}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}} + \frac{\left(b^{2/\right.}{}$$
$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}} +$$
$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab-a}{}$$

Mathematica [A] time = 0.115591, size = 154, normalized size = 0.93

$$\frac{2\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{3a^{4/3}A}{x^4} + \frac{12\sqrt[3]{a}(Ab-aB)}{x} + 4\sqrt[3]{b}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - 4\sqrt{3}\sqrt[3]{b}(Ab-aB)}{12a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b -
 a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a*B
)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))
```

Maple [A] time = 0.008, size = 216, normalized size = 1.3

$$-\frac{Ab}{3\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{B}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{Ab}{6\,a^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{B}{6\,a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a),x)
```

```
[Out] -1/3*b/a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))*B+1/6*b/a^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/a/(a/b)
^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3*b/a^2*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/4*A/a/x^4+1/a^2/x*A*b-1/a/x*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48209, size = 371, normalized size = 2.25

$$-\frac{4\sqrt{3}(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+4\,(Ba}{12\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*(B*a - A*b)*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(
1/3) + 1/3*sqrt(3)) - 2*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 4*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)
```

Sympy [A] time = 1.40904, size = 112, normalized size = 0.68

$$\operatorname{RootSum}\left(27t^3a^7+A^3b^4-3A^2Bab^3+3AB^2a^2b^2-B^3a^3b,\left(t\mapsto t\log\left(\frac{9t^2a^5}{A^2b^3-2ABab^2+B^2a^2b}+x\right)\right)\right)-\frac{Aa+x^3\,(-4A}{4a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 -
B**3*a**3*b, Lambda(_t, _t*log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B**
2*a**2*b) + x))) - (A*a + x**3*(-4*A*b + 4*B*a))/(4*a**2*x**4)
```

Giac [A] time = 1.14394, size = 266, normalized size = 1.61

$$\frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b}-\frac{\left(\left(-ab^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^3 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6*((-a*b^2)^(
2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^
3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)
```

3.68 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=168

$$-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}}+\frac{Ab-a}{2a^2x}$$

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(2*a^2*x^2) - (b^(2/3)*(A*b - a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) + (b^(2/3)*(A*b - a
*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)) - (b^(2/3)*(A*b - a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3))
```

Rubi [A] time = 0.118899, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {453, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}}+\frac{Ab-a}{2a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```


```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(2*a^2*x^2) - (b^(2/3)*(A*b - a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) + (b^(2/3)*(A*b - a
*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)) - (b^(2/3)*(A*b - a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3))
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{A}{5ax^5} - \frac{(5Ab-5aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{5a}$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{(b(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{(b(Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}} + \frac{(b(Ab-aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{8/3}}$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{8/3}} + \frac{(b(A\ldots}{\ldots}$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}} + \ldots$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} - \frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{b^{2/3}(Ab-\ldots}{\ldots}$$

Mathematica [A] time = 0.126023, size = 154, normalized size = 0.92

$$\frac{5b^{2/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{15a^{2/3}(Ab-aB)}{x^2}-\frac{6a^{5/3}A}{x^5}+10b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}b^{2/3}(Ab-a\ldots}{30a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A*
b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))
```

Maple [A] time = 0.007, size = 217, normalized size = 1.3

$$\frac{Ab}{3\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{B}{3\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{Ab}{6\,a^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B}{6\,a}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a),x)
```

```
[Out] 1/3*b/a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*B-1/6*b/a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/a/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3*b/a^2/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/5*A/a/x^5+1/2/a^2/x^2*A*b-1/2/a/x^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50861, size = 419, normalized size = 2.49

$$-\frac{10\,\sqrt{3}(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-5\,(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10\,(Ba-}{30\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x
*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(b^
```

```
2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(B*a - A*b)*x^5*(
b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 15*(B*a - A*b)*x^3 + 6*A*a)/(
a^2*x^5)
```

Sympy [A] time = 1.6943, size = 99, normalized size = 0.59

$$\operatorname{RootSum}\left(27t^3a^8 - A^3b^5 + 3A^2Bab^4 - 3AB^2a^2b^3 + B^3a^3b^2, \left(t \mapsto t\log\left(-\frac{3ta^3}{-Ab^2 + Bab} + x\right)\right)\right) - \frac{2Aa + x^3(-5Ab + 5Ba}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8 - A**3*b**5 + 3*A**2*B*a*b**4 - 3*A*B**2*a**2*b**3 +
B**3*a**3*b**2, Lambda(_t, _t*log(-3*_t*a**3/(-A*b**2 + B*a*b) + x))) - (2*
A*a + x**3*(-5*A*b + 5*B*a))/(10*a**2*x**5)
```

Giac [A] time = 1.13158, size = 238, normalized size = 1.42

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3} + \frac{\left(Bab - Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/a^3 - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 1/10*(5*B*a*x^3 - 5*A*b*x^
3 + 2*A*a)/(a^2*x^5)
```

3.69 $\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx$

Optimal. Leaf size=69

$$\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}-\frac{A}{6ax^6}$$

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(3*a^2*x^3) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A
*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.0640418, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(3*a^2*x^3) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A
*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{A}{ax^3}+\frac{-Ab+aB}{a^2x^2}-\frac{b(-Ab+aB)}{a^3x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx,x,x^3\right)\\
&= -\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}
\end{aligned}
$$

Mathematica [A] time = 0.0284331, size = 70, normalized size = 1.01

$$
\frac{-a\left(aA+2aBx^3-2Abx^3\right)+6bx^6\log(x)(Ab-aB)+2bx^6(aB-Ab)\log\left(a+bx^3\right)}{6a^3x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*Log[x] + 2*b*(-(A
*b) + a*B)*x^6*Log[a + b*x^3])/(6*a^3*x^6)
```

Maple [A] time = 0.007, size = 81, normalized size = 1.2

$$
-\frac{b^2\ln\left(bx^3+a\right)A}{3\,a^3}+\frac{b\ln\left(bx^3+a\right)B}{3\,a^2}-\frac{A}{6\,ax^6}+\frac{Ab}{3\,a^2x^3}-\frac{B}{3\,ax^3}+\frac{A\ln(x)\,b^2}{a^3}-\frac{bB\ln(x)}{a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a),x)
```

```
[Out] -1/3*b^2/a^3*ln(b*x^3+a)*A+1/3*b/a^2*ln(b*x^3+a)*B-1/6*A/a/x^6+1/3/a^2/x^3*
A*b-1/3/a/x^3*B+1/a^3*b^2*ln(x)*A-1/a^2*b*ln(x)*B
```

Maxima [A] time = 1.36767, size = 95, normalized size = 1.38

$$
\frac{\left(Bab-Ab^2\right)\log\left(bx^3+a\right)}{3\,a^3}-\frac{\left(Bab-Ab^2\right)\log\left(x^3\right)}{3\,a^3}-\frac{2\,(Ba-Ab)x^3+Aa}{6\,a^2x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*a*b - A*b^2)*log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*log(x^3)/a^3 -
 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)
```

Fricas [A] time = 1.58467, size = 158, normalized size = 2.29

$$\frac{2\left(Bab - Ab^2\right)x^6 \log\left(bx^3 + a\right) - 6\left(Bab - Ab^2\right)x^6 \log\left(x\right) - 2\left(Ba^2 - Aab\right)x^3 - Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a*b - A*b^2)*x^6*log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*log(x) -
2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)
```

Sympy [A] time = 2.77085, size = 61, normalized size = 0.88

$$-\frac{Aa + x^3\left(-2Ab + 2Ba\right)}{6a^2x^6} - \frac{b\left(-Ab + Ba\right)\log\left(x\right)}{a^3} + \frac{b\left(-Ab + Ba\right)\log\left(\frac{a}{b} + x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a),x)
```

```
[Out] -(A*a + x**3*(-2*A*b + 2*B*a))/(6*a**2*x**6) - b*(-A*b + B*a)*log(x)/a**3 +
 b*(-A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Giac [A] time = 1.12729, size = 134, normalized size = 1.94

$$-\frac{\left(Bab - Ab^2\right)\log\left(|x|\right)}{a^3} + \frac{\left(Bab^2 - Ab^3\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^3b} + \frac{3\,Babx^6 - 3\,Ab^2x^6 - 2\,Ba^2x^3 + 2\,Aabx^3 - Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*log(abs(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*log(abs(b*x^3 + a)
)/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 - A*
a^2)/(a^3*x^6)
```

3.70 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}}+\frac{Ab-}{4a^2}$$

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(a^3*x) + (b^(4/3)
*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(10/3)) + (b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)) - (b^
(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/
3))
```

Rubi [A] time = 0.133885, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}}+\frac{Ab-}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(a^3*x) + (b^(4/3)
*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(10/3)) + (b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)) - (b^
(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/
3))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx = -\frac{A}{7ax^7}-\frac{(7Ab-7aB)\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{7a}$$

$$= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}+\frac{(b(Ab-aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{a^2}$$

$$= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}-\frac{\left(b^2(Ab-aB)\right)\int \frac{x}{a+bx^3}\,dx}{a^3}$$

$$= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}}-\frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b}}{3a^{10/3}}$$

$$= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{\left(b^{4/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x-}}{6a^{10/3}}$$

$$= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{10/3}}$$

$$= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3a^{10/3}}$$

Mathematica [A] time = 0.135009, size = 173, normalized size = 0.94

$$\frac{14b^{4/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{21a^{4/3}(Ab-aB)}{x^4}-\frac{12a^{7/3}A}{x^7}+28b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}b^{4/3}(Ab}{84a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b)
 + a*B))/x + 28*Sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4/3)
*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(10/3
```

```
))
```

Maple [A] time = 0.007, size = 247, normalized size = 1.3

$$\frac{b^2A}{3a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{Bb}{3a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{b^2A}{6a^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{Bb}{6a^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a),x)
```

```
[Out] 1/3*b^2/a^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/3*b/a^2/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))*B-1/6*b^2/a^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*
b/a^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3*b^2/a^3*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/3*b/a^2*3^(1/2)/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/7*A/a/x^7+1/4/a^2/x^4*A
*b-1/4/a/x^4*B-1/a^3*b^2/x*A+1/a^2*b/x*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.702, size = 424, normalized size = 2.3

$$\frac{28\sqrt{3}\left(Bab-Ab^2\right)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\left(Bab-Ab^2\right)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\left(Bab\right.}{84a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)
^(1/3) - 1/3*sqrt(3)) + 14*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x^2 - a*x*
(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x +
 a*(b/a)^(2/3)) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*A*a^
2)/(a^3*x^7)
```

Sympy [A] time = 1.30245, size = 139, normalized size = 0.76

$$\text{RootSum}\left(27t^3a^{10} - A^3b^7 + 3A^2Bab^6 - 3AB^2a^2b^5 + B^3a^3b^4, \left(t \mapsto t\log\left(\frac{9t^2a^7}{A^2b^5 - 2ABab^4 + B^2a^2b^3} + x\right)\right)\right) + \frac{-4Aa^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5 +
 B**3*a**3*b**4, Lambda(_t, _t*log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4 +
 B**2*a**2*b**3) + x))) + (-4*A*a**2 + x**6*(-28*A*b**2 + 28*B*a*b) + x**3*
(7*A*a*b - 7*B*a**2))/(28*a**3*x**7)
```

Giac [A] time = 1.13866, size = 292, normalized size = 1.59

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^4} - \frac{\left(Bab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^4} + \frac{\left(\left(-a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 - 1/3*(B*a*b^2*(-a/b)^(1/3) - A*b^3*(
-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/6*((-a*b^2)^(2
/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4
+ 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*a^2)/
(a^3*x^7)
```

3.71 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=233

$$-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}}+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}}-\frac{a^{4/3}(7Ab-10aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}}$$

```
[Out] -(a*(7*A*b - 10*a*B)*x)/(3*b^4) + ((7*A*b - 10*a*B)*x^4)/(12*b^3) - ((7*A*b
 - 10*a*B)*x^7)/(21*a*b^2) + ((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - (a^(4
/3)*(7*A*b - 10*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*b^(13/3)) + (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*
b^(13/3)) - (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*b^(13/3))
```

Rubi [A] time = 0.138068, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}}+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}}-\frac{a^{4/3}(7Ab-10aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(7*A*b - 10*a*B)*x)/(3*b^4) + ((7*A*b - 10*a*B)*x^4)/(12*b^3) - ((7*A*b
 - 10*a*B)*x^7)/(21*a*b^2) + ((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - (a^(4
/3)*(7*A*b - 10*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*b^(13/3)) + (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*
b^(13/3)) - (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*b^(13/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx = \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{(-7Ab+10aB)\int\frac{x^9}{a+bx^3}dx}{3ab}$$

$$= \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{(-7Ab+10aB)\int\left(\frac{a^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{b}-\frac{a^3}{b^3\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{\left(a^2(7Ab-10aB)\right)\int\frac{}{a+}}{3b^4}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{\left(a^{4/3}(7Ab-10aB)\right)\int}{9b^4}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{a^{4/3}(7Ab-10aB)\log\left(\right.}{9b^{13/3}}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{a^{4/3}(7Ab-10aB)\log\left(\right.}{9b^{13/3}}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}-\frac{a^{4/3}(7Ab-10aB)\tan^{-}}{3\sqrt{3}b^{13/3}}$$

Mathematica [A] time = 0.13434, size = 203, normalized size = 0.87

$$\frac{14a^{4/3}(10aB-7Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{84a^2\sqrt[3]{b}x(aB-Ab)}{a+bx^3}-28a^{4/3}(10aB-7Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}a^{4/3}(10a}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7/
3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*Sqrt[3]*a^(4/
3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 28*a^(4/
3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10*a*B
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(252*b^(13/3))
```

Maple [A] time = 0.017, size = 288, normalized size = 1.2

$$\frac{Bx^7}{7\,b^2}+\frac{Ax^4}{4\,b^2}-\frac{Bx^4a}{2\,b^3}-2\,\frac{aAx}{b^3}+3\,\frac{a^2Bx}{b^4}-\frac{a^2Ax}{3\,b^3\left(bx^3+a\right)}+\frac{a^3xB}{3\,b^4\left(bx^3+a\right)}+\frac{7\,Aa^2}{9\,b^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{7\,Aa^2}{18\,b^4}\ln\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/7/b^2*B*x^7+1/4/b^2*A*x^4-1/2/b^3*B*x^4*a-2/b^3*a*A*x+3/b^4*a^2*B*x-1/3*a
^2/b^3*x/(b*x^3+a)*A+1/3*a^3/b^4*x/(b*x^3+a)*B+7/9*a^2/b^4*A/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))-7/18*a^2/b^4*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+7/9*a^2/b^4*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-10/9*a^3/b^5*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/9*a^3/b^5*B/(a/b)^(2/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-10/9*a^3/b^5*B/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80624, size = 636, normalized size = 2.73

$$\frac{36\,Bb^3x^{10} - 9\left(10\,Bab^2 - 7\,Ab^3\right)x^7 + 63\left(10\,Ba^2b - 7\,Aab^2\right)x^4 - 28\,\sqrt{3}\left(10\,Ba^3 - 7\,Aa^2b + \left(10\,Ba^2b - 7\,Aab^2\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} \text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A*
a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^
3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(
10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A
*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^2*b)*x
)/(b^5*x^3 + a*b^4)
```

Sympy [A] time = 1.58968, size = 153, normalized size = 0.66

$$\frac{Bx^7}{7b^2} + \frac{x\left(-Aa^2b + Ba^3\right)}{3ab^4 + 3b^5x^3} + \operatorname{RootSum}\left(729t^3b^{13} - 343A^3a^4b^3 + 1470A^2Ba^5b^2 - 2100AB^2a^6b + 1000B^3a^7, \left(t \mapsto t\log\left(-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum
(729*_t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2
*a**6*b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**
2) + x))) - x**4*(-A*b + 2*B*a)/(4*b**3) + x*(-2*A*a*b + 3*B*a**2)/b**4
```

Giac [A] time = 1.12912, size = 329, normalized size = 1.41

$$-\frac{\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - 7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} + \frac{\left(10\,Ba^3 - 7\,Aa^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{\left(10\left(-a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*b)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/3)*
B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 - 14*B
*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14
```

3.72 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^3\right)}+\frac{x^3(Ab-2aB)}{3b^3}-\frac{a(2Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{Bx^6}{6b^2}$$

```
[Out] ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^6)/(6*b^2) - (a^2*(A*b - a*B))/(3*b^4*(a
 + b*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rubi [A] time = 0.0905451, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^3\right)}+\frac{x^3(Ab-2aB)}{3b^3}-\frac{a(2Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^6)/(6*b^2) - (a^2*(A*b - a*B))/(3*b^4*(a
 + b*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^8 \left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^2}+\frac{a(-2Ab+3aB)}{b^3(a+bx)}\right)dx,x,x^3\right)$$
$$= \frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^6}{6b^2}-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^3\right)}-\frac{a(2Ab-3aB)\log\left(a+bx^3\right)}{3b^4}$$

Mathematica [A] time = 0.0654002, size = 72, normalized size = 0.88

$$\frac{\frac{2a^2(aB-Ab)}{a+bx^3}+2bx^3(Ab-2aB)+2a(3aB-2Ab)\log\left(a+bx^3\right)+b^2Bx^6}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)
```

Maple [A] time = 0.016, size = 97, normalized size = 1.2

$$\frac{Bx^6}{6\,b^2}+\frac{Ax^3}{3\,b^2}-\frac{2\,Bx^3a}{3\,b^3}-\frac{a^2A}{3\,b^3\left(bx^3+a\right)}+\frac{Ba^3}{3\,b^4\left(bx^3+a\right)}-\frac{2\,a\ln\left(bx^3+a\right)A}{3\,b^3}+\frac{a^2\ln\left(bx^3+a\right)B}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*B*x^6/b^2+1/3/b^2*A*x^3-2/3/b^3*B*x^3*a-1/3*a^2/b^3/(b*x^3+a)*A+1/3*a^3
/b^4/(b*x^3+a)*B-2/3*a/b^3*ln(b*x^3+a)*A+a^2/b^4*ln(b*x^3+a)*B
```

Maxima [A] time = 0.961575, size = 111, normalized size = 1.35

$$\frac{Ba^{3}-Aa^{2}b}{3\left(b^{5}x^{3}+ab^{4}\right)}+\frac{Bbx^{6}-2\,(2\,Ba-Ab)x^{3}}{6\,b^{3}}+\frac{\left(3\,Ba^{2}-2\,Aab\right)\log\left(bx^{3}+a\right)}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)/(b^5*x^3 + a*b^4) + 1/6*(B*b*x^6 - 2*(2*B*a - A*b)*x^
3)/b^3 + 1/3*(3*B*a^2 - 2*A*a*b)*log(b*x^3 + a)/b^4
```

Fricas [A] time = 1.72699, size = 251, normalized size = 3.06

$$\frac{Bb^{3}x^{9}-\left(3\,Bab^{2}-2\,Ab^{3}\right)x^{6}+2\,Ba^{3}-2\,Aa^{2}b-2\left(2\,Ba^{2}b-Aab^{2}\right)x^{3}+2\left(3\,Ba^{3}-2\,Aa^{2}b+\left(3\,Ba^{2}b-2\,Aab^{2}\right)x^{3}\right)\log (bx^{3}}{6\left(b^{5}x^{3}+ab^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^3*x^9 - (3*B*a*b^2 - 2*A*b^3)*x^6 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^3 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^3)
*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A] time = 1.5349, size = 78, normalized size = 0.95

$$\frac{Bx^{6}}{6b^{2}}+\frac{a\,(-2Ab+3Ba)\log\left(a+bx^{3}\right)}{3b^{4}}+\frac{-Aa^{2}b+Ba^{3}}{3ab^{4}+3b^{5}x^{3}}-\frac{x^{3}\,(-Ab+2Ba)}{3b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**3)/(3*b**4) + (-A*a**2*b
+ B*a**3)/(3*a*b**4 + 3*b**5*x**3) - x**3*(-A*b + 2*B*a)/(3*b**3)
```

Giac [A] time = 1.11033, size = 143, normalized size = 1.74

$$\frac{\left(3\,Ba^{2}-2\,Aab\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,b^{4}}+\frac{Bb^{2}x^{6}-4\,Babx^{3}+2\,Ab^{2}x^{3}}{6\,b^{4}}-\frac{3\,Ba^{2}bx^{3}-2\,Aab^{2}x^{3}+2\,Ba^{3}-Aa^{2}b}{3\,\left(bx^{3}+a\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(B*b^2*x^6 - 4*B*a*b*
x^3 + 2*A*b^2*x^3)/b^4 - 1/3*(3*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 2*B*a^3 - A*a
^2*b)/((b*x^3 + a)*b^4)
```

3.73 $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=215

$$-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}} -$$

```
[Out] ((5*A*b - 8*a*B)*x^2)/(6*b^3) - ((5*A*b - 8*a*B)*x^5)/(15*a*b^2) + ((A*b -
a*B)*x^8)/(3*a*b*(a + b*x^3)) + (a^(2/3)*(5*A*b - 8*a*B)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(11/3)) + (a^(2/3)*(5*A*b - 8
*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(11/3)) - (a^(2/3)*(5*A*b - 8*a*B)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(11/3))
```

Rubi [A] time = 0.139865, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}} -$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```


```
[Out] ((5*A*b - 8*a*B)*x^2)/(6*b^3) - ((5*A*b - 8*a*B)*x^5)/(15*a*b^2) + ((A*b -
a*B)*x^8)/(3*a*b*(a + b*x^3)) + (a^(2/3)*(5*A*b - 8*a*B)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(11/3)) + (a^(2/3)*(5*A*b - 8
*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(11/3)) - (a^(2/3)*(5*A*b - 8*a*B)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(11/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7 \left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{(-5Ab+8aB)\int \frac{x^7}{a+bx^3}\,dx}{3ab}$$

$$=\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{(-5Ab+8aB)\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$=\frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}-\frac{(a(5Ab-8aB))\int \frac{x}{a+bx^3}\,dx}{3b^3}$$

$$=\frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{\left(a^{2/3}(5Ab-8aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9b^{10/3}}-\frac{\left(a^{2/3}(5Ab-\right.}{}$$

$$=\frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}-\frac{\left(a^{2/3}(5Ab\right.}{}$$

$$=\frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}-\frac{a^{2/3}(5Ab-}{}$$

$$=\frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}}+\frac{a^{2/3}(5Ab}{}$$

Mathematica [A] time = 0.124517, size = 185, normalized size = 0.86

$$\frac{5a^{2/3}(8aB-5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-10a^{2/3}(8aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}a^{2/3}(8aB-5Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{90b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*B
)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b^(
```

```
1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^(1/
3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(90*b^(11/3))
```

Maple [A] time = 0.009, size = 266, normalized size = 1.2

$$\frac{Bx^5}{5\,b^2}+\frac{Ax^2}{2\,b^2}-\frac{Bx^2a}{b^3}+\frac{aAx^2}{3\,b^2\left(bx^3+a\right)}-\frac{Bx^2a^2}{3\,b^3\left(bx^3+a\right)}+\frac{5\,aA}{9\,b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{5\,aA}{18\,b^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/5/b^2*B*x^5+1/2/b^2*A*x^2-1/b^3*B*x^2*a+1/3*a/b^2*x^2/(b*x^3+a)*A-1/3*a^2
/b^3*x^2/(b*x^3+a)*B+5/9*a/b^3*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18*a/b^3*A
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9*a/b^3*A*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-8/9*a^2/b^4*B/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))+4/9*a^2/b^4*B/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+8/
9*a^2/b^4*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65707, size = 599, normalized size = 2.79

$$\frac{18\,Bb^2x^8-9\left(8\,Bab-5\,Ab^2\right)x^5-15\left(8\,Ba^2-5\,Aab\right)x^2+10\,\sqrt{3}\left(\left(8\,Bab-5\,Ab^2\right)x^3+8\,Ba^2-5\,Aab\right)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^2
 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)
*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b -
5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)
^(2/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a
*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^4*x^3 + a*b^3)
```

Sympy [A] time = 2.51983, size = 151, normalized size = 0.7

$$\frac{Bx^5}{5b^2} - \frac{x^2\left(-Aab+Ba^2\right)}{3ab^3+3b^4x^3} + \operatorname{RootSum}\left(729t^3b^{11} - 125A^3a^2b^3 + 600A^2Ba^3b^2 - 960AB^2a^4b + 512B^3a^5, \left(t \mapsto t\log\left(\frac{}{25A^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) - x**2*(-A*a*b + B*a**2)/(3*a*b**3 + 3*b**4*x**3) + RootSum
(729*_t**3*b**11 - 125*A**3*a**2*b**3 + 600*A**2*B*a**3*b**2 - 960*A*B**2*a
**4*b + 512*B**3*a**5, Lambda(_t, _t*log(81*_t**2*b**7/(25*A**2*a*b**2 - 80
*A*B*a**2*b + 64*B**2*a**3) + x))) - x**2*(-A*b + 2*B*a)/(2*b**3)
```

Giac [A] time = 1.10413, size = 319, normalized size = 1.48

$$-\frac{\left(8\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3} - \frac{\sqrt{3}\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2
```

```
/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3*(B
*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a - 5*(-
a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*(2*B*
b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10
```

3.74 $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}}-\frac{x^4(4Ab-7aB)}{12ab^2}+\frac{x(4Ab-7aB)}{3b^3}-\frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}}+\frac{\sqrt[3]{a}}{}$$

```
[Out] ((4*A*b - 7*a*B)*x)/(3*b^3) - ((4*A*b - 7*a*B)*x^4)/(12*a*b^2) + ((A*b - a*
B)*x^7)/(3*a*b*(a + b*x^3)) + (a^(1/3)*(4*A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(10/3)) - (a^(1/3)*(4*A*b - 7*a
*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(10/3)) + (a^(1/3)*(4*A*b - 7*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(10/3))
```

Rubi [A] time = 0.130409, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}}-\frac{x^4(4Ab-7aB)}{12ab^2}+\frac{x(4Ab-7aB)}{3b^3}-\frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}}+\frac{\sqrt[3]{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```


```
[Out] ((4*A*b - 7*a*B)*x)/(3*b^3) - ((4*A*b - 7*a*B)*x^4)/(12*a*b^2) + ((A*b - a*
B)*x^7)/(3*a*b*(a + b*x^3)) + (a^(1/3)*(4*A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(10/3)) - (a^(1/3)*(4*A*b - 7*a
*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(10/3)) + (a^(1/3)*(4*A*b - 7*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(10/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6 \left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{(-4Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{(-4Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{(a(4Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\left(\sqrt[3]{a}(4Ab-7aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9b^3} - \frac{\left(\sqrt[3]{a}(4Ab-7\right.}{}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} + \frac{\left(\sqrt[3]{a}(4Ab-\right.}{}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} + \frac{\sqrt[3]{a}(4Ab-7}{}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{\sqrt[3]{a}(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} - \frac{\sqrt[3]{a}(4Ab-}{}$$

Mathematica [A] time = 0.1235, size = 181, normalized size = 0.85

$$\frac{-2\sqrt[3]{a}(7aB-4Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{12a\sqrt[3]{b}x(Ab-aB)}{a+bx^3}+36\sqrt[3]{b}x(Ab-2aB)+4\sqrt[3]{a}(7aB-4Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*x
)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)*x
```

```
)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x] -
 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(36*b^(10/3))
```

Maple [A] time = 0.009, size = 257, normalized size = 1.2

$$\frac{Bx^4}{4b^2}+\frac{Ax}{b^2}-2\frac{Bax}{b^3}+\frac{aAx}{3b^2\left(bx^3+a\right)}-\frac{a^2Bx}{3b^3\left(bx^3+a\right)}-\frac{4Aa}{9b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2Aa}{9b^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/4/b^2*B*x^4+1/b^2*A*x-2/b^3*B*a*x+1/3*a/b^2*x/(b*x^3+a)*A-1/3*a^2/b^3*x/(
b*x^3+a)*B-4/9*a/b^3*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9*a/b^3*A/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*a/b^3*A/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+7/9*a^2/b^4*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-7/18*a^2/b^4*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+7/9*a^2/b^4*
B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77681, size = 551, normalized size = 2.59

$$9Bb^2x^7-9\left(7Bab-4Ab^2\right)x^4-4\sqrt{3}\left(\left(7Bab-4Ab^2\right)x^3+7Ba^2-4Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\left(\left(7Bab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(9*B*b^2*x^7 - 9*(7*B*a*b - 4*A*b^2)*x^4 - 4*sqrt(3)*((7*B*a*b - 4*A*b
^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^
(2/3) - sqrt(3)*a)/a) + 2*((7*B*a*b - 4*A*b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a
/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*((7*B*a*b - 4*A*b^2)
*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 12*(7*B*a^2
- 4*A*a*b)*x)/(b^4*x^3 + a*b^3)
```

Sympy [A] time = 1.43998, size = 124, normalized size = 0.58

$$\frac{Bx^4}{4b^2} - \frac{x\left(-Aab + Ba^2\right)}{3ab^3 + 3b^4x^3} + \operatorname{RootSum}\left(729t^3b^{10} + 64A^3ab^3 - 336A^2Ba^2b^2 + 588AB^2a^3b - 343B^3a^4, \left(t \mapsto t\log\left(\frac{9tb^3}{-4Ab +}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) - x*(-A*a*b + B*a**2)/(3*a*b**3 + 3*b**4*x**3) + RootSum(72
9*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2*b**2 + 588*A*B**2*a**3*b -
 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4*A*b + 7*B*a) + x))) - x*(-A
*b + 2*B*a)/b**3
```

Giac [A] time = 1.12706, size = 285, normalized size = 1.34

$$\frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba - 4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^4} - \frac{\left(7Ba^2 - 4Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9ab^3} + \frac{\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/9*(7*B*a^2 - 4*A*a*b)*(-a/b)^(
```

```
1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(7*(-a*b^2)^(1/3)*B*a - 4*(-
a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2
*x - A*a*b*x)/((b*x^3 + a)*b^3) + 1/4*(B*b^6*x^4 - 8*B*a*b^5*x + 4*A*b^6*x)
/b^8
```

3.75 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal. Leaf size=60

$$\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}+\frac{Bx^3}{3b^2}$$

```
[Out] (B*x^3)/(3*b^2) + (a*(A*b - a*B))/(3*b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[
a + b*x^3])/(3*b^3)
```

Rubi [A] time = 0.0581286, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (B*x^3)/(3*b^2) + (a*(A*b - a*B))/(3*b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[
a + b*x^3])/(3*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx=\frac{1}{3}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{B}{b^2}+\frac{a(-Ab+aB)}{b^2(a+bx)^2}+\frac{Ab-2aB}{b^2(a+bx)}\right)dx,x,x^3\right)$$
$$=\frac{Bx^3}{3b^2}+\frac{a(Ab-aB)}{3b^3\left(a+bx^3\right)}+\frac{(Ab-2aB)\log\left(a+bx^3\right)}{3b^3}$$

Mathematica [A] time = 0.0350622, size = 50, normalized size = 0.83

$$\frac{\frac{a(Ab-aB)}{a+bx^3}+(Ab-2aB)\log\left(a+bx^3\right)+bBx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*b
^3)
```

Maple [A] time = 0.01, size = 74, normalized size = 1.2

$$\frac{Bx^3}{3\,b^2}+\frac{aA}{3\,b^2\left(bx^3+a\right)}-\frac{a^2B}{3\,b^3\left(bx^3+a\right)}+\frac{\ln\left(bx^3+a\right)A}{3\,b^2}-\frac{2\,\ln\left(bx^3+a\right)Ba}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*B*x^3/b^2+1/3/b^2*a/(b*x^3+a)*A-1/3/b^3*a^2/(b*x^3+a)*B+1/3/b^2*ln(b*x^
3+a)*A-2/3/b^3*ln(b*x^3+a)*B*a
```

Maxima [A] time = 0.970366, size = 81, normalized size = 1.35

$$\frac{Bx^3}{3\,b^2} - \frac{Ba^2 - Aab}{3\left(b^4x^3 + ab^3\right)} - \frac{(2\,Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^2 - 1/3*(B*a^2 - A*a*b)/(b^4*x^3 + a*b^3) - 1/3*(2*B*a - A*b)*l
og(b*x^3 + a)/b^3
```

Fricas [A] time = 1.66241, size = 165, normalized size = 2.75

$$\frac{Bb^2x^6 + Babx^3 - Ba^2 + Aab - \left(\left(2\,Bab - Ab^2\right)x^3 + 2\,Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{3\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + B*a*b*x^3 - B*a^2 + A*a*b - ((2*B*a*b - A*b^2)*x^3 + 2*B*a
^2 - A*a*b)*log(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] time = 1.20801, size = 56, normalized size = 0.93

$$\frac{Bx^3}{3b^2} - \frac{-Aab + Ba^2}{3ab^3 + 3b^4x^3} - \frac{(-Ab + 2Ba)\log\left(a + bx^3\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) - (-A*a*b + B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*
a)*log(a + b*x**3)/(3*b**3)
```

Giac [A] time = 1.12053, size = 123, normalized size = 2.05

$$\frac{\frac{(bx^3+a)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^3+a}-\frac{Aab^2}{bx^3+a}}{b^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^3 + a) - A*a*b^2/(b*x^3 + a))/b^3)/b
```

3.76 $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}}-\frac{x^2(2Ab-5aB)}{6ab^2}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{\ldots}}{\sqrt{3}\sqrt[3]{\ldots}}\right.}{3\sqrt{3}\sqrt[3]{a}b^{8/3}}$$

```
[Out] -((2*A*b - 5*a*B)*x^2)/(6*a*b^2) + ((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3)) -
((2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(1/3)*b^(8/3)) - ((2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3
)*b^(8/3)) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(1/3)*b^(8/3))
```

Rubi [A] time = 0.110349, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 321, 292, 31, 634, 617, 204, 628}

$$\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}}-\frac{x^2(2Ab-5aB)}{6ab^2}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{\ldots}}{\sqrt{3}\sqrt[3]{\ldots}}\right.}{3\sqrt{3}\sqrt[3]{a}b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((2*A*b - 5*a*B)*x^2)/(6*a*b^2) + ((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3)) -
((2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(1/3)*b^(8/3)) - ((2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3
)*b^(8/3)) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(1/3)*b^(8/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)} + \frac{(-2Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)} + \frac{(2Ab-5aB)\int \frac{x}{a+bx^3}\,dx}{3b^2}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)} - \frac{(2Ab-5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{7/3}} + \frac{(2Ab-5aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,d$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{18\sqrt[3]{a}b^{8/3}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{18\sqrt[3]{a}b^{8/3}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)} - \frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}}$$

Mathematica [A] time = 0.11884, size = 165, normalized size = 0.84

$$\frac{\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}} - \frac{6b^{2/3}x^2(Ab-aB)}{a+bx^3} + \frac{2(5aB-2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}(5aB-2Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + 9b^{2/3}Bx^2}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-2
*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(-2
```

```
*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))
```

Maple [A] time = 0.01, size = 235, normalized size = 1.2

$$\frac{Bx^2}{2b^2} - \frac{x^2A}{3b\left(bx^3+a\right)} + \frac{Bx^2a}{3b^2\left(bx^3+a\right)} + \frac{5Ba}{9b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{5Ba}{18b^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{5Ba\sqrt{3}}{9b^3}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/2*B*x^2/b^2-1/3/b*x^2/(b*x^3+a)*A+1/3/b^2*x^2/(b*x^3+a)*B*a+5/9/b^3*B*a/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18/b^3*B*a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))-5/9/b^3*B*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-2/9/b^2*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*A/(a/b)^(1/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/b^2*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79822, size = 1314, normalized size = 6.7

$$\left[\frac{9\,Bab^3x^5+3\left(5\,Ba^2b^2-2\,Aab^3\right)x^2-3\,\sqrt{\frac{1}{3}}\left(5\,Ba^3b-2\,Aa^2b^2+\left(5\,Ba^2b^2-2\,Aab^3\right)x^3\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(\right.\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*a
^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*
log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)
^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B
*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b
^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*
A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4), 1/1
8*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5*B*a^3*b
 - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a
*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2
)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*
a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4)]
```

Sympy [A] time = 2.24371, size = 126, normalized size = 0.64

$$\frac{Bx^2}{2b^2}+\frac{x^2\left(-Ab+Ba\right)}{3ab^2+3b^3x^3}+\operatorname{RootSum}\left(729t^3ab^8+8A^3b^3-60A^2Bab^2+150AB^2a^2b-125B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a}{4A^2b^2-20AB}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*
_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B**
```

```
3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25*B*
*2*a**2) + x)))
```

Giac [A] time = 1.13116, size = 285, normalized size = 1.45

$$\frac{Bx^2}{2\,b^2} + \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax^2 - Abx^2}{3\left(bx^3 + a\right)b^2} + \frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{2}{3}}Ba - 2\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\text{arct}}{9\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^2 + 1/9*(5*B*a*(-a/b)^(1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b
^2) + 1/9*sqrt(3)*(5*(-a*b^2)^(2/3)*B*a - 2*(-a*b^2)^(2/3)*A*b)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/18*(5*(-a*b^2)^(2/3)
*B*a - 2*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^
4)
```

3.77 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal. Leaf size=190

$$-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{x(Ab-4aB)}{3ab^2}$$

```
[Out] -((A*b - 4*a*B)*x)/(3*a*b^2) + ((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((A*
b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^
(2/3)*b^(7/3)) + ((A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(7/3
)) - ((A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(
2/3)*b^(7/3))
```

Rubi [A] time = 0.106447, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{x(Ab-4aB)}{3ab^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 4*a*B)*x)/(3*a*b^2) + ((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((A*
b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^
(2/3)*b^(7/3)) + ((A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(7/3
)) - ((A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(
2/3)*b^(7/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx=\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(-Ab+4aB)\int\frac{x^3}{a+bx^3}dx}{3ab}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\int\frac{1}{a+bx^3}dx}{3b^2}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^{2/3}b^2}+\frac{(Ab-4aB)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{9a^{2/3}b^2}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18a^{2/3}b^{7/3}}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{18a^{2/3}b^{7/3}}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-$$

Mathematica [A] time = 0.121352, size = 160, normalized size = 0.84

$$\frac{\frac{(4aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{2(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{2\sqrt{3}(4aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{6\sqrt[3]{b}x(Ab-aB)}{a+bx^3}+18\sqrt[3]{b}Bx}{18b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*b
) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b -
 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3) -
```

```
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))
```

Maple [A] time = 0.01, size = 228, normalized size = 1.2

$$\frac{Bx}{b^2}-\frac{xA}{3b\left(bx^3+a\right)}+\frac{Bax}{3b^2\left(bx^3+a\right)}-\frac{4Ba}{9b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2Ba}{9b^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{4Ba\sqrt{3}}{9b^3}\mathrm{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] B/b^2*x-1/3/b*x/(b*x^3+a)*A+1/3/b^2*x/(b*x^3+a)*B*a-4/9/b^3*B*a/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))+2/9/b^3*B*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)-4/9/b^3*B*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1
/9/b^2*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*A/(a/b)^(2/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))+1/9/b^2*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83695, size = 1289, normalized size = 6.78

$$\left[\frac{18\,Ba^2b^2x^4 - 3\,\sqrt{\frac{1}{3}}\left(4\,Ba^3b - Aa^2b^2 + \left(4\,Ba^2b^2 - Aab^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\left(a^2b\right)^{\frac{1}{3}}ax - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^2 + \left(a^2b\right)^{\frac{2}{3}}x - \left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-}}{bx^3 + a}\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2
 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*
x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a
*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*a^2*b^2
*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)*x^3)*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2
*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a*b -
A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*
(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]
```

Sympy [A] time = 1.9163, size = 102, normalized size = 0.54

$$\frac{Bx}{b^2} + \frac{x\left(-Ab + Ba\right)}{3ab^2 + 3b^3x^3} + \operatorname{RootSum}\left(729t^3a^2b^7 - A^3b^3 + 12A^2Bab^2 - 48AB^2a^2b + 64B^3a^3, \left(t \mapsto t\log\left(-\frac{9tab^2}{-Ab + 4Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**2
*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, Lam
```

```
bda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))
```

Giac [A] time = 1.13746, size = 254, normalized size = 1.34

$$\frac{Bx}{b^2}+\frac{(4\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2}-\frac{\sqrt{3}\left(4\,\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3}+\frac{Bax-Abx}{3\left(bx^3+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
 - 1/9*sqrt(3)*(4*(-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/3*(B*a*x - A*b*x)/((b*x^3
 + a)*b^2) - 1/18*(4*(-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

3.78 $\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=41

$$\frac{B\log\left(a+bx^3\right)}{3b^2}-\frac{Ab-aB}{3b^2\left(a+bx^3\right)}$$

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.0380932, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 43}

$$\frac{B\log\left(a+bx^3\right)}{3b^2}-\frac{Ab-aB}{3b^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)dx, x, x^3\right)$$
$$= -\frac{Ab-aB}{3b^2\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^2}$$

Mathematica [A] time = 0.0138577, size = 41, normalized size = 1.

$$\frac{aB-Ab}{3b^2\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (-(A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Maple [A] time = 0.01, size = 47, normalized size = 1.2

$$-\frac{A}{3\,b\left(bx^3+a\right)}+\frac{Ba}{3\,b^2\left(bx^3+a\right)}+\frac{B\ln\left(bx^3+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/b/(b*x^3+a)*A+1/3/b^2/(b*x^3+a)*B*a+1/3*B*ln(b*x^3+a)/b^2
```

Maxima [A] time = 0.974642, size = 54, normalized size = 1.32

$$\frac{Ba-Ab}{3\left(b^3x^3+ab^2\right)}+\frac{B\log\left(bx^3+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2
```

Fricas [A] time = 1.61487, size = 92, normalized size = 2.24

$$\frac{Ba - Ab + \left(Bbx^3 + Ba\right)\log\left(bx^3 + a\right)}{3\left(b^3x^3 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

Sympy [A] time = 1.41959, size = 36, normalized size = 0.88

$$\frac{B\log\left(a + bx^3\right)}{3b^2} + \frac{-Ab + Ba}{3ab^2 + 3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**2) + (-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3)
```

Giac [A] time = 1.12752, size = 88, normalized size = 2.15

$$-\frac{B\left(\frac{\log\left(\frac{|bx^3+a|}{\left(bx^3+a\right)^2|b|}\right)}{b} - \frac{a}{\left(bx^3+a\right)b}\right)}{3\,b} - \frac{A}{3\left(bx^3 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b
 - 1/3*A/((b*x^3 + a)*b)
```

3.79 $\int \frac{x(A+Bx^3)}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=171

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[Out] ((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - ((A*b + 2*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(5/3)) + ((A*b + 2*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(5/3))
```

Rubi [A] time = 0.0879305, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {457, 292, 31, 634, 617, 204, 628}

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - ((A*b + 2*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(5/3)) + ((A*b + 2*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(5/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)} + \frac{(Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)} - \frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}b^{4/3}} + \frac{(Ab+2aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{4/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\int -}{6}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}} + \frac{(Ab+2}{}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)} - \frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\log\left(a^{2/3}\right.}{18a^{4}}$$

Mathematica [A] time = 0.0903798, size = 146, normalized size = 0.85

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{6\sqrt[3]{a}b^{2/3}x^2(aB-Ab)}{a+bx^3}-2(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(2aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{4/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(1/
3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(4/3)*b^(5/3))
```

Maple [A] time = 0.008, size = 223, normalized size = 1.3

$$\frac{(Ab-Ba)\,x^2}{3\,ab\left(bx^3+a\right)} - \frac{A}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} - \frac{2\,B}{9\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{A}{18\,ab}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}} + \frac{B}{9\,b^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9/a/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-2/9
/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1/18/a/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))*A+1/9/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+
1/9/a/b*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+2/9/b
^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79243, size = 1235, normalized size = 7.22

$$\left[-\frac{6\,\left(Ba^2b^2 - Aab^3\right)x^2 - 3\,\sqrt{\frac{1}{3}}\left(2\,Ba^3b + Aa^2b^2 + \left(2\,Ba^2b^2 + Aab^3\right)x^3\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\,\left(-ab^2\right)^{\frac{2}{3}}x^2 + \left(-ab^2\right)^{\frac{1}{3}}\right.}{bx^3 + a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 +
(2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b +
3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)
```

```
^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 + 2*B
*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x -
 (-a*b^2)^(1/3)))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a*b^3)*x
^2 - 6*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt
(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2
)^(1/3)/a)/b) - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((2*B*a*b + A*b^2)*x^3
 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x^3
+ a^3*b^3)]
```

Sympy [A] time = 1.04782, size = 117, normalized size = 0.68

$$-\frac{x^2(-Ab+Ba)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^4b^5+A^3b^3+6A^2Bab^2+12AB^2a^2b+8B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a^3b^3}{A^2b^2+4ABab+4B^2a^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] -x**2*(-A*b + B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5
 + A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t,
 _t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

Giac [A] time = 1.12135, size = 273, normalized size = 1.6

$$-\frac{\left(2\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}-\frac{Bax^2-Abx^2}{3\left(bx^3+a\right)ab}-\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba+\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\right.}{3\left(-\right.}\right.}{9\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*B*a*(-a/b)^(1/3) + A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/(a^2*b) - 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*a*b) - 1/9*sqrt(3)
*(2*(-a*b^2)^(2/3)*B*a + (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/18*(2*(-a*b^2)^(2/3)*B*a + (-a*b^2)^(
```

```
2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

3.80 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right.}$$

```
[Out] ((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*A*b + a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*A*b + a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rubi [A] time = 0.0862685, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^2,x]
```


```
[Out] ((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*A*b + a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*A*b + a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b} + \frac{(2Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\int \frac{\;}{a^{2/3}}}{6a^4}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(2Ab+aB}{}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\log\left(a^{2/3}\right.}{18a^{5/3}}$$

Mathematica [A] time = 0.0895817, size = 145, normalized size = 0.86

$$\frac{-(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{6a^{2/3}\sqrt[3]{b}x(aB-Ab)}{a+bx^3} + 2(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - 2\sqrt{3}(aB+2Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{}}{}}{\sqrt{3}}\right.}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

Maple [A] time = 0.007, size = 221, normalized size = 1.3

$$\frac{(Ab-Ba)\,x}{3\,ab\left(bx^3+a\right)} + \frac{2\,A}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{B}{9\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{A}{9\,ab}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{B}{18\,b^2}\ln\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+2/9/a/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/9/b
^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-1/9/a/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))*A-1/18/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+2/
9/a/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/9/b^2
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89835, size = 1233, normalized size = 7.3

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(Ba^3b+2\,Aa^2b^2+\left(Ba^2b^2+2\,Aab^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(\left(\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
```

```
(b*x^3 + a)) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B
*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a
^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^
2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b + 2
*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

Sympy [A] time = 1.25105, size = 97, normalized size = 0.57

$$-\frac{x\left(-Ab+Ba\right)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^5b^4-8A^3b^3-12A^2Bab^2-6AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{2Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] -x*(-A*b + B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 -
8*A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t
*log(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

Giac [A] time = 1.14177, size = 246, normalized size = 1.46

$$-\frac{\left(Ba+2\,Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba+2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}-\frac{Bax-Abx}{3\left(bx^3+a\right)ab}+\frac{\left(\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/9*sq
rt(3)*((-a*b^2)^(1/3)*B*a + 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a
*b) + 1/18*((-a*b^2)^(1/3)*B*a + 2*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(
```

```
1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

3.81 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^3)) + (A*Log[x])/a^2 - (A*Log[a + b*x^3])/(3*a^
2)
```

Rubi [A] time = 0.0458923, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^3)) + (A*Log[x])/a^2 - (A*Log[a + b*x^3])/(3*a^
2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{A}{a^2x} + \frac{-Ab+aB}{a(a+bx)^2} - \frac{Ab}{a^2(a+bx)}\right) dx, x, x^3\right)$$
$$= \frac{Ab-aB}{3ab\left(a+bx^3\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^3\right)}{3a^2}$$

Mathematica [A] time = 0.0302738, size = 46, normalized size = 0.9

$$\frac{\frac{a(Ab-aB)}{b\left(a+bx^3\right)} - A\log\left(a+bx^3\right) + 3A\log(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)
```

Maple [A] time = 0.014, size = 53, normalized size = 1.

$$\frac{A}{3\,a\left(bx^3+a\right)} - \frac{B}{3\,b\left(bx^3+a\right)} - \frac{A\ln\left(bx^3+a\right)}{3\,a^2} + \frac{A\ln\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a/(b*x^3+a)*A-1/3/b/(b*x^3+a)*B-1/3*A*ln(b*x^3+a)/a^2+A*ln(x)/a^2
```

Maxima [A] time = 1.03537, size = 69, normalized size = 1.35

$$-\frac{Ba-Ab}{3\left(ab^2x^3+a^2b\right)} - \frac{A\log\left(bx^3+a\right)}{3\,a^2} + \frac{A\log\left(x^3\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*log
(x^3)/a^2
```

Fricas [A] time = 1.712, size = 151, normalized size = 2.96

$$-\frac{Ba^2 - Aab + \left(Ab^2x^3 + Aab\right)\log\left(bx^3 + a\right) - 3\left(Ab^2x^3 + Aab\right)\log\left(x\right)}{3\left(a^2b^2x^3 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 + A
*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)
```

Sympy [A] time = 1.15689, size = 46, normalized size = 0.9

$$\frac{A\log\left(x\right)}{a^2} - \frac{A\log\left(\frac{a}{b} + x^3\right)}{3a^2} - \frac{-Ab + Ba}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**3)/(3*a**2) - (-A*b + B*a)/(3*a**2*b + 3*a*b
**2*x**3)
```

Giac [A] time = 1.11872, size = 82, normalized size = 1.61

$$-\frac{A\log\left(\left|bx^3 + a\right|\right)}{3\,a^2} + \frac{A\log\left(\left|x\right|\right)}{a^2} + \frac{Ab^2x^3 - Ba^2 + 2\,Aab}{3\left(bx^3 + a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^2
 + 2*A*a*b)/((b*x^3 + a)*a^2*b)
```

3.82 $\int \frac{A+Bx^3}{x^2(a+bx^3)^2} \, dx$

Optimal. Leaf size=196

$$-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}-\frac{4Ab-aB}{3a^2bx}+$$

```
[Out] -(4*A*b - a*B)/(3*a^2*b*x) + (A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b -
a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*
b^(2/3)) + ((4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(2/3)) - (
(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b
^(2/3))
```

Rubi [A] time = 0.105552, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}-\frac{4Ab-aB}{3a^2bx}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(4*A*b - a*B)/(3*a^2*b*x) + (A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b -
a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*
b^(2/3)) + ((4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(2/3)) - (
(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b
^(2/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}-\frac{(4Ab-aB)\int \frac{x}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{7/3}\sqrt[3]{b}}-\frac{(4Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{7/3}\sqrt[3]{b}}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}-\frac{(4Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}b^{2/3}}-$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^2\right.}{18a^{7/3}b^{2/3}}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}-\frac{(4A}{}$$

Mathematica [A] time = 0.123271, size = 164, normalized size = 0.84

$$\frac{\frac{(aB-4Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{2\sqrt{3}(4Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6\sqrt[3]{a}x^2(aB-Ab)}{a+bx^3}-\frac{18\sqrt[3]{a}A}{x}}{18a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3
]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(
```

```
4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))
```

Maple [A] time = 0.01, size = 241, normalized size = 1.2

$$-\frac{Ax^2b}{3\,a^2\left(bx^3+a\right)}+\frac{x^2B}{3\,a\left(bx^3+a\right)}+\frac{4\,A}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{2\,A}{9\,a^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{4\,A\sqrt{3}}{9\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^2/(b*x^3+a)*A*b+1/3/a*x^2/(b*x^3+a)*B+4/9/a^2*A/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))-2/9/a^2*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/a
^2*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a*B/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+1/9/a*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-A/a^2/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84136, size = 1296, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*sqrt(1/3)*((B*a^2*
b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*lo
g((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/
3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((B*a*b -
4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/
3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(
a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x), -1/18*(18
*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*sqrt(1/3)*((B*a^2*b^2 - 4*A*
a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((B*a*b - 4*A*b^2)*
x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x +
(a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2
/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x)]
```

Sympy [A] time = 1.36558, size = 122, normalized size = 0.62

$$\frac{-3Aa + x^3\left(-4Ab + Ba\right)}{3a^3x + 3a^2bx^4} + \operatorname{RootSum}\left(729t^3a^7b^2 - 64A^3b^3 + 48A^2Bab^2 - 12AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(\frac{81t^2a^5}{16A^2b^2 - 8AB}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t*
*3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*a*
*3, Lambda(_t, _t*log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a**2
) + x)))
```

Giac [A] time = 1.14221, size = 273, normalized size = 1.39

$$-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 4\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{Bax^3 - 4\,Abx^3 - 3\,Aa}{3\left(bx^4 + ax\right)a^2} - \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - 4\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan}{9\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(B*a*(-a/b)^(1/3) - 4*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b
)^(1/3)))/a^3 + 1/3*(B*a*x^3 - 4*A*b*x^3 - 3*A*a)/((b*x^4 + a*x)*a^2) - 1/9
*sqrt(3)*((-a*b^2)^(2/3)*B*a - 4*(-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*
x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/18*((-a*b^2)^(2/3)*B*a - 4*(-
a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

3.83 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{5Ab-2aB}{6a^2bx^2}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}$$

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*b*x^2) + (A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((5*
A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(8/3)*b^(1/3)) - ((5*A*b - 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^
(1/3)) + ((5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
18*a^(8/3)*b^(1/3))
```

Rubi [A] time = 0.102756, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{5Ab-2aB}{6a^2bx^2}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```


```
[Out] -(5*A*b - 2*a*B)/(6*a^2*b*x^2) + (A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((5*
A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(8/3)*b^(1/3)) - ((5*A*b - 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^
(1/3)) + ((5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
18*a^(8/3)*b^(1/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}-\frac{(5Ab-2aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{6a^{7/3}}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{18a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.12531, size = 163, normalized size = 0.83

$$\frac{\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{6a^{2/3}x(aB-Ab)}{a+bx^3}-\frac{9a^{2/3}A}{x^2}+\frac{2(2aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}(5Ab-2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*Sqrt[3]
*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*
```

```
(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))
```

Maple [A] time = 0.012, size = 237, normalized size = 1.2

$$-\frac{Abx}{3\,a^2\left(bx^3+a\right)}+\frac{xB}{3\,a\left(bx^3+a\right)}-\frac{5\,A}{9\,a^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,A}{18\,a^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5\,A\sqrt{3}}{9\,a^2}\arctan\left(\frac{\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x/(b*x^3+a)*A*b+1/3/a*x/(b*x^3+a)*B-5/9/a^2*A/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))+5/18/a^2*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9/a^2*
A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a*B/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a*B/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+2/9/a*B/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-1/2*A/a^2/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77428, size = 1396, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*sqrt(1/3)*((2*B*a^2
*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (
-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) +
((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*
x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (
2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^
2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 - 6*
sqrt(1/3)*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*s
qrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3
)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5
*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*
log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^2*x^5 + a^5*b*x^2)]
```

Sympy [A] time = 0.944601, size = 109, normalized size = 0.56

$$\frac{-3Aa + x^3\left(-5Ab + 2Ba\right)}{6a^3x^2 + 6a^2bx^5} + \operatorname{RootSum}\left(729t^3a^8b + 125A^3b^3 - 150A^2Bab^2 + 60AB^2a^2b - 8B^3a^3, \left(t \mapsto t\log\left(\frac{9ta^3}{-5Ab + 2B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(72
9*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8*B
**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))
```

Giac [A] time = 1.14669, size = 254, normalized size = 1.3

$$-\frac{\left(2\,Ba - 5\,Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}\,Ba - 5\,\left(-ab^2\right)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} + \frac{Bax - Abx}{3\left(bx^3 + a\right)a^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*B*a - 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/9*sqrt
(3)*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a^2
) + 1/18*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^3*b) - 1/2*A/(a^2*x^2)
```

3.84 $\int \frac{A+Bx^3}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{A}{3a^2x^3}$$

```
[Out] -A/(3*a^2*x^3) - (A*b - a*B)/(3*a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x])/a
^3 + ((2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.0758433, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -A/(3*a^2*x^3) - (A*b - a*B)/(3*a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x])/a
^3 + ((2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{A}{a^2x^2} + \frac{-2Ab+aB}{a^3x} - \frac{b(-Ab+aB)}{a^2(a+bx)^2} - \frac{b(-2Ab+aB)}{a^3(a+bx)}\right) dx, x, x^3\right)$$
$$= -\frac{A}{3a^2x^3} - \frac{Ab-aB}{3a^2\left(a+bx^3\right)} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

Mathematica [A] time = 0.0469824, size = 64, normalized size = 0.84

$$\frac{\frac{a(aB-Ab)}{a+bx^3} + (2Ab-aB)\log\left(a+bx^3\right) + 3\log(x)(aB-2Ab) - \frac{aA}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Maple [A] time = 0.014, size = 87, normalized size = 1.1

$$-\frac{Ab}{3\,a^2\left(bx^3+a\right)} + \frac{B}{3\,a\left(bx^3+a\right)} + \frac{2\,b\ln\left(bx^3+a\right)A}{3\,a^3} - \frac{\ln\left(bx^3+a\right)B}{3\,a^2} - \frac{A}{3\,a^2x^3} - 2\,\frac{A\ln(x)\,b}{a^3} + \frac{\ln(x)\,B}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*b/(b*x^3+a)*A+1/3/a/(b*x^3+a)*B+2/3/a^3*b*ln(b*x^3+a)*A-1/3/a^2*ln
(b*x^3+a)*B-1/3*A/a^2/x^3-2/a^3*ln(x)*A*b+1/a^2*ln(x)*B
```

Maxima [A] time = 0.948898, size = 103, normalized size = 1.36

$$\frac{(Ba-2\,Ab)x^3-Aa}{3\left(a^2bx^6+a^3x^3\right)}-\frac{(Ba-2\,Ab)\log\left(bx^3+a\right)}{3\,a^3}+\frac{(Ba-2\,Ab)\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*log
(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3
```

Fricas [A] time = 1.6387, size = 247, normalized size = 3.25

$$\frac{\left(Ba^2-2\,Aab\right)x^3-Aa^2-\left(\left(Bab-2\,Ab^2\right)x^6+\left(Ba^2-2\,Aab\right)x^3\right)\log\left(bx^3+a\right)+3\left(\left(Bab-2\,Ab^2\right)x^6+\left(Ba^2-2\,Aab\right)x^3\right)\mathrm{l}}{3\left(a^3bx^6+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*
a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x^3
)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

Sympy [A] time = 1.56255, size = 70, normalized size = 0.92

$$\frac{-Aa+x^3\,(-2Ab+Ba)}{3a^3x^3+3a^2bx^6}+\frac{(-2Ab+Ba)\log{(x)}}{a^3}-\frac{(-2Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)
```

```
[Out] (-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Giac [A] time = 1.10326, size = 108, normalized size = 1.42

$$\frac{(Ba-2\,Ab)\log\left(|x|\right)}{a^3}+\frac{Bax^3-2\,Abx^3-Aa}{3\left(bx^6+ax^3\right)a^2}-\frac{\left(Bab-2\,Ab^2\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 + a
*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)
```

3.85 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}}-\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}-\frac{\sqrt[3]{b}(7Ab-\dots}{}$$

```
[Out] -(7*A*b - 4*a*B)/(12*a^2*b*x^4) + (7*A*b - 4*a*B)/(3*a^3*x) + (A*b - a*B)/(
3*a*b*x^4*(a + b*x^3)) - (b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (b^(1/3)*(7*A*b - 4*a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)) + (b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3))
```

Rubi [A] time = 0.126611, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}}-\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}-\frac{\sqrt[3]{b}(7Ab-\dots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```


```
[Out] -(7*A*b - 4*a*B)/(12*a^2*b*x^4) + (7*A*b - 4*a*B)/(3*a^3*x) + (A*b - a*B)/(
3*a*b*x^4*(a + b*x^3)) - (b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (b^(1/3)*(7*A*b - 4*a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)) + (b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^4\left(a+bx^3\right)}+\frac{(7Ab-4aB)\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{(7Ab-4aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3a^2}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}+\frac{(b(7Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\left(b^{2/3}(7Ab-4aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{10/3}}+\frac{\left(b^{2/3}(7Ab-4a\ldots\right.}{\ldots}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}+\frac{\left(\sqrt[3]{b}(7Ab-4a\ldots\right.}{\ldots}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}+\frac{\sqrt[3]{b}(7Ab-4a\ldots}{\ldots}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{\sqrt[3]{b}(7Ab-4a\ldots}{9\ldots}$$

Mathematica [A] time = 0.136187, size = 185, normalized size = 0.86

$$\frac{2\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{9a^{4/3}A}{x^4}-\frac{12\sqrt[3]{a}bx^2(aB-Ab)}{a+bx^3}-\frac{36\sqrt[3]{a}(aB-2Ab)}{x}+4\sqrt[3]{b}(4aB-7Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\ldots}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b)
 + a*B)*x^2)/(a + b*x^3) - 4*Sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) + b^
(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(36*a^(10/3))
```

Maple [A] time = 0.013, size = 257, normalized size = 1.2

$$\frac{Ax^2b^2}{3\,a^3\left(bx^3+a\right)}-\frac{bBx^2}{3\,a^2\left(bx^3+a\right)}-\frac{7\,Ab}{9\,a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{7\,Ab}{18\,a^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{7\,Ab\sqrt{3}}{9\,a^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a^3*b^2*x^2/(b*x^3+a)*A-1/3/a^2*b*x^2/(b*x^3+a)*B-7/9/a^3*b*A/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))+7/18/a^3*b*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+7/9/a^3*b*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)+4/9/a^2*B/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*B/(a/b)^(1/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))-4/9/a^2*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))-1/4*A/a^2/x^4+2/a^3/x*A*b-1/a^2/x*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81541, size = 585, normalized size = 2.72

$$-\frac{12\left(4\,Bab-7\,Ab^2\right)x^6+9\left(4\,Ba^2-7\,Aab\right)x^3+9\,Aa^2+4\,\sqrt{3}\left(\left(4\,Bab-7\,Ab^2\right)x^7+\left(4\,Ba^2-7\,Aab\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 + 4
*sqrt(3)*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 2*((4*B*a*b - 7*A*b^2)*x^
7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*
(-b/a)^(1/3)) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a
)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^3*b*x^7 + a^4*x^4)
```

Sympy [A] time = 1.24492, size = 153, normalized size = 0.71

$$\mathrm{RootSum}\left(729t^3a^{10} + 343A^3b^4 - 588A^2Bab^3 + 336AB^2a^2b^2 - 64B^3a^3b, \left(t \mapsto t\log\left(\frac{81t^2a^7}{49A^2b^3 - 56ABab^2 + 16B^2a^2b} + x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a*
*2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 - 5
6*A*B*a*b**2 + 16*B**2*a**2*b) + x))) - (3*A*a**2 + x**6*(-28*A*b**2 + 16*B
*a*b) + x**3*(-21*A*a*b + 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)
```

Giac [A] time = 1.65287, size = 312, normalized size = 1.45

$$\frac{\left(4\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4} + \frac{\sqrt{3}\left(4\,\left(-ab^2\right)^{\frac{2}{3}}\,Ba - 7\,\left(-ab^2\right)^{\frac{2}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b} - \frac{B\ldots}{3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(4*B*a*b*(-a/b)^(1/3) - 7*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/a^4 + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)*A
```

```
*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/3*(B*
a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^(2/3)*B*a - 7*(-a
*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/4*(4*
B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)
```

3.86 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=215

$$-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}+$$

```
[Out] -(8*A*b - 5*a*B)/(15*a^2*b*x^5) + (8*A*b - 5*a*B)/(6*a^3*x^2) + (A*b - a*B)
/(3*a*b*x^5*(a + b*x^3)) - (b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)) + (b^(2/3)*(8*A*b - 5*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)) - (b^(2/3)*(8*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3))
```

Rubi [A] time = 0.127009, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```


```
[Out] -(8*A*b - 5*a*B)/(15*a^2*b*x^5) + (8*A*b - 5*a*B)/(6*a^3*x^2) + (A*b - a*B)
/(3*a*b*x^5*(a + b*x^3)) - (b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)) + (b^(2/3)*(8*A*b - 5*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)) - (b^(2/3)*(8*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{(8Ab-5aB)\int \frac{1}{x^6\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} - \frac{(8Ab-5aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{3a^2}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{(b(8Ab-5aB))\int \frac{1}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{(b(8Ab-5aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{11/3}} + \frac{(b(8Ab-5aB))\int}{9a}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} - \frac{\left(b^{2/3}(8Ab-5\right.}{}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} - \frac{b^{2/3}(8Ab-5a}{}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5\left(a+bx^3\right)} - \frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}} + \frac{b^{2/3}(8Ab-5}{}$$

Mathematica [A] time = 0.142564, size = 183, normalized size = 0.85

$$\frac{5b^{2/3}(5aB-8Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{30a^{2/3}bx(aB-Ab)}{a+bx^3} - \frac{45a^{2/3}(aB-2Ab)}{x^2} - \frac{18a^{5/3}A}{x^5} + 10b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(A
*b) + a*B)*x)/(a + b*x^3) - 10*Sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(90*a^(11/3))
```

Maple [A] time = 0.013, size = 252, normalized size = 1.2

$$\frac{b^2Ax}{3a^3(bx^3+a)}-\frac{bBx}{3a^2(bx^3+a)}+\frac{8Ab}{9a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{4Ab}{9a^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{8Ab\sqrt{3}}{9a^3}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a^3*b^2*x/(b*x^3+a)*A-1/3/a^2*b*x/(b*x^3+a)*B+8/9/a^3*b*A/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))-4/9/a^3*b*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
8/9/a^3*b*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/9
/a^2*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))-5/9/a^2*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))-1/5*A/a^2/x^5+1/a^3/x^2*A*b-1/2/a^2/x^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65282, size = 633, normalized size = 2.94

$$-\frac{15\left(5Bab-8Ab^2\right)x^6+9\left(5Ba^2-8Aab\right)x^3+18Aa^2+10\sqrt{3}\left(\left(5Bab-8Ab^2\right)x^8+\left(5Ba^2-8Aab\right)x^5\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 +
10*sqrt(3)*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*((5*B*a*b
 - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*((5*B*a*b - 8*A*b^2)*x^8 +
(5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)))/(a^3
*b*x^8 + a^4*x^5)
```

Sympy [A] time = 1.60649, size = 138, normalized size = 0.64

$$\operatorname{RootSum}\left(729t^3a^{11}-512A^3b^5+960A^2Bab^4-600AB^2a^2b^3+125B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{9ta^4}{-8Ab^2+5Bab}+x\right)\right)\right)-\frac{6Aa^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a*
*2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5*B
*a*b) + x))) - (6*A*a**2 + x**6*(-40*A*b**2 + 25*B*a*b) + x**3*(-24*A*a*b +
 15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)
```

Giac [A] time = 1.14912, size = 278, normalized size = 1.29

$$-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-8\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^4}+\frac{\left(5Bab-8Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4}-\frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/18*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b
^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4 - 1/3*(B*a*b*x
- A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(a^3*x
^5)
```

3.87 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{A}{6a^2x^6}$$

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(A*b - a*B))/(3*a^3*(a + b*
x^3)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^3])
/(3*a^4)
```

Rubi [A] time = 0.0994124, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```


```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(A*b - a*B))/(3*a^3*(a + b*
x^3)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^3])
/(3*a^4)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)}\right) dx, x,$$
$$= -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}$$

Mathematica [A] time = 0.0946432, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^6}+\frac{2ab(aB-Ab)}{a+bx^3}+\frac{2a(aB-2Ab)}{x^3}+2b(3Ab-2aB)\log\left(a+bx^3\right)-6b\log(x)(3Ab-2aB)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a + b*x^
3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])/(6*a^
4)
```

Maple [A] time = 0.016, size = 116, normalized size = 1.2

$$\frac{Ab^2}{3\,a^3\left(bx^3+a\right)}-\frac{Bb}{3\,a^2\left(bx^3+a\right)}-\frac{b^2\ln\left(bx^3+a\right)A}{a^4}+\frac{2\,b\ln\left(bx^3+a\right)B}{3\,a^3}-\frac{A}{6\,a^2x^6}+\frac{2\,Ab}{3\,a^3x^3}-\frac{B}{3\,a^2x^3}+3\,\frac{A\ln(x)\,b^2}{a^4}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a^3*b^2/(b*x^3+a)*A-1/3/a^2*b/(b*x^3+a)*B-1/a^4*b^2*ln(b*x^3+a)*A+2/3/a
^3*b*ln(b*x^3+a)*B-1/6*A/a^2/x^6+2/3/a^3/x^3*A*b-1/3/a^2/x^3*B+3*b^2/a^4*ln
```

```
(x)*A-2*b/a^3*ln(x)*B
```

Maxima [A] time = 0.966946, size = 143, normalized size = 1.47

$$-\frac{2\left(2\,Bab-3\,Ab^2\right)x^6+\left(2\,Ba^2-3\,Aab\right)x^3+Aa^2}{6\left(a^3bx^9+a^4x^6\right)}+\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(bx^3+a\right)}{3\,a^4}-\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*x
^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b -
 3*A*b^2)*log(x^3)/a^4
```

Fricas [A] time = 1.48455, size = 327, normalized size = 3.37

$$-\frac{2\left(2\,Ba^2b-3\,Aab^2\right)x^6+Aa^3+\left(2\,Ba^3-3\,Aa^2b\right)x^3-2\left(\left(2\,Bab^2-3\,Ab^3\right)x^9+\left(2\,Ba^2b-3\,Aab^2\right)x^6\right)\log\left(bx^3+a\right)+6\left(\left(\right.\right.}{6\left(a^4bx^9+a^5x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a) +
 6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(x))/(a^4*b
*x^9 + a^5*x^6)
```

Sympy [A] time = 2.59669, size = 100, normalized size = 1.03

$$-\frac{Aa^2+x^6\left(-6Ab^2+4Bab\right)+x^3\left(-3Aab+2Ba^2\right)}{6a^4x^6+6a^3bx^9}-\frac{b\left(-3Ab+2Ba\right)\log\left(x\right)}{a^4}+\frac{b\left(-3Ab+2Ba\right)\log\left(\frac{a}{b}+x^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**2,x)
```

```
[Out] -(A*a**2 + x**6*(-6*A*b**2 + 4*B*a*b) + x**3*(-3*A*a*b + 2*B*a**2))/(6*a**4
*x**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a
)*log(a/b + x**3)/(3*a**4)
```

Giac [A] time = 1.13623, size = 201, normalized size = 2.07

$$-\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(|x|\right)}{a^4}+\frac{\left(2\,Bab^2-3\,Ab^3\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^4b}-\frac{2\,Bab^2x^3-3\,Ab^3x^3+3\,Ba^2b-4\,Aab^2}{3\left(bx^3+a\right)a^4}+\frac{6\,Babx^6-9\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*log(abs(b*
x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*b^
2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*A*a
*b*x^3 - A*a^2)/(a^4*x^6)
```

3.88 $\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{a^2(3Ab-4aB)}{3b^5(a+bx^3)}+\frac{a^3(Ab-aB)}{6b^5(a+bx^3)^2}+\frac{x^3(Ab-3aB)}{3b^4}-\frac{a(Ab-2aB)\log(a+bx^3)}{b^5}+\frac{Bx^6}{6b^3}$$

```
[Out] ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^6)/(6*b^3) + (a^3*(A*b - a*B))/(6*b^5*(a
 + b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(3*b^5*(a + b*x^3)) - (a*(A*b - 2*a*B)
*Log[a + b*x^3])/b^5
```

Rubi [A] time = 0.140116, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{a^2(3Ab-4aB)}{3b^5(a+bx^3)}+\frac{a^3(Ab-aB)}{6b^5(a+bx^3)^2}+\frac{x^3(Ab-3aB)}{3b^4}-\frac{a(Ab-2aB)\log(a+bx^3)}{b^5}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^6)/(6*b^3) + (a^3*(A*b - a*B))/(6*b^5*(a
 + b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(3*b^5*(a + b*x^3)) - (a*(A*b - 2*a*B)
*Log[a + b*x^3])/b^5
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^3}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2}+\frac{3a(-Ab+2aB)}{b^4(a+bx)}\right)dx, x, x^3 \cdots\right)\\ &= \frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^6}{6b^3}+\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.0651171, size = 94, normalized size = 0.88

$$\frac{\frac{2a^2(4aB-3Ab)}{a+bx^3}+\frac{a^3(Ab-aB)}{\left(a+bx^3\right)^2}+2bx^3(Ab-3aB)+6a(2aB-Ab)\log\left(a+bx^3\right)+b^2Bx^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(6*
b^5)
```

Maple [A] time = 0.012, size = 134, normalized size = 1.3

$$\frac{Bx^6}{6\,b^3}+\frac{Ax^3}{3\,b^3}-\frac{Bx^3a}{b^4}-\frac{Aa^2}{b^4\left(bx^3+a\right)}+\frac{4\,Ba^3}{3\,b^5\left(bx^3+a\right)}-\frac{a\ln\left(bx^3+a\right)A}{b^4}+2\,\frac{a^2\ln\left(bx^3+a\right)B}{b^5}+\frac{a^3A}{6\,b^4\left(bx^3+a\right)^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*B*x^6/b^3+1/3/b^3*A*x^3-1/b^4*B*x^3*a-a^2/b^4/(b*x^3+a)*A+4/3*a^3/b^5/(
b*x^3+a)*B-a/b^4*ln(b*x^3+a)*A+2*a^2/b^5*ln(b*x^3+a)*B+1/6*a^3/b^4/(b*x^3+a
```

```
)^2*A-1/6*a^4/b^5/(b*x^3+a)^2*B
```

Maxima [A] time = 0.957149, size = 155, normalized size = 1.45

$$\frac{7\,Ba^4 - 5\,Aa^3b + 2\left(4\,Ba^3b - 3\,Aa^2b^2\right)x^3}{6\left(b^7x^6 + 2\,ab^6x^3 + a^2b^5\right)} + \frac{Bbx^6 - 2\,(3\,Ba - Ab)x^3}{6\,b^4} + \frac{\left(2\,Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^3)/(b^7*x^6 + 2*a*
b^6*x^3 + a^2*b^5) + 1/6*(B*b*x^6 - 2*(3*B*a - A*b)*x^3)/b^4 + (2*B*a^2 - A
*a*b)*log(b*x^3 + a)/b^5
```

Fricas [A] time = 1.43394, size = 362, normalized size = 3.38

$$\frac{Bb^4x^{12} - 2\left(2\,Bab^3 - Ab^4\right)x^9 - \left(11\,Ba^2b^2 - 4\,Aab^3\right)x^6 + 7\,Ba^4 - 5\,Aa^3b + 2\left(Ba^3b - 2\,Aa^2b^2\right)x^3 + 6\left(\left(2\,Ba^2b^2 - Aab^3\right)x^6\right.}{6\left(b^7x^6 + 2\,ab^6x^3 + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^4*x^12 - 2*(2*B*a*b^3 - A*b^4)*x^9 - (11*B*a^2*b^2 - 4*A*a*b^3)*x^
6 + 7*B*a^4 - 5*A*a^3*b + 2*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 6*((2*B*a^2*b^2 -
 A*a*b^3)*x^6 + 2*B*a^4 - A*a^3*b + 2*(2*B*a^3*b - A*a^2*b^2)*x^3)*log(b*x^
3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

Sympy [A] time = 3.09166, size = 112, normalized size = 1.05

$$\frac{Bx^6}{6b^3} + \frac{a\,(-Ab + 2Ba)\log\left(a + bx^3\right)}{b^5} + \frac{-5Aa^3b + 7Ba^4 + x^3\left(-6Aa^2b^2 + 8Ba^3b\right)}{6a^2b^5 + 12ab^6x^3 + 6b^7x^6} - \frac{x^3\,(-Ab + 3Ba)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*log(a + b*x**3)/b**5 + (-5*A*a**3*b + 7*
B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b))/(6*a**2*b**5 + 12*a*b**6*x**3
+ 6*b**7*x**6) - x**3*(-A*b + 3*B*a)/(3*b**4)
```

Giac [A] time = 1.11545, size = 177, normalized size = 1.65

$$\frac{\left(2\,Ba^2 - Aab\right)\log\left(\left|bx^3 + a\right|\right)}{b^5} + \frac{Bb^3x^6 - 6\,Bab^2x^3 + 2\,Ab^3x^3}{6\,b^6} - \frac{18\,Ba^2b^2x^6 - 9\,Aab^3x^6 + 28\,Ba^3bx^3 - 12\,Aa^2b^2x^3 + 11\,B}{6\left(bx^3 + a\right)^2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (2*B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^5 + 1/6*(B*b^3*x^6 - 6*B*a*b^2*x^3
+ 2*A*b^3*x^3)/b^6 - 1/6*(18*B*a^2*b^2*x^6 - 9*A*a*b^3*x^6 + 28*B*a^3*b*x^3
 - 12*A*a^2*b^2*x^3 + 11*B*a^4 - 4*A*a^3*b)/((b*x^3 + a)^2*b^5)
```

3.89 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal. Leaf size=88

$$-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2}+\frac{a(2Ab-3aB)}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{Bx^3}{3b^3}$$

```
[Out] (B*x^3)/(3*b^3) - (a^2*(A*b - a*B))/(6*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a
*B))/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rubi [A] time = 0.0878218, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2}+\frac{a(2Ab-3aB)}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) - (a^2*(A*b - a*B))/(6*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a
*B))/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^8 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{B}{b^3}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^3}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^2}+\frac{Ab-3aB}{b^3(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)}{6b^4\left(a+bx^3\right)^2}+\frac{a(2Ab-3aB)}{3b^4\left(a+bx^3\right)}+\frac{(Ab-3aB)\log\left(a+bx^3\right)}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.0372607, size = 92, normalized size = 1.05

$$\frac{2aAb-3a^2B}{3b^4\left(a+bx^3\right)}+\frac{a^3B-a^2Ab}{6b^4\left(a+bx^3\right)^2}+\frac{(Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) + (-(a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b - 3
*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Maple [A] time = 0.013, size = 110, normalized size = 1.3

$$\frac{Bx^3}{3\,b^3}+\frac{2\,aA}{3\,b^3\left(bx^3+a\right)}-\frac{a^2B}{b^4\left(bx^3+a\right)}+\frac{\ln\left(bx^3+a\right)A}{3\,b^3}-\frac{\ln\left(bx^3+a\right)Ba}{b^4}-\frac{a^2A}{6\,b^3\left(bx^3+a\right)^2}+\frac{Ba^3}{6\,b^4\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*B*x^3/b^3+2/3/b^3*a/(b*x^3+a)*A-1/b^4*a^2/(b*x^3+a)*B+1/3/b^3*ln(b*x^3+
a)*A-1/b^4*ln(b*x^3+a)*B*a-1/6/b^3*a^2/(b*x^3+a)^2*A+1/6/b^4*a^3/(b*x^3+a)^
```

```
2*B
```

Maxima [A] time = 0.985169, size = 127, normalized size = 1.44

$$\frac{Bx^3}{3\,b^3} - \frac{5\,Ba^3 - 3\,Aa^2b + 2\left(3\,Ba^2b - 2\,Aab^2\right)x^3}{6\left(b^6x^6 + 2\,ab^5x^3 + a^2b^4\right)} - \frac{(3\,Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^3 - 1/6*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/(
b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*B*a - A*b)*log(b*x^3 + a)/b^4
```

Fricas [A] time = 1.40629, size = 289, normalized size = 3.28

$$\frac{2\,Bb^3x^9 + 4\,Bab^2x^6 - 5\,Ba^3 + 3\,Aa^2b - 4\left(Ba^2b - Aab^2\right)x^3 - 2\left(\left(3\,Bab^2 - Ab^3\right)x^6 + 3\,Ba^3 - Aa^2b + 2\left(3\,Ba^2b - Aab^2\right)x^3\right)}{6\left(b^6x^6 + 2\,ab^5x^3 + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b
^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A
*a*b^2)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] time = 2.60514, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3b^3} - \frac{-3Aa^2b + 5Ba^3 + x^3\left(-4Aab^2 + 6Ba^2b\right)}{6a^2b^4 + 12ab^5x^3 + 6b^6x^6} - \frac{(-Ab + 3Ba)\log\left(a + bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) - (-3*A*a**2*b + 5*B*a**3 + x**3*(-4*A*a*b**2 + 6*B*a**2*b)
)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*log(a + b*x
**3)/(3*b**4)
```

Giac [A] time = 1.08668, size = 126, normalized size = 1.43

$$\frac{Bx^3}{3\,b^3} - \frac{(3\,Ba - Ab)\log\left(\left|bx^3 + a\right|\right)}{3\,b^4} + \frac{9\,Bab^2x^6 - 3\,Ab^3x^6 + 12\,Ba^2bx^3 - 2\,Aab^2x^3 + 4\,Ba^3}{6\left(bx^3 + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*x^3/b^3 - 1/3*(3*B*a - A*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(9*B*a*b^2*
x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a)^
2*b^4)
```

3.90 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=66

$$-\frac{Ab-2aB}{3b^3(a+bx^3)}+\frac{a(Ab-aB)}{6b^3(a+bx^3)^2}+\frac{B\log(a+bx^3)}{3b^3}$$

```
[Out] (a*(A*b - a*B))/(6*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(3*b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 0.0678781, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{Ab-2aB}{3b^3(a+bx^3)}+\frac{a(Ab-aB)}{6b^3(a+bx^3)^2}+\frac{B\log(a+bx^3)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (a*(A*b - a*B))/(6*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(3*b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/(3*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3} dx, x, x^3\right) \\ &= \frac{1}{3} \text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3} + \frac{Ab-2aB}{b^2(a+bx)^2} + \frac{B}{b^2(a+bx)}\right) dx, x, x^3\right) \\ &= \frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2} - \frac{Ab-2aB}{3b^3\left(a+bx^3\right)} + \frac{B\log\left(a+bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.0248096, size = 64, normalized size = 0.97

$$\frac{3a^2B - ab\left(A-4Bx^3\right) + 2B\left(a+bx^3\right)^2 \log\left(a+bx^3\right) - 2Ab^2x^3}{6b^3\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x^
3])/(6*b^3*(a + b*x^3)^2)
```

Maple [A] time = 0.009, size = 81, normalized size = 1.2

$$-\frac{A}{3\,b^2\left(bx^3+a\right)} + \frac{2\,Ba}{3\,b^3\left(bx^3+a\right)} + \frac{B\ln\left(bx^3+a\right)}{3\,b^3} + \frac{Aa}{6\,b^2\left(bx^3+a\right)^2} - \frac{a^2B}{6\,b^3\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] -1/3/b^2/(b*x^3+a)*A+2/3/b^3/(b*x^3+a)*B*a+1/3*B*ln(b*x^3+a)/b^3+1/6*a/b^2/
(b*x^3+a)^2*A-1/6*a^2/b^3/(b*x^3+a)^2*B
```

Maxima [A] time = 0.945317, size = 97, normalized size = 1.47

$$\frac{2\left(2\,Bab - Ab^2\right)x^3 + 3\,Ba^2 - Aab}{6\left(b^5x^6 + 2\,ab^4x^3 + a^2b^3\right)} + \frac{B\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a^
2*b^3) + 1/3*B*log(b*x^3 + a)/b^3
```

Fricas [A] time = 1.43763, size = 184, normalized size = 2.79

$$\frac{2\left(2\,Bab - Ab^2\right)x^3 + 3\,Ba^2 - Aab + 2\left(Bb^2x^6 + 2\,Babx^3 + Ba^2\right)\log\left(bx^3 + a\right)}{6\left(b^5x^6 + 2\,ab^4x^3 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^3
 + B*a^2)*log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [A] time = 1.98994, size = 70, normalized size = 1.06

$$\frac{B\log\left(a + bx^3\right)}{3b^3} + \frac{-Aab + 3Ba^2 + x^3\left(-2Ab^2 + 4Bab\right)}{6a^2b^3 + 12ab^4x^3 + 6b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**3) + (-A*a*b + 3*B*a**2 + x**3*(-2*A*b**2 + 4*B*a*b
))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
```

Giac [A] time = 1.13993, size = 82, normalized size = 1.24

$$\frac{B\log\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{2\,(2\,Ba-Ab)x^3+\frac{3\,Ba^2-Aab}{b}}{6\,\left(bx^3+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*log(abs(b*x^3 + a))/b^3 + 1/6*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*b
)/b)/((b*x^3 + a)^2*b^2)
```

3.91 $\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=32

$$-\frac{\left(A+Bx^3\right)^2}{6\left(a+bx^3\right)^2(Ab-aB)}$$

```
[Out] -(A + B*x^3)^2/(6*(A*b - a*B)*(a + b*x^3)^2)
```

Rubi [A] time = 0.022282, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {444, 37}

$$-\frac{\left(A+Bx^3\right)^2}{6\left(a+bx^3\right)^2(Ab-aB)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -(A + B*x^3)^2/(6*(A*b - a*B)*(a + b*x^3)^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^2 \left(A + Bx^3\right)}{\left(a + bx^3\right)^3} \, dx = \frac{1}{3} \operatorname{Subst}\left(\int \frac{A + Bx}{(a + bx)^3} \, dx, x, x^3\right)$$
$$= -\frac{\left(A + Bx^3\right)^2}{6(Ab - aB)\left(a + bx^3\right)^2}$$

Mathematica [A] time = 0.0134919, size = 30, normalized size = 0.94

$$-\frac{B\left(a + 2bx^3\right) + Ab}{6b^2\left(a + bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -(A*b + B*(a + 2*b*x^3))/(6*b^2*(a + b*x^3)^2)
```

Maple [A] time = 0.008, size = 39, normalized size = 1.2

$$-\frac{B}{3\,b^2\left(bx^3 + a\right)} - \frac{Ab - Ba}{6\,b^2\left(bx^3 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] -1/3*B/b^2/(b*x^3+a)-1/6*(A*b-B*a)/b^2/(b*x^3+a)^2
```

Maxima [A] time = 0.963658, size = 57, normalized size = 1.78

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Fricas [A] time = 1.3627, size = 86, normalized size = 2.69

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\,\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [A] time = 1.11282, size = 42, normalized size = 1.31

$$-\frac{Ab + Ba + 2Bbx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] -(A*b + B*a + 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Giac [A] time = 1.10107, size = 38, normalized size = 1.19

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\,\left(bx^3 + a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)
```

3.92 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=68

$$\frac{A}{3a^2\left(a+bx^3\right)}-\frac{A\log\left(a+bx^3\right)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{Ab-aB}{6ab\left(a+bx^3\right)^2}$$

```
[Out] (A*b - a*B)/(6*a*b*(a + b*x^3)^2) + A/(3*a^2*(a + b*x^3)) + (A*Log[x])/a^3
- (A*Log[a + b*x^3])/(3*a^3)
```

Rubi [A] time = 0.0583117, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{A}{3a^2\left(a+bx^3\right)}-\frac{A\log\left(a+bx^3\right)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{Ab-aB}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] (A*b - a*B)/(6*a*b*(a + b*x^3)^2) + A/(3*a^2*(a + b*x^3)) + (A*Log[x])/a^3
- (A*Log[a + b*x^3])/(3*a^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{A}{a^3x}+\frac{-Ab+aB}{a(a+bx)^3}-\frac{Ab}{a^2(a+bx)^2}-\frac{Ab}{a^3(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Ab-aB}{6ab\left(a+bx^3\right)^2}+\frac{A}{3a^2\left(a+bx^3\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.0450223, size = 59, normalized size = 0.87

$$\frac{\frac{a\left(-a^2B+3aAb+2Ab^2x^3\right)}{b\left(a+bx^3\right)^2}-2A\log\left(a+bx^3\right)+6A\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*L
og[a + b*x^3])/(6*a^3)
```

Maple [A] time = 0.013, size = 68, normalized size = 1.

$$\frac{A}{3\,a^2\left(bx^3+a\right)}-\frac{A\ln\left(bx^3+a\right)}{3\,a^3}+\frac{A}{6\,a\left(bx^3+a\right)^2}-\frac{B}{6\,b\left(bx^3+a\right)^2}+\frac{A\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^3,x)
```

```
[Out] 1/3*A/a^2/(b*x^3+a)-1/3*A*ln(b*x^3+a)/a^3+1/6/a/(b*x^3+a)^2*A-1/6/b/(b*x^3+
a)^2*B+A*ln(x)/a^3
```

Maxima [A] time = 0.961193, size = 104, normalized size = 1.53

$$\frac{2\,Ab^2x^3 - Ba^2 + 3\,Aab}{6\,\left(a^2b^3x^6 + 2\,a^3b^2x^3 + a^4b\right)} - \frac{A\log\left(bx^3 + a\right)}{3\,a^3} + \frac{A\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) -
 1/3*A*log(b*x^3 + a)/a^3 + 1/3*A*log(x^3)/a^3
```

Fricas [A] time = 1.49971, size = 250, normalized size = 3.68

$$\frac{2\,Aab^2x^3 - Ba^3 + 3\,Aa^2b - 2\,\left(Ab^3x^6 + 2\,Aab^2x^3 + Aa^2b\right)\log\left(bx^3 + a\right) + 6\,\left(Ab^3x^6 + 2\,Aab^2x^3 + Aa^2b\right)\log\left(x\right)}{6\,\left(a^3b^3x^6 + 2\,a^4b^2x^3 + a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a
^2*b)*log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*log(x))/(a^3
*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

Sympy [A] time = 1.10972, size = 75, normalized size = 1.1

$$\frac{A\log\left(x\right)}{a^3} - \frac{A\log\left(\frac{a}{b} + x^3\right)}{3a^3} + \frac{3Aab + 2Ab^2x^3 - Ba^2}{6a^4b + 12a^3b^2x^3 + 6a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**3)/(3*a**3) + (3*A*a*b + 2*A*b**2*x**3 - B*a
**2)/(6*a**4*b + 12*a**3*b**2*x**3 + 6*a**2*b**3*x**6)
```

Giac [A] time = 1.10443, size = 100, normalized size = 1.47

$$-\frac{A\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{A\log\left(|x|\right)}{a^3}+\frac{3\,Ab^3x^6+8\,Aab^2x^3-Ba^3+6\,Aa^2b}{6\left(bx^3+a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^3 + A*log(abs(x))/a^3 + 1/6*(3*A*b^3*x^6 + 8*A
*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)
```

3.93 $\int \frac{A+Bx^3}{x^4(a+bx^3)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{A}{3a^3x^3}$$

```
[Out] -A/(3*a^3*x^3) - (A*b - a*B)/(6*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(3*a^3*(
a + b*x^3)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3])/(
3*a^4)
```

Rubi [A] time = 0.0998586, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -A/(3*a^3*x^3) - (A*b - a*B)/(6*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(3*a^3*(
a + b*x^3)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3])/(
3*a^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{A}{a^3x^2}+\frac{-3Ab+aB}{a^4x}-\frac{b(-Ab+aB)}{a^2(a+bx)^3}-\frac{b(-2Ab+aB)}{a^3(a+bx)^2}-\frac{b(-3Ab+aB)}{a^4(a+bx)}\right)dx,x,x^3\right)\\ &= -\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}\end{aligned}$$

Mathematica [A] time = 0.054895, size = 87, normalized size = 0.86

$$\frac{\frac{a^2(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{2a(aB-2Ab)}{a+bx^3}+2(3Ab-aB)\log\left(a+bx^3\right)+6\log(x)(aB-3Ab)-\frac{2aA}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6*a
^4)
```

Maple [A] time = 0.015, size = 117, normalized size = 1.2

$$-\frac{2\,Ab}{3\,a^3\left(bx^3+a\right)}+\frac{B}{3\,a^2\left(bx^3+a\right)}+\frac{b\ln\left(bx^3+a\right)A}{a^4}-\frac{\ln\left(bx^3+a\right)B}{3\,a^3}-\frac{Ab}{6\,a^2\left(bx^3+a\right)^2}+\frac{B}{6\,a\left(bx^3+a\right)^2}-\frac{A}{3\,a^3x^3}-3\,\frac{A\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -2/3/a^3*b*A/(b*x^3+a)+1/3/a^2/(b*x^3+a)*B+1/a^4*b*ln(b*x^3+a)*A-1/3/a^3*ln
(b*x^3+a)*B-1/6/a^2*b/(b*x^3+a)^2*A+1/6/a/(b*x^3+a)^2*B-1/3*A/a^3/x^3-3/a^4
```

```
*ln(x)*A*b+1/a^3*ln(x)*B
```

Maxima [A] time = 1.00189, size = 147, normalized size = 1.46

$$\frac{2\left(Bab-3\,Ab^2\right)x^6+3\left(Ba^2-3\,Aab\right)x^3-2\,Aa^2}{6\left(a^3b^2x^9+2\,a^4bx^6+a^5x^3\right)}-\frac{(Ba-3\,Ab)\log\left(bx^3+a\right)}{3\,a^4}+\frac{(Ba-3\,Ab)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2*
x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*log(b*x^3 + a)/a^4 + 1/3*(
B*a - 3*A*b)*log(x^3)/a^4
```

Fricas [B] time = 1.46795, size = 412, normalized size = 4.08

$$\frac{2\left(Ba^2b-3\,Aab^2\right)x^6-2\,Aa^3+3\left(Ba^3-3\,Aa^2b\right)x^3-2\left(\left(Bab^2-3\,Ab^3\right)x^9+2\left(Ba^2b-3\,Aab^2\right)x^6+\left(Ba^3-3\,Aa^2b\right)x^3\right)\mathrm{l}}{6\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2*
((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2*b
)*x^3)*log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2
)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x
^3)
```

Sympy [A] time = 3.18212, size = 107, normalized size = 1.06

$$\frac{-2Aa^2+x^6\left(-6Ab^2+2Bab\right)+x^3\left(-9Aab+3Ba^2\right)}{6a^5x^3+12a^4bx^6+6a^3b^2x^9}+\frac{(-3Ab+Ba)\log(x)}{a^4}-\frac{(-3Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a*
*5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4 -
 (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)
```

Giac [A] time = 1.13708, size = 184, normalized size = 1.82

$$\frac{(Ba-3\,Ab)\log\left(|x|\right)}{a^4}-\frac{\left(Bab-3\,Ab^2\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^4b}+\frac{3\,Bab^2x^6-9\,Ab^3x^6+8\,Ba^2bx^3-22\,Aab^2x^3+6\,Ba^3-14\,Aa^2b}{6\left(bx^3+a\right)^2a^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (B*a - 3*A*b)*log(abs(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*log(abs(b*x^3 + a))/(
a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^3
+ 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 + A*
a)/(a^4*x^3)
```

3.94 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=122

$$\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{A}{6a^3x^6}$$

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(3*a^4*x^3) + (b*(A*b - a*B))/(6*a^3*(a + b*
x^3)^2) + (b*(3*A*b - 2*a*B))/(3*a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Log[
x])/a^5 - (b*(2*A*b - a*B)*Log[a + b*x^3])/a^5
```

Rubi [A] time = 0.129556, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 77}

$$\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(3*a^4*x^3) + (b*(A*b - a*B))/(6*a^3*(a + b*
x^3)^2) + (b*(3*A*b - 2*a*B))/(3*a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Log[
x])/a^5 - (b*(2*A*b - a*B)*Log[a + b*x^3])/a^5
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{A}{a^3x^3}+\frac{-3Ab+aB}{a^4x^2}-\frac{3b(-2Ab+aB)}{a^5x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^3}+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)^2}+\frac{3b^2(-2}{a^5(}\right.\right.$$
$$= -\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Ab-aB)\log}{a^5}$$

Mathematica [A] time = 0.0767724, size = 108, normalized size = 0.89

$$\frac{\frac{a^2b(Ab-aB)}{\left(a+bx^3\right)^2}-\frac{a^2A}{x^6}+\frac{2ab(3Ab-2aB)}{a+bx^3}-\frac{2a(aB-3Ab)}{x^3}+6b(aB-2Ab)\log\left(a+bx^3\right)+18b\log(x)(2Ab-aB)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*A)/x^6) - (2*a*(-3*A*b + a*B))/x^3 + (a^2*b*(A*b - a*B))/(a + b*x^3
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^3) + 18*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^3])/(6*a^5)
```

Maple [A] time = 0.014, size = 147, normalized size = 1.2

$$\frac{Ab^2}{a^4\left(bx^3+a\right)}-\frac{2\,Bb}{3\,a^3\left(bx^3+a\right)}-2\,\frac{b^2\ln\left(bx^3+a\right)A}{a^5}+\frac{b\ln\left(bx^3+a\right)B}{a^4}+\frac{Ab^2}{6\,a^3\left(bx^3+a\right)^2}-\frac{Bb}{6\,a^2\left(bx^3+a\right)^2}-\frac{A}{6\,a^3x^6}+\frac{Al}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^3,x)
```

```
[Out] 1/a^4*b^2*A/(b*x^3+a)-2/3/a^3*b/(b*x^3+a)*B-2/a^5*b^2*ln(b*x^3+a)*A+1/a^4*b
*ln(b*x^3+a)*B+1/6/a^3*b^2/(b*x^3+a)^2*A-1/6/a^2*b/(b*x^3+a)^2*B-1/6*A/a^3/
```

```
x^6+1/a^4/x^3*A*b-1/3/a^3/x^3*B+6*b^2/a^5*ln(x)*A-3*b/a^4*ln(x)*B
```

Maxima [A] time = 0.957756, size = 184, normalized size = 1.51

$$-\frac{6\left(Bab^2-2\,Ab^3\right)x^9+9\left(Ba^2b-2\,Aab^2\right)x^6+Aa^3+2\left(Ba^3-2\,Aa^2b\right)x^3}{6\left(a^4b^2x^{12}+2\,a^5bx^9+a^6x^6\right)}+\frac{\left(Bab-2\,Ab^2\right)\log\left(bx^3+a\right)}{a^5}-\frac{\left(Bab-2\,Ab\right.}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^12 + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b - 2
*A*b^2)*log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*log(x^3)/a^5
```

Fricas [B] time = 1.543, size = 478, normalized size = 3.92

$$-\frac{6\left(Ba^2b^2-2\,Aab^3\right)x^9+9\left(Ba^3b-2\,Aa^2b^2\right)x^6+Aa^4+2\left(Ba^4-2\,Aa^3b\right)x^3-6\left(\left(Bab^3-2\,Ab^4\right)x^{12}+2\left(Ba^2b^2-2\,Aab\right.\right.}{6\left(a^5b^2x^{12}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^4
 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 -
 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(b*x^3 + a) + 18*((B*a*b^
3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)
*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)
```

Sympy [A] time = 9.36646, size = 133, normalized size = 1.09

$$-\frac{Aa^3+x^9\left(-12Ab^3+6Bab^2\right)+x^6\left(-18Aab^2+9Ba^2b\right)+x^3\left(-4Aa^2b+2Ba^3\right)}{6a^6x^6+12a^5bx^9+6a^4b^2x^{12}}-\frac{3b\left(-2Ab+Ba\right)\log\left(x\right)}{a^5}+\frac{b\left(-2Ab+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**3,x)
```

```
[Out] -(A*a**3 + x**9*(-12*A*b**3 + 6*B*a*b**2) + x**6*(-18*A*a*b**2 + 9*B*a**2*b
) + x**3*(-4*A*a**2*b + 2*B*a**3))/(6*a**6*x**6 + 12*a**5*b*x**9 + 6*a**4*b
**2*x**12) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + b*(-2*A*b + B*a)*log(a/b + x*
*3)/a**5
```

Giac [A] time = 1.12672, size = 177, normalized size = 1.45

$$-\frac{3\left(Bab-2\,Ab^{2}\right)\log\left(|x|\right)}{a^{5}}+\frac{\left(Bab^{2}-2\,Ab^{3}\right)\log\left(\left|bx^{3}+a\right|\right)}{a^{5}b}-\frac{6\,Bab^{2}x^{9}-12\,Ab^{3}x^{9}+9\,Ba^{2}bx^{6}-18\,Aab^{2}x^{6}+2\,Ba^{3}x^{3}-4}{6\left(bx^{6}+ax^{3}\right)^{2}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + (B*a*b^2 - 2*A*b^3)*log(abs(b*x^3 +
a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*b^
2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)
```

3.95 $\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\tan^{-1}\left(\frac{\sqrt[3]{a}\cdots}{\sqrt{\cdots}}\right.}{9\sqrt{3}b^{14/3}}$$

```
[Out] (2*(5*A*b - 11*a*B)*x^2)/(9*b^4) - (4*(5*A*b - 11*a*B)*x^5)/(45*a*b^3) + ((
A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) + ((5*A*b - 11*a*B)*x^8)/(18*a*b^2*(
a + b*x^3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(14/3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*Log[a^
(1/3) + b^(1/3)*x])/(27*b^(14/3)) - (2*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*b^(14/3))
```

Rubi [A] time = 0.155731, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 288, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\tan^{-1}\left(\frac{\sqrt[3]{a}\cdots}{\sqrt{\cdots}}\right.}{9\sqrt{3}b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*(5*A*b - 11*a*B)*x^2)/(9*b^4) - (4*(5*A*b - 11*a*B)*x^5)/(45*a*b^3) + ((
A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) + ((5*A*b - 11*a*B)*x^8)/(18*a*b^2*(
a + b*x^3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(14/3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*Log[a^
(1/3) + b^(1/3)*x])/(27*b^(14/3)) - (2*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*b^(14/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(-5Ab+11aB)\int \frac{x^{10}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \frac{x^7}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx}{9ab^2}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4a(5Ab-11aB))}{9b^4}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{\left(4a^{2/3}(5Ab-11aB\right.}{27b^{13}}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-11aB)}{27b^1}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-11aB)}{27b^1}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-11aB)}{9\sqrt{3}b}$$

Mathematica [A] time = 0.166405, size = 216, normalized size = 0.88

$$\frac{\frac{45a^2b^{2/3}x^2(aB-Ab)}{(a+bx^3)^2}+20a^{2/3}(11aB-5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-40a^{2/3}(11aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-40\sqrt{3}a^{2/3}(11a}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(A*b - 3*a*B)*x^2 + 54*b^(5/3)*B*x^5 + (45*a^2*b^(2/3)*(-(A*b)
 + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^(2/3)*(7*A*b - 10*a*B)*x^2)/(a + b*x^3
) - 40*Sqrt[3]*a^(2/3)*(-5*A*b + 11*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] - 40*a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*a^(2
/3)*(-5*A*b + 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
b^(14/3))
```

Maple [A] time = 0.012, size = 308, normalized size = 1.3

$$\frac{Bx^5}{5b^3}+\frac{Ax^2}{2b^3}-\frac{3Bx^2a}{2b^4}+\frac{7aAx^5}{9b^2(bx^3+a)^2}-\frac{10a^2Bx^5}{9b^3(bx^3+a)^2}+\frac{11a^2Ax^2}{18b^3(bx^3+a)^2}-\frac{17Ba^3x^2}{18b^4(bx^3+a)^2}+\frac{20aA}{27b^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/5/b^3*B*x^5+1/2/b^3*A*x^2-3/2/b^4*B*x^2*a+7/9*a/b^2/(b*x^3+a)^2*A*x^5-10/
9*a^2/b^3/(b*x^3+a)^2*B*x^5+11/18*a^2/b^3/(b*x^3+a)^2*A*x^2-17/18*a^3/b^4/(
b*x^3+a)^2*B*x^2+20/27*a/b^4*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-10/27*a/b^4*A/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27*a/b^4*A*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-44/27*a^2/b^5*B/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+22/27*a^2/b^5*B/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+44/27*a^2/b^5*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4955, size = 838, normalized size = 3.41

$$\frac{54\,Bb^{3}x^{11}-27\left(11\,Bab^{2}-5\,Ab^{3}\right)x^{8}-96\left(11\,Ba^{2}b-5\,Aab^{2}\right)x^{5}-60\left(11\,Ba^{3}-5\,Aa^{2}b\right)x^{2}+40\,\sqrt{3}\left(\left(11\,Bab^{2}-5\,Ab^{3}\right)x^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270*(54*B*b^3*x^11 - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*A
*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*sqrt(3)*((11*B*a*b^2 - 5*A
*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 20*((11
*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)
*x^3)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3))
- 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5
*A*a*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^6*x^6 + 2*a
*b^5*x^3 + a^2*b^4)
```

Sympy [A] time = 3.72164, size = 189, normalized size = 0.77

$$\frac{Bx^{5}}{5b^{3}}-\frac{x^{5}\left(-14Aab^{2}+20Ba^{2}b\right)+x^{2}\left(-11Aa^{2}b+17Ba^{3}\right)}{18a^{2}b^{4}+36ab^{5}x^{3}+18b^{6}x^{6}}+\operatorname{RootSum}\left(19683t^{3}b^{14}-8000A^{3}a^{2}b^{3}+52800A^{2}Ba^{3}b^{2}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) - (x**5*(-14*A*a*b**2 + 20*B*a**2*b) + x**2*(-11*A*a**2*b +
 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*x**6) + RootSum(19683
```

```
*_t**3*b**14 - 8000*A**3*a**2*b**3 + 52800*A**2*B*a**3*b**2 - 116160*A*B**2
*a**4*b + 85184*B**3*a**5, Lambda(_t, _t*log(729*_t**2*b**9/(400*A**2*a*b**
2 - 1760*A*B*a**2*b + 1936*B**2*a**3) + x))) - x**2*(-A*b + 3*B*a)/(2*b**4)
```

Giac [A] time = 1.12666, size = 350, normalized size = 1.42

$$-\frac{4\left(11\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4}-\frac{4\sqrt{3}\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*(11*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^4) - 4/27*sqrt(3)*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2
)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 + 2/
27*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/b^6 - 1/18*(20*B*a^2*b*x^5 - 14*A*a*b^2*x^5 + 17*B*a^3*x^2
- 11*A*a^2*b*x^2)/((b*x^3 + a)^2*b^4) + 1/10*(2*B*b^12*x^5 - 15*B*a*b^11*x^
2 + 5*A*b^12*x^2)/b^15
```

3.96 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal. Leaf size=244

$$\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}}+\frac{x^7(2Ab-5aB)}{9ab^2\left(a+bx^3\right)}-\frac{7x^4(2Ab-5aB)}{36ab^3}+\frac{7x(2Ab-5aB)}{9b^4}-\frac{7\sqrt[3]{a}(2Ab-5a}{27}$$

```
[Out] (7*(2*A*b - 5*a*B)*x)/(9*b^4) - (7*(2*A*b - 5*a*B)*x^4)/(36*a*b^3) + ((A*b
- a*B)*x^10)/(6*a*b*(a + b*x^3)^2) + ((2*A*b - 5*a*B)*x^7)/(9*a*b^2*(a + b*
x^3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(13/3)) - (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x])/(27*b^(13/3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(13/3))
```

Rubi [A] time = 0.165375, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 288, 302, 200, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}}+\frac{x^7(2Ab-5aB)}{9ab^2\left(a+bx^3\right)}-\frac{7x^4(2Ab-5aB)}{36ab^3}+\frac{7x(2Ab-5aB)}{9b^4}-\frac{7\sqrt[3]{a}(2Ab-5a}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```


```
[Out] (7*(2*A*b - 5*a*B)*x)/(9*b^4) - (7*(2*A*b - 5*a*B)*x^4)/(36*a*b^3) + ((A*b
- a*B)*x^10)/(6*a*b*(a + b*x^3)^2) + ((2*A*b - 5*a*B)*x^7)/(9*a*b^2*(a + b*
x^3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(13/3)) - (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x])/(27*b^(13/3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(13/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(-4Ab+10aB)\int \frac{x^9}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{(7(2Ab-5aB))\int \frac{x^6}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{(7(2Ab-5aB))\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{9ab^2}$$

$$= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{(7a(2Ab-5aB))\int \frac{1}{a+bx}}{9b^4}$$

$$= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{\left(7\sqrt[3]{a}(2Ab-5aB)\right)\int \frac{}{\sqrt[3]{}}}{27b^4}$$

$$= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{}\right.}{27b^{13/3}}$$

$$= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{}\right.}{27b^{13/3}}$$

$$= \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}+\frac{7\sqrt[3]{a}(2Ab-5aB)\tan^{-1}}{9\sqrt{3}b^{13/3}}$$

Mathematica [A] time = 0.15317, size = 210, normalized size = 0.86

$$\frac{-14\sqrt[3]{a}(5aB-2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{18a^2\sqrt[3]{b}x(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{6a\sqrt[3]{b}x(13Ab-19aB)}{a+bx^3}+108\sqrt[3]{b}x(Ab-3aB)+28\sqrt[3]{a}(5aB-}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(A*b - 3*a*B)*x + 27*b^(4/3)*B*x^4 + (18*a^2*b^(1/3)*(-(A*b) +
 a*B)*x)/(a + b*x^3)^2 + (6*a*b^(1/3)*(13*A*b - 19*a*B)*x)/(a + b*x^3) - 28
*Sqrt[3]*a^(1/3)*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 28*a^(1/3)*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x] - 14*a^(1/3)*(-2*
A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*b^(13/3))
```

Maple [A] time = 0.013, size = 299, normalized size = 1.2

$$\frac{Bx^4}{4b^3}+\frac{Ax}{b^3}-3\frac{Bax}{b^4}+\frac{13\,Aax^4}{18\,b^2\left(bx^3+a\right)^2}-\frac{19\,a^2Bx^4}{18\,b^3\left(bx^3+a\right)^2}+\frac{5\,a^2Ax}{9\,b^3\left(bx^3+a\right)^2}-\frac{8\,Ba^3x}{9\,b^4\left(bx^3+a\right)^2}-\frac{14\,Aa}{27\,b^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/4/b^3*B*x^4+1/b^3*A*x-3/b^4*B*a*x+13/18*a/b^2/(b*x^3+a)^2*A*x^4-19/18*a^2
/b^3/(b*x^3+a)^2*B*x^4+5/9*a^2/b^3/(b*x^3+a)^2*A*x-8/9*a^3/b^4/(b*x^3+a)^2*
B*x-14/27*a/b^4*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27*a/b^4*A/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27*a/b^4*A/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+35/27*a^2/b^5*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-35/54*a^2/b^5*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+35/27*a^2/b
^5*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51259, size = 779, normalized size = 3.19

$$\frac{27\,Bb^3x^{10} - 54\left(5\,Bab^2 - 2\,Ab^3\right)x^7 - 147\left(5\,Ba^2b - 2\,Aab^2\right)x^4 - 28\,\sqrt{3}\left(\left(5\,Bab^2 - 2\,Ab^3\right)x^6 + 5\,Ba^3 - 2\,Aa^2b + 2\left(5\,Ba^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(27*B*b^3*x^10 - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A*
a*b^2)*x^4 - 28*sqrt(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b +
2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b
)^(2/3) - sqrt(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2
*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3)) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5
*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 84*(5*B*a^3
 - 2*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] time = 2.60956, size = 162, normalized size = 0.66

$$\frac{Bx^4}{4b^3} - \frac{x^4\left(-13Aab^2 + 19Ba^2b\right) + x\left(-10Aa^2b + 16Ba^3\right)}{18a^2b^4 + 36ab^5x^3 + 18b^6x^6} + \operatorname{RootSum}\left(19683t^3b^{13} + 2744A^3ab^3 - 20580A^2Ba^2b^2 + 514\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) - (x**4*(-13*A*a*b**2 + 19*B*a**2*b) + x*(-10*A*a**2*b + 16
*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*x**6) + RootSum(19683*_t
**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2*b**2 + 51450*A*B**2*a**3*b
 - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**4/(-14*A*b + 35*B*a) + x)))
- x*(-A*b + 3*B*a)/b**4
```

Giac [A] time = 1.14003, size = 316, normalized size = 1.3

$$\frac{7\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^5}-\frac{7\left(5Ba^2-2Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^4}+\frac{7\left(5\left(-ab^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 7/54*(5*(-a*b^2)^(1/3)*B*a - 2*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 - 1/18*(19
*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a)^2*b
^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^12
```

3.97 $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=222

$$\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}}+\frac{x^5(Ab-4aB)}{9ab^2\left(a+bx^3\right)}-\frac{5x^2(Ab-4aB)}{18ab^3}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}}-\frac{5(Ab-}{}$$

```
[Out] (-5*(A*b - 4*a*B)*x^2)/(18*a*b^3) + ((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2)
 + ((A*b - 4*a*B)*x^5)/(9*a*b^2*(a + b*x^3)) - (5*(A*b - 4*a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(11/3)) - (5*(
A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(11/3)) + (5*(A*b - 4*
a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(11/3))
```

Rubi [A] time = 0.139961, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 288, 321, 292, 31, 634, 617, 204, 628}

$$\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}}+\frac{x^5(Ab-4aB)}{9ab^2\left(a+bx^3\right)}-\frac{5x^2(Ab-4aB)}{18ab^3}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}}-\frac{5(Ab-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```


```
[Out] (-5*(A*b - 4*a*B)*x^2)/(18*a*b^3) + ((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2)
 + ((A*b - 4*a*B)*x^5)/(9*a*b^2*(a + b*x^3)) - (5*(A*b - 4*a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(11/3)) - (5*(
A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(11/3)) + (5*(A*b - 4*
a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(11/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(-2Ab+8aB)\int \frac{x^7}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{(5(Ab-4aB))\int \frac{x^4}{a+bx^3}\,dx}{9ab^2}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} + \frac{(5(Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{9b^3}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{(5(Ab-4aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27\sqrt[3]{a}b^{10/3}} + \frac{(5(Ab-4a}{}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} + \frac{(5(Ab-4}{}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} + \frac{5(Ab-4}{}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{11/3}} - \frac{5(Ab-4}{}$$

Mathematica [A] time = 0.154885, size = 194, normalized size = 0.87

$$\frac{\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}}-\frac{6b^{2/3}x^2(4Ab-7aB)}{a+bx^3}+\frac{9ab^{2/3}x^2(Ab-aB)}{\left(a+bx^3\right)^2}+\frac{10(4aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}+\frac{10\sqrt{3}(4aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+27b^{2}\cdots}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/3
)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (10*(-(A*b) + 4*a*B)*Log[a^(1
/3) + b^(1/3)*x])/a^(1/3) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/a^(1/3))/(54*b^(11/3))
```

Maple [A] time = 0.011, size = 275, normalized size = 1.2

$$\frac{Bx^2}{2b^3}-\frac{4Ax^5}{9b\left(bx^3+a\right)^2}+\frac{7Bx^5a}{9b^2\left(bx^3+a\right)^2}-\frac{5aAx^2}{18b^2\left(bx^3+a\right)^2}+\frac{11Bx^2a^2}{18b^3\left(bx^3+a\right)^2}-\frac{5A}{27b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{5A}{54b^3}\ln\left(x^2-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/2*B*x^2/b^3-4/9/b/(b*x^3+a)^2*A*x^5+7/9/b^2/(b*x^3+a)^2*B*x^5*a-5/18/b^2/
(b*x^3+a)^2*A*x^2*a+11/18/b^3/(b*x^3+a)^2*B*x^2*a^2-5/27/b^3*A/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+5/54/b^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
5/27/b^3*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+20/2
7/b^4*B*a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-10/27/b^4*B*a/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))-20/27/b^4*B*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57092, size = 1705, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
A*a^2*b^3)*x^2 - 15*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*
a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^
2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)
*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*((4*B*a*b^2
- A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A
*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5), 1/54*(27
*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 - A*a^2*b^3
)*x^2 - 30*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*a^3*b^2 +
 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*((4*B*a*b^2 - A*b^3
)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log
(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A*b^3)*x
^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)]
```

Sympy [A] time = 3.23634, size = 162, normalized size = 0.73

$$\frac{Bx^2}{2b^3} + \frac{x^5\left(-8Ab^2 + 14Bab\right) + x^2\left(-5Aab + 11Ba^2\right)}{18a^2b^3 + 36ab^4x^3 + 18b^5x^6} + \mathrm{RootSum}\left(19683t^3ab^{11} + 125A^3b^3 - 1500A^2Bab^2 + 6000AB^2a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**2
))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a*b
**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B**3*
a**3, Lambda(_t, _t*log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b + 400*
B**2*a**2) + x)))
```

Giac [A] time = 1.14368, size = 313, normalized size = 1.41

$$\frac{Bx^2}{2\,b^3}+\frac{5\left(4\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3}+\frac{5\,\sqrt{3}\left(4\,\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 + 5/27*(4*B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*l
og(abs(x - (-a/b)^(1/3)))/(a*b^3) + 5/27*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - (-
a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*
b^5) + 1/18*(14*B*a*b*x^5 - 8*A*b^2*x^5 + 11*B*a^2*x^2 - 5*A*a*b*x^2)/((b*x
^3 + a)^2*b^3) - 5/54*(4*(-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5)
```

3.98 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=220

$$-\frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}+\frac{x^4(Ab-}{18ab^2\left(a\right.}$$

```
[Out] (-2*(A*b - 7*a*B)*x)/(9*a*b^3) + ((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) +
((A*b - 7*a*B)*x^4)/(18*a*b^2*(a + b*x^3)) - (2*(A*b - 7*a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(10/3)) + (2*(A*
b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(10/3)) - ((A*b - 7*a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(2/3)*b^(10/3))
```

Rubi [A] time = 0.125802, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 288, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}+\frac{x^4(Ab-}{18ab^2\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-2*(A*b - 7*a*B)*x)/(9*a*b^3) + ((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) +
((A*b - 7*a*B)*x^4)/(18*a*b^2*(a + b*x^3)) - (2*(A*b - 7*a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(10/3)) + (2*(A*
b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(10/3)) - ((A*b - 7*a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(2/3)*b^(10/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2}+\frac{(-Ab+7aB)\int \frac{x^6}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)}-\frac{(2(Ab-7aB))\int \frac{x^3}{a+bx^3}\,dx}{9ab^2}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)}+\frac{(2(Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{9b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)}+\frac{(2(Ab-7aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{2/3}b^3}+\frac{(2(Ab-7a}{}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{(Ab-7a}{}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{(Ab-7a}{}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3}+\frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}+\frac{2(Ab-7}{}$$

Mathematica [A] time = 0.155371, size = 188, normalized size = 0.85

$$\frac{\frac{2(7aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{4(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{4\sqrt{3}(7aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{3\sqrt[3]{b}x(7Ab-13aB)}{a+bx^3}+\frac{9a\sqrt[3]{b}x(Ab-aB)}{\left(a+bx^3\right)^2}+54\sqrt[3]{b}B\ldots}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(7
*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b^(1
/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

Maple [A] time = 0.011, size = 268, normalized size = 1.2

$$\frac{Bx}{b^3}-\frac{7\,Ax^4}{18\,b\left(bx^3+a\right)^2}+\frac{13\,Bx^4a}{18\,b^2\left(bx^3+a\right)^2}-\frac{2\,aAx}{9\,b^2\left(bx^3+a\right)^2}+\frac{5\,a^2Bx}{9\,b^3\left(bx^3+a\right)^2}+\frac{2\,A}{27\,b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{A}{27\,b^3}\ln\left(x^2-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] B/b^3*x-7/18/b/(b*x^3+a)^2*A*x^4+13/18/b^2/(b*x^3+a)^2*B*x^4*a-2/9/b^2/(b*x
^3+a)^2*a*A*x+5/9/b^3/(b*x^3+a)^2*a^2*B*x+2/27/b^3*A/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))-1/27/b^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/b^3*A
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/27/b^4*B*a/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27/b^4*B*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))-14/27/b^4*B*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60266, size = 1712, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*((
7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2
*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(7*
B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*B*
a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/54*(54*B*a
^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*B*a^2*b^3
- A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*b^3)*x^3)*s
qrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*
b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*
x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(
7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*
B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A] time = 2.22335, size = 141, normalized size = 0.64

$$\frac{Bx}{b^3} + \frac{x^4\left(-7Ab^2 + 13Bab\right) + x\left(-4Aab + 10Ba^2\right)}{18a^2b^3 + 36ab^4x^3 + 18b^5x^6} + \text{RootSum}\Big(19683t^3a^2b^{10} - 8A^3b^3 + 168A^2Bab^2 - 1176AB^2a^2b + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a**
2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a**2*b**10 -
8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, Lamb
da(_t, _t*log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))
```

Giac [A] time = 1.13746, size = 282, normalized size = 1.28

$$\frac{Bx}{b^3} + \frac{2\,(7\,Ba - Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} - \frac{2\,\sqrt{3}\left(7\,\left(-ab^2\right)^{\frac{1}{3}}Ba - \left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^4} - \frac{\left(7\,\left(-ab^2\right)^{\frac{1}{3}}B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] B*x/b^3 + 2/27*(7*B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3
) - 2/27*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/27*(7*(-a*b^2)^(1/3)*B*
a - (-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) +
1/18*(13*B*a*b*x^4 - 7*A*b^2*x^4 + 10*B*a^2*x - 4*A*a*b*x)/((b*x^3 + a)^2*b
^3)
```

3.99 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal. Leaf size=201

$$\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}}-\frac{x^2(5aB+A}{18ab^2(a+bx}$$

```
[Out] ((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) - ((A*b + 5*a*B)*x^2)/(18*a*b^2*(a
+ b*x^3)) - ((A*b + 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(9*Sqrt[3]*a^(4/3)*b^(8/3)) - ((A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(
27*a^(4/3)*b^(8/3)) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(4/3)*b^(8/3))
```

Rubi [A] time = 0.106889, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 288, 292, 31, 634, 617, 204, 628}

$$\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}}-\frac{x^2(5aB+A}{18ab^2(a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```


```
[Out] ((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) - ((A*b + 5*a*B)*x^2)/(18*a*b^2*(a
+ b*x^3)) - ((A*b + 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(9*Sqrt[3]*a^(4/3)*b^(8/3)) - ((A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(
27*a^(4/3)*b^(8/3)) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(4/3)*b^(8/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2} + \frac{(Ab+5aB)\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)} + \frac{(Ab+5aB)\int \frac{x}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)} - \frac{(Ab+5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{4/3}b^{7/3}} + \frac{(Ab+5aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{4/3}b^{7/3}}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{54a^{4/3}b^{8/3}}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{54a^{4/3}b^{8/3}}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)} - \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}$$

Mathematica [A] time = 0.16759, size = 181, normalized size = 0.9

$$\frac{\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}} - \frac{2(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}} - \frac{2\sqrt{3}(5aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{6b^{2/3}x^2(Ab-4aB)}{a\left(a+bx^3\right)} - \frac{9b^{2/3}x^2(Ab-aB)}{\left(a+bx^3\right)^2}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

[Out] ((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2)/(a*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(54*b^(8/3))

Maple [A] time = 0.01, size = 241, normalized size = 1.2

$$\frac{1}{\left(bx^3+a\right)^2}\left(\frac{(Ab-4\,Ba)\,x^5}{9\,ab}-\frac{(Ab+5\,Ba)\,x^2}{18\,b^2}\right)-\frac{A}{27\,b^2a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{5\,B}{27\,b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{A}{54\,b^2a}\ln\left(x^2-\sqrt[3]{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] (1/9*(A*b-4*B*a)/a/b*x^5-1/18*(A*b+5*B*a)/b^2*x^2)/(b*x^3+a)^2-1/27/b^2/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1/54/b^2/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+5/54/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.77201, size = 1651, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 3
*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^
3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^
3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 +
A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b^3 - A*a*
b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^2*b^3 + A*
a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(
-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)
^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b
+ A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*
a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*
b^5*x^3 + a^4*b^4)]
```

Sympy [A] time = 2.27239, size = 153, normalized size = 0.76

$$-\frac{x^5\left(-2Ab^2+8Bab\right)+x^2\left(Aab+5Ba^2\right)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}+\operatorname{RootSum}\left(19683t^3a^4b^8+A^3b^3+15A^2Bab^2+75AB^2a^2b+125B^3a^3,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] -(x**5*(-2*A*b**2 + 8*B*a*b) + x**2*(A*a*b + 5*B*a**2))/(18*a**3*b**2 + 36*
a**2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**4*b**8 + A**3*b**
3 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, Lambda(_t, _t*log(
729*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))
```

Giac [A] time = 1.16679, size = 300, normalized size = 1.49

$$-\frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2}-\frac{\sqrt{3}\left(5\,\left(-ab^2\right)^{\frac{2}{3}}Ba+\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^4}-\frac{8\,Babx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(5*B*a*(-a/b)^(1/3) + A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a^2*b^2) - 1/27*sqrt(3)*(5*(-a*b^2)^(2/3)*B*a + (-a*b^2)^(2/3)*
A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) - 1/18
*(8*B*a*b*x^5 - 2*A*b^2*x^5 + 5*B*a^2*x^2 + A*a*b*x^2)/((b*x^3 + a)^2*a*b^2
) + 1/54*(5*(-a*b^2)^(2/3)*B*a + (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(a^2*b^4)
```

3.100 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=199

$$-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}+\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}-\frac{x(2aB+A}{9ab^2\left(a+b\right.}$$

```
[Out] ((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) - ((A*b + 2*a*B)*x)/(9*a*b^2*(a + b
*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(5/3)*b^(7/3)) + ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*
a^(5/3)*b^(7/3)) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(5/3)*b^(7/3))
```

Rubi [A] time = 0.113401, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {457, 288, 200, 31, 634, 617, 204, 628}

$$-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}+\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}-\frac{x(2aB+A}{9ab^2\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```


```
[Out] ((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) - ((A*b + 2*a*B)*x)/(9*a*b^2*(a + b
*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(5/3)*b^(7/3)) + ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*
a^(5/3)*b^(7/3)) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(5/3)*b^(7/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+4aB)\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\int \frac{1}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b^2} + \frac{(Ab+2aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{5/3}b^2}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{54a^{5/3}b^{7/3}}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{54a^{5/3}b^{7/3}}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} - \frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}$$

Mathematica [A] time = 0.158367, size = 178, normalized size = 0.89

$$\frac{-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}} + \frac{2(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{2\sqrt{3}(2aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{3\sqrt[3]{b}x(Ab-7aB)}{a\left(a+bx^3\right)} - \frac{9\sqrt[3]{b}x(Ab-aB)}{\left(a+bx^3\right)^2}}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a*
(a + b*x^3)) - (2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - ((
A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b
^(7/3))
```

Maple [A] time = 0.012, size = 239, normalized size = 1.2

$$\frac{1}{\left(bx^3+a\right)^2}\left(\frac{(Ab-7\,Ba)\,x^4}{18\,ab}-\frac{(Ab+2\,Ba)\,x}{9\,b^2}\right)+\frac{A}{27\,b^2a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,B}{27\,b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{A}{54\,b^2a}\ln\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/18*(A*b-7*B*a)/a/b*x^4-1/9*(A*b+2*B*a)/b^2*x)/(b*x^3+a)^2+1/27/b^2/a/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*A+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-1/54/
b^2/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/27/b^3/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/27/b^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97031, size = 1651, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b
^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-(a
^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*
a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
 + a)) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*
b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^
3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/
(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)
*x^4 - 6*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2
*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(
a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b^2
+ A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((2*B*a*b^2 + A*b^3
)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(
a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/(a^3*b^5*x^6 + 2*a^4*
b^4*x^3 + a^5*b^3)]
```

Sympy [A] time = 1.44404, size = 134, normalized size = 0.67

$$-\frac{x^4\left(-Ab^2+7Bab\right)+x\left(2Aab+4Ba^2\right)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}+\operatorname{RootSum}\left(19683t^3a^5b^7-A^3b^3-6A^2Bab^2-12AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] -(x**4*(-A*b**2 + 7*B*a*b) + x*(2*A*a*b + 4*B*a**2))/(18*a**3*b**2 + 36*a**
2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**5*b**7 - A**3*b**3 -
 6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(27*_t*
a**2*b**2/(A*b + 2*B*a) + x)))
```

Giac [A] time = 1.12087, size = 274, normalized size = 1.38

$$-\frac{(2\,Ba+Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{2}b^{2}}+\frac{\sqrt{3}\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}\,Ba+\left(-ab^{2}\right)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b^{3}}+\frac{\left(2\,\left(-ab^{2}\right)^{\frac{1}{3}}\,Ba+\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*B*a + A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) + 1/2
7*sqrt(3)*(2*(-a*b^2)^(1/3)*B*a + (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/54*(2*(-a*b^2)^(1/3)*B*a + (
-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3) - 1/1
8*(7*B*a*b*x^4 - A*b^2*x^4 + 4*B*a^2*x + 2*A*a*b*x)/((b*x^3 + a)^2*a*b^2)
```

3.101 $\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=201

$$\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}}+\frac{x^2(aB+2Ab)}{9a^2b\left(a+bx^3\right)}$$

```
[Out] ((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*x^2)/(9*a^2*b*(a +
 b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(9*Sqrt[3]*a^(7/3)*b^(5/3)) - ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(7/3)*b^(5/3)) + ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(7/3)*b^(5/3))
```

Rubi [A] time = 0.11535, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {457, 290, 292, 31, 634, 617, 204, 628}

$$\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}}+\frac{x^2(aB+2Ab)}{9a^2b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```


```
[Out] ((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*x^2)/(9*a^2*b*(a +
 b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(9*Sqrt[3]*a^(7/3)*b^(5/3)) - ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(7/3)*b^(5/3)) + ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(7/3)*b^(5/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(4Ab+2aB)\int \frac{x}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{x}{a+bx^3}\,dx}{9a^2b}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}b^{4/3}} + \frac{(2Ab+aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{7/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,d}{54a^{7/3}b^{5/3}}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{54a^{7/3}b^{5/3}}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} +$$

Mathematica [A] time = 0.129181, size = 178, normalized size = 0.89

$$\frac{-\frac{9a^{4/3}b^{2/3}x^2(aB-Ab)}{\left(a+bx^3\right)^2} + (aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) + \frac{6\sqrt[3]{a}b^{2/3}x^2(aB+2Ab)}{a+bx^3} - 2(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - 2\sqrt{3}(a}{54a^{7/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3)
*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*b^
```

```
(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] + (2*
A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(5
/3))
```

Maple [A] time = 0.01, size = 251, normalized size = 1.3

$$\frac{1}{\left(bx^3+a\right)^2}\left(\frac{(2\,Ab+Ba)\,x^5}{9\,a^2}+\frac{(7\,Ab-Ba)\,x^2}{18\,ab}\right)-\frac{2\,A}{27\,a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{B}{27\,ab^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{A}{27\,a^2b}\ln\left(x^2-\sqrt[3]{\vphantom{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2-2/27/a^2/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1
/27/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/54/a/b^2/(a/b)^
(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77298, size = 1648, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3*
sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b
^2 + 2*A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 +
 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*
b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A
*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/
3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 + 2*A*a*b^
4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*
b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x^3)*sqrt(-(
-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(
1/3)/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*
A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*
b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^
4*x^3 + a^5*b^3)]
```

Sympy [A] time = 1.19226, size = 153, normalized size = 0.76

$$\frac{x^5\left(4Ab^2+2Bab\right)+x^2\left(7Aab-Ba^2\right)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}+\mathrm{RootSum}\left(19683t^3a^7b^5+8A^3b^3+12A^2Bab^2+6AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3*
b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b**
3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(729*_
t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))
```

Giac [A] time = 1.12806, size = 301, normalized size = 1.5

$$-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba+2\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^3}+\frac{2\,Bab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(B*a*(-a/b)^(1/3) + 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/(a^3*b) - 1/27*sqrt(3)*((-a*b^2)^(2/3)*B*a + 2*(-a*b^2)^(2/3)*A*
b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^3) + 1/18*(
2*B*a*b*x^5 + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b)
+ 1/54*((-a*b^2)^(2/3)*B*a + 2*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/(a^3*b^3)
```

3.102 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=197

$$-\frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}+\frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}}+\frac{x(aB+5A}{18a^2b\left(a+\right.}$$

```
[Out] ((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*x)/(18*a^2*b*(a + b*
x^3)) - ((5*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(8/3)*b^(4/3)) + ((5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(8/3)*b^(4/3)) - ((5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(54*a^(8/3)*b^(4/3))
```

Rubi [A] time = 0.1026, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}+\frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}}+\frac{x(aB+5A}{18a^2b\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*x)/(18*a^2*b*(a + b*
x^3)) - ((5*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(8/3)*b^(4/3)) + ((5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(8/3)*b^(4/3)) - ((5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(54*a^(8/3)*b^(4/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)\int \frac{1}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\int \frac{1}{a+bx^3}\,dx}{9a^2b}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}b} + \frac{(5Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{8/3}b}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,d}{54a^{8/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{54a^{8/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} - \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} -$$

Mathematica [A] time = 0.122757, size = 175, normalized size = 0.89

$$\frac{-(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{9a^{5/3}\sqrt[3]{b}x(aB-Ab)}{\left(a+bx^3\right)^2} + \frac{3a^{2/3}\sqrt[3]{b}x(aB+5Ab)}{a+bx^3} + 2(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - 2\sqrt{3}(aB}{54a^{8/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*(
5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A*b
```

```
+ a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4/3))
```

Maple [A] time = 0.01, size = 249, normalized size = 1.3

$$\frac{1}{\left(bx^3+a\right)^2}\left(\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}\right)+\frac{5A}{27a^2b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{B}{27ab^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{5A}{54a^2b}\ln\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+5/27/a^2/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-5/5
4/a^2/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/54/a/b^2/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57998, size = 1648, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^
4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(a^
2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a
*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
+ a)) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b
^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2
*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3
)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(
a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x
^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(
B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b^2 + 5*
A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b^2 + 5*A*b^3)*
x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*
b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(a^4*b^4*x^6 + 2*a^5*b^
3*x^3 + a^6*b^2)]
```

Sympy [A] time = 0.986885, size = 133, normalized size = 0.68

$$\frac{x^4\left(5Ab^2+Bab\right)+x\left(8Aab-2Ba^2\right)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}+\operatorname{RootSum}\left(19683t^3a^8b^4-125A^3b^3-75A^2Bab^2-15AB^2a^2b-B^3a^3,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b**
2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b**3
 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27*_t
*a**3*b/(5*A*b + B*a) + x)))
```

Giac [A] time = 1.1472, size = 273, normalized size = 1.39

$$-\frac{(Ba+5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba+5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba+5\,\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/27*
sqrt(3)*((-a*b^2)^(1/3)*B*a + 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x
 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/54*((-a*b^2)^(1/3)*B*a + 5*(-a
*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2) + 1/18*
(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2*x + 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)
```

3.103 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}}+\frac{7Ab-}{18a^2bx(a}$$

```
[Out] (-2*(7*A*b - a*B))/(9*a^3*b*x) + (A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + (7*A
*b - a*B)/(18*a^2*b*x*(a + b*x^3)) + (2*(7*A*b - a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(2/3)) + (2*(7*A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(2/3)) - ((7*A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*b^(2/3))
```

Rubi [A] time = 0.12347, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}}+\frac{7Ab-}{18a^2bx(a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```


```
[Out] (-2*(7*A*b - a*B))/(9*a^3*b*x) + (A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + (7*A
*b - a*B)/(18*a^2*b*x*(a + b*x^3)) + (2*(7*A*b - a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(2/3)) + (2*(7*A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(2/3)) - ((7*A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*b^(2/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{(7Ab-aB)\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{(2(7Ab-aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} - \frac{(2(7Ab-aB))\int \frac{x}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{(2(7Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{10/3}\sqrt[3]{b}} - \frac{(2(7Ab-}{}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-}{}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-}{}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}} + \frac{2(7Ab}{}$$

Mathematica [A] time = 0.166219, size = 193, normalized size = 0.85

$$\frac{\frac{2(aB-7Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{9a^{4/3}x^2(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{4(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{4\sqrt{3}(7Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6\sqrt[3]{a}x^2(2aB-5Ab)}{a+bx^3}-\frac{54\sqrt[3]{a}A}{x}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1
/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*Sqrt[3]*(7*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(7*A*b - a*B)*Log[a^(1/3) +
 b^(1/3)*x])/b^(2/3) + (2*(-7*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(2/3))/(54*a^(10/3))
```

Maple [A] time = 0.012, size = 281, normalized size = 1.2

$$-\frac{5\,Ax^5b^2}{9\,a^3\left(bx^3+a\right)^2}+\frac{2\,bBx^5}{9\,a^2\left(bx^3+a\right)^2}-\frac{13\,Ax^2b}{18\,a^2\left(bx^3+a\right)^2}+\frac{7\,Bx^2}{18\,a\left(bx^3+a\right)^2}+\frac{14\,A}{27\,a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\frac{7\,A}{27\,a^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -5/9/a^3/(b*x^3+a)^2*A*x^5*b^2+2/9/a^2/(b*x^3+a)^2*B*x^5*b-13/18/a^2/(b*x^3
+a)^2*A*x^2*b+7/18/a/(b*x^3+a)^2*B*x^2+14/27/a^3*A/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))-7/27/a^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27/a^3*A*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/27/a^2*B/b/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27/a^2*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+2/27/a^2*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-A/a^3/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.66775, size = 1716, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*
a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*
A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b
^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*
sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2 - 7*
A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x), 1/54
*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*a^2*b
^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*A*a^
2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b^3)
*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x)]
```

Sympy [A] time = 1.55102, size = 162, normalized size = 0.71

$$\frac{-18Aa^2 + x^6\left(-28Ab^2 + 4Bab\right) + x^3\left(-49Aab + 7Ba^2\right)}{18a^5x + 36a^4bx^4 + 18a^3b^2x^7} + \mathrm{RootSum}\left(19683t^3a^{10}b^2 - 2744A^3b^3 + 1176A^2Bab^2 - 168A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**3,x)
```

```
[Out] (-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(1
8*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + RootSum(19683*_t**3*a**10*
b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*a**
3, Lambda(_t, _t*log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B**2*
a**2) + x)))
```

Giac [A] time = 1.13737, size = 305, normalized size = 1.34

$$-\frac{2\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{4}}-\frac{A}{a^{3}x}-\frac{2\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}Ba-7\left(-ab^{2}\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{4}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -2/27*(B*a*(-a/b)^(1/3) - 7*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^4 - A/(a^3*x) - 2/27*sqrt(3)*((-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^
(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b^2)
+ 1/18*(4*B*a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 +
a)^2*a^3) + 1/27*((-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b^2)
```

3.104 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=227

$$\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}-\frac{5(4Ab-aB)}{18a^3bx^2}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}}+\frac{5(4Ab}{}$$

```
[Out] (-5*(4*A*b - a*B))/(18*a^3*b*x^2) + (A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) +
 (4*A*b - a*B)/(9*a^2*b*x^2*(a + b*x^3)) + (5*(4*A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(1/3)) - (5*(4*A*
b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(1/3)) + (5*(4*A*b - a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3))
```

Rubi [A] time = 0.131091, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}-\frac{5(4Ab-aB)}{18a^3bx^2}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}}+\frac{5(4Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```


```
[Out] (-5*(4*A*b - a*B))/(18*a^3*b*x^2) + (A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) +
 (4*A*b - a*B)/(9*a^2*b*x^2*(a + b*x^3)) + (5*(4*A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(1/3)) - (5*(4*A*
b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(1/3)) + (5*(4*A*b - a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{(8Ab-2aB)\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{(5(4Ab-aB))\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{11/3}} - \frac{(5(4Ab -}{}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} - \frac{(5(4Ab}{}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{5(4Ab}{}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}} - \frac{5(4Ab}{}$$

Mathematica [A] time = 0.156448, size = 189, normalized size = 0.83

$$\frac{\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{9a^{5/3}x(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{3a^{2/3}x(5aB-11Ab)}{a+bx^3}-\frac{27a^{2/3}A}{x^2}+\frac{10(aB-4Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{10\sqrt{3}(4Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2
/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))
```

Maple [A] time = 0.013, size = 277, normalized size = 1.2

$$-\frac{11\,Ax^4b^2}{18\,a^3\left(bx^3+a\right)^2}+\frac{5\,bBx^4}{18\,a^2\left(bx^3+a\right)^2}-\frac{7\,Abx}{9\,a^2\left(bx^3+a\right)^2}+\frac{4\,Bx}{9\,a\left(bx^3+a\right)^2}-\frac{20\,A}{27\,a^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{10\,A}{27\,a^3}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -11/18/a^3/(b*x^3+a)^2*A*x^4*b^2+5/18/a^2/(b*x^3+a)^2*B*x^4*b-7/9/a^2/(b*x^
3+a)^2*b*A*x+4/9/a/(b*x^3+a)^2*B*x-20/27/a^3*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)+10/27/a^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27/a^3*A/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/27/a^2*B/b/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))-5/54/a^2*B/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))+5/27/a^2*B/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-1/2*A/a^3/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64808, size = 1785, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a^
3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4*A
*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*
a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((B*a*b^
2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(
-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a
*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2
)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5
+ a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a
^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*
a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(-(-a^2*b)^(1
/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2
*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^
5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
- (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)
*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))
/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

Sympy [A] time = 2.01203, size = 143, normalized size = 0.63

$$\frac{-9Aa^2 + x^6\left(-20Ab^2 + 5Bab\right) + x^3\left(-32Aab + 8Ba^2\right)}{18a^5x^2 + 36a^4bx^5 + 18a^3b^2x^8} + \operatorname{RootSum}\Big(19683t^3a^{11}b + 8000A^3b^3 - 6000A^2Bab^2 + 1500A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**3,x)
```

```
[Out] (-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(18
*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a**1
1*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**3*a
**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))
```

Giac [A] time = 1.14006, size = 282, normalized size = 1.24

$$-\frac{5\,(Ba-4\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4}+\frac{5\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\,\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}+\frac{5\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -5/27*(B*a - 4*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 5/27*sqrt
(3)*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) + 5/54*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^
(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 1/18*(5*B*a*b
*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 + a*x)^
2*a^3)
```

3.105 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=246

$$\frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}}+\frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)}-\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{7(5Ab-2aB)}{9a^4x}-\frac{7\sqrt[3]{b}(5Ab-2aB)}{27a}$$

```
[Out] (-7*(5*A*b - 2*a*B))/(36*a^3*b*x^4) + (7*(5*A*b - 2*a*B))/(9*a^4*x) + (A*b
- a*B)/(6*a*b*x^4*(a + b*x^3)^2) + (5*A*b - 2*a*B)/(9*a^2*b*x^4*(a + b*x^3)
) - (7*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(13/3)) - (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(1/3) + b^(1
/3)*x])/(27*a^(13/3)) + (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3))
```

Rubi [A] time = 0.158178, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}}+\frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)}-\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{7(5Ab-2aB)}{9a^4x}-\frac{7\sqrt[3]{b}(5Ab-2aB)}{27a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] (-7*(5*A*b - 2*a*B))/(36*a^3*b*x^4) + (7*(5*A*b - 2*a*B))/(9*a^4*x) + (A*b
- a*B)/(6*a*b*x^4*(a + b*x^3)^2) + (5*A*b - 2*a*B)/(9*a^2*b*x^4*(a + b*x^3)
) - (7*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(13/3)) - (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(1/3) + b^(1
/3)*x])/(27*a^(13/3)) + (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{(10Ab-4aB)\int \frac{1}{x^5\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7(5Ab-2aB))\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{(7(5Ab-2aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{9a^3}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7b(5Ab-2aB))\int \frac{x}{a+b}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{\left(7b^{2/3}(5Ab-2aB)\right)\int}{27a^{13/3}}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\right.}{27a^{13/3}}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\right.}{27a^{13/3}}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}(5Ab-2aB)\tan^{-1}}{9\sqrt{3}a^{13/3}}$$

<u>Mathematica</u> [A] time = 0.167024, size = 214, normalized size = 0.87

$$\frac{14\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{18a^{4/3}bx^2(aB-Ab)}{\left(a+bx^3\right)^2} - \frac{27a^{4/3}A}{x^4} - \frac{12\sqrt[3]{a}bx^2(5aB-8Ab)}{a+bx^3} - \frac{108\sqrt[3]{a}(aB-3Ab)}{x} + 28\sqrt[3]{b}(2aB}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*A)/x^4 - (108*a^(1/3)*(-3*A*b + a*B))/x - (18*a^(4/3)*b*(-(A*
b) + a*B)*x^2)/(a + b*x^3)^2 - (12*a^(1/3)*b*(-8*A*b + 5*a*B)*x^2)/(a + b*x
```

```
^3) - 28*Sqrt[3]*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] + 28*b^(1/3)*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(1/
3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*a^(
13/3))
```

Maple [A] time = 0.015, size = 299, normalized size = 1.2

$$\frac{8\,b^3Ax^5}{9\,a^4\left(bx^3+a\right)^2}-\frac{5\,b^2Bx^5}{9\,a^3\left(bx^3+a\right)^2}+\frac{19\,Ax^2b^2}{18\,a^3\left(bx^3+a\right)^2}-\frac{13\,bBx^2}{18\,a^2\left(bx^3+a\right)^2}-\frac{35\,Ab}{27\,a^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\frac{1}{\sqrt[3]{\frac{a}{b}}}+\frac{35\,Ab}{54\,a^4}\ln\left(x^2-\sqrt[3]{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^3,x)
```

```
[Out] 8/9/a^4*b^3/(b*x^3+a)^2*A*x^5-5/9/a^3*b^2/(b*x^3+a)^2*B*x^5+19/18/a^3*b^2/(
b*x^3+a)^2*A*x^2-13/18/a^2*b/(b*x^3+a)^2*B*x^2-35/27/a^4*b*A/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+35/54/a^4*b*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+35/27/a^4*b*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
14/27/a^3*B/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-7/27/a^3*B/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))-14/27/a^3*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))-1/4*A/a^3/x^4+3/a^4/x*A*b-1/a^3/x*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5678, size = 817, normalized size = 3.32

$$-\frac{84\left(2\,Bab^2-5\,Ab^3\right)x^9+147\left(2\,Ba^2b-5\,Aab^2\right)x^6+27\,Aa^3+54\left(2\,Ba^3-5\,Aa^2b\right)x^3+28\,\sqrt{3}\left(\left(2\,Bab^2-5\,Ab^3\right)x^{10}+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(84*(2*B*a*b^2 - 5*A*b^3)*x^9 + 147*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 27
*A*a^3 + 54*(2*B*a^3 - 5*A*a^2*b)*x^3 + 28*sqrt(3)*((2*B*a*b^2 - 5*A*b^3)*x
^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/a)^(1/
3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 14*((2*B*a*b^2 - 5*A*
b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/
a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 28*((2*B*a*b^2 -
5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*
(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x
^4)
```

Sympy [A] time = 3.87556, size = 189, normalized size = 0.77

$$\operatorname{RootSum}\left(19683t^3a^{13} + 42875A^3b^4 - 51450A^2Bab^3 + 20580AB^2a^2b^2 - 2744B^3a^3b, \left(t \mapsto t\log\left(\frac{729t^2a^9}{1225A^2b^3 - 980ABab^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*A
*B**2*a**2*b**2 - 2744*B**3*a**3*b, Lambda(_t, _t*log(729*_t**2*a**9/(1225*
A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) - (9*A*a**3 + x**9*(-1
40*A*b**3 + 56*B*a*b**2) + x**6*(-245*A*a*b**2 + 98*B*a**2*b) + x**3*(-90*A
*a**2*b + 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**10)
```

Giac [A] time = 1.15912, size = 343, normalized size = 1.39

$$\frac{7\left(2\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5} + \frac{7\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*(2*B*a*b*(-a/b)^(1/3) - 5*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/a^5 + 7/27*sqrt(3)*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)
*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 7/54*
(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^5*b) - 1/18*(10*B*a*b^2*x^5 - 16*A*b^3*x^5 + 13*B*a^2*b*x^2
- 19*A*a*b^2*x^2)/((b*x^3 + a)^2*a^4) - 1/4*(4*B*a*x^3 - 12*A*b*x^3 + A*a)/
(a^4*x^4)
```

3.106 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}}-\frac{4b^{2/3}(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}}\right.}{9\sqrt{3}a^{14/3}}$$

```
[Out] (-4*(11*A*b - 5*a*B))/(45*a^3*b*x^5) + (2*(11*A*b - 5*a*B))/(9*a^4*x^2) + (
A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + (11*A*b - 5*a*B)/(18*a^2*b*x^5*(a +
b*x^3)) - (4*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)) + (4*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3
) + b^(1/3)*x]])/(27*a^(14/3)) - (2*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(27*a^(14/3))
```

Rubi [A] time = 0.141659, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}}-\frac{4b^{2/3}(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}}\right.}{9\sqrt{3}a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```


```
[Out] (-4*(11*A*b - 5*a*B))/(45*a^3*b*x^5) + (2*(11*A*b - 5*a*B))/(9*a^4*x^2) + (
A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + (11*A*b - 5*a*B)/(18*a^2*b*x^5*(a +
b*x^3)) - (4*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)) + (4*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3
) + b^(1/3)*x]])/(27*a^(14/3)) - (2*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/(27*a^(14/3))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{(11Ab-5aB)\int\frac{1}{x^6\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}+\frac{(4(11Ab-5aB))\int\frac{1}{x^6\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}-\frac{(4(11Ab-5aB))\int\frac{1}{x^3\left(a+bx^3\right)}\,dx}{9a^3}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}+\frac{(4b(11Ab-5aB))}{9a^4}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}+\frac{(4b(11Ab-5aB))}{27a^{14/}}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}+\frac{4b^{2/3}(11Ab-5aB)}{27a}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}+\frac{4b^{2/3}(11Ab-5aB)}{27a}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}-\frac{4b^{2/3}(11Ab-5aB)}{9\sqrt{3}a}$$

Mathematica [A] time = 0.170137, size = 210, normalized size = 0.85

$$\frac{20b^{2/3}(5aB-11Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{45a^{5/3}bx(aB-Ab)}{\left(a+bx^3\right)^2}-\frac{15a^{2/3}bx(11aB-17Ab)}{a+bx^3}-\frac{135a^{2/3}(aB-3Ab)}{x^2}-\frac{54a^{5/3}A}{x^5}+40b^{2/3}(}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*A)/x^5 - (135*a^(2/3)*(-3*A*b + a*B))/x^2 - (45*a^(5/3)*b*(-(
A*b) + a*B)*x)/(a + b*x^3)^2 - (15*a^(2/3)*b*(-17*A*b + 11*a*B)*x)/(a + b*x
```

```
^3) - 40*Sqrt[3]*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]] + 40*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*b^(2
/3)*(-11*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
a^(14/3))
```

Maple [A] time = 0.016, size = 295, normalized size = 1.2

$$\frac{17\,b^3Ax^4}{18\,a^4\left(bx^3+a\right)^2}-\frac{11\,b^2Bx^4}{18\,a^3\left(bx^3+a\right)^2}+\frac{10\,b^2Ax}{9\,a^3\left(bx^3+a\right)^2}-\frac{7\,bBx}{9\,a^2\left(bx^3+a\right)^2}+\frac{44\,Ab}{27\,a^4}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{22\,Ab}{27\,a^4}\ln\left(x^2-\sqrt[3]{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^3,x)
```

```
[Out] 17/18/a^4*b^3/(b*x^3+a)^2*A*x^4-11/18/a^3*b^2/(b*x^3+a)^2*B*x^4+10/9/a^3*b^
2/(b*x^3+a)^2*A*x-7/9/a^2*b/(b*x^3+a)^2*B*x+44/27/a^4*b*A/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-22/27/a^4*b*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+44
/27/a^4*b*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-20/
27/a^3*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+10/27/a^3*B/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))-20/27/a^3*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))-1/5*A/a^3/x^5+3/2/a^4/x^2*A*b-1/2/a^3/x^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51152, size = 876, normalized size = 3.56

$$-\frac{60\left(5\,Bab^2-11\,Ab^3\right)x^9+96\left(5\,Ba^2b-11\,Aab^2\right)x^6+54\,Aa^3+27\left(5\,Ba^3-11\,Aa^2b\right)x^3+40\,\sqrt{3}\left(\left(5\,Bab^2-11\,Ab^3\right)x^{11}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/270*(60*(5*B*a*b^2 - 11*A*b^3)*x^9 + 96*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 5
4*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*sqrt(3)*((5*B*a*b^2 - 11*A*b^3
)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/
a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(
(5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 1
1*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(
b^2/a^2)^(2/3)) + 40*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b
^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)))/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5)
```

Sympy [A] time = 10.6843, size = 173, normalized size = 0.7

$$\operatorname{RootSum}\left(19683t^3a^{14}-85184A^3b^5+116160A^2Bab^4-52800AB^2a^2b^3+8000B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{27ta^5}{-44Ab^2+20Bab}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800*
A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, Lambda(_t, _t*log(-27*_t*a**5/(-44*
A*b**2 + 20*B*a*b) + x))) - (18*A*a**3 + x**9*(-220*A*b**3 + 100*B*a*b**2)
+ x**6*(-352*A*a*b**2 + 160*B*a**2*b) + x**3*(-99*A*a**2*b + 45*B*a**3))/(9
0*a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11)
```

Giac [A] time = 1.12628, size = 309, normalized size = 1.26

$$-\frac{4\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-11\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^5}+\frac{4\left(5Bab-11Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^5}-\frac{2\left(5\left(-a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 11*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 2/27*(5*(-a*b^2)^(1/3)*B*a - 11*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 - 1/18*(11
*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/((b*x^3 + a)^2*a
^4) - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)
```

3.107 $\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=70

$$\frac{a^2 \log \left(a+bx^3\right)}{3b^2(bc-ad)}-\frac{c^2 \log \left(c+dx^3\right)}{3d^2(bc-ad)}+\frac{x^3}{3bd}$$

```
[Out] x^3/(3*b*d) + (a^2*Log[a + b*x^3])/(3*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3
])/(3*d^2*(b*c - a*d))
```

Rubi [A] time = 0.0663375, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2 \log \left(a+bx^3\right)}{3b^2(bc-ad)}-\frac{c^2 \log \left(c+dx^3\right)}{3d^2(bc-ad)}+\frac{x^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^3/(3*b*d) + (a^2*Log[a + b*x^3])/(3*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3
])/(3*d^2*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right) dx, x, x^3\right)$$
$$= \frac{x^3}{3bd} + \frac{a^2\log\left(a+bx^3\right)}{3b^2(bc-ad)} - \frac{c^2\log\left(c+dx^3\right)}{3d^2(bc-ad)}$$

Mathematica [A] time = 0.0327186, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log\left(a+bx^3\right) - b\left(dx^3(ad-bc) + bc^2\log\left(c+dx^3\right)\right)}{3b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))/
(3*b^2*d^2*(b*c - a*d))
```

Maple [A] time = 0.009, size = 65, normalized size = 0.9

$$\frac{x^3}{3\,bd} + \frac{c^2\ln\left(dx^3+c\right)}{3\,d^2\,(ad-bc)} - \frac{a^2\ln\left(bx^3+a\right)}{3\,b^2\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*x^3/b/d+1/3*c^2/d^2/(a*d-b*c)*ln(d*x^3+c)-1/3*a^2/b^2/(a*d-b*c)*ln(b*x^
3+a)
```

Maxima [A] time = 0.930516, size = 92, normalized size = 1.31

$$\frac{a^2\log\left(bx^3+a\right)}{3\left(b^3c-ab^2d\right)} - \frac{c^2\log\left(dx^3+c\right)}{3\left(bcd^2-ad^3\right)} + \frac{x^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2
- a*d^3) + 1/3*x^3/(b*d)
```

Fricas [A] time = 2.69576, size = 142, normalized size = 2.03

$$\frac{a^2d^2\log\left(bx^3+a\right)-b^2c^2\log\left(dx^3+c\right)+\left(b^2cd-abd^2\right)x^3}{3\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)*
x^3)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [B] time = 7.52717, size = 201, normalized size = 2.87

$$-\frac{a^2\log\left(x^3+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{3b^2\left(ad-bc\right)}+\frac{c^2\log\left(x^3+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{3d^2\left(ad-bc\right)}+\frac{x^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] -a**2*log(x**3 + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a
**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(3
*b**2*(a*d - b*c)) + c**2*log(x**3 + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d
 + 2*a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*
d**2 + b**2*c**2))/(3*d**2*(a*d - b*c)) + x**3/(3*b*d)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.108 $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=301

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}+\ldots$$

```
[Out] x^2/(2*b*d) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(5/3)*(b*c - a*d)) + (c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)) - (a^(5/3)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)*(b*c - a*d)) + (c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(
5/3)*(b*c - a*d)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)*(b*c - a*d)) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*d^(5/3)*(b*c - a*d))
```

Rubi [A] time = 0.309725, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 584, 292, 31, 634, 617, 204, 628}

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)*(c + d*x^3)),x]
```


```
[Out] x^2/(2*b*d) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(5/3)*(b*c - a*d)) + (c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)) - (a^(5/3)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)*(b*c - a*d)) + (c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(
5/3)*(b*c - a*d)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)*(b*c - a*d)) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*d^(5/3)*(b*c - a*d))
```


Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{x^2}{2bd} - \frac{\int \frac{x(2ac+2(bc+ad)x^3)}{(a+bx^3)(c+dx^3)}\,dx}{2bd}$$

$$= \frac{x^2}{2bd} - \frac{\int \left(\frac{2a^2dx}{(-bc+ad)(a+bx^3)} + \frac{2bc^2x}{(bc-ad)(c+dx^3)}\right)dx}{2bd}$$

$$= \frac{x^2}{2bd} + \frac{a^2\int \frac{x}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{x}{c+dx^3}\,dx}{d(bc-ad)}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}(bc-ad)} + \frac{a^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{4/3}(bc-ad)} + \frac{c^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d^{4/3}(bc-ad)} - \frac{c^{5/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{\ldots}}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x\ldots}}{3d^{4/3}(bc-\ldots}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{5/3}(bc-ad)} + \frac{a^2\int \frac{\ldots}{a^{2/3}-\ldots}}{2b^{4\ldots}}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)} - \frac{c^{5\ldots}}{\ldots}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)} + \frac{c^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{5/3}(bc-ad)} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}\ldots\right.}{3d^{5/3}(bc-\ldots}$$

Mathematica [A] time = 0.143007, size = 242, normalized size = 0.8

$$\frac{\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{5/3}} - \frac{2a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{5/3}} - \frac{2\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} - \frac{3ax^2}{b} - \frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{5/3}} + \frac{2c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{5/3}} + \ldots}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-3*a*x^2)/b + (3*c*x^2)/d - (2*Sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/b^(5/3) + (2*Sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) +
(2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2])/d^(5/3))/(6*b*c - 6*a*d)
```

Maple [A] time = 0.008, size = 269, normalized size = 0.9

$$\frac{x^2}{2bd} - \frac{c^2}{3d^2(ad-bc)}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}} + \frac{c^2}{6d^2(ad-bc)}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}} + \frac{c^2\sqrt{3}}{3d^2(ad-bc)}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/2*x^2/b/d-1/3*c^2/d^2/(a*d-b*c)/(1/d*c)^(1/3)*ln(x+(1/d*c)^(1/3))+1/6*c^2
/d^2/(a*d-b*c)/(1/d*c)^(1/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))+1/3*c^2/
d^2/(a*d-b*c)*3^(1/2)/(1/d*c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1
))+1/3*a^2/b^2/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*a^2/b^2/(a*d-b*c
)/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*a^2/b^2/(a*d-b*c)*3^(1/
2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.93356, size = 659, normalized size = 2.19

$$\frac{2\sqrt{3}ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)-2\sqrt{3}bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}c}{3c}\right)+ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*d*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3
) - sqrt(3)*a)/a) - 2*sqrt(3)*b*c*(-c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*
x*(-c^2/d^2)^(1/3) + sqrt(3)*c)/c) + a*d*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a
^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + b*c*(-c^2/d^2)^(1/3)*log(c*x^2 - d*x*(
-c^2/d^2)^(2/3) - c*(-c^2/d^2)^(1/3)) - 2*a*d*(a^2/b^2)^(1/3)*log(a*x + b*(
a^2/b^2)^(2/3)) - 2*b*c*(-c^2/d^2)^(1/3)*log(c*x + d*(-c^2/d^2)^(2/3)) + 3*
(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)
```

Sympy [B] time = 167.128, size = 663, normalized size = 2.2

$$\operatorname{RootSum}\left(t^3\left(27a^3d^8-81a^2bcd^7+81ab^2c^2d^6-27b^3c^3d^5\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-243t^5a^8b^5d^{13}+1215t^5a^7b^6cd^{12}-24\cdots}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**8 - 81*a**2*b*c*d**7 + 81*a*b**2*c**2*d**6 - 27*b
**3*c**3*d**5) + c**5, Lambda(_t, _t*log(x + (-243*_t**5*a**8*b**5*d**13 +
1215*_t**5*a**7*b**6*c*d**12 - 2430*_t**5*a**6*b**7*c**2*d**11 + 2673*_t**5
*a**5*b**8*c**3*d**10 - 2430*_t**5*a**4*b**9*c**4*d**9 + 2673*_t**5*a**3*b*
*10*c**5*d**8 - 2430*_t**5*a**2*b**11*c**6*d**7 + 1215*_t**5*a*b**12*c**7*d
**6 - 243*_t**5*b**13*c**8*d**5 + 9*_t**2*a**10*d**10 - 18*_t**2*a**9*b*c*d
**9 + 9*_t**2*a**8*b**2*c**2*d**8 + 9*_t**2*a**2*b**8*c**8*d**2 - 18*_t**2*
a*b**9*c**9*d + 9*_t**2*b**10*c**10)/(a**8*c**3*d**5 + a**3*b**5*c**8)))) +
 RootSum(_t**3*(27*a**3*b**5*d**3 - 81*a**2*b**6*c*d**2 + 81*a*b**7*c**2*d
- 27*b**8*c**3) - a**5, Lambda(_t, _t*log(x + (-243*_t**5*a**8*b**5*d**13 +
 1215*_t**5*a**7*b**6*c*d**12 - 2430*_t**5*a**6*b**7*c**2*d**11 + 2673*_t**
5*a**5*b**8*c**3*d**10 - 2430*_t**5*a**4*b**9*c**4*d**9 + 2673*_t**5*a**3*b
```

```
**10*c**5*d**8 - 2430*_t**5*a**2*b**11*c**6*d**7 + 1215*_t**5*a*b**12*c**7*
d**6 - 243*_t**5*b**13*c**8*d**5 + 9*_t**2*a**10*d**10 - 18*_t**2*a**9*b*c*
d**9 + 9*_t**2*a**8*b**2*c**2*d**8 + 9*_t**2*a**2*b**8*c**8*d**2 - 18*_t**2
*a*b**9*c**9*d + 9*_t**2*b**10*c**10)/(a**8*c**3*d**5 + a**3*b**5*c**8))))
+ x**2/(2*b*d)
```

Giac [A] time = 1.17978, size = 436, normalized size = 1.45

$$\frac{\left(-ab^2\right)^{\frac{2}{3}} a^2 b \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^5c - a^2b^4d\right)} - \frac{\left(-cd^2\right)^{\frac{2}{3}} c^2 d \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2d^4 - acd^5\right)} - \frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c - a^2bd\right)} + \frac{c^2 \left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(-a*b^2)^(2/3)*a^2*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5*c
- a^2*b^4*d) - 1/6*(-c*d^2)^(2/3)*c^2*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(
2/3))/(b*c^2*d^4 - a*c*d^5) - 1/3*a^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)
))/(a*b^2*c - a^2*b*d) + 1/3*c^2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b
*c^2*d - a*c*d^2) - (-a*b^2)^(2/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(sqrt(3)*b^4*c - sqrt(3)*a*b^3*d) + (-c*d^2)^(2/3)*c*arctan
(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*
a*d^4) + 1/2*x^2/(b*d)
```

3.109 $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=296

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}-$$

```
[Out] x/(b*d) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*b^(4/3)*(b*c - a*d)) + (c^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]
*c^(1/3))])/(Sqrt[3]*d^(4/3)*(b*c - a*d)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(4/3)*(b*c - a*d)) - (c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(4/3)
*(b*c - a*d)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*b^(4/3)*(b*c - a*d)) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(6*d^(4/3)*(b*c - a*d))
```

Rubi [A] time = 0.266874, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)*(c + d*x^3)),x]
```


```
[Out] x/(b*d) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*b^(4/3)*(b*c - a*d)) + (c^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]
*c^(1/3))])/(Sqrt[3]*d^(4/3)*(b*c - a*d)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(4/3)*(b*c - a*d)) - (c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(4/3)
*(b*c - a*d)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*b^(4/3)*(b*c - a*d)) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(6*d^(4/3)*(b*c - a*d))
```


Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{bd}$$

$$= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^3}\,dx}{d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b(bc-ad)} + \frac{a^{4/3}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b(bc-ad)} - \frac{c^{4/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d(bc-ad)} - \frac{c^{4/3}\int \frac{2\sqrt[3]{c}-\sqrt[3]{}}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+}}{3d(bc-a}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}(bc-ad)} + \frac{a^{5/3}\int \frac{}{a^{2/3}-}}{2b(}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)} + \frac{c^{4/3}}{}$$

$$= \frac{x}{bd} - \frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)} + \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\right.}{3d^{4/3}(bc-}$$

Mathematica [A] time = 0.122466, size = 238, normalized size = 0.8

$$\frac{-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{4/3}} + \frac{2a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}} - \frac{2\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} - \frac{6ax}{b} + \frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{4/3}} - \frac{2c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{4/3}} + }{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/
```

```
3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (2*c
^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2])/d^(4/3))/(6*b*c - 6*a*d)
```

Maple [A] time = 0.009, size = 266, normalized size = 0.9

$$\frac{x}{bd}+\frac{c^2}{3\,d^2\,(ad-bc)}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{c^2}{6\,d^2\,(ad-bc)}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{c^2\sqrt{3}}{3\,d^2\,(ad-bc)}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] x/b/d+1/3/d^2*c^2/(a*d-b*c)/(1/d*c)^(2/3)*ln(x+(1/d*c)^(1/3))-1/6/d^2*c^2/(
a*d-b*c)/(1/d*c)^(2/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))+1/3/d^2*c^2/(a
*d-b*c)/(1/d*c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))-1/3
/b^2*a^2/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/b^2*a^2/(a*d-b*c)/(a/b
)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2*a^2/(a*d-b*c)/(a/b)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68465, size = 555, normalized size = 1.88

$$-\frac{2\sqrt{3}ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+2\sqrt{3}bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3\,c}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2cd-abd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a*d*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - s
qrt(3)*a)/a) + 2*sqrt(3)*b*c*(c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c/d)^(2
/3) - sqrt(3)*c)/c) - a*d*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3)) - b*c*(c/d)^(1/3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)) + 2*a*d*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 2*b*c*(c/d)^(1/3)*log(x + (c/d)^(1/3)) - 6*
(b*c - a*d)*x)/(b^2*c*d - a*b*d^2)
```

Sympy [A] time = 90.2173, size = 452, normalized size = 1.53

$$\operatorname{RootSum}\left(t^3\left(27a^3d^7 - 81a^2bcd^6 + 81ab^2c^2d^5 - 27b^3c^3d^4\right) - c^4, \left(t \mapsto t\log\left(x + \frac{81t^4a^5b^4d^9 - 243t^4a^4b^5cd^8 + 162t^4a^3b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**7 - 81*a**2*b*c*d**6 + 81*a*b**2*c**2*d**5 - 27*b
**3*c**3*d**4) - c**4, Lambda(_t, _t*log(x + (81*_t**4*a**5*b**4*d**9 - 243
*_t**4*a**4*b**5*c*d**8 + 162*_t**4*a**3*b**6*c**2*d**7 + 162*_t**4*a**2*b*
*7*c**3*d**6 - 243*_t**4*a*b**8*c**4*d**5 + 81*_t**4*b**9*c**5*d**4 + 3*_t*
a**6*d**6 - 3*_t*a**5*b*c*d**5 - 3*_t*a*b**5*c**5*d + 3*_t*b**6*c**6)/(a**5
*c*d**4 + a*b**4*c**5)))) + RootSum(_t**3*(27*a**3*b**4*d**3 - 81*a**2*b**5
*c*d**2 + 81*a*b**6*c**2*d - 27*b**7*c**3) + a**4, Lambda(_t, _t*log(x + (8
1*_t**4*a**5*b**4*d**9 - 243*_t**4*a**4*b**5*c*d**8 + 162*_t**4*a**3*b**6*c
**2*d**7 + 162*_t**4*a**2*b**7*c**3*d**6 - 243*_t**4*a*b**8*c**4*d**5 + 81*
_t**4*b**9*c**5*d**4 + 3*_t*a**6*d**6 - 3*_t*a**5*b*c*d**5 - 3*_t*a*b**5*c*
*5*d + 3*_t*b**6*c**6)/(a**5*c*d**4 + a*b**4*c**5)))) + x/(b*d)
```

Giac [A] time = 1.15153, size = 416, normalized size = 1.41

$$-\frac{a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\right.}{}\right.}{\sqrt{3}bcd^2-\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (-a*b^2)^
(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*
c - sqrt(3)*a*b^2*d) - (-c*d^2)^(1/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) + 1/6*(-a*b^2)^(1/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) - 1/6*(-c*d^2)^
(1/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3) + x/(b*d
)
```

3.110 $\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=53

$$\frac{c\log\left(c+dx^3\right)}{3d(bc-ad)}-\frac{a\log\left(a+bx^3\right)}{3b(bc-ad)}$$

```
[Out] -(a*Log[a + b*x^3])/(3*b*(b*c - a*d)) + (c*Log[c + d*x^3])/(3*d*(b*c - a*d)
)
```

Rubi [A] time = 0.0500404, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c\log\left(c+dx^3\right)}{3d(bc-ad)}-\frac{a\log\left(a+bx^3\right)}{3b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(a*Log[a + b*x^3])/(3*b*(b*c - a*d)) + (c*Log[c + d*x^3])/(3*d*(b*c - a*d)
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right)dx, x, x^3\right)$$
$$= -\frac{a\log\left(a+bx^3\right)}{3b(bc-ad)} + \frac{c\log\left(c+dx^3\right)}{3d(bc-ad)}$$

Mathematica [A] time = 0.0200565, size = 43, normalized size = 0.81

$$-\frac{ad\log\left(a+bx^3\right)-bc\log\left(c+dx^3\right)}{3b^2cd-3abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))
```

Maple [A] time = 0.007, size = 50, normalized size = 0.9

$$-\frac{c\ln\left(dx^3+c\right)}{(3\,ad-3\,bc)\,d}+\frac{a\ln\left(bx^3+a\right)}{(3\,ad-3\,bc)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*c/(a*d-b*c)/d*ln(d*x^3+c)+1/3*a/(a*d-b*c)/b*ln(b*x^3+a)
```

Maxima [A] time = 0.931464, size = 66, normalized size = 1.25

$$-\frac{a\log\left(bx^3+a\right)}{3\left(b^2c-abd\right)}+\frac{c\log\left(dx^3+c\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^2
)
```

Fricas [A] time = 1.79347, size = 92, normalized size = 1.74

$$-\frac{ad\log\left(bx^3+a\right)-bc\log\left(dx^3+c\right)}{3\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] time = 2.89661, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^3+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{3b\left(ad-bc\right)}-\frac{c\log\left(x^3+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(3*d*(a*d - b*c))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.111 $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}-$$

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3
)*(b*c - a*d)) - (c^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
2/3)*(b*c - a*d)) - (c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(2/3)*(b*c - a*
d)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(
b*c - a*d)) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(2/3)*(b*c - a*d))
```

Rubi [A] time = 0.15416, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3
)*(b*c - a*d)) - (c^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
2/3)*(b*c - a*d)) - (c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(2/3)*(b*c - a*
d)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(
b*c - a*d)) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(2/3)*(b*c - a*d))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
```

```
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\int \frac{x}{a+bx^3}\,dx}{bc-ad} + \frac{c\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{b}(bc-ad)} - \frac{a^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{b}(bc-ad)} - \frac{c^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3\sqrt[3]{d}(bc-ad)} + \frac{c^{2/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3\sqrt[3]{d}(bc-ad)}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{2/3}(bc-ad)} - \frac{a\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}(bc-ad)}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)} + \frac{c^{2/3}\log\left(\ldots\right)}{\ldots}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)}$$

Mathematica [A] time = 0.0898973, size = 224, normalized size = 0.78

$$\frac{-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{2/3}} - \frac{2c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}} - \frac{2\sqrt{3}c^{2/3}\ldots}{\ldots}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) -
(2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) + (
2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(2/3))
/(6*b*c - 6*a*d)
```

Maple [A] time = 0.008, size = 246, normalized size = 0.9

$$\frac{c}{(3ad-3bc)d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}} - \frac{c}{(6ad-6bc)d}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}} - \frac{c\sqrt{3}}{(3ad-3bc)d}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*c/(a*d-b*c)/d/(1/d*c)^(1/3)*ln(x+(1/d*c)^(1/3))-1/6*c/(a*d-b*c)/d/(1/d*
c)^(1/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))-1/3*c/(a*d-b*c)*3^(1/2)/d/(1
/d*c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))-1/3*a/(a*d-b*c)/b/(a/
b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*a/(a*d-b*c)/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*a/(a*d-b*c)*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67034, size = 590, normalized size = 2.05

$$-\frac{2\sqrt{3}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-2\sqrt{3}\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}c}{3c}\right)-\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)\right.}{6\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3)
 + sqrt(3)*a)/a) - 2*sqrt(3)*(c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c^2
/d^2)^(1/3) - sqrt(3)*c)/c) - (-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(
2/3) - a*(-a^2/b^2)^(1/3)) - (c^2/d^2)^(1/3)*log(c*x^2 - d*x*(c^2/d^2)^(2/3
) + c*(c^2/d^2)^(1/3)) + 2*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)) +
 2*(c^2/d^2)^(1/3)*log(c*x + d*(c^2/d^2)^(2/3)))/(b*c - a*d)
```

Sympy [B] time = 11.714, size = 573, normalized size = 1.99

$$\mathrm{RootSum}\left(t^3\left(27a^3d^5-81a^2bcd^4+81ab^2c^2d^3-27b^3c^3d^2\right)-c^2,\left(t\mapsto t\log\left(x+\frac{243t^5a^6b^2d^8-1458t^5a^5b^3cd^7+3645t^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**5 - 81*a**2*b*c*d**4 + 81*a*b**2*c**2*d**3 - 27*b
**3*c**3*d**2) - c**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 14
58*_t**5*a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**
3*b**5*c**3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*
d**3 + 243*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**
4 + 9*_t**2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*
b**4*c**4*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3)))) + RootSum(_
t**3*(27*a**3*b**2*d**3 - 81*a**2*b**3*c*d**2 + 81*a*b**4*c**2*d - 27*b**5*
c**3) + a**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 1458*_t**5*
a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**3*b**5*c*
*3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*d**3 + 24
3*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**4 + 9*_t*
*2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*b**4*c**4
*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3))))
```

Giac [A] time = 1.13683, size = 386, normalized size = 1.34

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right.\right.}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right.}{\sqrt{3}bcd^2-\sqrt{3}ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(2/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b
^2*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3)
```

```
)/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^(2/3)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3)
```

3.112 $\int \frac{x^3}{(a+bx^3)(c+dx^3)} dx$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}+$$

```
[Out] (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3
)*(b*c - a*d)) - (c^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(1/3)*(b*c - a*d)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
1/3)*(b*c - a*d)) + (c^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(1/3)*(b*c - a*
d)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)*(
b*c - a*d)) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(1/3)*(b*c - a*d))
```

Rubi [A] time = 0.148388, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3
)*(b*c - a*d)) - (c^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(1/3)*(b*c - a*d)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
1/3)*(b*c - a*d)) + (c^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(1/3)*(b*c - a*
d)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)*(
b*c - a*d)) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(1/3)*(b*c - a*d))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
```

```
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{a\int \frac{1}{a+bx^3}\,dx}{bc-ad} + \frac{c\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3(bc-ad)} - \frac{\sqrt[3]{a}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3(bc-ad)} + \frac{\sqrt[3]{c}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3(bc-ad)} + \frac{\sqrt[3]{c}\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,\text{(}}{3(bc-ad)}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)} - \frac{a^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2(bc-ad)} + \frac{\sqrt[3]{a}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}}{6\sqrt[3]{b}(bc-}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\log\left(c\right.}{}$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)} -$$

Mathematica [A] time = 0.085183, size = 224, normalized size = 0.78

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{d}} - \frac{2\sqrt{3}\sqrt[3]{c}\tan^{-}}{\sqrt[3]{}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) -
(2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) - (
2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(1/3))
/(6*b*c - 6*a*d)
```

Maple [A] time = 0.009, size = 246, normalized size = 0.9

$$-\frac{c}{(3ad-3bc)d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{c}{(6ad-6bc)d}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{c\sqrt{3}}{(3ad-3bc)d}\arctan\left(\frac{\sqrt{3}}{3}\left(2x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*c/(a*d-b*c)/d/(1/d*c)^(2/3)*ln(x+(1/d*c)^(1/3))+1/6*c/(a*d-b*c)/d/(1/d
*c)^(2/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))-1/3*c/(a*d-b*c)/d/(1/d*c)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))+1/3*a/(a*d-b*c)/b/(a
/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*a/(a*d-b*c)/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))+1/3*a/(a*d-b*c)/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62498, size = 487, normalized size = 1.69

$$-\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log}{6(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)
*a)/a) + 2*sqrt(3)*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(-c/d)^(2/3) - sq
rt(3)*c)/c) - (a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - (-c/d)^(
1/3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)) + 2*(a/b)^(1/3)*log(x + (a/b)
^(1/3)) + 2*(-c/d)^(1/3)*log(x - (-c/d)^(1/3)))/(b*c - a*d)
```

Sympy [A] time = 18.3284, size = 342, normalized size = 1.19

$$\mathrm{RootSum}\left(t^3\left(27a^3d^4-81a^2bcd^3+81ab^2c^2d^2-27b^3c^3d\right)+c,\left(t\mapsto t\log\left(x+\frac{162t^4a^4bd^5-648t^4a^3b^2cd^4+972t^4a^2b^3c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*b
**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4*a
**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*d**
2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c**2)
/(a*d + b*c)))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2 + 81*
a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*
d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t*
*4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d
 - 3*_t*b**2*c**2)/(a*d + b*c))))
```

Giac [A] time = 1.16067, size = 375, normalized size = 1.3

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd}+\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right.\right.}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right.}{\sqrt{3}bcd-\sqrt{3}ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(1/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b
*d) + (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/
(sqrt(3)*b*c*d - sqrt(3)*a*d^2) - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(b^2*c - a*b*d) + 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)^
(1/3) + (-c/d)^(2/3))/(b*c*d - a*d^2)
```

3.113 $\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

```
[Out] Log[a + b*x^3]/(3*(b*c - a*d)) - Log[c + d*x^3]/(3*(b*c - a*d))
```

Rubi [A] time = 0.0308165, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[a + b*x^3]/(3*(b*c - a*d)) - Log[c + d*x^3]/(3*(b*c - a*d))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^3\right)}{3(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^3\right)}{3(bc-ad)}$$
$$= \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

Mathematica [A] time = 0.0175543, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^3)-\log(c+dx^3)}{3bc-3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)
```

Maple [A] time = 0.006, size = 42, normalized size = 0.9

$$\frac{\ln(dx^3+c)}{3\,ad-3\,bc} - \frac{\ln(bx^3+a)}{3\,ad-3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3/(a*d-b*c)*ln(d*x^3+c)-1/3/(a*d-b*c)*ln(b*x^3+a)
```

Maxima [A] time = 0.924533, size = 55, normalized size = 1.22

$$\frac{\log(bx^3+a)}{3\,(bc-ad)} - \frac{\log(dx^3+c)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)
```

Fricas [A] time = 1.52765, size = 69, normalized size = 1.53

$$\frac{\log\left(bx^3+a\right)-\log\left(dx^3+c\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)
```

Sympy [B] time = 2.50961, size = 138, normalized size = 3.07

$$\frac{\log\left(x^3+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3\,(ad-bc)}-\frac{\log\left(x^3+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(3*(a*d - b*c))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.114 $\int \frac{x}{(a+bx^3)(c+dx^3)} dx$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\,\text{ta}}{\sqrt{3}}$$

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1
/3)*(b*c - a*d))) + (d^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*(b*c - a*d)) + (d^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c -
 a*d)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3
)*(b*c - a*d)) - (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(1/3)*(b*c - a*d))
```

Rubi [A] time = 0.143002, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {482, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\,\text{ta}}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1
/3)*(b*c - a*d))) + (d^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*(b*c - a*d)) + (d^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c -
 a*d)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3
)*(b*c - a*d)) - (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(1/3)*(b*c - a*d))
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
```

```
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b\int \frac{x}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}(bc-ad)} + \frac{d^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3\sqrt[3]{c}(bc-ad)} - \frac{d^{2/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}}{3\sqrt[3]{c}(bc-ad)}$$

$$= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x-}}{2(bc-ad}$$

$$= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c^2\right.}{6}$$

$$= -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}$$

Mathematica [A] time = 0.0991425, size = 224, normalized size = 0.78

$$\frac{-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{\sqrt[3]{c}} - \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{c}} - \frac{2\sqrt{3}\sqrt[3]{d}\tan^{-1}}{\sqrt[3]{}}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) -
(2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(1/3) + (
2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(1/3))
/(-6*b*c + 6*a*d)
```

Maple [A] time = 0.007, size = 222, normalized size = 0.8

$$-\frac{1}{3\,ad-3\,bc}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}} + \frac{1}{6\,ad-6\,bc}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}} + \frac{\sqrt{3}}{3\,ad-3\,bc}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\frac{1}{\sqrt[3]{\frac{c}{d}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/(a*d-b*c)/(1/d*c)^(1/3)*ln(x+(1/d*c)^(1/3))+1/6/(a*d-b*c)/(1/d*c)^(1/3
)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))+1/3/(a*d-b*c)*3^(1/2)/(1/d*c)^(1/3)
*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))+1/3/(a*d-b*c)/(a/b)^(1/3)*ln(x+(
a/b)^(1/3))-1/6/(a*d-b*c)/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3
/(a*d-b*c)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62779, size = 497, normalized size = 1.73

$$\frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right.\right.}{6\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3))
- 2*sqrt(3)*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3)) +
 (b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + (-d/c)^(1/3)*lo
g(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 2*(b/a)^(1/3)*log(b*x + a*(b
/a)^(2/3)) - 2*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)))/(b*c - a*d)
```

Sympy [A] time = 11.1138, size = 515, normalized size = 1.79

$$\mathrm{RootSum}\left(t^3\left(27a^4d^3-81a^3bcd^2+81a^2b^2c^2d-27ab^3c^3\right)-b,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5a^6bc^2d^5+3645t^5a^5b^2\cdots}{\cdots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**4*d**3 - 81*a**3*b*c*d**2 + 81*a**2*b**2*c**2*d - 27*a
*b**3*c**3) - b, Lambda(_t, _t*log(x + (243*_t**5*a**7*c*d**6 - 1458*_t**5*
a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860*_t**5*a**4*b**3*c*
*4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a**2*b**5*c**6*d + 24
3*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*b*c*d**3 + 18*_t**2
*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*b**4*c**4)/(a*b*d**
2 + b**2*c*d)))) + RootSum(_t**3*(27*a**3*c*d**3 - 81*a**2*b*c**2*d**2 + 81
*a*b**2*c**3*d - 27*b**3*c**4) + d, Lambda(_t, _t*log(x + (243*_t**5*a**7*c
*d**6 - 1458*_t**5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860
*_t**5*a**4*b**3*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a*
*2*b**5*c**6*d + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*
b*c*d**3 + 18*_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*
b**4*c**4)/(a*b*d**2 + b**2*c*d))))
```

Giac [A] time = 1.15834, size = 392, normalized size = 1.36

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c-\sqrt{3}a^2bd}+\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\cdots\right)\right)}{3\left(-\frac{c}{d}\right)^{\cdots}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(2/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c - sqrt(3)*
a^2*b*d) + (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^(2/3)*log(x^
2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2*d - a*c*d^2)
```

3.115 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-$$

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rubi [A] time = 0.142317, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)} + \frac{b\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\ldots}}{2\sqrt[3]{a}(bc-a\ldots}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(\ldots\right.}{\ldots}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}{3c^{2/3}(bc-ad)}$$

Mathematica [A] time = 0.100221, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} - \frac{2\sqrt{3}d^{2/3}\,\mathrm{ta}\ldots}{\ldots}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

Maple [A] time = 0.007, size = 222, normalized size = 0.8

$$\frac{1}{3\,ad-3\,bc}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{1}{6\,ad-6\,bc}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}}{3\,ad-3\,bc}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3/(a*d-b*c)/(1/d*c)^(2/3)*ln(x+(1/d*c)^(1/3))-1/6/(a*d-b*c)/(1/d*c)^(2/3)
*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))+1/3/(a*d-b*c)/(1/d*c)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))-1/3/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a
/b)^(1/3))+1/6/(a*d-b*c)/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/
(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93631, size = 606, normalized size = 2.1

$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\right.\right.}{6(bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

Sympy [A] time = 46.4953, size = 447, normalized size = 1.55

$$\text{RootSum}\left(t^3\left(27a^5d^3-81a^4bcd^2+81a^3b^2c^2d-27a^2b^3c^3\right)+b^2,\left(t\mapsto t\log\left(x+\frac{81t^4a^7c^2d^5-243t^4a^6bc^3d^4+162t^4a^5b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*a
**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243
*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b*
*3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*
a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2
*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c*
*3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x + (8
1*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c
**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*
_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c*
*3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

Giac [A] time = 1.15826, size = 375, normalized size = 1.3

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\right.\right.}{3\left(-\frac{c}{d}\right.}\right.}{\sqrt{3}bc^2-\sqrt{3}acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

3.116 $\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=62

$$-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^3])/(3*a*(b*c - a*d)) + (d*Log[c + d*x^3])/(3
*c*(b*c - a*d))
```

Rubi [A] time = 0.0604353, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^3])/(3*a*(b*c - a*d)) + (d*Log[c + d*x^3])/(3
*c*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{acx} + \frac{b^2}{a(-bc+ad)(a+bx)} + \frac{d^2}{c(bc-ad)(c+dx)}\right)dx, x, x^3\right)$$
$$= \frac{\log(x)}{ac} - \frac{b\log\left(a+bx^3\right)}{3a(bc-ad)} + \frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}$$

Mathematica [A] time = 0.0286106, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^3\right)+ad\log\left(c+dx^3\right)-3ad\log(x)+3bc\log(x)}{3abc^2-3a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3*
a*b*c^2 - 3*a^2*c*d)
```

Maple [A] time = 0.01, size = 59, normalized size = 1.

$$-\frac{d\ln\left(dx^3+c\right)}{3\,c\,(ad-bc)} + \frac{b\ln\left(bx^3+a\right)}{3\,a\,(ad-bc)} + \frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*d/c/(a*d-b*c)*ln(d*x^3+c)+1/3*b/a/(a*d-b*c)*ln(b*x^3+a)+ln(x)/a/c
```

Maxima [A] time = 0.949285, size = 82, normalized size = 1.32

$$-\frac{b\log\left(bx^3+a\right)}{3\left(abc-a^2d\right)} + \frac{d\log\left(dx^3+c\right)}{3\left(bc^2-acd\right)} + \frac{\log\left(x^3\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*d
) + 1/3*log(x^3)/(a*c)
```

Fricas [A] time = 3.51428, size = 123, normalized size = 1.98

$$-\frac{bc\log\left(bx^3+a\right)-ad\log\left(dx^3+c\right)-3\left(bc-ad\right)\log\left(x\right)}{3\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.117 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=299

$$-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}-$$

```
[Out] -(1/(a*c*x)) + (b^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(4/3)*(b*c - a*d)) - (d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)) + (b^(4/3)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(4/3)*(b*c - a*d)) - (d^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^
(4/3)*(b*c - a*d)) - (b^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(6*a^(4/3)*(b*c - a*d)) + (d^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(6*c^(4/3)*(b*c - a*d))
```

Rubi [A] time = 0.271776, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(4/3)*(b*c - a*d)) - (d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)) + (b^(4/3)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(4/3)*(b*c - a*d)) - (d^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^
(4/3)*(b*c - a*d)) - (b^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(6*a^(4/3)*(b*c - a*d)) + (d^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(6*c^(4/3)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{acx}+\frac{\int \frac{x\left(-bc-ad-bdx^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{ac}$$

$$= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx}{(bc-ad)\left(a+bx^3\right)}-\frac{ad^2x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{ac}$$

$$= -\frac{1}{acx}-\frac{b^2\int \frac{x}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x}{c+dx^3}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{acx}+\frac{b^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}(bc-ad)}-\frac{b^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}(bc-ad)}-\frac{d^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{4/3}(bc-ad)}+$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}(bc-ad)}-$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-$$

Mathematica [A] time = 0.136827, size = 244, normalized size = 0.82

$$\frac{\frac{b^{4/3}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}}-\frac{2b^{4/3}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}-\frac{2\sqrt{3}b^{4/3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{6b}{a}-\frac{d^{4/3}x\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{4/3}}+\frac{2d^{4/3}x\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{4/3}}+}{6adx-6bcx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (2*Sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(4/3) + (2*Sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3
))/Sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (2*
d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)
```

Maple [A] time = 0.01, size = 257, normalized size = 0.9

$$-\frac{1}{acx}+\frac{d}{3\,c\,(ad-bc)}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}-\frac{d}{6\,c\,(ad-bc)}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}-\frac{d\sqrt{3}}{3\,c\,(ad-bc)}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/a/c/x+1/3*d/c/(a*d-b*c)/(1/d*c)^(1/3)*ln(x+(1/d*c)^(1/3))-1/6*d/c/(a*d-b
*c)/(1/d*c)^(1/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))-1/3*d/c/(a*d-b*c)*3
^(1/2)/(1/d*c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))-1/3*b/a/(a*d
-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*b/a/(a*d-b*c)/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3*b/a/(a*d-b*c)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89678, size = 585, normalized size = 1.96

$$-\frac{2\,\sqrt{3}bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-2\,\sqrt{3}adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)-bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*
sqrt(3)) - 2*sqrt(3)*a*d*x*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(d/c)^(1/3) - 1
/3*sqrt(3)) - b*c*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1
/3)) - a*d*x*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 2*b
*c*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 2*a*d*x*(d/c)^(1/3)*log(d*x +
 c*(d/c)^(2/3)) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)
```

Sympy [B] time = 39.3303, size = 661, normalized size = 2.21

$$\text{RootSum}\left(t^3\left(27a^7d^3 - 81a^6bcd^2 + 81a^5b^2c^2d - 27a^4b^3c^3\right) + b^4, \left(t \mapsto t\log\left(x + \frac{-243t^5a^{12}c^4d^8 + 1215t^5a^{11}bc^5d^7 - 2430}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**7*d**3 - 81*a**6*b*c*d**2 + 81*a**5*b**2*c**2*d - 27*a
**4*b**3*c**3) + b**4, Lambda(_t, _t*log(x + (-243*_t**5*a**12*c**4*d**8 +
1215*_t**5*a**11*b*c**5*d**7 - 2430*_t**5*a**10*b**2*c**6*d**6 + 2673*_t**5
*a**9*b**3*c**7*d**5 - 2430*_t**5*a**8*b**4*c**8*d**4 + 2673*_t**5*a**7*b**
5*c**9*d**3 - 2430*_t**5*a**6*b**6*c**10*d**2 + 1215*_t**5*a**5*b**7*c**11*
d - 243*_t**5*a**4*b**8*c**12 + 9*_t**2*a**9*d**9 - 18*_t**2*a**8*b*c*d**8
+ 9*_t**2*a**7*b**2*c**2*d**7 + 9*_t**2*a**2*b**7*c**7*d**2 - 18*_t**2*a*b*
*8*c**8*d + 9*_t**2*b**9*c**9)/(a**4*b**3*d**7 + b**7*c**4*d**3)))) + RootS
um(_t**3*(27*a**3*c**4*d**3 - 81*a**2*b*c**5*d**2 + 81*a*b**2*c**6*d - 27*b
**3*c**7) - d**4, Lambda(_t, _t*log(x + (-243*_t**5*a**12*c**4*d**8 + 1215*
_t**5*a**11*b*c**5*d**7 - 2430*_t**5*a**10*b**2*c**6*d**6 + 2673*_t**5*a**9
*b**3*c**7*d**5 - 2430*_t**5*a**8*b**4*c**8*d**4 + 2673*_t**5*a**7*b**5*c**
9*d**3 - 2430*_t**5*a**6*b**6*c**10*d**2 + 1215*_t**5*a**5*b**7*c**11*d - 2
43*_t**5*a**4*b**8*c**12 + 9*_t**2*a**9*d**9 - 18*_t**2*a**8*b*c*d**8 + 9*_
t**2*a**7*b**2*c**2*d**7 + 9*_t**2*a**2*b**7*c**7*d**2 - 18*_t**2*a*b**8*c*
*8*d + 9*_t**2*b**9*c**9)/(a**4*b**3*d**7 + b**7*c**4*d**3)))) - 1/(a*c*x)
```

<u>Giac</u> [A] time = 1.15925, size = 412, normalized size = 1.38

$$\frac{b^{2}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^{2}bc-a^{3}d\right)}-\frac{d^{2}\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{3}-ac^{2}d\right)}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^{2}bc-\sqrt{3}a^{3}d}-\frac{\left(-cd^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\right.}{3\left(-\right.}\right.}{\sqrt{3}bc^{3}-\sqrt{3}ac^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) + (-a*b^2)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c - s
qrt(3)*a^3*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(2/3)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(2/3)*log(x
^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/(a*c*x)
```

3.118 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=301

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}+ d^{5/$$

```
[Out] -1/(2*a*c*x^2) + (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(5/3)*(b*c - a*d)) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(
Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)) - (b^(5/3)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(5/3)*(b*c - a*d)) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*
c^(5/3)*(b*c - a*d)) + (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(5/3)*(b*c - a*d)) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2])/(6*c^(5/3)*(b*c - a*d))
```

Rubi [A] time = 0.254838, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 522, 200, 31, 634, 617, 204, 628}

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}+ d^{5/$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(2*a*c*x^2) + (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(5/3)*(b*c - a*d)) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(
Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)) - (b^(5/3)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(5/3)*(b*c - a*d)) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*
c^(5/3)*(b*c - a*d)) + (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(5/3)*(b*c - a*d)) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2])/(6*c^(5/3)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{2acx^2}+\frac{\int \frac{-2(bc+ad)-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{2ac}$$

$$= -\frac{1}{2acx^2}-\frac{b^2\int \frac{1}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^3}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{2acx^2}-\frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}(bc-ad)}-\frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}(bc-ad)}+\frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{5/3}(bc-ad)}+\frac{d^2\int \frac{2\sqrt[3]{c}-\sqrt[3]{\cdots}}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x\cdots}}{3c^{5/3}(bc-\cdots}$$

$$= -\frac{1}{2acx^2}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)}+\frac{b^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}(bc-ad)}-\frac{b^2\int\cdots}{\cdots}$$

$$= -\frac{1}{2acx^2}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}\cdots$$

$$= -\frac{1}{2acx^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\cdots}{3c^{5/3}\cdots}$$

Mathematica [A] time = 0.151707, size = 259, normalized size = 0.86

$$\frac{\frac{2b^{5/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{b^{5/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}-\frac{2\sqrt{3}b^{5/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{3b}{a}-\frac{2d^{5/3}x^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}}+\frac{d^{5/3}x^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}}\cdots}{6x^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (2*Sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(5/3) + (2*Sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/c^
```

```
(1/3))/Sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5/3)
 - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)
```

Maple [A] time = 0.008, size = 257, normalized size = 0.9

$$-\frac{d}{3c(ad-bc)}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{d}{6c(ad-bc)}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{d\sqrt{3}}{3c(ad-bc)}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/c*d/(a*d-b*c)/(1/d*c)^(2/3)*ln(x+(1/d*c)^(1/3))+1/6/c*d/(a*d-b*c)/(1/d
*c)^(2/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))-1/3/c*d/(a*d-b*c)/(1/d*c)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))-1/2/a/c/x^2+1/3/a*b/
(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/a*b/(a*d-b*c)/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/a*b/(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.55102, size = 711, normalized size = 2.36

$$-\frac{2\sqrt{3}bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}adx^2\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(-\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)
^(2/3) - sqrt(3)*b)/b) + 2*sqrt(3)*a*d*x^2*(-d^2/c^2)^(1/3)*arctan(1/3*(2*s
qrt(3)*c*x*(-d^2/c^2)^(2/3) - sqrt(3)*d)/d) - b*c*x^2*(b^2/a^2)^(1/3)*log(b
^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - a*d*x^2*(-d^2/c^2)^
(1/3)*log(d^2*x^2 + c*d*x*(-d^2/c^2)^(1/3) + c^2*(-d^2/c^2)^(2/3)) + 2*b*c*
x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*a*d*x^2*(-d^2/c^2)^(1/
3)*log(d*x - c*(-d^2/c^2)^(1/3)) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*d)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13891, size = 417, normalized size = 1.39

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}-\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x\cdots\right.}{3\left(\cdots\right.}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) - (-a*b^2)^(1/3)
*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c -
 sqrt(3)*a^3*d) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/
(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(1/3)*b*log(
```

```
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(1/3)
*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/2/(a*c*x^
2)
```

3.119 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

```
[Out] -1/(3*a*c*x^3) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^3])/(3*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Rubi [A] time = 0.0922362, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(3*a*c*x^3) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^3])/(3*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx,x,x^3\right)$$
$$= -\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}$$

Mathematica [A] time = 0.0396203, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^3\right)}{3a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(3*a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3])/(
3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Maple [A] time = 0.01, size = 87, normalized size = 1.

$$\frac{d^2\ln\left(dx^3+c\right)}{3\,c^2\,(ad-bc)}-\frac{b^2\ln\left(bx^3+a\right)}{3\,a^2\,(ad-bc)}-\frac{1}{3\,acx^3}-\frac{\ln(x)\,d}{ac^2}-\frac{\ln(x)\,b}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*d^2/c^2/(a*d-b*c)*ln(d*x^3+c)-1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)-1/3/a/c
/x^3-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b
```

Maxima [A] time = 0.959482, size = 117, normalized size = 1.34

$$\frac{b^2\log\left(bx^3+a\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^3+c\right)}{3\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^3\right)}{3\,a^2c^2}-\frac{1}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 -
a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)
```

Fricas [A] time = 11.2672, size = 200, normalized size = 2.3

$$\frac{b^2c^2x^3\log\left(bx^3+a\right)-a^2d^2x^3\log\left(dx^3+c\right)-3\left(b^2c^2-a^2d^2\right)x^3\log\left(x\right)-abc^2+a^2cd}{3\left(a^2bc^3-a^3c^2d\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2 -
 a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.120 $\int \frac{1}{x^5(a+bx^3)(c+dx^3)} \, dx$

Optimal. Leaf size=318

$$\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2\ldots}\right.}{6c^{7/3}(bc-ad)}$$

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(a^2*c^2*x) - (b^(7/3)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*(b*c - a*d)) + (d^(7/3)*ArcTan
[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d))
- (b^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*(b*c - a*d)) + (d^(7/3)*Log
[c^(1/3) + d^(1/3)*x])/(3*c^(7/3)*(b*c - a*d)) + (b^(7/3)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*(b*c - a*d)) - (d^(7/3)*Log[c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(7/3)*(b*c - a*d))
```

Rubi [A] time = 0.379855, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2\ldots}\right.}{6c^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(a^2*c^2*x) - (b^(7/3)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*(b*c - a*d)) + (d^(7/3)*ArcTan
[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d))
- (b^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*(b*c - a*d)) + (d^(7/3)*Log
[c^(1/3) + d^(1/3)*x])/(3*c^(7/3)*(b*c - a*d)) + (b^(7/3)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*(b*c - a*d)) - (d^(7/3)*Log[c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(7/3)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{4acx^4}+\frac{\int \frac{-4(bc+ad)-4bdx^3}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{4ac}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{x\left(-4\left(b^2c^2+abcd+a^2d^2\right)-4bd(bc+ad)x^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \left(-\frac{4b^3c^2x}{(bc-ad)\left(a+bx^3\right)}-\frac{4a^2d^3x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{x}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{x}{c+dx^3}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{8/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}(bc-ad)}+\frac{b^{8/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{7/3}(bc-ad)}+\frac{d^{8/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{7/3}(bc-ad)}- \cdots$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2 \cdots}}{6a^{7/3}(bc-ad)} \cdots$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b} \cdots\right.}{6a^{7/3}(bc-a \cdots} \cdots$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)}+\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)} \cdots$$

Mathematica [A] time = 0.182195, size = 282, normalized size = 0.89

$$\frac{-\frac{12b^2x^3}{a^2}+\frac{4b^{7/3}x^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}}-\frac{2b^{7/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{7/3}}+\frac{4\sqrt{3}b^{7/3}x^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}+\frac{3b}{a}+\frac{12d^2x^3}{c^2}-\frac{4d^{7/3}x^4\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{7/3}}+\frac{2d}{}}{12x^4(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*Sqrt[3]*b^(7/
3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (4*Sqrt[3]*d^
(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(7/3) + (4*b^(7/3)
*x^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) - (4*d^(7/3)*x^4*Log[c^(1/3) + d^(1/
3)*x])/c^(7/3) - (2*b^(7/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/a^(7/3) + (2*d^(7/3)*x^4*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
])/c^(7/3))/(12*(-(b*c) + a*d)*x^4)
```

Maple [A] time = 0.012, size = 291, normalized size = 0.9

$$-\frac{d^2}{3c^2(ad-bc)}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}+\frac{d^2}{6c^2(ad-bc)}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}+\frac{d^2\sqrt{3}}{3c^2(ad-bc)}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*d^2/c^2/(a*d-b*c)/(1/d*c)^(1/3)*ln(x+(1/d*c)^(1/3))+1/6*d^2/c^2/(a*d-b
*c)/(1/d*c)^(1/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))+1/3*d^2/c^2/(a*d-b*
c)*3^(1/2)/(1/d*c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))+1/3*b^2/
a^2/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*b^2/a^2/(a*d-b*c)/(a/b)^(1/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*b^2/a^2/(a*d-b*c)*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/4/a/c/x^4+1/a/c^2/x*d+1/a^2/c
/x*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.76087, size = 697, normalized size = 2.19

$$4\sqrt{3}b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b^2*c^2*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) -
1/3*sqrt(3)) - 4*sqrt(3)*a^2*d^2*x^4*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/
c)^(1/3) + 1/3*sqrt(3)) + 2*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(
2/3) + a*(b/a)^(1/3)) + 2*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x^2 - c*x*(-d/c)^(
2/3) - c*(-d/c)^(1/3)) - 4*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3))
 - 4*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)) - 3*a*b*c^2 + 3*a^2
*c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12395, size = 443, normalized size = 1.39

$$-\frac{b^3\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}-\frac{\left(-ab^2\right)^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d}+\frac{\left(-cd^2\right)^{\frac{2}{3}}d\arctan\left(\frac{\sqrt{3}}{-}\right.}{\sqrt{3}bc^4-\sqrt{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) - (-a*b^2)^(2/3
)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*c
- sqrt(3)*a^4*d) + (-c*d^2)^(2/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))
/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(2/3)*b*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2)^(2/3
)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/4*(4*b*c
*x^3 + 4*a*d*x^3 - a*c)/(a^2*c^2*x^4)
```

3.121 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=321

$$-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\right.}{6c^{8/3}(bc-ad)}$$

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(2*a^2*c^2*x^2) - (b^(8/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Ar
cTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*
d)) + (b^(8/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*(b*c - a*d)) - (d^(8/3)
*Log[c^(1/3) + d^(1/3)*x])/(3*c^(8/3)*(b*c - a*d)) - (b^(8/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Log[c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(8/3)*(b*c - a*d))
```

Rubi [A] time = 0.455683, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\right.}{6c^{8/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```


```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(2*a^2*c^2*x^2) - (b^(8/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Ar
cTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*
d)) + (b^(8/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*(b*c - a*d)) - (d^(8/3)
*Log[c^(1/3) + d^(1/3)*x])/(3*c^(8/3)*(b*c - a*d)) - (b^(8/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Log[c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(8/3)*(b*c - a*d))
```


Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^3}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{5ac}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\int \frac{-10\left(b^2c^2+abcd+a^2d^2\right)-10bd(bc+ad)x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{10a^2c^2}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{1}{c+dx^3}\,dx}{c^2(bc-ad)}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}(bc-ad)}+\frac{b^3\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{8/3}(bc-ad)}-\frac{d^3\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{8/3}(bc-ad)}-\frac{d^3\int \cdots}{\cdots}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}-\frac{b^{8/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\cdots}{6a^{8/3}(bc-ad)}\cdots\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots\right.}{6a^{8/3}(bc-a\cdots}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}+\frac{d^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\cdots}{3a^{8/3}(bc-ad)}\cdots
\end{aligned}
$$

Mathematica [A] time = 0.179111, size = 282, normalized size = 0.88

$$\frac{-\frac{15b^2x^3}{a^2}-\frac{10b^{8/3}x^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}+\frac{5b^{8/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}}+\frac{10\sqrt{3}b^{8/3}x^5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}+\frac{6b}{a}+\frac{15d^2x^3}{c^2}+\frac{10d^{8/3}x^5\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{8/3}}-}{30x^5(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*Sqrt[3]*b^(8
/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) - (10*Sqrt[3]*
d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (10*b^(8
/3)*x^5*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3) + d
^(1/3)*x])/c^(8/3) + (5*b^(8/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(8/3) - (5*d^(8/3)*x^5*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/c^(8/3))/(30*(-(b*c) + a*d)*x^5)
```

Maple [A] time = 0.011, size = 293, normalized size = 0.9

$$\frac{d^2}{3c^2(ad-bc)}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{d^2}{6c^2(ad-bc)}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{d^2\sqrt{3}}{3c^2(ad-bc)}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3/c^2*d^2/(a*d-b*c)/(1/d*c)^(2/3)*ln(x+(1/d*c)^(1/3))-1/6/c^2*d^2/(a*d-b*
c)/(1/d*c)^(2/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))+1/3/c^2*d^2/(a*d-b*c
)/(1/d*c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))-1/3/a^2*b
^2/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/a^2*b^2/(a*d-b*c)/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/a^2*b^2/(a*d-b*c)/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/5/a/c/x^5+1/2/a/c^2/x^2*d+1/2/
a^2/c/x^2*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21296, size = 811, normalized size = 2.53

$$-\frac{10\sqrt{3}b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+10\sqrt{3}a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-5b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-
b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 10*sqrt(3)*a^2*d^2*x^5*(d^2/c^2)^(1/3)*arc
tan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - 5*b^2*c^2*x^5*(-b^
2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) -
 5*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d
^2/c^2)^(2/3)) + 10*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/
3)) + 10*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*a*b*c
^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16629, size = 454, normalized size = 1.41

$$-\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}d^2\arctan\left(\right.}{\sqrt{3}bc^4-\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) + (-a*b^2)^(1/3
)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*
c - sqrt(3)*a^4*d) - (-c*d^2)^(1/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(1/3)*b
^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2
)^(1/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/
10*(5*b*c*x^3 + 5*a*d*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

3.122 $\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=119

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(3*a^3*(b*c - a*d)) + (d^3*Log[
c + d*x^3])/(3*c^3*(b*c - a*d))
```

Rubi [A] time = 0.129099, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(3*a^3*(b*c - a*d)) + (d^3*Log[
c + d*x^3])/(3*c^3*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{acx^3} + \frac{-bc-ad}{a^2c^2x^2} + \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^4}{a^3(-bc+ad)(a+bx)} + \frac{}{c^3(bc-}\right.\right.$$
$$= -\frac{1}{6acx^6} + \frac{bc+ad}{3a^2c^2x^3} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\log(x)}{a^3c^3} - \frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)} + \frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}$$

Mathematica [A] time = 0.0540109, size = 119, normalized size = 1.

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3} + \frac{b^3\log\left(a+bx^3\right)}{3a^3(ad-bc)} + \frac{ad+bc}{3a^2c^2x^3} + \frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)} - \frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^3])/(3*a^3*(-(b*c) + a*d)) + (d^3*L
og[c + d*x^3])/(3*c^3*(b*c - a*d))
```

Maple [A] time = 0.013, size = 124, normalized size = 1.

$$-\frac{d^3\ln\left(dx^3+c\right)}{3\,c^3\,(ad-bc)} + \frac{b^3\ln\left(bx^3+a\right)}{3\,a^3\,(ad-bc)} - \frac{1}{6\,acx^6} + \frac{d}{3\,ac^2x^3} + \frac{b}{3\,a^2cx^3} + \frac{\ln(x)\,d^2}{ac^3} + \frac{\ln(x)\,bd}{a^2c^2} + \frac{\ln(x)\,b^2}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*d^3/c^3/(a*d-b*c)*ln(d*x^3+c)+1/3*b^3/a^3/(a*d-b*c)*ln(b*x^3+a)-1/6/a/
c/x^6+1/3/a/c^2/x^3*d+1/3/a^2/c/x^3*b+1/a/c^3*ln(x)*d^2+1/a^2/c^2*ln(x)*b*d
+1/a^3/c*ln(x)*b^2
```

Maxima [A] time = 0.948244, size = 158, normalized size = 1.33

$$-\frac{b^3\log\left(bx^3+a\right)}{3\left(a^3bc-a^4d\right)} + \frac{d^3\log\left(dx^3+c\right)}{3\left(bc^4-ac^3d\right)} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^3\right)}{3\,a^3c^3} + \frac{2\,(bc+ad)x^3-ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(b*x^3 + a)/(a^3*b*c - a^4*d) + 1/3*d^3*log(d*x^3 + c)/(b*c^4 -
 a*c^3*d) + 1/3*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^3)/(a^3*c^3) + 1/6*(2*(
b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^6)
```

Fricas [A] time = 35.0137, size = 255, normalized size = 2.14

$$-\frac{2\,b^3c^3x^6\log\left(bx^3+a\right)-2\,a^3d^3x^6\log\left(dx^3+c\right)-6\left(b^3c^3-a^3d^3\right)x^6\log\left(x\right)+a^2bc^3-a^3c^2d-2\left(ab^2c^3-a^3cd^2\right)x^3}{6\left(a^3bc^4-a^4c^3d\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3*x^6*log(b*x^3 + a) - 2*a^3*d^3*x^6*log(d*x^3 + c) - 6*(b^3*
c^3 - a^3*d^3)*x^6*log(x) + a^2*b*c^3 - a^3*c^2*d - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.123 $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=352

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}+\cdots$$

```
[Out] -1/(7*a*c*x^7) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(a^3*c^3*x) + (b^(10/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(10/3)*(b*c - a*d)) - (d^(10/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)
/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(10/3)*(b*c - a*d)) + (b^(10/3)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(10/3)*(b*c - a*d)) - (d^(10/3)*Log[c^(1/3) + d^(1/3)*x
])/(3*c^(10/3)*(b*c - a*d)) - (b^(10/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(10/3)*(b*c - a*d)) + (d^(10/3)*Log[c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2])/(6*c^(10/3)*(b*c - a*d))
```

Rubi [A] time = 0.500987, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(7*a*c*x^7) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(a^3*c^3*x) + (b^(10/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(10/3)*(b*c - a*d)) - (d^(10/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)
/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(10/3)*(b*c - a*d)) + (b^(10/3)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(10/3)*(b*c - a*d)) - (d^(10/3)*Log[c^(1/3) + d^(1/3)*x
])/(3*c^(10/3)*(b*c - a*d)) - (b^(10/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(10/3)*(b*c - a*d)) + (d^(10/3)*Log[c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2])/(6*c^(10/3)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{7acx^7}+\frac{\int \frac{-7(bc+ad)-7bdx^3}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{7ac}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{\int \frac{-28\left(b^2c^2+abcd+a^2d^2\right)-28bd(bc+ad)x^3}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{28a^2c^2}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \frac{x\left(-28(bc+ad)\left(b^2c^2+a^2d^2\right)-28bd\left(b^2c^2+abcd+a^2d^2\right)x^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,d}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \left(-\frac{28b^4c^3x}{(bc-ad)\left(a+bx^3\right)}-\frac{28a^3d^4x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^4\int \frac{x}{a+bx^3}\,dx}{a^3(bc-ad)}+\frac{d^4\int \frac{x}{c+dx^3}\,dx}{c^3(bc-ad)}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{11/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}(bc-ad)}-\frac{b^{11/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}}{3a^{10/3}(bc-ad)}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}-\frac{d^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}}{\sqrt{3}\sqrt[3]{c}}\right.}{\sqrt{3}c^{10/3}(bc-ad)}$$

Mathematica [A] time = 0.194873, size = 304, normalized size = 0.86

$$\frac{\frac{84b^3x^6}{a^3}-\frac{21b^2x^3}{a^2}-\frac{28b^{10/3}x^7\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{10/3}}+\frac{14b^{10/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{10/3}}-\frac{28\sqrt{3}b^{10/3}x^7\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{10/3}}+\frac{12b}{a}-\frac{84d^3x^6}{c^3}+\frac{21d^2x^3}{c^2}+-}{84x^7(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((12*b)/a - (12*d)/c - (21*b^2*x^3)/a^2 + (21*d^2*x^3)/c^2 + (84*b^3*x^6)/a
^3 - (84*d^3*x^6)/c^3 - (28*Sqrt[3]*b^(10/3)*x^7*ArcTan[(1 - (2*b^(1/3)*x)/
```

```
a^(1/3))/Sqrt[3]])/a^(10/3) + (28*Sqrt[3]*d^(10/3)*x^7*ArcTan[(1 - (2*d^(1/
3)*x)/c^(1/3))/Sqrt[3]])/c^(10/3) - (28*b^(10/3)*x^7*Log[a^(1/3) + b^(1/3)*
x])/a^(10/3) + (28*d^(10/3)*x^7*Log[c^(1/3) + d^(1/3)*x])/c^(10/3) + (14*b^
(10/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(10/3) - (14*d
^(10/3)*x^7*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(10/3))/(84*(
-(b*c) + a*d)*x^7)
```

Maple [A] time = 0.014, size = 334, normalized size = 1.

$$\frac{d^3}{3c^3(ad-bc)}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}-\frac{d^3}{6c^3(ad-bc)}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{\frac{c}{d}}}-\frac{d^3\sqrt{3}}{3c^3(ad-bc)}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*d^3/c^3/(a*d-b*c)/(1/d*c)^(1/3)*ln(x+(1/d*c)^(1/3))-1/6*d^3/c^3/(a*d-b*
c)/(1/d*c)^(1/3)*ln(x^2-(1/d*c)^(1/3)*x+(1/d*c)^(2/3))-1/3*d^3/c^3/(a*d-b*c
)*3^(1/2)/(1/d*c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/d*c)^(1/3)*x-1))-1/3*b^3/a
^3/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*b^3/a^3/(a*d-b*c)/(a/b)^(1/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b^3/a^3/(a*d-b*c)*3^(1/2)/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/7/a/c/x^7+1/4/a/c^2/x^4*d+1/4/
a^2/c/x^4*b-1/a/c^3/x*d^2-1/a^2/c^2/x*b*d-1/a^3/c/x*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63884, size = 759, normalized size = 2.16

$$-\frac{28\sqrt{3}b^3c^3x^7\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-28\sqrt{3}a^3d^3x^7\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-14\,b^3c^3x^7\left(-\frac{b}{a}\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^3*c^3*x^7*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3
) + 1/3*sqrt(3)) - 28*sqrt(3)*a^3*d^3*x^7*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*
(d/c)^(1/3) - 1/3*sqrt(3)) - 14*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x^2 - a*x*(-
b/a)^(2/3) - a*(-b/a)^(1/3)) - 14*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x^2 - c*x*(
d/c)^(2/3) + c*(d/c)^(1/3)) + 28*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 28*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x + c*(d/c)^(2/3)) + 84*(b^3*c^
3 - a^3*d^3)*x^6 + 12*a^2*b*c^3 - 12*a^3*c^2*d - 21*(a*b^2*c^3 - a^3*c*d^2)
*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13945, size = 509, normalized size = 1.45

$$\frac{b^4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^4bc-a^5d\right)}-\frac{d^4\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^5-ac^4d\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^4bc-\sqrt{3}a^5d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}d^2\arctan\left(\cdots\right)}{\sqrt{3}bc^5-\sqrt{3}a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^4*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*c - a^5*d) - 1/3*d^4
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^5 - a*c^4*d) + (-a*b^2)^(2/3)
*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^4*b*c
 - sqrt(3)*a^5*d) - (-c*d^2)^(2/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(sqrt(3)*b*c^5 - sqrt(3)*a*c^4*d) - 1/6*(-a*b^2)^(2/3)*b^
2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b*c - a^5*d) + 1/6*(-c*d^2)
^(2/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^5 - a*c^4*d) - 1/2
8*(28*b^2*c^2*x^6 + 28*a*b*c*d*x^6 + 28*a^2*d^2*x^6 - 7*a*b*c^2*x^3 - 7*a^2
*c*d*x^3 + 4*a^2*c^2)/(a^3*c^3*x^7)
```

3.124 $\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=148

$$\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{a^5Ax^{m+1}}{m+1}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^4x^{m+16}(5}{m+}$$

```
[Out] (a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3*
b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/(1
0 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B)*x
^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

Rubi [A] time = 0.0992086, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{a^5Ax^{m+1}}{m+1}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^4x^{m+16}(5}{m+}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3*
b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/(1
0 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B)*x
^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx = \int \left(a^5Ax^m+a^4(5Ab+aB)x^{3+m}+5a^3b(2Ab+aB)x^{6+m}+10a^2b^2(Ab+aB)x^{9+m}+5a\right.$$
$$=\frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+aB)x^{10+m}}{10+m}+\frac{5}{}$$

Mathematica [A] time = 0.172627, size = 137, normalized size = 0.93

$$x^{m+1}\left(\frac{10a^2b^2x^9(aB+Ab)}{m+10}+\frac{5a^3bx^6(aB+2Ab)}{m+7}+\frac{a^4x^3(aB+5Ab)}{m+4}+\frac{a^5A}{m+1}+\frac{b^4x^{15}(5aB+Ab)}{m+16}+\frac{5ab^3x^{12}(2aB+Ab)}{m+13}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^5*A)/(1 + m) + (a^4*(5*A*b + a*B)*x^3)/(4 + m) + (5*a^3*b*(2*
A*b + a*B)*x^6)/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^9)/(10 + m) + (5*a*b^3*
(A*b + 2*a*B)*x^12)/(13 + m) + (b^4*(A*b + 5*a*B)*x^15)/(16 + m) + (b^5*B*x
^18)/(19 + m))
```

Maple [B] time = 0.026, size = 1078, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] x^(1+m)*(B*b^5*m^6*x^18+51*B*b^5*m^5*x^18+1005*B*b^5*m^4*x^18+A*b^5*m^6*x^1
5+5*B*a*b^4*m^6*x^15+9605*B*b^5*m^3*x^18+54*A*b^5*m^5*x^15+270*B*a*b^4*m^5*
x^15+45474*B*b^5*m^2*x^18+1110*A*b^5*m^4*x^15+5550*B*a*b^4*m^4*x^15+95064*B
*b^5*m*x^18+5*A*a*b^4*m^6*x^12+10940*A*b^5*m^3*x^15+10*B*a^2*b^3*m^6*x^12+5
4700*B*a*b^4*m^3*x^15+58240*B*b^5*x^18+285*A*a*b^4*m^5*x^12+52929*A*b^5*m^2
*x^15+570*B*a^2*b^3*m^5*x^12+264645*B*a*b^4*m^2*x^15+6165*A*a*b^4*m^4*x^12+
112206*A*b^5*m*x^15+12330*B*a^2*b^3*m^4*x^12+561030*B*a*b^4*m*x^15+10*A*a^2
*b^3*m^6*x^9+63355*A*a*b^4*m^3*x^12+69160*A*b^5*x^15+10*B*a^3*b^2*m^6*x^9+1
26710*B*a^2*b^3*m^3*x^12+345800*B*a*b^4*x^15+600*A*a^2*b^3*m^5*x^9+316230*A
*a*b^4*m^2*x^12+600*B*a^3*b^2*m^5*x^9+632460*B*a^2*b^3*m^2*x^12+13740*A*a^2
*b^3*m^4*x^9+684360*A*a*b^4*m*x^12+13740*B*a^3*b^2*m^4*x^9+1368720*B*a^2*b^
3*m*x^12+10*A*a^3*b^2*m^6*x^6+149600*A*a^2*b^3*m^3*x^9+425600*A*a*b^4*x^12+
5*B*a^4*b*m^6*x^6+149600*B*a^3*b^2*m^3*x^9+851200*B*a^2*b^3*x^12+630*A*a^3*
b^2*m^5*x^6+783690*A*a^2*b^3*m^2*x^9+315*B*a^4*b*m^5*x^6+783690*B*a^3*b^2*m
^2*x^9+15330*A*a^3*b^2*m^4*x^6+1753800*A*a^2*b^3*m*x^9+7665*B*a^4*b*m^4*x^6
+1753800*B*a^3*b^2*m*x^9+5*A*a^4*b*m^6*x^3+179690*A*a^3*b^2*m^3*x^6+1106560
*A*a^2*b^3*x^9+B*a^5*m^6*x^3+89845*B*a^4*b*m^3*x^6+1106560*B*a^3*b^2*x^9+33
0*A*a^4*b*m^5*x^3+1021860*A*a^3*b^2*m^2*x^6+66*B*a^5*m^5*x^3+510930*B*a^4*b
*m^2*x^6+8550*A*a^4*b*m^4*x^3+2437680*A*a^3*b^2*m*x^6+1710*B*a^5*m^4*x^3+12
18840*B*a^4*b*m*x^6+A*a^5*m^6+109300*A*a^4*b*m^3*x^3+1580800*A*a^3*b^2*x^6+
```

```
21860*B*a^5*m^3*x^3+790400*B*a^4*b*x^6+69*A*a^5*m^5+702645*A*a^4*b*m^2*x^3+
140529*B*a^5*m^2*x^3+1905*A*a^5*m^4+1984770*A*a^4*b*m*x^3+396954*B*a^5*m*x^
3+26795*A*a^5*m^3+1383200*A*a^4*b*x^3+276640*B*a^5*x^3+201174*A*a^5*m^2+757
896*A*a^5*m+1106560*A*a^5)/(1+m)/(4+m)/(7+m)/(10+m)/(13+m)/(16+m)/(19+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.67311, size = 2048, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")

[Out] ((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5*
m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^19 + ((5*B*a*b^4 + A*b^5)*m^6 + 345800
*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 + A*b
^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2 + 1
12206*(5*B*a*b^4 + A*b^5)*m)*x^16 + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 170240
*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2*B*a^2
*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B*a^2*b^
3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^13 + 10*((B*a^3*b^2
+ A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3*b^2 + A*
a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^2 + A*a^2*
b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2 + A*a^2*b^
3)*m)*x^10 + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b + 316160*A*a^3
*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^3*b^2)*m^4 +
17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3*b^2)*m^2 + 24
3768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m^6 + 276640*B*a
^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B*a^5 + 5*A*a^4*b
)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*A*a^4*b)*m^2 + 39
```

```
6954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m^5 + 1905*A*a^5*m^
4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m + 1106560*A*a^5)*x)
*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 + 959070*m^2 + 18644
56*m + 1106560)
```

Sympy [A] time = 35.1889, size = 5418, normalized size = 36.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**12
) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a**
5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**2*b
**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*log(x), Eq(m, -19)), (-A*a**5/(
15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2*b**3
/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*log(x) - B*a**5/(12*x**12) - 5*B*a
**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) + 5*B*a*b
**4*log(x) + B*b**5*x**3/3, Eq(m, -16)), (-A*a**5/(12*x**12) - 5*A*a**4*b/(
9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a*b**4*log
(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*B*a**3*b**
2/(3*x**3) + 10*B*a**2*b**3*log(x) + 5*B*a*b**4*x**3/3 + B*b**5*x**6/6, Eq(
m, -13)), (-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b**2/(3*x**3)
 + 10*A*a**2*b**3*log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 - B*a**5/(6*x*
*6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*log(x) + 10*B*a**2*b**3*x**3/3 +
 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, Eq(m, -10)), (-A*a**5/(6*x**6) - 5*A*a*
*4*b/(3*x**3) + 10*A*a**3*b**2*log(x) + 10*A*a**2*b**3*x**3/3 + 5*A*a*b**4*
x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*log(x) + 10*B*a**3*b*
*2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*b**5*x**12/12, Eq(
m, -7)), (-A*a**5/(3*x**3) + 5*A*a**4*b*log(x) + 10*A*a**3*b**2*x**3/3 + 5*
A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**9/9 + A*b**5*x**12/12 + B*a**5*log(x) +
5*B*a**4*b*x**3/3 + 5*B*a**3*b**2*x**6/3 + 10*B*a**2*b**3*x**9/9 + 5*B*a*b*
*4*x**12/12 + B*b**5*x**15/15, Eq(m, -4)), (A*a**5*log(x) + 5*A*a**4*b*x**3
/3 + 5*A*a**3*b**2*x**6/3 + 10*A*a**2*b**3*x**9/9 + 5*A*a*b**4*x**12/12 + A
*b**5*x**15/15 + B*a**5*x**3/3 + 5*B*a**4*b*x**6/6 + 10*B*a**3*b**2*x**9/9
+ 5*B*a**2*b**3*x**12/6 + B*a*b**4*x**15/3 + B*b**5*x**18/18, Eq(m, -1)), (
A*a**5*m**6*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 69*A*a**5*m**5*x*x**m/(m**7 + 70*m**6
 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560
) + 1905*A*a**5*m**4*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
```

69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 26795*A*a**5*m**3*x*x**m/(m*
*7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456
*m + 1106560) + 201174*A*a**5*m**2*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 757896*A*a**5*
m*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 1106560*A*a**5*x*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*A*a*
*4*b*m**6*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3
+ 959070*m**2 + 1864456*m + 1106560) + 330*A*a**4*b*m**5*x**4*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 8550*A*a**4*b*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700
*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 109300*A*a**4*b*
m**3*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959
070*m**2 + 1864456*m + 1106560) + 702645*A*a**4*b*m**2*x**4*x**m/(m**7 + 70
*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 11
06560) + 1984770*A*a**4*b*m*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
4 + 227969*m3 + 959070*m**2 + 1864456*m + 1106560) + 1383200*A*a**4*b*x
4*xm/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*A*a**3*b**2*m**6*x**7*x**m/(m**7 + 70*m**6 +
1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
+ 630*A*a**3*b**2*m**5*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 15330*A*a**3*b**2*m**4*
x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m
2 + 1864456*m + 1106560) + 179690*A*a3*b**2*m**3*x**7*x**m/(m**7 + 70*m
6 + 1974*m5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106
560) + 1021860*A*a**3*b**2*m**2*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 2437680*A*a**3
*b**2*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 1580800*A*a**3*b**2*x**7*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 10*A*a**2*b**3*m**6*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 600*A*a**2*b**
3*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 13740*A*a**2*b**3*m**4*x**10*x**m/(m**
7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*
m + 1106560) + 149600*A*a**2*b**3*m**3*x**10*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 783690
*A*a**2*b**3*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*A*a**2*b**3*m*x**10
*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2
+ 1864456*m + 1106560) + 1106560*A*a**2*b**3*x**10*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
5*A*a*b**4*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 22796
9*m**3 + 959070*m**2 + 1864456*m + 1106560) + 285*A*a*b**4*m**5*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864

456*m + 1106560) + 6165*A*a*b**4*m**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**
5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 63355*A
*a*b**4*m**3*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
3 + 959070*m2 + 1864456*m + 1106560) + 316230*A*a*b**4*m**2*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864
456*m + 1106560) + 684360*A*a*b**4*m*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 425600*A
*a*b**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + A*b**5*m**6*x**16*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 54*A*b**5*m**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2
27969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1110*A*b**5*m**4*x**16*x*
*m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1
864456*m + 1106560) + 10940*A*b**5*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m
5 + 28700*m4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 52929
*A*b**5*m**2*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
3 + 959070*m2 + 1864456*m + 1106560) + 112206*A*b**5*m*x**16*x**m/(m**7
 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
 + 1106560) + 69160*A*b**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
4 + 227969*m3 + 959070*m**2 + 1864456*m + 1106560) + B*a**5*m**6*x**4*x
m/(m7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
1864456*m + 1106560) + 66*B*a**5*m**5*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1710*B*a
5*m4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 21860*B*a**5*m**3*x**4*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 140529*B*a**5*m**2*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 396954*B*a**5*m*x
4*xm/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 276640*B*a**5*x**4*x**m/(m**7 + 70*m**6 + 1974*
m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*B*
a**4*b*m**6*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**
3 + 959070*m**2 + 1864456*m + 1106560) + 315*B*a**4*b*m**5*x**7*x**m/(m**7
+ 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
+ 1106560) + 7665*B*a**4*b*m**4*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 89845*B*a**4*b
*m**3*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95
9070*m**2 + 1864456*m + 1106560) + 510930*B*a**4*b*m**2*x**7*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 1218840*B*a**4*b*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 790400*B*a**4*b*x
7*xm/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*B*a**3*b**2*m**6*x**10*x**m/(m**7 + 70*m**6
+ 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
 + 600*B*a**3*b**2*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4

```
 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 13740*B*a**3*b**2*m**
4*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95907
0*m**2 + 1864456*m + 1106560) + 149600*B*a**3*b**2*m**3*x**10*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 783690*B*a**3*b**2*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 +
 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*B*
a**3*b**2*m*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m*
*3 + 959070*m**2 + 1864456*m + 1106560) + 1106560*B*a**3*b**2*x**10*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 10*B*a**2*b**3*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 570*B*a*
*2*b**3*m**5*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 12330*B*a**2*b**3*m**4*x**13*x**
m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 18
64456*m + 1106560) + 126710*B*a**2*b**3*m**3*x**13*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
632460*B*a**2*b**3*m**2*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4
 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1368720*B*a**2*b**3*m
*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070
*m**2 + 1864456*m + 1106560) + 851200*B*a**2*b**3*x**13*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 5*B*a*b**4*m**6*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 270*B*a*b**4*m**5*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
 1864456*m + 1106560) + 5550*B*a*b**4*m**4*x**16*x**m/(m**7 + 70*m**6 + 197
4*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 54
700*B*a*b**4*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 264645*B*a*b**4*m**2*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
 1864456*m + 1106560) + 561030*B*a*b**4*m*x**16*x**m/(m**7 + 70*m**6 + 1974
*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 345
800*B*a*b**4*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + B*b**5*m**6*x**19*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 51*B*b**5*m**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**
4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1005*B*b**5*m**4*x**
19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**
2 + 1864456*m + 1106560) + 9605*B*b**5*m**3*x**19*x**m/(m**7 + 70*m**6 + 19
74*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 4
5474*B*b**5*m**2*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 95064*B*b**5*m*x**19*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 58240*B*b**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 2870
0*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560), True))
```

Giac [B] time = 1.22102, size = 1797, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^5*m^6*x^19*x^m + 51*B*b^5*m^5*x^19*x^m + 1005*B*b^5*m^4*x^19*x^m + 5*B
*a*b^4*m^6*x^16*x^m + A*b^5*m^6*x^16*x^m + 9605*B*b^5*m^3*x^19*x^m + 270*B*
a*b^4*m^5*x^16*x^m + 54*A*b^5*m^5*x^16*x^m + 45474*B*b^5*m^2*x^19*x^m + 555
0*B*a*b^4*m^4*x^16*x^m + 1110*A*b^5*m^4*x^16*x^m + 95064*B*b^5*m*x^19*x^m +
 10*B*a^2*b^3*m^6*x^13*x^m + 5*A*a*b^4*m^6*x^13*x^m + 54700*B*a*b^4*m^3*x^1
6*x^m + 10940*A*b^5*m^3*x^16*x^m + 58240*B*b^5*x^19*x^m + 570*B*a^2*b^3*m^5
*x^13*x^m + 285*A*a*b^4*m^5*x^13*x^m + 264645*B*a*b^4*m^2*x^16*x^m + 52929*
A*b^5*m^2*x^16*x^m + 12330*B*a^2*b^3*m^4*x^13*x^m + 6165*A*a*b^4*m^4*x^13*x
^m + 561030*B*a*b^4*m*x^16*x^m + 112206*A*b^5*m*x^16*x^m + 10*B*a^3*b^2*m^6
*x^10*x^m + 10*A*a^2*b^3*m^6*x^10*x^m + 126710*B*a^2*b^3*m^3*x^13*x^m + 633
55*A*a*b^4*m^3*x^13*x^m + 345800*B*a*b^4*x^16*x^m + 69160*A*b^5*x^16*x^m +
600*B*a^3*b^2*m^5*x^10*x^m + 600*A*a^2*b^3*m^5*x^10*x^m + 632460*B*a^2*b^3*
m^2*x^13*x^m + 316230*A*a*b^4*m^2*x^13*x^m + 13740*B*a^3*b^2*m^4*x^10*x^m +
 13740*A*a^2*b^3*m^4*x^10*x^m + 1368720*B*a^2*b^3*m*x^13*x^m + 684360*A*a*b
^4*m*x^13*x^m + 5*B*a^4*b*m^6*x^7*x^m + 10*A*a^3*b^2*m^6*x^7*x^m + 149600*B
*a^3*b^2*m^3*x^10*x^m + 149600*A*a^2*b^3*m^3*x^10*x^m + 851200*B*a^2*b^3*x^
13*x^m + 425600*A*a*b^4*x^13*x^m + 315*B*a^4*b*m^5*x^7*x^m + 630*A*a^3*b^2*
m^5*x^7*x^m + 783690*B*a^3*b^2*m^2*x^10*x^m + 783690*A*a^2*b^3*m^2*x^10*x^m
 + 7665*B*a^4*b*m^4*x^7*x^m + 15330*A*a^3*b^2*m^4*x^7*x^m + 1753800*B*a^3*b
^2*m*x^10*x^m + 1753800*A*a^2*b^3*m*x^10*x^m + B*a^5*m^6*x^4*x^m + 5*A*a^4*
b*m^6*x^4*x^m + 89845*B*a^4*b*m^3*x^7*x^m + 179690*A*a^3*b^2*m^3*x^7*x^m +
1106560*B*a^3*b^2*x^10*x^m + 1106560*A*a^2*b^3*x^10*x^m + 66*B*a^5*m^5*x^4*
x^m + 330*A*a^4*b*m^5*x^4*x^m + 510930*B*a^4*b*m^2*x^7*x^m + 1021860*A*a^3*
b^2*m^2*x^7*x^m + 1710*B*a^5*m^4*x^4*x^m + 8550*A*a^4*b*m^4*x^4*x^m + 12188
40*B*a^4*b*m*x^7*x^m + 2437680*A*a^3*b^2*m*x^7*x^m + A*a^5*m^6*x*x^m + 2186
0*B*a^5*m^3*x^4*x^m + 109300*A*a^4*b*m^3*x^4*x^m + 790400*B*a^4*b*x^7*x^m +
 1580800*A*a^3*b^2*x^7*x^m + 69*A*a^5*m^5*x*x^m + 140529*B*a^5*m^2*x^4*x^m
+ 702645*A*a^4*b*m^2*x^4*x^m + 1905*A*a^5*m^4*x*x^m + 396954*B*a^5*m*x^4*x^
m + 1984770*A*a^4*b*m*x^4*x^m + 26795*A*a^5*m^3*x*x^m + 276640*B*a^5*x^4*x^
m + 1383200*A*a^4*b*x^4*x^m + 201174*A*a^5*m^2*x*x^m + 757896*A*a^5*m*x*x^m
 + 1106560*A*a^5*x*x^m)/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 +
 959070*m^2 + 1864456*m + 1106560)
```

3.125 $\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

Optimal. Leaf size=71

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b +
 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

Rubi [A] time = 0.0400606, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b +
 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx &= \int \left(a^2Ax^m+a(2Ab+aB)x^{3+m}+b(Ab+2aB)x^{6+m}+b^2Bx^{9+m}\right) dx\\ &= \frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{4+m}}{4+m}+\frac{b(Ab+2aB)x^{7+m}}{7+m}+\frac{b^2Bx^{10+m}}{10+m}\end{aligned}$$

Mathematica [A] time = 0.0513311, size = 66, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2A}{m+1}+\frac{bx^6(2aB+Ab)}{m+7}+\frac{ax^3(aB+2Ab)}{m+4}+\frac{b^2Bx^9}{m+10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(4 + m) + (b*(A*b + 2*a*
B)*x^6)/(7 + m) + (b^2*B*x^9)/(10 + m))
```

Maple [B] time = 0.008, size = 262, normalized size = 3.7

$$\frac{x^{1+m}\left(Bb^2m^3x^9+12\,Bb^2m^2x^9+39\,Bb^2mx^9+Ab^2m^3x^6+2\,Babm^3x^6+28\,Bb^2x^9+15\,Ab^2m^2x^6+30\,Babm^2x^6+54\,Ab^2m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] x^(1+m)*(B*b^2*m^3*x^9+12*B*b^2*m^2*x^9+39*B*b^2*m*x^9+A*b^2*m^3*x^6+2*B*a*
b*m^3*x^6+28*B*b^2*x^9+15*A*b^2*m^2*x^6+30*B*a*b*m^2*x^6+54*A*b^2*m*x^6+108
*B*a*b*m*x^6+2*A*a*b*m^3*x^3+40*A*b^2*x^6+B*a^2*m^3*x^3+80*B*a*b*x^6+36*A*a
*b*m^2*x^3+18*B*a^2*m^2*x^3+174*A*a*b*m*x^3+87*B*a^2*m*x^3+A*a^2*m^3+140*A*
a*b*x^3+70*B*a^2*x^3+21*A*a^2*m^2+138*A*a^2*m+280*A*a^2)/(10+m)/(7+m)/(4+m)
/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.56837, size = 497, normalized size = 7.

$$\frac{\left(\left(Bb^2m^3+12\,Bb^2m^2+39\,Bb^2m+28\,Bb^2\right)x^{10}+\left(\left(2\,Bab+Ab^2\right)m^3+80\,Bab+40\,Ab^2+15\left(2\,Bab+Ab^2\right)m^2+54\left(2\,B\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 12*B*b^2*m^2 + 39*B*b^2*m + 28*B*b^2)*x^10 + ((2*B*a*b + A*b^
2)*m^3 + 80*B*a*b + 40*A*b^2 + 15*(2*B*a*b + A*b^2)*m^2 + 54*(2*B*a*b + A*b
^2)*m)*x^7 + ((B*a^2 + 2*A*a*b)*m^3 + 70*B*a^2 + 140*A*a*b + 18*(B*a^2 + 2*
A*a*b)*m^2 + 87*(B*a^2 + 2*A*a*b)*m)*x^4 + (A*a^2*m^3 + 21*A*a^2*m^2 + 138*
A*a^2*m + 280*A*a^2)*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Sympy [A] time = 3.78817, size = 1057, normalized size = 14.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6*
x**6) - 2*B*a*b/(3*x**3) + B*b**2*log(x), Eq(m, -10)), (-A*a**2/(6*x**6) -
2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*b**
2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/3 +
B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*log(x)
+ 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b**2*x*
*9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m +
280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 13
8*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*A*a**2*x*
x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**4*x**m/(m*
*4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m/(m**4 + 22
*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 + 22*m**3 + 1
59*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 28
0) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 5
4*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 40*A*b**2*
x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a**2*m**3*x**4*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m**2*x**4*x**m/(m**4
 + 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*x**m/(m**4 + 22*m**
```

```
3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**
2 + 418*m + 280) + 2*B*a*b*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*
m + 280) + 30*B*a*b*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280
) + 108*B*a*b*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 80*B*
a*b*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*b**2*m**3*x**10
*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 12*B*b**2*m**2*x**10*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 39*B*b**2*m*x**10*x**m/(m**4 +
 22*m**3 + 159*m**2 + 418*m + 280) + 28*B*b**2*x**10*x**m/(m**4 + 22*m**3 +
 159*m**2 + 418*m + 280), True))
```

Giac [B] time = 1.12885, size = 448, normalized size = 6.31

$$\frac{Bb^2m^3x^{10}x^m + 12\,Bb^2m^2x^{10}x^m + 39\,Bb^2mx^{10}x^m + 2\,Babm^3x^7x^m + Ab^2m^3x^7x^m + 28\,Bb^2x^{10}x^m + 30\,Babm^2x^7x^m + 15\,Ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^10*x^m + 12*B*b^2*m^2*x^10*x^m + 39*B*b^2*m*x^10*x^m + 2*B*a*b
*m^3*x^7*x^m + A*b^2*m^3*x^7*x^m + 28*B*b^2*x^10*x^m + 30*B*a*b*m^2*x^7*x^m
 + 15*A*b^2*m^2*x^7*x^m + 108*B*a*b*m*x^7*x^m + 54*A*b^2*m*x^7*x^m + B*a^2*
m^3*x^4*x^m + 2*A*a*b*m^3*x^4*x^m + 80*B*a*b*x^7*x^m + 40*A*b^2*x^7*x^m + 1
8*B*a^2*m^2*x^4*x^m + 36*A*a*b*m^2*x^4*x^m + 87*B*a^2*m*x^4*x^m + 174*A*a*b
*m*x^4*x^m + A*a^2*m^3*x*x^m + 70*B*a^2*x^4*x^m + 140*A*a*b*x^4*x^m + 21*A*
a^2*m^2*x*x^m + 138*A*a^2*m*x*x^m + 280*A*a^2*x*x^m)/(m^4 + 22*m^3 + 159*m^
2 + 418*m + 280)
```

3.126 $\int x^m \left(a+bx^3\right) \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=45

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m))
/(7 + m)
```

Rubi [A] time = 0.0206881, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m))
/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^3\right) \left(A+Bx^3\right) \, dx &= \int \left(aAx^m+(Ab+aB)x^{3+m}+bBx^{6+m}\right) \, dx\\ &= \frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.0293848, size = 42, normalized size = 0.93

$$x^{m+1}\left(\frac{x^3(aB+Ab)}{m+4}+\frac{aA}{m+1}+\frac{bBx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^3)/(4 + m) + (b*B*x^6)/(7 + m))
```

Maple [B] time = 0.004, size = 110, normalized size = 2.4

$$\frac{x^{1+m}\left(Bbm^2x^6+5\,Bbmx^6+4\,bBx^6+Abm^2x^3+Bam^2x^3+8\,Abmx^3+8\,Bamx^3+7\,Ax^3b+7\,Bx^3a+Aam^2+11\,Aam+\right.}{(7+m)\,(4+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] x^(1+m)*(B*b*m^2*x^6+5*B*b*m*x^6+4*B*b*x^6+A*b*m^2*x^3+B*a*m^2*x^3+8*A*b*m*
x^3+8*B*a*m*x^3+7*A*b*x^3+7*B*a*x^3+A*a*m^2+11*A*a*m+28*A*a)/(7+m)/(4+m)/(1
+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.52469, size = 213, normalized size = 4.73

$$\frac{\left(\left(Bbm^2+5\,Bbm+4\,Bb\right)x^7+\left((Ba+Ab)m^2+7\,Ba+7\,Ab+8\,(Ba+Ab)m\right)x^4+\left(Aam^2+11\,Aam+28\,Aa\right)x\right)x^m}{m^3+12\,m^2+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B*
a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39*m
 + 28)
```

Sympy [A] time = 1.46232, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{6x^6} - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} + Bb\log(x) \\ -\frac{Aa}{3x^3} + Ab\log(x) + Ba\log(x) + \frac{Bbx^3}{3} \\ Aa\log(x) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} \\ \frac{Aam^2xx^m}{m^3+12m^2+39m+28} + \frac{11Aamxx^m}{m^3+12m^2+39m+28} + \frac{28Aaxx^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^4x^m}{m^3+12m^2+39m+28} + \frac{7Abx^4x^m}{m^3+12m^2+39m+28} + \frac{Bam^2}{m^3+12m^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m,
-7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)), (A
*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 28*A*a*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m/(m**
3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) +
 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(m**3 +
12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*B
*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m**3 + 12*m
**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*B*b*x
**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

Giac [B] time = 1.12567, size = 193, normalized size = 4.29

$$\frac{Bbm^2x^7x^m + 5\,Bbmx^7x^m + 4\,Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8\,Bamx^4x^m + 8\,Abmx^4x^m + 7\,Bax^4x^m + 7\,Abx^4x^m +}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b*
m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x^4
```

```
*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39*m
+ 28)
```

3.127 $\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*b*(1 + m))
```

Rubi [A] time = 0.0350864, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {459, 364}

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*b*(1 + m))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m \left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^{1+m}}{b(1+m)} - \frac{(-Ab(1+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{b(1+m)}$$
$$= \frac{Bx^{1+m}}{b(1+m)} + \frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ab(1+m)}$$

Mathematica [A] time = 0.0587332, size = 55, normalized size = 0.83

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+aB\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -(
(b*x^3)/a)]))/(a*b*(1 + m))
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \frac{x^m \left(Bx^3+A\right)}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)x^m}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Sympy [C] time = 26.9433, size = 190, normalized size = 2.88

$$\frac{Amxx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Axx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Bmx^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] A*m*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3
)/(9*a*gamma(m/3 + 4/3)) + A*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m
/3 + 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3)) + B*m*x**4*x**m*lerchphi(
b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*gamma(m/3 + 7
/3)) + 4*B*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma
(m/3 + 4/3)/(9*a*gamma(m/3 + 7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

3.128 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+Ab(2-m))\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[Out] ((A*b - a*B)*x^(1 + m))/(3*a*b*(a + b*x^3)) + ((A*b*(2 - m) + a*B*(1 + m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a^2*
b*(1 + m))
```

Rubi [A] time = 0.0415492, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {457, 364}

$$\frac{x^{m+1}(aB(m+1)+Ab(2-m))\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(3*a*b*(a + b*x^3)) + ((A*b*(2 - m) + a*B*(1 + m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a^2*
b*(1 + m))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x^{1+m}}{3ab\left(a+bx^3\right)}+\frac{(-Ab(-2+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{3ab}\\ &= \frac{(Ab-aB)x^{1+m}}{3ab\left(a+bx^3\right)}+\frac{(Ab(2-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3a^2b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0575832, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)\right)}{a^2b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^2
*b*(1 + m))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{x^m\left(Bx^3+A\right)}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^m}{(bx^3 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)x^m}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^m}{(bx^3 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

3.129 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+Ab(5-m))\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{6a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

```
[Out] ((A*b - a*B)*x^(1 + m))/(6*a*b*(a + b*x^3)^2) + ((A*b*(5 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(6*a^
3*b*(1 + m))
```

Rubi [A] time = 0.0407562, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {457, 364}

$$\frac{x^{m+1}(aB(m+1)+Ab(5-m))\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{6a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(6*a*b*(a + b*x^3)^2) + ((A*b*(5 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(6*a^
3*b*(1 + m))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{1+m}}{6ab\left(a+bx^3\right)^2}+\frac{(-Ab(-5+m)+aB(1+m))\int \frac{x^m}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{1+m}}{6ab\left(a+bx^3\right)^2}+\frac{(Ab(5-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{6a^3b(1+m)}$$

Mathematica [A] time = 0.0559169, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)\right)}{a^3b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^3
*b*(1 + m))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x^m \left(Bx^3+A\right)}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^m}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)x^m}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)x^m}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

3.130 $\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=112

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/
(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3
, (4 + m)/3, -((d*x^3)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rubi [A] time = 0.0525424, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {482, 364}

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/
(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3
, (4 + m)/3, -((d*x^3)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\frac{b\int \frac{(ex)^m}{a+bx^3}\,dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{c+dx^3}\,dx}{bc-ad}$$
$$=\frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}$$

Mathematica [A] time = 0.0625725, size = 86, normalized size = 0.77

$$\frac{x(ex)^m\left(ad\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{dx^3}{c}\right)-bc\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])
 + a*d*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((d*x^3)/c)]))/(a*c*(-(b
*c) + a*d)*(1 + m))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^3+a\right)\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^3+a\right)\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{bdx^6 + (bc + ad)x^3 + ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b*d*x^6 + (b*c + a*d)*x^3 + a*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^3 + a)(dx^3 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

3.131 $\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21
```

Rubi [A] time = 0.0154295, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{7/2}+(Ab+aB)x^{13/2}+bBx^{19/2}\right)dx\\&=\frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.0150327, size = 33, normalized size = 0.85

$$\frac{2}{315}x^{9/2}\left(21x^3(aB+Ab)+35aA+15bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(35*a*A + 21*(A*b + a*B)*x^3 + 15*b*B*x^6))/315
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$\frac{30\,bB{x}^{6}+42\,A{x}^{3}b+42\,B{x}^{3}a+70\,Aa}{315}{x}^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/315*x^(9/2)*(15*B*b*x^6+21*A*b*x^3+21*B*a*x^3+35*A*a)
```

Maxima [A] time = 0.956209, size = 36, normalized size = 0.92

$$\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,(Ba+Ab)x^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b*x^(21/2) + 2/15*(B*a + A*b)*x^(15/2) + 2/9*A*a*x^(9/2)
```

Fricas [A] time = 1.50193, size = 85, normalized size = 2.18

$$\frac{2}{315}\left(15\,Bbx^{10}+21\,(Ba+Ab)x^{7}+35\,Aax^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b*x^10 + 21*(B*a + A*b)*x^7 + 35*A*a*x^4)*sqrt(x)
```

Sympy [A] time = 35.8397, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9} + \frac{2Abx^{\frac{15}{2}}}{15} + \frac{2Bax^{\frac{15}{2}}}{15} + \frac{2Bbx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(15/2)/15 + 2*B*a*x**(15/2)/15 + 2*B*b*x**(21/2
)/21
```

Giac [A] time = 1.11723, size = 39, normalized size = 1.

$$\frac{2}{21} Bbx^{\frac{21}{2}} + \frac{2}{15} Bax^{\frac{15}{2}} + \frac{2}{15} Abx^{\frac{15}{2}} + \frac{2}{9} Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b*x^(21/2) + 2/15*B*a*x^(15/2) + 2/15*A*b*x^(15/2) + 2/9*A*a*x^(9/2)
```

3.132 $\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

Rubi [A] time = 0.0158456, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int\left(aAx^{5/2}+(Ab+aB)x^{11/2}+bBx^{17/2}\right)dx\\ &= \frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.0138206, size = 33, normalized size = 0.85

$$\frac{2x^{7/2}\left(133x^3(aB+Ab)+247aA+91bBx^6\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(7/2)*(247*a*A + 133*(A*b + a*B)*x^3 + 91*b*B*x^6))/1729
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$\frac{182\,bBx^{6}+266\,Ax^{3}b+266\,Bx^{3}a+494\,Aa}{1729}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/1729*x^(7/2)*(91*B*b*x^6+133*A*b*x^3+133*B*a*x^3+247*A*a)
```

Maxima [A] time = 0.934875, size = 36, normalized size = 0.92

$$\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/19*B*b*x^(19/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/7*A*a*x^(7/2)
```

Fricas [A] time = 1.44092, size = 88, normalized size = 2.26

$$\frac{2}{1729}\left(91\,Bbx^{9}+133\,(Ba+Ab)x^{6}+247\,Aax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)
```

Sympy [A] time = 17.5613, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7} + \frac{2Abx^{\frac{13}{2}}}{13} + \frac{2Bax^{\frac{13}{2}}}{13} + \frac{2Bbx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(19/2
)/19
```

Giac [A] time = 1.10463, size = 39, normalized size = 1.

$$\frac{2}{19}\,Bbx^{\frac{19}{2}} + \frac{2}{13}\,Bax^{\frac{13}{2}} + \frac{2}{13}\,Abx^{\frac{13}{2}} + \frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2)
```

3.133 $\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

Rubi [A] time = 0.0153684, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aAx^{3/2}+(Ab+aB)x^{9/2}+bBx^{15/2}\right)\,dx\\ &= \frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0132942, size = 33, normalized size = 0.85

$$\frac{2}{935}x^{5/2}\left(85x^3(aB+Ab)+187aA+55bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(5/2)*(187*a*A + 85*(A*b + a*B)*x^3 + 55*b*B*x^6))/935
```

Maple [A] time = 0.006, size = 32, normalized size = 0.8

$$\frac{110\,bBx^{6}+170\,Ax^{3}b+170\,Bx^{3}a+374\,Aa}{935}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/935*x^(5/2)*(55*B*b*x^6+85*A*b*x^3+85*B*a*x^3+187*A*a)
```

Maxima [A] time = 0.934817, size = 36, normalized size = 0.92

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/5*A*a*x^(5/2)
```

Fricas [A] time = 1.64699, size = 85, normalized size = 2.18

$$\frac{2}{935}\left(55\,Bbx^{8}+85\,(Ba+Ab)x^{5}+187\,Aax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)
```

Sympy [A] time = 5.73434, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5} + \frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Bax^{\frac{11}{2}}}{11} + \frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(17/2
)/17
```

Giac [A] time = 1.11139, size = 39, normalized size = 1.

$$\frac{2}{17} Bbx^{\frac{17}{2}} + \frac{2}{11} Bax^{\frac{11}{2}} + \frac{2}{11} Abx^{\frac{11}{2}} + \frac{2}{5} Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2)
```

3.134 $\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

```
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

Rubi [A] time = 0.0160388, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aA\sqrt{x}+(Ab+aB)x^{7/2}+bBx^{13/2}\right)dx\\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0131893, size = 33, normalized size = 0.85

$$\frac{2}{45}x^{3/2}\left(5x^3(aB+Ab)+15aA+3bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(15*a*A + 5*(A*b + a*B)*x^3 + 3*b*B*x^6))/45
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$\frac{6\,bBx^{6}+10\,Ax^{3}b+10\,Bx^{3}a+30\,Aa}{45}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/45*x^(3/2)*(3*B*b*x^6+5*A*b*x^3+5*B*a*x^3+15*A*a)
```

Maxima [A] time = 0.960754, size = 36, normalized size = 0.92

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/3*A*a*x^(3/2)
```

Fricas [A] time = 1.69488, size = 77, normalized size = 1.97

$$\frac{2}{45}\left(3\,Bbx^{7}+5\,(Ba+Ab)x^{4}+15\,Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)
```

Sympy [A] time = 2.52409, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{3}{2}}}{3} + \frac{2Abx^{\frac{9}{2}}}{9} + \frac{2Bax^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a*x**(3/2)/3 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(15/2)/15
```

Giac [A] time = 1.09066, size = 39, normalized size = 1.

$$\frac{2}{15}\,Bbx^{\frac{15}{2}} + \frac{2}{9}\,Bax^{\frac{9}{2}} + \frac{2}{9}\,Abx^{\frac{9}{2}} + \frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)
```

3.135 $\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

Rubi [A] time = 0.0148383, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx &= \int \left(\frac{aA}{\sqrt{x}}+(Ab+aB)x^{5/2}+bBx^{11/2}\right)dx\\ &= 2aA\sqrt{x}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{13}bBx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0134174, size = 33, normalized size = 0.89

$$\frac{2}{91}\sqrt{x}\left(13x^3(aB+Ab)+91aA+7bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(91*a*A + 13*(A*b + a*B)*x^3 + 7*b*B*x^6))/91
```

Maple [A] time = 0.005, size = 32, normalized size = 0.9

$$\frac{14\,bBx^{6}+26\,Ax^{3}b+26\,Bx^{3}a+182\,Aa}{91}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/91*x^(1/2)*(7*B*b*x^6+13*A*b*x^3+13*B*a*x^3+91*A*a)
```

Maxima [A] time = 0.947247, size = 36, normalized size = 0.97

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)
```

Fricas [A] time = 1.70982, size = 76, normalized size = 2.05

$$\frac{2}{91}\left(7\,Bbx^{6}+13\,(Ba+Ab)x^{3}+91\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*sqrt(x)
```

Sympy [A] time = 2.07037, size = 44, normalized size = 1.19

$$2Aa\sqrt{x} + \frac{2Abx^{\frac{7}{2}}}{7} + \frac{2Bax^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13
```

Giac [A] time = 1.10385, size = 39, normalized size = 1.05

$$\frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,Bax^{\frac{7}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2*A*a*sqrt(x)
```

3.136 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

Rubi [A] time = 0.0148993, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{aA}{x^{3/2}}+(Ab+aB)x^{3/2}+bBx^{9/2}\right)dx\\ &= -\frac{2aA}{\sqrt{x}}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.0099154, size = 35, normalized size = 0.95

$$\frac{2\left(-55aA+11aBx^3+11Abx^3+5bBx^6\right)}{55\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (2*(-55*a*A + 11*A*b*x^3 + 11*a*B*x^3 + 5*b*B*x^6))/(55*Sqrt[x])
```

Maple [A] time = 0.004, size = 32, normalized size = 0.9

$$-\frac{-10\,bBx^{6}-22\,Ax^{3}b-22\,Bx^{3}a+110\,Aa}{55}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(3/2),x)
```

```
[Out] -2/55*(-5*B*b*x^6-11*A*b*x^3-11*B*a*x^3+55*A*a)/x^(1/2)
```

Maxima [A] time = 0.947561, size = 36, normalized size = 0.97

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)
```

Fricas [A] time = 1.70209, size = 76, normalized size = 2.05

$$\frac{2\left(5\,Bbx^{6}+11\,(Ba+Ab)x^{3}-55\,Aa\right)}{55\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/sqrt(x)
```

Sympy [A] time = 2.52094, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}}+\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Bax^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11
```

Giac [A] time = 1.09866, size = 39, normalized size = 1.05

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,Bax^{\frac{5}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) - 2*A*a/sqrt(x)
```

3.137 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

```
[Out] (-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

Rubi [A] time = 0.0150596, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{aA}{x^{5/2}}+(Ab+aB)\sqrt{x}+bBx^{7/2}\right)dx\\ &= -\frac{2aA}{3x^{3/2}}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{9}bBx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.0111206, size = 34, normalized size = 0.87

$$\frac{2\left(-3aA+3aBx^3+3Abx^3+bBx^6\right)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$-\frac{-2\,bBx^{6}-6\,Ax^{3}b-6\,Bx^{3}a+6\,Aa}{9}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/9*(-B*b*x^6-3*A*b*x^3-3*B*a*x^3+3*A*a)/x^(3/2)
```

Maxima [A] time = 0.936978, size = 36, normalized size = 0.92

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)
```

Fricas [A] time = 1.71192, size = 69, normalized size = 1.77

$$\frac{2\left(Bbx^{6}+3\,(Ba+Ab)x^{3}-3\,Aa\right)}{9\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)
```

Sympy [A] time = 3.10627, size = 46, normalized size = 1.18

$$-\frac{2Aa}{3x^{\frac{3}{2}}}+\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Bax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)/
9
```

Giac [A] time = 1.12935, size = 39, normalized size = 1.

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{3}\,Bax^{\frac{3}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)
```

3.138 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

```
[Out] (-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

Rubi [A] time = 0.0161729, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {448}

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{aA}{x^{7/2}}+\frac{Ab+aB}{\sqrt{x}}+bBx^{5/2}\right)dx\\ &= -\frac{2aA}{5x^{5/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{7}bBx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0110783, size = 36, normalized size = 0.97

$$\frac{2\left(5bx^3\left(7A+Bx^3\right)-7a\left(A-5Bx^3\right)\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (2*(-7*a*(A - 5*B*x^3) + 5*b*x^3*(7*A + B*x^3)))/(35*x^(5/2))
```

Maple [A] time = 0.004, size = 32, normalized size = 0.9

$$-\frac{-10\,bBx^6-70\,Ax^3b-70\,Bx^3a+14\,Aa}{35}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(7/2),x)
```

```
[Out] -2/35*(-5*B*b*x^6-35*A*b*x^3-35*B*a*x^3+7*A*a)/x^(5/2)
```

Maxima [A] time = 0.983294, size = 36, normalized size = 0.97

$$\frac{2}{7}\,Bbx^{\frac{7}{2}}+2\,(Ba+Ab)\sqrt{x}-\frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)
```

Fricas [A] time = 1.74647, size = 74, normalized size = 2.

$$\frac{2\left(5\,Bbx^6+35\,(Ba+Ab)x^3-7\,Aa\right)}{35\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)
```

Sympy [A] time = 4.23594, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}}+2Ab\sqrt{x}+2Ba\sqrt{x}+\frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7
```

Giac [A] time = 1.10656, size = 39, normalized size = 1.05

$$\frac{2}{7}\,Bbx^{\frac{7}{2}}+2\,Ba\sqrt{x}+2\,Ab\sqrt{x}-\frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)
```

3.139 $\int x^{7/2} \left(a + bx^3\right)^2 \left(A + Bx^3\right) dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2} + \frac{2}{21}bx^{21/2}(2aB + Ab) + \frac{2}{15}ax^{15/2}(aB + 2Ab) + \frac{2}{27}b^2Bx^{27/2}$$

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)*
x^(21/2))/21 + (2*b^2*B*x^(27/2))/27
```

Rubi [A] time = 0.0306103, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2Ax^{9/2} + \frac{2}{21}bx^{21/2}(2aB + Ab) + \frac{2}{15}ax^{15/2}(aB + 2Ab) + \frac{2}{27}b^2Bx^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)*
x^(21/2))/21 + (2*b^2*B*x^(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2} \left(a + bx^3\right)^2 \left(A + Bx^3\right) dx &= \int \left(a^2Ax^{7/2} + a(2Ab + aB)x^{13/2} + b(Ab + 2aB)x^{19/2} + b^2Bx^{25/2}\right) dx \\ &= \frac{2}{9}a^2Ax^{9/2} + \frac{2}{15}a(2Ab + aB)x^{15/2} + \frac{2}{21}b(Ab + 2aB)x^{21/2} + \frac{2}{27}b^2Bx^{27/2}\end{aligned}$$

Mathematica [A] time = 0.030353, size = 53, normalized size = 0.84

$$\frac{2}{945}x^{9/2}\left(105a^2A + 45bx^6(2aB + Ab) + 63ax^3(aB + 2Ab) + 35b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(105*a^2*A + 63*a*(2*A*b + a*B)*x^3 + 45*b*(A*b + 2*a*B)*x^6 + 3
5*b^2*B*x^9))/945
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$\frac{70\,B b^2 x^9 + 90\,A b^2 x^6 + 180\,B x^6 a b + 252\,a A b x^3 + 126\,B x^3 a^2 + 210\,a^2 A}{945} x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/945*x^(9/2)*(35*B*b^2*x^9+45*A*b^2*x^6+90*B*a*b*x^6+126*A*a*b*x^3+63*B*a^
2*x^3+105*A*a^2)
```

Maxima [A] time = 0.945678, size = 69, normalized size = 1.1

$$\frac{2}{27}\,B b^2 x^{\frac{27}{2}} + \frac{2}{21}\left(2\,B a b + A b^2\right)x^{\frac{21}{2}} + \frac{2}{15}\left(B a^2 + 2\,A a b\right)x^{\frac{15}{2}} + \frac{2}{9}\,A a^2 x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 2/21*(2*B*a*b + A*b^2)*x^(21/2) + 2/15*(B*a^2 + 2*A*a
*b)*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

Fricas [A] time = 1.67165, size = 138, normalized size = 2.19

$$\frac{2}{945}\left(35\,B b^2 x^{13} + 45\left(2\,B a b + A b^2\right)x^{10} + 63\left(B a^2 + 2\,A a b\right)x^7 + 105\,A a^2 x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^2*x^13 + 45*(2*B*a*b + A*b^2)*x^10 + 63*(B*a^2 + 2*A*a*b)*x^7
 + 105*A*a^2*x^4)*sqrt(x)
```

Sympy [A] time = 51.8472, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9} + \frac{4Aabx^{\frac{15}{2}}}{15} + \frac{2Ab^2x^{\frac{21}{2}}}{21} + \frac{2Ba^2x^{\frac{15}{2}}}{15} + \frac{4Babx^{\frac{21}{2}}}{21} + \frac{2Bb^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(15/2)/15 + 2*A*b**2*x**(21/2)/21 + 2*B*a*
*2*x**(15/2)/15 + 4*B*a*b*x**(21/2)/21 + 2*B*b**2*x**(27/2)/27
```

Giac [A] time = 1.07986, size = 72, normalized size = 1.14

$$\frac{2}{27}Bb^2x^{\frac{27}{2}} + \frac{4}{21}Babx^{\frac{21}{2}} + \frac{2}{21}Ab^2x^{\frac{21}{2}} + \frac{2}{15}Ba^2x^{\frac{15}{2}} + \frac{4}{15}Aabx^{\frac{15}{2}} + \frac{2}{9}Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 4/21*B*a*b*x^(21/2) + 2/21*A*b^2*x^(21/2) + 2/15*B*a^
2*x^(15/2) + 4/15*A*a*b*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

3.140 $\int x^{5/2} \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Rubi [A] time = 0.0309277, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int\left(a^2Ax^{5/2}+a(2Ab+aB)x^{11/2}+b(Ab+2aB)x^{17/2}+b^2Bx^{23/2}\right)dx\\ &= \frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{19}b(Ab+2aB)x^{19/2}+\frac{2}{25}b^2Bx^{25/2}\end{aligned}$$

Mathematica [A] time = 0.0280438, size = 63, normalized size = 1.

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Maple [A] time = 0.009, size = 56, normalized size = 0.9

$$\frac{3458\,Bb^2x^9 + 4550\,Ab^2x^6 + 9100\,Bx^6ab + 13300\,aAbx^3 + 6650\,Bx^3a^2 + 12350\,a^2A}{43225}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/43225*x^(7/2)*(1729*B*b^2*x^9+2275*A*b^2*x^6+4550*B*a*b*x^6+6650*A*a*b*x^
3+3325*B*a^2*x^3+6175*A*a^2)
```

Maxima [A] time = 0.959429, size = 69, normalized size = 1.1

$$\frac{2}{25}\,Bb^2x^{\frac{25}{2}} + \frac{2}{19}\left(2\,Bab + Ab^2\right)x^{\frac{19}{2}} + \frac{2}{13}\left(Ba^2 + 2\,Aab\right)x^{\frac{13}{2}} + \frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 2/19*(2*B*a*b + A*b^2)*x^(19/2) + 2/13*(B*a^2 + 2*A*a
*b)*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

Fricas [A] time = 1.6829, size = 149, normalized size = 2.37

$$\frac{2}{43225}\left(1729\,Bb^2x^{12} + 2275\left(2\,Bab + Ab^2\right)x^9 + 3325\left(Ba^2 + 2\,Aab\right)x^6 + 6175\,Aa^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^2*x^12 + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*a
*b)*x^6 + 6175*A*a^2*x^3)*sqrt(x)
```

Sympy [A] time = 46.9283, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{19}{2}}}{19}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{19}{2}}}{19}+\frac{2Bb^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25
```

Giac [A] time = 1.1303, size = 72, normalized size = 1.14

$$\frac{2}{25}Bb^2x^{\frac{25}{2}}+\frac{4}{19}Babx^{\frac{19}{2}}+\frac{2}{19}Ab^2x^{\frac{19}{2}}+\frac{2}{13}Ba^2x^{\frac{13}{2}}+\frac{4}{13}Aabx^{\frac{13}{2}}+\frac{2}{7}Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 4/19*B*a*b*x^(19/2) + 2/19*A*b^2*x^(19/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

3.141 $\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{23}b^2Bx^{23/2}$$

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

Rubi [A] time = 0.0299877, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{23}b^2Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2Ax^{3/2}+a(2Ab+aB)x^{9/2}+b(Ab+2aB)x^{15/2}+b^2Bx^{21/2}\right)\,dx\\ &= \frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{23}b^2Bx^{23/2}\end{aligned}$$

Mathematica [A] time = 0.027907, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(4301a^2A+1265bx^6(2aB+Ab)+1955ax^3(aB+2Ab)+935b^2Bx^9\right)}{21505}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(5/2)*(4301*a^2*A + 1955*a*(2*A*b + a*B)*x^3 + 1265*b*(A*b + 2*a*B)*x^
6 + 935*b^2*B*x^9))/21505
```

Maple [A] time = 0.009, size = 56, normalized size = 0.9

$$\frac{1870\,Bb^{2}x^{9}+2530\,Ab^{2}x^{6}+5060\,Bx^{6}ab+7820\,aAbx^{3}+3910\,Bx^{3}a^{2}+8602\,a^{2}A}{21505}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/21505*x^(5/2)*(935*B*b^2*x^9+1265*A*b^2*x^6+2530*B*a*b*x^6+3910*A*a*b*x^3
+1955*B*a^2*x^3+4301*A*a^2)
```

Maxima [A] time = 0.937444, size = 69, normalized size = 1.1

$$\frac{2}{23}\,Bb^{2}x^{\frac{23}{2}}+\frac{2}{17}\left(2\,Bab+Ab^{2}\right)x^{\frac{17}{2}}+\frac{2}{11}\left(Ba^{2}+2\,Aab\right)x^{\frac{11}{2}}+\frac{2}{5}\,Aa^{2}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/11*(B*a^2 + 2*A*a
*b)*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

Fricas [A] time = 1.63661, size = 147, normalized size = 2.33

$$\frac{2}{21505}\left(935\,Bb^{2}x^{11}+1265\left(2\,Bab+Ab^{2}\right)x^{8}+1955\left(Ba^{2}+2\,Aab\right)x^{5}+4301\,Aa^{2}x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^2*x^11 + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a*
b)*x^5 + 4301*A*a^2*x^2)*sqrt(x)
```

Sympy [A] time = 18.7176, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5} + \frac{4Aabx^{\frac{11}{2}}}{11} + \frac{2Ab^2x^{\frac{17}{2}}}{17} + \frac{2Ba^2x^{\frac{11}{2}}}{11} + \frac{4Babx^{\frac{17}{2}}}{17} + \frac{2Bb^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23
```

Giac [A] time = 1.12972, size = 72, normalized size = 1.14

$$\frac{2}{23}Bb^2x^{\frac{23}{2}} + \frac{4}{17}Babx^{\frac{17}{2}} + \frac{2}{17}Ab^2x^{\frac{17}{2}} + \frac{2}{11}Ba^2x^{\frac{11}{2}} + \frac{4}{11}Aabx^{\frac{11}{2}} + \frac{2}{5}Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

3.142 $\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

Rubi [A] time = 0.0290315, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx&=\int \left(a^2A\sqrt{x}+a(2Ab+aB)x^{7/2}+b(Ab+2aB)x^{13/2}+b^2Bx^{19/2}\right)\,dx\\&=\frac{2}{3}a^2Ax^{3/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{21}b^2Bx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.0273809, size = 53, normalized size = 0.84

$$\frac{2}{315}x^{3/2}\left(105a^2A+21bx^6(2aB+Ab)+35ax^3(aB+2Ab)+15b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(105*a^2*A + 35*a*(2*A*b + a*B)*x^3 + 21*b*(A*b + 2*a*B)*x^6 + 1
5*b^2*B*x^9))/315
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$\frac{30\,B b^2 x^9 + 42\,A b^2 x^6 + 84\,B x^6 a b + 140\,a A b x^3 + 70\,B x^3 a^2 + 210\,a^2 A}{315} x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/315*x^(3/2)*(15*B*b^2*x^9+21*A*b^2*x^6+42*B*a*b*x^6+70*A*a*b*x^3+35*B*a^2
*x^3+105*A*a^2)
```

Maxima [A] time = 0.935667, size = 69, normalized size = 1.1

$$\frac{2}{21}\,B b^2 x^{\frac{21}{2}} + \frac{2}{15}\left(2\,B a b + A b^2\right) x^{\frac{15}{2}} + \frac{2}{9}\left(B a^2 + 2\,A a b\right) x^{\frac{9}{2}} + \frac{2}{3}\,A a^2 x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/9*(B*a^2 + 2*A*a*
b)*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

Fricas [A] time = 1.70579, size = 134, normalized size = 2.13

$$\frac{2}{315}\left(15\,B b^2 x^{10} + 21\left(2\,B a b + A b^2\right) x^7 + 35\left(B a^2 + 2\,A a b\right) x^4 + 105\,A a^2 x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^2*x^10 + 21*(2*B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2*A*a*b)*x^4
+ 105*A*a^2*x)*sqrt(x)
```

Sympy [A] time = 4.46744, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21
```

Giac [A] time = 1.1311, size = 72, normalized size = 1.14

$$\frac{2}{21}Bb^2x^{\frac{21}{2}}+\frac{4}{15}Babx^{\frac{15}{2}}+\frac{2}{15}Ab^2x^{\frac{15}{2}}+\frac{2}{9}Ba^2x^{\frac{9}{2}}+\frac{4}{9}Aabx^{\frac{9}{2}}+\frac{2}{3}Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

3.143 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} \, dx$

Optimal. Leaf size=61

$$2a^2A\sqrt{x} + \frac{2}{13}bx^{13/2}(2aB + Ab) + \frac{2}{7}ax^{7/2}(aB + 2Ab) + \frac{2}{19}b^2Bx^{19/2}$$

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13/
2))/13 + (2*b^2*B*x^(19/2))/19
```

Rubi [A] time = 0.0291588, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2A\sqrt{x} + \frac{2}{13}bx^{13/2}(2aB + Ab) + \frac{2}{7}ax^{7/2}(aB + 2Ab) + \frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13/
2))/13 + (2*b^2*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a + bx^3)^2 (A + Bx^3)}{\sqrt{x}} \, dx &= \int \left(\frac{a^2A}{\sqrt{x}} + a(2Ab + aB)x^{5/2} + b(Ab + 2aB)x^{11/2} + b^2Bx^{17/2}\right) dx \\ &= 2a^2A\sqrt{x} + \frac{2}{7}a(2Ab + aB)x^{7/2} + \frac{2}{13}b(Ab + 2aB)x^{13/2} + \frac{2}{19}b^2Bx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.0292543, size = 53, normalized size = 0.87

$$\frac{2\sqrt{x}\left(1729a^2A+133bx^6(2aB+Ab)+247ax^3(aB+2Ab)+91b^2Bx^9\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(1729*a^2*A + 247*a*(2*A*b + a*B)*x^3 + 133*b*(A*b + 2*a*B)*x^6
+ 91*b^2*B*x^9))/1729
```

Maple [A] time = 0.007, size = 56, normalized size = 0.9

$$\frac{182\,Bb^2x^9+266\,Ab^2x^6+532\,Bx^6ab+988\,aAbx^3+494\,Bx^3a^2+3458\,a^2A}{1729}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/1729*x^(1/2)*(91*B*b^2*x^9+133*A*b^2*x^6+266*B*a*b*x^6+494*A*a*b*x^3+247*
B*a^2*x^3+1729*A*a^2)
```

Maxima [A] time = 0.948468, size = 69, normalized size = 1.13

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{2}{13}\left(2\,Bab+Ab^2\right)x^{\frac{13}{2}}+\frac{2}{7}\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}+2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/7*(B*a^2 + 2*A*a*
b)*x^(7/2) + 2*A*a^2*sqrt(x)
```

Fricas [A] time = 1.6437, size = 135, normalized size = 2.21

$$\frac{2}{1729}\left(91\,Bb^2x^9+133\left(2\,Bab+Ab^2\right)x^6+247\left(Ba^2+2\,Aab\right)x^3+1729\,Aa^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x^
3 + 1729*A*a^2)*sqrt(x)
```

Sympy [A] time = 7.22549, size = 78, normalized size = 1.28

$$2Aa^2\sqrt{x}+\frac{4Aabx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{2Ba^2x^{\frac{7}{2}}}{7}+\frac{4Babx^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x*
*(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19
```

Giac [A] time = 1.10846, size = 72, normalized size = 1.18

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{13}\,Babx^{\frac{13}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)
```

3.144 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}} \, dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(
11/2))/11 + (2*b^2*B*x^(17/2))/17
```

Rubi [A] time = 0.0303508, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(
11/2))/11 + (2*b^2*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}} \, dx &= \int \left(\frac{a^2A}{x^{3/2}}+a(2Ab+aB)x^{3/2}+b(Ab+2aB)x^{9/2}+b^2Bx^{15/2}\right) dx\\ &= -\frac{2a^2A}{\sqrt{x}}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{17}b^2Bx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.0189261, size = 60, normalized size = 0.98

$$\frac{-374a^2(5A-Bx^3)+68abx^3(11A+5Bx^3)+10b^2x^6(17A+11Bx^3)}{935\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-374*a^2*(5*A - B*x^3) + 68*a*b*x^3*(11*A + 5*B*x^3) + 10*b^2*x^6*(17*A +
11*B*x^3))/(935*Sqrt[x])
```

Maple [A] time = 0.006, size = 56, normalized size = 0.9

$$-\frac{-110\,Bb^2x^9-170\,Ab^2x^6-340\,Bx^6ab-748\,aAbx^3-374\,Bx^3a^2+1870\,a^2A}{935}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x)
```

```
[Out] -2/935*(-55*B*b^2*x^9-85*A*b^2*x^6-170*B*a*b*x^6-374*A*a*b*x^3-187*B*a^2*x^
3+935*A*a^2)/x^(1/2)
```

Maxima [A] time = 0.940555, size = 69, normalized size = 1.13

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{2}{11}\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{5}\left(Ba^2+2\,Aab\right)x^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/5*(B*a^2 + 2*A*a*
b)*x^(5/2) - 2*A*a^2/sqrt(x)
```

Fricas [A] time = 1.68896, size = 131, normalized size = 2.15

$$\frac{2\left(55\,Bb^2x^9+85\left(2\,Bab+Ab^2\right)x^6+187\left(Ba^2+2\,Aab\right)x^3-935\,Aa^2\right)}{935\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3
- 935*A*a^2)/sqrt(x)
```

Sympy [A] time = 7.95219, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Ba^2x^{\frac{5}{2}}}{5}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*x
**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17
```

Giac [A] time = 1.1013, size = 72, normalized size = 1.18

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{5}\,Ba^2x^{\frac{5}{2}}+\frac{4}{5}\,Aabx^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^2
*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)
```

3.145 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)
*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

Rubi [A] time = 0.0301588, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)
*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^{5/2}}\,dx &= \int \left(\frac{a^2A}{x^{5/2}}+a(2Ab+aB)\sqrt{x}+b(Ab+2aB)x^{7/2}+b^2Bx^{13/2}\right)dx\\ &= -\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{15}b^2Bx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.0156948, size = 57, normalized size = 0.9

$$\frac{-30a^2\left(A-Bx^3\right)+20abx^3\left(3A+Bx^3\right)+2b^2x^6\left(5A+3Bx^3\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-30*a^2*(A - B*x^3) + 20*a*b*x^3*(3*A + B*x^3) + 2*b^2*x^6*(5*A + 3*B*x^3)
)/(45*x^(3/2))
```

Maple [A] time = 0.008, size = 56, normalized size = 0.9

$$-\frac{-6\,Bb^2x^9-10\,Ab^2x^6-20\,Bx^6ab-60\,aAbx^3-30\,Bx^3a^2+30\,a^2A}{45}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/45*(-3*B*b^2*x^9-5*A*b^2*x^6-10*B*a*b*x^6-30*A*a*b*x^3-15*B*a^2*x^3+15*A
*a^2)/x^(3/2)
```

Maxima [A] time = 0.948136, size = 69, normalized size = 1.1

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{9}\left(2\,Bab+Ab^2\right)x^{\frac{9}{2}}+\frac{2}{3}\left(Ba^2+2\,Aab\right)x^{\frac{3}{2}}-\frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b)
*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

Fricas [A] time = 1.74359, size = 124, normalized size = 1.97

$$\frac{2\left(3\,Bb^2x^9+5\left(2\,Bab+Ab^2\right)x^6+15\left(Ba^2+2\,Aab\right)x^3-15\,Aa^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 15
*A*a^2)/x^(3/2)
```

Sympy [A] time = 9.44909, size = 80, normalized size = 1.27

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a**
2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15
```

Giac [A] time = 1.10721, size = 72, normalized size = 1.14

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{9}\,Babx^{\frac{9}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}+\frac{2}{3}\,Ba^2x^{\frac{3}{2}}+\frac{4}{3}\,Aabx^{\frac{3}{2}}-\frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

3.146 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} \, dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

```
[Out] (-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(13/2))/13
```

Rubi [A] time = 0.0310831, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} \, dx &= \int \left(\frac{a^2A}{x^{7/2}}+\frac{a(2Ab+aB)}{\sqrt{x}}+b(Ab+2aB)x^{5/2}+b^2Bx^{11/2}\right) dx\\ &= -\frac{2a^2A}{5x^{5/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{13}b^2Bx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.0161413, size = 57, normalized size = 0.93

$$\frac{2\left(-91a^2\left(A-5Bx^3\right)+130abx^3\left(7A+Bx^3\right)+5b^2x^6\left(13A+7Bx^3\right)\right)}{455x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (2*(-91*a^2*(A - 5*B*x^3) + 130*a*b*x^3*(7*A + B*x^3) + 5*b^2*x^6*(13*A + 7
*B*x^3)))/(455*x^(5/2))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.9

$$-\frac{-70\,Bb^2x^9-130\,Ab^2x^6-260\,Bx^6ab-1820\,aAbx^3-910\,Bx^3a^2+182\,a^2A}{455}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x)
```

```
[Out] -2/455*(-35*B*b^2*x^9-65*A*b^2*x^6-130*B*a*b*x^6-910*A*a*b*x^3-455*B*a^2*x^
3+91*A*a^2)/x^(5/2)
```

Maxima [A] time = 0.943316, size = 69, normalized size = 1.13

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{2}{7}\left(2\,Bab+Ab^2\right)x^{\frac{7}{2}}+2\left(Ba^2+2\,Aab\right)\sqrt{x}-\frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2*(B*a^2 + 2*A*a*b)*s
qrt(x) - 2/5*A*a^2/x^(5/2)
```

Fricas [A] time = 1.72558, size = 130, normalized size = 2.13

$$\frac{2\left(35\,Bb^{2}x^{9}+65\left(2\,Bab+Ab^{2}\right)x^{6}+455\left(Ba^{2}+2\,Aab\right)x^{3}-91\,Aa^{2}\right)}{455\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3
- 91*A*a^2)/x^(5/2)
```

Sympy [A] time = 12.0103, size = 76, normalized size = 1.25

$$-\frac{2Aa^{2}}{5x^{\frac{5}{2}}}+4Aab\sqrt{x}+\frac{2Ab^{2}x^{\frac{7}{2}}}{7}+2Ba^{2}\sqrt{x}+\frac{4Babx^{\frac{7}{2}}}{7}+\frac{2Bb^{2}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13
```

Giac [A] time = 1.16006, size = 72, normalized size = 1.18

$$\frac{2}{13}\,Bb^{2}x^{\frac{13}{2}}+\frac{4}{7}\,Babx^{\frac{7}{2}}+\frac{2}{7}\,Ab^{2}x^{\frac{7}{2}}+2\,Ba^{2}\sqrt{x}+4\,Aab\sqrt{x}-\frac{2\,Aa^{2}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt(
x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)
```

3.147 $\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*B
)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33
```

Rubi [A] time = 0.0436307, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*B
)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\int \left(a^3Ax^{7/2}+a^2(3Ab+aB)x^{13/2}+3ab(Ab+aB)x^{19/2}+b^2(Ab+3aB)x^{25/2}+b^3Bx^{31/2}\right.$$
$$=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2(3Ab+aB)x^{15/2}+\frac{2}{7}ab(Ab+aB)x^{21/2}+\frac{2}{27}b^2(Ab+3aB)x^{27/2}+\frac{2}{33}b$$

Mathematica [A] time = 0.0414161, size = 71, normalized size = 0.84

$$\frac{2x^{9/2}\left(693a^2x^3(aB+3Ab)+1155a^3A+385b^2x^9(3aB+Ab)+1485abx^6(aB+Ab)+315b^3Bx^{12}\right)}{10395}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(1155*a^3*A + 693*a^2*(3*A*b + a*B)*x^3 + 1485*a*b*(A*b + a*B)*x
^6 + 385*b^2*(A*b + 3*a*B)*x^9 + 315*b^3*B*x^12))/10395
```

Maple [A] time = 0.004, size = 80, normalized size = 0.9

$$\frac{630\,Bb^{3}x^{12}+770\,x^{9}Ab^{3}+2310\,x^{9}Bab^{2}+2970\,x^{6}Aab^{2}+2970\,x^{6}Ba^{2}b+4158\,x^{3}Aa^{2}b+1386\,x^{3}Ba^{3}+2310\,Aa^{3}}{10395}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/10395*x^(9/2)*(315*B*b^3*x^12+385*A*b^3*x^9+1155*B*a*b^2*x^9+1485*A*a*b^2
*x^6+1485*B*a^2*b*x^6+2079*A*a^2*b*x^3+693*B*a^3*x^3+1155*A*a^3)
```

Maxima [A] time = 0.945578, size = 99, normalized size = 1.16

$$\frac{2}{33}\,Bb^{3}x^{\frac{33}{2}}+\frac{2}{27}\left(3\,Bab^{2}+Ab^{3}\right)x^{\frac{27}{2}}+\frac{2}{7}\left(Ba^{2}b+Aab^{2}\right)x^{\frac{21}{2}}+\frac{2}{9}\,Aa^{3}x^{\frac{9}{2}}+\frac{2}{15}\left(Ba^{3}+3\,Aa^{2}b\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/27*(3*B*a*b^2 + A*b^3)*x^(27/2) + 2/7*(B*a^2*b + A*
a*b^2)*x^(21/2) + 2/9*A*a^3*x^(9/2) + 2/15*(B*a^3 + 3*A*a^2*b)*x^(15/2)
```

Fricas [A] time = 1.62614, size = 194, normalized size = 2.28

$$\frac{2}{10395}\left(315\,Bb^{3}x^{16}+385\left(3\,Bab^{2}+Ab^{3}\right)x^{13}+1485\left(Ba^{2}b+Aab^{2}\right)x^{10}+1155\,Aa^{3}x^{4}+693\left(Ba^{3}+3\,Aa^{2}b\right)x^{7}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/10395*(315*B*b^3*x^16 + 385*(3*B*a*b^2 + A*b^3)*x^13 + 1485*(B*a^2*b + A*
a*b^2)*x^10 + 1155*A*a^3*x^4 + 693*(B*a^3 + 3*A*a^2*b)*x^7)*sqrt(x)
```

Sympy [A] time = 114.892, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9} + \frac{2Aa^2bx^{\frac{15}{2}}}{5} + \frac{2Aab^2x^{\frac{21}{2}}}{7} + \frac{2Ab^3x^{\frac{27}{2}}}{27} + \frac{2Ba^3x^{\frac{15}{2}}}{15} + \frac{2Ba^2bx^{\frac{21}{2}}}{7} + \frac{2Bab^2x^{\frac{27}{2}}}{9} + \frac{2Bb^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 2*A*a**2*b*x**(15/2)/5 + 2*A*a*b**2*x**(21/2)/7 + 2*A
*b**3*x**(27/2)/27 + 2*B*a**3*x**(15/2)/15 + 2*B*a**2*b*x**(21/2)/7 + 2*B*a
*b**2*x**(27/2)/9 + 2*B*b**3*x**(33/2)/33
```

Giac [A] time = 1.13504, size = 104, normalized size = 1.22

$$\frac{2}{33}Bb^3x^{\frac{33}{2}} + \frac{2}{9}Bab^2x^{\frac{27}{2}} + \frac{2}{27}Ab^3x^{\frac{27}{2}} + \frac{2}{7}Ba^2bx^{\frac{21}{2}} + \frac{2}{7}Aab^2x^{\frac{21}{2}} + \frac{2}{15}Ba^3x^{\frac{15}{2}} + \frac{2}{5}Aa^2bx^{\frac{15}{2}} + \frac{2}{9}Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/9*B*a*b^2*x^(27/2) + 2/27*A*b^3*x^(27/2) + 2/7*B*a^
2*b*x^(21/2) + 2/7*A*a*b^2*x^(21/2) + 2/15*B*a^3*x^(15/2) + 2/5*A*a^2*b*x^(
15/2) + 2/9*A*a^3*x^(9/2)
```

3.148 $\int x^{5/2} \left(a+bx^3\right)^3 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

Rubi [A] time = 0.0412137, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx &= \int \left(a^3Ax^{5/2}+a^2(3Ab+aB)x^{11/2}+3ab(Ab+aB)x^{17/2}+b^2(Ab+3aB)x^{23/2}+b^3Bx^{29/}\right.\\ &= \frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{19}ab(Ab+aB)x^{19/2}+\frac{2}{25}b^2(Ab+3aB)x^{25/2}+\frac{2}{3}\end{aligned}$$

Mathematica [A] time = 0.0373498, size = 85, normalized size = 1.

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

Maple [A] time = 0.006, size = 80, normalized size = 0.9

$$\frac{86450\,Bb^3x^{12}+107198\,x^9Ab^3+321594\,x^9Bab^2+423150\,x^6Aab^2+423150\,x^6Ba^2b+618450\,x^3Aa^2b+206150\,x^3Ba^3+}{1339975}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/1339975*x^(7/2)*(43225*B*b^3*x^12+53599*A*b^3*x^9+160797*B*a*b^2*x^9+2115
75*A*a*b^2*x^6+211575*B*a^2*b*x^6+309225*A*a^2*b*x^3+103075*B*a^3*x^3+19142
5*A*a^3)
```

Maxima [A] time = 0.949983, size = 99, normalized size = 1.16

$$\frac{2}{31}\,Bb^3x^{\frac{31}{2}}+\frac{2}{25}\left(3\,Bab^2+Ab^3\right)x^{\frac{25}{2}}+\frac{6}{19}\left(Ba^2b+Aab^2\right)x^{\frac{19}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}+\frac{2}{13}\left(Ba^3+3\,Aa^2b\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 2/25*(3*B*a*b^2 + A*b^3)*x^(25/2) + 6/19*(B*a^2*b + A
*a*b^2)*x^(19/2) + 2/7*A*a^3*x^(7/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

Fricas [A] time = 1.66221, size = 211, normalized size = 2.48

$$\frac{2}{1339975}\left(43225\,Bb^3x^{15}+53599\left(3\,Bab^2+Ab^3\right)x^{12}+211575\left(Ba^2b+Aab^2\right)x^9+191425\,Aa^3x^3+103075\left(Ba^3+3\,Aa^2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1339975*(43225*B*b^3*x^15 + 53599*(3*B*a*b^2 + A*b^3)*x^12 + 211575*(B*a^
2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*sqr
t(x)
```

Sympy [A] time = 69.1066, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{25}{2}}}{25}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{19}{2}}}{19}+\frac{6Bab^2x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 + 2
*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 + 6*
B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31
```

Giac [A] time = 1.11192, size = 104, normalized size = 1.22

$$\frac{2}{31}Bb^3x^{\frac{31}{2}}+\frac{6}{25}Bab^2x^{\frac{25}{2}}+\frac{2}{25}Ab^3x^{\frac{25}{2}}+\frac{6}{19}Ba^2bx^{\frac{19}{2}}+\frac{6}{19}Aab^2x^{\frac{19}{2}}+\frac{2}{13}Ba^3x^{\frac{13}{2}}+\frac{6}{13}Aa^2bx^{\frac{13}{2}}+\frac{2}{7}Aa^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 6/25*B*a*b^2*x^(25/2) + 2/25*A*b^3*x^(25/2) + 6/19*B*
a^2*b*x^(19/2) + 6/19*A*a*b^2*x^(19/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b
*x^(13/2) + 2/7*A*a^3*x^(7/2)
```

3.149 $\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

Rubi [A] time = 0.041957, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx &= \int \left(a^3Ax^{3/2}+a^2(3Ab+aB)x^{9/2}+3ab(Ab+aB)x^{15/2}+b^2(Ab+3aB)x^{21/2}+b^3Bx^{27/2}\right)\\ &= \frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{23}b^2(Ab+3aB)x^{23/2}+\frac{2}{29}\end{aligned}$$

Mathematica [A] time = 0.040925, size = 85, normalized size = 1.

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

Maple [A] time = 0.009, size = 80, normalized size = 0.9

$$\frac{43010\,Bb^3x^{12}+54230\,x^9Ab^3+162690\,x^9Bab^2+220110\,x^6Aab^2+220110\,x^6Ba^2b+340170\,x^3Aa^2b+113390\,x^3Ba^3+}{623645}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/623645*x^(5/2)*(21505*B*b^3*x^12+27115*A*b^3*x^9+81345*B*a*b^2*x^9+110055
*A*a*b^2*x^6+110055*B*a^2*b*x^6+170085*A*a^2*b*x^3+56695*B*a^3*x^3+124729*A
*a^3)
```

Maxima [A] time = 0.936533, size = 99, normalized size = 1.16

$$\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{2}{23}\left(3\,Bab^2+Ab^3\right)x^{\frac{23}{2}}+\frac{6}{17}\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2}{11}\left(Ba^3+3\,Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 2/23*(3*B*a*b^2 + A*b^3)*x^(23/2) + 6/17*(B*a^2*b + A
*a*b^2)*x^(17/2) + 2/5*A*a^3*x^(5/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

Fricas [A] time = 1.73349, size = 208, normalized size = 2.45

$$\frac{2}{623645}\left(21505\,Bb^3x^{14}+27115\left(3\,Bab^2+Ab^3\right)x^{11}+110055\left(Ba^2b+Aab^2\right)x^8+124729\,Aa^3x^2+56695\left(Ba^3+3\,Aa^2b\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/623645*(21505*B*b^3*x^14 + 27115*(3*B*a*b^2 + A*b^3)*x^11 + 110055*(B*a^2
*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(
x)
```

Sympy [A] time = 45.0458, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5} + \frac{6Aa^2bx^{\frac{11}{2}}}{11} + \frac{6Aab^2x^{\frac{17}{2}}}{17} + \frac{2Ab^3x^{\frac{23}{2}}}{23} + \frac{2Ba^3x^{\frac{11}{2}}}{11} + \frac{6Ba^2bx^{\frac{17}{2}}}{17} + \frac{6Bab^2x^{\frac{23}{2}}}{23} + \frac{2Bb^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 + 6*
B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29
```

Giac [A] time = 1.1227, size = 104, normalized size = 1.22

$$\frac{2}{29}Bb^3x^{\frac{29}{2}} + \frac{6}{23}Bab^2x^{\frac{23}{2}} + \frac{2}{23}Ab^3x^{\frac{23}{2}} + \frac{6}{17}Ba^2bx^{\frac{17}{2}} + \frac{6}{17}Aab^2x^{\frac{17}{2}} + \frac{2}{11}Ba^3x^{\frac{11}{2}} + \frac{6}{11}Aa^2bx^{\frac{11}{2}} + \frac{2}{5}Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 6/23*B*a*b^2*x^(23/2) + 2/23*A*b^3*x^(23/2) + 6/17*B*
a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b
*x^(11/2) + 2/5*A*a^3*x^(5/2)
```

3.150 $\int \sqrt{x} \left(a+b x^3\right)^3 \left(A+B x^3\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{9} a^2 x^{9/2} (a B+3 A b)+\frac{2}{3} a^3 A x^{3/2}+\frac{2}{21} b^2 x^{21/2} (3 a B+A b)+\frac{2}{5} a b x^{15/2} (a B+A b)+\frac{2}{27} b^3 B x^{27/2}$$

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)*
x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

Rubi [A] time = 0.0405446, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9} a^2 x^{9/2} (a B+3 A b)+\frac{2}{3} a^3 A x^{3/2}+\frac{2}{21} b^2 x^{21/2} (3 a B+A b)+\frac{2}{5} a b x^{15/2} (a B+A b)+\frac{2}{27} b^3 B x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)*
x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x} \left(a+b x^3\right)^3 \left(A+B x^3\right) \, dx &= \int \left(a^3 A \sqrt{x}+a^2 (3 A b+a B) x^{7/2}+3 a b (A b+a B) x^{13/2}+b^2 (A b+3 a B) x^{19/2}+b^3 B x^{25/2}\right) \\ &= \frac{2}{3} a^3 A x^{3/2}+\frac{2}{9} a^2 (3 A b+a B) x^{9/2}+\frac{2}{5} a b (A b+a B) x^{15/2}+\frac{2}{21} b^2 (A b+3 a B) x^{21/2}+\frac{2}{27} b^3 \end{aligned}$$

Mathematica [A] time = 0.0382633, size = 71, normalized size = 0.84

$$\frac{2}{945} x^{3/2} \left(105 a^2 x^3 (a B+3 A b)+315 a^3 A+45 b^2 x^9 (3 a B+A b)+189 a b x^6 (a B+A b)+35 b^3 B x^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(315*a^3*A + 105*a^2*(3*A*b + a*B)*x^3 + 189*a*b*(A*b + a*B)*x^6
 + 45*b^2*(A*b + 3*a*B)*x^9 + 35*b^3*B*x^12))/945
```

Maple [A] time = 0.007, size = 80, normalized size = 0.9

$$\frac{70\,b^{3}Bx^{12}+90\,x^{9}b^{3}A+270\,x^{9}ab^{2}B+378\,x^{6}ab^{2}A+378\,x^{6}a^{2}bB+630\,x^{3}Aa^{2}b+210\,x^{3}Ba^{3}+630\,a^{3}A}{945}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/945*x^(3/2)*(35*B*b^3*x^12+45*A*b^3*x^9+135*B*a*b^2*x^9+189*A*a*b^2*x^6+1
89*B*a^2*b*x^6+315*A*a^2*b*x^3+105*B*a^3*x^3+315*A*a^3)
```

Maxima [A] time = 0.991515, size = 99, normalized size = 1.16

$$\frac{2}{27}\,Bb^{3}x^{\frac{27}{2}}+\frac{2}{21}\left(3\,Bab^{2}+Ab^{3}\right)x^{\frac{21}{2}}+\frac{2}{5}\left(Ba^{2}b+Aab^{2}\right)x^{\frac{15}{2}}+\frac{2}{3}\,Aa^{3}x^{\frac{3}{2}}+\frac{2}{9}\left(Ba^{3}+3\,Aa^{2}b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/3*A*a^3*x^(3/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

Fricas [A] time = 1.72516, size = 182, normalized size = 2.14

$$\frac{2}{945}\left(35\,Bb^{3}x^{13}+45\left(3\,Bab^{2}+Ab^{3}\right)x^{10}+189\left(Ba^{2}b+Aab^{2}\right)x^{7}+315\,Aa^{3}x+105\left(Ba^{3}+3\,Aa^{2}b\right)x^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^3*x^13 + 45*(3*B*a*b^2 + A*b^3)*x^10 + 189*(B*a^2*b + A*a*b^2
)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A] time = 7.63712, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A*
b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*b*
*2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27
```

Giac [A] time = 1.12638, size = 104, normalized size = 1.22

$$\frac{2}{27}Bb^3x^{\frac{27}{2}}+\frac{2}{7}Bab^2x^{\frac{21}{2}}+\frac{2}{21}Ab^3x^{\frac{21}{2}}+\frac{2}{5}Ba^2bx^{\frac{15}{2}}+\frac{2}{5}Aab^2x^{\frac{15}{2}}+\frac{2}{9}Ba^3x^{\frac{9}{2}}+\frac{2}{3}Aa^2bx^{\frac{9}{2}}+\frac{2}{3}Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 2/5*B*a^
2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(9/
2) + 2/3*A*a^3*x^(3/2)
```

3.151 $\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal. Leaf size=83

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Rubi [A] time = 0.0427669, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx &= \int \left(\frac{a^3A}{\sqrt{x}}+a^2(3Ab+aB)x^{5/2}+3ab(Ab+aB)x^{11/2}+b^2(Ab+3aB)x^{17/2}+b^3Bx^{23/2}\right)dx\\ &= 2a^3A\sqrt{x}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}+\frac{2}{25}b^3Bx^{25/2}\end{aligned}$$

Mathematica [A] time = 0.038728, size = 83, normalized size = 1.

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Maple [A] time = 0.009, size = 80, normalized size = 1.

$$\frac{3458\,b^3Bx^{12}+4550\,x^9b^3A+13650\,x^9ab^2B+19950\,x^6ab^2A+19950\,x^6a^2bB+37050\,x^3Aa^2b+12350\,x^3Ba^3+86450\,a}{43225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/43225*x^(1/2)*(1729*B*b^3*x^12+2275*A*b^3*x^9+6825*B*a*b^2*x^9+9975*A*a*b
^2*x^6+9975*B*a^2*b*x^6+18525*A*a^2*b*x^3+6175*B*a^3*x^3+43225*A*a^3)
```

Maxima [A] time = 0.941228, size = 99, normalized size = 1.19

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{2}{19}\left(3\,Bab^2+Ab^3\right)x^{\frac{19}{2}}+\frac{6}{13}\left(Ba^2b+Aab^2\right)x^{\frac{13}{2}}+2\,Aa^3\sqrt{x}+\frac{2}{7}\left(Ba^3+3\,Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2*A*a^3*sqrt(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

Fricas [A] time = 1.67248, size = 192, normalized size = 2.31

$$\frac{2}{43225}\left(1729\,Bb^3x^{12}+2275\left(3\,Bab^2+Ab^3\right)x^9+9975\left(Ba^2b+Aab^2\right)x^6+43225\,Aa^3+6175\left(Ba^3+3\,Aa^2b\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^3*x^12 + 2275*(3*B*a*b^2 + A*b^3)*x^9 + 9975*(B*a^2*b + A
*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A] time = 18.321, size = 112, normalized size = 1.35

$$2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{7}{2}}}{7} + \frac{6Aab^2x^{\frac{13}{2}}}{13} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{7}{2}}}{7} + \frac{6Ba^2bx^{\frac{13}{2}}}{13} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b*
*3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b**
2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25
```

Giac [A] time = 1.13561, size = 104, normalized size = 1.25

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{6}{19}\,Bab^2x^{\frac{19}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{6}{13}\,Ba^2bx^{\frac{13}{2}} + \frac{6}{13}\,Aab^2x^{\frac{13}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + 2\,Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^
(7/2) + 2*A*a^3*sqrt(x)
```

3.152 $\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{2}{5}a^2x^{5/2}(aB+3Ab)-\frac{2a^3A}{\sqrt{x}}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x
^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

Rubi [A] time = 0.040886, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2x^{5/2}(aB+3Ab)-\frac{2a^3A}{\sqrt{x}}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x
^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{x^{3/2}}\,dx &= \int \left(\frac{a^3A}{x^{3/2}}+a^2(3Ab+aB)x^{3/2}+3ab(Ab+aB)x^{9/2}+b^2(Ab+3aB)x^{15/2}+b^3Bx^{21/2}\right)dx\\ &= -\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/2}+\frac{2}{23}b^3Bx^{23/2}\end{aligned}$$

Mathematica [A] time = 0.0239988, size = 81, normalized size = 0.98

$$\frac{2346a^2bx^3\left(11A+5Bx^3\right)-8602a^3\left(5A-Bx^3\right)+690ab^2x^6\left(17A+11Bx^3\right)+110b^3x^9\left(23A+17Bx^3\right)}{21505\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-8602*a^3*(5*A - B*x^3) + 2346*a^2*b*x^3*(11*A + 5*B*x^3) + 690*a*b^2*x^6*
(17*A + 11*B*x^3) + 110*b^3*x^9*(23*A + 17*B*x^3))/(21505*Sqrt[x])
```

Maple [A] time = 0.006, size = 80, normalized size = 1.

$$-\frac{-1870\,b^3Bx^{12}-2530\,x^9b^3A-7590\,x^9ab^2B-11730\,x^6ab^2A-11730\,x^6a^2bB-25806\,x^3Aa^2b-8602\,x^3Ba^3+43010\,a^3A}{21505}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x)
```

```
[Out] -2/21505*(-935*B*b^3*x^12-1265*A*b^3*x^9-3795*B*a*b^2*x^9-5865*A*a*b^2*x^6-
5865*B*a^2*b*x^6-12903*A*a^2*b*x^3-4301*B*a^3*x^3+21505*A*a^3)/x^(1/2)
```

Maxima [A] time = 0.942053, size = 99, normalized size = 1.19

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{2}{17}\left(3\,Bab^2+Ab^3\right)x^{\frac{17}{2}}+\frac{6}{11}\left(Ba^2b+Aab^2\right)x^{\frac{11}{2}}-\frac{2\,Aa^3}{\sqrt{x}}+\frac{2}{5}\left(Ba^3+3\,Aa^2b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/11*(B*a^2*b + A
*a*b^2)*x^(11/2) - 2*A*a^3/sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

Fricas [A] time = 1.72498, size = 190, normalized size = 2.29

$$\frac{2\left(935\,Bb^{3}x^{12}+1265\left(3\,Bab^{2}+Ab^{3}\right)x^{9}+5865\left(Ba^{2}b+Aab^{2}\right)x^{6}-21505\,Aa^{3}+4301\left(Ba^{3}+3\,Aa^{2}b\right)x^{3}\right)}{21505\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A*
a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)
```

Sympy [A] time = 26.7273, size = 112, normalized size = 1.35

$$-\frac{2Aa^{3}}{\sqrt{x}}+\frac{6Aa^{2}bx^{\frac{5}{2}}}{5}+\frac{6Aab^{2}x^{\frac{11}{2}}}{11}+\frac{2Ab^{3}x^{\frac{17}{2}}}{17}+\frac{2Ba^{3}x^{\frac{5}{2}}}{5}+\frac{6Ba^{2}bx^{\frac{11}{2}}}{11}+\frac{6Bab^{2}x^{\frac{17}{2}}}{17}+\frac{2Bb^{3}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*b
**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*b*
*2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23
```

Giac [A] time = 1.12058, size = 104, normalized size = 1.25

$$\frac{2}{23}\,Bb^{3}x^{\frac{23}{2}}+\frac{6}{17}\,Bab^{2}x^{\frac{17}{2}}+\frac{2}{17}\,Ab^{3}x^{\frac{17}{2}}+\frac{6}{11}\,Ba^{2}bx^{\frac{11}{2}}+\frac{6}{11}\,Aab^{2}x^{\frac{11}{2}}+\frac{2}{5}\,Ba^{3}x^{\frac{5}{2}}+\frac{6}{5}\,Aa^{2}bx^{\frac{5}{2}}-\frac{2\,Aa^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B*
a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^
(5/2) - 2*A*a^3/sqrt(x)
```

3.153 $\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} dx$

Optimal. Leaf size=85

$$\frac{2}{3}a^2x^{3/2}(aB+3Ab)-\frac{2a^3A}{3x^{3/2}}+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a*
B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

Rubi [A] time = 0.0448236, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2x^{3/2}(aB+3Ab)-\frac{2a^3A}{3x^{3/2}}+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a*
B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{a^3A}{x^{5/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{13/2}+b^3Bx^{19/2}\right)dx\\ &= -\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{21}b^3Bx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.0227491, size = 77, normalized size = 0.91

$$\frac{2\left(35a^2bx^3\left(3A+Bx^3\right)-35a^3\left(A-Bx^3\right)+7ab^2x^6\left(5A+3Bx^3\right)+b^3x^9\left(7A+5Bx^3\right)\right)}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A + 3
*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))
```

Maple [A] time = 0.008, size = 80, normalized size = 0.9

$$-\frac{-10\,b^3Bx^{12}-14\,x^9b^3A-42\,x^9ab^2B-70\,x^6ab^2A-70\,x^6a^2bB-210\,x^3Aa^2b-70\,x^3Ba^3+70\,a^3A}{105}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/105*(-5*B*b^3*x^12-7*A*b^3*x^9-21*B*a*b^2*x^9-35*A*a*b^2*x^6-35*B*a^2*b*
x^6-105*A*a^2*b*x^3-35*B*a^3*x^3+35*A*a^3)/x^(3/2)
```

Maxima [A] time = 0.966754, size = 99, normalized size = 1.16

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{15}\left(3\,Bab^2+Ab^3\right)x^{\frac{15}{2}}+\frac{2}{3}\left(Ba^2b+Aab^2\right)x^{\frac{9}{2}}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}+\frac{2}{3}\left(Ba^3+3\,Aa^2b\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) - 2/3*A*a^3/x^(3/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

Fricas [A] time = 1.84283, size = 171, normalized size = 2.01

$$\frac{2\left(5\,Bb^3x^{12}+7\left(3\,Bab^2+Ab^3\right)x^9+35\left(Ba^2b+Aab^2\right)x^6-35\,Aa^3+35\left(Ba^3+3\,Aa^2b\right)x^3\right)}{105\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x^
6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)
```

Sympy [A] time = 35.2066, size = 112, normalized size = 1.32

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}}+2Aa^2bx^{\frac{3}{2}}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Ba^3x^{\frac{3}{2}}}{3}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A*
b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b**
2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21
```

Giac [A] time = 1.11934, size = 104, normalized size = 1.22

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{5}\,Bab^2x^{\frac{15}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{3}\,Ba^2bx^{\frac{9}{2}}+\frac{2}{3}\,Aab^2x^{\frac{9}{2}}+\frac{2}{3}\,Ba^3x^{\frac{3}{2}}+2\,Aa^2bx^{\frac{3}{2}}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a^
2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) -
 2/3*A*a^3/x^(3/2)
```

3.154 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx$

Optimal. Leaf size=83

$$2a^2\sqrt{x}(aB+3Ab)-\frac{2a^3A}{5x^{5/2}}+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

```
[Out] (-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

Rubi [A] time = 0.040558, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2\sqrt{x}(aB+3Ab)-\frac{2a^3A}{5x^{5/2}}+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx &= \int\left(\frac{a^3A}{x^{7/2}}+\frac{a^2(3Ab+aB)}{\sqrt{x}}+3ab(Ab+aB)x^{5/2}+b^2(Ab+3aB)x^{11/2}+b^3Bx^{17/2}\right)dx\\ &= -\frac{2a^3A}{5x^{5/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{19}b^3Bx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.0237802, size = 78, normalized size = 0.94

$$\frac{7410a^2bx^3\left(7A+Bx^3\right)-3458a^3\left(A-5Bx^3\right)+570ab^2x^6\left(13A+7Bx^3\right)+70b^3x^9\left(19A+13Bx^3\right)}{8645x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(13
*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))
```

Maple [A] time = 0.006, size = 80, normalized size = 1.

$$-\frac{-910\,b^3Bx^{12}-1330\,x^9b^3A-3990\,x^9ab^2B-7410\,x^6ab^2A-7410\,x^6a^2bB-51870\,Aa^2bx^3-17290\,Ba^3x^3+3458\,a^3A}{8645}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x)
```

```
[Out] -2/8645*(-455*B*b^3*x^12-665*A*b^3*x^9-1995*B*a*b^2*x^9-3705*A*a*b^2*x^6-37
05*B*a^2*b*x^6-25935*A*a^2*b*x^3-8645*B*a^3*x^3+1729*A*a^3)/x^(5/2)
```

Maxima [A] time = 0.936169, size = 99, normalized size = 1.19

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{2}{13}\left(3\,Bab^2+Ab^3\right)x^{\frac{13}{2}}+\frac{6}{7}\left(Ba^2b+Aab^2\right)x^{\frac{7}{2}}-\frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}+2\left(Ba^3+3\,Aa^2b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2/5*A*a^3/x^(5/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

Fricas [A] time = 1.67592, size = 186, normalized size = 2.24

$$\frac{2\left(455\,Bb^{3}x^{12}+665\left(3\,Bab^{2}+Ab^{3}\right)x^{9}+3705\left(Ba^{2}b+Aab^{2}\right)x^{6}-1729\,Aa^{3}+8645\left(Ba^{3}+3\,Aa^{2}b\right)x^{3}\right)}{8645\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a*
b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)
```

Sympy [A] time = 39.641, size = 110, normalized size = 1.33

$$-\frac{2Aa^{3}}{5x^{\frac{5}{2}}}+6Aa^{2}b\sqrt{x}+\frac{6Aab^{2}x^{\frac{7}{2}}}{7}+\frac{2Ab^{3}x^{\frac{13}{2}}}{13}+2Ba^{3}\sqrt{x}+\frac{6Ba^{2}bx^{\frac{7}{2}}}{7}+\frac{6Bab^{2}x^{\frac{13}{2}}}{13}+\frac{2Bb^{3}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b
**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(19/2)/19
```

Giac [A] time = 1.11192, size = 104, normalized size = 1.25

$$\frac{2}{19}\,Bb^{3}x^{\frac{19}{2}}+\frac{6}{13}\,Bab^{2}x^{\frac{13}{2}}+\frac{2}{13}\,Ab^{3}x^{\frac{13}{2}}+\frac{6}{7}\,Ba^{2}bx^{\frac{7}{2}}+\frac{6}{7}\,Aab^{2}x^{\frac{7}{2}}+2\,Ba^{3}\sqrt{x}+6\,Aa^{2}b\sqrt{x}-\frac{2\,Aa^{3}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) -
2/5*A*a^3/x^(5/2)
```

3.155 $\int \frac{x^{7/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

Optimal. Leaf size=73

$$\frac{2x^{3/2}(Ab-aB)}{3b^2}-\frac{2\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2Bx^{9/2}}{9b}$$

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(9/2))/(9*b) - (2*Sqrt[a]*(A*b - a
*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Rubi [A] time = 0.0476554, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 321, 329, 275, 205}

$$\frac{2x^{3/2}(Ab-aB)}{3b^2}-\frac{2\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(9/2))/(9*b) - (2*Sqrt[a]*(A*b - a
*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{2Bx^{9/2}}{9b}-\frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{9b}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{2\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0795324, size = 67, normalized size = 0.92

$$\frac{2x^{3/2}\left(-3aB+3Ab+bBx^3\right)}{9b^2}+\frac{2\sqrt{a}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Maple [A] time = 0.013, size = 78, normalized size = 1.1

$$\frac{2B}{9b}x^{\frac{9}{2}}+\frac{2A}{3b}x^{\frac{3}{2}}-\frac{2Ba}{3b^2}x^{\frac{3}{2}}-\frac{2Aa}{3b}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{2a^2B}{3b^2}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/9*B*x^(9/2)/b+2/3/b*A*x^(3/2)-2/3/b^2*B*x^(3/2)*a-2/3*a/b/(a*b)^(1/2)*arc
tan(b*x^(3/2)/(a*b)^(1/2))*A+2/3*a^2/b^2/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)
^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7725, size = 321, normalized size = 4.4

$$\left[-\frac{3(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^3-2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^3+a}\right)-2\left(Bbx^4-3(Ba-Ab)x\right)\sqrt{x}}{9b^2},\ \frac{2\left(3(Ba-Ab)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{a}\right)+\left(Bbx^4-3(B\right.\right.}{9b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(b
*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*b)
*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*sqrt
(x))/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11999, size = 86, normalized size = 1.18

$$\frac{2\left(Ba^2 - Aab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}b^2} + \frac{2\left(Bb^2x^{\frac{9}{2}} - 3\,Babx^{\frac{3}{2}} + 3\,Ab^2x^{\frac{3}{2}}\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b^
2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3
```

3.156 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

Optimal. Leaf size=288

$$\frac{2\sqrt{x}(Ab-aB)}{b^2}+\frac{\sqrt[6]{a}(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}-\frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}+\frac{\sqrt[6]{a}(A\cdots}{\cdots}$$

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(7/2))/(7*b) + (a^(1/6)*(A*b - a*B)*Ar
cTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (a^(1/6)*(A*b -
 a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (2*a^(1
/6)*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + (a^(1/6)*
(A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*
Sqrt[3]*b^(13/6)) - (a^(1/6)*(A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6))
```

Rubi [A] time = 0.519036, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 209, 634, 618, 204, 628, 205}

$$\frac{2\sqrt{x}(Ab-aB)}{b^2}+\frac{\sqrt[6]{a}(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}-\frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}+\frac{\sqrt[6]{a}(A\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(7/2))/(7*b) + (a^(1/6)*(A*b - a*B)*Ar
cTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (a^(1/6)*(A*b -
 a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (2*a^(1
/6)*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + (a^(1/6)*
(A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*
Sqrt[3]*b^(13/6)) - (a^(1/6)*(A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{7b}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{\left(2\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3b^2} - \frac{\left(2\sqrt[6]{a}(Ab -\right.}{}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\left(\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x}\right.}{2\sqrt{3}b^{13/6}}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}\right.}{2\sqrt{3}b^{13/6}}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{}}{\sqrt[6]{a}}\right.}{3b^{13/6}}$$

Mathematica [C] time = 0.0493863, size = 54, normalized size = 0.19

$$\frac{2\sqrt{x}\left((7aB-7Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)-7aB+7Ab+bBx^3\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3 + (-7*A*b + 7*a*B)*Hypergeometric2F1[1/
6, 1, 7/6, -((b*x^3)/a)]))/(7*b^2)
```

Maple [A] time = 0.115, size = 371, normalized size = 1.3

$$\frac{2B}{7b}x^{\frac{7}{2}}+2\,\frac{A\sqrt{x}}{b}-2\,\frac{Ba\sqrt{x}}{b^2}-\frac{2A}{3b}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{2Ba}{3b^2}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{\sqrt{3}A}{6b}\sqrt[6]{\frac{a}{b}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/7*B*x^(7/2)/b+2/b*A*x^(1/2)-2/b^2*B*a*x^(1/2)-2/3/b*(a/b)^(1/6)*arctan(x^
(1/2)/(a/b)^(1/6))*A+2/3*a/b^2*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B+1/
6/b*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-1/6
*a/b^2*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-
1/3/b*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*A+1/3*a/b^2*(a/b)^(
1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*B-1/6/b*3^(1/2)*(a/b)^(1/6)*ln(x
+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+1/6*a/b^2*3^(1/2)*(a/b)^(1/6)*l
n(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-1/3/b*(a/b)^(1/6)*arctan(2*x
^(1/2)/(a/b)^(1/6)+3^(1/2))*A+1/3*a/b^2*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^
(1/6)+3^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34292, size = 5104, normalized size = 17.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/42*(28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*
A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4
*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*
a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a*b^2 - A*b^3)*
sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^
3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6))*b^11*(-(
B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*
B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/6) + 2*sqrt(3)*(B*a*b^1
1 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) - sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4
*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6))/(B^6*a^7 - 6*A*B^
5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*
A^5*B*a^2*b^5 + A^6*a*b^6)) + 28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 1
5*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b
^5 + A^6*a*b^6)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*
b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6
)/b^13)^(1/6))*b^11*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) + 2*sqrt(3)*(B*a*b^11 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*
A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5
 + A^6*a*b^6)/b^13)^(5/6) + sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a
^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*
b^6))/(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 +
15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)) - 7*b^2*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*
B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x +
4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2
 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b
^13)^(1/6)) + 7*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
```

```
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/
6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a
^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7
- 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*
b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) + 14*b^2*(-(B^6*a^7 - 6*A*B
^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6
*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b
+ 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^
2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 14*b^2*(-(B^6*a^7 -
 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b
^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-b^2*(-(B^6*a^7 - 6*A*B^5
*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A
^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 12*(B*b*x^3
- 7*B*a + 7*A*b)*sqrt(x))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16075, size = 390, normalized size = 1.35

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}} Ba - \left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}} Ba - \left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3*((
```

```
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(
x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-
(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/6)*B*a
 - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x^(7/2)
- 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7
```

3.157 $\int \frac{x^{3/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

Optimal. Leaf size=270

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\ldots}{\sqrt[6]{a}}\right.}{3\sqrt[6]{a}b^{11/6}}$$

```
[Out] (2*B*x^(5/2))/(5*b) - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt
[x])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6)) - ((A*b - a*
B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a
^(1/6)*b^(11/6))
```

Rubi [A] time = 0.550781, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\ldots}{\sqrt[6]{a}}\right.}{3\sqrt[6]{a}b^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```


```
[Out] (2*B*x^(5/2))/(5*b) - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt
[x])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6)) - ((A*b - a*
B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a
^(1/6)*b^(11/6))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{\left(4\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3b^{5/3}} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x}\right.}{3\sqrt[6]{a}b^{5/3}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} - \frac{(Ab}{}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} - \frac{(Ab-aB)\,\mathrm{lo}}{}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{2(Ab-aB)\tan^{-}}{3\sqrt[6]{a}b^{11/6}}$$

Mathematica [C] time = 0.0413152, size = 45, normalized size = 0.17

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\right)}{5ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(5/2)*(a*B + (A*b - a*B)*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)]
))/(5*a*b)
```

Maple [A] time = 0.043, size = 350, normalized size = 1.3

$$\frac{2B}{5b}x^{\frac{5}{2}}+\frac{2A}{3b}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}-\frac{2Ba}{3b^2}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}+\frac{\sqrt{3}A}{6a}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)-\frac{\sqrt{3}B}{6b}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/5*B*x^(5/2)/b+2/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*A-2/3/b^2/(a/
b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B*a+1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x-3^(
1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-1/6/b*3^(1/2)*(a/b)^(5/6)*ln(x-3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/3/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(
a/b)^(1/6)-3^(1/2))*A-1/3/b^2/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1
/2))*B*a-1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(
1/3))*A+1/6/b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1
/3))*B+1/3/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*A-1/3/b^2/(a
/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75195, size = 7606, normalized size = 28.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(12*B*x^(5/2) - 20*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^6*b^9 - 5*A*B^4*a^5*b^10
 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^1
4)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + (B^1
0*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^
4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3
*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^7*b^7 -
6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3
*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*
B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6
*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^1
1))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 -
 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5
*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A
^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b +
15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^
5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 20*sqrt(3)*b*(-(B^
6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^
2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*arctan(1/3*(sqrt(3)*sq
rt(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^
3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b
+ 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*
b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8
*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5
+ 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B
*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b
^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b
^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2
 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 - 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^
6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1
/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a
^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5
*B*a*b^5 + A^6*b^6)) + 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^
11))^(1/6)*log(a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5
```

```
/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 +
5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A
^6*b^6)/(a*b^11))^(1/6)*log(-a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*
a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6
)/(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*
B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*b*(-(B^6*a^6 - 6*A*B^5*
a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^
5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10
+ 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^
10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A
^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^
3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7
 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*
a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a*b^11))^(2/3)) + 5*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a*b^11))^(1/6)*log(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11
 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^
9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*
A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^
2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 +
15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^
2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2
/3)))/b
```

Sympy [A] time = 79.262, size = 867, normalized size = 3.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(5/2)/5 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)
```

```
/5)/b, Eq(a, 0)), (-(-1)**(5/6)*A*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) +
sqrt(x))/(3*a**(1/6)*b**30*(1/b)**(175/6)) + (-1)**(5/6)*A*log((-1)**(1/6)*
a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b**30*(1/b)**(175/6)) - (-1)**
(5/6)*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a*
*(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b**30*(1/b)**(175/6)) - (-1)**(5/6)*
sqrt(3)*A*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)*
*(1/6)))/(3*a**(1/6)*b**30*(1/b)**(175/6)) + (-1)**(5/6)*sqrt(3)*A*atan(sqr
t(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/
6)*b**30*(1/b)**(175/6)) + (-1)**(5/6)*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)
*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b**3
2*(1/b)**(187/6)) + (-1)**(5/6)*B*a**(5/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)*
*(1/6) + sqrt(x))/(3*b**31*(1/b)**(175/6)) - (-1)**(5/6)*B*a**(5/6)*log((-1
)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**31*(1/b)**(175/6)) + (-1)**
(5/6)*B*a**(5/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)*
*(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**31*(1/b)**(175/6)) + (-1)**(5/6)*
sqrt(3)*B*a**(5/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/
6)*(1/b)**(1/6)))/(3*b**31*(1/b)**(175/6)) - (-1)**(5/6)*sqrt(3)*B*a**(5/6)
*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/
(3*b**31*(1/b)**(175/6)) - (-1)**(5/6)*B*a**(5/6)*log(4*(-1)**(1/6)*a**(1/6
)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**3
3*(1/b)**(187/6)) + 2*B*x**(5/2)/(5*b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.158 $\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=53

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

```
[Out] (2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*
Sqrt[a]*b^(3/2))
```

Rubi [A] time = 0.0357264, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 329, 275, 205}

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*
Sqrt[a]*b^(3/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{2Bx^{3/2}}{3b}-\frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3b}\\ &= \frac{2Bx^{3/2}}{3b}-\frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\operatorname{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3b}\\ &= \frac{2Bx^{3/2}}{3b}+\frac{(2(Ab-aB))\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b}\\ &= \frac{2Bx^{3/2}}{3b}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0450268, size = 52, normalized size = 0.98

$$\frac{2}{3}\left(\frac{Bx^{3/2}}{b}-\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*((B*x^(3/2))/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(Sqr
t[a]*b^(3/2))))/3
```

Maple [A] time = 0.007, size = 53, normalized size = 1.

$$\frac{2B}{3b}x^{\frac{3}{2}}+\frac{2A}{3}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{2Ba}{3b}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/3*B*x^(3/2)/b+2/3/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))*A-2/3/b/(a*b)
^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76878, size = 261, normalized size = 4.92

$$\left[\frac{2\,Babx^{\frac{3}{2}}+(Ba-Ab)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)}{3\,ab^2},\frac{2\left(Babx^{\frac{3}{2}}-(Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)\right)}{3\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^
(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a*b
)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.10648, size = 53, normalized size = 1.

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba - Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

3.159 $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)}\,dx$

Optimal. Leaf size=268

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\cdots}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}$$

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)
])/(3*a^(5/6)*b^(7/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(5/6)*b^(7/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(5/6)*b^(7/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(
1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6)) + ((A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b
^(7/6))
```

Rubi [A] time = 0.472592, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\cdots}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```


```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)
])/(3*a^(5/6)*b^(7/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(5/6)*b^(7/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(5/6)*b^(7/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(
1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6)) + ((A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b
^(7/6))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)}\,dx = \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{b}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(2(Ab-aB))\operatorname{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{5/6}b} + \frac{(2(Ab-aB))\operatorname{Subst}\left(\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\cdots}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\cdots}\cdots\right.}{3a^{5/6}b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\operatorname{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{(Ab-a\cdots}{\cdots}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{(Ab-aB)\log\left(\cdots\right.}{2\cdots}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{2(Ab-aB)\tan^{-1}\left(\cdots\right.}{3a^{5/6}b^{7/6}}$$

Mathematica [C] time = 0.0262235, size = 43, normalized size = 0.16

$$\frac{2\sqrt{x}\left((Ab-aB)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+aB\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[x]*(a*B + (A*b - a*B)*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)])
)/(a*b)
```

Maple [A] time = 0.03, size = 347, normalized size = 1.3

$$2\frac{B\sqrt{x}}{b}+\frac{2A}{3a}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)-\frac{2B}{3b}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)-\frac{\sqrt{3}A}{6a}\sqrt[6]{\frac{a}{b}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)+\frac{\sqrt{3}B}{6b}\sqrt[6]{\frac{a}{b}}\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)/x^(1/2),x)
```

```
[Out] 2*B*x^(1/2)/b+2/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*A-2/3/b*(a/b)^(
1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(
a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+1/6/b*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a
/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/3/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(
1/6)-3^(1/2))*A-1/3/b*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*B+1
/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-1/
6/b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/3
/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*A-1/3/b*(a/b)^(1/6)*ar
ctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30493, size = 5096, normalized size = 19.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="fricas")
```

[Out] 1/6*(4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6))*a^4*b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + 2*sqrt(3)*(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - (B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6))*a^4*b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + 2*sqrt(3)*(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + 4*(B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) + b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) + 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*

```
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqrt(x))
- 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3
+ 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-a*b*(
-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^
4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqr
t(x)) + 12*B*sqrt(x))/b
```

Sympy [A] time = 39.5693, size = 864, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/
7)/a, Eq(b, 0)), (-(-1)**(1/6)*A*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + s
qrt(x))/(3*a**(5/6)*b**33*(1/b)**(197/6)) + (-1)**(1/6)*A*log((-1)**(1/6)*a
**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(5/6)*b**33*(1/b)**(197/6)) - (-1)**(
1/6)*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**
(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(5/6)*b**33*(1/b)**(197/6)) + (-1)**(1/6)*A
*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(
1/b)**(1/3) + 4*x)/(6*a**(5/6)*b**33*(1/b)**(197/6)) + (-1)**(1/6)*sqrt(3)*
A*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))
/(3*a**(5/6)*b**33*(1/b)**(197/6)) - (-1)**(1/6)*sqrt(3)*A*atan(sqrt(3)/3 +
 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(5/6)*b**33
*(1/b)**(197/6)) + (-1)**(1/6)*B*a**(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**
(1/6) + sqrt(x))/(3*b**34*(1/b)**(197/6)) - (-1)**(1/6)*B*a**(1/6)*log((-1)
**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**34*(1/b)**(197/6)) + (-1)**(
1/6)*B*a**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**
(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**34*(1/b)**(197/6)) - (-1)**(1/6)*B
*a**(1/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a
**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**34*(1/b)**(197/6)) - (-1)**(1/6)*sqrt(3)*
B*a**(1/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)
**(1/6)))/(3*b**34*(1/b)**(197/6)) + (-1)**(1/6)*sqrt(3)*B*a**(1/6)*atan(sq
rt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**34
*(1/b)**(197/6)) + 2*B*sqrt(x)/b, True))
```

Giac [A] time = 1.14175, size = 378, normalized size = 1.41

$$\frac{2\,B\sqrt{x}}{b} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^2} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x\right.}{6\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqr
t(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5)^
(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(
3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a -
 (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/
(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^
(1/6))/(a*b^2)
```

3.160 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

Optimal. Leaf size=268

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{}}{}\right.}{3a^{7/6}b^{5/6}}$$

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(7/6)*b^(5/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6)*b^(5/6)) + ((A*b - a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6
)*b^(5/6))
```

Rubi [A] time = 0.552016, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{}}{}\right.}{3a^{7/6}b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(7/6)*b^(5/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6)*b^(5/6)) + ((A*b - a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6
)*b^(5/6))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{7/6}b^{2/3}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}}\right.}{3a^{7/6}b^{2/3}}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{7/6}b^{5/6}} + \frac{(Ab-}{}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}} + \frac{(Ab-aB)\log}{}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{2(Ab-aB)\tan^{-1}\left(\right.}{3a^{7/6}b^{5/6}}$$

Mathematica [C] time = 0.0131668, size = 46, normalized size = 0.17

$$\frac{2\left(x^3(aB-Ab)\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)-5aA\right)}{5a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (2*(-5*a*A + (-(A*b) + a*B)*x^3*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a
)]))/(5*a^2*Sqrt[x])
```

Maple [A] time = 0.034, size = 349, normalized size = 1.3

$$-2\frac{A}{a\sqrt{x}}-\frac{2A}{3a}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}+\frac{2B}{3b}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}-\frac{\sqrt{3}Ab}{6a^2}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)+\frac{\sqrt{3}B}{6a}\left(\frac{a}{b}\right)^{\frac{5}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a),x)
```

```
[Out] -2*A/a/x^(1/2)-2/3/a/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*A+2/3/b/(a/b)^
(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B-1/6/a^2*3^(1/2)*(a/b)^(5/6)*ln(x-3^(1/2
)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A*b+1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x-3^(1/
2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-1/3/a/(a/b)^(1/6)*arctan(2*x^(1/2)/(a
/b)^(1/6)-3^(1/2))*A+1/3/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2)
)*B+1/6/a^2*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3
))*A*b-1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/
3))*B-1/3/a/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*A+1/3/b/(a/b)
^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.068, size = 7648, normalized size = 28.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3
*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/
6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3
*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B
^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B
^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) + (B^10*a^10 - 10*A*
B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4
- 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*
B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^11*b^3 - 6*A*B^5*a^10*
b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*
B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5
))^(2/3))*a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) +
2*sqrt(3)*(B^5*a^6*b - 5*A*B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^
3*b^4 + 5*A^4*B*a^2*b^5 - A^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^
4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 1
5*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*arctan(1/3*(sqrt(3)*sqrt(-4*(
B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 +
 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2
*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^
6*b^6)/(a^7*b^5))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^
2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A
^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9
+ A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 -
20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-
(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4
*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3))*a*b*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^6*b - 5*A*
B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^3*b^4 + 5*A^4*B*a^2*b^5 - A
^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6
) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
```

```
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b
^5))^(1/6)*log(a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20
*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))
^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3
 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 -
 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(4*(B^5*a^11*b^4 - 5*A*B^4
*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5
*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6
) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^
3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A
^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^
6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 1
5*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3)) - a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(1/6)*log(-4*(B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*
B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(
-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^
4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) + 4*(B^10*a^10 -
10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6
*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45
*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^
5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 -
 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a^7*b^5))^(2/3)) - 12*A*sqrt(x))/(a*x)
```

Sympy [A] time = 54.1643, size = 867, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)
/5)/a, Eq(b, 0)), (-2*A/(a*sqrt(x)) - (-1)**(5/6)*A*b**5*(1/b)**(29/6)*log(
(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)) - (-1)**(5/6)*A*l
og(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/
b)**(1/3) + 4*x)/(6*a**(7/6)*b**3*(1/b)**(19/6)) + (-1)**(5/6)*A*log(-(-1)*
*(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)*b**4*(1/b)**(25/6)) + (
-1)**(5/6)*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/
3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(7/6)*b**4*(1/b)**(25/6)) + (-1)**(5/
6)*sqrt(3)*A*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/
b)**(1/6)))/(3*a**(7/6)*b**4*(1/b)**(25/6)) - (-1)**(5/6)*sqrt(3)*A*atan(sq
rt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(7
/6)*b**4*(1/b)**(25/6)) + (-1)**(5/6)*B*b**4*(1/b)**(29/6)*log((-1)**(1/6)*
a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)) + (-1)**(5/6)*B*log(4*(-1)**(
1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) +
4*x)/(6*a**(1/6)*b**4*(1/b)**(19/6)) - (-1)**(5/6)*B*log(-(-1)**(1/6)*a**(1
/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b**5*(1/b)**(25/6)) - (-1)**(5/6)*B
*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*
(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b**5*(1/b)**(25/6)) - (-1)**(5/6)*sqrt(3)*B
*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/
(3*a**(1/6)*b**5*(1/b)**(25/6)) + (-1)**(5/6)*sqrt(3)*B*atan(sqrt(3)/3 + 2*
(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b**5*(1/
b)**(25/6)), True))
```

Giac [A] time = 1.37481, size = 378, normalized size = 1.41

$$-\frac{2A}{a\sqrt{x}} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba - \left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2b^5} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba - \left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + \right.}{6a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^5) + 1/6*sqrt(3)*((a*
b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x +
(a/b)^(1/3))/(a^2*b^5) + 1/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan
((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^5) + 1/3*((a*b^5)^(5
/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b
)^(1/6))/(a^2*b^5) + 2/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(sqr
```

```
t(x)/(a/b)^(1/6))/(a^2*b^5)
```

3.161 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=53

$$-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

```
[Out] (-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*a^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.0370199, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 329, 275, 205}

$$-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*a^(3/2)*Sqrt[b])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx &= -\frac{2A}{3ax^{3/2}}-\frac{\left(2\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3a}\\ &= -\frac{2A}{3ax^{3/2}}-\frac{\left(4\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3a}\\ &= -\frac{2A}{3ax^{3/2}}-\frac{(2(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3a}\\ &= -\frac{2A}{3ax^{3/2}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0284999, size = 53, normalized size = 1.

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*a^(3/2)*Sqrt[b])
```

Maple [A] time = 0.01, size = 53, normalized size = 1.

$$-\frac{2Ab}{3a}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{2B}{3}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{2A}{3a}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a),x)
```

```
[Out] -2/3/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))*A*b+2/3/(a*b)^(1/2)*arctan
(b*x^(3/2)/(a*b)^(1/2))*B-2/3*A/a/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08949, size = 282, normalized size = 5.32

$$\left[\frac{(Ba-Ab)\sqrt{-ab}x^2\log\left(\frac{bx^3+2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2Aab\sqrt{x}}{3a^2bx^2}, \frac{2\left((Ba-Ab)\sqrt{ab}x^2\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)-Aab\sqrt{x}\right)}{3a^2bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*((B*a - A*b)*sqrt(-a*b)*x^2*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*
x^3 + a)) - 2*A*a*b*sqrt(x))/(a^2*b*x^2), 2/3*((B*a - A*b)*sqrt(a*b)*x^2*ar
ctan(sqrt(a*b)*x^(3/2)/a) - A*a*b*sqrt(x))/(a^2*b*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09644, size = 53, normalized size = 1.

$$\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a}-\frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

3.162 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=270

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\cdots}{\sqrt[6]{a}}\right.}{3a^{11/6}\sqrt[6]{b}}$$

```
[Out] (-2*A)/(5*a*x^(5/2)) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^
(1/6)])/(3*a^(11/6)*b^(1/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqr
t[x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(11/6)*b^(1/6)) - ((A*b - a
*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*
a^(11/6)*b^(1/6))
```

Rubi [A] time = 0.484734, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\cdots}{\sqrt[6]{a}}\right.}{3a^{11/6}\sqrt[6]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^
(1/6)])/(3*a^(11/6)*b^(1/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqr
t[x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(11/6)*b^(1/6)) - ((A*b - a
*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*
a^(11/6)*b^(1/6))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{5ax^{5/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{\left(4\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{11/6}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}+}{\sqrt[3]{a}+\sqrt{3}}\right.}{3a^{11/6}}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6a^{5/3}} - \frac{(Ab}{}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\,\text{lo}}{}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{2(Ab-aB)\tan^{-}}{3a^{11/6}\sqrt[6]{b}}$$

Mathematica [C] time = 0.0145714, size = 47, normalized size = 0.17

$$\frac{10x^3(aB-Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)-2aA}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*a*A + 10*(-(A*b) + a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)
])/(5*a^2*x^(5/2))
```

Maple [A] time = 0.035, size = 352, normalized size = 1.3

$$-\frac{2\,Ab}{3\,a^2}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{2\,B}{3\,a}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{\sqrt{3}Ab}{6\,a^2}\sqrt[6]{\frac{a}{b}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)-\frac{\sqrt{3}B}{6\,a}\sqrt[6]{\frac{a}{b}}\ln\left(x-\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a),x)
```

```
[Out] -2/3/a^2*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*A*b+2/3/a*(a/b)^(1/6)*arct
an(x^(1/2)/(a/b)^(1/6))*B+1/6/a^2*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1
/6)*x^(1/2)+(a/b)^(1/3))*A*b-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(
1/6)*x^(1/2)+(a/b)^(1/3))*B-1/3/a^2*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6
)-3^(1/2))*A*b+1/3/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*B-1/
6/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A*b
+1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-
1/3/a^2*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*A*b+1/3/a*(a/b)^(
1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))*B-2/5*A/a/x^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38083, size = 5083, normalized size = 18.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))
^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^3 - A*a^2*b
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6))*a^9*b*
(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A
^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5/6) + 2*sqrt(3)*(B*a^
10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*
B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
6*A^5*B*a*b^5 + A^6*b^6)) + 20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^11*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
^11*b))^(1/6))*a^9*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5
/6) + 2*sqrt(3)*(B*a^10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b +
 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^11*b))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 5*a*x^3*(-(B^6*a^6 - 6*A*B
^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6
*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + 4*
(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(1/6)) + 5*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/
6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a
^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 10*a*x^3*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(a^2*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
```

```
a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) - 10*a*x^3*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-a^2*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) + 12*A*sqr
t(x))/(a*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12632, size = 378, normalized size = 1.4

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b
^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b)
+ 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) +
 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 2/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) -
 2/5*A/(a*x^(5/2))
```

3.163 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{x^{3/2}(Ab-3aB)}{3ab^2}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}+\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[Out] -((A*b - 3*a*B)*x^(3/2))/(3*a*b^2) + ((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x^
3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0550496, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 321, 329, 275, 205}

$$-\frac{x^{3/2}(Ab-3aB)}{3ab^2}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}+\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 3*a*B)*x^(3/2))/(3*a*b^2) + ((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x^
3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{\left(-\frac{3Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{3ab}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.103562, size = 77, normalized size = 0.81

$$\frac{\frac{\sqrt{b}x^{3/2}(3aB-Ab+2bBx^3)}{a+bx^3}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}}}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((Sqrt[b]*x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(a + b*x^3) + ((A*b - 3*a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[a])/(3*b^(5/2))
```

Maple [A] time = 0.016, size = 93, normalized size = 1.

$$\frac{2B}{3b^2}x^{\frac{3}{2}}-\frac{A}{3b\left(bx^3+a\right)}x^{\frac{3}{2}}+\frac{Ba}{3b^2\left(bx^3+a\right)}x^{\frac{3}{2}}+\frac{A}{3b}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{Ba}{b^2}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 2/3*B*x^(3/2)/b^2-1/3/b*x^(3/2)/(b*x^3+a)*A+1/3/b^2*x^(3/2)/(b*x^3+a)*B*a+1
/3/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))*A-1/b^2/(a*b)^(1/2)*arctan(b
*x^(3/2)/(a*b)^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76392, size = 478, normalized size = 5.03

$$\left[\frac{\left(\left(3\,Bab - Ab^2\right)x^3 + 3\,Ba^2 - Aab\right)\sqrt{-ab}\log\left(\frac{bx^3 - 2\,\sqrt{-ab}x^{\frac{3}{2}} - a}{bx^3 + a}\right) + 2\left(2\,Bab^2x^4 + \left(3\,Ba^2b - Aab^2\right)x\right)\sqrt{x}}{6\left(ab^4x^3 + a^2b^3\right)}, -\frac{\left(\left(3\,Bab - Ab^2\right)x^3 +\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*s
qrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a*b
^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*B*a
^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3*B*a
^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11211, size = 92, normalized size = 0.97

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba - Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2} + \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{3\left(bx^3 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b
)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)
```

3.164 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=312

$$-\frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}-\frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{5/6}b^{13/6}}$$

```
[Out] -((A*b - 7*a*B)*Sqrt[x])/(3*a*b^2) + ((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x^
3)) - ((A*b - 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(
5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6
)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)
])/(9*a^(5/6)*b^(13/6)) - ((A*b - 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/
6)*b^(13/6))
```

Rubi [A] time = 0.501587, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}-\frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{5/6}b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 7*a*B)*Sqrt[x])/(3*a*b^2) + ((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x^
3)) - ((A*b - 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(
5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6
)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)
])/(9*a^(5/6)*b^(13/6)) - ((A*b - 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/
6)*b^(13/6))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{\left(-\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{6b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{5/6}b^2} + \frac{(Ab-7}{}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{}}\right.}{12\sqrt{3}a^{5/6}b^{13/6}}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\right.}{12\sqrt{3}a^{5/6}b^{13/6}}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} - \frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}+\right.}{18a^{5/6}b^{13/6}}$$

Mathematica [C] time = 0.0761386, size = 76, normalized size = 0.24

$$\frac{\sqrt{x}\left(\left(a+bx^3\right)(Ab-7aB)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+a\left(7aB-Ab+6bBx^3\right)\right)}{3ab^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[x]*(a*(-(A*b) + 7*a*B + 6*b*B*x^3) + (A*b - 7*a*B)*(a + b*x^3)*Hyperg
eometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(3*a*b^2*(a + b*x^3))
```

Maple [A] time = 0.039, size = 399, normalized size = 1.3

$$2\frac{B\sqrt{x}}{b^2}-\frac{A}{3b\left(bx^3+a\right)}\sqrt{x}+\frac{Ba}{3b^2\left(bx^3+a\right)}\sqrt{x}-\frac{7B}{9b^2}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{7B\sqrt{3}}{36b^2}\sqrt[6]{\frac{a}{b}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)-\frac{7}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 2*B/b^2*x^(1/2)-1/3/b*x^(1/2)/(b*x^3+a)*A+1/3/b^2*x^(1/2)/(b*x^3+a)*B*a-7/9
/b^2*B*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+7/36/b^2*B*3^(1/2)*(a/b)^(1/
6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-7/18/b^2*B*(a/b)^(1/6)*arc
tan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-7/36/b^2*B*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-7/18/b^2*B*(a/b)^(1/6)*arctan(2*x^(1/2
)/(a/b)^(1/6)+3^(1/2))+1/9/b*A/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/
36/b*A/a*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+
1/18/b*A/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))+1/36/b*A/a*3^(
1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/18/b*A/a*(
a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52257, size = 5975, normalized size = 19.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b +
36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5
*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-
(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3
*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3
) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)
*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*
B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(
1/6))*a^4*b^11*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b
^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^13))^(5/6) + 2*sqrt(3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(11764
9*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b
^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) - sq
rt(3)*(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A
^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B
^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3
+ 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*(b^3*x^3 + a
*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860
*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^1
3))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B
^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b
^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b
 + A^2*b^2)*x - (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6))*a^4*b^11*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + 2*sqrt(
3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5
*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 4
2*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 - 1008
42*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2
*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B^6*a^6 - 100842*A*B^5*a^5*b
+ 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A
^5*B*a*b^5 + A^6*b^6)) - (b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5
*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4
 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^
6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735
*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^
2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + (b^3*x
^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2
- 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a
^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*
A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b
```

```
^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x - (
7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 360
15*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*
a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6
 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*
A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a*b^2*(-(
117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*
a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)
 - (7*B*a - A*b)*sqrt(x)) - 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-a*b^2*(-(117649*B^
6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a
 - A*b)*sqrt(x)) + 12*(6*B*b*x^3 + 7*B*a - A*b)*sqrt(x))/(b^3*x^3 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20197, size = 423, normalized size = 1.36

$$\frac{2B\sqrt{x}}{b^2} - \frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} + \frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)\right.}{36\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/36*sqrt(3)*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*lo
g(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^3) + 1/36*sqrt(3)*(7*
(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x
```

```
 + (a/b)^(1/3))/(a*b^3) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*b^2)
 - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) -
 1/9*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/
(a*b^3)
```

3.165 $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{bx}\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{bx}\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{7/6}b^{11/6}}$$

```
[Out] ((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(7/6)*b^(11/6)) + ((A*b + 5*a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(7/6)*b^(11/6)) - ((A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6))
```

Rubi [A] time = 0.549336, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{bx}\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{bx}\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{7/6}b^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```


```
[Out] ((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(7/6)*b^(11/6)) + ((A*b + 5*a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(7/6)*b^(11/6)) - ((A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)}+\frac{\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\int\frac{x^{3/2}}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)}+\frac{\left(2\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int\frac{x^4}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+5aB)\,\text{Subst}\left(\int\frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{9a^{7/6}b^{5/3}}+\frac{(Ab+5aB)\,\text{Subst}\left(\int\frac{}{\sqrt[3]{a}}\right.}{9a^{7/}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}}+\frac{(Ab+5aB)\,\text{Subst}\left(\int\frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right.}{12\sqrt{3}a^{7/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}}+\frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(A}{}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)}-\frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(Ab+}{}$$

Mathematica [C] time = 0.0513197, size = 62, normalized size = 0.21

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (A*b - a*B)
*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)]))/(5*a^2*b)
```

Maple [A] time = 0.037, size = 381, normalized size = 1.3

$$\frac{Ab-Ba}{3\,ab\left(bx^{3}+a\right)}x^{\frac{5}{2}}+\frac{A}{9\,ab}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}+\frac{5\,B}{9\,b^{2}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}+\frac{\sqrt{3}A}{36\,a^{2}}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)+\frac{5}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/9/a/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b
)^(1/6))*A+5/9/b^2/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B+1/36/a^2*3^(1/
2)*(a/b)^(5/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+5/36/a/b*3^(
1/2)*(a/b)^(5/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/18/a/b/(
a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*A+5/18/b^2/(a/b)^(1/6)*arc
tan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*B-1/36/a^2*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-5/36/a/b*3^(1/2)*(a/b)^(5/6)*ln(x+3^
(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/18/a/b/(a/b)^(1/6)*arctan(2*x^(1
/2)/(a/b)^(1/6)+3^(1/2))*A+5/18/b^2/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6
)+3^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69405, size = 8817, normalized size = 30.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(B*a - A*b)*x^(5/2) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*
a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375
*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9
*b^11 + 250*A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) +
 (9765625*B^10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 937
5000*A^3*B^7*a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 1
31250*A^6*B^4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A
^9*B*a*b^9 + A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 93
75*A^2*B^4*a^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*
B*a^6*b^12 + A^6*a^5*b^13)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*
B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 +
 A^6*b^6)/(a^7*b^11))^(2/3))*a*b^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9
375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*
B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6
 + A^5*a*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a
^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*
b^6)/(a^7*b^11))^(1/6) + sqrt(3)*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*
A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b
^5 + A^6*b^6))/(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) + 4
*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^
2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^11*b^
9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*A^3*B^2*a^8*b^12 + 2
5*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5
*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30
*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^10*a^10 + 19531250*A
*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A
^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^
7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (156
25*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^
3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(-(15
625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b
^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(2/3))*a*b
^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B
^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1
```

/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b
^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6 + A^5*a*b^7)*sqrt(x)*(-(15625*B
^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 +
375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - sqrt(3)
*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a
^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6))/(15625*B^6*a^6 +
18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B
^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) - 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^
6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 3
75*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(a^6*b^
9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^
3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/
6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*
a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(-(156
25*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-
a^6*b^9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*
A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11
))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^
3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*l
og((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*
A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a
^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*
A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^
10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7
*a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^
4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 +
A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a
^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 +
A^6*a^5*b^13)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2
+ 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a
^7*b^11))^(2/3)) + (a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b
+ 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A
^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-(3125*B^5*a^11*b^9 + 3125*A*B^
4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b
^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2
*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5
+ A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^10*a^10 + 19531250*A*B^9*a^9*b +
17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A^4*B^6*a^6*b^
4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7
+ 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (15625*B^6*a^11*b
^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^3*a^8*b^10 +
375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(-(15625*B^6*a^6 +

```
18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*
B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(2/3)))/(a*b^2*x^3 + a^
2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.166 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=71

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0418958, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 329, 275, 205}

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0515127, size = 71, normalized size = 1.

$$
\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Maple [A] time = 0.015, size = 74, normalized size = 1.

$$\frac{Ab-Ba}{3\,ab\left(bx^{3}+a\right)}x^{\frac{3}{2}}+\frac{A}{3\,a}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{3\,b}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b
)^(1/2))*A+1/3/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82273, size = 414, normalized size = 5.83

$$\left[-\frac{2\left(Ba^{2}b-Aab^{2}\right)x^{\frac{3}{2}}+\left(\left(Bab+Ab^{2}\right)x^{3}+Ba^{2}+Aab\right)\sqrt{-ab}\log\left(\frac{bx^{3}-2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^{3}+a}\right)}{6\left(a^{2}b^{3}x^{3}+a^{3}b^{2}\right)},-\frac{\left(Ba^{2}b-Aab^{2}\right)x^{\frac{3}{2}}-\left(\left(Bab+Ab^{2}\right)x^{3}\right.}{3\left(a^{2}b^{3}x^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(B*a^2*b - A*a*b^2)*x^(3/2) + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*b
)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a^2*b^3*
x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^(3/2) - ((B*a*b + A*b^2)*x^3 +
```

```
B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a^2*b^3*x^3 + a^3*b^
2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63099, size = 85, normalized size = 1.2

$$\frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}ab}-\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\left(bx^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^(3
/2) - A*b*x^(3/2))/((b*x^3 + a)*a*b)
```

3.167 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}-\frac{(aB+5Ab)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{11/6}b^{7/6}}$$

```
[Out] ((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b +
 a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(11/6)*b^(7/6)) - ((5*A*b + a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6))
```

Rubi [A] time = 0.468368, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}-\frac{(aB+5Ab)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{11/6}b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```


```
[Out] ((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b +
 a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(11/6)*b^(7/6)) - ((5*A*b + a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)}+\frac{\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\int\frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)}+\frac{\left(2\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int\frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)}+\frac{(5Ab+aB)\,\text{Subst}\left(\int\frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{9a^{11/6}b}+\frac{(5Ab+aB)\,\text{Subst}\left(\int\frac{\sqrt[6]{}}{\sqrt[3]{a}+\sqrt{}}\right.}{9a^{11/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)}+\frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}}-\frac{(5Ab+aB)\,\text{Subst}\left(\int\frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)}+\frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}}-\frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(5Ab}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)}-\frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}}+\frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}}+\frac{(5Ab+a}{9}$$

Mathematica [C] time = 0.0295547, size = 68, normalized size = 0.24

$$\frac{\sqrt{x}\left(\left(a+bx^3\right)(aB+5Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+a(Ab-aB)\right)}{3a^2b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] (Sqrt[x]*(a*(A*b - a*B) + (5*A*b + a*B)*(a + b*x^3)*Hypergeometric2F1[1/6,
1, 7/6, -((b*x^3)/a)]))/(3*a^2*b*(a + b*x^3))
```

Maple [A] time = 0.037, size = 381, normalized size = 1.3

$$\frac{Ab-Ba}{3\,ab\left(bx^{3}+a\right)}\sqrt{x}+\frac{5\,A}{9\,a^{2}}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{B}{9\,ab}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)-\frac{5\sqrt{3}A}{36\,a^{2}}\sqrt[6]{\frac{a}{b}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x)
```

```
[Out] 1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+5/9/a^2*(a/b)^(1/6)*arctan(x^(1/2)/(a/b
)^(1/6))*A+1/9/a/b*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B-5/36/a^2*3^(1/
2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-1/36/a/b*3^(
1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/18/a^2*(
a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*A+1/18/a/b*(a/b)^(1/6)*arc
tan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*B+5/36/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+1/36/a/b*3^(1/2)*(a/b)^(1/6)*ln(x+3^
(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/18/a^2*(a/b)^(1/6)*arctan(2*x^(1
/2)/(a/b)^(1/6)+3^(1/2))*A+1/18/a/b*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6
)+3^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33607, size = 5873, normalized size = 20.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B
^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^
5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^2*(-(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) +
(B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6))*a
^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3
*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)
)^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sqrt(x)*(-(B^6*a^6 + 30*A*B^
5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4
 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(5/6) + sqrt(3)*(B^6*a^6
+ 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^
2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(B^6*a^6 + 30*A*B^5*a^5*b +
 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*
A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 +
30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*
a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x - (B*a^3*b
+ 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^7))^(1/6))*a^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*
a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 +
 15625*A^6*b^6)/(a^11*b^7))^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sq
rt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*
b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))
^(5/6) - sqrt(3)*(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3
*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)) + (a*b^2*x^3 + a^2*
b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1
/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^
3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(
a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a
^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^
3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(
```

```
a^11*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A
^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*
a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a
*b + 25*A^2*b^2)*x - (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)) + 2*(a*b^2*x^3 + a^2
*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(
1/6)*log(a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3
*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a
^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) - 2*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6
 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B
^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(-a^2*
b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
+ 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/
6) + (B*a + 5*A*b)*sqrt(x)) - 12*(B*a - A*b)*sqrt(x))/(a*b^2*x^3 + a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18149, size = 408, normalized size = 1.41

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\right.\right.}{36\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*
(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a
+ 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/
(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((a*b^
5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x)
)/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*a
rctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/9*((a*b
^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^2)
```

3.168 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{13/6}b^{5/6}}$$

```
[Out] -(7*A*b - a*B)/(3*a^2*b*Sqrt[x]) + (A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3))
+ ((7*A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(13/6
)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(18*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(
9*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*
Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*
b^(5/6))
```

Rubi [A] time = 0.686408, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\right.}{18a^{13/6}b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] -(7*A*b - a*B)/(3*a^2*b*Sqrt[x]) + (A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3))
+ ((7*A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(13/6
)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(18*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(
9*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*
Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*
b^(5/6))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx=\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}+\frac{\left(\frac{7Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$=-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}-\frac{(7Ab-aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{6a^2}$$

$$=-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}-\frac{(7Ab-aB)\,\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3a^2}$$

$$=-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}-\frac{(7Ab-aB)\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{9a^{13/6}b^{2/3}}-\frac{(7Ab-aB}{}$$

$$=-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}-\frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x}\right.}{12\sqrt{3}a^{13/6}b^{5/6}}$$

$$=-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}-\frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}\right.}{12\sqrt{3}a^{13/6}b^{5/6}}$$

$$=-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}+\frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{}}{}\right.}{18a^{13/6}b^{5/6}}$$

Mathematica [C] time = 0.0737763, size = 70, normalized size = 0.22

$$\frac{2\left(x^3(aB-Ab)\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)-Abx^3\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)-5aA\right)}{5a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] (2*(-5*a*A - A*b*x^3*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (-(A*b
) + a*B)*x^3*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)]))/(5*a^3*Sqrt[x]
)
```

Maple [A] time = 0.042, size = 395, normalized size = 1.2

$$-\frac{Ab}{3\,a^{2}\left(bx^{3}+a\right)}x^{\frac{5}{2}}+\frac{B}{3\,a\left(bx^{3}+a\right)}x^{\frac{5}{2}}-\frac{7\,A}{9\,a^{2}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}-\frac{7\,Ab\sqrt{3}}{36\,a^{3}}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)-\frac{7\,A}{18\,a^{2}}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(5/2)/(b*x^3+a)*A*b+1/3/a*x^(5/2)/(b*x^3+a)*B-7/9/a^2*A/(a/b)^(1
/6)*arctan(x^(1/2)/(a/b)^(1/6))-7/36/a^3*A*b*3^(1/2)*(a/b)^(5/6)*ln(x-3^(1/
2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-7/18/a^2*A/(a/b)^(1/6)*arctan(2*x^(1/2)
/(a/b)^(1/6)-3^(1/2))+7/36/a^3*A*b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(
1/6)*x^(1/2)+(a/b)^(1/3))-7/18/a^2*A/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/
6)+3^(1/2))+1/9/a*B/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/36/a^2*B*3^
(1/2)*(a/b)^(5/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/18/a*B/b/
(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-1/36/a^2*B*3^(1/2)*(a/b)^
(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/18/a*B/b/(a/b)^(1/6)*
arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-2*A/a^2/x^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7451, size = 9007, normalized size = 28.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B
^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*
b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^1
6*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 +
12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^
5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 -
100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*
B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a
^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*
B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249
*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 -
6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 11
7649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*
b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 1
17649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^2
+ 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 16807*
A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 68
60*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^
6*b^6)/(a^13*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*
a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5
 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)) + 4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*
B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(B^5*a
^16*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7
+ 12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*
a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4
- 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*
A*B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6
*a^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^
7*B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 2824752
49*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5
- 6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 +
117649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^
2 + 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 1680
7*A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 -
6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*
A^6*b^6)/(a^13*b^5))^(1/6) + sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^
```

```
4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b
^5 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)) - 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(a^11*b^4*(-(B^6*a^6 - 42*A*B^5*a^5*b
 + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100
842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4*a
^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 168
07*A^5*b^5)*sqrt(x)) + 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b +
735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-a^11*b^4*(-(B^6*a^6 -
 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^
2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a
^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^
4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A
*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2
*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log((B^5*a^16
*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 1
2005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5
*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 1
00842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*B
^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^
6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B
^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*
A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 - 6
860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 117
649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A
^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^
6)/(a^13*b^5))^(2/3)) - (a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 7
35*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*
A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-(B^5*a^16*b^4 - 35*A*B
^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 12005*A^4*B*a^
12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*
B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*B^9*a^9*b + 22
05*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^6*b^4 - 42353
64*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B^3*a^3*b^7 +
259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*A^10*b^10)*x
- (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 - 6860*A^3*B^3*a
^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 117649*A^6*a^9*b
^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^
3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5)
)^(2/3)) + 12*((B*a - 7*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34675, size = 414, normalized size = 1.3

$$\frac{Bax^3 - 7\,Abx^3 - 6\,Aa}{3\left(bx^{\frac{7}{2}} + a\sqrt{x}\right)a^2} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba - 7\,\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba - 7\,\left(ab^5\right)^{\frac{5}{6}}Ab\right)\mathrm{lo}}{36\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*A*a)/((b*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt
(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/
6) + x + (a/b)^(1/3))/(a^3*b^5) + 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^
5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5)
 + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^5) + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5
)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^
5) + 1/9*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/
6))/(a^3*b^5)
```

3.169 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{3Ab-aB}{3a^2bx^{3/2}}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

```
[Out] -(3*A*b - a*B)/(3*a^2*b*x^(3/2)) + (A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3))
- ((3*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Rubi [A] time = 0.0571234, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 325, 329, 275, 205}

$$-\frac{3Ab-aB}{3a^2bx^{3/2}}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] -(3*A*b - a*B)/(3*a^2*b*x^(3/2)) + (A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3))
- ((3*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx &= \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}+\frac{\left(\frac{9Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{3ab}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.116151, size = 79, normalized size = 0.81

$$\frac{\frac{\sqrt{a}\left(-2aA+aBx^3-3Abx^3\right)}{x^{3/2}\left(a+bx^3\right)}+\frac{(aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{b}}}{3a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] ((Sqrt[a]*(-2*a*A - 3*A*b*x^3 + a*B*x^3))/(x^(3/2)*(a + b*x^3)) + ((-3*A*b
+ a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[b])/(3*a^(5/2))
```

Maple [A] time = 0.019, size = 93, normalized size = 1.

$$-\frac{Ab}{3\,a^2\left(bx^3+a\right)}x^{\frac{3}{2}}+\frac{B}{3\,a\left(bx^3+a\right)}x^{\frac{3}{2}}-\frac{Ab}{a^2}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{3\,a}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{2\,A}{3\,a^2}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(3/2)/(b*x^3+a)*A*b+1/3/a*x^(3/2)/(b*x^3+a)*B-1/a^2/(a*b)^(1/2)*
arctan(b*x^(3/2)/(a*b)^(1/2))*A*b+1/3/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^
(1/2))*B-2/3*A/a^2/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92989, size = 498, normalized size = 5.13

$$\left[\frac{\left(\left(Bab - 3\,Ab^{2}\right)x^{5} + \left(Ba^{2} - 3\,Aab\right)x^{2}\right)\sqrt{-ab}\log\left(\frac{bx^{3}+2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^{3}+a}\right) - 2\left(2\,Aa^{2}b - \left(Ba^{2}b - 3\,Aab^{2}\right)x^{3}\right)\sqrt{x}}{6\left(a^{3}b^{2}x^{5} + a^{4}bx^{2}\right)}, \frac{\left(\left(Bab - 3\,Ab^{2}\right)x^{5}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^3
 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a
*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x^5
+ (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a^2*b
 - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10318, size = 89, normalized size = 0.92

$$\frac{(Ba - 3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^{2}} + \frac{Bax^{3} - 3\,Abx^{3} - 2\,Aa}{3\left(bx^{\frac{9}{2}} + ax^{\frac{3}{2}}\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x^
3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)
```

3.170 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}+(11Ab$$

```
[Out] -(11*A*b - 5*a*B)/(15*a^2*b*x^(5/2)) + (A*b - a*B)/(3*a*b*x^(5/2)*(a + b*x^
3)) + ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*
a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(18*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x]
)/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(17/6)*b^(1/6)) - ((11*
A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(1
2*Sqrt[3]*a^(17/6)*b^(1/6))
```

Rubi [A] time = 0.50449, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}+(11Ab$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```


```
[Out] -(11*A*b - 5*a*B)/(15*a^2*b*x^(5/2)) + (A*b - a*B)/(3*a*b*x^(5/2)*(a + b*x^
3)) + ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*
a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(18*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x]
)/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(17/6)*b^(1/6)) - ((11*
A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(1
2*Sqrt[3]*a^(17/6)*b^(1/6))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{\left(\frac{11Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{6a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\,\mathrm{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{17/6}} - \frac{(11\ldots}{\ldots}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\,\mathrm{Subst}\left(\int \frac{\ldots}{\sqrt[3]{a}-\ldots}\right.}{36a^{8\ldots}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\ldots\right.}{12\sqrt{3}a^{17/6}\ldots}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\tan^{-1}\left(\ldots\right.}{18a^{17/6}\ldots}$$

Mathematica [C] time = 0.0860647, size = 74, normalized size = 0.23

$$\frac{5x^3(5aB-11Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+\frac{a\left(-6aA+5aBx^3-11Abx^3\right)}{a+bx^3}}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] ((a*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(a + b*x^3) + 5*(-11*A*b + 5*a*B)*x^
3*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)])/(15*a^3*x^(5/2))
```

Maple [A] time = 0.039, size = 389, normalized size = 1.2

$$-\frac{Ab}{3a^{2}\left(bx^{3}+a\right)}\sqrt{x}+\frac{B}{3a\left(bx^{3}+a\right)}\sqrt{x}-\frac{11\,Ab}{9\,a^{3}}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{11\,Ab\sqrt{3}}{36\,a^{3}}\sqrt[6]{\frac{a}{b}}\ln\left(x-\sqrt{3}\sqrt[6]{\frac{a}{b}}\sqrt{x}+\sqrt[3]{\frac{a}{b}}\right)-\frac{11\,Ab}{18\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(1/2)/(b*x^3+a)*A*b+1/3/a*x^(1/2)/(b*x^3+a)*B-11/9/a^3*A*b*(a/b)
^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(x-3
^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan(2*
x^(1/2)/(a/b)^(1/6)-3^(1/2))-11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)
*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan(2*x^(1/2
)/(a/b)^(1/6)+3^(1/2))+5/9/a^2*B*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-5/
36/a^2*B*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+
5/18/a^2*B*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))+5/36/a^2*B*3^(
1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+5/18/a^2*B*(
a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-2/5*A/a^2/x^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32402, size = 6660, normalized size = 20.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

[Out] -1/180*(20*sqrt(3)*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^
5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a
^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4
*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*
B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 12
1*A^2*b^2)*x + (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B
^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*
B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*
b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500
*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(
-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6
*b^6)/(a^17*b))^(5/6) - sqrt(3)*(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11343
75*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 48
31530*A^5*B*a*b^5 + 1771561*A^6*b^6))/(15625*B^6*a^6 - 206250*A*B^5*a^5*b +
 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^
4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b*x^6 + a^3*x
^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 33275
00*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 177156
1*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6
- 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 +
5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^
(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)
sqrt(x)(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 -
3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1
771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5
*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^
2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*
(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b
+ 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b
^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + sqrt(3)*(1562
5*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*
a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6))
/(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6
b^6)) - 5(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11
34375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 -
 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^6*(-(15625*B^
6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*
b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^1
7*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x + (5*B*a^4 - 11*A*
a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*
b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b

```
^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 5*(a^2*b*x^6 + a^3*x^3)*(-(15625*B
^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3
*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^
17*b))^(1/6)*log(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^
4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5
*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 1
21*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*
B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4
*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 10
*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*
B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A
^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^3*(-(15625*B^6*a^6 - 20
6250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490
375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6
) - (5*B*a - 11*A*b)*sqrt(x)) - 10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 -
 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5
490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(
1/6)*log(-a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b
^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^
5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 12*((5*B
*a - 11*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^6 + a^3*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17935, size = 423, normalized size = 1.33

$$\frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b}-\frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x\right.}{36\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B
*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*(5*(
a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/9*(
5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^
3*b) - 2/5*A/(a^2*x^(5/2))
```

3.171 $\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=104

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}(3aB+Ab)}{12ab^2\left(a+bx^3\right)}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

```
[Out] ((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) - ((A*b + 3*a*B)*x^(3/2))/(12*a
*b^2*(a + b*x^3)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(3/2)*b^(5/2))
```

Rubi [A] time = 0.0571408, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 288, 329, 275, 205}

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}(3aB+Ab)}{12ab^2\left(a+bx^3\right)}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) - ((A*b + 3*a*B)*x^(3/2))/(12*a
*b^2*(a + b*x^3)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(3/2)*b^(5/2))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2} + \frac{\left(\frac{3Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)} + \frac{(Ab+3aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8ab^2}$$

$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)} + \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{4ab^2}$$

$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)} + \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{12ab^2}$$

$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)} + \frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.141612, size = 93, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\sqrt{b}x^{3/2}\left(-3a^2B-ab\left(A+5Bx^3\right)+Ab^2x^3\right)}{\left(a+bx^3\right)^2}+(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - a*b*(A + 5*B*x^3)))/(a +
b*x^3)^2 + (A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(
5/2))
```

Maple [A] time = 0.016, size = 96, normalized size = 0.9

$$\frac{2}{3\left(bx^3+a\right)^2}\left(\frac{Ab-5\,Ba}{8\,ab}x^{\frac{9}{2}}-\frac{Ab+3\,Ba}{8\,b^2}x^{\frac{3}{2}}\right)+\frac{A}{12\,ab}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{4\,b^2}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2/3*(1/8*(A*b-5*B*a)/a/b*x^(9/2)-1/8*(A*b+3*B*a)/b^2*x^(3/2))/(b*x^3+a)^2+1
/12/b/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))*A+1/4/b^2/(a*b)^(1/2)*arc
tan(b*x^(3/2)/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74468, size = 664, normalized size = 6.38

$$\left[-\frac{\left(\left(3\,Bab^2+Ab^3\right)x^6+3\,Ba^3+Aa^2b+2\left(3\,Ba^2b+Aab^2\right)x^3\right)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\left(\left(5\,Ba^2b^2-Aab^3\right)x^4+\left(3\,B\right.\right.}{24\left(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2
*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2*b^
5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a
^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11753, size = 113, normalized size = 1.09

$$\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}ab^2}-\frac{5\,Babx^{\frac{9}{2}}-Ab^2x^{\frac{9}{2}}+3\,Ba^2x^{\frac{3}{2}}+Aabx^{\frac{3}{2}}}{12\left(bx^3+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/12*(5*
B*a*b*x^(9/2) - A*b^2*x^(9/2) + 3*B*a^2*x^(3/2) + A*a*b*x^(3/2))/((b*x^3 +
a)^2*a*b^2)
```

3.172 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=327

$$-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}-\frac{(7aB+5Ab)\tan^{-1}(}{216a^{11/6}b^{1}}$$

```
[Out] ((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) - ((5*A*b + 7*a*B)*Sqrt[x])/(36
*a*b^2*(a + b*x^3)) - ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) - ((5*A*b + 7*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11
/6)*b^(13/6)) + ((5*A*b + 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6))
```

Rubi [A] time = 0.493356, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 288, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}-\frac{(7aB+5Ab)\tan^{-1}(}{216a^{11/6}b^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```


```
[Out] ((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) - ((5*A*b + 7*a*B)*Sqrt[x])/(36
*a*b^2*(a + b*x^3)) - ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) - ((5*A*b + 7*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11
/6)*b^(13/6)) + ((5*A*b + 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} + \frac{\left(\frac{5Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{108a^{11/6}b^2} + \frac{(5Ab}{}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{}}{\sqrt[3]{a}-\sqrt{3}}\right.}{144\sqrt{3}a^{11/6}b}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\right.}{144\sqrt{3}a^{11/6}b^{13/6}}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} - \frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}\right.}{216a^{11/6}b^{13/6}}$$

Mathematica [C] time = 0.0964715, size = 92, normalized size = 0.28

$$\frac{\sqrt{x}\left(a\left(-7a^2B-ab\left(5A+13Bx^3\right)+Ab^2x^3\right)+\left(a+bx^3\right)^2(7aB+5Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{36a^2b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (Sqrt[x]*(a*(-7*a^2*B + A*b^2*x^3 - a*b*(5*A + 13*B*x^3)) + (5*A*b + 7*a*B)
*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^2*b^2*(
a + b*x^3)^2)
```

Maple [A] time = 0.043, size = 416, normalized size = 1.3

$$2\frac{1}{\left(bx^{3}+a\right)^{2}}\left(\frac{\left(Ab-13\,Ba\right)x^{7/2}}{72\,ab}-\frac{\left(5\,Ab+7\,Ba\right)\sqrt{x}}{72\,b^{2}}\right)+\frac{5\,A}{108\,ba^{2}}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{7\,B}{108\,b^{2}a}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)^
2+5/108/b/a^2*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*A+7/108/b^2/a*(a/b)^(
1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B-5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x-3^(
1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-7/432/b^2/a*3^(1/2)*(a/b)^(1/6)*ln(
x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*arctan
(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2*x^(1/2)/
(a/b)^(1/6)-3^(1/2))*B+5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(
1/6)*x^(1/2)+(a/b)^(1/3))*A+7/432/b^2/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a
/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*arctan(2*x^(1/2)/(
a/b)^(1/6)+3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+
3^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56046, size = 6793, normalized size = 20.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(4*sqrt(3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 +
504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4593
75*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
+ 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
+ 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70
*A*B*a*b + 25*A^2*b^2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/6))*a^9*b^11*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*
a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*
b^5 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^
9*b^12)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 + 504210*A*B
^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2
*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*
B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^
2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^4*x^6 + 2
*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2
*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5
*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4
*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^
6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B
*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 90
0375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 13
1250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6))*a^9*b^11*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^9*b^12)*sqrt(x)*(-(117649*B^6*a
^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 +
 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^
(5/6) - sqrt(3)*(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b
^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 +
 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6
 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4
59375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1
/6)*log(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
```

```
 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^
2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5
*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a
^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)) - (a*b^4*x
^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 9003
75*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 1312
50*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(a^4*b^4*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B*a^3*b^2 + 5*A*a^
2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)/(a^11*b^13))^(1/6)) + 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b
^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^13))^(1/6)*log(a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
 + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)
*sqrt(x)) - 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504
210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*
A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*lo
g(-a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2
+ 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 12*((13*B*a*b
- A*b^2)*x^3 + 7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3
*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15888, size = 443, normalized size = 1.35

$$\frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}-\frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\right.}{432\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1/6
)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(
1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(
(sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a*b^5)
^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))
/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)
*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A*b^2*x^(
7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)
```

3.173 $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=327

$$\frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\tan^{-1}\left(\sqrt{\right.}}{216a^{13/6}b^{11/6}}$$

```
[Out] ((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*x^(5/2))/(36
*a^2*b*(a + b*x^3)) - ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13
/6)*b^(11/6)) - ((7*A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13/6)*b^(11/6))
```

Rubi [A] time = 0.597, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\tan^{-1}\left(\sqrt{\right.}}{216a^{13/6}b^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*x^(5/2))/(36
*a^2*b*(a + b*x^3)) - ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13
/6)*b^(11/6)) - ((7*A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13/6)*b^(11/6))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{\left(\frac{7Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{72a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\,\mathrm{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\,\mathrm{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{108a^{13/6}b^{5/3}} + \frac{(7Ab \cdots}{\cdots}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\,\mathrm{Subst}\left(\int \frac{-\sqrt{3}\cdots}{\sqrt[3]{a}-\sqrt{3}\cdots}\right.}{144\sqrt{3}a^{13/6}\cdots}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\cdots\right.}{144\sqrt{3}a^{13/6}b^{11/\cdots}}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} - \frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{\cdots}\right.}{216a^{13/6}b^{11/\cdots}}$$

Mathematica [C] time = 0.0538124, size = 62, normalized size = 0.19

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},3;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)] + (A*b - a*B)
*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3)/a)]))/(5*a^3*b)
```

Maple [A] time = 0.042, size = 411, normalized size = 1.3

$$2\frac{1}{\left(bx^3+a\right)^2}\left(\frac{(7\,Ab+5\,Ba)\,x^{11/2}}{72\,a^2}+\frac{(13\,Ab-Ba)\,x^{5/2}}{72\,ab}\right)+\frac{7\,A}{108\,a^2b}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}+\frac{5\,B}{108\,ab^2}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2*(1/72*(7*A*b+5*B*a)/a^2*x^(11/2)+1/72*(13*A*b-B*a)/a/b*x^(5/2))/(b*x^3+a)
^2+7/108/a^2/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*A+5/108/a/b^2/(a/b)^
(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B+7/432/a^3*3^(1/2)*(a/b)^(5/6)*ln(x-3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+5/432/a^2/b*3^(1/2)*(a/b)^(5/6)*ln(x
-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+7/216/a^2/b/(a/b)^(1/6)*arctan(
2*x^(1/2)/(a/b)^(1/6)-3^(1/2))*A+5/216/a/b^2/(a/b)^(1/6)*arctan(2*x^(1/2)/(
a/b)^(1/6)-3^(1/2))*B-7/432/a^3*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6
)*x^(1/2)+(a/b)^(1/3))*A-5/432/a^2/b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)+(a/b)^(1/3))*B+7/216/a^2/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b
)^(1/6)+3^(1/2))*A+5/216/a/b^2/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(
1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.24809, size = 10242, normalized size = 31.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/432*(4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9003
75*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*sqrt((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 6
1250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 +
16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 13
6718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^
3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*
A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8
+ 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 1
31250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 +
 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(
-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*
B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)
/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^7
*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*b^5
+ 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 13125
0*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^
4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) + s
qrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^
6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A
^6*b^6)) + 4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6
 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9
00375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10
 + 61250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^1
3 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10
+ 136718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^
7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431
250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*
b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7
 + 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^
10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^1
3)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*
A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*
b^6)/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
```

4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5
*a^7*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*
b^5 + 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
 - sqrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8
57500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 11764
9*A^6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 1176
49*A^6*b^6)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
*log(a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^
2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 +
117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61
250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5
*b^5)*sqrt(x)) + 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900
375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/
6)*log(-a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5
 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b +
 61250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*
A^5*b^5)*sqrt(x)) - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6
+ 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90
0375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1
/6)*log((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 61250*A^2*B^3*a^14*b^1
1 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 + 16807*A^5*a^11*b^14)*
sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85
7500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649
*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 136718750*A*B^9*a^9*b +
 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^3 + 7878281250*A^4*B^
6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*A^6*B^4*a^4*b^6 + 123
53145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8 + 2017680350*A^9*B*a*
b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 131250*A*B^5*a^14*b^8
+ 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 + 900375*A^4*B^2*a^11*
b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(-(15625*B^6*a^6 + 131
250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*
A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(2/3))
+ (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5
*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b
^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(-(3125*B^5
*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 61250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2
*a^13*b^12 + 60025*A^4*B*a^12*b^13 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*
B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*

b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b
^11))^(5/6) + (9765625*B^10*a^10 + 136718750*A*B^9*a^9*b + 861328125*A^2*B^
8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235
512500*A^5*B^5*a^5*b^5 + 15441431250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*
a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A
^10*b^10)*x - (15625*B^6*a^15*b^7 + 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*
a^13*b^9 + 857500*A^3*B^3*a^12*b^10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5
*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b +
459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 +
504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(2/3)) - 12*((5*B*a*b +
7*A*b^2)*x^5 - (B*a^2 - 13*A*a*b)*x^2)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^
3 + a^4*b)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] Timed out

Giac [A] time = 1.295, size = 443, normalized size = 1.35

$$\frac{5\,Babx^{\frac{11}{2}}+7\,Ab^{2}x^{\frac{11}{2}}-Ba^{2}x^{\frac{5}{2}}+13\,Aabx^{\frac{5}{2}}}{36\,(bx^{3}+a)^{2}a^{2}b}-\frac{\sqrt{3}\left(5\,(ab^{5})^{\frac{5}{6}}Ba+7\,(ab^{5})^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^{3}b^{6}}+\sqrt{3}\Big(5\,(ab^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/36*(5*B*a*b*x^(11/2) + 7*A*b^2*x^(11/2) - B*a^2*x^(5/2) + 13*A*a*b*x^(5/2
))/((b*x^3 + a)^2*a^2*b) - 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(
5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/
432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/216*(5*(a*b^5)^(5/6)*B*a + 7

```
*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(
a^3*b^6) + 1/216*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*arctan(-(sqrt(
3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^6) + 1/108*(5*(a*b^5)^(5/6)
*B*a + 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b^6)
```

3.174 $\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=104

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}+\frac{x^{3/2}(aB+3Ab)}{12a^2b\left(a+bx^3\right)}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

```
[Out] ((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*x^(3/2))/(12*a
^2*b*(a + b*x^3)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0595053, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 290, 329, 275, 205}

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}+\frac{x^{3/2}(aB+3Ab)}{12a^2b\left(a+bx^3\right)}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*x^(3/2))/(12*a
^2*b*(a + b*x^3)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(5/2)*b^(3/2))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2} + \frac{\left(\frac{9Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2} + \frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)} + \frac{(3Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2} + \frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{4a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2} + \frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{12a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2} + \frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)} + \frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

Mathematica [A] time = 0.0730823, size = 94, normalized size = 0.9

$$\frac{\frac{\sqrt{a}\sqrt{b}x^{3/2}\left(-a^2B+ab\left(5A+Bx^3\right)+3Ab^2x^3\right)}{\left(a+bx^3\right)^2}+(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-(a^2*B) + 3*A*b^2*x^3 + a*b*(5*A + B*x^3)))/(a
+ b*x^3)^2 + (3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b
^(3/2))
```

Maple [A] time = 0.014, size = 97, normalized size = 0.9

$$\frac{2}{3\left(bx^3+a\right)^2}\left(\frac{3\,Ab+Ba}{8\,a^2}x^{\frac{9}{2}}+\frac{5\,Ab-Ba}{8\,ab}x^{\frac{3}{2}}\right)+\frac{A}{4\,a^2}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{B}{12\,ab}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x)
```

```
[Out] 2/3*(1/8*(3*A*b+B*a)/a^2*x^(9/2)+1/8*(5*A*b-B*a)/a/b*x^(3/2))/(b*x^3+a)^2+1
/4/a^2/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))*A+1/12/a/b/(a*b)^(1/2)*arc
tan(b*x^(3/2)/(a*b)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11275, size = 664, normalized size = 6.38

$$\left[-\frac{\left(\left(Bab^2+3\,Ab^3\right)x^6+Ba^3+3\,Aa^2b+2\left(Ba^2b+3\,Aab^2\right)x^3\right)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\left(\left(Ba^2b^2+3\,Aab^3\right)x^4-\left(Ba\right.\right.}{24\left(a^3b^4x^6+2\,a^4b^3x^3+a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2
*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 +
3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a
^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13459, size = 113, normalized size = 1.09

$$\frac{(Ba + 3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}a^2b} + \frac{Babx^{\frac{9}{2}} + 3\,Ab^2x^{\frac{9}{2}} - Ba^2x^{\frac{3}{2}} + 5\,Aabx^{\frac{3}{2}}}{12\,\left(bx^3 + a\right)^2 a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/12*(B*
a*b*x^(9/2) + 3*A*b^2*x^(9/2) - B*a^2*x^(3/2) + 5*A*a*b*x^(3/2))/((b*x^3 +
a)^2*a^2*b)
```

3.175 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=321

$$-\frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}-\frac{5(aB+11Ab)\tan^{-}}{216a^{17/}}$$

```
[Out] ((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*Sqrt[x])/(36*
a^2*b*(a + b*x^3)) - (5*(11*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17
/6)*b^(7/6)) + (5*(11*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6))
```

Rubi [A] time = 0.522086, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}-\frac{5(aB+11Ab)\tan^{-}}{216a^{17/}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```


```
[Out] ((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*Sqrt[x])/(36*
a^2*b*(a + b*x^3)) - (5*(11*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17
/6)*b^(7/6)) + (5*(11*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6))
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)^2}\,dx}{12ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{(5(11Ab+aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{(5(11Ab+aB))\,\mathrm{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{(5(11Ab+aB))\,\mathrm{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{108a^{17/6}b} + \frac{(5(1\cdots}{\cdots}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{(5(11Ab+aB))\,\mathrm{Subst}\left(\int \frac{-\cdots}{\sqrt[3]{a}-\cdots}\cdots\right)}{144\sqrt{3}a^{17/\cdots}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{\cdots}\right.}{144\sqrt{3}a^{17/6}b^{7/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} - \frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} + \frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{\cdots}\right.}{216a^{17/6}b^{7/6}}$$

Mathematica [C] time = 0.0439971, size = 91, normalized size = 0.28

$$\frac{\sqrt{x}\left(a\left(-5a^2B+ab\left(17A+Bx^3\right)+11Ab^2x^3\right)+5\left(a+bx^3\right)^2(aB+11Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{36a^3b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] (Sqrt[x]*(a*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3)) + 5*(11*A*b + a*
B)*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^3*b*(
a + b*x^3)^2)
```

Maple [A] time = 0.041, size = 401, normalized size = 1.3

$$2\frac{1}{\left(bx^{3}+a\right)^{2}}\left(\frac{(11\,Ab+Ba)\,x^{7/2}}{72\,a^{2}}+\frac{(17\,Ab-5\,Ba)\,\sqrt{x}}{72\,ab}\right)+\frac{55\,A}{108\,a^{3}}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)+\frac{5\,B}{108\,a^{2}b}\sqrt[6]{\frac{a}{b}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x)
```

```
[Out] 2*(1/72*(11*A*b+B*a)/a^2*x^(7/2)+1/72*(17*A*b-5*B*a)/a/b*x^(1/2))/(b*x^3+a)
^2+55/108/a^3*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*A+5/108/a^2/b*(a/b)^(
1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B-55/432/a^3*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-5/432/a^2/b*3^(1/2)*(a/b)^(1/6)*ln(x
-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+55/216/a^3*(a/b)^(1/6)*arctan(2
*x^(1/2)/(a/b)^(1/6)-3^(1/2))*A+5/216/a^2/b*(a/b)^(1/6)*arctan(2*x^(1/2)/(a
/b)^(1/6)-3^(1/2))*B+55/432/a^3*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6
)*x^(1/2)+(a/b)^(1/3))*A+5/432/a^2/b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)+(a/b)^(1/3))*B+55/216/a^3*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)
^(1/6)+3^(1/2))*A+5/216/a^2/b*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1
/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.61839, size = 6384, normalized size = 19.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*(20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B
^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^
2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2
+ 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 177
1561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B*a*b + 121*A^2*b^2)*x +
(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*(-(B^6*a^6 + 66*A*B^5*a^
5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4
 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - 2*sqrt(3)*(B*a
^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) + sqrt(3)*(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^
2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5
*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3
*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b
 + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B
*a*b + 121*A^2*b^2)*x - (B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*
(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3
+ 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7)
)^(5/6) - 2*sqrt(3)*(B*a^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - sqrt(3)*
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 +
66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*
B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 5*(a^2*b^3*x^6 + 2*a
^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26
620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561
*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 18
15*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 96630
6*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*a^2 + 22*A*B*a
*b + 121*A^2*b^2)*x + 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A
*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*
```

```
a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)) - 5*(a^2
*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^
4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a
*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*
a^2 + 22*A*B*a*b + 121*A^2*b^2)*x - 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(
B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2
19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(
1/6)) + 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*a^3*b*(-(B^6
*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2196
15*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6
) + 5*(B*a + 11*A*b)*sqrt(x)) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^
(1/6)*log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 266
20*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b^7))^(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) + 12*((B*a*b + 11*A*
b^2)*x^3 - 5*B*a^2 + 17*A*a*b)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*
b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20399, size = 435, normalized size = 1.36

$$\frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2}-\frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+\right.}{432\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 5/432*sqrt(3)*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^2) - 5/432*sqrt(3)*((a*b^5)^(1/6)*B
*a + 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b^2) + 5/216*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^2) + 5/216*((a*b^5)^(1/6
)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/
b)^(1/6))/(a^3*b^2) + 5/108*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arct
an(sqrt(x)/(a/b)^(1/6))/(a^3*b^2) + 1/36*(B*a*b*x^(7/2) + 11*A*b^2*x^(7/2)
- 5*B*a^2*sqrt(x) + 17*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^2*b)
```

3.176 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=351

$$-\frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\tan^{-}}{216a^{19/}}$$

```
[Out] (-7*(13*A*b - a*B))/(36*a^3*b*Sqrt[x]) + (A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*
x^3)^2) + (13*A*b - a*B)/(36*a^2*b*Sqrt[x]*(a + b*x^3)) + (7*(13*A*b - a*B)
*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7
*(13*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6
)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(1
9/6)*b^(5/6)) - (7*(13*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqr
t[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6)) + (7*(13*A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6
)*b^(5/6))
```

Rubi [A] time = 0.616117, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\tan^{-}}{216a^{19/}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```


```
[Out] (-7*(13*A*b - a*B))/(36*a^3*b*Sqrt[x]) + (A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*
x^3)^2) + (13*A*b - a*B)/(36*a^2*b*Sqrt[x]*(a + b*x^3)) + (7*(13*A*b - a*B)
*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7
*(13*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6
)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(1
9/6)*b^(5/6)) - (7*(13*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqr
t[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6)) + (7*(13*A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6
)*b^(5/6))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{\left(\frac{13Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} + \frac{(7(13Ab-aB))\int \frac{1}{x^{3/2}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{(7(13Ab-aB))\int \frac{x^{3/2}}{a+bx^3}\,dx}{72a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{(7(13Ab-aB))\,\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x\right.}{36a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{(7(13Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{}}\right.}{108a^{19/6}b^{2/3}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} - (7(13$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} - 7(13$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} + \frac{7(13Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}$$

Mathematica [C] time = 0.0895798, size = 113, normalized size = 0.32

$$2\left(-\frac{x^{5/2}(Ab-aB)\,{}_2F_1\left(\frac{5}{6},3;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4} - \frac{Abx^{5/2}\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4} - \frac{Abx^{5/2}\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4} - \frac{A}{a^3\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] 2*(-(A/(a^3*Sqrt[x])) - (A*b*x^(5/2)*Hypergeometric2F1[5/6, 1, 11/6, -((b*x
^3)/a)])/(5*a^4) - (A*b*x^(5/2)*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a
)])/(5*a^4) - ((A*b - a*B)*x^(5/2)*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3
)/a)])/(5*a^4))
```

Maple [A] time = 0.045, size = 435, normalized size = 1.2

$$-\frac{19\,Ab^{2}}{36\,a^{3}\left(bx^{3}+a\right)^{2}}x^{\frac{11}{2}}+\frac{7\,Bb}{36\,a^{2}\left(bx^{3}+a\right)^{2}}x^{\frac{11}{2}}-\frac{25\,Ab}{36\,a^{2}\left(bx^{3}+a\right)^{2}}x^{\frac{5}{2}}+\frac{13\,B}{36\,a\left(bx^{3}+a\right)^{2}}x^{\frac{5}{2}}-\frac{91\,A}{108\,a^{3}}\arctan\left(\sqrt{x}\frac{1}{\sqrt[6]{\frac{a}{b}}}\right)\frac{1}{\sqrt[6]{\frac{a}{b}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x)
```

```
[Out] -19/36/a^3/(b*x^3+a)^2*x^(11/2)*A*b^2+7/36/a^2/(b*x^3+a)^2*x^(11/2)*b*B-25/
36/a^2/(b*x^3+a)^2*A*x^(5/2)*b+13/36/a/(b*x^3+a)^2*B*x^(5/2)-91/108/a^3*A/(
a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-91/432/a^4*A*b*3^(1/2)*(a/b)^(5/6)*l
n(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-91/216/a^3*A/(a/b)^(1/6)*arcta
n(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))+91/432/a^4*A*b*3^(1/2)*(a/b)^(5/6)*ln(x+3^
(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-91/216/a^3*A/(a/b)^(1/6)*arctan(2*x^
(1/2)/(a/b)^(1/6)+3^(1/2))+7/108/a^2*B/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(
1/6))+7/432/a^3*B*3^(1/2)*(a/b)^(5/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b
)^(1/3))+7/216/a^2*B/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-7/
432/a^3*B*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))
+7/216/a^2*B/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-2*A/a^3/x^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.899, size = 9719, normalized size = 27.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5
*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*
b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^6 -
21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*sqrt(
x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b
^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*
b^5))^(5/6) + (B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8*b^2 - 263640*
A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*a^5*b^5 + 1013
629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 36707882445*A^8*B^2*a
^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*x - (B^6*a^19*b
^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 4
28415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(
B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 4
28415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^
(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3
*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b
^6)/(a^19*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*
B^3*a^6*b^3 - 21970*A^3*B^2*a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3
*b^6)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a
^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b
^5 + 4826809*A^6*b^6))/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 4
3940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48268
09*A^6*b^6)) + 28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 -
78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*
B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arct
an(1/50421*(2*sqrt(3)*sqrt(-282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1
690*A^2*B^3*a^19*b^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371
293*A^5*a^16*b^9)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^
2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 +
4826809*A^6*b^6)/(a^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b
 + 7605*A^2*B^8*a^8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4
- 93565836*A^5*B^5*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^
3*a^3*b^7 + 36707882445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858
```

```
491849*A^10*b^10)*x - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^
2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758
*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*
A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*
A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B
^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^
2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 33614*sq
rt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*B^3*a^6*b^3 - 21970*A^3*B^2*
a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3*b^6)*sqrt(x)*(-(B^6*a^6 - 7
8*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B
^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 168
07*sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3
*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6))
/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 +
 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)) - 14*(a^3
*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*
a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*
b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807*a^16*b^4*(-(B^6*a^6 - 78
*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^
2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 1680
7*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3
+ 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 14*(a^3*b^2*x^7 + 2*a^4*b
*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6)*log(-16807*a^16*b^4*(-(B^6*a^6 - 78*A*B^5*a^5*b + 25
35*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 22277
58*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A
*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*
b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B
^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 42
8415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(
1/6)*log(282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^
6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*s
qrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a
^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a
^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8
*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*
a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 3670788
2445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*x
 - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 43
940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 + 4
826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 4
3940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48268
09*A^6*b^6)/(a^19*b^5))^(2/3)) - 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B
^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 42
```

```
8415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(
1/6)*log(-282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b
^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*
sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*
a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(
a^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^
8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5
*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 367078
82445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*
x - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 4
3940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 +
4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 -
43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826
809*A^6*b^6)/(a^19*b^5))^(2/3)) + 12*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2
- 13*A*a*b)*x^3 - 72*A*a^2)*sqrt(x))/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40314, size = 444, normalized size = 1.26

$$-\frac{2A}{a^3\sqrt{x}}+\frac{7\,Babx^{\frac{11}{2}}-19\,Ab^2x^{\frac{11}{2}}+13\,Ba^2x^{\frac{5}{2}}-25\,Aabx^{\frac{5}{2}}}{36\left(bx^3+a\right)^2a^3}-\frac{7\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-13\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{432\,a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -2*A/(a^3*sqrt(x)) + 1/36*(7*B*a*b*x^(11/2) - 19*A*b^2*x^(11/2) + 13*B*a^2*
x^(5/2) - 25*A*a*b*x^(5/2))/((b*x^3 + a)^2*a^3) - 7/432*sqrt(3)*((a*b^5)^(5
```

```
/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a^4*b^5) + 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b
)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) + 7/216*((a
*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqr
t(x))/(a/b)^(1/6))/(a^4*b^5) + 7/216*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^4*b^5) + 7/1
08*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(
a^4*b^5)
```

3.177 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=130

$$\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{5Ab-aB}{4a^3bx^{3/2}}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

```
[Out] -(5*A*b - a*B)/(4*a^3*b*x^(3/2)) + (A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)^2
) + (5*A*b - a*B)/(12*a^2*b*x^(3/2)*(a + b*x^3)) - ((5*A*b - a*B)*ArcTan[(S
qrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])
```

Rubi [A] time = 0.0718134, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 290, 325, 329, 275, 205}

$$\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{5Ab-aB}{4a^3bx^{3/2}}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] -(5*A*b - a*B)/(4*a^3*b*x^(3/2)) + (A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)^2
) + (5*A*b - a*B)/(12*a^2*b*x^(3/2)*(a + b*x^3)) - ((5*A*b - a*B)*ArcTan[(S
qrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{\left(\frac{15Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} + \frac{(3(5Ab-aB))\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{8a^2b}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(3(5Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(3(5Ab-aB))\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right.}{4a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(5Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{4a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.153326, size = 102, normalized size = 0.78

$$\frac{a^2\left(5Bx^3-8A\right)+a\left(3bBx^6-25Abx^3\right)-15Ab^2x^6}{12a^3x^{3/2}\left(a+bx^3\right)^2} + \frac{(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] (-15*A*b^2*x^6 + a^2*(-8*A + 5*B*x^3) + a*(-25*A*b*x^3 + 3*b*B*x^6))/(12*a^
3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]
])/(4*a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.02, size = 133, normalized size = 1.

$$-\frac{7\,Ab^2}{12\,a^3\left(bx^3+a\right)^2}x^{\frac{9}{2}} + \frac{Bb}{4\,a^2\left(bx^3+a\right)^2}x^{\frac{9}{2}} - \frac{3\,Ab}{4\,a^2\left(bx^3+a\right)^2}x^{\frac{3}{2}} + \frac{5\,B}{12\,a\left(bx^3+a\right)^2}x^{\frac{3}{2}} - \frac{5\,Ab}{4\,a^3}\arctan\left(bx^{\frac{3}{2}}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} + \frac{B}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x)
```

```
[Out] -7/12/a^3/(b*x^3+a)^2*x^(9/2)*A*b^2+1/4/a^2/(b*x^3+a)^2*x^(9/2)*b*B-3/4/a^2
/(b*x^3+a)^2*A*x^(3/2)*b+5/12/a/(b*x^3+a)^2*B*x^(3/2)-5/4/a^3/(a*b)^(1/2)*a
rctan(b*x^(3/2)/(a*b)^(1/2))*A*b+1/4/a^2/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)
^(1/2))*B-2/3*A/a^3/x^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93026, size = 738, normalized size = 5.68

$$\left[\frac{3\left(\left(Bab^2-5\,Ab^3\right)x^8+2\left(Ba^2b-5\,Aab^2\right)x^5+\left(Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^3+2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\left(3\left(Ba^2b^2-5\,Aab^3\right)x^6\cdots}{24\left(a^4b^3x^8+2\,a^5b^2x^5+a^6bx^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 -
5*A*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 +
a)) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b
^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((B*a*
b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8
*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2
*x^5 + a^6*b*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11905, size = 119, normalized size = 0.92

$$\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}-\frac{2\,A}{3\,a^3x^{\frac{3}{2}}}+\frac{3\,Babx^{\frac{9}{2}}-7\,Ab^2x^{\frac{9}{2}}+5\,Ba^2x^{\frac{3}{2}}-9\,Aabx^{\frac{3}{2}}}{12\,\left(bx^3+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(B*a - 5*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/3*A/(a^3*
x^(3/2)) + 1/12*(3*B*a*b*x^(9/2) - 7*A*b^2*x^(9/2) + 5*B*a^2*x^(3/2) - 9*A*
a*b*x^(3/2))/((b*x^3 + a)^2*a^3)
```

3.178 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=351

$$\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\log\left(\sqrt{3}\right.}{144\sqrt{3}a^{2}}$$

```
[Out] (-11*(17*A*b - 5*a*B))/(180*a^3*b*x^(5/2)) + (A*b - a*B)/(6*a*b*x^(5/2)*(a
+ b*x^3)^2) + (17*A*b - 5*a*B)/(36*a^2*b*x^(5/2)*(a + b*x^3)) + (11*(17*A*b
 - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1
/6)) - (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/(108*a^(23/6)*b^(1/6)) + (11*(17*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6)) - (11*(
17*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])
/(144*Sqrt[3]*a^(23/6)*b^(1/6))
```

Rubi [A] time = 0.541375, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 209, 634, 618, 204, 628, 205}

$$\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\log\left(\sqrt{3}\right.}{144\sqrt{3}a^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] (-11*(17*A*b - 5*a*B))/(180*a^3*b*x^(5/2)) + (A*b - a*B)/(6*a*b*x^(5/2)*(a
+ b*x^3)^2) + (17*A*b - 5*a*B)/(36*a^2*b*x^(5/2)*(a + b*x^3)) + (11*(17*A*b
 - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1
/6)) - (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/(108*a^(23/6)*b^(1/6)) + (11*(17*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6)) - (11*(
17*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])
/(144*Sqrt[3]*a^(23/6)*b^(1/6))
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{\left(\frac{17Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} + \frac{(11(17Ab-5aB))\int \frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} - \frac{(11(17Ab-5aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72a^3}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} - \frac{(11(17Ab-5aB))\,\mathrm{Subst}\left(\int \frac{1}{a+bx}\right.}{36a^3}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} - \frac{(11(17Ab-5aB))\,\mathrm{Subst}\left(\int \frac{}{\sqrt[3]{a}-}\right.}{108a^{23/6}}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} - \frac{11(17Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}} -$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} - \frac{11(17Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}} +$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} + \frac{11(17Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{}}{}\right.}{216a^{23/6}\sqrt[6]{b}}$$

Mathematica [C] time = 0.110109, size = 96, normalized size = 0.27

$$\frac{\frac{a\left(a^2\left(85Bx^3-72A\right)+a\left(55bBx^6-289Abx^3\right)-187Ab^2x^6\right)}{\left(a+bx^3\right)^2}+55x^3(5aB-17Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)}{180a^4x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] ((a*(-187*A*b^2*x^6 + a^2*(-72*A + 85*B*x^3) + a*(-289*A*b*x^3 + 55*b*B*x^6
)))/(a + b*x^3)^2 + 55*(-17*A*b + 5*a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6,
```

```
-((b*x^3)/a)])/(180*a^4*x^(5/2))
```

Maple [A] time = 0.043, size = 429, normalized size = 1.2

$$-\frac{23\,A b^{2}}{36\,a^{3}\left(b x^{3}+a\right)^{2}} x^{\frac{7}{2}}+\frac{11\,B b}{36\,a^{2}\left(b x^{3}+a\right)^{2}} x^{\frac{7}{2}}-\frac{29\,A b}{36\,a^{2}\left(b x^{3}+a\right)^{2}} \sqrt{x}+\frac{17\,B}{36\,a\left(b x^{3}+a\right)^{2}} \sqrt{x}-\frac{187\,A b}{108\,a^{4}} \sqrt[6]{\frac{a}{b}} \arctan\left(\sqrt{x} \frac{1}{\sqrt[6]{\frac{a}{b}}}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x)
```

```
[Out] -23/36/a^3/(b*x^3+a)^2*x^(7/2)*A*b^2+11/36/a^2/(b*x^3+a)^2*x^(7/2)*b*B-29/3
6/a^2/(b*x^3+a)^2*A*x^(1/2)*b+17/36/a/(b*x^3+a)^2*B*x^(1/2)-187/108/a^4*A*b
*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+187/432/a^4*A*b*3^(1/2)*(a/b)^(1/6
)*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1/6)
*arctan(2*x^(1/2)/(a/b)^(1/6)-3^(1/2))-187/432/a^4*A*b*3^(1/2)*(a/b)^(1/6)*
ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1/6)*a
rctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+55/108/a^3*B*(a/b)^(1/6)*arctan(x^(1/2
)/(a/b)^(1/6))-55/432/a^3*B*3^(1/2)*(a/b)^(1/6)*ln(x-3^(1/2)*(a/b)^(1/6)*x^
(1/2)+(a/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)-3^
(1/2))+55/432/a^3*B*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a
/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))-2
/5*A/a^3/x^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40146, size = 7052, normalized size = 20.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2160*(220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6
 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3
+ 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23
*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a
^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^
2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + (25*
B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x + (5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(
15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3
*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A
^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 270
9375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
- 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt(3)*(5*B
*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27
09375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) - sqrt(3)*(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 122825
00*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 2413
7569*A^6*b^6)) + 220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625
*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*
a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^
6)/(a^23*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750
*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132037
5*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3
) + (25*B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x - (5*B*a^5 - 17*A*a^4*b)*sqr
t(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 1228
2500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24
137569*A^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5
*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*
a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt
(3)*(5*B*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^
5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2
*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + sqrt(
3)*(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500
*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 241375
69*A^6*b^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2
- 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^
5 + 24137569*A^6*b^6)) - 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
```

```
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27093
75*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 -
42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2
- 170*A*B*a*b + 289*A^2*b^2)*x + 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6)/(a^23*b))^(1/6)) + 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^
6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3
*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/
(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375
*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42
595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2 -
170*A*B*a*b + 289*A^2*b^2)*x - 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)) + 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6
*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*
b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(
a^23*b))^(1/6)*log(11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A
^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 4259
5710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*
sqrt(x)) - 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318
750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132
0375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(
1/6)*log(-11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^
4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*
B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x))
- 12*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2)
*sqrt(x))/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17512, size = 451, normalized size = 1.28

$$\frac{11\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba - 17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b} - \frac{11\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba - 17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right.\right.}{432\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqr
t(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(1/
6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arcta
n((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(a*b^5
)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x
))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*
b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 23*A*b^2*
x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3) - 2/5*A/
(a^3*x^(5/2))
```

3.179 $\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=103

$$\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b
*x^3)^(5/2))/(15*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*B
*(a + b*x^3)^(9/2))/(27*b^4)
```

Rubi [A] time = 0.08424, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b
*x^3)^(5/2))/(15*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*B
*(a + b*x^3)^(9/2))/(27*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx}(A+Bx)\,dx,x,x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}+\frac{(Ab-3aB)(a+bx)}{b^3}\right.\right.$$
$$= \frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2}{}$$

Mathematica [A] time = 0.0554779, size = 75, normalized size = 0.73

$$\frac{2\left(a+bx^3\right)^{3/2}\left(24a^2b\left(A+Bx^3\right)-16a^3B-6ab^2x^3\left(6A+5Bx^3\right)+5b^3x^6\left(9A+7Bx^3\right)\right)}{945b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A +
 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)
```

Maple [A] time = 0.007, size = 77, normalized size = 0.8

$$\frac{70\,Bx^9b^3+90\,Ab^3x^6-60\,Bab^2x^6-72\,Aab^2x^3+48\,Ba^2bx^3+48\,Aa^2b-32\,Ba^3}{945\,b^4}\left(bx^3+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/945*(b*x^3+a)^(3/2)*(35*B*b^3*x^9+45*A*b^3*x^6-30*B*a*b^2*x^6-36*A*a*b^2*
x^3+24*B*a^2*b*x^3+24*A*a^2*b-16*B*a^3)/b^4
```

Maxima [A] time = 0.928928, size = 159, normalized size = 1.54

$$\frac{2}{945}B\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(bx^3+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(bx^3+a\right)^{\frac{3}{2}}a^3}{b^4}\right)+\frac{2}{315}A\left(\frac{15\left(bx^3+a\right)^{\frac{7}{2}}}{b^3}-\frac{42\left(bx^3+\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x^
3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^3
+ a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b^3)
```

Fricas [A] time = 1.71534, size = 220, normalized size = 2.14

$$\frac{2\left(35\,Bb^4x^{12}+5\left(Bab^3+9\,Ab^4\right)x^9-3\left(2\,Ba^2b^2-3\,Aab^3\right)x^6-16\,Ba^4+24\,Aa^3b+4\left(2\,Ba^3b-3\,Aa^2b^2\right)x^3\right)\sqrt{bx^3+a}}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*b
^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b*x
^3 + a)/b^4
```

Sympy [A] time = 3.67128, size = 219, normalized size = 2.13

$$\begin{cases}\frac{16Aa^3\sqrt{a+bx^3}}{315b^3}-\frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Aax^6\sqrt{a+bx^3}}{105b}+\frac{2Ax^9\sqrt{a+bx^3}}{21}-\frac{32Ba^4\sqrt{a+bx^3}}{945b^4}+\frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3}-\frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^9\sqrt{a+bx^3}}{189b}+\frac{2Bx}{} \\ \sqrt{a}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a + b
*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a +
```

```
 b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqrt(a
 + b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x
**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (s
qrt(a)*(A*x**9/9 + B*x**12/12), True))
```

Giac [A] time = 1.11406, size = 144, normalized size = 1.4

$$\frac{2\left(\frac{3\left(15\left(bx^3+a\right)^{\frac{7}{2}}-42\left(bx^3+a\right)^{\frac{5}{2}}a+35\left(bx^3+a\right)^{\frac{3}{2}}a^2\right)A}{b^2}+\frac{\left(35\left(bx^3+a\right)^{\frac{9}{2}}-135\left(bx^3+a\right)^{\frac{7}{2}}a+189\left(bx^3+a\right)^{\frac{5}{2}}a^2-105\left(bx^3+a\right)^{\frac{3}{2}}a^3\right)B}{b^3}\right)}{945\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/945*(3*(15*(b*x^3 + a)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)^(3
/2)*a^2)*A/b^2 + (35*(b*x^3 + a)^(9/2) - 135*(b*x^3 + a)^(7/2)*a + 189*(b*x
^3 + a)^(5/2)*a^2 - 105*(b*x^3 + a)^(3/2)*a^3)*B/b^3)/b
```

3.180 $\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)
^(5/2))/(15*b^3) + (2*B*(a + b*x^3)^(7/2))/(21*b^3)
```

Rubi [A] time = 0.0598291, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)
^(5/2))/(15*b^3) + (2*B*(a + b*x^3)^(7/2))/(21*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rubi steps

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)dx=\frac{1}{3}\operatorname{Subst}\left(\int x\sqrt{a+bx}(A+Bx)\,dx,x,x^3\right)$$
$$=\frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2}+\frac{(Ab-2aB)(a+bx)^{3/2}}{b^2}+\frac{B(a+bx)^{5/2}}{b^2}\right)dx,x,x^3\right)$$
$$=-\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

Mathematica [A] time = 0.0387175, size = 57, normalized size = 0.78

$$\frac{2\left(a+bx^3\right)^{3/2}\left(8a^2B-2ab\left(7A+6Bx^3\right)+3b^2x^3\left(7A+5Bx^3\right)\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(8*a^2*B + 3*b^2*x^3*(7*A + 5*B*x^3) - 2*a*b*(7*A + 6*
B*x^3)))/(315*b^3)
```

Maple [A] time = 0.005, size = 53, normalized size = 0.7

$$-\frac{-30\,b^2Bx^6-42\,Ax^3b^2+24\,Bx^3ab+28\,abA-16\,a^2B}{315\,b^3}\left(bx^3+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] -2/315*(b*x^3+a)^(3/2)*(-15*B*b^2*x^6-21*A*b^2*x^3+12*B*a*b*x^3+14*A*a*b-8*
B*a^2)/b^3
```

Maxima [A] time = 0.937005, size = 113, normalized size = 1.55

$$\frac{2}{315}\,B\left(\frac{15\left(bx^3+a\right)^{\frac{7}{2}}}{b^3}-\frac{42\left(bx^3+a\right)^{\frac{5}{2}}a}{b^3}+\frac{35\left(bx^3+a\right)^{\frac{3}{2}}a^2}{b^3}\right)+\frac{2}{45}\,A\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3
+ a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)
*a/b^2)
```

Fricas [A] time = 1.69199, size = 166, normalized size = 2.27

$$\frac{2\left(15\,Bb^3x^9+3\left(Bab^2+7\,Ab^3\right)x^6+8\,Ba^3-14\,Aa^2b-\left(4\,Ba^2b-7\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3
```

Sympy [A] time = 1.61222, size = 168, normalized size = 2.3

$$\begin{cases} -\frac{4Aa^2\sqrt{a+bx^3}}{45b^2}+\frac{2Aax^3\sqrt{a+bx^3}}{45b}+\frac{2Ax^6\sqrt{a+bx^3}}{15}+\frac{16Ba^3\sqrt{a+bx^3}}{315b^3}-\frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^6\sqrt{a+bx^3}}{105b}+\frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x**
3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(315*
b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a + b*x
**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**6/6
+ B*x**9/9), True))
```

Giac [A] time = 1.10447, size = 107, normalized size = 1.47

$$\frac{2\left(\frac{7\left(3\left(bx^3+a\right)^{\frac{5}{2}}-5\left(bx^3+a\right)^{\frac{3}{2}}a\right)A}{b}+\frac{\left(15\left(bx^3+a\right)^{\frac{7}{2}}-42\left(bx^3+a\right)^{\frac{5}{2}}a+35\left(bx^3+a\right)^{\frac{3}{2}}a^2\right)B}{b^2}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(7*(3*(b*x^3 + a)^(5/2) - 5*(b*x^3 + a)^(3/2)*a)*A/b + (15*(b*x^3 + a
)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)^(3/2)*a^2)*B/b^2)/b
```

3.181 $\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=46

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^2) + (2*B*(a + b*x^3)^(5/2))/(15*b^2
)
```

Rubi [A] time = 0.0399213, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^2) + (2*B*(a + b*x^3)^(5/2))/(15*b^2
)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)dx=\frac{1}{3}\text{Subst}\left(\int\sqrt{a+bx}(A+Bx)\,dx,x,x^3\right)$$
$$=\frac{1}{3}\text{Subst}\left(\int\left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)dx,x,x^3\right)$$
$$=\frac{2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

Mathematica [A] time = 0.0229571, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-2aB+5Ab+3bBx^3\right)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

Maple [A] time = 0.007, size = 31, normalized size = 0.7

$$\frac{6\,bBx^3+10\,Ab-4\,Ba}{45\,b^2}\left(bx^3+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/45*(b*x^3+a)^(3/2)*(3*B*b*x^3+5*A*b-2*B*a)/b^2
```

Maxima [A] time = 0.922227, size = 66, normalized size = 1.43

$$\frac{2}{45}B\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}a}{b^2}\right)+\frac{2\left(bx^3+a\right)^{\frac{3}{2}}A}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3 +
 a)^(3/2)*A/b
```

Fricas [A] time = 1.63958, size = 113, normalized size = 2.46

$$\frac{2\left(3\,Bb^2x^6 + \left(Bab + 5\,Ab^2\right)x^3 - 2\,Ba^2 + 5\,Aab\right)\sqrt{bx^3 + a}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3 +
 a)/b^2
```

Sympy [A] time = 0.628639, size = 117, normalized size = 2.54

$$\begin{cases} \frac{2Aa\sqrt{a+bx^3}}{9b} + \frac{2Ax^3\sqrt{a+bx^3}}{9} - \frac{4Ba^2\sqrt{a+bx^3}}{45b^2} + \frac{2Bax^3\sqrt{a+bx^3}}{45b} + \frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*B
*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*B
*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), True)
)
```

Giac [A] time = 1.12564, size = 63, normalized size = 1.37

$$\frac{2\left(5\left(bx^3 + a\right)^{\frac{3}{2}}A + \frac{\left(3\left(bx^3+a\right)^{\frac{5}{2}} - 5\left(bx^3+a\right)^{\frac{3}{2}}a\right)B}{b}\right)}{45\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*(b*x^3 + a)^(3/2)*A + (3*(b*x^3 + a)^(5/2) - 5*(b*x^3 + a)^(3/2)*a)
*B/b)/b
```

3.182 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx$

Optimal. Leaf size=64

$$\frac{2}{3}A\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}$$

```
[Out] (2*A*Sqrt[a + b*x^3])/3 + (2*B*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*A*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rubi [A] time = 0.0431833, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$\frac{2}{3}A\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*A*Sqrt[a + b*x^3])/3 + (2*B*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*A*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx,x,x^3\right)\\
&= \frac{2B\left(a+bx^3\right)^{3/2}}{9b}+\frac{1}{3}A\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}+\frac{1}{3}(aA)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}+\frac{(2aA)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2}{3}A\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}-\frac{2}{3}\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0471751, size = 60, normalized size = 0.94

$$\frac{2}{9}\left(\frac{\sqrt{a+bx^3}\left(B\left(a+bx^3\right)+3Ab\right)}{b}-3\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*((Sqrt[a + b*x^3]*(3*A*b + B*(a + b*x^3)))/b - 3*Sqrt[a]*A*ArcTanh[Sqrt[
a + b*x^3]/Sqrt[a]]))/9
```

Maple [A] time = 0.184, size = 50, normalized size = 0.8

$$\frac{2B}{9b}\left(bx^3+a\right)^{\frac{3}{2}}+A\left(\frac{2}{3}\sqrt{bx^3+a}-\frac{2}{3}\sqrt{a}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x,x)
```

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b+A*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a
)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77271, size = 301, normalized size = 4.7

$$\left[\frac{3\,A\sqrt{ab}\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(Bbx^3+Ba+3\,Ab\right)\sqrt{bx^3+a}}{9\,b},\ \frac{2\left(3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\left(Bbx^3+Ba+3\,Ab\right)\sqrt{bx^3+a}\right)}{9\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/9*(3*A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*
(B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b, 2/9*(3*A*sqrt(-a)*b*arctan(sqrt
(b*x^3 + a)*sqrt(-a)/a) + (B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b]
```

Sympy [A] time = 18.8294, size = 76, normalized size = 1.19

$$-\frac{A\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+bx^3}\right)}{3}-\frac{B\left(\begin{cases}-\sqrt{a}x^3 & \text{for } b=0\\ -\frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -A*(-2*a*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*x**3))/3 -
 B*Piecewise((-sqrt(a)*x**3, Eq(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), True
))/3
```

Giac [A] time = 1.09711, size = 82, normalized size = 1.28

$$\frac{2\,Aa\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^2+3\,\sqrt{bx^3+a}\,Ab^3\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*((b*x^3 + a)^(3/2)*
B*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3
```

3.183 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^3}(2aB+Ab)}{3a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}$$

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(3*a) - (A*(a + b*x^3)^(3/2))/(3*a*x^3) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rubi [A] time = 0.063156, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{a+bx^3}(2aB+Ab)}{3a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(3*a) - (A*(a + b*x^3)^(3/2))/(3*a*x^3) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx,x,x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{(Ab+2aB)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^3\right)}{6a}\\
&= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{1}{6}(Ab+2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{(Ab+2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}\\
&= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}-\frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.0397215, size = 63, normalized size = 0.75

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\left(2Bx^3-A\right)}{x^3}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] ((Sqrt[a + b*x^3]*(-A + 2*B*x^3))/x^3 - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x
^3]/Sqrt[a]])/Sqrt[a])/3
```

Maple [A] time = 0.021, size = 72, normalized size = 0.9

$$A\left(-\frac{1}{3x^3}\sqrt{bx^3+a}-\frac{b}{3}\text{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}\right)+B\left(\frac{2}{3}\sqrt{bx^3+a}-\frac{2}{3}\sqrt{a}\text{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] A*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
+B*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82532, size = 333, normalized size = 3.96

$$\left[\frac{(2\,Ba+Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(2\,Bax^3-Aa\right)\sqrt{bx^3+a}}{6\,ax^3},\ \frac{(2\,Ba+Ab)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\left(2\,Bax\right.}{3\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*
a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b)*
sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*a*x^3 - A*a)*sqrt(b*
x^3 + a))/(a*x^3)]
```

Sympy [A] time = 23.2387, size = 134, normalized size = 1.6

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}-\frac{2B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2B\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*x
**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2
*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3*sq
rt(a/(b*x**3) + 1))
```

Giac [A] time = 1.13968, size = 92, normalized size = 1.1

$$\frac{2\,\sqrt{bx^3+a}Bb+\frac{\left(2\,Bab+Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^3+a}Ab}{x^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(b*x^3 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^3 + a)*A*b/x^3)/b
```

3.184 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7} dx$

Optimal. Leaf size=88

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{\sqrt{a+bx^3}(Ab-4aB)}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}$$

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a*x^3) - (A*(a + b*x^3)^(3/2))/(6*a*x^6
) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rubi [A] time = 0.0696097, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{\sqrt{a+bx^3}(Ab-4aB)}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a*x^3) - (A*(a + b*x^3)^(3/2))/(6*a*x^6
) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx,x,x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}+\frac{\left(-\frac{Ab}{2}+2aB\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx,x,x^3\right)}{6a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}-\frac{(b(Ab-4aB))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{24a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}-\frac{(Ab-4aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{12a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}+\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0683401, size = 93, normalized size = 1.06

$$\frac{-\left(a+bx^3\right)\left(2a\left(A+2Bx^3\right)+Abx^3\right)-bx^6\sqrt{\frac{bx^3}{a}+1}(4aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{12ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] (-((a + b*x^3)*(A*b*x^3 + 2*a*(A + 2*B*x^3))) - b*(-(A*b) + 4*a*B)*x^6*Sqrt
[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]])/(12*a*x^6*Sqrt[a + b*x^3])
```

Maple [A] time = 0.022, size = 96, normalized size = 1.1

$$A\left(-\frac{1}{6x^6}\sqrt{bx^3+a}-\frac{b}{12ax^3}\sqrt{bx^3+a}+\frac{b^2}{12}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}\right)+B\left(-\frac{1}{3x^3}\sqrt{bx^3+a}-\frac{b}{3}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] A*(-1/6*(b*x^3+a)^(1/2)/x^6-1/12*b/a*(b*x^3+a)^(1/2)/x^3+1/12*b^2/a^(3/2)*a
rctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh(
(b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62361, size = 397, normalized size = 4.51

$$\left[-\frac{\left(4\,Bab - Ab^2\right)\sqrt{a}x^6\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right) + 2\left(\left(4\,Ba^2 + Aab\right)x^3 + 2\,Aa^2\right)\sqrt{bx^3 + a}}{24\,a^2x^6}, \frac{\left(4\,Bab - Ab^2\right)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{b}}{\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - A*b^2)*sqrt(a)*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a
) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x
^6), 1/12*((4*B*a*b - A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a
) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

Sympy [B] time = 50.8912, size = 160, normalized size = 1.82

$$-\frac{Aa}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*sqrt(b)/(4*x**(9/2)*sqr
t(a/(b*x**3) + 1)) - A*b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + A*b*
*2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x*
*3) + 1)/(3*x**(3/2)) - B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

Giac [A] time = 1.11189, size = 162, normalized size = 1.84

$$\frac{\frac{\left(4\,Bab^2-Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}Ba^2b^2+\left(bx^3+a\right)^{\frac{3}{2}}Ab^3+\sqrt{bx^3+a}Aab^3}{ab^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - (
4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^(3/
2)*A*b^3 + sqrt(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b
```

3.185 $\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=303

$$-\frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ax\sqrt{a+}}{}$$

```
[Out] (6*a*(17*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(935*b^2) + (2*(17*A*b - 8*a*B)*x^
4*Sqrt[a + b*x^3])/(187*b) + (2*B*x^4*(a + b*x^3)^(3/2))/(17*b) - (4*3^(3/4
)*Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.150994, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 279, 321, 218}

$$-\frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ax\sqrt{a+bx^3}(17}{935b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(17*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(935*b^2) + (2*(17*A*b - 8*a*B)*x^
4*Sqrt[a + b*x^3])/(187*b) + (2*B*x^4*(a + b*x^3)^(3/2))/(17*b) - (4*3^(3/4
)*Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)dx=\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}-\frac{\left(2\left(-\frac{17Ab}{2}+4aB\right)\right)\int x^3\sqrt{a+bx^3}\,dx}{17b}$$

$$=\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}+\frac{(3a(17Ab-8aB))\int\frac{x^3}{\sqrt{a+bx^3}}dx}{187b}$$

$$=\frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2}+\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}-\frac{\left(6a^2(17A\right.}{}$$

$$=\frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2}+\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}-\frac{4\,3^{3/4}\sqrt{2}}{}$$

Mathematica [C] time = 0.132576, size = 89, normalized size = 0.29

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a(8aB-17Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(a+bx^3\right)\left(8aB-17Ab-11bBx^3\right)\right)}{187b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-17*A*b + 8*a*B - 11*b*B*x^3)) + (a*(-
17*A*b + 8*a*B)*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (
b*x^3)/a]))/(187*b^2)
```

Maple [B] time = 0.021, size = 658, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/17*x^7*(b*x^3+a)^(1/2)+6/187*a/b*x^4*(b*x^3+a)^(1/2)-48/935*a^2/b^2*x*
(b*x^3+a)^(1/2)-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)))+A*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a/b*x*(b*x^3+a)^(1/2)
+4/55*I*a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bx^6 + Ax^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.26851, size = 83, normalized size = 0.27

$$\frac{A\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt{a}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + B*sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

3.186 $\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=268

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+b}}{}$$

```
[Out] (2*(11*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(55*b) + (2*B*x*(a + b*x^3)^(3/2))/(
11*b) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*b^(4/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rubi [A] time = 0.0911127, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 218}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+bx^3}(11A}{55b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(11*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(55*b) + (2*B*x*(a + b*x^3)^(3/2))/(
11*b) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*b^(4/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+aB\right)\right)\int\sqrt{a+bx^3}\,dx}{11b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} + \frac{(3a(11Ab-2aB))\int\frac{1}{\sqrt{a+bx^3}}\,dx}{55b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\cdots}{55b^{4/3}\sqrt{\frac{\cdots}{((\cdots}}}$$

Mathematica [C] time = 0.0629002, size = 75, normalized size = 0.28

$$\frac{2x\sqrt{a+bx^3}\left(\frac{(11Ab-2aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}+B\left(a+bx^3\right)\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3) + ((11*A*b - 2*a*B)*Hypergeometric2F1[-
1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(11*b)
```

Maple [B] time = 0.014, size = 618, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/11*x^4*(b*x^3+a)^(1/2)+6/55*a/b*x*(b*x^3+a)^(1/2)+4/55*I*a^2/b^2*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/5*x*(b*x^3+a
)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bx^3 + A\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 1.90089, size = 82, normalized size = 0.31

$$\frac{A\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{B\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + B*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

3.187 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^3}\,dx$

Optimal. Leaf size=269

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x\sqrt{a+bx^3}(4aB}{10a}$$

```
[Out] ((5*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/(10*a) - (A*(a + b*x^3)^(3/2))/(2*a*x^2
) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0994994, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 195, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x\sqrt{a+bx^3}(4aB+5Ab)}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

```
[Out] ((5*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/(10*a) - (A*(a + b*x^3)^(3/2))/(2*a*x^2
) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^3}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{2ax^2} - \frac{\left(-\frac{5Ab}{2}-2aB\right)\int \sqrt{a+bx^3}\,dx}{2a}$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A\left(a+bx^3\right)^{3/2}}{2ax^2} + \frac{1}{20}(3(5Ab+4aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A\left(a+bx^3\right)^{3/2}}{2ax^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.}}}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}$$

Mathematica [C] time = 0.0624979, size = 81, normalized size = 0.3

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3(4aB+5Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-A\left(a+bx^3\right)\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) + ((5*A*b + 4*a*B)*x^3*Hypergeometric2F1
[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x^2)
```

Maple [B] time = 0.02, size = 596, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] B*(2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*(
(x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)))+A*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

Sympy [A] time = 2.2229, size = 85, normalized size = 0.32

$$\frac{A\sqrt{a}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{B\sqrt{a}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] A*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + B*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

3.188 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^6}\,dx$

Optimal. Leaf size=272

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}}{2}$$

```
[Out] ((A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a*x^2) - (A*(a + b*x^3)^(3/2))/(5*a*x^
5) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*a*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.102572, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 277, 218}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}(Ab-1}{20ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] ((A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a*x^2) - (A*(a + b*x^3)^(3/2))/(5*a*x^
5) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*a*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^6}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{5ax^5} - \frac{\left(\frac{Ab}{2}-5aB\right)\int \frac{\sqrt{a+bx^3}}{x^3}\,dx}{5a}$$

$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A\left(a+bx^3\right)^{3/2}}{5ax^5} - \frac{(3b(Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a}$$

$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A\left(a+bx^3\right)^{3/2}}{5ax^5} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.}}}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}$$

Mathematica [C] time = 0.0826477, size = 80, normalized size = 0.29

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{Ab}{2}-5aB\right){}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-2A\left(a+bx^3\right)\right)}{10ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + (((A*b)/2 - 5*a*B)*x^3*Hypergeometric2
F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(10*a*x^5)
```

Maple [B] time = 0.021, size = 616, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x)
```

```
[Out] B*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))+A*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/
x^2+1/20*I*b/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Sympy [A] time = 2.87325, size = 94, normalized size = 0.35

$$\frac{A\sqrt{a}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] A*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + B*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

3.189 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^9}\,dx$

Optimal. Leaf size=305

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-16aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3b\sqrt{a+bx}}{32}$$

```
[Out] ((7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(80*a*x^5) + (3*b*(7*A*b - 16*a*B)*Sqrt[
a + b*x^3])/(320*a^2*x^2) - (A*(a + b*x^3)^(3/2))/(8*a*x^8) + (3^(3/4)*Sqrt
[2 + Sqrt[3]]*b^(5/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.138148, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 277, 325, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3b\sqrt{a+bx^3}(7Ab-}{320a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

```
[Out] ((7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(80*a*x^5) + (3*b*(7*A*b - 16*a*B)*Sqrt[
a + b*x^3])/(320*a^2*x^2) - (A*(a + b*x^3)^(3/2))/(8*a*x^8) + (3^(3/4)*Sqrt
[2 + Sqrt[3]]*b^(5/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^9}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} - \frac{\left(\frac{7Ab}{2}-8aB\right)\int \frac{\sqrt{a+bx^3}}{x^6}\,dx}{8a}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} - \frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} - \frac{(3b(7Ab-16aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{160a}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} + \frac{\left(3b^2(7Ab-16aB)\right)}{640a^2}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}(7}{}$$

Mathematica [C] time = 0.0904265, size = 80, normalized size = 0.26

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{7Ab}{2}-8aB\right){}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-5A\left(a+bx^3\right)\right)}{40ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + (((7*A*b)/2 - 8*a*B)*x^3*Hypergeometri
c2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*a*x^8)
```

Maple [B] time = 0.023, size = 660, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x)
```

```
[Out] A*(-1/8*(b*x^3+a)^(1/2)/x^8-3/80*b/a*(b*x^3+a)^(1/2)/x^5+21/320*b^2/a^2*(b*
x^3+a)^(1/2)/x^2-7/320*I*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
```

```
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)))+B*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b/a*(b*x^3+a)^(1/2)/x
^2+1/20*I*b/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Sympy [A] time = 4.83166, size = 97, normalized size = 0.32

$$\frac{A\sqrt{a}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{B\sqrt{a}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] A*sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**8*gamma(-5/3)) + B*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

3.190 $\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=581

$$-\frac{8\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{24a^2\sqrt{a+\ldots}}{1729b^{8/3}\left(\left(\ldots\right.\right.}$$

```
[Out] (6*a*(19*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2*(19*A*b - 10*a*
B)*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*A*b - 10*a*B)*Sqrt[a + b*x^3]
)/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^5*(a + b*x^3)
^(3/2))/(19*b) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 10*a*B)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 10*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.355192, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 279, 321, 303, 218, 1877}

$$-\frac{24a^2\sqrt{a+bx^3}(19Ab-10aB)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{8\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|\ldots\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(19*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2*(19*A*b - 10*a*
B)*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*A*b - 10*a*B)*Sqrt[a + b*x^3]
```

```
)/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^5*(a + b*x^3)
^(3/2))/(19*b) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 10*a*B)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 10*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b} - \frac{\left(2\left(-\frac{19Ab}{2}+5aB\right)\right)\int x^4\sqrt{a+bx^3}\,dx}{19b}$$

$$= \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b} + \frac{(3a(19Ab-10aB))\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{247b}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b} - \frac{(12a\ldots}{}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b} - \frac{(12a\ldots}{}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} - \frac{24a^2(19Ab-10aB)\sqrt{a\ldots}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\ldots\right.}$$

Mathematica [C] time = 0.131995, size = 91, normalized size = 0.16

$$\frac{2x^2\sqrt{a+bx^3}\left(\frac{a(10aB-19Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(a+bx^3\right)\left(10aB-19Ab-13bBx^3\right)\right)}{247b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-19*A*b + 10*a*B - 13*b*B*x^3)) + (a
*(-19*A*b + 10*a*B)*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(247*b^2)
```

Maple [B] time = 0.021, size = 966, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/19*x^8*(b*x^3+a)^(1/2)+6/247*a/b*x^5*(b*x^3+a)^(1/2)-60/1729*a^2/b^2*x
^2*(b*x^3+a)^(1/2)-80/1729*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))))+A*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a/b*x^2*(b*x^3+a)^(1/2)+8/91*I*a^2/b^
2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
```

```
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bx^7 + Ax^4\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^7 + A*x^4)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.82605, size = 83, normalized size = 0.14

$$\frac{A\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(8/3)) + B*sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

3.191 $\int x\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=548

$$\frac{2\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a\ldots}{\ldots}$$

[Out] (2*(13*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*A*b - 4*a*B)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^2*(a + b*x^3)^(3/2))/(13*b) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.241307, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {459, 279, 303, 218, 1877}

$$\frac{2\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a + b*x^3]*(A + B*x^3),x]

[Out] (2*(13*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*A*b - 4*a*B)*Sqrt[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^2*(

```
a + b*x^3)^(3/2))/(13*b) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b - 4
*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b - 4*a*B)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)dx=\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}-\frac{\left(2\left(-\frac{13Ab}{2}+2aB\right)\right)\int x\sqrt{a+bx^3}\,dx}{13b}$$

$$=\frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}+\frac{(3a(13Ab-4aB))\int\frac{x}{\sqrt{a+bx^3}}\,dx}{91b}$$

$$=\frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}+\frac{(3a(13Ab-4aB))\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{91b^{4/3}}$$

$$=\frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}-\frac{3\sqrt[4]{3}\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 0.0899134, size = 75, normalized size = 0.14

$$\frac{x^2\sqrt{a+bx^3}\left(\frac{(13Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}+4B\left(a+bx^3\right)\right)}{26b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3) + ((13*A*b - 4*a*B)*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(26*b)
```

Maple [B] time = 0.016, size = 926, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/13*x^5*(b*x^3+a)^(1/2)+6/91*a/b*x^2*(b*x^3+a)^(1/2)+8/91*I*a^2/b^2*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bx^4 + Ax\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.04844, size = 83, normalized size = 0.15

$$\frac{A\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + B*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

3.192 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}\,dx$

Optimal. Leaf size=545

$$\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\right.}{}$$

```
[Out] ((7*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/(7*a) + (3*(7*A*b + 2*a*B)*Sqrt[a + b
*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/
2))/(a*x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*3^(3/4)*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rubi [A] time = 0.24853, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 279, 303, 218, 1877}

$$\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] ((7*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/(7*a) + (3*(7*A*b + 2*a*B)*Sqrt[a + b
*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/
```

```
2))/(a*x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*3^(3/4)*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{ax} - \frac{\left(-\frac{7Ab}{2}-aB\right)\int x\sqrt{a+bx^3}\,dx}{a}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A\left(a+bx^3\right)^{3/2}}{ax} + \frac{1}{14}(3(7Ab+2aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A\left(a+bx^3\right)^{3/2}}{ax} + \frac{(3(7Ab+2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{14\sqrt[3]{b}} + \frac{\left(3\sqrt{\frac{1}{2}\left(2\right.}\right.}{}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} + \frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A\left(a+bx^3\right)^{3/2}}{ax} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}}{}$$

Mathematica [C] time = 0.0683148, size = 81, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3(2aB+7Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-2A\left(a+bx^3\right)\right)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + ((7*A*b + 2*a*B)*x^3*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x)
```

Maple [B] time = 0.017, size = 902, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] B*(2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))))+A*(-1/x*(b*x^3+a)^(1/2)-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1
/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Sympy [A] time = 2.22075, size = 85, normalized size = 0.16

$$\frac{A\sqrt{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] A*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3)) + B*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

3.193 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^5}\,dx$

Optimal. Leaf size=546

$$\frac{3^{3/4}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] -((A*b + 8*a*B)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(A*b + 8*a*B)*Sqrt[a
+ b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))
/(4*a*x^4) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3]) + (3^(3/4)*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.251191, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 303, 218, 1877}

$$\frac{3^{3/4}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] -((A*b + 8*a*B)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(A*b + 8*a*B)*Sqrt[a
+ b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))
/(4*a*x^4) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) +
```

```
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3]) + (3^(3/4)*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^5}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} - \frac{\left(-\frac{Ab}{2}-4aB\right)\int \frac{\sqrt{a+bx^3}}{x^2}\,dx}{4a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} + \frac{(3b(Ab+8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} + \frac{\left(3b^{2/3}(Ab+8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{16a} + \frac{\left(3\sqrt{\frac{1}{2}\left(2\right.}\right.}{}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(Ab}{}$$

Mathematica [C] time = 0.0804254, size = 80, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(-\frac{x^3(8aB+Ab)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - A\left(a+bx^3\right)\right)}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((A*b + 8*a*B)*x^3*Hypergeometric2F1[-
1/2, -1/3, 2/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(4*a*x^4)
```

Maple [B] time = 0.02, size = 920, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x)
```

```
[Out] B*(-1/x*(b*x^3+a)^(1/2)-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-
a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/
4/x^4*(b*x^3+a)^(1/2)-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I*b/a*3^(1/2)*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Sympy [A] time = 2.40324, size = 92, normalized size = 0.17

$$\frac{A\sqrt{a}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\sqrt{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] A*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + B*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

3.194 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}\,dx$

Optimal. Leaf size=581

$$-\frac{3^{3/4} b^{4/3} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-14aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3b^{4/3}\sqrt{a+bx^3}(5A}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}$$

```
[Out] ((5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a*x^4) + (3*b*(5*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(7*a*x^7)
 + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (3^(3/4)*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.320445, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 325, 303, 218, 1877}

$$-\frac{3b^{4/3}\sqrt{a+bx^3}(5Ab-14aB)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3^{3/4} b^{4/3} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right.}{56\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] ((5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a*x^4) + (3*b*(5*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112
```

```
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(7*a*x^7)
 + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (3^(3/4)*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{\left(\frac{5Ab}{2}-7aB\right)\int \frac{\sqrt{a+bx^3}}{x^5}\,dx}{7a}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} - \frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{(3b(5Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{112a}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{\left(3b^2(5Ab-14aB)\right)}{224a^2}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{\left(3b^{5/3}(5Ab-14aB\right)}{22}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5Ab-14aB)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A\left(a\right.}{7}$$

Mathematica [C] time = 0.0834389, size = 80, normalized size = 0.14

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{5Ab}{2}-7aB\right){}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-4A\left(a+bx^3\right)\right)}{28ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-4*A*(a + b*x^3) + (((5*A*b)/2 - 7*a*B)*x^3*Hypergeometri
c2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(28*a*x^7)
```

Maple [B] time = 0.023, size = 964, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x)
```

```
[Out] B*(-1/4/x^4*(b*x^3+a)^(1/2)-3/8*b/a*(b*x^3+a)^(1/2)/x-1/8*I*b/a*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/7/x^7*(b*x^3+a)^(1/2)-3/56*b/a*(b*x^3
+a)^(1/2)/x^4+15/112*b^2/a^2*(b*x^3+a)^(1/2)/x+5/112*I*b^2/a^2*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
```

```
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Sympy [A] time = 3.03668, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] A*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**7*gamma(-4/3)) + B*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

3.195 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx$

Optimal. Leaf size=614

$$\frac{3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-20aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3b^{7/3}\sqrt{a+bx^3}(11A}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}$$

```
[Out] ((11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(140*a*x^7) + (3*b*(11*A*b - 20*a*B)*Sq
rt[a + b*x^3])/(1120*a^2*x^4) - (3*b^2*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(
448*a^3*x) + (3*b^(7/3)*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(448*a^3*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(10*a*x^10) - (3*3^(1
/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
3^(3/4)*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(224*Sqrt[2]*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.378772, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 325, 303, 218, 1877}

$$\frac{3b^{7/3}\sqrt{a+bx^3}(11Ab-20aB)}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3b^2\sqrt{a+bx^3}(11Ab-20aB)}{448a^3x}+\frac{3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-20aB)F\left(\text{s}\right.}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

```
[Out] ((11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(140*a*x^7) + (3*b*(11*A*b - 20*a*B)*Sq
rt[a + b*x^3])/(1120*a^2*x^4) - (3*b^2*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(
448*a^3*x) + (3*b^(7/3)*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(448*a^3*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(10*a*x^10) - (3*3^(1
/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
3^(3/4)*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(224*Sqrt[2]*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
```

```
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}} - \frac{\left(\frac{11Ab}{2}-10aB\right)\int \frac{\sqrt{a+bx^3}}{x^8}\,dx}{10a}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} - \frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}} - \frac{(3b(11Ab-20aB))\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{280a}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}} + \frac{\left(3b^2(11Ab-20\right.}{4}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x} - \frac{A}{}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x} - \frac{A}{}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x} + \frac{3b}{4}$$

Mathematica [C] time = 0.0968264, size = 80, normalized size = 0.13

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{11Ab}{2}-10aB\right){}_2F_1\left(-\frac{7}{3},-\frac{1}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - 7A\left(a+bx^3\right)\right)}{70ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-7*A*(a + b*x^3) + (((11*A*b)/2 - 10*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(70*a*x^10)
```

Maple [B] time = 0.024, size = 1006, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x)
```

```
[Out] A*(-1/10*(b*x^3+a)^(1/2)/x^10-3/140*b/a*(b*x^3+a)^(1/2)/x^7+33/1120*b^2/a^2
*(b*x^3+a)^(1/2)/x^4-33/448*b^3/a^3*(b*x^3+a)^(1/2)/x-11/448*I*b^3/a^3*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+B*(-1/7/x^7*(b*x^3+a)^(1/2)-3/56*b/a
*(b*x^3+a)^(1/2)/x^4+15/112*b^2/a^2*(b*x^3+a)^(1/2)/x+5/112*I*b^2/a^2*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Sympy [A] time = 4.23243, size = 97, normalized size = 0.16

$$\frac{A\sqrt{a}\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)}+\frac{B\sqrt{a}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**11,x)
```

```
[Out] A*sqrt(a)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**10*gamma(-7/3)) + B*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

3.196 $\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=103

$$\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
b*x^3)^(7/2))/(21*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(27*b^4) + (2*
B*(a + b*x^3)^(11/2))/(33*b^4)
```

Rubi [A] time = 0.0789919, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
b*x^3)^(7/2))/(21*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(27*b^4) + (2*
B*(a + b*x^3)^(11/2))/(33*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3} + \frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3} + \frac{(Ab-3aB)(}{b^3}\right.\right.$$
$$= \frac{2a^2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^4} - \frac{2a(2Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{9/2}}{27b^4}$$

Mathematica [A] time = 0.0598628, size = 78, normalized size = 0.76

$$\frac{2\left(a+bx^3\right)^{5/2}\left(8a^2b\left(11A+15Bx^3\right)-48a^3B-10ab^2x^3\left(22A+21Bx^3\right)+35b^3x^6\left(11A+9Bx^3\right)\right)}{10395b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(-48*a^3*B + 35*b^3*x^6*(11*A + 9*B*x^3) + 8*a^2*b*(11
*A + 15*B*x^3) - 10*a*b^2*x^3*(22*A + 21*B*x^3)))/(10395*b^4)
```

Maple [A] time = 0.007, size = 77, normalized size = 0.8

$$\frac{630\,Bx^9b^3+770\,Ab^3x^6-420\,Bab^2x^6-440\,Aab^2x^3+240\,Ba^2bx^3+176\,Aa^2b-96\,Ba^3}{10395\,b^4}\left(bx^3+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] 2/10395*(b*x^3+a)^(5/2)*(315*B*b^3*x^9+385*A*b^3*x^6-210*B*a*b^2*x^6-220*A*
a*b^2*x^3+120*B*a^2*b*x^3+88*A*a^2*b-48*B*a^3)/b^4
```

Maxima [A] time = 0.949675, size = 159, normalized size = 1.54

$$\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)A+\frac{2}{3465}\left(\frac{105\left(bx^3+a\right)^{\frac{11}{2}}}{b^4}-\frac{385\left(bx^3+a\right)^{\frac{9}{2}}a}{b^4}+\frac{495\left(bx^3+a\right.}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 +
a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a)^
(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/b^4
)*B
```

Fricas [A] time = 1.6951, size = 286, normalized size = 2.78

$$\frac{2\left(315\,Bb^5x^{15}+35\left(12\,Bab^4+11\,Ab^5\right)x^{12}+5\left(3\,Ba^2b^3+110\,Aab^4\right)x^9-3\left(6\,Ba^3b^2-11\,Aa^2b^3\right)x^6-48\,Ba^5+88\,Aa^4b+\right.}{10395\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3
+ 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A*a
^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [A] time = 8.34159, size = 267, normalized size = 2.59

$$\begin{cases}\frac{16Aa^4\sqrt{a+bx^3}}{945b^3}-\frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Aa^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Aax^9\sqrt{a+bx^3}}{189}+\frac{2Abx^{12}\sqrt{a+bx^3}}{27}-\frac{32Ba^5\sqrt{a+bx^3}}{3465b^4}+\frac{16Ba^4x^3\sqrt{a+bx^3}}{3465b^3}-\frac{4Ba^3x^6\sqrt{a+bx^3}}{1155b^2}+ \\ a^{\frac{3}{2}}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((16*A*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*A*a**3*x**3*sqrt(a + b
*x**3)/(945*b**2) + 2*A*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*A*a*x**9*sq
rt(a + b*x**3)/189 + 2*A*b*x**12*sqrt(a + b*x**3)/27 - 32*B*a**5*sqrt(a + b
*x**3)/(3465*b**4) + 16*B*a**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4*B*a**3
*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*B*a**2*x**9*sqrt(a + b*x**3)/(693*b)
 + 8*B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33, Ne(b,
 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))
```

Giac [B] time = 1.1598, size = 323, normalized size = 3.14

$$\frac{2\left(\frac{33\left(15\left(bx^{3}+a\right)^{\frac{7}{2}}-42\left(bx^{3}+a\right)^{\frac{5}{2}}a+35\left(bx^{3}+a\right)^{\frac{3}{2}}a^{2}\right)Aa}{b^{2}}+\frac{11\left(35\left(bx^{3}+a\right)^{\frac{9}{2}}-135\left(bx^{3}+a\right)^{\frac{7}{2}}a+189\left(bx^{3}+a\right)^{\frac{5}{2}}a^{2}-105\left(bx^{3}+a\right)^{\frac{3}{2}}a^{3}\right)Ba}{b^{3}}+\frac{11\left(35\left(bx^{3}+a\right)^{\frac{9}{2}}-13\right.}{}\right.}{10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/10395*(33*(15*(b*x^3 + a)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)
^(3/2)*a^2)*A*a/b^2 + 11*(35*(b*x^3 + a)^(9/2) - 135*(b*x^3 + a)^(7/2)*a +
189*(b*x^3 + a)^(5/2)*a^2 - 105*(b*x^3 + a)^(3/2)*a^3)*B*a/b^3 + 11*(35*(b*
x^3 + a)^(9/2) - 135*(b*x^3 + a)^(7/2)*a + 189*(b*x^3 + a)^(5/2)*a^2 - 105*
(b*x^3 + a)^(3/2)*a^3)*A/b^2 + (315*(b*x^3 + a)^(11/2) - 1540*(b*x^3 + a)^(
9/2)*a + 2970*(b*x^3 + a)^(7/2)*a^2 - 2772*(b*x^3 + a)^(5/2)*a^3 + 1155*(b*
x^3 + a)^(3/2)*a^4)*B/b^3)/b
```

3.197 $\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{7/2}(Ab-2aB)}{21b^3}-\frac{2a\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3
)^(7/2))/(21*b^3) + (2*B*(a + b*x^3)^(9/2))/(27*b^3)
```

Rubi [A] time = 0.0575149, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{7/2}(Ab-2aB)}{21b^3}-\frac{2a\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3
)^(7/2))/(21*b^3) + (2*B*(a + b*x^3)^(9/2))/(27*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \frac{1}{3}\text{Subst}\left(\int x(a+bx)^{3/2}(A+Bx)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2} + \frac{(Ab-2aB)(a+bx)^{5/2}}{b^2} + \frac{B(a+bx)^{7/2}}{b^2}\right) dx, x$$
$$= -\frac{2a(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^3} + \frac{2(Ab-2aB)\left(a+bx^3\right)^{7/2}}{21b^3} + \frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

Mathematica [A] time = 0.0377418, size = 57, normalized size = 0.78

$$\frac{2\left(a+bx^3\right)^{5/2}\left(8a^2B-2ab\left(9A+10Bx^3\right)+5b^2x^3\left(9A+7Bx^3\right)\right)}{945b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(8*a^2*B + 5*b^2*x^3*(9*A + 7*B*x^3) - 2*a*b*(9*A + 10
*B*x^3)))/(945*b^3)
```

Maple [A] time = 0.006, size = 53, normalized size = 0.7

$$-\frac{-70\,b^2Bx^6-90\,Ax^3b^2+40\,Bx^3ab+36\,abA-16\,a^2B}{945\,b^3}\left(bx^3+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] -2/945*(b*x^3+a)^(5/2)*(-35*B*b^2*x^6-45*A*b^2*x^3+20*B*a*b*x^3+18*A*a*b-8*
B*a^2)/b^3
```

Maxima [A] time = 0.927493, size = 113, normalized size = 1.55

$$\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)A+\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*(
b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^
2/b^3)*B
```

Fricas [A] time = 1.71325, size = 219, normalized size = 3.

$$\frac{2\left(35\,Bb^4x^{12}+5\left(10\,Bab^3+9\,Ab^4\right)x^9+3\left(Ba^2b^2+24\,Aab^3\right)x^6+8\,Ba^4-18\,Aa^3b-\left(4\,Ba^3b-9\,Aa^2b^2\right)x^3\right)\sqrt{bx^3+a}}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*a
*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*x^
3 + a)/b^3
```

Sympy [A] time = 5.31557, size = 216, normalized size = 2.96

$$\begin{cases} -\frac{4Aa^3\sqrt{a+bx^3}}{105b^2}+\frac{2Aa^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Aax^6\sqrt{a+bx^3}}{105}+\frac{2Abx^9\sqrt{a+bx^3}}{21}+\frac{16Ba^4\sqrt{a+bx^3}}{945b^3}-\frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Ba^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Bax^9\sqrt{a+bx^3}}{189}+\frac{2B \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a + b
*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b*x
**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a + b*
x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9*sqr
t(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*(
A*x**6/6 + B*x**9/9), True))
```

Giac [B] time = 1.20729, size = 247, normalized size = 3.38

$$\frac{2\left(\frac{21\left(3\left(bx^3+a\right)^{\frac{5}{2}}-5\left(bx^3+a\right)^{\frac{3}{2}}a\right)Aa}{b}+\frac{3\left(15\left(bx^3+a\right)^{\frac{7}{2}}-42\left(bx^3+a\right)^{\frac{5}{2}}a+35\left(bx^3+a\right)^{\frac{3}{2}}a^2\right)Ba}{b^2}+\frac{3\left(15\left(bx^3+a\right)^{\frac{7}{2}}-42\left(bx^3+a\right)^{\frac{5}{2}}a+35\left(bx^3+a\right)^{\frac{3}{2}}a^2\right)A}{b}+\frac{\left(35\left(b\right.\right.}{}\right.}{945b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/945*(21*(3*(b*x^3 + a)^(5/2) - 5*(b*x^3 + a)^(3/2)*a)*A*a/b + 3*(15*(b*x^
3 + a)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)^(3/2)*a^2)*B*a/b^2 +
 3*(15*(b*x^3 + a)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)^(3/2)*a^
2)*A/b + (35*(b*x^3 + a)^(9/2) - 135*(b*x^3 + a)^(7/2)*a + 189*(b*x^3 + a)^
(5/2)*a^2 - 105*(b*x^3 + a)^(3/2)*a^3)*B/b^2)/b
```

3.198 $\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=46

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^2) + (2*B*(a + b*x^3)^(7/2))/(21*b^
2)
```

Rubi [A] time = 0.0399532, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^2) + (2*B*(a + b*x^3)^(7/2))/(21*b^
2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{1}{3}\,\text{Subst}\left(\int(a+bx)^{3/2}(A+Bx)\,dx,x,x^3\right)$$
$$=\frac{1}{3}\,\text{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)dx,x,x^3\right)$$
$$=\frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

Mathematica [A] time = 0.0252422, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{5/2}\left(-2aB+7Ab+5bBx^3\right)}{105b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

Maple [A] time = 0.005, size = 31, normalized size = 0.7

$$\frac{10\,bBx^3+14\,Ab-4\,Ba}{105\,b^2}\left(bx^3+a\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] 2/105*(b*x^3+a)^(5/2)*(5*B*b*x^3+7*A*b-2*B*a)/b^2
```

Maxima [A] time = 0.924079, size = 66, normalized size = 1.43

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}A}{15\,b}+\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)
^(5/2)*a/b^2)*B
```

Fricas [A] time = 1.58608, size = 162, normalized size = 3.52

$$\frac{2\left(5\,Bb^{3}x^{9} + \left(8\,Bab^{2} + 7\,Ab^{3}\right)x^{6} - 2\,Ba^{3} + 7\,Aa^{2}b + \left(Ba^{2}b + 14\,Aab^{2}\right)x^{3}\right)\sqrt{bx^{3} + a}}{105\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*a
^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2
```

Sympy [A] time = 2.78142, size = 165, normalized size = 3.59

$$\begin{cases} \frac{2Aa^{2}\sqrt{a+bx^{3}}}{15b} + \frac{4Aax^{3}\sqrt{a+bx^{3}}}{15} + \frac{2Abx^{6}\sqrt{a+bx^{3}}}{15} - \frac{4Ba^{3}\sqrt{a+bx^{3}}}{105b^{2}} + \frac{2Ba^{2}x^{3}\sqrt{a+bx^{3}}}{105b} + \frac{16Bax^{6}\sqrt{a+bx^{3}}}{105} + \frac{2Bbx^{9}\sqrt{a+bx^{3}}}{21} & \text{for}\: b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^{3}}{3} + \frac{Bx^{6}}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/1
5 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2) +
 2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/105
+ 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**6/6
), True))
```

Giac [B] time = 1.18997, size = 162, normalized size = 3.52

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{3}{2}}Aa+7\left(3\left(bx^3+a\right)^{\frac{5}{2}}-5\left(bx^3+a\right)^{\frac{3}{2}}a\right)A+\frac{7\left(3\left(bx^3+a\right)^{\frac{5}{2}}-5\left(bx^3+a\right)^{\frac{3}{2}}a\right)Ba}{b}+\frac{\left(15\left(bx^3+a\right)^{\frac{7}{2}}-42\left(bx^3+a\right)^{\frac{5}{2}}a+35\left(bx^3+a\right)^{\frac{3}{2}}\right.}{b}\right.}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x^3 + a)^(3/2)*A*a + 7*(3*(b*x^3 + a)^(5/2) - 5*(b*x^3 + a)^(3
/2)*a)*A + 7*(3*(b*x^3 + a)^(5/2) - 5*(b*x^3 + a)^(3/2)*a)*B*a/b + (15*(b*x
^3 + a)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)^(3/2)*a^2)*B/b)/b
```

3.199 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx$

Optimal. Leaf size=81

$$-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}$$

```
[Out] (2*a*A*Sqrt[a + b*x^3])/3 + (2*A*(a + b*x^3)^(3/2))/9 + (2*B*(a + b*x^3)^(5
/2))/(15*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rubi [A] time = 0.0566899, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*Sqrt[a + b*x^3])/3 + (2*A*(a + b*x^3)^(3/2))/9 + (2*B*(a + b*x^3)^(5
/2))/(15*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx, x, x^3\right)\\
&= \frac{2B\left(a+bx^3\right)^{5/2}}{15b} + \frac{1}{3}A\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{9}A\left(a+bx^3\right)^{3/2} + \frac{2B\left(a+bx^3\right)^{5/2}}{15b} + \frac{1}{3}(aA)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A\left(a+bx^3\right)^{3/2} + \frac{2B\left(a+bx^3\right)^{5/2}}{15b} + \frac{1}{3}\left(a^2A\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.\\
&= \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A\left(a+bx^3\right)^{3/2} + \frac{2B\left(a+bx^3\right)^{5/2}}{15b} + \frac{\left(2a^2A\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a\right.}{3b}\\
&= \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A\left(a+bx^3\right)^{3/2} + \frac{2B\left(a+bx^3\right)^{5/2}}{15b} - \frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0671314, size = 80, normalized size = 0.99

$$\frac{2\left(-15a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+5Ab\left(a+bx^3\right)^{3/2}+15aAb\sqrt{a+bx^3}+3B\left(a+bx^3\right)^{5/2}\right)}{45b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*(15*a*A*b*Sqrt[a + b*x^3] + 5*A*b*(a + b*x^3)^(3/2) + 3*B*(a + b*x^3)^(5
/2) - 15*a^(3/2)*A*b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/(45*b)
```

Maple [A] time = 0.017, size = 66, normalized size = 0.8

$$\frac{2B}{15b}\left(bx^3+a\right)^{\frac{5}{2}}+A\left(\frac{2bx^3}{9}\sqrt{bx^3+a}+\frac{8a}{9}\sqrt{bx^3+a}-\frac{2}{3}a^{\frac{3}{2}}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x)
```

```
[Out] 2/15*B*(b*x^3+a)^(5/2)/b+A*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)
-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74219, size = 409, normalized size = 5.05

$$\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Bb^2x^6+\left(6\,Bab+5\,Ab^2\right)x^3+3\,Ba^2+20\,Aab\right)\sqrt{bx^3+a}}{45\,b}, \frac{2\left(15\,A\sqrt{-a}ab\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] [1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 +
 a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*B*b
^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b]
```

Sympy [A] time = 39.0241, size = 82, normalized size = 1.01

$$\frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2Aa\sqrt{a+bx^3}}{3}+\frac{2A\left(a+bx^3\right)^{\frac{3}{2}}}{9}+\frac{2B\left(a+bx^3\right)^{\frac{5}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

```
[Out] 2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*sqrt(-a)) + 2*A*a*sqrt(a + b*x*
*3)/3 + 2*A*(a + b*x**3)**(3/2)/9 + 2*B*(a + b*x**3)**(5/2)/(15*b)
```

Giac [A] time = 1.26816, size = 108, normalized size = 1.33

$$\frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}Bb^4+5\left(bx^3+a\right)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx^3+a}Aab^5\right)}{45\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^(
5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5
```

3.200 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^4}\,dx$

Optimal. Leaf size=110

$$\frac{\left(a+bx^3\right)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}(2aB+3Ab)-\frac{1}{3}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}$$

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^3])/3 + ((3*A*b + 2*a*B)*(a + b*x^3)^(3/2))/(
9*a) - (A*(a + b*x^3)^(5/2))/(3*a*x^3) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

Rubi [A] time = 0.0840313, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(a+bx^3\right)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}(2aB+3Ab)-\frac{1}{3}\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^3])/3 + ((3*A*b + 2*a*B)*(a + b*x^3)^(3/2))/(
9*a) - (A*(a + b*x^3)^(5/2))/(3*a*x^3) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^4}dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}dx,x,x^3\right)$$
$$= -\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx,x,x^3\right)}{3a}$$
$$= \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{1}{6}(3Ab+2aB)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}dx,x,x\right.$$
$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{1}{6}(a(3Ab+2aB$$
$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{(a(3Ab+2aB)}{}$$
$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} - \frac{1}{3}\sqrt{a}(3Ab+2a$$

Mathematica [A] time = 0.0473543, size = 80, normalized size = 0.73

$$\frac{1}{9}\left(\frac{\sqrt{a+bx^3}\left(-3aA+8aBx^3+6Abx^3+2bBx^6\right)}{x^3} - 3\sqrt{a}(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/x^3 - 3*Sqr
t[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/9
```

Maple [A] time = 0.02, size = 101, normalized size = 0.9

$$A\left(-\frac{a}{3x^3}\sqrt{bx^3+a} + \frac{2b}{3}\sqrt{bx^3+a} - \sqrt{a}b\text{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)\right) + B\left(\frac{2bx^3}{9}\sqrt{bx^3+a} + \frac{8a}{9}\sqrt{bx^3+a} - \frac{2}{3}a^{\frac{3}{2}}\text{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x)
```

```
[Out] A*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-a^(1/2)*b*arctanh((b*x^
3+a)^(1/2)/a^(1/2)))+B*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3
*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81138, size = 405, normalized size = 3.68

$$\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(2\,Bbx^6+2\,(4\,Ba+3\,Ab)x^3-3\,Aa\right)\sqrt{bx^3+a}}{18\,x^3},\ \frac{3\,(2\,Ba+3\,Ab)\sqrt{-a}x^3\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a)
 + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a
))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]
```

Sympy [A] time = 28.6349, size = 223, normalized size = 2.03

$$-A\sqrt{a}b\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}-\frac{2Ba^{\frac{3}{2}}\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba^2}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ba}{3\sqrt{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - A*a*sqrt(b)*sqrt(a/(b*x**3
) + 1)/(3*x**(3/2)) + 2*A*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*A
*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - 2*B*a**(3/2)*asinh(sqrt(a)/(s
qrt(b)*x**(3/2)))/3 + 2*B*a**2/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) +
2*B*a*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + B*b*Piecewise((sqrt(a)*x*
*3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Giac [A] time = 1.22006, size = 139, normalized size = 1.26

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}Bb+6\sqrt{bx^3+a}Bab+6\sqrt{bx^3+a}Ab^2+\frac{3\left(2Ba^2b+3Aab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\sqrt{bx^3+a}Aab}{x^3}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(2*(b*x^3 + a)^(3/2)*B*b + 6*sqrt(b*x^3 + a)*B*a*b + 6*sqrt(b*x^3 + a)*
A*b^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a)
 - 3*sqrt(b*x^3 + a)*A*a*b/x^3)/b
```

3.201 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^7}\,dx$

Optimal. Leaf size=115

$$-\frac{\left(a+bx^3\right)^{3/2}(4aB+Ab)}{12ax^3}+\frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a}-\frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6}$$

```
[Out] (b*(A*b + 4*a*B)*Sqrt[a + b*x^3])/(4*a) - ((A*b + 4*a*B)*(a + b*x^3)^(3/2))
/(12*a*x^3) - (A*(a + b*x^3)^(5/2))/(6*a*x^6) - (b*(A*b + 4*a*B)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rubi [A] time = 0.0868472, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{\left(a+bx^3\right)^{3/2}(4aB+Ab)}{12ax^3}+\frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a}-\frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] (b*(A*b + 4*a*B)*Sqrt[a + b*x^3])/(4*a) - ((A*b + 4*a*B)*(a + b*x^3)^(3/2))
/(12*a*x^3) - (A*(a + b*x^3)^(5/2))/(6*a*x^6) - (b*(A*b + 4*a*B)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^7}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(Ab+4aB)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^3\right)}{12a}$$
$$= -\frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(b(Ab+4aB))\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)}{8a}$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{8}(b(Ab+4aB))\operatorname{Su}$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{4}(Ab+4aB)\operatorname{Subst}$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} - \frac{b(Ab+4aB)\tanh^{-}}{4\sqrt{a}}$$

Mathematica [C] time = 0.026296, size = 59, normalized size = 0.51

$$\frac{\left(a+bx^3\right)^{5/2}\left(bx^6(4aB+Ab)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\right)-5a^2A\right)}{30a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] ((a + b*x^3)^(5/2)*(-5*a^2*A + b*(A*b + 4*a*B)*x^6*Hypergeometric2F1[2, 5/2
, 7/2, 1 + (b*x^3)/a]))/(30*a^3*x^6)
```

Maple [A] time = 0.023, size = 107, normalized size = 0.9

$$A\left(-\frac{a}{6\,x^6}\sqrt{bx^3+a}-\frac{5\,b}{12\,x^3}\sqrt{bx^3+a}-\frac{b^2}{4}\operatorname{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}\right)+B\left(-\frac{a}{3\,x^3}\sqrt{bx^3+a}+\frac{2\,b}{3}\sqrt{bx^3+a}-\sqrt{a}b\operatorname{Artanl}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x)
```

```
[Out] A*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b
*x^3+a)^(1/2)/a^(1/2))/a^(1/2))+B*(-1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+
a)^(1/2)-a^(1/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81928, size = 439, normalized size = 3.82

$$\left[\frac{3\left(4\,Bab+Ab^2\right)\sqrt{a}x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(8\,Babx^6-\left(4\,Ba^2+5\,Aab\right)x^3-2\,Aa^2\right)\sqrt{bx^3+a}}{24\,ax^6}, \frac{3\left(4\,Bab+Ab^2\right)\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(
a) + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt(b
*x^3 + a))/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^
3 + a)*sqrt(-a)/a) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt
(b*x^3 + a))/(a*x^6)]
```

Sympy [B] time = 62.9708, size = 243, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Aa\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}-B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)-\frac{Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/2
)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*
b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - B
*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

Giac [A] time = 1.15565, size = 177, normalized size = 1.54

$$\frac{8\sqrt{bx^3+a}Bb^2 + \frac{3\left(4\,Bab^2+Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2 - 4\sqrt{bx^3+a}Ba^2b^2 + 5\left(bx^3+a\right)^{\frac{3}{2}}Ab^3 - 3\sqrt{bx^3+a}Aab^3}{b^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(8*sqrt(b*x^3 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^3 + a
)/sqrt(-a))/sqrt(-a) - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a
^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^3 + a)*A*a*b^3)/(b^2*x^6))/
b
```

3.202 $\int x^3 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=336

$$-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(23Ab-8aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2x}{}$$

```
[Out] (54*a^2*(23*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(21505*b^2) + (18*a*(23*A*b - 8
*a*B)*x^4*Sqrt[a + b*x^3])/(4301*b) + (2*(23*A*b - 8*a*B)*x^4*(a + b*x^3)^(
3/2))/(391*b) + (2*B*x^4*(a + b*x^3)^(5/2))/(23*b) - (36*3^(3/4)*Sqrt[2 + S
qrt[3]]*a^3*(23*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(21505*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.175004, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 279, 321, 218}

$$\frac{54a^2x\sqrt{a+bx^3}(23Ab-8aB)}{21505b^2}-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(23Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (54*a^2*(23*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(21505*b^2) + (18*a*(23*A*b - 8
*a*B)*x^4*Sqrt[a + b*x^3])/(4301*b) + (2*(23*A*b - 8*a*B)*x^4*(a + b*x^3)^(
3/2))/(391*b) + (2*B*x^4*(a + b*x^3)^(5/2))/(23*b) - (36*3^(3/4)*Sqrt[2 + S
qrt[3]]*a^3*(23*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(21505*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
```

```
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b} - \frac{\left(2\left(-\frac{23Ab}{2}+4aB\right)\right)\int x^3\left(a+bx^3\right)^{3/2}dx}{23b}$$
$$= \frac{2(23Ab-8aB)x^4\left(a+bx^3\right)^{3/2}}{391b} + \frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b} + \frac{(9a(23Ab-8aB))\int x^3\sqrt{a+bx}}{391b}$$
$$= \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4\left(a+bx^3\right)^{3/2}}{391b} + \frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b} +$$
$$= \frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2} + \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4\left(a\right.}{391b}$$
$$= \frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2} + \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4\left(a\right.}{391b}$$

Mathematica [C] time = 0.131918, size = 93, normalized size = 0.28

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a^2(8aB-23Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(a+bx^3\right)^2\left(8aB-23Ab-17bBx^3\right)\right)}{391b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-23*A*b + 8*a*B - 17*b*B*x^3)) + (a^
2*(-23*A*b + 8*a*B)*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(391*b^2)
```

Maple [B] time = 0.023, size = 694, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301*a^2/b*x
^4*(b*x^3+a)^(1/2)-432/21505*a^3/b^2*x*(b*x^3+a)^(1/2)-288/21505*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/17*b*x^
7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2/b*x*(b*x^3+a)^(1/
2)+36/935*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbx^9 + (Ba + Ab)x^6 + Aax^3\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^9 + (B*a + A*b)*x^6 + A*a*x^3)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 6.06084, size = 172, normalized size = 0.51

$$\frac{Aa^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{A\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{B\sqrt{a}bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F}{3\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + A*sqrt(a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*a**(3/2)*x**7*gamma(7/3)*h
yper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*sq
rt(a)*b*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(13/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

3.203 $\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=299

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-2aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\left(a+b\right.}{}$$

```
[Out] (18*a*(17*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(935*b) + (2*(17*A*b - 2*a*B)*x*(
a + b*x^3)^(3/2))/(187*b) + (2*B*x*(a + b*x^3)^(5/2))/(17*b) + (18*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.123566, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 218}

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\left(a+bx^3\right)^{3/2}(}{187b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(17*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(935*b) + (2*(17*A*b - 2*a*B)*x*(
a + b*x^3)^(3/2))/(187*b) + (2*B*x*(a + b*x^3)^(5/2))/(17*b) + (18*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} - \frac{\left(2\left(-\frac{17Ab}{2}+aB\right)\right)\int \left(a+bx^3\right)^{3/2}dx}{17b}$$

$$= \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} + \frac{(9a(17Ab-2aB))\int \sqrt{a+bx^3}\,dx}{187b}$$

$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} + \frac{(27a^{\cdots}}{\cdots}$$

$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} + \frac{18\ 3^{\cdots}}{\cdots}$$

Mathematica [C] time = 0.056973, size = 77, normalized size = 0.26

$$\frac{2x\sqrt{a+bx^3}\left(B\left(a+bx^3\right)^2-\frac{a\left(aB-\frac{17Ab}{2}\right){}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2 - (a*((-17*A*b)/2 + a*B)*Hypergeometr
ic2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

Maple [B] time = 0.014, size = 654, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2/b*x*(
b*x^3+a)^(1/2)+36/935*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-
1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)))+A*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)
-18/55*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 4.0534, size = 170, normalized size = 0.57

$$\frac{Aa^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{A\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{B\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\right.}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + A*sqrt(a)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*a**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*sqrt(a)*b
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

3.204 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$

Optimal. Leaf size=295

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+11Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x\left(a+bx\right.}{}$$

```
[Out] (9*(11*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/110 + ((11*A*b + 4*a*B)*x*(a + b*x^3
)^(3/2))/(22*a) - (A*(a + b*x^3)^(5/2))/(2*a*x^2) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*a*(11*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(110*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.126661, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 195, 218}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+11Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x\left(a+bx^3\right)^{3/2}(4a}{22a}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```


```
[Out] (9*(11*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/110 + ((11*A*b + 4*a*B)*x*(a + b*x^3
)^(3/2))/(22*a) - (A*(a + b*x^3)^(5/2))/(2*a*x^2) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*a*(11*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(110*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^3}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{2ax^2} - \frac{\left(-\frac{11Ab}{2}-2aB\right)\int \left(a+bx^3\right)^{3/2}\,dx}{2a}$$

$$= \frac{(11Ab+4aB)x\left(a+bx^3\right)^{3/2}}{22a} - \frac{A\left(a+bx^3\right)^{5/2}}{2ax^2} + \frac{1}{44}(9(11Ab+4aB))\int \sqrt{a+bx^3}\,dx$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x\left(a+bx^3\right)^{3/2}}{22a} - \frac{A\left(a+bx^3\right)^{5/2}}{2ax^2} + \frac{1}{220}(27a(1$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x\left(a+bx^3\right)^{3/2}}{22a} - \frac{A\left(a+bx^3\right)^{5/2}}{2ax^2} + \frac{9\ 3^{3/4}\sqrt{2}}{}$$

Mathematica [C] time = 0.0486247, size = 83, normalized size = 0.28

$$-\frac{x\sqrt{a+bx^3}\left(-2aB-\frac{11Ab}{2}\right){}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{A\left(a+bx^3\right)^{5/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

```
[Out] -(A*(a + b*x^3)^(5/2))/(2*a*x^2) - (((-11*A*b)/2 - 2*a*B)*x*Sqrt[a + b*x^3]
*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.02, size = 629, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x)
```

```
[Out] B*(2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)
/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/2*a*(b*x^3+
a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^3, x)
```

Sympy [A] time = 3.93437, size = 172, normalized size = 0.58

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{A\sqrt{a}bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{Ba^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{B\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(-\right.}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**3,x)
```

```
[Out] A*a**(3/2)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + A*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*a**(3/2)*x*gamma(1/3)*hyper((
-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*sqrt(a)*b*
x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
```

```
ma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

3.205 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx$

Optimal. Leaf size=297

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\left(a+bx^3\right)^{3/}}{4}$$

```
[Out] (9*b*(A*b + 2*a*B)*x*Sqrt[a + b*x^3])/(20*a) - ((A*b + 2*a*B)*(a + b*x^3)^(
3/2))/(4*a*x^2) - (A*(a + b*x^3)^(5/2))/(5*a*x^5) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*b^(2/3)*(A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(20*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.120987, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 277, 195, 218}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\left(a+bx^3\right)^{3/2}(2aB+}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```


```
[Out] (9*b*(A*b + 2*a*B)*x*Sqrt[a + b*x^3])/(20*a) - ((A*b + 2*a*B)*(a + b*x^3)^(
3/2))/(4*a*x^2) - (A*(a + b*x^3)^(5/2))/(5*a*x^5) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*b^(2/3)*(A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(20*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^6}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{5ax^5} + -\frac{\left(-\frac{5Ab}{2}-5aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^3}\,dx}{5a}$$

$$= -\frac{(Ab+2aB)\left(a+bx^3\right)^{3/2}}{4ax^2} - \frac{A\left(a+bx^3\right)^{5/2}}{5ax^5} + \frac{(9b(Ab+2aB))\int \sqrt{a+bx^3}\,dx}{8a}$$

$$= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a} - \frac{(Ab+2aB)\left(a+bx^3\right)^{3/2}}{4ax^2} - \frac{A\left(a+bx^3\right)^{5/2}}{5ax^5} + \frac{1}{40}(27b(Ab+2a$$

$$= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a} - \frac{(Ab+2aB)\left(a+bx^3\right)^{3/2}}{4ax^2} - \frac{A\left(a+bx^3\right)^{5/2}}{5ax^5} + \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b^2}{}$$

Mathematica [C] time = 0.0710639, size = 82, normalized size = 0.28

$$\frac{\sqrt{a+bx^3}\left(-\frac{5x^3(2aB+Ab)\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{2A\left(a+bx^3\right)^2}{a}\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-2*A*(a + b*x^3)^2)/a - (5*(A*b + 2*a*B)*x^3*Hypergeomet
ric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(10*x^5)
```

Maple [B] time = 0.022, size = 626, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x)
```

```
[Out] B*(-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
```

```
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/5*a*(b*x^3+a)^(1/2
)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^6, x)
```

Sympy [A] time = 4.30118, size = 184, normalized size = 0.62

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{A\sqrt{a}b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{B\sqrt{a}bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\right.}{3\Gamma\left(\frac{4}{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**6,x)
```

```
[Out] A*a**(3/2)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + A*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + B*a**(3/2)*gamma(-2/3)*h
yper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
B*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

3.206 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^9}\,dx$

Optimal. Leaf size=302

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-16aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\left(a+b\right.}{}$$

```
[Out] (9*b*(A*b - 16*a*B)*Sqrt[a + b*x^3])/(320*a*x^2) + ((A*b - 16*a*B)*(a + b*x
^3)^(3/2))/(80*a*x^5) - (A*(a + b*x^3)^(5/2))/(8*a*x^8) - (9*3^(3/4)*Sqrt[2
 + Sqrt[3]]*b^(5/3)*(A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.126118, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 277, 218}

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\left(a+bx^3\right)^{3/2}(A}{80ax}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

```
[Out] (9*b*(A*b - 16*a*B)*Sqrt[a + b*x^3])/(320*a*x^2) + ((A*b - 16*a*B)*(a + b*x
^3)^(3/2))/(80*a*x^5) - (A*(a + b*x^3)^(5/2))/(8*a*x^8) - (9*3^(3/4)*Sqrt[2
 + Sqrt[3]]*b^(5/3)*(A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^9}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{\left(\frac{Ab}{2}-8aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^6}\,dx}{8a}$$

$$= \frac{(Ab-16aB)\left(a+bx^3\right)^{3/2}}{80ax^5} - \frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{(9b(Ab-16aB))\int \frac{\sqrt{a+bx^3}}{x^3}\,dx}{160a}$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)\left(a+bx^3\right)^{3/2}}{80ax^5} - \frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{\left(27b^2(Ab-16a\right.}{640}$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)\left(a+bx^3\right)^{3/2}}{80ax^5} - \frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{9\ 3^{3/4}\sqrt{2+\sqrt{3}}b}{}$$

Mathematica [C] time = 0.0784211, size = 82, normalized size = 0.27

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{Ab}{2}-8aB\right){}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\frac{5A\left(a+bx^3\right)^2}{a}\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + (((A*b)/2 - 8*a*B)*x^3*Hypergeom
etric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*x^8)
```

Maple [B] time = 0.024, size = 653, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x)
```

```
[Out] A*(-1/8*a*(b*x^3+a)^(1/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320*b^2/a*(b*x
^3+a)^(1/2)/x^2+9/320*I*b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)))+B*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2
-9/20*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^9} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^9, x)
```

Sympy [A] time = 5.50204, size = 196, normalized size = 0.65

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3}, -\frac{1}{2} \\ -\frac{5}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\right.}{3x^2\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**9,x)
```

```
[Out] A*a**(3/2)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + A*sqrt(a)*b*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + B*a**(3/2)*gamma(-5/3)
*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)
) + B*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*
```

```
pi)/a)/(3*x**2*gamma(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^9} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

3.207 $\int x^4 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=614

$$-\frac{72\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2x^2\sqrt{a+}}{86}$$

```
[Out] (54*a^2*(5*A*b - 2*a*B)*x^2*Sqrt[a + b*x^3])/(8645*b^2) + (18*a*(5*A*b - 2*
a*B)*x^5*Sqrt[a + b*x^3])/(1235*b) - (216*a^3*(5*A*b - 2*a*B)*Sqrt[a + b*x^
3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(5*A*b - 2*a*B)
*x^5*(a + b*x^3)^(3/2))/(95*b) + (2*B*x^5*(a + b*x^3)^(5/2))/(25*b) + (108*
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 - (72*Sqrt[2]*3^(3/4)*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.370186, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 279, 321, 303, 218, 1877}

$$\frac{54a^2x^2\sqrt{a+bx^3}(5Ab-2aB)}{8645b^2}-\frac{216a^3\sqrt{a+bx^3}(5Ab-2aB)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{72\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2a}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (54*a^2*(5*A*b - 2*a*B)*x^2*Sqrt[a + b*x^3])/(8645*b^2) + (18*a*(5*A*b - 2*
a*B)*x^5*Sqrt[a + b*x^3])/(1235*b) - (216*a^3*(5*A*b - 2*a*B)*Sqrt[a + b*x^
```

```
3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(5*A*b - 2*a*B)
*x^5*(a + b*x^3)^(3/2))/(95*b) + (2*B*x^5*(a + b*x^3)^(5/2))/(25*b) + (108*
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 - (72*Sqrt[2]*3^(3/4)*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b} - \frac{\left(2\left(-\frac{25Ab}{2}+5aB\right)\right)\int x^4\left(a+bx^3\right)^{3/2}dx}{25b}$$

$$= \frac{2(5Ab-2aB)x^5\left(a+bx^3\right)^{3/2}}{95b} + \frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b} + \frac{(9a(5Ab-2aB))\int x^4\sqrt{a+bx^3}\,dx}{95b}$$

$$= \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} + \frac{2(5Ab-2aB)x^5\left(a+bx^3\right)^{3/2}}{95b} + \frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b} + \frac{(27 \ldots}{\ldots}$$

$$= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} + \frac{2(5Ab-2aB)x^5\left(a+bx \ldots\right.}{95b}$$

$$= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} + \frac{2(5Ab-2aB)x^5\left(a+bx \ldots\right.}{95b}$$

$$= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} - \frac{216a^3(5Ab-2aB)\sqrt{a \ldots}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} \ldots\right.}$$

Mathematica [C] time = 0.135269, size = 96, normalized size = 0.16

$$\frac{2x^2\sqrt{a+bx^3}\left(\frac{5a^2(2aB-5Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(a+bx^3\right)^2\left(10aB-25Ab-19bBx^3\right)\right)}{475b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-25*A*b + 10*a*B - 19*b*B*x^3)) +
(5*a^2*(-5*A*b + 2*a*B)*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sq
rt[1 + (b*x^3)/a]))/(475*b^2)
```

Maple [B] time = 0.023, size = 1002, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/25*b*x^11*(b*x^3+a)^(1/2)+56/475*a*x^8*(b*x^3+a)^(1/2)+54/6175*a^2/b*x
^5*(b*x^3+a)^(1/2)-108/8645*a^3/b^2*x^2*(b*x^3+a)^(1/2)-144/8645*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/19*b*x^8*(b*x^3+a)^(1/2)+44
/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2/b*x^2*(b*x^3+a)^(1/2)+72/1729*I*a^3/
b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
```

```
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbx^{10} + (Ba + Ab)x^7 + Aax^4\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^10 + (B*a + A*b)*x^7 + A*a*x^4)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 4.83317, size = 172, normalized size = 0.28

$$\frac{Aa^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{A\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{B\sqrt{a}bx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1}{3\Gamma\left(\frac{14}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(8/3)) + A*sqrt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*a**(3/2)*x**8*gamma(8/3)*h
yper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*sq
rt(a)*b*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(14/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

3.208 $\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=581

$$\frac{18\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2\sqrt{a+bx}}{1729b^{5/3}\left(\left(1+\right.\right.}$$

```
[Out] (18*a*(19*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*A*b - 4*
a*B)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) +
(2*(19*A*b - 4*a*B)*x^2*(a + b*x^3)^(3/2))/(247*b) + (2*B*x^2*(a + b*x^3)^(
5/2))/(19*b) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 4*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3]) + (18*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 4*a*B)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rubi [A] time = 0.292301, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {459, 279, 303, 218, 1877}

$$\frac{54a^2\sqrt{a+bx^3}(19Ab-4aB)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{18\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(19*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*A*b - 4*
a*B)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) +
```

```
(2*(19*A*b - 4*a*B)*x^2*(a + b*x^3)^(3/2))/(247*b) + (2*B*x^2*(a + b*x^3)^(
5/2))/(19*b) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 4*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3]) + (18*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 4*a*B)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx=\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}-\frac{\left(2\left(-\frac{19Ab}{2}+2aB\right)\right)\int x\left(a+bx^3\right)^{3/2}dx}{19b}$$

$$=\frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}+\frac{(9a(19Ab-4aB))\int x\sqrt{a+bx^3}\,dx}{247b}$$

$$=\frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b}+\frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}+\frac{(2}{}$$

$$=\frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b}+\frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}+\frac{(2}{}$$

$$=\frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b}+\frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2(19Ab-4aB)x^2\left(a+\right.}{247b}$$

Mathematica [C] time = 0.0988664, size = 78, normalized size = 0.13

$$\frac{x^2\sqrt{a+bx^3}\left(\frac{a(19Ab-4aB)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}+4B\left(a+bx^3\right)^2\right)}{38b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3)^2 + (a*(19*A*b - 4*a*B)*Hypergeometri
c2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(38*b)
```

Maple [B] time = 0.014, size = 962, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2/b*x^
2*(b*x^3+a)^(1/2)+72/1729*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)))+A*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbx^7 + (Ba + Ab)x^4 + Aax\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^7 + (B*a + A*b)*x^4 + A*a*x)*sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.59926, size = 172, normalized size = 0.3

$$\frac{Aa^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{A\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\cdots\right.}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + A*sqrt(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*a**(3/2)*x**5*gamma(5/3)*hyp
er((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*sqrt(a
)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(11/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

3.209 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$

Optimal. Leaf size=573

$$\frac{9\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+13Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a\cdots}{\cdots}$$

```
[Out] (9*(13*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/91 + (27*a*(13*A*b + 2*a*B)*Sqrt[a
 + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((13*A*b + 2*
a*B)*x^2*(a + b*x^3)^(3/2))/(13*a) - (A*(a + b*x^3)^(5/2))/(a*x) - (27*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9
*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.294175, antiderivative size = 573, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 279, 303, 218, 1877}

$$\frac{9\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+13Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] (9*(13*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/91 + (27*a*(13*A*b + 2*a*B)*Sqrt[a
 + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((13*A*b + 2*
```

```
a*B)*x^2*(a + b*x^3)^(3/2))/(13*a) - (A*(a + b*x^3)^(5/2))/(a*x) - (27*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9
*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^2}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)\int x\left(a+bx^3\right)^{3/2}\,dx}{a}$$

$$= \frac{(13Ab+2aB)x^2\left(a+bx^3\right)^{3/2}}{13a} - \frac{A\left(a+bx^3\right)^{5/2}}{ax} + \frac{1}{26}(9(13Ab+2aB))\int x\sqrt{a+bx^3}\,dx$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2\left(a+bx^3\right)^{3/2}}{13a} - \frac{A\left(a+bx^3\right)^{5/2}}{ax} + \frac{1}{182}(27a($$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2\left(a+bx^3\right)^{3/2}}{13a} - \frac{A\left(a+bx^3\right)^{5/2}}{ax} + \frac{(27a(13A}{}$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(13Ab+2aB)x^2\left(a+bx^3\right)^{3}}{13a}$$

Mathematica [C] time = 0.0359826, size = 83, normalized size = 0.14

$$-\frac{x^2\sqrt{a+bx^3}\left(-aB-\frac{13Ab}{2}\right){}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - \frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] -((A*(a + b*x^3)^(5/2))/(a*x)) - (((-13*A*b)/2 - a*B)*x^2*Sqrt[a + b*x^3]*H
ypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.017, size = 937, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x)
```

```
[Out] B*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(
b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-
I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^2, x)
```

Sympy [A] time = 3.97357, size = 173, normalized size = 0.3

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{A\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\right.}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**2,x)
```

```
[Out] A*a**(3/2)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) + A*sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*a**(3/2)*x**2*gamma(2/3)*hype
r((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*sqrt(a)
*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

3.210 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^5}\,dx$

Optimal. Leaf size=578

$$\frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+7Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\ldots}{}$$

```
[Out] (9*b*(7*A*b + 8*a*B)*x^2*Sqrt[a + b*x^3])/(56*a) + (27*b^(1/3)*(7*A*b + 8*a
*B)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((7*A*b + 8
*a*B)*(a + b*x^3)^(3/2))/(8*a*x) - (A*(a + b*x^3)^(5/2))/(4*a*x^4) - (27*3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
9*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.291731, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 279, 303, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

```
[Out] (9*b*(7*A*b + 8*a*B)*x^2*Sqrt[a + b*x^3])/(56*a) + (27*b^(1/3)*(7*A*b + 8*a
*B)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((7*A*b + 8
```

```
*a*B)*(a + b*x^3)^(3/2))/(8*a*x) - (A*(a + b*x^3)^(5/2))/(4*a*x^4) - (27*3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
9*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^5}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} - \frac{\left(-\frac{7Ab}{2}-4aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^2}\,dx}{4a}$$

$$= -\frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax} - \frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} + \frac{(9b(7Ab+8aB))\int x\sqrt{a+bx^3}\,dx}{16a}$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax} - \frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} + \frac{1}{112}(27b(7Ab$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax} - \frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} + \frac{1}{112}\left(27b^{2/3}(7\right.$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} + \frac{27\sqrt[3]{b}(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax} -$$

Mathematica [C] time = 0.074026, size = 85, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(-4aB-\frac{7Ab}{2}\right){}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{4x\sqrt{\frac{bx^3}{a}+1}}-\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

```
[Out] -(A*(a + b*x^3)^(5/2))/(4*a*x^4) + (((-7*A*b)/2 - 4*a*B)*Sqrt[a + b*x^3]*Hy
pergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(4*x*Sqrt[1 + (b*x^3)/a])
```

Maple [B] time = 0.021, size = 932, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x)
```

```
[Out] B*(-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))))+A*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1
/2)/x-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
```

```
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bbx^6+(Ba+Ab)x^3+Aa)\sqrt{bx^3+a}}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^5, x)
```

Sympy [A] time = 6.09603, size = 182, normalized size = 0.31

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{A\sqrt{a}b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{B\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**5,x)
```

```
[Out] A*a**(3/2)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + A*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*a**(3/2)*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*sqrt
(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

3.211 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^8}\,dx$

Optimal. Leaf size=576

$$\frac{9\, 3^{3/4} b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14aB+Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}}{}$$

```
[Out] (-9*b*(A*b + 14*a*B)*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(A*b + 14*a*B
)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((A*b + 14
*a*B)*(a + b*x^3)^(3/2))/(56*a*x^4) - (A*(a + b*x^3)^(5/2))/(7*a*x^7) - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) +
 (9*3^(3/4)*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.29661, antiderivative size = 576, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 303, 218, 1877}

$$\frac{9\, 3^{3/4} b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (-9*b*(A*b + 14*a*B)*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(A*b + 14*a*B
)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((A*b + 14
```

```
*a*B)*(a + b*x^3)^(3/2))/(56*a*x^4) - (A*(a + b*x^3)^(5/2))/(7*a*x^7) - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) +
 (9*3^(3/4)*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^8}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7} - \frac{\left(-\frac{Ab}{2}-7aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^5}\,dx}{7a}$$

$$= -\frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4} - \frac{A\left(a+bx^3\right)^{5/2}}{7ax^7} + \frac{(9b(Ab+14aB))\int \frac{\sqrt{a+bx^3}}{x^2}\,dx}{112a}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} - \frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4} - \frac{A\left(a+bx^3\right)^{5/2}}{7ax^7} + \frac{\left(27b^2(Ab+14 \cdots\right.}{2 \cdots}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} - \frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4} - \frac{A\left(a+bx^3\right)^{5/2}}{7ax^7} + \frac{\left(27b^{5/3}(Ab+1 \cdots\right.}{\cdots}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4} - \cdots$$

Mathematica [C] time = 0.0754099, size = 82, normalized size = 0.14

$$\frac{\sqrt{a+bx^3}\left(-\frac{x^3(14aB+Ab)\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{4A\left(a+bx^3\right)^2}{a}\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-4*A*(a + b*x^3)^2)/a - ((A*b + 14*a*B)*x^3*Hypergeometr
ic2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(28*x^7)
```

Maple [B] time = 0.025, size = 957, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x)
```

```
[Out] B*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3
+a)^(1/2)/x^4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-9/112*I*b^2/a*3^(1/2)*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^8, x)
```

Sympy [A] time = 4.95087, size = 194, normalized size = 0.34

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3}, -\frac{1}{2}\\ -\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, -\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, -\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\right.}{3x\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**8,x)
```

```
[Out] A*a**(3/2)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + A*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + B*a**(3/2)*gamma(-4/3)
*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)
) + B*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

3.212 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx$

Optimal. Leaf size=608

$$-\frac{9\,3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{224\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27b^2\sqrt{a+bx^3}(Ab}{448a^2x}$$

```
[Out] (9*b*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(224*a*x^4) + (27*b^2*(A*b - 4*a*B)*Sqr
t[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(448
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((A*b - 4*a*B)*(a + b*x^3)^(3/2
))/(28*a*x^7) - (A*(a + b*x^3)^(5/2))/(10*a*x^10) + (27*3^(1/4)*Sqrt[2 - Sq
rt[3]]*b^(7/3)*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*b^(7/3)*(
A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(224*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.354608, antiderivative size = 608, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 325, 303, 218, 1877}

$$\frac{27b^2\sqrt{a+bx^3}(Ab-4aB)}{448a^2x}-\frac{27b^{7/3}\sqrt{a+bx^3}(Ab-4aB)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{9\,3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)F\left(\sin^{-1}\right.}{224\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

```
[Out] (9*b*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(224*a*x^4) + (27*b^2*(A*b - 4*a*B)*Sqr
t[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(448
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((A*b - 4*a*B)*(a + b*x^3)^(3/2
))/(28*a*x^7) - (A*(a + b*x^3)^(5/2))/(10*a*x^10) + (27*3^(1/4)*Sqrt[2 - Sq
rt[3]]*b^(7/3)*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*b^(7/3)*(
A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(224*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
```

```
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}} - \frac{\left(\frac{5Ab}{2}-10aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^8}\,dx}{10a}$$

$$= \frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7} - \frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}} - \frac{(9b(Ab-4aB))\int \frac{\sqrt{a+bx^3}}{x^5}\,dx}{56a}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7} - \frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}} - \frac{\left(27b^2(Ab-4aB\right.}{448}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} + \frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7} - \frac{A(a}{10}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} + \frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7} - \frac{A(a}{10}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} - \frac{27b^{7/3}(Ab-4aB)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \cdots$$

Mathematica [C] time = 0.0748919, size = 82, normalized size = 0.13

$$\frac{\sqrt{a+bx^3}\left(\frac{5x^3(Ab-4aB)\,{}_2F_1\left(-\frac{7}{3},-\frac{3}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - \frac{7A\left(a+bx^3\right)^2}{a}\right)}{70x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-7*A*(a + b*x^3)^2)/a + (5*(A*b - 4*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(70*x^10)
```

Maple [B] time = 0.023, size = 1002, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x)
```

```
[Out] A*(-1/10*a*(b*x^3+a)^(1/2)/x^10-23/140*b*(b*x^3+a)^(1/2)/x^7-27/1120*b^2/a*
(b*x^3+a)^(1/2)/x^4+27/448*b^3/a^2*(b*x^3+a)^(1/2)/x+9/448*I*b^3/a^2*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+B*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*
(b*x^3+a)^(1/2)/x^4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-9/112*I*b^2/a*3^(1/2)*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^11, x)
```

Sympy [A] time = 6.48285, size = 199, normalized size = 0.33

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix} -\frac{10}{3}, -\frac{1}{2} \\ -\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{4}{3}\right)2}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**11,x)
```

```
[Out] A*a**(3/2)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + A*sqrt(a)*b*gamma(-7/3)*hyper((-7/3, -1/2), (-
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + B*a**(3/2)*gamma(-7
/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4
/3)) + B*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

3.213 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=103

$$\frac{2a^2\sqrt{a+bx^3}(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4}-\frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

```
[Out] (2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x
^3)^(3/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*B*(a
 + b*x^3)^(7/2))/(21*b^4)
```

Rubi [A] time = 0.073733, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\sqrt{a+bx^3}(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4}-\frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x
^3)^(3/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*B*(a
 + b*x^3)^(7/2))/(21*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}} + \frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3} + \frac{(Ab-3aB)(a+bx)^{3/2}}{b^3} + \frac{B(a+bx)^{5/2}}{b^3}\right.\right.$$
$$= \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4} - \frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4} + \frac{2B\left(a+bx^3\right.}{21b^4}$$

Mathematica [A] time = 0.0547454, size = 78, normalized size = 0.76

$$\frac{2\sqrt{a+bx^3}\left(8a^2b\left(7A+3Bx^3\right)-48a^3B-2ab^2x^3\left(14A+9Bx^3\right)+3b^3x^6\left(7A+5Bx^3\right)\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x^3) + 3*b^3*x^6*(7*A +
5*B*x^3) - 2*a*b^2*x^3*(14*A + 9*B*x^3)))/(315*b^4)
```

Maple [A] time = 0.006, size = 77, normalized size = 0.8

$$\frac{30\,Bx^9b^3+42\,Ab^3x^6-36\,Bab^2x^6-56\,Aab^2x^3+48\,Ba^2bx^3+112\,Aa^2b-96\,Ba^3}{315\,b^4}\sqrt{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/315*(b*x^3+a)^(1/2)*(15*B*b^3*x^9+21*A*b^3*x^6-18*B*a*b^2*x^6-28*A*a*b^2*
x^3+24*B*a^2*b*x^3+56*A*a^2*b-48*B*a^3)/b^4
```

Maxima [A] time = 0.93774, size = 159, normalized size = 1.54

$$\frac{2}{105}\,B\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^4}-\frac{21\left(bx^3+a\right)^{\frac{5}{2}}a}{b^4}+\frac{35\left(bx^3+a\right)^{\frac{3}{2}}a^2}{b^4}-\frac{35\sqrt{bx^3+a}\,a^3}{b^4}\right)+\frac{2}{45}\,A\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^3}-\frac{10\left(bx^3+a\right)^{\frac{3}{2}}a}{b^3}+15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*B*(5*(b*x^3 + a)^(7/2)/b^4 - 21*(b*x^3 + a)^(5/2)*a/b^4 + 35*(b*x^3 +
 a)^(3/2)*a^2/b^4 - 35*sqrt(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^(5/
2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3)
```

Fricas [A] time = 1.72196, size = 173, normalized size = 1.68

$$\frac{2\left(15\,Bb^3x^9-3\left(6\,Bab^2-7\,Ab^3\right)x^6-48\,Ba^3+56\,Aa^2b+4\left(6\,Ba^2b-7\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [A] time = 3.36788, size = 175, normalized size = 1.7

$$\begin{cases} \frac{16Aa^2\sqrt{a+bx^3}}{45b^3}-\frac{8Aax^3\sqrt{a+bx^3}}{45b^2}+\frac{2Ax^6\sqrt{a+bx^3}}{15b}-\frac{32Ba^3\sqrt{a+bx^3}}{105b^4}+\frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3}-\frac{4Bax^6\sqrt{a+bx^3}}{35b^2}+\frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x**
3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x**3
```

```
)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*sqrt
(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), ((A*x
**9/9 + B*x**12/12)/sqrt(a), True))
```

Giac [A] time = 1.13856, size = 140, normalized size = 1.36

$$\frac{2\left(15\left(bx^3+a\right)^{\frac{7}{2}}B-63\left(bx^3+a\right)^{\frac{5}{2}}Ba+105\left(bx^3+a\right)^{\frac{3}{2}}Ba^2-105\sqrt{bx^3+a}Ba^3+21\left(bx^3+a\right)^{\frac{5}{2}}Ab-70\left(bx^3+a\right)^{\frac{3}{2}}Aab+\right.}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(15*(b*x^3 + a)^(7/2)*B - 63*(b*x^3 + a)^(5/2)*B*a + 105*(b*x^3 + a)^
(3/2)*B*a^2 - 105*sqrt(b*x^3 + a)*B*a^3 + 21*(b*x^3 + a)^(5/2)*A*b - 70*(b*
x^3 + a)^(3/2)*A*a*b + 105*sqrt(b*x^3 + a)*A*a^2*b)/b^4
```

3.214 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-2aB)}{9b^3}-\frac{2a\sqrt{a+bx^3}(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

```
[Out] (-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(
3/2))/(9*b^3) + (2*B*(a + b*x^3)^(5/2))/(15*b^3)
```

Rubi [A] time = 0.0546994, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-2aB)}{9b^3}-\frac{2a\sqrt{a+bx^3}(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(
3/2))/(9*b^3) + (2*B*(a + b*x^3)^(5/2))/(15*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}}+\frac{(Ab-2aB)\sqrt{a+bx}}{b^2}+\frac{B(a+bx)^{3/2}}{b^2}\right)dx,x,x^3\right)\\ &= -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.039873, size = 56, normalized size = 0.77

$$\frac{2\sqrt{a+bx^3}\left(8a^2B-2ab\left(5A+2Bx^3\right)+b^2x^3\left(5A+3Bx^3\right)\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(8*a^2*B - 2*a*b*(5*A + 2*B*x^3) + b^2*x^3*(5*A + 3*B*x^
3)))/(45*b^3)
```

Maple [A] time = 0.008, size = 53, normalized size = 0.7

$$-\frac{-6\,b^2Bx^6-10\,Ax^3b^2+8\,Bx^3ab+20\,abA-16\,a^2B}{45\,b^3}\sqrt{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/45*(b*x^3+a)^(1/2)*(-3*B*b^2*x^6-5*A*b^2*x^3+4*B*a*b*x^3+10*A*a*b-8*B*a^
2)/b^3
```

Maxima [A] time = 0.929006, size = 112, normalized size = 1.53

$$\frac{2}{45}\,B\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^3}-\frac{10\left(bx^3+a\right)^{\frac{3}{2}}a}{b^3}+\frac{15\,\sqrt{bx^3+a}a^2}{b^3}\right)+\frac{2}{9}\,A\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{bx^3+a}a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^
3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)
```

Fricas [A] time = 1.6849, size = 117, normalized size = 1.6

$$\frac{2\left(3\,Bb^2x^6-\left(4\,Bab-5\,Ab^2\right)x^3+8\,Ba^2-10\,Aab\right)\sqrt{bx^3+a}}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x^
3 + a)/b^3
```

Sympy [A] time = 1.59172, size = 124, normalized size = 1.7

$$\begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2}+\frac{2Ax^3\sqrt{a+bx^3}}{9b}+\frac{16Ba^2\sqrt{a+bx^3}}{45b^3}-\frac{8Bax^3\sqrt{a+bx^3}}{45b^2}+\frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for}\ b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9*
b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(45
*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**9/9
)/sqrt(a), True))
```

Giac [A] time = 1.11859, size = 99, normalized size = 1.36

$$\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}B-10\left(bx^3+a\right)^{\frac{3}{2}}Ba+15\sqrt{bx^3+a}Ba^2+5\left(bx^3+a\right)^{\frac{3}{2}}Ab-15\sqrt{bx^3+a}Aab\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*(b*x^3 + a)^(5/2)*B - 10*(b*x^3 + a)^(3/2)*B*a + 15*sqrt(b*x^3 + a)
*B*a^2 + 5*(b*x^3 + a)^(3/2)*A*b - 15*sqrt(b*x^3 + a)*A*a*b)/b^3
```

3.215 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}} dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{a+bx^3}(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}$$

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^2) + (2*B*(a + b*x^3)^(3/2))/(9*b^2)
```

Rubi [A] time = 0.0378856, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\sqrt{a+bx^3}(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^2) + (2*B*(a + b*x^3)^(3/2))/(9*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}} + \frac{B\sqrt{a+bx}}{b}\right) dx, x, x^3\right)$$
$$= \frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2} + \frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}$$

Mathematica [A] time = 0.0221966, size = 33, normalized size = 0.72

$$\frac{2\sqrt{a+bx^3}\left(-2aB+3Ab+bBx^3\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)
```

Maple [A] time = 0.006, size = 30, normalized size = 0.7

$$\frac{2\,bBx^3+6\,Ab-4\,Ba}{9\,b^2}\sqrt{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/9*(b*x^3+a)^(1/2)*(B*b*x^3+3*A*b-2*B*a)/b^2
```

Maxima [A] time = 0.938191, size = 65, normalized size = 1.41

$$\frac{2}{9}\,B\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3+a}a}{b^2}\right) + \frac{2\,\sqrt{bx^3+a}A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 +
a)*A/b
```

Fricas [A] time = 1.74681, size = 69, normalized size = 1.5

$$\frac{2\left(Bbx^3 - 2\,Ba + 3\,Ab\right)\sqrt{bx^3 + a}}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2
```

Sympy [A] time = 0.768402, size = 75, normalized size = 1.63

$$\begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) + 2
*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a),
True))
```

Giac [A] time = 1.11092, size = 58, normalized size = 1.26

$$\frac{2\left(\left(bx^3 + a\right)^{\frac{3}{2}}B - 3\,\sqrt{bx^3 + a}Ba + 3\,\sqrt{bx^3 + a}Ab\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*((b*x^3 + a)^(3/2)*B - 3*sqrt(b*x^3 + a)*B*a + 3*sqrt(b*x^3 + a)*A*b)/b
^2
```

3.216 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}} dx$

Optimal. Leaf size=48

$$\frac{2B\sqrt{a+bx^3}}{3b} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Rubi [A] time = 0.0316857, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{2B\sqrt{a+bx^3}}{3b} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{1}{3}A\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{(2A)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.025787, size = 48, normalized size = 1.

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Maple [A] time = 0.016, size = 37, normalized size = 0.8

$$-\frac{2A}{3}\operatorname{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}+\frac{2B}{3b}\sqrt{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8653, size = 252, normalized size = 5.25

$$\left[\frac{A\sqrt{a}b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\sqrt{bx^3+a}Ba}{3ab},\frac{2\left(A\sqrt{-a}b\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\sqrt{bx^3+a}Ba\right)}{3ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*sq
rt(b*x^3 + a)*B*a)/(a*b), 2/3*(A*sqrt(-a)*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + sqrt(b*x^3 + a)*B*a)/(a*b)]
```

Sympy [A] time = 6.97342, size = 65, normalized size = 1.35

$$\frac{2A\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+bx^3}}\right)}{3a\sqrt{-\frac{1}{a}}}-\frac{B\left(\begin{cases}-\frac{x^3}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**3)))/(3*a*sqrt(-1/a)) - B*Piecewise((-
x**3/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**3)/b, True))/3
```

Giac [A] time = 1.14264, size = 54, normalized size = 1.12

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2\sqrt{bx^3+a}B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)*B/b
```

3.217 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=58

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

```
[Out] -(A*Sqrt[a + b*x^3])/(3*a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2))
```

Rubi [A] time = 0.0457008, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(3*a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a}\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{\left(2\left(-\frac{Ab}{2}+aB\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3ab}\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0388455, size = 57, normalized size = 0.98

$$\frac{1}{3}\left(\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] (-((A*Sqrt[a + b*x^3])/(a*x^3)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/a^(3/2))/3
```

Maple [A] time = 0.02, size = 62, normalized size = 1.1

$$A\left(-\frac{1}{3\,ax^{3}}\sqrt{bx^{3}+a}+\frac{b}{3}\mathrm{Artanh}\left(\sqrt{bx^{3}+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}\right)-\frac{2\,B}{3}\mathrm{Artanh}\left(\sqrt{bx^{3}+a}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x)
```

```
[Out] A*(-1/3/a*(b*x^3+a)^(1/2)/x^3+1/3*b/a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)
))-2/3*B*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78028, size = 302, normalized size = 5.21

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}x^{3}\log\left(\frac{bx^{3}+2\,\sqrt{bx^{3}+a}\sqrt{a}+2\,a}{x^{3}}\right)+2\,\sqrt{bx^{3}+a}Aa}{6\,a^{2}x^{3}},\ \frac{(2\,Ba-Ab)\sqrt{-a}x^{3}\arctan\left(\frac{\sqrt{bx^{3}+a}\sqrt{-a}}{a}\right)-\sqrt{bx^{3}+a}Aa}{3\,a^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*B*a - A*b)*sqrt(a)*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) + 2*sqrt(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*sqrt(-a)*x^
3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^3 + a)*A*a)/(a^2*x^3)]
```

Sympy [A] time = 16.3022, size = 80, normalized size = 1.38

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}-\frac{2B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)
)
```

Giac [A] time = 1.15022, size = 84, normalized size = 1.45

$$\frac{\frac{\left(2\,Bab-Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{\sqrt{bx^3+a}Ab}{ax^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^3 + a)*A*b/(a*x^3))/b
```

3.218 $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a+bx^3}(3Ab-4aB)}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

```
[Out] -(A*Sqrt[a + b*x^3])/(6*a*x^6) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a^2*
x^3) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Rubi [A] time = 0.0695414, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}(3Ab-4aB)}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(6*a*x^6) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a^2*
x^3) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{\left(-\frac{3Ab}{2}+2aB\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right)}{6a}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(b(3Ab-4aB))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{24a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(3Ab-4aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{12a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.205633, size = 81, normalized size = 0.9

$$\frac{\sqrt{a+bx^3}\left(b\left(2aB-\frac{3Ab}{2}\right)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\frac{a}{bx^3}\right)-\frac{a^2A}{x^6}\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-((a^2*A)/x^6) + b*((-3*A*b)/2 + 2*a*B)*(-(a/(b*x^3)) + A
rcTanh[Sqrt[1 + (b*x^3)/a]]/Sqrt[1 + (b*x^3)/a])))/(6*a^3)
```

Maple [A] time = 0.022, size = 102, normalized size = 1.1

$$A\left(-\frac{1}{6\,ax^6}\sqrt{bx^3+a}+\frac{b}{4\,a^2x^3}\sqrt{bx^3+a}-\frac{b^2}{4}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{5}{2}}\right)+B\left(-\frac{1}{3\,ax^3}\sqrt{bx^3+a}+\frac{b}{3}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x)
```

```
[Out] A*(-1/6/a*(b*x^3+a)^(1/2)/x^6+1/4/a^2*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2/a^(5/2)
*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*(-1/3/a*(b*x^3+a)^(1/2)/x^3+1/3*b/a^(3
/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82213, size = 409, normalized size = 4.54

$$\left[-\frac{\left(4\,Bab-3\,Ab^{2}\right)\sqrt{a}x^{6}\log\left(\frac{bx^{3}-2\,\sqrt{bx^{3}+a}\sqrt{a}+2\,a}{x^{3}}\right)+2\,\left(\left(4\,Ba^{2}-3\,Aab\right)x^{3}+2\,Aa^{2}\right)\sqrt{bx^{3}+a}}{24\,a^{3}x^{6}},-\frac{\left(4\,Bab-3\,Ab^{2}\right)\sqrt{-a}x^{6}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a
^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqr
t(-a)/a) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6)]
```

Sympy [B] time = 31.3192, size = 163, normalized size = 1.81

$$-\frac{A}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{Ab^{\frac{3}{2}}}{4a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{Ab^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{3}}+1}}{3ax^{\frac{3}{2}}}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + A*sqrt(b)/(12*a*x**(9/2)*sq
rt(a/(b*x**3) + 1)) + A*b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A
*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*
x**3) + 1)/(3*a*x**(3/2)) + B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a**(3/
2))
```

Giac [A] time = 1.1216, size = 163, normalized size = 1.81

$$-\frac{\frac{\left(4\,Bab^{2}-3\,Ab^{3}\right)\arctan\left(\frac{\sqrt{bx^{3}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{4\,\left(bx^{3}+a\right)^{\frac{3}{2}}Bab^{2}-4\,\sqrt{bx^{3}+a}Ba^{2}b^{2}-3\,\left(bx^{3}+a\right)^{\frac{3}{2}}Ab^{3}+5\,\sqrt{bx^{3}+a}Aab^{3}}{a^{2}b^{2}x^{6}}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3 +
 a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b
```

3.219 $\int \frac{x^3\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=270

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+bx^3}(}{55}$$

```
[Out] (2*(11*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(55*b^2) + (2*B*x^4*Sqrt[a + b*x^3])
/(11*b) - (4*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rubi [A] time = 0.101476, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {459, 321, 218}

$$-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+bx^3}(11Ab-}{55b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(11*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(55*b^2) + (2*B*x^4*Sqrt[a + b*x^3])
/(11*b) - (4*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+4aB\right)\right)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{11b}$$

$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{(2a(11Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{55b^2}$$

$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{4\sqrt{2+\sqrt{3}}a(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}\cdots}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}}\cdots}$$

Mathematica [C] time = 0.0771953, size = 89, normalized size = 0.33

$$\frac{2x\left(a\sqrt{\frac{bx^3}{a}+1}(8aB-11Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\left(8aB-11Ab-5bBx^3\right)\right)}{55b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*(-((a + b*x^3)*(-11*A*b + 8*a*B - 5*b*B*x^3)) + a*(-11*A*b + 8*a*B)*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^2*
Sqrt[a + b*x^3])
```

Maple [B] time = 0.023, size = 624, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/11/b*x^4*(b*x^3+a)^(1/2)-16/55*a/b^2*x*(b*x^3+a)^(1/2)-32/165*I*a^2/b^
3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/5/b*x
*(b*x^3+a)^(1/2)+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^3}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^6 + Ax^3}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 3.57547, size = 80, normalized size = 0.3

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)x^3}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

3.220 $\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=239

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2Bx\sqrt{a+bx^3}}{5b}$$

```
[Out] (2*B*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^
(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.062515, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {388, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2Bx\sqrt{a+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/Sqrt[a + b*x^3],x]
```


```
[Out] (2*B*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^
(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```


Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx\sqrt{a+bx^3}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{2Bx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0367592, size = 74, normalized size = 0.31

$$\frac{x\sqrt{\frac{bx^3}{a}+1}(5Ab-2aB)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+2Bx\left(a+bx^3\right)}{5b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x*(a + b*x^3) + (5*A*b - 2*a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F
1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(5*b*Sqrt[a + b*x^3])
```

Maple [B] time = 0.016, size = 586, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/5/b*x*(b*x^3+a)^(1/2)+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))-2/3*I*A*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^3 + A}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.24463, size = 78, normalized size = 0.33

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

3.221 $\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=243

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

```
[Out] -(A*Sqrt[a + b*x^3])/(2*a*x^2) - (Sqrt[2 + Sqrt[3]]*(A*b - 4*a*B)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0689875, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 218}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(2*a*x^2) - (Sqrt[2 + Sqrt[3]]*(A*b - 4*a*B)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\left(\frac{Ab}{2}-2aB\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\sqrt{2+\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0314648, size = 78, normalized size = 0.32

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(4aB-Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-2A\left(a+bx^3\right)}{4ax^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (-(A*b) + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(4*a*x^2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.019, size = 587, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*B*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+A*
(-1/2/a*(b*x^3+a)^(1/2)/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{bx^6 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^6 + a*x^3), x)
```

Sympy [A] time = 1.90012, size = 82, normalized size = 0.34

$$\frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

3.222 $\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}(7Ab}{20a^2x}$$

```
[Out] -(A*Sqrt[a + b*x^3])/(5*a*x^5) + ((7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a^2
*x^2) + (Sqrt[2 + Sqrt[3]]*b^(2/3)*(7*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*3^(1/4)*a^2*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rubi [A] time = 0.101739, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 325, 218}

$$\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}(7Ab-10aB}{20a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(5*a*x^5) + ((7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a^2
*x^2) + (Sqrt[2 + Sqrt[3]]*b^(2/3)*(7*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*3^(1/4)*a^2*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx &= -\frac{A\sqrt{a+bx^3}}{5ax^5}-\frac{\left(\frac{7Ab}{2}-5aB\right)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{5a}\\
&= -\frac{A\sqrt{a+bx^3}}{5ax^5}+\frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2}+\frac{(b(7Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{5ax^5}+\frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2}+\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right.}}}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}
\end{aligned}
$$

Mathematica [C] time = 0.0332985, size = 78, normalized size = 0.28

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(7Ab-10aB)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)-4A\left(a+bx^3\right)}{20ax^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*A*(a + b*x^3) + (7*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(20*a*x^5*Sqrt[a + b*x^3])
```

Maple [B] time = 0.022, size = 625, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(-1/2/a*(b*x^3+a)^(1/2)/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/5/a*(b*x^3+a)^(1/2)/x^5+7/20/a^2*b*(b*x^3+a
)^(1/2)/x^2-7/60*I*b/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + ax^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{bx^9 + ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^9 + a*x^6), x)
```

Sympy [A] time = 2.44721, size = 90, normalized size = 0.33

$$\frac{A\Gamma\left(-\frac{5}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{5}{3}, \frac{1}{2} \\ -\frac{2}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{1}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x**2*gamma(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + ax^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

3.223 $\int \frac{x^4\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=548

$$-\frac{8\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a\cdots}{\cdots}$$

```
[Out] (2*(13*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(91*b^2) + (2*B*x^5*Sqrt[a + b*x^
3])/(13*b) - (8*a*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(91*b^(8/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b -
 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(91*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*a^(4/3)*(13*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)
*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.245451, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 321, 303, 218, 1877}

$$-\frac{8\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(13*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(91*b^2) + (2*B*x^5*Sqrt[a + b*x^
3])/(13*b) - (8*a*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(91*b^(8/3)*((1 + Sqrt
```

```
[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b -
 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(91*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*a^(4/3)*(13*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)
*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{\left(2\left(-\frac{13Ab}{2}+5aB\right)\right)\int\frac{x^4}{\sqrt{a+bx^3}}\,dx}{13b}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{(4a(13Ab-10aB))\int\frac{x}{\sqrt{a+bx^3}}\,dx}{91b^2}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{(4a(13Ab-10aB))\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{91b^{7/3}} - \frac{\left(4\sqrt{2\left(\right.}\right.}{}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}}{}$$

Mathematica [C] time = 0.0771294, size = 91, normalized size = 0.17

$$\frac{2x^2\left(a\sqrt{\frac{bx^3}{a}+1}(10aB-13Ab)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\left(10aB-13Ab-7bBx^3\right)\right)}{91b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*(-((a + b*x^3)*(-13*A*b + 10*a*B - 7*b*B*x^3)) + a*(-13*A*b + 10*a*B
)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(91*
b^2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.022, size = 932, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/13/b*x^5*(b*x^3+a)^(1/2)-20/91*a/b^2*x^2*(b*x^3+a)^(1/2)-80/273*I*a^2/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/7/b*x^2*(b*x^3+a)^(1/2)
+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^4}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^7 + Ax^4}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^7 + A*x^4)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.45576, size = 80, normalized size = 0.15

$$\frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(11/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^4}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

3.224 $\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=517

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] (2*B*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*A*b - 4*a*B)*Sqrt[a + b*x^3])/(7*b^
(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(7*A*b - 4*
a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7
*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.18611, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {459, 303, 218, 1877}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*A*b - 4*a*B)*Sqrt[a + b*x^3])/(7*b^
(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
```

```
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(7*A*b - 4*
a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7
*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{2Bx^2\sqrt{a+bx^3}}{7b}-\frac{\left(2\left(-\frac{7Ab}{2}+2aB\right)\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b}\\
&= \frac{2Bx^2\sqrt{a+bx^3}}{7b}+\frac{(7Ab-4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}}+\frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt[3]{a}(7Ab-4aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}}\\
&= \frac{2Bx^2\sqrt{a+bx^3}}{7b}+\frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{\cdots}}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.0538732, size = 75, normalized size = 0.15

$$
\frac{x^2\left(\sqrt{\frac{bx^3}{a}+1}(7Ab-4aB)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4B\left(a+bx^3\right)\right)}{14b\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x^2*(4*B*(a + b*x^3) + (7*A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(14*b*Sqrt[a + b*x^3])
```

Maple [B] time = 0.013, size = 892, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/7/b*x^2*(b*x^3+a)^(1/2)+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
```

```
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))))-2/3*I*A*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^4 + Ax}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)/sqrt(b*x^3 + a), x)
```

Sympy [A] time = 2.09443, size = 80, normalized size = 0.15

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(Bx^3+A)x}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

3.225 $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=509

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

```
[Out] -((A*Sqrt[a + b*x^3])/(a*x)) + ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(a*b^(2/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (Sqrt[2]*(A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.19513, antiderivative size = 509, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 303, 218, 1877}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-}{\left(\left(1+\right.\right.}}}{2a^{2/}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((A*Sqrt[a + b*x^3])/(a*x)) + ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(a*b^(2/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
```

```
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (Sqrt[2]*(A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{ax} - \frac{\left(-\frac{Ab}{2}-aB\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{a}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt[3]{b}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}(Ab+2aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}}}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}}}$$

Mathematica [C] time = 0.0307633, size = 77, normalized size = 0.15

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(2aB+Ab)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-4A\left(a+bx^3\right)}{4ax\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*A*(a + b*x^3) + (A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[1/2, 2/3, 5/3, -((b*x^3)/a)])/(4*a*x*Sqrt[a + b*x^3])
```

Maple [B] time = 0.021, size = 891, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*B*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/a*(b*x^3+a)^(1/2
)/x-1/3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{bx^5 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^5 + a*x^2), x)
```

Sympy [A] time = 1.75708, size = 82, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

3.226 $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=550

$$-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] -(A*Sqrt[a + b*x^3])/(4*a*x^4) + ((5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*x
) - (b^(1/3)*(5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(5/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]) - (b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rubi [A] time = 0.251813, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 325, 303, 218, 1877}

$$-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(4*a*x^4) + ((5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*x
) - (b^(1/3)*(5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*((1 + Sqrt[3])*a^(1/3)
```

```
 + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(5/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]) - (b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx &= -\frac{A\sqrt{a+bx^3}}{4ax^4}-\frac{\left(\frac{5Ab}{2}-4aB\right)\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{4a}\\ &= -\frac{A\sqrt{a+bx^3}}{4ax^4}+\frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x}-\frac{(b(5Ab-8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a^2}\\ &= -\frac{A\sqrt{a+bx^3}}{4ax^4}+\frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x}-\frac{\left(b^{2/3}(5Ab-8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{16a^2}-\frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}b^{2/3}(\ldots\right.}{\ldots}\\ &= -\frac{A\sqrt{a+bx^3}}{4ax^4}+\frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x}-\frac{\sqrt[3]{b}(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(5Ab-8aB)\left(\ldots\right.}{\ldots}\end{aligned}$$

Mathematica [C] time = 0.0351921, size = 78, normalized size = 0.14

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(5Ab-8aB)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)-2A\left(a+bx^3\right)}{8ax^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (5*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(8*a*x^4*Sqrt[a + b*x^3])
```

Maple [B] time = 0.02, size = 929, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(-1/a*(b*x^3+a)^(1/2)/x-1/3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
)+A*(-1/4/a*(b*x^3+a)^(1/2)/x^4+5/8/a^2*b*(b*x^3+a)^(1/2)/x+5/24*I*b/a^2*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{bx^8+ax^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^8 + a*x^5), x)
```

Sympy [A] time = 2.17487, size = 88, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

3.227 $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=581

$$\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{5b^{4/3}\sqrt{a+bx^3}(11Ab-}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}$$

```
[Out] -(A*Sqrt[a + b*x^3])/(7*a*x^7) + ((11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a^
2*x^4) - (5*b*(11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*x) + (5*b^(4/3)*(
11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(8/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + (5*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rubi [A] time = 0.307602, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 325, 303, 218, 1877}

$$\frac{5b^{4/3}\sqrt{a+bx^3}(11Ab-14aB)}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(7*a*x^7) + ((11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a^
2*x^4) - (5*b*(11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*x) + (5*b^(4/3)*(
```

```
11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(8/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + (5*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx &= -\frac{A\sqrt{a+bx^3}}{7ax^7}-\frac{\left(\frac{11Ab}{2}-7aB\right)\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{7a}\\
&= -\frac{A\sqrt{a+bx^3}}{7ax^7}+\frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4}+\frac{(5b(11Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{112a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{7ax^7}+\frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4}-\frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x}+\frac{\left(5b^2(11Ab-14aB)\right)\int \frac{x}{\sqrt{a+b}}}{224a^3}\\
&= -\frac{A\sqrt{a+bx^3}}{7ax^7}+\frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4}-\frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x}+\frac{\left(5b^{5/3}(11Ab-14aB)\right)\int \frac{(1-}{}}{224a^3}\\
&= -\frac{A\sqrt{a+bx^3}}{7ax^7}+\frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4}-\frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x}+\frac{5b^{4/3}(11Ab-14aB)\sqrt{a+bx}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right.}
\end{aligned}
$$

Mathematica [C] time = 0.0341074, size = 78, normalized size = 0.13

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(11Ab-14aB)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)-8A\left(a+bx^3\right)}{56ax^7\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] (-8*A*(a + b*x^3) + (11*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)])/(56*a*x^7*Sqrt[a + b*x^3])
```

Maple [B] time = 0.023, size = 970, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x)
```

```
[Out] A*(-1/7/a*(b*x^3+a)^(1/2)/x^7+11/56/a^2*b*(b*x^3+a)^(1/2)/x^4-55/112/a^3*b^
2*(b*x^3+a)^(1/2)/x-55/336*I*b^2/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))))+B*(-1/4/a*(b*x^3+a)^(1/2)/x^4+5/8/a^2*b*(b*x^3+a)^(1/2)/x+5/24*I*b/a^2
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + ax^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{bx^{11}+ax^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^11 + a*x^8), x)
```

Sympy [A] time = 2.95047, size = 94, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*x**4*gamma(-1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bx^3+A}{\sqrt{bx^3+a}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

3.228 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\left(a+bx^3\right)^{3/2}(Ab-3aB)}{9b^4}-\frac{2a\sqrt{a+bx^3}(2Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

```
[Out] (-2*a^2*(A*b - a*B))/(3*b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*B*(a +
 b*x^3)^(5/2))/(15*b^4)
```

Rubi [A] time = 0.0767542, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\left(a+bx^3\right)^{3/2}(Ab-3aB)}{9b^4}-\frac{2a\sqrt{a+bx^3}(2Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(3*b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*B*(a +
 b*x^3)^(5/2))/(15*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}}+\frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}}+\frac{(Ab-3aB)\sqrt{a+bx}}{b^3}+\frac{B(a+bx)^{3/2}}{b^3}\right)dx,x,x^3\right)\\
&= -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}-\frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}
\end{aligned}$$

Mathematica [A] time = 0.0515247, size = 77, normalized size = 0.75

$$\frac{2\left(-8a^2b\left(5A-3Bx^3\right)+48a^3B-2ab^2x^3\left(10A+3Bx^3\right)+b^3x^6\left(5A+3Bx^3\right)\right)}{45b^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2*
x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])
```

Maple [A] time = 0.007, size = 77, normalized size = 0.8

$$-\frac{-6\,Bx^9b^3-10\,Ab^3x^6+12\,Bab^2x^6+40\,Aab^2x^3-48\,Ba^2bx^3+80\,Aa^2b-96\,Ba^3}{45\,b^4}\frac{1}{\sqrt{bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/45/(b*x^3+a)^(1/2)*(-3*B*b^3*x^9-5*A*b^3*x^6+6*B*a*b^2*x^6+20*A*a*b^2*x^
3-24*B*a^2*b*x^3+40*A*a^2*b-48*B*a^3)/b^4
```

Maxima [A] time = 0.943429, size = 157, normalized size = 1.52

$$\frac{2}{15}\,B\left(\frac{\left(bx^3+a\right)^{\frac{5}{2}}}{b^4}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}a}{b^4}+\frac{15\,\sqrt{bx^3+a}a^2}{b^4}+\frac{5\,a^3}{\sqrt{bx^3+a}b^4}\right)+\frac{2}{9}\,A\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^3}-\frac{6\,\sqrt{bx^3+a}a}{b^3}-\frac{3\,a^2}{\sqrt{bx^3+a}b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*((b*x^3 + a)^(5/2)/b^4 - 5*(b*x^3 + a)^(3/2)*a/b^4 + 15*sqrt(b*x^3 +
 a)*a^2/b^4 + 5*a^3/(sqrt(b*x^3 + a)*b^4)) + 2/9*A*((b*x^3 + a)^(3/2)/b^3 -
 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3 + a)*b^3))
```

Fricas [A] time = 1.75572, size = 186, normalized size = 1.81

$$\frac{2\left(3\,Bb^3x^9-\left(6\,Bab^2-5\,Ab^3\right)x^6+48\,Ba^3-40\,Aa^2b+4\left(6\,Ba^2b-5\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{45\left(b^5x^3+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^3*x^9 - (6*B*a*b^2 - 5*A*b^3)*x^6 + 48*B*a^3 - 40*A*a^2*b + 4*(
6*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^5*x^3 + a*b^4)
```

Sympy [A] time = 3.70818, size = 175, normalized size = 1.7

$$\begin{cases} -\frac{16Aa^2}{9b^3\sqrt{a+bx^3}}-\frac{8Aax^3}{9b^2\sqrt{a+bx^3}}+\frac{2Ax^6}{9b\sqrt{a+bx^3}}+\frac{32Ba^3}{15b^4\sqrt{a+bx^3}}+\frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}}-\frac{4Bax^6}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^9}{15b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(a
 + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a +
 b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b**2
*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/
9 + B*x**12/12)/a**(3/2), True))
```

Giac [A] time = 1.14203, size = 131, normalized size = 1.27

$$\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}B-15\left(bx^3+a\right)^{\frac{3}{2}}Ba+45\sqrt{bx^3+a}Ba^2+5\left(bx^3+a\right)^{\frac{3}{2}}Ab-30\sqrt{bx^3+a}Aab+\frac{15\left(Ba^3-Aa^2b\right)}{\sqrt{bx^3+a}}\right)}{45\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*(b*x^3 + a)^(5/2)*B - 15*(b*x^3 + a)^(3/2)*B*a + 45*sqrt(b*x^3 + a)
*B*a^2 + 5*(b*x^3 + a)^(3/2)*A*b - 30*sqrt(b*x^3 + a)*A*a*b + 15*(B*a^3 - A
*a^2*b)/sqrt(b*x^3 + a))/b^4
```

3.229 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{a+bx^3}(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}$$

```
[Out] (2*a*(A*b - a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/(3*b^3) + (2*B*(a + b*x^3)^(3/2))/(9*b^3)
```

Rubi [A] time = 0.0548598, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\sqrt{a+bx^3}(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/(3*b^3) + (2*B*(a + b*x^3)^(3/2))/(9*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{1}{3}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}}+\frac{Ab-2aB}{b^2\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^2}\right)dx,x,x^3\right)$$
$$=\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}$$

Mathematica [A] time = 0.033604, size = 55, normalized size = 0.75

$$\frac{2\left(-8a^2B+a\left(6Ab-4bBx^3\right)+b^2x^3\left(3A+Bx^3\right)\right)}{9b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-8*a^2*B + b^2*x^3*(3*A + B*x^3) + a*(6*A*b - 4*b*B*x^3)))/(9*b^3*Sqrt[
a + b*x^3])
```

Maple [A] time = 0.007, size = 52, normalized size = 0.7

$$\frac{2\,b^2Bx^6+6\,Ax^3b^2-8\,Bx^3ab+12\,abA-16\,a^2B}{9\,b^3}\frac{1}{\sqrt{bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/9/(b*x^3+a)^(1/2)*(B*b^2*x^6+3*A*b^2*x^3-4*B*a*b*x^3+6*A*a*b-8*B*a^2)/b^3
```

Maxima [A] time = 0.936323, size = 109, normalized size = 1.49

$$\frac{2}{9}B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3}-\frac{6\sqrt{bx^3+a}a}{b^3}-\frac{3a^2}{\sqrt{bx^3+a}b^3}\right)+\frac{2}{3}A\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^3 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3
+ a)*b^3)) + 2/3*A*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2))
```

Fricas [A] time = 1.73956, size = 131, normalized size = 1.79

$$\frac{2\left(Bb^2x^6-(4\,Bab-3\,Ab^2)x^3-8\,Ba^2+6\,Aab\right)\sqrt{bx^3+a}}{9\left(b^4x^3+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^2*x^6 - (4*B*a*b - 3*A*b^2)*x^3 - 8*B*a^2 + 6*A*a*b)*sqrt(b*x^3 +
a)/(b^4*x^3 + a*b^3)
```

Sympy [A] time = 1.8081, size = 124, normalized size = 1.7

$$\begin{cases} \frac{4Aa}{3b^2\sqrt{a+bx^3}}+\frac{2Ax^3}{3b\sqrt{a+bx^3}}-\frac{16Ba^2}{9b^3\sqrt{a+bx^3}}-\frac{8Bax^3}{9b^2\sqrt{a+bx^3}}+\frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for}\ b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3)
) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x**
```

```
3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a*
*(3/2), True))
```

Giac [A] time = 1.12547, size = 88, normalized size = 1.21

$$\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}B-6\sqrt{bx^3+a}Ba+3\sqrt{bx^3+a}Ab-\frac{3\left(Ba^2-Aab\right)}{\sqrt{bx^3+a}}\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/9*((b*x^3 + a)^(3/2)*B - 6*sqrt(b*x^3 + a)*B*a + 3*sqrt(b*x^3 + a)*A*b -
3*(B*a^2 - A*a*b)/sqrt(b*x^3 + a))/b^3
```

3.230 $\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

```
[Out] (-2*(A*b - a*B))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^2)
```

Rubi [A] time = 0.0365055, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(A*b - a*B))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)dx, x, x^3\right)$$
$$= -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}$$

Mathematica [A] time = 0.0221462, size = 33, normalized size = 0.72

$$\frac{2\left(2aB-Ab+bBx^3\right)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

Maple [A] time = 0.006, size = 30, normalized size = 0.7

$$-\frac{-2\,bBx^3+2\,Ab-4\,Ba}{3\,b^2}\frac{1}{\sqrt{bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3/(b*x^3+a)^(1/2)*(-B*b*x^3+A*b-2*B*a)/b^2
```

Maxima [A] time = 0.942499, size = 63, normalized size = 1.37

$$\frac{2}{3}\,B\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}b^2}\right)-\frac{2\,A}{3\,\sqrt{bx^3+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3 +
 a)*b)
```

Fricas [A] time = 1.75286, size = 85, normalized size = 1.85

$$\frac{2\left(Bbx^3 + 2\,Ba - Ab\right)\sqrt{bx^3 + a}}{3\left(b^3x^3 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

Sympy [A] time = 0.884662, size = 75, normalized size = 1.63

$$\begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}} + \frac{4Ba}{3b^2\sqrt{a+bx^3}} + \frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) +
2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/2)
, True))
```

Giac [A] time = 1.1392, size = 47, normalized size = 1.02

$$\frac{2\left(\sqrt{bx^3 + a}B + \frac{Ba-Ab}{\sqrt{bx^3+a}}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(b*x^3 + a)*B + (B*a - A*b)/sqrt(b*x^3 + a))/b^2
```

3.231 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Rubi [A] time = 0.0397789, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}+\frac{(2A)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab}\\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0521013, size = 58, normalized size = 1.

$$\frac{1}{3}\left(-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2(aB-Ab)}{ab\sqrt{a+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] ((-2*(-(A*b) + a*B))/(a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/a^(3/2))/3
```

Maple [A] time = 0.026, size = 57, normalized size = 1.

$$-\frac{2B}{3b}\frac{1}{\sqrt{bx^3+a}}+A\left(\frac{2}{3a}\frac{1}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2}{3}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*B/b/(b*x^3+a)^(1/2)+A*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3/a^(3/2)*arctanh((
b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81584, size = 371, normalized size = 6.4

$$\left[\frac{\left(Ab^2x^3+Aab\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}\left(Ba^2-Aab\right)}{3\left(a^2b^2x^3+a^3b\right)},\frac{2\left(\left(Ab^2x^3+Aab\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)-\sqrt{bx^3\cdots}\right.}{3\left(a^2b^2x^3+a^3b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) +
2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3*(
(A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^
3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]
```

Sympy [A] time = 9.94424, size = 56, normalized size = 0.97

$$\frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a\sqrt{-a}}-\frac{2\,(-Ab+Ba)}{3ab\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - 2*(-A*b + B*a)/(3*a*b*
sqrt(a + b*x**3))
```

Giac [A] time = 1.09715, size = 72, normalized size = 1.24

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a}-\frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqrt
(b*x^3 + a)*a*b)
```

3.232 $\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

```
[Out] -(3*A*b - 2*a*B)/(3*a^2*Sqrt[a + b*x^3]) - A/(3*a*x^3*Sqrt[a + b*x^3]) + ((
3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rubi [A] time = 0.0650946, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```


```
[Out] -(3*A*b - 2*a*B)/(3*a^2*Sqrt[a + b*x^3]) - A/(3*a*x^3*Sqrt[a + b*x^3]) + ((
3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx,x,x^3\right)\\
&= -\frac{A}{3ax^3\sqrt{a+bx^3}}+\frac{\left(-\frac{3Ab}{2}+aB\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{3a}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}-\frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{6a^2}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}-\frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3a^2b}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0180701, size = 57, normalized size = 0.66

$$\frac{x^3(2aB-3Ab)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)-aA}{3a^2x^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a*A) + (-3*A*b + 2*a*B)*x^3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/
a])/(3*a^2*x^3*Sqrt[a + b*x^3])
```

Maple [A] time = 0.022, size = 100, normalized size = 1.2

$$A\left(-\frac{2b}{3a^2}\frac{1}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{1}{3a^2x^3}\sqrt{bx^3+a}+b\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{5}{2}}\right)+B\left(\frac{2}{3a}\frac{1}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2}{3}\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x)
```

```
[Out] A*(-2/3*b/a^2/((x^3+a/b)*b)^(1/2)-1/3/a^2*(b*x^3+a)^(1/2)/x^3+1/a^(5/2)*b*a
rctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3/a^(3/2)*a
rctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72835, size = 508, normalized size = 5.91

$$\left[-\frac{\left(\left(2\,Bab-3\,Ab^{2}\right)x^{6}+\left(2\,Ba^{2}-3\,Aab\right)x^{3}\right)\sqrt{a}\log\left(\frac{bx^{3}+2\,\sqrt{bx^{3}+a}\sqrt{a}+2\,a}{x^{3}}\right)-2\left(\left(2\,Ba^{2}-3\,Aab\right)x^{3}-Aa^{2}\right)\sqrt{bx^{3}+a}}{6\left(a^{3}bx^{6}+a^{4}x^{3}\right)},\ \frac{\left((2\,Bab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(a)*log((b*x
^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*((2*B*a^2 - 3*A*a*b)*x^3 - A
*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), 1/3*(((2*B*a*b - 3*A*b^2)*x^6
 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (
(2*B*a^2 - 3*A*a*b)*x^3 - A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [B] time = 78.4772, size = 264, normalized size = 3.07

$$A\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{\sqrt{b}}{a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)+B\left(\frac{2a^{3}\sqrt{1+\frac{bx^{3}}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}+\frac{a^{3}\log\left(\frac{bx^{3}}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}-\frac{2a^{3}\log\left(\sqrt{1+\frac{bx^{3}}{a}}+\right.}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*
sqrt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)) + B*(
2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**
3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/
(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) +
3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2)
+ 3*a**(7/2)*b*x**3))
```

Giac [A] time = 1.12342, size = 134, normalized size = 1.56

$$\frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^2}+\frac{2\left(bx^3+a\right)Ba-2\,Ba^2-3\left(bx^3+a\right)Ab+2\,Aab}{3\left(\left(bx^3+a\right)^{\frac{3}{2}}-\sqrt{bx^3+a}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 3*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(
2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^(3
/2) - sqrt(b*x^3 + a)*a)*a^2)
```

3.233 $\int \frac{A+Bx^3}{x^7 \left(a+bx^3\right)^{3/2}} \, dx$

Optimal. Leaf size=118

$$\frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}}+\frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

```
[Out] (b*(5*A*b - 4*a*B))/(4*a^3*Sqrt[a + b*x^3]) - A/(6*a*x^6*Sqrt[a + b*x^3]) +
 (5*A*b - 4*a*B)/(12*a^2*x^3*Sqrt[a + b*x^3]) - (b*(5*A*b - 4*a*B)*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] time = 0.0899382, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}(5Ab-4aB)}{4a^3x^3}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(6*a*x^6*Sqrt[a + b*x^3]) - (5*A*b - 4*a*B)/(6*a^2*x^3*Sqrt[a + b*x^3])
+ ((5*A*b - 4*a*B)*Sqrt[a + b*x^3])/(4*a^3*x^3) - (b*(5*A*b - 4*a*B)*ArcTan
h[Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx,x,x^3\right)$$
$$=-\frac{A}{6ax^6\sqrt{a+bx^3}}+\frac{\left(-\frac{5Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^3\right)}{6a}$$
$$=-\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}-\frac{(5Ab-4aB)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)}{4a^2}$$
$$=-\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}+\frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3}+\frac{(b(5Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{8a^3}$$
$$=-\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}+\frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3}+\frac{(5Ab-4aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{4a^3}$$
$$=-\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}+\frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3}-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.0199701, size = 60, normalized size = 0.51

$$\frac{bx^6(5Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^3}{a}+1\right)-a^2A}{6a^3x^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a^2*A) + b*(5*A*b - 4*a*B)*x^6*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^
3)/a])/(6*a^3*x^6*Sqrt[a + b*x^3])
```

Maple [A] time = 0.026, size = 141, normalized size = 1.2

$$A\left(\frac{2\,b^2}{3\,a^3}\frac{1}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{1}{6\,a^2x^6}\sqrt{bx^3+a}+\frac{7\,b}{12\,a^3x^3}\sqrt{bx^3+a}-\frac{5\,b^2}{4}\text{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{7}{2}}\right)+B\left(-\frac{2\,b}{3\,a^2}\frac{1}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x)

[Out] A*(2/3*b^2/a^3/((x^3+a/b)*b)^(1/2)-1/6/a^2*(b*x^3+a)^(1/2)/x^6+7/12*b/a^3*(b*x^3+a)^(1/2)/x^3-5/4/a^(7/2)*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*(-2/3*b/a^2/((x^3+a/b)*b)^(1/2)-1/3/a^2*(b*x^3+a)^(1/2)/x^3+1/a^(5/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.79482, size = 630, normalized size = 5.34

$$\left[-\frac{3\left(\left(4\,Bab^2-5\,Ab^3\right)x^9+\left(4\,Ba^2b-5\,Aab^2\right)x^6\right)\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(3\left(4\,Ba^2b-5\,Aab^2\right)x^6+2\,Aa^3+\left(4\,Ba^3\right.\right.}{24\left(a^4bx^9+a^5x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] [-1/24*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*(4*B*a^2*b - 5*A*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^9 + a^5*x^6), -1/12*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(4*B*a^2*b - 5*A*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^9 + a^5*x^6)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1264, size = 185, normalized size = 1.57

$$-\frac{\left(4\,Bab-5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3}-\frac{2\left(Bab-Ab^2\right)}{3\,\sqrt{bx^3+a}a^3}-\frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^3+a}Ba^2b-7\left(bx^3+a\right)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^3}}{12\,a^3b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
2/3*(B*a*b - A*b^2)/(sqrt(b*x^3 + a)*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a*b
 - 4*sqrt(b*x^3 + a)*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^3 + a
)*A*a*b^2)/(a^3*b^2*x^6)
```

3.234 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{32\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{165\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x^4(11Ab-}{33b^2\sqrt{a+}}$$

```
[Out] (-2*(11*A*b - 14*a*B)*x^4)/(33*b^2*Sqrt[a + b*x^3]) + (2*B*x^7)/(11*b*Sqrt[
a + b*x^3]) + (16*(11*A*b - 14*a*B)*x*Sqrt[a + b*x^3])/(165*b^3) - (32*Sqrt
[2 + Sqrt[3]]*a*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(165*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.139273, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 288, 321, 218}

$$-\frac{32\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{165\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x^4(11Ab-14aB)}{33b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(11*A*b - 14*a*B)*x^4)/(33*b^2*Sqrt[a + b*x^3]) + (2*B*x^7)/(11*b*Sqrt[
a + b*x^3]) + (16*(11*A*b - 14*a*B)*x*Sqrt[a + b*x^3])/(165*b^3) - (32*Sqrt
[2 + Sqrt[3]]*a*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(165*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
```

```
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2Bx^7}{11b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{11Ab}{2}+7aB\right)\right)\int \frac{x^6}{\left(a+bx^3\right)^{3/2}}\,dx}{11b}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{(8(11Ab-14aB))\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{33b^2}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} - \frac{(16a(11Ab-14aB))\int \frac{1}{\sqrt{a+bx^3}}}{165b^3}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} - \frac{32\sqrt{2+\sqrt{3}}a(11Ab-14aB)\left(\right.}{}$$

Mathematica [C] time = 0.0987876, size = 103, normalized size = 0.34

$$\frac{2x\left(-112a^2B+8a\sqrt{\frac{bx^3}{a}+1}(14aB-11Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a\left(88Ab-42bBx^3\right)+3b^2x^3\left(11A+5Bx^3\right)\right)}{165b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-112*a^2*B + 3*b^2*x^3*(11*A + 5*B*x^3) + a*(88*A*b - 42*b*B*x^3) + 8
*a*(-11*A*b + 14*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(165*b^3*Sqrt[a + b*x^3])
```

Maple [B] time = 0.027, size = 666, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-2/3/b^3*a^2*x/((x^3+a/b)*b)^(1/2)+2/11/b^2*x^4*(b*x^3+a)^(1/2)-38/55*a/
b^3*x*(b*x^3+a)^(1/2)-448/495*I*a^2/b^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/3/b^2*a*x/((x^3+a/b)*b)^(1/2)+2/5/b^2*x*(b
*x^3+a)^(1/2)+32/45*I*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^6}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(Bx^9 + Ax^6)\sqrt{bx^3 + a}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^9 + A*x^6)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [A] time = 51.4509, size = 80, normalized size = 0.27

$$\frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((3/2, 10/3), (13/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(13/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^3+A\right)x^6}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

3.235 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(5Ab-8aB)}{15b^2\sqrt{a+bx^3}}$$

```
[Out] (-2*(5*A*b - 8*a*B)*x)/(15*b^2*Sqrt[a + b*x^3]) + (2*B*x^4)/(5*b*Sqrt[a + b
*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rubi [A] time = 0.0995218, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {459, 288, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt[3]{b}x+(1+\sqrt{3})\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(5Ab-8aB)}{15b^2\sqrt{a+bx^3}}+\frac{2B}{5b\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(5*A*b - 8*a*B)*x)/(15*b^2*Sqrt[a + b*x^3]) + (2*B*x^4)/(5*b*Sqrt[a + b
*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
```

)

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2Bx^4}{5b\sqrt{a+bx^3}}-\frac{\left(2\left(-\frac{5Ab}{2}+4aB\right)\right)\int \frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx}{5b}$$

$$=-\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}+\frac{(2(5Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{15b^2}$$

$$=-\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}+\frac{4\sqrt{2+\sqrt{3}}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx}}$$

Mathematica [C] time = 0.0763849, size = 78, normalized size = 0.29

$$\frac{2x\left(\sqrt{\frac{bx^3}{a}+1}(5Ab-8aB)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+8aB-5Ab+3bBx^3\right)}{15b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-5*A*b + 8*a*B + 3*b*B*x^3 + (5*A*b - 8*a*B)*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.018, size = 627, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(2/3/b^2*a*x/((x^3+a/b)*b)^(1/2)+2/5/b^2*x*(b*x^3+a)^(1/2)+32/45*I*a/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-2/3/b*x/
((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^3}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^6 + Ax^3)\sqrt{bx^3 + a}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [A] time = 16.3946, size = 80, normalized size = 0.3

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^3}{(bx^3 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

3.236 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=251

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

```
[Out] (2*(A*b - a*B)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*
3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0631136, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {385, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```


```
[Out] (2*(A*b - a*B)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*
3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{\left(2\left(\frac{Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.032788, size = 73, normalized size = 0.29

$$\frac{x\left(\sqrt{\frac{bx^3}{a}+1}(2aB+Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-2aB+2Ab\right)}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(2*A*b - 2*a*B + (A*b + 2*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3
, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*b*Sqrt[a + b*x^3])
```

Maple [B] time = 0.018, size = 613, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2
)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [A] time = 9.66712, size = 78, normalized size = 0.31

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

3.237 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{x(7Ab-4aB)}{6a^2\sqrt{a+bx^3}}-$$

```
[Out] -A/(2*a*x^2*Sqrt[a + b*x^3]) - ((7*A*b - 4*a*B)*x)/(6*a^2*Sqrt[a + b*x^3])
- (Sqrt[2 + Sqrt[3]]*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(6*3^(1/4)*a^2*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.100817, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 199, 218}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{x(7Ab-4aB)}{6a^2\sqrt{a+bx^3}}-\frac{A}{2ax^2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```


```
[Out] -A/(2*a*x^2*Sqrt[a + b*x^3]) - ((7*A*b - 4*a*B)*x)/(6*a^2*Sqrt[a + b*x^3])
- (Sqrt[2 + Sqrt[3]]*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(6*3^(1/4)*a^2*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{\left(\frac{7Ab}{2}-2aB\right)\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx}{2a}$$

$$= -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{12a^2}$$

$$= -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin\right.}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

Mathematica [C] time = 0.0348464, size = 86, normalized size = 0.32

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(4aB-7Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-6aA+8aBx^3-14Abx^3}{12a^2x^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-6*a*A - 14*A*b*x^3 + 8*a*B*x^3 + (-7*A*b + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(12*a^2*x^2*Sqrt[a + b*x^3
])
```

Maple [B] time = 0.021, size = 631, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/2/a^2*(b*x^3+a)^(1/2)/x^2-2/3*b/a^2*x/((
x^3+a/b)*b)^(1/2)+7/18*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^2x^9 + 2\,abx^6 + a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^9 + 2*a*b*x^6 + a^2*x^3), x)
```

Sympy [A] time = 39.287, size = 82, normalized size = 0.3

$$\frac{A\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{Bx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

3.238 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=304

$$\frac{7\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{7\sqrt{a+bx^3}}{6}$$

```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^3]) - (13*A*b - 10*a*B)/(15*a^2*x^2*Sqrt[a + b*x^3
]) + (7*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(60*a^3*x^2) + (7*Sqrt[2 + Sqrt[
3]]*b^(2/3)*(13*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 - 4*Sqrt[3]])/(60*3^(1/4)*a^3*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.135799, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 290, 325, 218}

$$\frac{7\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{7\sqrt{a+bx^3}(13Ab-}{60a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```


```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^3]) - (13*A*b - 10*a*B)/(15*a^2*x^2*Sqrt[a + b*x^3
]) + (7*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(60*a^3*x^2) + (7*Sqrt[2 + Sqrt[
3]]*b^(2/3)*(13*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 - 4*Sqrt[3]])/(60*3^(1/4)*a^3*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{\left(\frac{13Ab}{2}-5aB\right)\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} - \frac{(7(13Ab-10aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{30a^2}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{(7b(13Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}}{120a^3}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{7\sqrt{2+\sqrt{3}}b^{2/3}(13Ab-10aB)}{}$$

Mathematica [C] time = 0.0337289, size = 72, normalized size = 0.24

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(13Ab-10aB)\,{}_2F_1\left(-\frac{2}{3},\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a}\right)-4aA}{20a^2x^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-4*a*A + (13*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3,
 3/2, 1/3, -((b*x^3)/a)])/(20*a^2*x^5*Sqrt[a + b*x^3])
```

Maple [B] time = 0.026, size = 667, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-1/2/a^2*(b*x^3+a)^(1/2)/x^2-2/3*b/a^2*x/((x^3+a/b)*b)^(1/2)+7/18*I/a^2*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
```

```
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/5/a^2*
(b*x^3+a)^(1/2)/x^5+17/20/a^3*b*(b*x^3+a)^(1/2)/x^2+2/3*b^2/a^3*x/((x^3+a/b
)*b)^(1/2)-91/180*I/a^3*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^2x^{12} + 2\,abx^9 + a^2x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^12 + 2*a*b*x^9 + a^2*x^6), x)
```

Sympy [A] time = 139.688, size = 90, normalized size = 0.3

$$\frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x**2*gamma(1/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

3.239 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=547

$$\frac{8\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{}$$

[Out] (-2*(7*A*b - 10*a*B)*x^2)/(21*b^2*Sqrt[a + b*x^3]) + (2*B*x^5)/(7*b*Sqrt[a + b*x^3]) + (8*(7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (8*Sqrt[2]*a^(1/3)*(7*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rubi [A] time = 0.254766, antiderivative size = 547, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 303, 218, 1877}

$$\frac{8\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]

[Out] (-2*(7*A*b - 10*a*B)*x^2)/(21*b^2*Sqrt[a + b*x^3]) + (2*B*x^5)/(7*b*Sqrt[a + b*x^3]) + (8*(7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])

```
*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*
b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (8*Sqrt[2]*a^(1/3)*(7*A*b - 10*a*B)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2Bx^5}{7b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{7Ab}{2}+5aB\right)\right)\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{7b}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{21b^2}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{21b^{7/3}} + \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}\sqrt[3]{a}(7A\ldots\right.}{21\ldots}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-10aB)\left(\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.0690726, size = 79, normalized size = 0.14

$$\frac{2x^2\left(\sqrt{\frac{bx^3}{a}+1}(10aB-7Ab)\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-10aB+7Ab+bBx^3\right)}{7b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*(7*A*b - 10*a*B + b*B*x^3 + (-7*A*b + 10*a*B)*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

Maple [B] time = 0.026, size = 937, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(2/3/b^2*a*x^2/((x^3+a/b)*b)^(1/2)+2/7/b^2*x^2*(b*x^3+a)^(1/2)+80/63*I*a/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-2/3/b*x^2/((x^3+a/b)*b)^
(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)x^4}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^{7}+Ax^{4}\right)\sqrt{bx^{3}+a}}{b^{2}x^{6}+2\,abx^{3}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^7 + A*x^4)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [A] time = 22.5103, size = 80, normalized size = 0.15

$$\frac{Ax^{5}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^{8}\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^{3}+A\right)x^{4}}{\left(bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

3.240 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=524

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\cdots}{\cdots}$$

```
[Out] (2*(A*b - a*B)*x^2)/(3*a*b*Sqrt[a + b*x^3]) - (2*(A*b - 4*a*B)*Sqrt[a + b*x
^3])/(3*a*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]
*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(A*b - 4*a*
B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3
^(1/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.194962, antiderivative size = 524, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {457, 303, 218, 1877}

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}{3^{3/4}a^{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(3*a*b*Sqrt[a + b*x^3]) - (2*(A*b - 4*a*B)*Sqrt[a + b*x
^3])/(3*a*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]
```

```
*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(A*b - 4*a*
B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3
^(1/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
```

```
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}}+\frac{\left(2\left(-\frac{Ab}{2}+2aB\right)\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3ab^{4/3}}-\frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}(Ab-4aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a^{2/3}b^{4/3}}$$

$$= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}}-\frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt{2-\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}}}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}$$

Mathematica [C] time = 0.0588465, size = 71, normalized size = 0.14

$$\frac{x^2\left(\sqrt{\frac{bx^3}{a}+1}(Ab-4aB)\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+4aB\right)}{2ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x^2*(4*a*B + (A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2,
 5/3, -((b*x^3)/a)]))/(2*a*b*Sqrt[a + b*x^3])
```

Maple [B] time = 0.017, size = 921, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))))+A*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)x}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^4 + Ax\right)\sqrt{bx^3 + a}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [A] time = 10.1911, size = 80, normalized size = 0.15

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(Bx^3+A)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

3.241 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=548

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{a+bx^3}(5Ab-2a}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

```
[Out] -(A/(a*x*Sqrt[a + b*x^3])) - ((5*A*b - 2*a*B)*x^2)/(3*a^2*Sqrt[a + b*x^3])
+ ((5*A*b - 2*a*B)*Sqrt[a + b*x^3])/(3*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (Sqrt[2 - Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rubi [A] time = 0.243851, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 290, 303, 218, 1877}

$$\frac{\sqrt{a+bx^3}(5Ab-2aB)}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^3])) - ((5*A*b - 2*a*B)*x^2)/(3*a^2*Sqrt[a + b*x^3])
+ ((5*A*b - 2*a*B)*Sqrt[a + b*x^3])/(3*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
```

```
 b^(1/3)*x)) - (Sqrt[2 - Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{ax\sqrt{a+bx^3}} - \frac{\left(\frac{5Ab}{2}-aB\right)\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{a}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{6a^2}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{6a^2\sqrt[3]{b}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}(5Ab-2aB\right.}{3a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt{2-\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\right.}{2\ 3}$$

Mathematica [C] time = 0.0312392, size = 72, normalized size = 0.13

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(2aB-5Ab)\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-4aA}{4a^2x\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-4*a*A + (-5*A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3
/2, 5/3, -((b*x^3)/a)])/(4*a^2*x*Sqrt[a + b*x^3])
```

Maple [B] time = 0.023, size = 939, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+A*(-1/a^2*(b*x^3+a)^(1/2)/x-2/3*b/a^2*x^2/((x^3+a/b)*b)^(1/2)-5/
9*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^2x^8 + 2\,abx^5 + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^8 + 2*a*b*x^5 + a^2*x^2), x)
```

Sympy [A] time = 28.518, size = 82, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

3.242 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=580

$$-\frac{5\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\ldots\right.}{\ldots}$$

```
[Out] -A/(4*a*x^4*Sqrt[a + b*x^3]) - (11*A*b - 8*a*B)/(12*a^2*x*Sqrt[a + b*x^3])
+ (5*(11*A*b - 8*a*B)*Sqrt[a + b*x^3])/(24*a^3*x) - (5*b^(1/3)*(11*A*b - 8*
a*B)*Sqrt[a + b*x^3])/(24*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqr
t[2 - Sqrt[3]]*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (5
*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.303617, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 290, 325, 303, 218, 1877}

$$-\frac{5\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(4*a*x^4*Sqrt[a + b*x^3]) - (11*A*b - 8*a*B)/(12*a^2*x*Sqrt[a + b*x^3])
+ (5*(11*A*b - 8*a*B)*Sqrt[a + b*x^3])/(24*a^3*x) - (5*b^(1/3)*(11*A*b - 8*
```

```
a*B)*Sqrt[a + b*x^3])/(24*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqr
t[2 - Sqrt[3]]*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (5
*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{\left(\frac{11Ab}{2}-4aB\right)\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx}{4a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} - \frac{(5(11Ab-8aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{24a^2}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{(5b(11Ab-8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{48a^3}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{\left(5b^{2/3}(11Ab-8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\ldots}{\sqrt{a+b\ldots}}}{48a^3}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{5\sqrt[3]{b}(11Ab-8aB)\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \ldots$$

Mathematica [C] time = 0.0331659, size = 72, normalized size = 0.12

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(11Ab-8aB)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a}\right)-2aA}{8a^2x^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*a*A + (11*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3,
3/2, 2/3, -((b*x^3)/a)])/(8*a^2*x^4*Sqrt[a + b*x^3])
```

Maple [B] time = 0.024, size = 975, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-1/a^2*(b*x^3+a)^(1/2)/x-2/3*b/a^2*x^2/((x^3+a/b)*b)^(1/2)-5/9*I/a^2*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/4/a^2*(b*x^3+a)^(1/2)/x^4+13/
8/a^3*b*(b*x^3+a)^(1/2)/x+2/3*b^2/a^3*x^2/((x^3+a/b)*b)^(1/2)+55/72*I/a^3*b
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
```

```
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^{11}+2\,abx^8+a^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^11 + 2*a*b*x^8 + a^2*x^5), x)
```

Sympy [A] time = 107.752, size = 88, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

3.243 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=611

$$\frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{55b^{4/3}\sqrt{a+bx^3}(17}{336a^4\left(\left(1+\sqrt{3}\right)\right.}$$

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^3]) - (17*A*b - 14*a*B)/(21*a^2*x^4*Sqrt[a + b*x^3
]) + (11*(17*A*b - 14*a*B)*Sqrt[a + b*x^3])/(168*a^3*x^4) - (55*b*(17*A*b -
 14*a*B)*Sqrt[a + b*x^3])/(336*a^4*x) + (55*b^(4/3)*(17*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(336*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (55*Sqrt[2 - Sq
rt[3]]*b^(4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(224*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (55*b^(
4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(168*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.361824, antiderivative size = 611, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 290, 325, 303, 218, 1877}

$$\frac{55b^{4/3}\sqrt{a+bx^3}(17Ab-14aB)}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^3]) - (17*A*b - 14*a*B)/(21*a^2*x^4*Sqrt[a + b*x^3
]) + (11*(17*A*b - 14*a*B)*Sqrt[a + b*x^3])/(168*a^3*x^4) - (55*b*(17*A*b -
 14*a*B)*Sqrt[a + b*x^3])/(336*a^4*x) + (55*b^(4/3)*(17*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(336*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (55*Sqrt[2 - Sq
rt[3]]*b^(4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(224*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (55*b^(
4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(168*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
```

```
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{\left(\frac{17Ab}{2}-7aB\right)\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} - \frac{(11(17Ab-14aB))\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{42a^2}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} + \frac{(55b(17Ab-14aB))\int \frac{1}{x^2\sqrt{a+b}}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x} +$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x} +$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x} +$$

Mathematica [C] time = 0.0340703, size = 72, normalized size = 0.12

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}(17Ab-14aB)\,{}_2F_1\left(-\frac{4}{3},\frac{3}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)-8aA}{56a^2x^7\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-8*a*A + (17*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3,
 3/2, -1/3, -((b*x^3)/a)])/(56*a^2*x^7*Sqrt[a + b*x^3])
```

Maple [B] time = 0.025, size = 1018, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-1/4/a^2*(b*x^3+a)^(1/2)/x^4+13/8/a^3*b*(b*x^3+a)^(1/2)/x+2/3*b^2/a^3*x^
2/((x^3+a/b)*b)^(1/2)+55/72*I/a^3*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)))+A*(-1/7/a^2*(b*x^3+a)^(1/2)/x^7+25/56/a^3*b*(b*x^3+a)^(1/2)/x^4-237/112
*b^2/a^4*(b*x^3+a)^(1/2)/x-2/3*b^3/a^4*x^2/((x^3+a/b)*b)^(1/2)-935/1008*I*b
^2/a^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(
b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^{14}+2\,abx^{11}+a^2x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^14 + 2*a*b*x^11 + a^2*x^8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

3.244 $\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}$$

```
[Out] (-2*a^2*(A*b - a*B))/(9*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(3*b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*B*(a +
 b*x^3)^(3/2))/(9*b^4)
```

Rubi [A] time = 0.0779481, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(9*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(3*b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*B*(a +
 b*x^3)^(3/2))/(9*b^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}}+\frac{Ab-3aB}{b^3\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^3}\right)dx,x,x^3\right)\\ &= -\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.0592669, size = 73, normalized size = 0.71

$$\frac{2\left(8a^2b\left(A-3Bx^3\right)-16a^3B-6ab^2x^3\left(Bx^3-2A\right)+b^3x^6\left(3A+Bx^3\right)\right)}{9b^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x^
6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))
```

Maple [A] time = 0.008, size = 76, normalized size = 0.7

$$\frac{2\,Bx^9b^3+6\,Ab^3x^6-12\,Bab^2x^6+24\,Aab^2x^3-48\,Ba^2bx^3+16\,Aa^2b-32\,Ba^3}{9\,b^4}\left(bx^3+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] 2/9/(b*x^3+a)^(3/2)*(B*b^3*x^9+3*A*b^3*x^6-6*B*a*b^2*x^6+12*A*a*b^2*x^3-24*
B*a^2*b*x^3+8*A*a^2*b-16*B*a^3)/b^4
```

Maxima [A] time = 0.929296, size = 157, normalized size = 1.52

$$\frac{2}{9}\,B\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^4}-\frac{9\,\sqrt{bx^3+a}a}{b^4}-\frac{9\,a^2}{\sqrt{bx^3+a}b^4}+\frac{a^3}{\left(bx^3+a\right)^{\frac{3}{2}}b^4}\right)+\frac{2}{9}\,A\left(\frac{3\,\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}b^3}-\frac{a^2}{\left(bx^3+a\right)^{\frac{3}{2}}b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^4 - 9*sqrt(b*x^3 + a)*a/b^4 - 9*a^2/(sqrt(b*x^3
+ a)*b^4) + a^3/((b*x^3 + a)^(3/2)*b^4)) + 2/9*A*(3*sqrt(b*x^3 + a)/b^3 + 6
*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)^(3/2)*b^3))
```

Fricas [A] time = 1.73032, size = 201, normalized size = 1.95

$$\frac{2\left(Bb^3x^9-3\left(2\,Bab^2-Ab^3\right)x^6-16\,Ba^3+8\,Aa^2b-12\left(2\,Ba^2b-Aab^2\right)x^3\right)\sqrt{bx^3+a}}{9\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^3*x^9 - 3*(2*B*a*b^2 - A*b^3)*x^6 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] time = 6.09598, size = 338, normalized size = 3.28

$$\begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}+\frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}+\frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}-\frac{32Ba^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}}-\frac{48Ba^2bx^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*
x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x
**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))
```

Giac [A] time = 1.12799, size = 124, normalized size = 1.2

$$\frac{2\left((bx^3+a)^{\frac{3}{2}}B - 9\sqrt{bx^3+a}Ba + 3\sqrt{bx^3+a}Ab - \frac{9(bx^3+a)Ba^2 - Ba^3 - 6(bx^3+a)Aab + Aa^2b}{(bx^3+a)^{\frac{3}{2}}}\right)}{9\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*((b*x^3 + a)^(3/2)*B - 9*sqrt(b*x^3 + a)*B*a + 3*sqrt(b*x^3 + a)*A*b -
(9*(b*x^3 + a)*B*a^2 - B*a^3 - 6*(b*x^3 + a)*A*a*b + A*a^2*b)/(b*x^3 + a)^(
3/2))/b^4
```

3.245 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

```
[Out] (2*a*(A*b - a*B))/(9*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(3*b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^3)
```

Rubi [A] time = 0.0555171, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(9*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(3*b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^5 \left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}}+\frac{Ab-2aB}{b^2(a+bx)^{3/2}}+\frac{B}{b^2\sqrt{a+bx}}\right)dx,x,x^3\right)$$
$$= \frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

Mathematica [A] time = 0.0374402, size = 54, normalized size = 0.74

$$\frac{16a^2B-4ab\left(A-6Bx^3\right)+6b^2x^3\left(Bx^3-A\right)}{9b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (16*a^2*B - 4*a*b*(A - 6*B*x^3) + 6*b^2*x^3*(-A + B*x^3))/(9*b^3*(a + b*x^3
)^(3/2))
```

Maple [A] time = 0.006, size = 53, normalized size = 0.7

$$-\frac{-6\,b^2Bx^6+6\,Ax^3b^2-24\,Bx^3ab+4\,abA-16\,a^2B}{9\,b^3}\left(bx^3+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9/(b*x^3+a)^(3/2)*(-3*B*b^2*x^6+3*A*b^2*x^3-12*B*a*b*x^3+2*A*a*b-8*B*a^2
)/b^3
```

Maxima [A] time = 0.925425, size = 113, normalized size = 1.55

$$\frac{2}{9}\,B\left(\frac{3\,\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}b^3}-\frac{a^2}{\left(bx^3+a\right)^{\frac{3}{2}}b^3}\right)-\frac{2}{9}\,A\left(\frac{3}{\sqrt{bx^3+a}b^2}-\frac{a}{\left(bx^3+a\right)^{\frac{3}{2}}b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*(3*sqrt(b*x^3 + a)/b^3 + 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)
^(3/2)*b^3)) - 2/9*A*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2))
```

Fricas [A] time = 1.7427, size = 155, normalized size = 2.12

$$\frac{2\left(3\,Bb^2x^6+3\left(4\,Bab-Ab^2\right)x^3+8\,Ba^2-2\,Aab\right)\sqrt{bx^3+a}}{9\left(b^5x^6+2\,ab^4x^3+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*B*b^2*x^6 + 3*(4*B*a*b - A*b^2)*x^3 + 8*B*a^2 - 2*A*a*b)*sqrt(b*x^3
+ a)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [A] time = 2.41853, size = 240, normalized size = 3.29

$$\begin{cases}-\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}-\frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{24Babx^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{6Bb^2x^6}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+}} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{5}{2}}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
```

```
3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)),
 Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))
```

Giac [A] time = 1.13762, size = 82, normalized size = 1.12

$$\frac{2\left(3\sqrt{bx^3+a}B+\frac{6\left(bx^3+a\right)Ba-Ba^2-3\left(bx^3+a\right)Ab+Aab}{\left(bx^3+a\right)^{\frac{3}{2}}}\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*(3*sqrt(b*x^3 + a)*B + (6*(b*x^3 + a)*B*a - B*a^2 - 3*(b*x^3 + a)*A*b +
 A*a*b)/(b*x^3 + a)^(3/2))/b^3
```

3.246 $\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

```
[Out] (-2*(A*b - a*B))/(9*b^2*(a + b*x^3)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0359458, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b - a*B))/(9*b^2*(a + b*x^3)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^3])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)dx,x,x^3\right)$$
$$= -\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.0228209, size = 33, normalized size = 0.72

$$-\frac{2\left(2aB+Ab+3bBx^3\right)}{9b^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

Maple [A] time = 0.006, size = 30, normalized size = 0.7

$$-\frac{6\,bBx^3+2\,Ab+4\,Ba}{9\,b^2}\left(bx^3+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9/(b*x^3+a)^(3/2)*(3*B*b*x^3+A*b+2*B*a)/b^2
```

Maxima [A] time = 0.93893, size = 66, normalized size = 1.43

$$-\frac{2}{9}B\left(\frac{3}{\sqrt{bx^3+a}b^2}-\frac{a}{\left(bx^3+a\right)^{\frac{3}{2}}b^2}\right)-\frac{2\,A}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x^
3 + a)^(3/2)*b)
```

Fricas [A] time = 1.68313, size = 111, normalized size = 2.41

$$-\frac{2\left(3\,Bbx^3+2\,Ba+Ab\right)\sqrt{bx^3+a}}{9\left(b^4x^6+2\,ab^3x^3+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2
*b^2)
```

Sympy [A] time = 1.16649, size = 144, normalized size = 3.13

$$\begin{cases} -\frac{2Ab}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{4Ba}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{6Bbx^3}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} & \text{for}\: b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)
) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*B*
b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b, 0)
), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))
```

Giac [A] time = 1.14252, size = 43, normalized size = 0.93

$$-\frac{2\left(3\left(bx^3+a\right)B-Ba+Ab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)
```

3.247 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

```
[Out] (2*(A*b - a*B))/(9*a*b*(a + b*x^3)^(3/2)) + (2*A)/(3*a^2*Sqrt[a + b*x^3]) -
 (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rubi [A] time = 0.0504637, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B))/(9*a*b*(a + b*x^3)^(3/2)) + (2*A)/(3*a^2*Sqrt[a + b*x^3]) -
 (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx, x, x^3\right)$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{A\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)}{3a}$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2A}{3a^2\sqrt{a+bx^3}} + \frac{A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a^2}$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2A}{3a^2\sqrt{a+bx^3}} + \frac{(2A)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3a^2b}$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2A}{3a^2\sqrt{a+bx^3}} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

Mathematica [C] time = 0.0221555, size = 62, normalized size = 0.81

$$\frac{2a(Ab-aB)+6Ab\left(a+bx^3\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)}{9a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*a*(A*b - a*B) + 6*A*b*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b
*x^3)/a])/(9*a^2*b*(a + b*x^3)^(3/2))
```

Maple [A] time = 0.027, size = 85, normalized size = 1.1

$$-\frac{2B}{9b}\left(bx^3+a\right)^{-\frac{3}{2}} + A\left(\frac{2}{9ab^2}\sqrt{bx^3+a}\left(x^3+\frac{a}{b}\right)^{-2} + \frac{2}{3a^2}\frac{1}{\sqrt{\left(x^3+\frac{a}{b}\right)b}} - \frac{2}{3}\operatorname{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{5}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9*B/b/(b*x^3+a)^(3/2)+A*(2/9/a/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/3/a^2/(
(x^3+a/b)*b)^(1/2)-2/3/a^(5/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82575, size = 522, normalized size = 6.78

$$\left[\frac{3\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Aab^2x^3-Ba^3+4\,Aa^2b\right)\sqrt{bx^3+a}}{9\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}, \frac{2\left(3\left(Ab^3x^6+2\,Aab^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b
*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(
b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2*A*
a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*A*a*b
^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 +
 a^5*b)]
```

Sympy [A] time = 17.1417, size = 76, normalized size = 0.99

$$\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}}-\frac{2\left(-Ab+Ba\right)}{9ab\left(a+bx^3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

```
[Out] 2*A/(3*a**2*sqrt(a + b*x**3)) + 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a**2
*sqrt(-a)) - 2*(-A*b + B*a)/(9*a*b*(a + b*x**3)**(3/2))
```

Giac [A] time = 1.1286, size = 90, normalized size = 1.17

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^2} - \frac{2\left(Ba^2 - 3\left(bx^3 + a\right)Ab - Aab\right)}{9\left(bx^3 + a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/9*(B*a^2 - 3*(b*x
^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a^2*b)
```

3.248 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

```
[Out] -(5*A*b - 2*a*B)/(9*a^2*(a + b*x^3)^(3/2)) - A/(3*a*x^3*(a + b*x^3)^(3/2))
- (5*A*b - 2*a*B)/(3*a^3*Sqrt[a + b*x^3]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*a^(7/2))
```

Rubi [A] time = 0.0869774, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(5*A*b - 2*a*B)/(9*a^2*(a + b*x^3)^(3/2)) - A/(3*a*x^3*(a + b*x^3)^(3/2))
- (5*A*b - 2*a*B)/(3*a^3*Sqrt[a + b*x^3]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*a^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx, x, x^3\right)$$

$$= -\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} + \frac{\left(-\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)}{6a^2}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{6a^3}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{3a^3b}$$

$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

Mathematica [C] time = 0.0203189, size = 57, normalized size = 0.5

$$\frac{x^3(2aB-5Ab)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^3}{a}+1\right)-3aA}{9a^2x^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-3*a*A + (-5*A*b + 2*a*B)*x^3*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^3)
/a])/(9*a^2*x^3*(a + b*x^3)^(3/2))
```

Maple [A] time = 0.028, size = 157, normalized size = 1.4

$$A\left(-\frac{2}{9\,a^2b}\sqrt{bx^3+a}\left(x^3+\frac{a}{b}\right)^{-2}-\frac{4\,b}{3\,a^3}\frac{1}{\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{1}{3\,a^3x^3}\sqrt{bx^3+a}+\frac{5\,b}{3}\text{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)a^{-\frac{7}{2}}\right)+B\left(\frac{2}{9\,ab^2}\sqrt{bx^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x)
```

```
[Out] A*(-2/9/a^2/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-4/3*b/a^3/((x^3+a/b)*b)^(1/2)-1/3
/a^3*(b*x^3+a)^(1/2)/x^3+5/3/a^(7/2)*b*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*
(2/9/a/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/3/a^2/((x^3+a/b)*b)^(1/2)-2/3/a^(5
/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77859, size = 757, normalized size = 6.7

$$\left[-\frac{3\left(\left(2\,Bab^2-5\,Ab^3\right)x^9+2\left(2\,Ba^2b-5\,Aab^2\right)x^6+\left(2\,Ba^3-5\,Aa^2b\right)x^3\right)\sqrt{a}\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)-2\left(3\left(2\,Ba^2b-5\,A\right.\right.}{18\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B
*a^3 - 5*A*a^2*b)*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a
)/x^3) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*
b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*((2*
B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B*a^3 - 5*A*a^2
*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(2*B*a^2*b - 5*A*
a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17357, size = 136, normalized size = 1.2

$$\frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^3}+\frac{2\left(3\left(bx^3+a\right)Ba+Ba^2-6\left(bx^3+a\right)Ab-Aab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx^3+a}A}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 5*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/9*(
3*(b*x^3 + a)*B*a + B*a^2 - 6*(b*x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a
^3) - 1/3*sqrt(b*x^3 + a)*A/(a^3*x^3)
```

3.249 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=299

$$\frac{32\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-14aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x^4(5Ab-14}{45b^2\left(a+bx^3\right.}$$

```
[Out] (-2*(5*A*b - 14*a*B)*x^4)/(45*b^2*(a + b*x^3)^(3/2)) + (2*B*x^7)/(5*b*(a +
b*x^3)^(3/2)) - (16*(5*A*b - 14*a*B)*x)/(135*b^3*Sqrt[a + b*x^3]) + (32*Sqr
t[2 + Sqrt[3]]*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(135*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.131619, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {459, 288, 218}

$$\frac{32\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x^4(5Ab-14aB)}{45b^2\left(a+bx^3\right)^{3/2}}-1$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(5*A*b - 14*a*B)*x^4)/(45*b^2*(a + b*x^3)^(3/2)) + (2*B*x^7)/(5*b*(a +
b*x^3)^(3/2)) - (16*(5*A*b - 14*a*B)*x)/(135*b^3*Sqrt[a + b*x^3]) + (32*Sqr
t[2 + Sqrt[3]]*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(135*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
```

```
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x^6 \left(A + Bx^3\right)}{\left(a + bx^3\right)^{5/2}} dx = \frac{2Bx^7}{5b \left(a + bx^3\right)^{3/2}} - \frac{\left(2 \left(-\frac{5Ab}{2} + 7aB\right)\right) \int \frac{x^6}{\left(a+bx^3\right)^{5/2}} dx}{5b}$$

$$= -\frac{2(5Ab - 14aB)x^4}{45b^2 \left(a + bx^3\right)^{3/2}} + \frac{2Bx^7}{5b \left(a + bx^3\right)^{3/2}} + \frac{(8(5Ab - 14aB)) \int \frac{x^3}{\left(a+bx^3\right)^{3/2}} dx}{45b^2}$$

$$= -\frac{2(5Ab - 14aB)x^4}{45b^2 \left(a + bx^3\right)^{3/2}} + \frac{2Bx^7}{5b \left(a + bx^3\right)^{3/2}} - \frac{16(5Ab - 14aB)x}{135b^3 \sqrt{a + bx^3}} + \frac{(16(5Ab - 14aB)) \int \frac{1}{\sqrt{a+bx^3}} dx}{135b^3}$$

$$= -\frac{2(5Ab - 14aB)x^4}{45b^2 \left(a + bx^3\right)^{3/2}} + \frac{2Bx^7}{5b \left(a + bx^3\right)^{3/2}} - \frac{16(5Ab - 14aB)x}{135b^3 \sqrt{a + bx^3}} + \frac{32\sqrt{2 + \sqrt{3}}(5Ab - 14aB) \left(\sqrt[3]{a} + \sqrt[3]{b}\right.}{135\sqrt[4]{3}b^{1}}$$

Mathematica [C] time = 0.128289, size = 108, normalized size = 0.36

$$\frac{2x \left(112a^2B + 8 \left(a + bx^3\right) \sqrt{\frac{bx^3}{a} + 1}(5Ab - 14aB)\, {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{bx^3}{a}\right) + a \left(154bBx^3 - 40Ab\right) + b^2x^3 \left(27Bx^3 - 55A\right)\right)}{135b^3 \left(a + bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(112*a^2*B + b^2*x^3*(-55*A + 27*B*x^3) + a*(-40*A*b + 154*b*B*x^3) +
8*(5*A*b - 14*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)]))/(135*b^3*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.035, size = 683, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*a^2*x/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^2+40/27/b^3*a*x/((x^3+a/b)*b)^(
1/2)+2/5/b^3*x*(b*x^3+a)^(1/2)+448/405*I*a/b^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(2/9*a*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)
^2-22/27/b^2*x/((x^3+a/b)*b)^(1/2)-32/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)x^6}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^9 + Ax^6\right)\sqrt{bx^3 + a}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^9 + A*x^6)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x
^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^6}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

3.250 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=283

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(8aB+Ab)}{27ab^2\sqrt{a+bx^3}}+\ldots$$

```
[Out] (2*(A*b - a*B)*x^4)/(9*a*b*(a + b*x^3)^(3/2)) - (2*(A*b + 8*a*B)*x)/(27*a*b
^2*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a*b^(7/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3])
```

Rubi [A] time = 0.103012, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {457, 288, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(8aB+Ab)}{27ab^2\sqrt{a+bx^3}}+\frac{2x^4(Ab\ldots}{9ab\left(a+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^4)/(9*a*b*(a + b*x^3)^(3/2)) - (2*(A*b + 8*a*B)*x)/(27*a*b
^2*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a*b^(7/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
```

```
rt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(\frac{Ab}{2}+4aB\right)\right)\int \frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{(2(Ab+8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-}\right.}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

Mathematica [C] time = 0.112292, size = 99, normalized size = 0.35

$$\frac{2x\left(-8a^2B+\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(8aB+Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-ab\left(A+11Bx^3\right)+2Ab^2x^3\right)}{27ab^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(-8*a^2*B + 2*A*b^2*x^3 - a*b*(A + 11*B*x^3) + (A*b + 8*a*B)*(a + b*x^
3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(27
*a*b^2*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.022, size = 669, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9*a*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-22/27/b^2*x/((x^3+a/b)*b)^(1/2)-
32/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
```

```
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*
b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+
A*(-2/9*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/
81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)x^3}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(Bx^6 + Ax^3\right)\sqrt{bx^3 + a}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x
^3 + a^3), x)
```

Sympy [A] time = 171.476, size = 80, normalized size = 0.28

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)x^3}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

3.251 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=283

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(2aB+7Ab)}{27a^2b\sqrt{a+bx^3}}$$

```
[Out] (2*(A*b - a*B)*x)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(7*A*b + 2*a*B)*x)/(27*a^2
*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(4/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.09851, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 199, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(2aB+7Ab)}{27a^2b\sqrt{a+bx^3}}+\frac{2x(}{9ab(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```


```
[Out] (2*(A*b - a*B)*x)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(7*A*b + 2*a*B)*x)/(27*a^2
*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(4/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(\frac{7Ab}{2}+aB\right)\right)\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{(7Ab+2aB)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)x}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-}\right.}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

Mathematica [C] time = 0.0545086, size = 103, normalized size = 0.36

$$\frac{-2a^2Bx + x\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(2aB+7Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right) + 4abx\left(5A+Bx^3\right) + 14Ab^2x^4}{27a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*B*x + 14*A*b^2*x^4 + 4*a*b*x*(5*A + B*x^3) + (7*A*b + 2*a*B)*x*(a +
 b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])
/(27*a^2*b*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.018, size = 674, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/
81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A
*(2/9/a*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)*b)^(1/2)-1
4/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 +
 a^3), x)
```

Sympy [A] time = 170.077, size = 78, normalized size = 0.28

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/2)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

3.252 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{7\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{7x(13Ab-4aB}{54a^3\sqrt{a+bx^3}}$$

```
[Out] -A/(2*a*x^2*(a + b*x^3)^(3/2)) - ((13*A*b - 4*a*B)*x)/(18*a^2*(a + b*x^3)^(
3/2)) - (7*(13*A*b - 4*a*B)*x)/(54*a^3*Sqrt[a + b*x^3]) - (7*Sqrt[2 + Sqrt[
3]]*(13*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(54*3^(1/4)*a^3*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.129629, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 199, 218}

$$-\frac{7\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{7x(13Ab-4aB)}{54a^3\sqrt{a+bx^3}}-\frac{x(1}{18a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```


```
[Out] -A/(2*a*x^2*(a + b*x^3)^(3/2)) - ((13*A*b - 4*a*B)*x)/(18*a^2*(a + b*x^3)^(
3/2)) - (7*(13*A*b - 4*a*B)*x)/(54*a^3*Sqrt[a + b*x^3]) - (7*Sqrt[2 + Sqrt[
3]]*(13*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(54*3^(1/4)*a^3*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{13Ab}{2}-2aB\right)\int \frac{1}{\left(a+bx^3\right)^{5/2}}\,dx}{2a}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}} - \frac{(7(13Ab-4aB))\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx}{36a^2}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}} - \frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}} - \frac{(7(13Ab-4aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{108a^3}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}} - \frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}} - \frac{7\sqrt{2+\sqrt{3}}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{54\sqrt[4]{3}a^3}$$

Mathematica [C] time = 0.0582123, size = 116, normalized size = 0.39

$$\frac{a^2\left(80Bx^3-54A\right)+7x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(4aB-13Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a\left(56bBx^6-260Abx^3\right)-182Ab^2x^6}{108a^3x^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-182*A*b^2*x^6 + a^2*(-54*A + 80*B*x^3) + a*(-260*A*b*x^3 + 56*b*B*x^6) +
7*(-13*A*b + 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/3, 1/2, 4/3, -((b*x^3)/a)])/(108*a^3*x^2*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.026, size = 689, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9/a*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)*b)^(1/2)-
14/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
)+A*(-1/2/a^3*(b*x^3+a)^(1/2)/x^2-2/9/a^2*x/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-3
2/27*b/a^3*x/((x^3+a/b)*b)^(1/2)+91/162*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^3x^{12} + 3\,ab^2x^9 + 3\,a^2bx^6 + a^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^12 + 3*a*b^2*x^9 + 3*a^2*b*x^6
+ a^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

3.253 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=334

$$\frac{91\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{540\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{91\sqrt{a+b}}{}$$

```
[Out] -A/(5*a*x^5*(a + b*x^3)^(3/2)) - (19*A*b - 10*a*B)/(45*a^2*x^2*(a + b*x^3)^
(3/2)) - (13*(19*A*b - 10*a*B))/(135*a^3*x^2*Sqrt[a + b*x^3]) + (91*(19*A*b
 - 10*a*B)*Sqrt[a + b*x^3])/(540*a^4*x^2) + (91*Sqrt[2 + Sqrt[3]]*b^(2/3)*(
19*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(540*3^(1/4)*a^4*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.166023, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 290, 325, 218}

$$\frac{91\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{540\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{91\sqrt{a+bx^3}(19A}{540a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^3)^(3/2)) - (19*A*b - 10*a*B)/(45*a^2*x^2*(a + b*x^3)^
(3/2)) - (13*(19*A*b - 10*a*B))/(135*a^3*x^2*Sqrt[a + b*x^3]) + (91*(19*A*b
 - 10*a*B)*Sqrt[a + b*x^3])/(540*a^4*x^2) + (91*Sqrt[2 + Sqrt[3]]*b^(2/3)*(
19*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
```

```
Sqrt[3]])/(540*3^(1/4)*a^4*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{19Ab}{2}-5aB\right)\int \frac{1}{x^3\left(a+bx^3\right)^{5/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{(13(19Ab-10aB))\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx}{90a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} - \frac{(91(19Ab-10aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,d}{270a^3}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} + \frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2} + \frac{(}{-}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} + \frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2} + -$$

Mathematica [C] time = 0.0412599, size = 83, normalized size = 0.25

$$\frac{x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(\frac{19Ab}{2}-5aB\right){}_2F_1\left(-\frac{2}{3},\frac{5}{2};\frac{1}{3};-\frac{bx^3}{a}\right)-2a^2A}{10a^3x^5\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*a^2*A + ((19*A*b)/2 - 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-2/3, 5/2, 1/3, -((b*x^3)/a)])/(10*a^3*x^5*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.029, size = 722, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-1/2/a^3*(b*x^3+a)^(1/2)/x^2-2/9/a^2*x/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-32/
27*b/a^3*x/((x^3+a/b)*b)^(1/2)+91/162*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+A*(-1/5/a^3*(b*x^3+a)^(1/2)/x^5+27/20*b/a^4
*(b*x^3+a)^(1/2)/x^2+2/9/a^3*x*(b*x^3+a)^(1/2)/(x^3+a/b)^2+50/27*b^2/a^4*x/
((x^3+a/b)*b)^(1/2)-1729/1620*I*b/a^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^3x^{15} + 3\,ab^2x^{12} + 3\,a^2bx^9 + a^3x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^15 + 3*a*b^2*x^12 + 3*a^2*b*x^9
 + a^3*x^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

3.254 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=577

$$\frac{80\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-16aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{40\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{\ldots}\right.}{\ldots}$$

```
[Out] (-2*(7*A*b - 16*a*B)*x^5)/(63*b^2*(a + b*x^3)^(3/2)) + (2*B*x^8)/(7*b*(a +
b*x^3)^(3/2)) - (20*(7*A*b - 16*a*B)*x^2)/(189*b^3*Sqrt[a + b*x^3]) + (80*(
7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(189*b^(11/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (40*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(63*3^(3/4)*b^(11/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]) + (80*Sqrt[2]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(189*3^(1/4)*b^(11/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rubi [A] time = 0.317913, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 303, 218, 1877}

$$\frac{80\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{40\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(7*A*b - 16*a*B)*x^5)/(63*b^2*(a + b*x^3)^(3/2)) + (2*B*x^8)/(7*b*(a +
b*x^3)^(3/2)) - (20*(7*A*b - 16*a*B)*x^2)/(189*b^3*Sqrt[a + b*x^3]) + (80*(
7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(189*b^(11/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (40*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(63*3^(3/4)*b^(11/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]) + (80*Sqrt[2]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(189*3^(1/4)*b^(11/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{\left(2\left(-\frac{7Ab}{2}+8aB\right)\right)\int \frac{x^7}{\left(a+bx^3\right)^{5/2}}\,dx}{7b}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} + \frac{(10(7Ab-16aB))\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{63b^2}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{189b^3}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{189b^{10/3}}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{80(7Ab-16aB)\sqrt{a+bx^3}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{40}{}$$

Mathematica [C] time = 0.104084, size = 109, normalized size = 0.19

$$\frac{2x^2\left(-32a^2B+2\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(16aB-7Ab)\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+2ab\left(7A-8Bx^3\right)+b^2x^3\left(7A+Bx^3\right)\right)}{7b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x^2*(-32*a^2*B + 2*a*b*(7*A - 8*B*x^3) + b^2*x^3*(7*A + B*x^3) + 2*(-7*A
*b + 16*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/
3, -((b*x^3)/a)]))/(7*b^3*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.036, size = 997, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*a^2*x^2/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^2+44/27/b^3*a*x^2/((x^3+a/b)*
b)^(1/2)+2/7/b^3*x^2*(b*x^3+a)^(1/2)+1280/567*I*a/b^4*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))))+A*(2/9*a*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-26/27/b^
2*x^2/((x^3+a/b)*b)^(1/2)-80/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^7}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(Bx^{10} + Ax^7)\sqrt{bx^3 + a}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^10 + A*x^7)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*
x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)x^7}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

3.255 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=559

$$-\frac{8\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

```
[Out] (2*(A*b - a*B)*x^5)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(A*b - 10*a*B)*x^2)/(27*
a*b^2*Sqrt[a + b*x^3]) - (8*(A*b - 10*a*B)*Sqrt[a + b*x^3])/(27*a*b^(8/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*Sqrt[2 - Sqrt[3]]*(A*b - 10*a*B)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/
4)*a^(2/3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*(A*b - 10*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(2/
3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.265025, antiderivative size = 559, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 288, 303, 218, 1877}

$$-\frac{8\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-}{((1+}}}{9\ 3^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^5)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(A*b - 10*a*B)*x^2)/(27*
a*b^2*Sqrt[a + b*x^3]) - (8*(A*b - 10*a*B)*Sqrt[a + b*x^3])/(27*a*b^(8/3)*(
```

```
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*Sqrt[2 - Sqrt[3]]*(A*b - 10*a*B)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/
4)*a^(2/3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*(A*b - 10*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(2/
3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(-\frac{Ab}{2}+5aB\right)\right)\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{(4(Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{(4(Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{27ab^{7/3}} - \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}(Ab-1\right.}{27a^{2/3}b}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{4\sqrt{2-\sqrt{3}}(Ab-10aB)\left(\sqrt[3]{a}\right.}{9}$$

Mathematica [C] time = 0.0939175, size = 92, normalized size = 0.16

$$\frac{2x^2\left(\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(Ab-10aB)\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-aAb+5aB\left(2a+bx^3\right)\right)}{5ab^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x^2*(-(a*A*b) + 5*a*B*(2*a + b*x^3) + (A*b - 10*a*B)*(a + b*x^3)*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(5*a*b^2*(a +
 b*x^3)^(3/2))
```

Maple [B] time = 0.022, size = 981, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9*a*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-26/27/b^2*x^2/((x^3+a/b)*b)^(1
/2)-80/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-2/9*x^2/b^3*
(b*x^3+a)^(1/2)/(x^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1/2)+8/81*I/b^2/a*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^4}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bx^7 + Ax^4)\sqrt{bx^3 + a}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^7 + A*x^4)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x
^3 + a^3), x)
```

Sympy [A] time = 173.245, size = 80, normalized size = 0.14

$$\frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{5}{2}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*ex
```

```
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(11/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)x^4}{(bx^3 + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

3.256 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=563

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}(4aB+5}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}$$

```
[Out] (2*(A*b - a*B)*x^2)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(5*A*b + 4*a*B)*x^2)/(27
*a^2*b*Sqrt[a + b*x^3]) - (2*(5*A*b + 4*a*B)*Sqrt[a + b*x^3])/(27*a^2*b^(5/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(5*A*b + 4*a*B
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^
(3/4)*a^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(5*A*b + 4*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a
^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.248184, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {457, 290, 303, 218, 1877}

$$-\frac{2\sqrt{a+bx^3}(4aB+5Ab)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(5*A*b + 4*a*B)*x^2)/(27
*a^2*b*Sqrt[a + b*x^3]) - (2*(5*A*b + 4*a*B)*Sqrt[a + b*x^3])/(27*a^2*b^(5/
```

```
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(5*A*b + 4*a*B
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^
(3/4)*a^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(5*A*b + 4*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a
^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}}+\frac{\left(2\left(\frac{5Ab}{2}+2aB\right)\right)\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}}-\frac{(5Ab+4aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}}-\frac{(5Ab+4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{27a^2b^{4/3}}-\frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}(5Ab+4aB\right.}{27a^{5/3}b^{4/3}}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}}-\frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt{2-\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}\right.}{9}$$

Mathematica [C] time = 0.0709847, size = 81, normalized size = 0.14

$$\frac{x^2\left(\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(4aB+5Ab)\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-4a^2B\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x^2*(-4*a^2*B + (5*A*b + 4*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.019, size = 986, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*x^2/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1/2
)+8/81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(2/9/a*x^2/b^2*
(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)*b)^(1/2)+10/81*I/a^2*3
^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)x}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^4 + Ax\right)\sqrt{bx^3 + a}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3
 + a^3), x)
```

Sympy [A] time = 170.762, size = 80, normalized size = 0.14

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)x}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

3.257 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=578

$$\frac{5\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{a+bx^3}(11Ab-2}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}$$

```
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - ((11*A*b - 2*a*B)*x^2)/(9*a^2*(a + b*x^3)^(3
/2)) - (5*(11*A*b - 2*a*B)*x^2)/(27*a^3*Sqrt[a + b*x^3]) + (5*(11*A*b - 2*a
*B)*Sqrt[a + b*x^3])/(27*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (5*Sqrt[2 - Sqrt[3]]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) + (5*Sqrt[2]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.299794, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 290, 303, 218, 1877}

$$\frac{5\sqrt{a+bx^3}(11Ab-2aB)}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{5\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - ((11*A*b - 2*a*B)*x^2)/(9*a^2*(a + b*x^3)^(3
/2)) - (5*(11*A*b - 2*a*B)*x^2)/(27*a^3*Sqrt[a + b*x^3]) + (5*(11*A*b - 2*a
```

```
*B)*Sqrt[a + b*x^3])/(27*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (5*Sqrt[2 - Sqrt[3]]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) + (5*Sqrt[2]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{ax\left(a+bx^3\right)^{3/2}}-\frac{\left(\frac{11Ab}{2}-aB\right)\int \frac{x}{\left(a+bx^3\right)^{5/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}}-\frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{(5(11Ab-2aB))\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{18a^2}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}}-\frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}}+\frac{(5(11Ab-2aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{54a^3}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}}-\frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}}+\frac{(5(11Ab-2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{54a^3\sqrt[3]{b}}+$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}}-\frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}}+\frac{5(11Ab-2aB)\sqrt{a+bx^3}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{5\sqrt{2}}{}$$

Mathematica [C] time = 0.0384735, size = 86, normalized size = 0.15

$$-\frac{x^2\sqrt{\frac{bx^3}{a}+1}\left(\frac{11Ab}{2}-aB\right){}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a^3\sqrt{a+bx^3}}-\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - (((11*A*b)/2 - a*B)*x^2*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(2*a^3*Sqrt[a + b*x^3])
```

Maple [B] time = 0.026, size = 1001, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9/a*x^2/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)*b)^(1
/2)+10/81*I/a^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))+A*(-1/a^3*(b*x
^3+a)^(1/2)/x-2/9/a^2*x^2/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-28/27*b/a^3*x^2/((x
^3+a/b)*b)^(1/2)-55/81*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)
^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{b^3x^{11} + 3\,ab^2x^8 + 3\,a^2bx^5 + a^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^11 + 3*a*b^2*x^8 + 3*a^2*b*x^5
+ a^3*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

3.258 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=610

$$-\frac{55\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] -A/(4*a*x^4*(a + b*x^3)^(3/2)) - (17*A*b - 8*a*B)/(36*a^2*x*(a + b*x^3)^(3/
2)) - (11*(17*A*b - 8*a*B))/(108*a^3*x*Sqrt[a + b*x^3]) + (55*(17*A*b - 8*a
*B)*Sqrt[a + b*x^3])/(216*a^4*x) - (55*b^(1/3)*(17*A*b - 8*a*B)*Sqrt[a + b*
x^3])/(216*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (55*Sqrt[2 - Sqrt[3]]
*b^(1/3)*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(144*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (55*b^(1/3)*(1
7*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(108*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.3556, antiderivative size = 610, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 290, 325, 303, 218, 1877}

$$-\frac{55\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

```
[Out] -A/(4*a*x^4*(a + b*x^3)^(3/2)) - (17*A*b - 8*a*B)/(36*a^2*x*(a + b*x^3)^(3/
2)) - (11*(17*A*b - 8*a*B))/(108*a^3*x*Sqrt[a + b*x^3]) + (55*(17*A*b - 8*a
*B)*Sqrt[a + b*x^3])/(216*a^4*x) - (55*b^(1/3)*(17*A*b - 8*a*B)*Sqrt[a + b*
x^3])/(216*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (55*Sqrt[2 - Sqrt[3]]
*b^(1/3)*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(144*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (55*b^(1/3)*(1
7*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(108*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
```

```
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{17Ab}{2}-4aB\right)\int \frac{1}{x^2\left(a+bx^3\right)^{5/2}}\,dx}{4a}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{(11(17Ab-8aB))\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx}{72a^2}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} - \frac{(55(17Ab-8aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{216a^3}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x} -$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x} -$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x} -$$

Mathematica [C] time = 0.0397292, size = 83, normalized size = 0.14

$$\frac{x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(\frac{17Ab}{2}-4aB\right){}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};-\frac{bx^3}{a}\right)-a^2A}{4a^3x^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-(a^2*A) + ((17*A*b)/2 - 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-1/3, 5/2, 2/3, -((b*x^3)/a)])/(4*a^3*x^4*(a + b*x^3)^(3/2))
```

Maple [B] time = 0.028, size = 1034, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-1/a^3*(b*x^3+a)^(1/2)/x-2/9/a^2*x^2/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-28/27
*b/a^3*x^2/((x^3+a/b)*b)^(1/2)-55/81*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))))+A*(-1/4/a^3*(b*x^3+a)^(1/2)/x^4+21/8*b/a^4*(b*x^3+a)^(1/2)/x+2/9
/a^3*x^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+46/27*b^2/a^4*x^2/((x^3+a/b)*b)^(1/2)+
935/648*I*b/a^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^3x^{14}+3\,ab^2x^{11}+3\,a^2bx^8+a^3x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^14 + 3*a*b^2*x^11 + 3*a^2*b*x^8
 + a^3*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

3.259 $\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=97

$$\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

```
[Out] (32*c^2*Sqrt[c + d*x^3])/(3*d^3) - (10*c*(c + d*x^3)^(3/2))/(9*d^3) + (2*(c
 + d*x^3)^(5/2))/(15*d^3) - (32*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])])/(Sqrt[3]*d^3)
```

Rubi [A] time = 0.093991, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 88, 50, 63, 203}

$$\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```


```
[Out] (32*c^2*Sqrt[c + d*x^3])/(3*d^3) - (10*c*(c + d*x^3)^(3/2))/(9*d^3) + (2*(c
 + d*x^3)^(5/2))/(15*d^3) - (32*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])])/(Sqrt[3]*d^3)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{5c\sqrt{c+dx}}{d^2} + \frac{(c+dx)^{3/2}}{d^2} + \frac{16c^2\sqrt{c+dx}}{d^2(4c+dx)}\right)dx, x, x^3\right)$$
$$= -\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{\left(16c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)}{3d^2}$$
$$= \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{\left(16c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{d^2}$$
$$= \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{\left(32c^3\right)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^3}$$
$$= \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

Mathematica [A] time = 0.0618643, size = 77, normalized size = 0.79

$$\frac{2\sqrt{c+dx^3}\left(218c^2 - 19cdx^3 + 3d^2x^6\right) - 480\sqrt{3}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{45d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6) - 480*Sqrt[3]*c^(5/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(45*d^3)
```

Maple [C] time = 0.187, size = 506, normalized size = 5.2

$$\frac{1}{d^2}\left(d\left(\frac{2\,x^6}{15}\sqrt{dx^3+c} + \frac{2\,cx^3}{45\,d}\sqrt{dx^3+c} - \frac{4\,c^2}{45\,d^2}\sqrt{dx^3+c}\right) - \frac{8\,c}{9\,d}\left(dx^3+c\right)^{\frac{3}{2}}\right) + 16\,\frac{c^2}{d^2}\left(2/3\,\frac{\sqrt{dx^3+c}}{d} + \frac{i/3\sqrt{2}}{d^3}\sum_{\_alpha=RootOf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 1/d^2*(d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45/d*c*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d
*x^3+c)^(1/2)/d^2)-8/9*c/d*(d*x^3+c)^(3/2))+16*c^2/d^2*(2/3*(d*x^3+c)^(1/2)
/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/
(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^
2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d+4*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10169, size = 397, normalized size = 4.09

$$\left[\frac{2\left(120\sqrt{3}\sqrt{-c}c^2\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\left(3d^2x^6-19cdx^3+218c^2\right)\sqrt{dx^3+c}\right)}{45d^3},-\frac{2\left(240\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [2/45*(120*sqrt(3)*sqrt(-c)*c^2*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt
(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3
```

```
+ c))/d^3, -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sq
rt(c)) - (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] time = 55.3091, size = 85, normalized size = 0.88

$$\frac{2\left(-\frac{16\sqrt{3}c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{16c^2\sqrt{c+dx^3}}{3}-\frac{5c\left(c+dx^3\right)^{\frac{3}{2}}}{9}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-16*sqrt(3)*c**(5/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + 16*c
**2*sqrt(c + d*x**3)/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x**3)**(5/2)/15
)/d**3
```

Giac [A] time = 1.11766, size = 111, normalized size = 1.14

$$-\frac{32\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}-25\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+240\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/4
5*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^3
+ c)*c^2*d^12)/d^15
```

3.260 $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=76

$$\frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

```
[Out] (-8*c*Sqrt[c + d*x^3])/(3*d^2) + (2*(c + d*x^3)^(3/2))/(9*d^2) + (8*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Rubi [A] time = 0.0623822, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 50, 63, 203}

$$\frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (-8*c*Sqrt[c + d*x^3])/(3*d^2) + (2*(c + d*x^3)^(3/2))/(9*d^2) + (8*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{(4c)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(4c^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(8c^2\right)\operatorname{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}
\end{aligned}
$$

Mathematica [A] time = 0.0283623, size = 65, normalized size = 0.86

$$\frac{24\sqrt{3}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+2\left(dx^3-11c\right)\sqrt{c+dx^3}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(-11*c + d*x^3)*Sqrt[c + d*x^3] + 24*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x
^3]/(Sqrt[3]*Sqrt[c])])/(9*d^2)
```

Maple [C] time = 0.01, size = 446, normalized size = 5.9

$$\frac{2}{9\,d^2}\left(dx^3+c\right)^{\frac{3}{2}}-4\,\frac{c}{d}\left(2/3\,\frac{\sqrt{dx^3+c}}{d}+\frac{i/3\sqrt{2}}{d^3}\sum_{\_alpha=RootOf\left(\_Z^3d+4c\right)}\frac{\sqrt[3]{-d^2c}\left(i\sqrt[3]{-d^2c}\_alpha\sqrt{3}d-i\left(-d^2c\right)^{2/3}\sqrt{3}+2\_a\right.}{\sqrt{dx^3+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^2-4/d*c*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum(
(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3))
)/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1
/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d
-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.09525, size = 332, normalized size = 4.37

$$\left[\frac{2\left(6\sqrt{3}\sqrt{-c}c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}\left(dx^3-11c\right)\right)}{9d^2}, \frac{2\left(12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}\left(dx^3-11c\right)\right.}{9d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")

[Out] [2/9*(6*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, 2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2]

Sympy [A] time = 19.2291, size = 68, normalized size = 0.89

$$\frac{2\left(\frac{4\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}-\frac{4c\sqrt{c+dx^3}}{3}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)

[Out] 2*(4*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 - 4*c*sqrt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**2

Giac [A] time = 1.13672, size = 92, normalized size = 1.21

$$\frac{2\left(\frac{12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d}+\frac{\left(dx^3+c\right)^{\frac{3}{2}}d^2-12\sqrt{dx^3+c}cd^2}{d^3}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + ((d
*x^3 + c)^(3/2)*d^2 - 12*sqrt(d*x^3 + c)*c*d^2)/d^3)/d
```

3.261 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

```
[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d)
```

Rubi [A] time = 0.045898, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 50, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3d} - c\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3d} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\
&= \frac{2\sqrt{c+dx^3}}{3d} - \frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}
\end{aligned}$$

Mathematica [A] time = 0.0142963, size = 54, normalized size = 0.95

$$\frac{2\left(\sqrt{c+dx^3}-\sqrt{3}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])]))/(3*d)
```

Maple [C] time = 0.006, size = 425, normalized size = 7.5

$$\frac{2}{3d}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{d^3}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3d+4c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1
/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^
(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d
^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0047, size = 281, normalized size = 4.93

$$\left[\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+2\sqrt{dx^3+c}}{3d},-\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c)) + 2*sqrt(d*x^3 + c))/d, -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*
sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*x^3 + c))/d]
```

Sympy [A] time = 4.40482, size = 51, normalized size = 0.89

$$\frac{2\left(-\frac{\sqrt{3}\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + sqrt(c +
 d*x**3)/3)/d
```

Giac [A] time = 1.12275, size = 59, normalized size = 1.04

$$-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d}+\frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + 2/3*sq
rt(d*x^3 + c)/d
```

3.262 $\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])
```

Rubi [A] time = 0.056413, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 83, 63, 208, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(4c+dx)}\,dx,x,x^3\right)\\
&= \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)+\frac{1}{4}d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{6d}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0162407, size = 59, normalized size = 0.91

$$
\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)-\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(6*Sqrt[c])
```

Maple [C] time = 0.019, size = 468, normalized size = 7.2

$$-\frac{d}{4c}\left(\frac{2}{3d}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{d^3}\sum_{\_alpha=RootOf\left(d\_Z^3+4c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x)
```

```
[Out] -1/4*d/c*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/
2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))+1/4/c*(2/3*(d*x^3+c)^(
1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)
```

Fricas [A] time = 1.86965, size = 382, normalized size = 5.88

$$\left[\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{12c}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right.}{12c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)*s
qrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c
)) - 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c]
```

Sympy [A] time = 8.85035, size = 66, normalized size = 1.02

$$\frac{2\left(\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}+\frac{\sqrt{3}d\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)
```

```
[Out] 2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*atan(sqrt(3)
*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d
```

Giac [A] time = 1.13242, size = 68, normalized size = 1.05

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/sqrt(c) + 1/6*arcta
n(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)
```

3.263 $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}$$

```
[Out] -Sqrt[c + d*x^3]/(12*c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])
/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Rubi [A] time = 0.0769426, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 99, 156, 63, 208, 203}

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])
/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
```

```
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(4c+dx)}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{cd-\frac{d^2x}{2}}{x\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{12c}$$
$$= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{48c} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{16c}$$
$$= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{24c} - \frac{d\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{8c}$$
$$= -\frac{\sqrt{c+dx^3}}{12cx^3} - \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

Mathematica [A] time = 0.0399331, size = 88, normalized size = 1.

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}} - \frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])
/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Maple [C] time = 0.022, size = 511, normalized size = 5.8

$$\frac{d^2}{16\,c^2}\left(\frac{2}{3\,d}\sqrt{dx^3+c} + \frac{\frac{i}{3}\sqrt{2}}{d^3}\sum_{\_alpha=\text{RootOf}\left(d\_Z^3+4\,c\right)} \sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x)
```

```
[Out] 1/16*d^2/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_a
lpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_al
pha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))+1/4/c*(-1/3*(d*x^3
+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-1/16*d/c^2*(2
/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

Fricas [A] time = 1.89225, size = 497, normalized size = 5.65

$$\left[-\frac{2\,\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-\sqrt{c}dx^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+4\,\sqrt{dx^3+c}c}{48\,c^2x^3},-\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3+2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 - sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 4*sqr
t(d*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*sqr
```

```
t(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*arct
an(sqrt(d*x^3 + c)*sqrt(-c)/c) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)
```

Giac [A] time = 1.10534, size = 101, normalized size = 1.15

$$-\frac{1}{24}\,d\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{2\sqrt{dx^3+c}}{cdx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -1/24*d*(sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) - arct
an(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) + 2*sqrt(d*x^3 + c)/(c*d*x^3))
```

3.264 $\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=689

$$-\frac{50\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{\ldots}\right.}{\sqrt{c+dx^3}}\right.}{\sqrt{3}d^{5/3}}$$

```
[Out] (2*x^2*Sqrt[c + d*x^3])/(7*d) - (50*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (2*2^(1/3)*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(5/3)) + (2*2^(1/
3)*c^(7/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(5/3)) - (
2*2^(1/3)*c^(7/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c +
d*x^3]])/d^(5/3) + (2*2^(1/3)*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*
d^(5/3)) + (25*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5
0*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.477017, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {478, 584, 303, 218, 1877, 484}

$$-\frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}}+\frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}}-\frac{2\sqrt[3]{2}c^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{2\sqrt[3]{2}c^{7/6}\tanh^{-1}\left(\ldots\right.}{3d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*x^2*Sqrt[c + d*x^3])/(7*d) - (50*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (2*2^(1/3)*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(5/3)) + (2*2^(1/
3)*c^(7/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(5/3)) - (
2*2^(1/3)*c^(7/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c +
d*x^3]])/d^(5/3) + (2*2^(1/3)*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*
d^(5/3)) + (25*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5
0*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \frac{x\left(8c^2+\frac{25}{2}cdx^3\right)}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{7d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \left(\frac{25cx}{2\sqrt{c+dx^3}} - \frac{42c^2x}{\sqrt{c+dx^3}(4c+dx^3)}\right)dx}{7d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(25c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{(12c^2)\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}c^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{c}(\cdots}{\cdots}\right)}{d^{5/3}}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\cdots\right)}{\sqrt{3}d^{5/3}}$$

Mathematica [C] time = 0.0759278, size = 133, normalized size = 0.19

$$\frac{-5dx^5\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) - 8cx^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) + 8x^2\left(c+dx^3\right)}{28d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (8*x^2*(c + d*x^3) - 8*c*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), -(d*x^3)/(4*c)] - 5*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), -(d*x^3)/(4*c)])/(28*d*Sqrt[c + d*x^3])
```

Maple [C] time = 0.034, size = 1309, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 1/d*(2/7*x^2*(d*x^3+c)^(1/2)-2/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2))))-4/d*c*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2
*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/
3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2
*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3*
2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*
(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-
d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*d+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x^4}{dx^3 + 4c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}\,x^4}{dx^3 + 4\,c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c + dx^3}}{4c + dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}\,x^4}{dx^3 + 4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

3.265 $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=659

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2/3}\sqrt{\cdots}}$$

```
[Out] (2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/
(2^(2/3)*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
]])])/(2^(2/3)*Sqrt[3]*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(2^(2/3)*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(3*2^(2/3)*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/
3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.167457, antiderivative size = 659, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {489, 303, 218, 1877, 484}

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/
(2^(2/3)*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(2^(2/3)*Sqrt[3]*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(2^(2/3)*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(3*2^(2/3)*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/
3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
```

```
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\left((3c)\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{c+dx^3}}\,dx$$

$$= \frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}-\frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}+\frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}}-\frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+\cdots}}{\sqrt{\cdots}}\right)}{3\ 2^{2/3}d^{2/3}}\cdots$$

$$= \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}-\frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}+\frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{\cdots}\right)}{\sqrt{\cdots}}\right)}{2^{2/3}d^{2/3}}\cdots$$

Mathematica [C] time = 0.0295845, size = 63, normalized size = 0.1

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -(d*x^3)
/(4*c)])/(8*Sqrt[c + d*x^3])
```

Maple [C] time = 0.007, size = 848, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] -2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/
_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)
+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_
alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(
-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4
*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x}{dx^3 + 4c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+cx}}{dx^3+4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+cx}}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

3.266 $\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$

Optimal. Leaf size=697

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\cdots}{\left(\left(1+\cdots\right.\right.}}\cdots}$$

```
[Out] -Sqrt[c + d*x^3]/(4*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c*((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d
^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) + (d^(1/3)*ArcTan[
Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) - (d^(1/3)*
ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3
)*c^(5/6)) + (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(12*2^(2/3)*c^(5/6
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)
*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^
(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d
^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[2]
*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.383887, antiderivative size = 697, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {475, 584, 303, 218, 1877, 484}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)\cdots}}\cdots}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(4*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c*((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d
^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) + (d^(1/3)*ArcTan[
Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) - (d^(1/3)*
ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3
)*c^(5/6)) + (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(12*2^(2/3)*c^(5/6)
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)
*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^
(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d
^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[2]
*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{4cx}+\frac{\int \frac{x\left(5cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx}+\frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}}+\frac{3cdx}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\right)dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx}+\frac{1}{4}(3d)\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx+\frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}-\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}c^{5/6}}+$$

$$= -\frac{\sqrt{c+dx^3}}{4cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}-$$

Mathematica [C] time = 0.0714126, size = 136, normalized size = 0.2

$$\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+25cdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-40c\left(c+dx^3\right)}{160c^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

```
[Out] (-40*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -(d*x^3)/(4*c)] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -(d*x^3)/(4*c)])/(160*c^2*x*Sqrt[c + d*x^3])
```

Maple [C] time = 0.02, size = 1306, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x)
```

```
[Out] 1/4/c*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*
(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*
(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/4*
d/c*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*s
um(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^
(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1
/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(
2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(
1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*d+4*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 + 4\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{dx^5+4\,cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d*x^5 + 4*c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

3.267 $\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -(d*x^3)/(4*c), -((d*x^3)/
c)])/(16*c*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0537595, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -(d*x^3)/(4*c), -((d*x^3)/
c)])/(16*c*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.274762, size = 236, normalized size = 3.58

$$\frac{x\left(32\left(\frac{64c^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{d(4c+dx^3)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)-16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)}+\frac{c}{d}+x^3\right)-17x^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1\right.}{80\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x*(-17*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
d*x^3)/(4*c)] + 32*(c/d + x^3 + (64*c^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), -(d*x^3)/(4*c)])/(d*(4*c + d*x^3)*(-16*c*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), -(d*x^3)/(4*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3
)/c), -(d*x^3)/(4*c)] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -(d*x^3)
/(4*c)]))))))/(80*Sqrt[c + d*x^3])
```

Maple [C] time = 0.033, size = 1003, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 1/d*(2/5*x*(d*x^3+c)^(1/2)-2/5*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)))-4/d*c*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/
3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/
(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*
_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x^3}{dx^3 + 4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}x^3}{dx^3 + 4\,c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}\,x^3}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

3.268 $\int \frac{\sqrt{c+dx^3}}{4c+dx^3} dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -(d*x^3)/(4*c), -((d*x^3)/c)
])/(4*c*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0280723, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {430, 429}

$$\frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -(d*x^3)/(4*c), -((d*x^3)/c)
])/(4*c*Sqrt[1 + (d*x^3)/c])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.141994, size = 165, normalized size = 2.58

$$\frac{16cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\left(4c+dx^3\right)\left(16cF_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};-\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-2F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (16*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -(d*x^3)/
(4*c)])/((4*c + d*x^3)*(16*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -(d*
x^3)/(4*c)] - 3*d*x^3*(AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -(d*x^3)/(
4*c)] - 2*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(d*x^3)/(4*c)])))
```

Maple [C] time = 0.006, size = 696, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] -2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
```

```
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*
I/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/
3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^
2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{dx^3+4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

3.269 $\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -(d*x^3)/(4*c), -((d*x^3)/c)
])/(8*c*x^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.056457, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -(d*x^3)/(4*c), -((d*x^3)/c)
])/(8*c*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3\left(4c+dx^3\right)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.126923, size = 244, normalized size = 3.7

$$\frac{\frac{2048c^3dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\left(4c+dx^3\right)\left(16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}-d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-}{256c^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] (-32*c*(c + d*x^3) - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
 -((d*x^3)/c), -(d*x^3)/(4*c)] + (2048*c^3*d*x^3*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), -(d*x^3)/(4*c)])/((4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1,
4/3, -((d*x^3)/c), -(d*x^3)/(4*c)] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -(
(d*x^3)/c), -(d*x^3)/(4*c)] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -(
d*x^3)/(4*c)]))))/(256*c^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.02, size = 1002, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x)
```

```
[Out] 1/4/c*(-1/2/x^2*(d*x^3+c)^(1/2)-1/2*I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-
d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2))-1/4*d/c*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{(dx^3 + 4c)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{dx^6+4\,cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d*x^6 + 4*c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**3*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

3.270 $\int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)} dx$

Optimal. Leaf size=78

$$\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

```
[Out] (-10*c*Sqrt[c + d*x^3])/(3*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (32*c^(3/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

Rubi [A] time = 0.0741554, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 88, 63, 203}

$$\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-10*c*Sqrt[c + d*x^3])/(3*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (32*c^(3/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{5c}{d^2\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{d^2}+\frac{16c^2}{d^2\sqrt{c+dx}(4c+dx)}\right)dx, x, x^3\right)\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(16c^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(32c^2\right)\operatorname{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}
\end{aligned}$$

Mathematica [A] time = 0.0538965, size = 65, normalized size = 0.83

$$\frac{32\sqrt{3}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+2\left(dx^3-14c\right)\sqrt{c+dx^3}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x
^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)
```

Maple [C] time = 0.035, size = 467, normalized size = 6.

$$\frac{1}{d^2}\left(d\left(\frac{2x^3}{9d}\sqrt{dx^3+c}-\frac{4c}{9d^2}\sqrt{dx^3+c}\right)-\frac{8c}{3d}\sqrt{dx^3+c}\right)-\frac{\frac{16i}{9}c\sqrt{2}}{d^5}\sum_{\_alpha=RootOf(d\_Z^3+4c)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/d^2*(d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)-8/3*c*(d*x^3
+c)^(1/2)/d)-16/9*I*c/d^5*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d
^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+
2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53041, size = 332, normalized size = 4.26

$$\left[\frac{2\left(8\sqrt{3}\sqrt{-c}c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}\left(dx^3-14c\right)\right)}{9d^3}, \frac{2\left(16\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}\left(dx^3-14\right.\right.}{9d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(8*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 14*c))/d^3, 2/9*(16*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 14*c))/d^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [A] time = 1.10584, size = 86, normalized size = 1.1

$$\frac{32\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6-15\sqrt{dx^3+c}\,cd^6\right)}{9d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 32/9*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/9*
((d*x^3 + c)^(3/2)*d^6 - 15*sqrt(d*x^3 + c)*c*d^6)/d^9
```

3.271 $\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])])/(3*Sqrt[3]*d^2)
```

Rubi [A] time = 0.0477036, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 80, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])])/(3*Sqrt[3]*d^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{3d}\\
&= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(8c)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}\\
&= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}
\end{aligned}
$$

Mathematica [A] time = 0.0199415, size = 56, normalized size = 0.95

$$
\frac{6\sqrt{c+dx^3} - 8\sqrt{3}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(9*d^2)
```

Maple [C] time = 0.011, size = 425, normalized size = 7.2

$$\frac{2}{3\,d^{2}}\sqrt{d x^{3}+c}+\frac{\frac{4\,i}{9}\sqrt{2}}{d^{4}}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^{3}d+4\,c\right)}\sqrt[3]{-d^{2}c}\,\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\,\sqrt[3]{-d^{2}c}+\sqrt[3]{-d^{2}c}\right)\right)\frac{1}{\sqrt[3]{-d^{2}c}}}\,\sqrt{d\left(x-\frac{1}{d}\,\sqrt[3]{-d^{2}c}\right)\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^2+4/9*I/d^4*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x
-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c
)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57338, size = 294, normalized size = 4.98

$$\left[\frac{2\left(2\,\sqrt{3}\sqrt{-c}\log\left(\frac{dx^{3}-2\,\sqrt{3}\sqrt{dx^{3}+c}\sqrt{-c}-2\,c}{dx^{3}+4\,c}\right)+3\,\sqrt{dx^{3}+c}\right)}{9\,d^{2}},-\frac{2\left(4\,\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^{3}+c}}{3\,\sqrt{c}}\right)-3\,\sqrt{dx^{3}+c}\right)}{9\,d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(2*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) -
2*c)/(d*x^3 + 4*c)) + 3*sqrt(d*x^3 + c))/d^2, -2/9*(4*sqrt(3)*sqrt(c)*arcta
n(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] time = 16.4857, size = 65, normalized size = 1.1

$$\begin{cases} \frac{2\left(-\frac{4\sqrt{3}\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d}+\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for } d \neq 0 \\ \frac{x^6}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-4*sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))
/(9*d) + sqrt(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(24*c**(3/2)), True))
```

Giac [A] time = 1.13233, size = 66, normalized size = 1.12

$$-\frac{2\left(\frac{4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d}-\frac{3\sqrt{dx^3+c}}{d}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*s
qrt(d*x^3 + c)/d)/d
```

3.272 $\int \frac{x^2}{\sqrt{c+dx^3}(4c+dx^3)} \, dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

```
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Rubi [A] time = 0.0357859, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {444, 63, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\ &= \frac{2\operatorname{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}\end{aligned}$$

Mathematica [A] time = 0.0091939, size = 40, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Maple [C] time = 0.098, size = 413, normalized size = 10.3

$$\frac{-\frac{i}{9}\sqrt{2}}{d^3c}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3d+4c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}+i\sqrt{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/
(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^
2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d+4*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56121, size = 232, normalized size = 5.8

$$\left[-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)}{9cd}, \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c))/(c*d), 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqr
t(c))/(sqrt(c)*d)]
```

Sympy [A] time = 7.88747, size = 37, normalized size = 0.92

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)*d)
```

Giac [A] time = 1.11347, size = 39, normalized size = 0.98

$$\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{9\,\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

3.273 $\int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)} dx$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

```
[Out] -ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(6*Sqrt[3]*c^(3/2)) - ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))
```

Rubi [A] time = 0.0553441, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 86, 63, 208, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(6*Sqrt[3]*c^(3/2)) - ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{12c} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)}{12c}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)}{6c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{6cd}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0184914, size = 59, normalized size = 0.91

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] + 3*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]])/(18*c^(3/2))
```

Maple [C] time = 0.018, size = 433, normalized size = 6.7

$$\frac{\frac{i}{36}\sqrt{2}}{c^2d^2}\sum_{\_alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}+i\sqrt{3}\sqrt[3]{\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/36*I/d^2/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3)
)/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*
d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d+4*c))-1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+4c\right)\sqrt{dx^3+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)
```

Fricas [A] time = 1.53335, size = 392, normalized size = 6.03

$$\left[-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{36c^2},-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-6\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3}}{}\right.}{36c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*s
qrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sqrt
(3)*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3
+ 4*c)) - 6*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]
```

Sympy [A] time = 12.594, size = 63, normalized size = 0.97

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{6c\sqrt{-c}}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**3)/sqrt(-c))/(6*c*sqrt(-c)) - sqrt(3)*atan(sqrt(3)*sqrt(
c + d*x**3)/(3*sqrt(c)))/(18*c**(3/2))
```

Giac [A] time = 1.11447, size = 72, normalized size = 1.11

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18c^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*arc
tan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

3.274 $\int \frac{1}{x^4\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

```
[Out] -Sqrt[c + d*x^3]/(12*c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))
```

Rubi [A] time = 0.0769034, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 103, 156, 63, 208, 203}

$$-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3}-\frac{\operatorname{Subst}\left(\int \frac{3cd+\frac{d^2x}{2}}{x\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)}{12c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3}-\frac{d\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{16c^2}+\frac{d^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)}{48c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3}-\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{8c^2}+\frac{d\operatorname{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)}{24c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0319086, size = 88, normalized size = 1.

$$
-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))
```

Maple [C] time = 0.02, size = 477, normalized size = 5.4

$$
\frac{-\frac{i}{144}\sqrt{2}}{dc^3}\sum_{\_alpha=\operatorname{RootOf}\left(d\_Z^3+4c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}+i\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/144*I/d/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3)
)/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*
d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d+4*c))+1/4/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(3/2))+1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3 + 4\,c\right)\sqrt{dx^3 + c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)
```

Fricas [A] time = 1.60369, size = 505, normalized size = 5.74

$$\left[\frac{2\,\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right) + 9\,\sqrt{c}dx^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - 12\,\sqrt{dx^3+c}c}{144\,c^3x^3}, -\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3-2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 12*
sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 - 2
```

```
*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d*x^3
*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [A] time = 1.12235, size = 101, normalized size = 1.15

$$\frac{1}{72}\,d\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{9\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}-\frac{6\sqrt{dx^3+c}}{c^2dx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*d*(sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 9*arc
tan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 6*sqrt(d*x^3 + c)/(c^2*d*x^3
))
```

3.275 $\int \frac{x^4}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=667

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}}{\left(\left(1\right.\right.}}}{d^{5/3}\sqrt{\left(\right.}}$$

```
[Out] (2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (2*2^(1
/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (2*2^(1/3)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(
Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^(5/3)) + (2*2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6
)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*d^(5/3)) - (2*2^(1/3)
*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(9*d^(5/3)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3
)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c
^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
 + d*x^3])
```

Rubi [A] time = 0.235014, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {483, 303, 218, 1877, 484}

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}}}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (2*2^(1
/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (2*2^(1/3)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(
Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^(5/3)) + (2*2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6
)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*d^(5/3)) - (2*2^(1/3)
*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(9*d^(5/3)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3
)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c
^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
 + d*x^3])
```

<u>Rule 483</u>

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

<u>Rule 303</u>

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

<u>Rule 218</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

<u>Rule 1877</u>

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```

```
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d} - \frac{(4c)\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx}{d}$$

$$= \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right.}{\sqrt{c+dx^3}}\right.}{3d^{5/3}}$$

$$= \frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} +$$

Mathematica [C] time = 0.0316698, size = 67, normalized size = 0.1

$$\frac{x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{20c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -(d*x^3)/
(4*c)])/(20*c*Sqrt[c + d*x^3])
```

Maple [C] time = 0.033, size = 848, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/d^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+4/9*I/d^4*2^(1/2)*sum(
1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/
3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)
*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I
*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^4}{d^2x^6+5\,cdx^3+4\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d^2*x^6 + 5*c*d*x^3 + 4*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.276 $\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=206

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3
*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c]
)]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3
)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```

Rubi [A] time = 0.0324488, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```


```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3
*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c]
)]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3
)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```


Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
```

```
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c}}\right.}{9\ 2^{2/3}c^{5/6}d^{2/}}$$

Mathematica [C] time = 0.0260982, size = 67, normalized size = 0.33

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -(d*x^3)/
(4*c)])/(8*c*Sqrt[c + d*x^3])
```

Maple [C] time = 0.007, size = 416, normalized size = 2.

$$\frac{-\frac{i}{9}\sqrt{2}}{d^3c}\sum_{\_alpha=RootOf\left(\_Z^3d+4c\right)}\frac{1}{\_alpha}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c
)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_a
```

```
lpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [B] time = 7.15857, size = 5685, normalized size = 27.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt
(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c
^5*d^4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^
5*d^4))^(5/6))*sqrt(d*x^3 + c) + (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*
d)*(-1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) + 3*(sqrt(3)*sqrt(1/3)*c^3*
d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(
1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5
/6))*sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3
)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/
2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3)
 + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*
d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c
*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c)
)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) + 1/9*s
```

```
qrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)
*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^
4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^5*d^4
))^(5/6))*sqrt(d*x^3 + c) - (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*d)*(-
1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) - 3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x
*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6)
- 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*
sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3)*(c^
4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1
/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) - 12
*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*d^4*x
^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*
x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) - 1/36*(1/43
2)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/
2)^(2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) +
 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4)
)^(1/3) + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3
)*(c^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(
1/6)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x
^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/36*(1/432)^(
1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(
2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*
(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1
/3) - 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c
^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)
*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 +
 c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/18*(1/432)^(1/6)
*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 +
 48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^(2
/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5
*d^4))^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + sqr
t(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + 2
*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6
))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 1/1
8*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d
*x^3 - 32*c^3 + 48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-
1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^
2*x^2)*(-1/(c^5*d^4))^(1/3) - 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^
4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1
/(c^5*d^4)) + 2*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/
(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 +
 64*c^3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.277 $\int \frac{1}{x^2\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=697

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\ldots}{\left(\left(1+\ldots\right.\right.}}}{8c^{5/3}\sqrt{\ldots}}$$

```
[Out] -Sqrt[c + d*x^3]/(4*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c^2*((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)) + (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) - (d^(1/3)*A
rcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) + (
d^(1/3)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(
12*2^(2/3)*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(36*2^(2/
3)*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (
d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x
^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])
/(2*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.388117, antiderivative size = 697, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {480, 584, 303, 218, 1877, 484}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2\ldots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\ldots\right.}}}{8c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\ldots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\ldots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(4*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c^2*((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)) + (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) - (d^(1/3)*A
rcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) + (
d^(1/3)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(
12*2^(2/3)*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(36*2^(2/
3)*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (
d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x
^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])
/(2*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\int \frac{x\left(cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{4c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}}-\frac{cdx}{\sqrt{c+dx^3}(4c+dx^3)}\right)dx}{4c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{8c^2}-\frac{d\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\right.}{\sqrt{c+dx}}\right.}{12\ 2^{2/3}c^{11/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+}}{\sqrt{3}}\right.}{12\ 2^{2/3}\sqrt{3}c^{11}}$$

Mathematica [C] time = 0.074898, size = 136, normalized size = 0.2

$$\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+5cdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-40c\left(c+dx^3\right)}{160c^3x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-40*c*(c + d*x^3) + 5*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), -(d*x^3)/(4*c)] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), -(d*x^3)/(4*c)])/(160*c^3*x*Sqrt[c + d*x^3])
```

Maple [C] time = 0.019, size = 874, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/4/c*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)))+1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(
-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3
)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^
(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^8+5\,cdx^5+4\,c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^8 + 5*c*d*x^5 + 4*c^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

3.278 $\int \frac{x^3}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -(d*x^3)/(4*c), -((d*x^
3)/c)])/(16*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0525761, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -(d*x^3)/(4*c), -((d*x^
3)/c)])/(16*c*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(4c+dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

Mathematica [A] time = 0.0317753, size = 67, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{16c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(d*x^3)/
(4*c)])/(16*c*Sqrt[c + d*x^3])
```

Maple [C] time = 0.032, size = 696, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/d^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
```

```
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+4/
9*I/d^4*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(
1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alph
a^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(dx^3+4c)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}x^3}{d^2x^6+5\,cdx^3+4\,c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d^2*x^6 + 5*c*d*x^3 + 4*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.279 $\int \frac{1}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -(d*x^3)/(4*c), -((d*x^3)
/c)])/(4*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0284523, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -(d*x^3)/(4*c), -((d*x^3)
/c)])/(4*c*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{c+dx^3}\left(4c+dx^3\right)}dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(4c+dx^3\right)\sqrt{1+\frac{dx^3}{c}}}dx}{\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.0466243, size = 165, normalized size = 2.58

$$\frac{16cxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\sqrt{c+dx^3}\left(4c+dx^3\right)\left(16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (16*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(d*x^3)/(4*c)])/(Sqrt[c +
 d*x^3]*(4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(d*x^
3)/(4*c)] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -(d*x^3)/(4*c
)] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -(d*x^3)/(4*c)])))
```

Maple [C] time = 0.006, size = 416, normalized size = 6.5

$$\frac{-\frac{i}{9}\sqrt{2}}{d^3c}\sum_{\_alpha=RootOf\left(\_Z^3d+4c\right)}\frac{1}{\_alpha^2}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2
*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*
```

```
x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*
_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+4c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^6+5\,cdx^3+4\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^6 + 5*c*d*x^3 + 4*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3 + 4\,c\right)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

3.280 $\int \frac{1}{x^3\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -(d*x^3)/(4*c), -((d*x^3)
/c)])/(8*c*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0563244, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -(d*x^3)/(4*c), -((d*x^3)
/c)])/(8*c*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(4c+dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.157965, size = 243, normalized size = 3.68

$$\frac{-\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{c^3}+\frac{2048dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\left(4c+dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)-16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)}-\frac{32\left(c+dx^3\right)}{c^2}}{256x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] ((-32*(c + d*x^3))/c^2 - (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), -(d*x^3)/(4*c)])/c^3 + (2048*d*x^3*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -(d*x^3)/(4*c)])/((4*c + d*x^3)*(-16*c*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -(d*x^3)/(4*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -(d*x^3)/(4*c)] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -(d*x^3)/(4*c)]))))/(256*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.02, size = 722, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/4/c*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
```

```
(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3)))^(1/2))+1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*(-d^2*c
)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2
*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d
^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(
1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/
2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*
c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/6/d*(2*I*(-d^2*c)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3 + 4\,c\right)\sqrt{dx^3 + c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{d^2x^9 + 5\,cdx^6 + 4\,c^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^9 + 5*c*d*x^6 + 4*c^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

3.281 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rubi [A] time = 0.0193199, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Mathematica [C] time = 0.0167214, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

Maple [C] time = 0.247, size = 164, normalized size = 1.3

$$\frac{i}{36}\sqrt{2}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\_alpha^2\left(-2\,\_alpha^2+\_alpha+1+i\sqrt{3}\,(1-\_alpha)\right)\sqrt{\frac{i}{2}\left(1+2\,x-i\sqrt{3}\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(1+2*x-I*3^(1/2)))^(1/2)*((-1+x)/(I*3^(1
/2)-3))^(1/2)*(-1/2*I*(1+2*x+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+
_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/
2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3
-4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Fricas [B] time = 3.35207, size = 3299, normalized size = 25.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
```

```
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)

[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

3.282 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=111

$$-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}+\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}$$

```
[Out] (-1024*c^3*Sqrt[c + d*x^3])/(3*d^4) - (38*c^2*(c + d*x^3)^(3/2))/(3*d^4) -
(4*c*(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(21*d^4) + (1024*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rubi [A] time = 0.0967581, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}+\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-1024*c^3*Sqrt[c + d*x^3])/(3*d^4) - (38*c^2*(c + d*x^3)^(3/2))/(3*d^4) -
(4*c*(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(21*d^4) + (1024*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{57c^2\sqrt{c+dx}}{d^3}+\frac{512c^3\sqrt{c+dx}}{d^3(8c-dx)}-\frac{6c(c+dx)^{3/2}}{d^3}-\frac{(c+dx)^{5/2}}{d^3}\right)dx,x,x^3\right)\\
&= -\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(512c^3\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{3d^3}\\
&= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(1536c^4\right)\text{Subst}\left(\int \frac{}{(8c-d\cdots}\right.}{d^3}\\
&= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(3072c^4\right)\text{Subst}\left(\int \frac{1}{9c-x^2\cdots}\right.}{d^4}\\
&= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right.}{d^4}
\end{aligned}
$$

Mathematica [A] time = 0.0749603, size = 81, normalized size = 0.73

$$\frac{107520c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(764c^2dx^3+18632c^3+57cd^2x^6+5d^3x^9\right)}{105d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9)
+ 107520*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(105*d^4)
```

Maple [C] time = 0.056, size = 582, normalized size = 5.2

$$-\frac{1}{d}\left(\frac{2x^9}{21}\sqrt{dx^3+c}+\frac{2cx^6}{105d}\sqrt{dx^3+c}-\frac{8c^2x^3}{315d^2}\sqrt{dx^3+c}+\frac{16c^3}{315d^3}\sqrt{dx^3+c}\right)-8\frac{c}{d^2}\left(2/15x^6\sqrt{dx^3+c}+\frac{2cx^3\sqrt{dx^3+c}}{45d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/21*x^9*(d*x^3+c)^(1/2)+2/105/d*c*x^6*(d*x^3+c)^(1/2)-8/315*c^2/d^2*
x^3*(d*x^3+c)^(1/2)+16/315*c^3*(d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/15*x^6*(d*x^
3+c)^(1/2)+2/45/d*c*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d*x^3+c)^(1/2)/d^2)-128/9
*c^2*(d*x^3+c)^(3/2)/d^4-512*c^3/d^3*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/
2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)
+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_
alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I
*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80048, size = 420, normalized size = 3.78

$$\left[\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3\right)\sqrt{dx^3+c}\right)}{105\,d^4},-\frac{2\left(53760\,\sqrt{-c}c^3\arctan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^
3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c
))/d^4]
```

Sympy [A] time = 155.865, size = 99, normalized size = 0.89

$$\frac{2\left(-\frac{512c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{512c^3\sqrt{c+dx^3}}{3}-\frac{19c^2\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx^3\right)^{\frac{5}{2}}}{5}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-512*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 512*c**3*sqrt(c
 + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3)**(5/2)/5 -
```

```
(c + d*x**3)**(7/2)/21)/d**4
```

Giac [A] time = 1.09088, size = 135, normalized size = 1.22

$$-\frac{1024\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^4}-\frac{2\left(5\left(dx^3+c\right)^{\frac{7}{2}}d^{24}+42\left(dx^3+c\right)^{\frac{5}{2}}cd^{24}+665\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{24}+17920\,\sqrt{dx^3+c}c^3d^{24}\right)}{105\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(d
*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2)*
c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28
```

3.283 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=90

$$-\frac{128c^2\sqrt{c+dx^3}}{3d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

```
[Out] (-128*c^2*Sqrt[c + d*x^3])/(3*d^3) - (14*c*(c + d*x^3)^(3/2))/(9*d^3) - (2*
(c + d*x^3)^(5/2))/(15*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

Rubi [A] time = 0.0850978, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{128c^2\sqrt{c+dx^3}}{3d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-128*c^2*Sqrt[c + d*x^3])/(3*d^3) - (14*c*(c + d*x^3)^(3/2))/(9*d^3) - (2*
(c + d*x^3)^(5/2))/(15*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{7c\sqrt{c+dx}}{d^2}+\frac{64c^2\sqrt{c+dx}}{d^2(8c-dx)}-\frac{(c+dx)^{3/2}}{d^2}\right)dx, x, x^3\right)$$

$$= -\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(64c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(192c^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^2}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(384c^3\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^3}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

Mathematica [A] time = 0.0579725, size = 70, normalized size = 0.78

$$\frac{5760c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(998c^2+41cdx^3+3d^2x^6\right)}{45d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6) + 5760*c^(5/2)*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^3)
```

Maple [C] time = 0.011, size = 507, normalized size = 5.6

$$-\frac{1}{d^2}\left(d\left(\frac{2x^6}{15}\sqrt{dx^3+c}+\frac{2cx^3}{45d}\sqrt{dx^3+c}-\frac{4c^2}{45d^2}\sqrt{dx^3+c}\right)+\frac{16c}{9d}\left(dx^3+c\right)^{\frac{3}{2}}\right)-64\frac{c^2}{d^2}\left(2/3\frac{\sqrt{dx^3+c}}{d}+\frac{i/3\sqrt{2}}{d^3}\sum_{\_alpha=Root}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45/d*c*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(
d*x^3+c)^(1/2)/d^2)+16/9*c/d*(d*x^3+c)^(3/2))-64*c^2/d^2*(2/3*(d*x^3+c)^(1/
2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3)
)/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*
d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77396, size = 360, normalized size = 4.

$$\left[\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3},-\frac{2\left(2880\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(3\right.\right.}{45\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(28
80*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 41*c*
d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] time = 47.6439, size = 82, normalized size = 0.91

$$\frac{2\left(-\frac{64c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{64c^2\sqrt{c+dx^3}}{3}-\frac{7c\left(c+dx^3\right)^{\frac{3}{2}}}{9}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-64*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 64*c**2*sqrt(c +
 d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/15)/d**3
```

Giac [A] time = 1.12081, size = 112, normalized size = 1.24

$$-\frac{128\,c^{3}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^{3}}-\frac{2\left(3\left(dx^{3}+c\right)^{\frac{5}{2}}d^{12}+35\left(dx^{3}+c\right)^{\frac{3}{2}}cd^{12}+960\,\sqrt{dx^{3}+c}c^{2}d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^2*
d^12)/d^15
```

3.284 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=69

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

```
[Out] (-16*c*Sqrt[c + d*x^3])/(3*d^2) - (2*(c + d*x^3)^(3/2))/(9*d^2) + (16*c^(3/
2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rubi [A] time = 0.0571967, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 80, 50, 63, 206}

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-16*c*Sqrt[c + d*x^3])/(3*d^2) - (2*(c + d*x^3)^(3/2))/(9*d^2) + (16*c^(3/
2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{(8c)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{\left(24c^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{\left(48c^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.0323542, size = 58, normalized size = 0.84

$$\frac{144c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(25c+dx^3\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3) + 144*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^2)
```

Maple [C] time = 0.007, size = 446, normalized size = 6.5

$$-\frac{2}{9\,d^2}\left(dx^3+c\right)^{\frac{3}{2}}-8\,\frac{c}{d}\left(2/3\,\frac{\sqrt{dx^3+c}}{d}+\frac{i/3\sqrt{2}}{d^3}\sum_{\_alpha=RootOf\left(\_Z^3d-8\,c\right)}\frac{\sqrt[3]{-d^2c}\left(i\sqrt[3]{-d^2c}\_alpha\,\sqrt{3}d-i\left(-d^2c\right)^{2/3}\sqrt{3}+2\,\_\right.}{\sqrt{dx^3+\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^2-8/d*c*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum
((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(
1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*
d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2
*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81016, size = 296, normalized size = 4.29

$$\left[\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(dx^3+25\,c\right)\sqrt{dx^3+c}\right)}{9\,d^2},\,-\frac{2\left(72\,\sqrt{-c}c\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(dx^3+25\,c\right)\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] time = 14.7207, size = 65, normalized size = 0.94

$$\frac{2\left(-\frac{8c^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{8c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-8*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 8*c*sqrt(c + d*x*
*3)/3 - (c + d*x**3)**(3/2)/9)/d**2
```

Giac [A] time = 1.1145, size = 93, normalized size = 1.35

$$-\frac{2\left(\frac{72\,c^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{\left(dx^{3}+c\right)^{\frac{3}{2}}d^{2}+24\,\sqrt{dx^{3}+c}cd^{2}}{d^{3}}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -2/9*(72*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + ((d*x^3 +
c)^(3/2)*d^2 + 24*sqrt(d*x^3 + c)*c*d^2)/d^3)/d
```

3.285 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d
```

Rubi [A] time = 0.0432402, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 50, 63, 206}

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2\sqrt{c+dx^3}}{3d} + (3c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{(6c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\
&= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0165303, size = 47, normalized size = 0.94

$$
-\frac{2\left(\sqrt{c+dx^3}-3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3*
d)
```

Maple [C] time = 0.007, size = 425, normalized size = 8.5

$$-\frac{2}{3d}\sqrt{dx^3+c}-\frac{\frac{i}{3}\sqrt{2}}{d^3}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3d-8c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/3*(d*x^3+c)^(1/2)/d-1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)
^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76107, size = 247, normalized size = 4.94

$$\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-2\sqrt{dx^3+c}}{3d},-\frac{2\left(3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\sqrt{dx^3+c}\right)}{3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 2*sqrt(d*x^3 + c))/d, -2/3*(3*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + sqrt(d*x^3 + c))/d]
```

Sympy [A] time = 4.45561, size = 46, normalized size = 0.92

$$\frac{2\left(-\frac{c\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d*x**3)/3)/d
```

Giac [A] time = 1.13745, size = 58, normalized size = 1.16

$$-\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{2\,\sqrt{dx^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -2*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/3*sqrt(d*x^3 + c
)/d
```

3.286 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]/(12*Sqrt[c])
```

Rubi [A] time = 0.0540572, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 83, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]/(12*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)}\,dx,x,x^3\right)\\
&= \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)+\frac{1}{8}(3d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{12d}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0167401, size = 53, normalized size = 0.91

$$
\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(12*Sqrt[c])
```

Maple [C] time = 0.011, size = 468, normalized size = 8.1

$$-\frac{d}{8c}\left(\frac{2}{3d}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{d^3}\sum_{\_alpha=RootOf\left(\_Z^3d-8c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x)
```

```
[Out] -1/8*d/c*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/
2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alph
a-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(2/3*(d*x^3+c)
^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{dx^3+c}}{\left(dx^3-8c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x), x)
```

Fricas [A] time = 1.56793, size = 342, normalized size = 5.9

$$\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{24c}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{12c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, 1/12*(
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c))/c]
```

Sympy [A] time = 8.1773, size = 60, normalized size = 1.03

$$\frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)
```

```
[Out] 2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan(sqrt(c + d*
x**3)/sqrt(-c))/(24*sqrt(-c)))/d
```

Giac [A] time = 1.11777, size = 65, normalized size = 1.12

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3
+ c)/sqrt(-c))/sqrt(-c)
```

3.287 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx$

Optimal. Leaf size=81

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

```
[Out] -Sqrt[c + d*x^3]/(24*c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Rubi [A] time = 0.0722102, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{\text{Subst}\left(\int \frac{5cd+\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$
$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{64c}$$
$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c} + \frac{(3d)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{32c}$$
$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

Mathematica [A] time = 0.0294411, size = 81, normalized size = 1.

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Maple [C] time = 0.012, size = 511, normalized size = 6.3

$$-\frac{d^2}{64\,c^2}\left(\frac{2}{3\,d}\sqrt{dx^3+c} + \frac{\frac{i}{3}\sqrt{2}}{d^3}\sum_{\_alpha=\text{RootOf}\left(\_Z^3d-8\,c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x)
```

```
[Out] -1/64*d^2/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3)
)^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*
_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(-1/3*(d*
x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/64*d/c^2
*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)
```

Fricas [A] time = 1.63788, size = 462, normalized size = 5.7

$$\left[\frac{3\,\sqrt{c}dx^3\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+5\,\sqrt{c}dx^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\,\sqrt{dx^3+c}c}{192\,c^2x^3},\frac{5\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-3\,\sqrt{-}}{96}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), 1/96*(5*sqrt(-c)*d*x^3*arctan(sqrt(d
```

```
*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c
)/c) - 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

Giac [A] time = 1.14159, size = 104, normalized size = 1.28

$$\frac{1}{96}\,d\left(\frac{5\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{3\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{4\sqrt{dx^3+c}}{cdx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/96*d*(5*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3*arctan(1/3*sqrt
(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 4*sqrt(d*x^3 + c)/(c*d*x^3))
```

3.288 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=107

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

```
[Out] -Sqrt[c + d*x^3]/(48*c*x^6) - (d*Sqrt[c + d*x^3])/(64*c^2*x^3) + (d^2*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(256*c^(5/2))
```

Rubi [A] time = 0.0940667, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c*x^6) - (d*Sqrt[c + d*x^3])/(64*c^2*x^3) + (d^2*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(256*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
```

```
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} + \frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\text{Subst}\left(\int \frac{6c^2d^2-3cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^2} + \frac{\left(3d^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^2} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+\right.}{256c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

Mathematica [A] time = 0.0522811, size = 96, normalized size = 0.9

$$\frac{3d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 3d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - 4\sqrt{c}\sqrt{c+dx^3}\left(4c+3dx^3\right)}{768c^{5/2}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 3*d*x^3) + 3*d^2*x^6*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])] + 3*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(5/2
)*x^6)
```

Maple [C] time = 0.024, size = 574, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x)
```

```
[Out] -1/512*d^3/c^3*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(-1/6*(d
*x^3+c)^(1/2)/x^6-1/12*d*(d*x^3+c)^(1/2)/c/x^3+1/12*d^2*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(3/2))+1/64*d/c^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/512*d^2/c^3*(2/3*(d*x^3+c)^(1/2)-2/3*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

Fricas [A] time = 1.51803, size = 439, normalized size = 4.1

$$\left[\frac{3\,\sqrt{c}d^2x^6\log\left(\frac{d^2x^6+24\,cdx^3+8\,\left(dx^3+4\,c\right)\sqrt{dx^3+c}\sqrt{c}+32\,c^2}{dx^6-8\,cx^3}\right)-8\left(3\,cdx^3+4\,c^2\right)\sqrt{dx^3+c}}{1536\,c^3x^6}, -\frac{3\,\sqrt{-c}d^2x^6\arctan\left(\frac{\left(dx^3+4\,c\right)\sqrt{dx^3+c}\sqrt{-c}}{4\,\left(cdx^3+c^2\right)}\right)}{768\,c^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*sqrt(c)*d^2*x^6*log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c)*sqrt
(d*x^3 + c)*sqrt(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^3*x^6), -1/768*(3*sqrt(-c)*d^2*x^6*arctan(1/4*(d*x^3 + 4*
c)*sqrt(d*x^3 + c)*sqrt(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*sqrt(d
*x^3 + c))/(c^3*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

Giac [A] time = 1.11543, size = 126, normalized size = 1.18

$$-\frac{1}{768}\,d^2\left(\frac{3\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{3\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{4\left(3\left(dx^3+c\right)^{\frac{3}{2}}+\sqrt{dx^3+c}c\right)}{c^2d^2x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1/768*d^2*(3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 3*arctan(1/
3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 4*(3*(d*x^3 + c)^(3/2) + sqrt(
d*x^3 + c)*c)/(c^2*d^2*x^6))
```

3.289 $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=648

$$-\frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{c}\right.}$$

```
[Out] (-214*c*x^2*Sqrt[c + d*x^3])/(91*d^2) - (2*x^5*Sqrt[c + d*x^3])/(13*d) - (1
2248*c^2*Sqrt[c + d*x^3])/(91*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- (32*Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(8/3) + (32*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1
/6)*Sqrt[c + d*x^3])])/d^(8/3) - (32*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/d^(8/3) + (6124*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(8/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3]) - (12248*Sqrt[2]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.887867, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {478, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{32\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{32c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{32c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+}}{3\sqrt{}}\right.}{d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-214*c*x^2*Sqrt[c + d*x^3])/(91*d^2) - (2*x^5*Sqrt[c + d*x^3])/(13*d) - (1
2248*c^2*Sqrt[c + d*x^3])/(91*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- (32*Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(8/3) + (32*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1
/6)*Sqrt[c + d*x^3])])/d^(8/3) - (32*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/d^(8/3) + (6124*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(8/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3]) - (12248*Sqrt[2]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2x^5\sqrt{c+dx^3}}{13d} + \frac{2\int \frac{x^4\left(40c^2+\frac{107}{2}cdx^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}dx}{13d}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} + \frac{4\int \frac{x\left(856c^3d+1531c^2d^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}dx}{91d^3}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} + \frac{4\int \left(-\frac{1531c^2dx}{\sqrt{c+dx^3}} + \frac{13104c^3dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{91d^3}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{(6124c^2)\int \frac{x}{\sqrt{c+dx^3}}dx}{91d^2} + \frac{(576c^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}dx}{d^2}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{(48c^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{d^3} - \frac{(6124c^2)\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,d}{91d^{7/3}}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{91}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{\sqrt{c+dx^3}}\right.}{d^{8/3}}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{\sqrt{c+dx^3}}\right.}{d^{8/3}}$$

Mathematica [C] time = 0.0875009, size = 150, normalized size = 0.23

$$\frac{2140c^2x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+1531cdx^5\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20\left(107c^2x^2+114cdx^5+7d^2x\right.}{910d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-20*(107*c^2*x^2 + 114*c*d*x^5 + 7*d^2*x^8) + 2140*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 1531*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(910*d^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.037, size = 1788, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/13*x^5*(d*x^3+c)^(1/2)+6/91*c*x^2*(d*x^3+c)^(1/2)/d+8/91*I/d^2
*c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))+8*c*(2/7*x^2*(d*x^3+c)^(1/2
)-2/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-64*c^2/d^2*(-2/3*I*
3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-
```

```
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*
(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3))
)/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1
/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d
-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*
c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3 + c}x^7}{dx^3 - 8c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{dx^3+c}\,x^7}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

3.290 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=624

$$-\frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{4\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right.}{d^{5/3}}$$

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d) - (118*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (4*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(5/3) + (4*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(5/3) - (4*c^(7/6)*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(5/3) + (59*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^
(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) - (118*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3
)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3])
```

Rubi [A] time = 0.746765, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {478, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{4\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{4c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{}}{\left((1+\sqrt{}\right.}}}{7\sqrt[4]{3}d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d) - (118*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (4*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(5/3) + (4*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(5/3) - (4*c^(7/6)*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(5/3) + (59*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^
(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) - (118*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3
)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \frac{x\left(16c^2+\frac{59}{2}cdx^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \left(-\frac{59cx}{2\sqrt{c+dx^3}} + \frac{252c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(59c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{(72c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(6c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2} - \frac{(59c)\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{7d^{4/3}} + \frac{(6c^{4/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+\cdots}}}{d^{4/3}}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\cdots}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\cdots}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{\cdots}\right.}{3\sqrt[6]{\cdots}}\cdots\right)}{d^{5/3}}\cdots$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{\cdots}\right.}{3\sqrt[6]{\cdots}}\cdots\right)}{d^{5/3}}\cdots$$

Mathematica [C] time = 0.0770859, size = 130, normalized size = 0.21

$$\frac{x^2\left(59dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80c\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80\left(c+dx^3\right)\right)}{280d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-80*(c + d*x^3) + 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 59*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(280*d*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 1310, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/7*x^2*(d*x^3+c)^(1/2)-2/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/
2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))
^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2))))-8/d*c*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^
2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^
2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3
*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)
^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3
*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(
-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
```

```
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3+c}x^4}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}x^4}{dx^3-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{dx^3+c}x^4}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

3.291 $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=601

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{2/3}\cdots}$$

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[
3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(2*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
 d*x^3])])/(2*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*
d^(2/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt
[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.531586, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {489, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\cdots\right.}{\left(\left(1+\sqrt{\cdots}\right.\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[
3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(2*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
 d*x^3])])/(2*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*
d^(2/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt
[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = (9c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx - \int \frac{x}{\sqrt{c+dx^3}}\,dx$$

$$= -\frac{3\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{4d} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{\sqrt[3]{d}} + \frac{\left(3\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{d}} - \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt[3]{c}\right)\dots}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\dots\right.\right.}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\dots}{\dots}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \frac{\sqrt[6]{c}\tanh^{-1}\left(\dots\right.}{2d^{2/3}\dots}$$

Mathematica [C] time = 0.0326667, size = 63, normalized size = 0.1

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(16*Sqrt[c + d*x^3])
```

Maple [C] time = 0.007, size = 848, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] 2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1/2)*sum(1/_
alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+
I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_a
lpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*
(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
```

```
8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3+cx}}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+cx}}{dx^3-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{dx^3+c}\,x}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

3.292 $\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=632

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\ldots}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}{16c^{2/3}\sqrt{\frac{\sqrt[3]{\ldots}}{\left(\left(1+\ldots\right.\right.}}}$$

```
[Out] -Sqrt[c + d*x^3]/(8*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c*((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]])/(16*c^(5/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(
1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*c^(5/6)) - (d^(1/3)*ArcTanh[Sqr
t[c + d*x^3]/(3*Sqrt[c])])/(16*c^(5/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3
)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3
) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*c
^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c
^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.717123, antiderivative size = 632, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {475, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)\ldots}}}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\ldots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(8*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c*((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]])/(16*c^(5/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(
1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(16*c^(5/6)) - (d^(1/3)*ArcTanh[Sqr
t[c + d*x^3]/(3*Sqrt[c])]])/(16*c^(5/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3
)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3
) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*c
^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c
^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \frac{x\left(13cd-\frac{d^2x^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{9cdx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{1}{8}(9d)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{16c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} - \frac{3\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{32c} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{16c} + \frac{\left(3d^{2/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{32c^{2/3}} +$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\right.\right.}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+d}}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}}$$

Mathematica [C] time = 0.0824002, size = 137, normalized size = 0.22

$$\frac{-d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+65cdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(c+dx^3\right)}{640c^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-80*c*(c + d*x^3) + 65*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^2*x*Sqrt[c + d*x^3])
```

Maple [C] time = 0.014, size = 1306, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*
(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*
(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/8*
d/c*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*s
um(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^
(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1
/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
```

```
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d
*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3+c}}{(dx^3-8c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{dx^5-8\,cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d*x^5 - 8*c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

3.293 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$

Optimal. Leaf size=654

$$\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{4/3}\ldots}{\ldots}$$

```
[Out] -Sqrt[c + d*x^3]/(32*c*x^4) - (d*Sqrt[c + d*x^3])/(16*c^2*x) + (d^(4/3)*Sqr
t[c + d*x^3])/(16*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
(11/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^
3])])/(128*c^(11/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*
c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^
(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
8*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.837512, antiderivative size = 654, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {475, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}}+\frac{d^{4/3}\left(\ldots\right)}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(32*c*x^4) - (d*Sqrt[c + d*x^3])/(16*c^2*x) + (d^(4/3)*Sqr
t[c + d*x^3])/(16*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
(11/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^
3])])/(128*c^(11/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*
c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^
(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
8*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{32cx^4}+\frac{\int \frac{16cd+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}-\frac{\int \frac{x\left(-100c^2d^2+8cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}-\frac{\int \left(-\frac{8cd^2x}{\sqrt{c+dx^3}}-\frac{36c^2d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}+\frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^2}+\frac{\left(9d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{64c}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}-\frac{(3d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c^2}+\frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c^2}+\frac{\left(3d^{5/3}\right)\int}{25}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}}}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\right.}{\left(\left(1+\sqrt{3}\right)\right.}}}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4/3}\tanh}{1}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4/3}\tanh}{1}$$

Mathematica [C] time = 0.093574, size = 153, normalized size = 0.23

$$\frac{125cd^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\left(d^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+40c\left(c^2+3cdx^3+2d^2x^6\right)\right)}{5120c^3x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (125*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 3*c*d*x^3 + 2*d^2*x^6) + d^3*x^9*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(5120*
c^3*x^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.022, size = 1782, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x)
```

```
[Out] 1/64*d/c^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x
-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))
-1/64*d^2/c^2*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^
2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^
2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3
*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)
^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3
*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
```

```
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(
-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c))+1/8/c*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d*x^3+c)^(1/2
)/c/x-1/8*I*d/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/
3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3+c}}{(dx^3-8c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{dx^8-8cx^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d*x^8 - 8*c*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

3.294 $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=678

$$-\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{d^2\sqrt{c}}{112}$$

```
[Out] -Sqrt[c + d*x^3]/(56*c*x^7) - (19*d*Sqrt[c + d*x^3])/(1792*c^2*x^4) + (d^2*
Sqrt[c + d*x^3])/(112*c^3*x) - (d^(7/3)*Sqrt[c + d*x^3])/(112*c^3*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(17/6)) + (d^(7/3)*ArcTanh[(c^
(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(17/6)) - (d^(7/
3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(17/6)) + (3^(1/4)*Sqrt[2
- Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (d^(7/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*c^(8/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.950718, antiderivative size = 678, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {475, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\tanh^{-1}\left(\right.}{1024c^{17/}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(56*c*x^7) - (19*d*Sqrt[c + d*x^3])/(1792*c^2*x^4) + (d^2*
Sqrt[c + d*x^3])/(112*c^3*x) - (d^(7/3)*Sqrt[c + d*x^3])/(112*c^3*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(17/6)) + (d^(7/3)*ArcTanh[(c^
(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(17/6)) - (d^(7/
3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(17/6)) + (3^(1/4)*Sqrt[2
- Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (d^(7/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*c^(8/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*Sqrt[c + d*x^3])
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{56cx^7} + \frac{\int \frac{19cd+\frac{11d^2x^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{56c}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} - \frac{\int \frac{128c^2d^2-\frac{95}{2}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1792c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} + \frac{\int \frac{x\left(-260c^3d^3+64c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} + \frac{\int \left(-\frac{64c^2d^3x}{\sqrt{c+dx^3}} + \frac{252c^3d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^3\int \frac{x}{\sqrt{c+dx^3}}\,dx}{224c^3} + \frac{\left(9d^3\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{512c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{\left(3d^2\right)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^3} - \frac{d^{8/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}}{224c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\right.}{}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+d}}\right.}{1024c^{17/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}\right.}{\sqrt{c+d}}\right.}{1024c^{17/6}}$$

Mathematica [C] time = 0.0929053, size = 164, normalized size = 0.24

$$\frac{32d^4x^{12}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-325cd^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-160c\left(51c^2dx^3+32c^3+3cd^2x^6\right.}{286720c^4x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (-160*c*(32*c^3 + 51*c^2*d*x^3 + 3*c*d^2*x^6 - 16*d^3*x^9) - 325*c*d^3*x^9*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
 + 32*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(286720*c^4*x^7*Sqrt[c + d*x^3])
```

Maple [C] time = 0.026, size = 2280, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x)
```

```
[Out] 1/512*d^2/c^3*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)))-1/512*d^3/c^3*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*
(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*
```

```
(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I
/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))
/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d
^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/x^7-3/56*d*(d*x^3+c)
^(1/2)/c/x^4+15/112*d^2*(d*x^3+c)^(1/2)/c^2/x+5/112*I*d^2/c^2*3^(1/2)*(-d^2
*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2))))+1/64*d/c^2*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d
*x^3+c)^(1/2)/c/x-1/8*I*d/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*
(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*
(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{dx^{11} - 8\,cx^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d*x^11 - 8*c*x^8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

3.295 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=130

$$-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}+\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}$$

```
[Out] (-3072*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(9*d^4) - (3
8*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (4*c*(c + d*x^3)^(7/2))/(7*d^4) - (2*(c
+ d*x^3)^(9/2))/(27*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^4
```

Rubi [A] time = 0.112546, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}+\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```



```
[Out] (-3072*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(9*d^4) - (3
8*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (4*c*(c + d*x^3)^(7/2))/(7*d^4) - (2*(c
+ d*x^3)^(9/2))/(27*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^4
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{57c^2(c+dx)^{3/2}}{d^3} + \frac{512c^3(c+dx)^{3/2}}{d^3(8c-dx)} - \frac{6c(c+dx)^{5/2}}{d^3} - \frac{(c+dx)^{7/2}}{d^3}\right)dx, x, x^3\right)$$
$$= -\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4} - \frac{4c\left(c+dx^3\right)^{7/2}}{7d^4} - \frac{2\left(c+dx^3\right)^{9/2}}{27d^4} + \frac{\left(512c^3\right)\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d^3}$$
$$= -\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4} - \frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4} - \frac{4c\left(c+dx^3\right)^{7/2}}{7d^4} - \frac{2\left(c+dx^3\right)^{9/2}}{27d^4} + \frac{\left(1536c^4\right)\operatorname{Subst}\left(\int\right.}{d^3}$$
$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4} - \frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4} - \frac{4c\left(c+dx^3\right)^{7/2}}{7d^4} - \frac{2\left(c+dx^3\right)^{9/2}}{27d^4} +$$
$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4} - \frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4} - \frac{4c\left(c+dx^3\right)^{7/2}}{7d^4} - \frac{2\left(c+dx^3\right)^{9/2}}{27d^4} +$$
$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4} - \frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4} - \frac{4c\left(c+dx^3\right)^{7/2}}{7d^4} - \frac{2\left(c+dx^3\right)^{9/2}}{27d^4} +$$

Mathematica [A] time = 0.0962615, size = 93, normalized size = 0.72

$$\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} - \frac{2\sqrt{c+dx^3}\left(4611c^2d^2x^6 + 61892c^3dx^3 + 1509176c^4 + 410cd^3x^9 + 35d^4x^{12}\right)}{945d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410
*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^4
```

Maple [C] time = 0.043, size = 634, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/27*d*x^12*(d*x^3+c)^(1/2)+20/189*c*x^9*(d*x^3+c)^(1/2)+2/315/d*c^2*
x^6*(d*x^3+c)^(1/2)-8/945/d^2*c^3*x^3*(d*x^3+c)^(1/2)+16/945/d^3*c^4*(d*x^3
+c)^(1/2))-8*c/d^2*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)
+2/105/d*c^2*x^3*(d*x^3+c)^(1/2)-4/105/d^2*c^3*(d*x^3+c)^(1/2))-128/15*c^2*
(d*x^3+c)^(5/2)/d^4-512*c^3/d^3*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(
1/2)/d+3*I/d^3*c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/
3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^
2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3802, size = 495, normalized size = 3.81

$$\left[\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+61892\,c^3dx^3+1509176\,c^4\right)\sqrt{dx^3+c}\right)}{945\,d^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*
x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*arcta
n(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2
*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13079, size = 158, normalized size = 1.22

$$-\frac{9216\,c^5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4}-\frac{2\left(35\left(dx^3+c\right)^{\frac{9}{2}}d^{32}+270\left(dx^3+c\right)^{\frac{7}{2}}cd^{32}+3591\left(dx^3+c\right)^{\frac{5}{2}}c^2d^{32}+53760\left(dx^3+c\right)^{\frac{3}{2}}c^3d^{32}+\right.}{945\,d^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -9216*c^5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/945*(35*(
d*x^3 + c)^(9/2)*d^32 + 270*(d*x^3 + c)^(7/2)*c*d^32 + 3591*(d*x^3 + c)^(5/
2)*c^2*d^32 + 53760*(d*x^3 + c)^(3/2)*c^3*d^32 + 1451520*sqrt(d*x^3 + c)*c^
4*d^32)/d^36
```

3.296 $\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=109

$$-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}$$

```
[Out] (-384*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(9*d^3) - (14*
c*(c + d*x^3)^(5/2))/(15*d^3) - (2*(c + d*x^3)^(7/2))/(21*d^3) + (1152*c^(7
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rubi [A] time = 0.102259, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-384*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(9*d^3) - (14*
c*(c + d*x^3)^(5/2))/(15*d^3) - (2*(c + d*x^3)^(7/2))/(21*d^3) + (1152*c^(7
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{7c(c+dx)^{3/2}}{d^2} + \frac{64c^2(c+dx)^{3/2}}{d^2(8c-dx)} - \frac{(c+dx)^{5/2}}{d^2}\right) dx, x, x^3\right)$$
$$= -\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(64c^2\right)\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d^2}$$
$$= -\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(192c^3\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d^2}$$
$$= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(1728c^4\right)\operatorname{Subst}\left(\int\right.}{}$$
$$= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(3456c^4\right)\operatorname{Subst}\left(\int\right.}{}$$
$$= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{1152c^{7/2}\tanh^{-1}\left(\right.}{d^3}$$

Mathematica [A] time = 0.0699023, size = 81, normalized size = 0.74

$$\frac{362880c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3}\left(2579c^2dx^3 + 62882c^3 + 192cd^2x^6 + 15d^3x^9\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^
9) + 362880*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(315*d^3)
```

Maple [C] time = 0.012, size = 541, normalized size = 5.

$$-\frac{1}{d^2}\left(d\left(\frac{2\,dx^9}{21}\sqrt{dx^3+c} + \frac{16\,cx^6}{105}\sqrt{dx^3+c} + \frac{2\,c^2x^3}{105\,d}\sqrt{dx^3+c} - \frac{4\,c^3}{105\,d^2}\sqrt{dx^3+c}\right) + \frac{16\,c}{15\,d}\left(dx^3+c\right)^{\frac{5}{2}}\right) - 64\,\frac{c^2}{d^2}\left(2/9\,x^3\sqrt{dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105/d*
c^2*x^3*(d*x^3+c)^(1/2)-4/105/d^2*c^3*(d*x^3+c)^(1/2))+16/15*c/d*(d*x^3+c)^
(5/2))-64*c^2/d^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3
*c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*
c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2
*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)
^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/1
8/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38414, size = 429, normalized size = 3.94

$$\left[\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3\right)\sqrt{dx^3+c}\right)}{315\,d^3},-\frac{2\left(181440\,\sqrt{-c}c^3\,\mathrm{arc}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d
```

```
*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*
x^3 + c))/d^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12501, size = 135, normalized size = 1.24

$$-\frac{1152\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3}-\frac{2\left(15\left(dx^3+c\right)^{\frac{7}{2}}d^{18}+147\left(dx^3+c\right)^{\frac{5}{2}}cd^{18}+2240\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{18}+60480\sqrt{dx^3+c}c^3d^{18}\right)}{315\,d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1152*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/315*(15*(
d*x^3 + c)^(7/2)*d^18 + 147*(d*x^3 + c)^(5/2)*c*d^18 + 2240*(d*x^3 + c)^(3/
2)*c^2*d^18 + 60480*sqrt(d*x^3 + c)*c^3*d^18)/d^21
```

3.297 $\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=88

$$-\frac{48c^2\sqrt{c+dx^3}}{d^2}+\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}$$

```
[Out] (-48*c^2*Sqrt[c + d*x^3])/d^2 - (16*c*(c + d*x^3)^(3/2))/(9*d^2) - (2*(c +
d*x^3)^(5/2))/(15*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/d^2
```

Rubi [A] time = 0.0716315, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 80, 50, 63, 206}

$$-\frac{48c^2\sqrt{c+dx^3}}{d^2}+\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-48*c^2*Sqrt[c + d*x^3])/d^2 - (16*c*(c + d*x^3)^(3/2))/(9*d^2) - (2*(c +
d*x^3)^(5/2))/(15*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/d^2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$
$$= -\frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{(8c)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d}$$
$$= -\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{\left(24c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d}$$
$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{\left(216c^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d}$$
$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{\left(432c^3\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}$$
$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Mathematica [A] time = 0.0359215, size = 70, normalized size = 0.8

$$\frac{6480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3}\left(1123c^2+46cdx^3+3d^2x^6\right)}{45d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6) + 6480*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^2)
```

Maple [C] time = 0.01, size = 462, normalized size = 5.3

$$-\frac{2}{15\,d^2}\left(dx^3+c\right)^{\frac{5}{2}} - 8\,\frac{c}{d}\left(2/9\,x^3\sqrt{dx^3+c} + \frac{56\,c\sqrt{dx^3+c}}{9\,d} + \frac{3\,ic\sqrt{2}}{d^3}\sum_{\_alpha=RootOf\left(\_Z^3d-8\,c\right)} \frac{\sqrt[3]{-d^2c}\left(i\sqrt[3]{-d^2c}\_alpha\,\sqrt{3}d - i\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/15*(d*x^3+c)^(5/2)/d^2-8/d*c*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(
1/2)/d+3*I/d^3*c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/
3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^
2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35718, size = 363, normalized size = 4.12

$$\left[\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(3\,d^2x^6+46\,cdx^3+1123\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^2},-\frac{2\left(3240\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(3\right.\right.}{45\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*(3
240*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 46*c
```

```
*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] time = 78.0689, size = 90, normalized size = 1.02

$$-\frac{144c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{\frac{3}{2}}}{9d^2}-\frac{2\left(c+dx^3\right)^{\frac{5}{2}}}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -144*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**2*sqrt(-c)) - 48*c**2*sqr
t(c + d*x**3)/d**2 - 16*c*(c + d*x**3)**(3/2)/(9*d**2) - 2*(c + d*x**3)**(5
/2)/(15*d**2)
```

Giac [A] time = 1.13788, size = 112, normalized size = 1.27

$$-\frac{144\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^2}-\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^8+40\left(dx^3+c\right)^{\frac{3}{2}}cd^8+1080\,\sqrt{dx^3+c}c^2d^8\right)}{45\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2*d
^8)/d^10
```

3.298 $\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=67

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}$$

```
[Out] (-6*c*Sqrt[c + d*x^3])/d - (2*(c + d*x^3)^(3/2))/(9*d) + (18*c^(3/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rubi [A] time = 0.0569293, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 50, 63, 206}

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-6*c*Sqrt[c + d*x^3])/d - (2*(c + d*x^3)^(3/2))/(9*d) + (18*c^(3/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2\left(c+dx^3\right)^{3/2}}{9d} + (3c)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2\left(c+dx^3\right)^{3/2}}{9d} + \left(27c^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2\left(c+dx^3\right)^{3/2}}{9d} + \frac{\left(54c^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\
&= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2\left(c+dx^3\right)^{3/2}}{9d} + \frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.034809, size = 58, normalized size = 0.87

$$\frac{162c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(28c+dx^3\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3) + 162*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d)
```

Maple [C] time = 0.01, size = 441, normalized size = 6.6

$$-\frac{2x^3}{9}\sqrt{dx^3+c}-\frac{56c}{9d}\sqrt{dx^3+c}-\frac{3ic\sqrt{2}}{d^3}\sum_{\_alpha=RootOf(\_Z^3d-8c)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/9*x^3*(d*x^3+c)^(1/2)-56/9*c*(d*x^3+c)^(1/2)/d-3*I/d^3*c*2^(1/2)*sum((-d
^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)
*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-
d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38863, size = 293, normalized size = 4.37

$$\left[\frac{81\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\left(dx^3+28\,c\right)\sqrt{dx^3+c}}{9\,d},-\frac{2\left(81\,\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(dx^3+28\,c\right)\sqrt{dx^3+c}\right)}{9\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]
```

Sympy [A] time = 28.1262, size = 65, normalized size = 0.97

$$-\frac{18c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d\sqrt{-c}}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{\frac{3}{2}}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -18*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) - 6*c*sqrt(c + d*
x**3)/d - 2*(c + d*x**3)**(3/2)/(9*d)
```

Giac [A] time = 1.16134, size = 88, normalized size = 1.31

$$-\frac{18\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^2+27\,\sqrt{dx^3+c}cd^2\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 + c
)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3
```

3.299 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)} dx$

Optimal. Leaf size=73

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Rubi [A] time = 0.0696853, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 84, 156, 63, 208, 206}

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)}\,dx, x, x^3\right)\\
&= -\frac{2}{3}\sqrt{c+dx^3}-\frac{\operatorname{Subst}\left(\int \frac{-c^2d-10cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}\\
&= -\frac{2}{3}\sqrt{c+dx^3}+\frac{1}{24}c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)+\frac{1}{8}(27cd)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{2}{3}\sqrt{c+dx^3}+\frac{1}{4}(27c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)+\frac{c\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12d}\\
&= -\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0260791, size = 73, normalized size = 1.

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Maple [C] time = 0.021, size = 500, normalized size = 6.9

$$-\frac{d}{8c}\left(\frac{2x^3}{9}\sqrt{dx^3+c}+\frac{56c}{9d}\sqrt{dx^3+c}+\frac{3ic\sqrt{2}}{d^3}\sum_{\_alpha=\mathrm{RootOf}(d\_Z^3-8c)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x)
```

```
[Out] -1/8*d/c*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3*c*2^(1/2
)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+
I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_a
lpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*
(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c)))+1/8/c*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*a
rctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)
```

Fricas [A] time = 1.43051, size = 396, normalized size = 5.42

$$\left[\frac{9}{8}\,\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+\frac{1}{24}\,\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-\frac{2}{3}\,\sqrt{dx^3+c},\frac{1}{12}\,\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))
+ 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*sqr
t(d*x^3 + c), 1/12*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9/4*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 2/3*sqrt(d*x^3 + c)]
```

Sympy [A] time = 21.3097, size = 73, normalized size = 1.

$$-\frac{9c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{2\sqrt{c+dx^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)
```

```
[Out] -9*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(4*sqrt(-c)) + c*atan(sqrt(c + d*x
**3)/sqrt(-c))/(12*sqrt(-c)) - 2*sqrt(c + d*x**3)/3
```

Giac [A] time = 1.13432, size = 82, normalized size = 1.12

$$\frac{c \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{9\,c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}} - \frac{2}{3}\sqrt{dx^3+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d*
x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)
```

3.300 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] -Sqrt[c + d*x^3]/(24*x^3) + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Rubi [A] time = 0.0720884, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 98, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*x^3) + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} - \frac{\text{Subst}\left(\int \frac{-13c^2d-\frac{17}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{1}{192}(13d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right) + \frac{1}{64}\left(27d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx\right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{13}{96}\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right) + \frac{1}{32}(27d)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Mathematica [A] time = 0.0418725, size = 78, normalized size = 1.

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*x^3) + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Maple [C] time = 0.023, size = 556, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x)
```

```
[Out] -1/64*d^2/c^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3*c*2
^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
```

```
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(
1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c
)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/
3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*
(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c)))+1/8/c*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)
*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/64*d/c^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+
8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)
```

Fricas [A] time = 1.44545, size = 462, normalized size = 5.92

$$\left[\frac{27\,\sqrt{c}dx^3\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+13\,\sqrt{c}dx^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\,\sqrt{dx^3+c}c}{192\,cx^3},\ \frac{13\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), 1/96*(13*sqrt(-c)*d*x^3*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(
```

```
-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

Giac [A] time = 1.1014, size = 92, normalized size = 1.18

$$\frac{1}{96}\,d\left(\frac{13\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{27\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{dx^3+c}}{dx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/96*d*(13*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 27*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(d*x^3 + c)/(d*x^3))
```

3.301 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx$

Optimal. Leaf size=104

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

```
[Out] -Sqrt[c + d*x^3]/(48*x^6) - (11*d*Sqrt[c + d*x^3])/(192*c*x^3) + (9*d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(768*c^(3/2))
```

Rubi [A] time = 0.0946632, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*x^6) - (11*d*Sqrt[c + d*x^3])/(192*c*x^3) + (9*d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(768*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{48x^6}-\frac{\mathrm{Subst}\left(\int \frac{-22c^2d-\frac{35}{2}cd^2x}{x^2(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{48c}\\
&= -\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{\mathrm{Subst}\left(\int \frac{74c^3d^2+11c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{384c^3}\\
&= -\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{\left(37d^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{1536c}+\frac{\left(27d^3\right)\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,d\right.}{512c}\\
&= -\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{(37d)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{768c}+\frac{\left(27d^2\right)\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx\right.}{256c}\\
&= -\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0516253, size = 96, normalized size = 0.92

$$
\frac{27d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-37d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)-4\sqrt{c}\sqrt{c+dx^3}\left(4c+11dx^3\right)}{768c^{3/2}x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 11*d*x^3) + 27*d^2*x^6*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])] - 37*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(
3/2)*x^6)
```

Maple [C] time = 0.026, size = 617, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x)
```

```
[Out] -1/512*d^3/c^3*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3*c*
2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^
(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1
/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d
*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))+1/8/c*(-1/6*c*(d*x^3+c)^(1/2)/x^6-5/12*d*(d*x^3+c)^(1/2)/x^3-1
/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/64*d/c^2*(-1/3*c*(d*x^3+
c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2
)))+1/512*d^2/c^3*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3
/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

Fricas [A] time = 1.43754, size = 532, normalized size = 5.12

$$\left[\frac{27\,\sqrt{c}d^2x^6\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+37\,\sqrt{c}d^2x^6\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\left(11\,cdx^3+4\,c^2\right)\sqrt{dx^3+c}}{1536\,c^2x^6},\frac{37\,\sqrt{-c}d^2x^6\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), 1/768*(37*
sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d^2*x^6*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 +
 c))/(c^2*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10344, size = 127, normalized size = 1.22

$$\frac{1}{768}d^2\left(\frac{37\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{27\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{4\left(11\left(dx^3+c\right)^{\frac{3}{2}}-7\sqrt{dx^3+c}c\right)}{cd^2x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/768*d^2*(37*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 27*arctan(1/3
*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 4*(11*(d*x^3 + c)^(3/2) - 7*sqrt(
d*x^3 + c)*c)/(c*d^2*x^6))
```

3.302 $\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=669

$$-\frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}$$

```
[Out] (-36534*c^2*x^2*Sqrt[c + d*x^3])/(1729*d^2) - (348*c*x^5*Sqrt[c + d*x^3])/(
247*d) - (2*x^8*Sqrt[c + d*x^3])/19 - (2094648*c^3*Sqrt[c + d*x^3])/(1729*d
^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (288*Sqrt[3]*c^(19/6)*ArcTan[
(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) + (288*c^
(19/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/3
) - (288*c^(19/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) + (1047324*
3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (698216*Sqrt[
2]*3^(3/4)*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.966234, antiderivative size = 669, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {477, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}-\frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{288\sqrt{3}c^{19/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{288c^{19/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{3\sqrt[6]{c}\sqrt{c+d}}\right.}{d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-36534*c^2*x^2*Sqrt[c + d*x^3])/(1729*d^2) - (348*c*x^5*Sqrt[c + d*x^3])/(
247*d) - (2*x^8*Sqrt[c + d*x^3])/19 - (2094648*c^3*Sqrt[c + d*x^3])/(1729*d
^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (288*Sqrt[3]*c^(19/6)*ArcTan[
(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) + (288*c^
(19/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/3
) - (288*c^(19/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) + (1047324*
3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (698216*Sqrt[
2]*3^(3/4)*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
```

```
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^7\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2\int \frac{x^7\left(-\frac{147c^2d}{2}-87cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{19d}$$

$$= -\frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{4\int \frac{x^4\left(-3480c^3d^2-\frac{18267}{4}c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{247d^3}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int \frac{x\left(-73068c^4d^3-\frac{261831}{2}c^3d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1729d^5}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int \left(\frac{261831c^3d^3x}{2\sqrt{c+dx^3}} - \frac{1120392c^4d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{1729d^5}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{\left(1047324c^3\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1729d^2} + \frac{\left(5184c^4\right)}{}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{\left(432c^3\right)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^3} -$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{10}{}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{28}{}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{28}{}$$

Mathematica [C] time = 0.108191, size = 163, normalized size = 0.24

$$\frac{261831c^2dx^5\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+365340c^3x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20x^2\left(19485c^2dx^3+182\right.}{17290d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-20*x^2*(18267*c^3 + 19485*c^2*d*x^3 + 1309*c*d^2*x^6 + 91*d^3*x^9) + 3653
40*c^3*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*
x^3)/(8*c)] + 261831*c^2*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17290*d^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.042, size = 1840, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/19*d*x^8*(d*x^3+c)^(1/2)+44/247*c*x^5*(d*x^3+c)^(1/2)+54/1729/
d*c^2*x^2*(d*x^3+c)^(1/2)+72/1729*I/d^2*c^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2))))+8*c*(2/13*x^5*d*(d*x^3+c)^(1/2)+32/91*c*x^2*(d*x^3+c)^(1/2)-18/
91*I*c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
```

```
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))))-64*c^2/d^2*(2/7*x^2*
(d*x^3+c)^(1/2)-44/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(
-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(
-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(
1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/
d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/
(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^
2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^7}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}x^7}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

3.303 $\int \frac{x^4\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=645

$$-\frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[Out] (-240*c*x^2*Sqrt[c + d*x^3])/(91*d) - (2*x^5*Sqrt[c + d*x^3])/13 - (13782*c
^2*Sqrt[c + d*x^3])/(91*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (36*
Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/d^(5/3) + (36*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sq
rt[c + d*x^3])])/d^(5/3) - (36*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d^(5/3) + (6891*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (4594*Sqrt[2]*3^(3/4)*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.827206, antiderivative size = 645, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {477, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{36\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{36c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{36c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-240*c*x^2*Sqrt[c + d*x^3])/(91*d) - (2*x^5*Sqrt[c + d*x^3])/13 - (13782*c
^2*Sqrt[c + d*x^3])/(91*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (36*
Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/d^(5/3) + (36*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sq
rt[c + d*x^3])])/d^(5/3) - (36*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d^(5/3) + (6891*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (4594*Sqrt[2]*3^(3/4)*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{2\int \frac{x^4\left(-\frac{93c^2d}{2}-60cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{13d}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{4\int \frac{x\left(-960c^3d^2-\frac{6891}{4}c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{91d^3}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{4\int \left(\frac{6891c^2d^2x}{4\sqrt{c+dx^3}}-\frac{14742c^3d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{91d^3}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{\left(6891c^2\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{91d}+\frac{\left(648c^3\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{\left(54c^2\right)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2}-\frac{\left(6891c^2\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\cdots}{\sqrt{c+dx^3}}\cdots}{91d^{4/3}}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\cdots\right.}{\cdots}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{36\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\cdots\right)}{\sqrt{c+dx^3}}\cdots\right.}{d^{5/3}}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d}-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{36\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\cdots\right)}{\sqrt{c+dx^3}}\cdots\right.}{d^{5/3}}$$

Mathematica [C] time = 0.0796552, size = 150, normalized size = 0.23

$$\frac{9600c^2x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+6891cdx^5\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80\left(120c^2x^2+127cdx^5+7d^2x^8\cdots\right.}{3640d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-80*(120*c^2*x^2 + 127*c*d*x^5 + 7*d^2*x^8) + 9600*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 6891*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(3640*d*Sqrt[c + d*x^3])
```

Maple [C] time = 0.012, size = 1344, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/13*x^5*d*(d*x^3+c)^(1/2)+32/91*c*x^2*(d*x^3+c)^(1/2)-18/91*I*c^2*3^
(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-8/d*c*(2/7*x^2*(d*x^3+c)^(1/2)-4
4/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1
/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c
)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3
```

```
)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(
1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/
3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*
_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*
I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/
2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(dx^7+cx^4\right)\sqrt{dx^3+c}}{dx^3-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^7 + c*x^4)*sqrt(d*x^3 + c)/(d*x^3 - 8*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{cx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx-\int \frac{dx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**7*sqr
t(c + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^3 + c\right)^{\frac{3}{2}} x^4}{dx^3 - 8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

3.304 $\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=627

$$-\frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{\cdots}\right.}{\sqrt{c+dx^3}}\right.}{2d^{2/3}}$$

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/7 - (132*c*Sqrt[c + d*x^3])/(7*d^(2/3)*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (9*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (9*c^(7/6)*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) + (66*3^(1/4)*Sqrt[2 - Sqr
t[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (44*Sqrt[2]*3^(3/4)*c^(4/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rubi [A] time = 0.722607, antiderivative size = 627, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {477, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}-\frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left((1+\cdots\right.}}}{7d^{2/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/7 - (132*c*Sqrt[c + d*x^3])/(7*d^(2/3)*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (9*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (9*c^(7/6)*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) + (66*3^(1/4)*Sqrt[2 - Sqr
t[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (44*Sqrt[2]*3^(3/4)*c^(4/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \frac{x\left(-\frac{39c^2d}{2}-33cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \left(\frac{33cdx}{\sqrt{c+dx^3}} - \frac{567c^2dx}{2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{1}{7}(66c)\int \frac{x}{\sqrt{c+dx^3}}\,dx + \left(81c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{(27c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{4d} - \frac{(66c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{7\sqrt[3]{d}} + \frac{\left(27c^{4/3}\right)\int \frac{1+\cdots}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\cdots}}{4\sqrt[3]{d}}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\cdots}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\cdots}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\cdots\right.}{3\sqrt[6]{\cdots}}\cdots\right)}{2d^{2/3}}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\cdots\right.}{3\sqrt[6]{\cdots}}\cdots\right)}{2d^{2/3}}$$

Mathematica [C] time = 0.0785141, size = 127, normalized size = 0.2

$$\frac{x^2\left(132dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+195c\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-160\left(c+dx^3\right)\right)}{560\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-160*(c + d*x^3) + 195*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] + 132*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(560*Sqrt[c + d*x^3])
```

Maple [C] time = 0.009, size = 864, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/7*x^2*(d*x^3+c)^(1/2)+44/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
))-3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c
)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_a
lpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^{3}+c\right)^{\frac{3}{2}}x}{dx^{3}-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(dx^{4}+cx\right)\sqrt{dx^{3}+c}}{dx^{3}-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^4 + c*x)*sqrt(d*x^3 + c)/(d*x^3 - 8*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{cx\sqrt{c+dx^{3}}}{-8c+dx^{3}}\,dx-\int \frac{dx^{4}\sqrt{c+dx^{3}}}{-8c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**4*sqrt(c
 + d*x**3)/(-8*c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

3.305 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=626

$$-\frac{5\,3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\right.}{}$$

```
[Out] -Sqrt[c + d*x^3]/(8*x) - (15*d^(1/3)*Sqrt[c + d*x^3])/(8*((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(1/6)*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/16 + (9*c^(1/6)*d^(1/3)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/16 - (9*c^(1/6)*d^(1/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/16 + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1
/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(16*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (5*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3
])
```

Rubi [A] time = 0.718377, antiderivative size = 626, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {474, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{5\,3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{16\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(8*x) - (15*d^(1/3)*Sqrt[c + d*x^3])/(8*((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(1/6)*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/16 + (9*c^(1/6)*d^(1/3)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/16 - (9*c^(1/6)*d^(1/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/16 + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1
/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(16*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (5*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3
])
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{8x}+\frac{\int \frac{x\left(21c^2d+\frac{15}{2}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x}+\frac{\int \left(-\frac{15cdx}{2\sqrt{c+dx^3}}+\frac{81c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x}-\frac{1}{16}(15d)\int \frac{x}{\sqrt{c+dx^3}}\,dx+\frac{1}{8}(81cd)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{8x}-\frac{27}{32}\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx-\frac{1}{16}\left(15d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx+\frac{1}{32}$$

$$= -\frac{\sqrt{c+dx^3}}{8x}-\frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin\right.}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+d}}$$

$$= -\frac{\sqrt{c+dx^3}}{8x}-\frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)+\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\tanh$$

$$= -\frac{\sqrt{c+dx^3}}{8x}-\frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)+\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\tanh$$

Mathematica [C] time = 0.0762011, size = 137, normalized size = 0.22

$$\frac{3d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+21cdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16c\left(c+dx^3\right)}{128cx\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-16*c*(c + d*x^3) + 21*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(128*c*x*Sqrt[c + d*x^3])
```

Maple [C] time = 0.022, size = 1339, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/2)*(-d^2
*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2))))-1/8*d/c*(2/7*x^2*(d*x^3+c)^(1/2)-44/7*I*c*3^(
1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-d
^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)
*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-
d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^
```

```
(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

3.306 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx$

Optimal. Leaf size=651

$$\frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+$$

```
[Out] -Sqrt[c + d*x^3]/(32*x^4) - (3*d*Sqrt[c + d*x^3])/(16*c*x) + (3*d^(4/3)*Sqr
t[c + d*x^3])/(16*c*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
(5/6)) + (9*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])])/(128*c^(5/6)) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(12
8*c^(5/6)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) +
(3^(3/4)*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(8*Sqrt[2]*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.844219, antiderivative size = 651, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {474, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}}+\frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}\right.}}}{8\sqrt{2}c^{2/3}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(32*x^4) - (3*d*Sqrt[c + d*x^3])/(16*c*x) + (3*d^(4/3)*Sqr
t[c + d*x^3])/(16*c*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
(5/6)) + (9*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])])/(128*c^(5/6)) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(12
8*c^(5/6)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) +
(3^(3/4)*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(8*Sqrt[2]*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{32x^4}+\frac{\int \frac{48c^2d+\frac{69}{2}cd^2x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\int \frac{x\left(-516c^3d^2+24c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\int \left(-\frac{24c^2d^2x}{\sqrt{c+dx^3}}-\frac{324c^3d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{1}{64}\left(81d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx+\frac{\left(3d^2\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{(27d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c}+\frac{\left(3d^{5/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c}+\frac{\left(27d^{5/3}\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{\cdots}}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\cdots\right.\right.}}}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d^{4/3}\tan\cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d^{4/3}\tan\cdots}{\cdots}$$

Mathematica [C] time = 0.0910361, size = 154, normalized size = 0.24

$$\frac{645cd^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\left(3d^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+40c\left(c^2+7cdx^3+6d^2x^6\right)\right)}{5120c^2x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (645*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 7*c*d*x^3 + 6*d^2*x^6) + 3*d^3*x^9*Sqrt[1
+ (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(512
0*c^2*x^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.025, size = 1810, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x)
```

```
[Out] 1/64*d/c^2*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/2)*
(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/64*d^2/c^2*(2/7*x^2*(d*x^3+c)^(1/2)-44
/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/
_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)
```

```
+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_
alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I
*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))+1/8/c*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d*
3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{dx^8-8\,cx^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^3 + c)^(3/2)/(d*x^8 - 8*c*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

3.307 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx$

Optimal. Leaf size=675

$$\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3d^2}{5}$$

```
[Out] -Sqrt[c + d*x^3]/(56*x^7) - (75*d*Sqrt[c + d*x^3])/(1792*c*x^4) - (3*d^2*Sq
rt[c + d*x^3])/(56*c^2*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(7/3)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)) - (9*d
^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3*3^(1/4)*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(112*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*c^
(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.93572, antiderivative size = 675, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {474, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}-\frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{7/3}\tanh^{-1}}{1024c^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(56*x^7) - (75*d*Sqrt[c + d*x^3])/(1792*c*x^4) - (3*d^2*Sq
rt[c + d*x^3])/(56*c^2*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(7/3)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(1024*c^(11/6)) - (9*d
^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(1024*c^(11/6)) - (3*3^(1/4)*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(112*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]]])/(28*Sqrt[2]*c^
(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{56x^7} + \frac{\int \frac{75c^2d+\frac{123}{2}cd^2x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{56c}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{\int \frac{-768c^3d^2-\frac{375}{2}c^2d^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1792c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{\int \frac{x\left(5340c^4d^3-384c^3d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{\int \left(\frac{384c^3d^3x}{\sqrt{c+dx^3}} + \frac{2268c^4d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{\left(3d^3\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^2} + \frac{\left(81d^3\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{512c}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} - \frac{\left(27d^2\right)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^2} + \frac{\left(3d^{8/3}\right)\int \frac{\left(1-\sqrt{3}\right)\cdots}{\sqrt{c+\cdots}}\cdots}{112c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\cdots\right.}{\sqrt{c+\cdots}}\cdots\right.}{1024c^{11/6}}\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\cdots\right.}{\sqrt{c+\cdots}}\cdots\right.}{1024c^{11/6}}\cdots$$

Mathematica [C] time = 0.100431, size = 167, normalized size = 0.25

$$\frac{6675cd^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(6d^4x^{12}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5c\left(107c^2dx^3+32c^3+171c\right.\right.}{286720c^3x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (6675*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 + 107*c^2*d*x^3 + 171*c*d^2*x^6 + 96*d^3
*x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)
/c), (d*x^3)/(8*c)]))/(286720*c^3*x^7*Sqrt[c + d*x^3])
```

Maple [C] time = 0.024, size = 2306, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x)
```

```
[Out] 1/512*d^2/c^3*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/
2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/512*d^3/c^3*(2/7*x^2*(d*x^3+c)^(1/2
)-44/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
```

```
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*su
m(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(
1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c
)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/
3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*
(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c)))+1/8/c*(-1/7*c*(d*x^3+c)^(1/2)/x^7-17/56*d*(d*x^3+c)^(1/2)/x^4-2
7/112*d^2/c*(d*x^3+c)^(1/2)/x-9/112*I*d^2/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2))))+1/64*d/c^2*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x
-9/8*I*d*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{dx^{11}-8\,cx^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^3 + c)^(3/2)/(d*x^11 - 8*c*x^8), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

3.308 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=90

$$-\frac{38c^2\sqrt{c+dx^3}}{d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}$$

```
[Out] (-38*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/(3*d^4) - (2*(c + d
*x^3)^(5/2))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(9*d^4)
```

Rubi [A] time = 0.0806504, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 88, 63, 206}

$$-\frac{38c^2\sqrt{c+dx^3}}{d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-38*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/(3*d^4) - (2*(c + d
*x^3)^(5/2))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(9*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(-\frac{57c^2}{d^3\sqrt{c+dx}}+\frac{512c^3}{d^3(8c-dx)\sqrt{c+dx}}-\frac{6c\sqrt{c+dx}}{d^3}-\frac{(c+dx)^{3/2}}{d^3}\right)dx, x, x^3\right)\\
&= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}+\frac{\left(512c^3\right)\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^3}\\
&= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}+\frac{\left(1024c^3\right)\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^4}\\
&= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}
\end{aligned}$$

Mathematica [A] time = 0.0698614, size = 69, normalized size = 0.77

$$\frac{5120c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-6\sqrt{c+dx^3}\left(296c^2+12cdx^3+d^2x^6\right)}{45d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)
```

Maple [C] time = 0.036, size = 528, normalized size = 5.9

$$-\frac{1}{d}\left(\frac{2x^6}{15d}\sqrt{dx^3+c}-\frac{8cx^3}{45d^2}\sqrt{dx^3+c}+\frac{16c^2}{45d^3}\sqrt{dx^3+c}\right)-8\frac{c}{d^2}\left(2/9\frac{x^3\sqrt{dx^3+c}}{d}-4/9\frac{c\sqrt{dx^3+c}}{d^2}\right)-\frac{128c^2}{3d^4}\sqrt{dx^3+c}--$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d*(2/15/d*x^6*(d*x^3+c)^(1/2)-8/45*c/d^2*x^3*(d*x^3+c)^(1/2)+16/45*c^2*(
d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2
)/d^2)-128/3*c^2*(d*x^3+c)^(1/2)/d^4-512/27*I*c^2/d^6*2^(1/2)*sum((-d^2*c)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2
*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32712, size = 360, normalized size = 4.

$$\left[\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(d^2x^6+12\,cdx^3+296\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^4}, -\frac{2\left(2560\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\left(d^2\right.\right.}{45\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(25
60*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^2*x^6 + 12*c*
d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11246, size = 111, normalized size = 1.23

$$-\frac{1024\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d^4}-\frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16}+10\left(dx^3+c\right)^{\frac{3}{2}}cd^{16}+285\,\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d*
x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c^2
*d^16)/d^20
```

3.309 $\int \frac{x^8}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

```
[Out] (-14*c*Sqrt[c + d*x^3])/(3*d^3) - (2*(c + d*x^3)^(3/2))/(9*d^3) + (128*c^(3
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)
```

Rubi [A] time = 0.0689688, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 88, 63, 206}

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```


```
[Out] (-14*c*Sqrt[c + d*x^3])/(3*d^3) - (2*(c + d*x^3)^(3/2))/(9*d^3) + (128*c^(3
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{7c}{d^2\sqrt{c+dx}}+\frac{64c^2}{d^2(8c-dx)\sqrt{c+dx}}-\frac{\sqrt{c+dx}}{d^2}\right)dx, x, x^3\right)\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(64c^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(128c^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}
\end{aligned}
$$

Mathematica [A] time = 0.0493628, size = 58, normalized size = 0.82

$$
\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(22c+dx^3\right)}{9d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^3)
```

Maple [C] time = 0.013, size = 468, normalized size = 6.6

$$-\frac{1}{d^2}\left(d\left(\frac{2x^3}{9d}\sqrt{dx^3+c}-\frac{4c}{9d^2}\sqrt{dx^3+c}\right)+\frac{16c}{3d}\sqrt{dx^3+c}\right)-\frac{\frac{64i}{27}c\sqrt{2}}{d^5}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3d-8c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d^2*(d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)+16/3*c*(d*x
^3+c)^(1/2)/d)-64/27*I*c/d^5*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*
(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/
3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^
2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31304, size = 296, normalized size = 4.17

$$\left[\frac{2\left(32c^{\frac{3}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-\left(dx^3+22c\right)\sqrt{dx^3+c}\right)}{9d^3},-\frac{2\left(64\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(dx^3+22c\right)\sqrt{dx^3+c}\right)}{9d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(32*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3, -2/9*(64*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^8}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**8/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.10726, size = 88, normalized size = 1.24

$$-\frac{128\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d^3}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6+21\,\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -128/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/9*((d*x^
3 + c)^(3/2)*d^6 + 21*sqrt(d*x^3 + c)*c*d^6)/d^9
```

3.310 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=52

$$\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(9*d^2)
```

Rubi [A] time = 0.0442747, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 80, 63, 206}

$$\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```


```
[Out] (-2*Sqrt[c + d*x^3])/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(9*d^2)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{(8c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d}\\
&= -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{(16c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^2}\\
&= -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}
\end{aligned}$$

Mathematica [A] time = 0.0182125, size = 49, normalized size = 0.94

$$-\frac{2\left(3\sqrt{c+dx^3}-8\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(
9*d^2)
```

Maple [C] time = 0.012, size = 425, normalized size = 8.2

$$-\frac{2}{3d^2}\sqrt{dx^3+c}-\frac{\frac{8i}{27}\sqrt{2}}{d^4}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3d-8c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*(d*x^3+c)^(1/2)/d^2-8/27*I/d^4*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*
(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2
*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*
d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32343, size = 255, normalized size = 4.9

$$\left[\frac{2\left(4\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\sqrt{dx^3+c}\right)}{9d^2},\ -\frac{2\left(8\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+c}\right)}{9d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(4*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 3*sqrt(d*x^3 + c))/d^2, -2/9*(8*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sq
rt(-c)/c) + 3*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] time = 17.7507, size = 61, normalized size = 1.17

$$\begin{cases} \frac{2\left(-\frac{8c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)) - sqr
t(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(48*c**(3/2)), True))
```

Giac [A] time = 1.10945, size = 65, normalized size = 1.25

$$-\frac{2\left(\frac{8\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(8*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
+ c)/d)/d
```

3.311 $\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Rubi [A] time = 0.0327539, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}\end{aligned}$$

Mathematica [A] time = 0.0095423, size = 33, normalized size = 1.

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Maple [C] time = 0.008, size = 413, normalized size = 12.5

$$\frac{-\frac{i}{27}\sqrt{2}}{d^3c}\sum_{\_alpha=RootOf\left(\_Z^3d-8c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}+i\sqrt{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))
/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d
^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3471, size = 193, normalized size = 5.85

$$\left[\frac{\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)}{9\sqrt{c}d}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)}{9\,cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c)
*d), -2/9*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)/(c*d)]
```

Sympy [A] time = 7.5725, size = 32, normalized size = 0.97

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c))
```

Giac [A] time = 1.13444, size = 36, normalized size = 1.09

$$-\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)
```

3.312 $\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]]/(12*c^(3/2))
```

Rubi [A] time = 0.0520377, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 86, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]]/(12*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}+\frac{d\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}\\
&= \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{12c}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{12cd}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0163623, size = 51, normalized size = 0.88

$$
\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(36*c^(3/2))
```

Maple [C] time = 0.01, size = 433, normalized size = 7.5

$$-\frac{\frac{i}{216}\sqrt{2}}{c^2d^2}\sum_{\_alpha=RootOf(\_Z^3d-8c)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}+i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/216*I/d^2/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/
3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^
2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))-1/12*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x), x)
```

Fricas [A] time = 1.30533, size = 347, normalized size = 5.98

$$\left[\frac{\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{72c^2}, \frac{3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{36c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)
) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, 1/36
*(3*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c))/c^2]
```

Sympy [A] time = 10.7737, size = 58, normalized size = 1.

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{36c\sqrt{-c}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(36*c*sqrt(-c)) + atan(sqrt(c + d*x**3
)/sqrt(-c))/(12*c*sqrt(-c))
```

Giac [A] time = 1.11872, size = 73, normalized size = 1.26

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{36\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

3.313 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=81

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

```
[Out] -Sqrt[c + d*x^3]/(24*c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2
88*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Rubi [A] time = 0.0731036, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 103, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2
88*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$
$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{64c^2} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2}$$
$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{32c^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{96c^2}$$
$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Mathematica [A] time = 0.0296759, size = 81, normalized size = 1.

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2
88*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Maple [C] time = 0.01, size = 477, normalized size = 5.9

$$\frac{-\frac{i}{1728}\sqrt{2}}{dc^3} \sum_{\_alpha=RootOf\left(\_Z^3d-8c\right)} \sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}+i\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/1728*I/d/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3
))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2
*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/96*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/
c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)
```

Fricas [A] time = 1.37201, size = 462, normalized size = 5.7

$$\left[\frac{\sqrt{c}dx^3\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\,\sqrt{c}dx^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}c}{576\,c^3x^3},-\frac{9\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\cdots}{2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^
3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(9*sqrt(-c)*d*x^3*arctan(sqrt(
```

```
d*x^3 + c)*sqrt(-c)/c) + sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-8cx^4\sqrt{c+dx^3}+dx^7\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.12406, size = 103, normalized size = 1.27

$$-\frac{1}{288}\,d\left(\frac{9\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{12\,\sqrt{dx^3+c}}{c^2dx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/288*d*(9*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 12*sqrt(d*x^3 + c)/(c^2*d*x^3))
```

3.314 $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=107

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

```
[Out] -Sqrt[c + d*x^3]/(48*c^2*x^6) + (5*d*Sqrt[c + d*x^3])/(192*c^3*x^3) + (d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(256*c^(7/2))
```

Rubi [A] time = 0.0977468, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c^2*x^6) + (5*d*Sqrt[c + d*x^3])/(192*c^3*x^3) + (d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(256*c^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6}-\frac{\text{Subst}\left(\int \frac{10cd-\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{48c^2}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}+\frac{\text{Subst}\left(\int \frac{42c^2d^2-5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{384c^4}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}+\frac{\left(7d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{512c^3}+\frac{d^3\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+d}}\right.}{1536c^3}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}+\frac{(7d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{256c^3}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{9c-x^2}\right.}{768}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0611427, size = 95, normalized size = 0.89

$$
\frac{d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-63d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)+12\sqrt{c}\sqrt{c+dx^3}\left(5dx^3-4c\right)}{2304c^{7/2}x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (12*Sqrt[c]*Sqrt[c + d*x^3]*(-4*c + 5*d*x^3) + d^2*x^6*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])] - 63*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2304*c^(7/
2)*x^6)
```

Maple [C] time = 0.024, size = 540, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/13824*I/c^4*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3)
)/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*
d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*d*(d*x^3+c
)^(1/2)/c^2/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/64*d/c^
2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2
))-1/768*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)
```

Fricas [A] time = 1.34174, size = 525, normalized size = 4.91

$$\left[\frac{\sqrt{c}d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+63\,\sqrt{c}d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+24\left(5\,cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{4608\,c^4x^6},\ \frac{63\,\sqrt{-c}d^2x^6\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), 1/2304*(63*sq
rt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*d^2*x^6*arctan
(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/
(c^4*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11786, size = 127, normalized size = 1.19

$$\frac{1}{2304}\,d^2\left(\frac{63\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{12\left(5\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c\right)}{c^3d^2x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2304*d^2*(63*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - arctan(1/3
*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 12*(5*(d*x^3 + c)^(3/2) - 9*sqr
t(d*x^3 + c)*c)/(c^3*d^2*x^6))
```

3.315 $\int \frac{x^7}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=630

$$-\frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}$$

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d^2) - (104*c*Sqrt[c + d*x^3])/(7*d^(8/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(8/3)) + (32*c^(7/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) - (32*
c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(8/3)) + (52*3^(1/4)*Sqr
t[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (104*Sqrt[2]*c^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(
8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.723149, antiderivative size = 630, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {479, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}+\frac{32c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}-\frac{32c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}-\frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{7\sqrt[4]{3}d^{8/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d^2) - (104*c*Sqrt[c + d*x^3])/(7*d^(8/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(8/3)) + (32*c^(7/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) - (32*
c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(8/3)) + (52*3^(1/4)*Sqr
t[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (104*Sqrt[2]*c^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(
8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^7}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d^2} + \frac{2\int \frac{x\left(16c^2+26cdx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{7d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} + \frac{2\int \left(-\frac{26cx}{\sqrt{c+dx^3}} + \frac{224c^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{7d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{(52c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d^2} + \frac{\left(64c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{(16c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3} - \frac{(52c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{7d^{7/3}} + \frac{\left(16c^{4/3}\right)\int \cdots}{3\cdots}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} + \frac{32c^{7/6}\tanh\cdots}{9\cdots}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} + \frac{32c^{7/6}\tanh\cdots}{9\cdots}$$

Mathematica [C] time = 0.0823164, size = 130, normalized size = 0.21

$$\frac{x^2\left(13dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+20c\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20\left(c+dx^3\right)\right)}{70d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*(-20*(c + d*x^3) + 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 13*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(70*d^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.033, size = 1311, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d^2*(d*(2/7*x^2*(d*x^3+c)^(1/2)/d+8/21*I/d^2*c*3^(1/2)*(-d^2*c)^(1/3)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-
d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2))))-16/3*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d
*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*
c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d
*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-64/
27*I*c/d^5*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^
(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alp
ha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
```

```
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^7}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**7/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.316 $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=601

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{5/3}\sqrt{\cdots}}$$

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*c^(
1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sq
rt[3]*d^(5/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt
[c + d*x^3])])/(9*d^(5/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(9*d^(5/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (
2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.581432, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {483, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{d^{5/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*c^(
1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sq
rt[3]*d^(5/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt
[c + d*x^3])]])/(9*d^(5/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(9*d^(5/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (
2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d}+\frac{(8c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= -\frac{2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^2}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{d^{4/3}}+\frac{\left(2\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{3d^{4/3}}-\frac{\left(\sqrt{2\left(2\cdots\right.}\right.}{\cdots}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1\cdots}{(1\cdots}\right.\right.}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\cdots$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{4\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}+\frac{4\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}+\cdots$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{4\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}+\frac{4\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}-\cdots$$

Mathematica [C] time = 0.0324154, size = 67, normalized size = 0.11

$$\frac{x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{40c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(40*c*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 848, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-8/27*I/d^4*2^(1/2)*sum(
1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/
3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)
*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2
*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
```

```
*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}x^4}{d^2x^6-7\,cdx^3-8\,c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^4/(d^2*x^6 - 7*c*d*x^3 - 8*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**4/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.317 $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=141

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(6*Sqrt[3]
*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*c^(5/
6)*d^(2/3))
```

Rubi [A] time = 0.413438, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(6*Sqrt[3]
*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*c^(5/
6)*d^(2/3))
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{12cd} + \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{12c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[3]{d}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{c}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{6\sqrt[3]{c}d^{2/3}} - \frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{12\sqrt[3]{c}} + \frac{d^{4/3}\,\text{Subst}\left(\int \frac{}{-\frac{2d}{c}}\right.}{3}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{6\sqrt[3]{c}d^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

Mathematica [C] time = 0.026365, size = 67, normalized size = 0.48

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(16*c*Sqrt[c + d*x^3])
```

Maple [C] time = 0.007, size = 416, normalized size = 3.

$$\frac{-\frac{i}{27}\sqrt{2}}{d^3c}\sum_{\_alpha=RootOf\left(\_Z^3d-8c\right)}\frac{1}{\_alpha}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*
c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_
alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/
2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [B] time = 9.37557, size = 5659, normalized size = 40.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*
d^4))^(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d
^4))^(5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqr
t(d*x^3 + c) + (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3)
+ 12*sqrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*
sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5
```

```
*x^6 + 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*
d^3*x^4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x
^2)*(1/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 -
1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(
2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6
) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^
3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^
8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 1/54*
sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*d^4))^
(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^(
5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqrt(d*x^
3 + c) - (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3) + 12*s
qrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*sqrt(3
)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5*x^6 +
 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*d^3*x^
4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x^2)*(1
/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c
^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) -
 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*
(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7
- 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20
*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 1/108*(1/(c
^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18
*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) + 6*sq
rt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*
c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7
+ 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38
*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 1/108*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4
+ 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 +
 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 6
4*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(
1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1
/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/
216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 108
8*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3
) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) -
 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x
^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 +
 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + 1/216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 2
76*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4
```

```
- 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 +
c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^
5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c
^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5
*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.318 $\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=632

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left(\left(1+\cdots\right.\right.}}}{16c^{5/3}\sqrt{\cdots}}$$

```
[Out] -Sqrt[c + d*x^3]/(8*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c^2*((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/
3)*x))/Sqrt[c + d*x^3]])/(48*Sqrt[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3)
+ d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(144*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3
]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(16*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3])
```

Rubi [A] time = 0.72339, antiderivative size = 632, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {480, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}}}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(8*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c^2*((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/
3)*x))/Sqrt[c + d*x^3]])/(48*Sqrt[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3)
+ d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(144*c^(11/6)) - (d^(1/3)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(144*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3
]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(16*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3])
```

<u>Rule 480</u>

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

<u>Rule 584</u>

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

<u>Rule 303</u>

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

<u>Rule 218</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\int \frac{x\left(5cd-\frac{d^2x^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{8c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{cdx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{8c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{16c^2} + \frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{16c^2} + \frac{d^{2/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}} \cdots}{96c^{5/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\cdots\right)}}\cdots}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\cdots\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right)}}}\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\cdots\right)}{144c^{11/6}}\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\cdots\right)}{144c^{11/6}}\cdots$$

Mathematica [C] time = 0.0779246, size = 137, normalized size = 0.22

$$\frac{-d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+25cdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(c+dx^3\right)}{640c^3x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^3*x*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 874, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)))-1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*
(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/
3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^
2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^8-7\,cdx^5-8\,c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^8 - 7*c*d*x^5 - 8*c^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-8cx^2\sqrt{c+dx^3}+dx^5\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**2*sqrt(c + d*x**3) + d*x**5*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

3.319 $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=654

$$-\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{4/3}\ldots}{\ldots}$$

```
[Out] -Sqrt[c + d*x^3]/(32*c^2*x^4) + (d*Sqrt[c + d*x^3])/(16*c^3*x) - (d^(4/3)*S
qrt[c + d*x^3])/(16*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*Arc
Tan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(384*Sqrt[3]*
c^(17/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])])/(1152*c^(17/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1
152*c^(17/6)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(8/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) -
 (d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]
])/(8*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.830333, antiderivative size = 654, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {480, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\left(\sqrt[3]{c}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(32*c^2*x^4) + (d*Sqrt[c + d*x^3])/(16*c^3*x) - (d^(4/3)*S
qrt[c + d*x^3])/(16*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*Arc
Tan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(384*Sqrt[3]*
c^(17/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])])/(1152*c^(17/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1
152*c^(17/6)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(8/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) -
 (d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]
])/(8*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{\int \frac{-16cd+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{32c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{\int \frac{x\left(60c^2d^2-8cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{256c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{\int \left(\frac{8cd^2x}{\sqrt{c+dx^3}} - \frac{4c^2d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{256c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^3} + \frac{d^2\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{64c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{768c^3} - \frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c^3} + \frac{d^{5/3}\cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\cdots}{32c^{\cdots}}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}} + \cdots$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}} + \cdots$$

Mathematica [C] time = 0.0911575, size = 152, normalized size = 0.23

$$\frac{4d^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 75cd^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 160c\left(-c^2+cdx^3+2d^2x^6\right)}{5120c^4x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (160*c*(-c^2 + c*d*x^3 + 2*d^2*x^6) - 75*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 4*d^3*x^9*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(5120*c^4
*x^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.025, size = 1351, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64*d/c^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2))))-1/1728*I/d/c^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)
^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/4*(d*x^3+c)^(1/2)/c/
x^4+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I*d/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
```

```
^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^
(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^{11}-7\,cdx^8-8\,c^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^11 - 7*c*d*x^8 - 8*c^2*x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-8cx^5\sqrt{c+dx^3}+dx^8\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**5*sqrt(c + d*x**3) + d*x**8*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

3.320 $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=678

$$\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3d}{}$$

```
[Out] -Sqrt[c + d*x^3]/(56*c^2*x^7) + (37*d*Sqrt[c + d*x^3])/(1792*c^3*x^4) - (3*
d^2*Sqrt[c + d*x^3])/(56*c^4*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^4*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)
 + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(23/6)) + (d^(7/3)*ArcTanh
[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(23/6)) - (d
^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(23/6)) - (3*3^(1/4)*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(112*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/
3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*c
^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.92948, antiderivative size = 678, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {480, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+}}{3\sqrt{}}\right)}{9216c^{23/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(56*c^2*x^7) + (37*d*Sqrt[c + d*x^3])/(1792*c^3*x^4) - (3*
d^2*Sqrt[c + d*x^3])/(56*c^4*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^4*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)
 + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(23/6)) + (d^(7/3)*ArcTanh
[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(23/6)) - (d
^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(23/6)) - (3*3^(1/4)*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(112*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/
3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*c
^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{\int \frac{-37cd+\frac{11d^2x^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{56c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{\int \frac{-768c^2d^2+\frac{185}{2}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1792c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\int \frac{x\left(3100c^3d^3-384c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{14336c^6}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\int \left(\frac{384c^2d^3x}{\sqrt{c+dx^3}}+\frac{28c^3d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{14336c^6}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\left(3d^3\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^4}+\frac{d^3\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,d}{512c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}-\frac{d^2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6144c^4}+\frac{\left(3d^{8/3}\right)\int}{}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3\sqrt[4]{3}\sqrt{2-}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{7/3}\tan^{-1}\left(\right.}{3072}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{7/3}\tan^{-1}\left(\right.}{3072}$$

Mathematica [C] time = 0.107502, size = 167, normalized size = 0.25

$$\frac{3875cd^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(6d^4x^{12}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5c\left(-5c^2dx^3+32c^3+59cd^2x\right.\right.}{286720c^5x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (3875*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 - 5*c^2*d*x^3 + 59*c*d^2*x^6 + 96*d^3*x^
9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c)
, (d*x^3)/(8*c)]))/(286720*c^5*x^7*Sqrt[c + d*x^3])
```

Maple [C] time = 0.024, size = 1849, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/512*d^2/c^3*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2))))-1/13824*I/c^4*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(
x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*
c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
```

```
/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d
)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/
c/x^7+11/56*d*(d*x^3+c)^(1/2)/c^2/x^4-55/112*d^2*(d*x^3+c)^(1/2)/c^3/x-55/3
36*I*d^2/c^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))+1/64*d/c^2*(-1/4*(d
*x^3+c)^(1/2)/c/x^4+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I*d/c^2*3^(1/2)*(-d^2*
c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{dx^3 + c}\left(dx^3 - 8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

3.321 $\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.05414, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(8c-dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

Mathematica [A] time = 0.0361884, size = 67, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(32*c*Sqrt[c + d*x^3])
```

Maple [C] time = 0.03, size = 696, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
```

```
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*
c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))-8/2
7*I/d^4*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(
1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alph
a^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}x^3}{d^2x^6-7\,cdx^3-8\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^3/(d^2*x^6 - 7*c*d*x^3 - 8*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**3/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.322 $\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0282031, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} = \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.146211, size = 166, normalized size = 2.59

$$\frac{32cxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\sqrt{c+dx^3}\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (32*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*
x^3)*Sqrt[c + d*x^3]*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)
/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]
- 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
```

Maple [C] time = 0.006, size = 416, normalized size = 6.5

$$\frac{-\frac{i}{27}\sqrt{2}}{d^3c}\sum_{\_alpha=RootOf(\_Z^3d-8c)}\frac{1}{\_alpha^2}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^
2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d
```

```
*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2
*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^6-7\,cdx^3-8\,c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^6 - 7*c*d*x^3 - 8*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{dx^3 + c}\left(dx^3 - 8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

3.323 $\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(16*c*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0566818, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```


```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(16*c*x^2*Sqrt[c + d*x^3])
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.154552, size = 242, normalized size = 3.67

$$\frac{\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{4096dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(dx^3-8c\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-\frac{64\left(c+dx^3\right)}{c^2}}{1024x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3))/c^2 + (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^3 + (4096*d*x^3*AppellF1[1/3, 1/2, 1,
 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((-8*c + d*x^3)*(32*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)]))))/(1024*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 722, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
```

```
(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3)))^(1/2))-1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*(-d^2*
c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^
2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-
d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2
*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2
*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^9-7\,cdx^6-8\,c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^9 - 7*c*d*x^6 - 8*c^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-8cx^3\sqrt{c+dx^3}+dx^6\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**3*sqrt(c + d*x**3) + d*x**6*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

3.324 $\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 1/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(40*c*x^5*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0589708, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 1/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(40*c*x^5*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.19014, size = 261, normalized size = 3.95

$$\frac{64c\left(\frac{3264c^2d^2x^6F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-16c^2+7cdx^3+23d^2x^6\right)-23d^3x^9\sqrt{\frac{dx^3}{c}+}}{40960c^4x^5\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-23*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 64*c*(-16*c^2 + 7*c*d*x^3 + 23*d^2*x^6 + (3264*c^2*d^2*x^6*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32
*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(Appel
lF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(40960*c^4*x^5*Sqrt[c + d*x^3])
```

Maple [C] time = 0.025, size = 1047, normalized size = 15.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64*d/c^2*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
```

```
-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-1/1728*I/d/c^3*2^(1/2)*sum(1/_alpha^2*(-
d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/
(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2
)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2
*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(
-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/
8/c*(-1/5/c/x^5*(d*x^3+c)^(1/2)+7/20*d/c^2/x^2*(d*x^3+c)^(1/2)-7/60*I*d/c^2
*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-8cx^6\sqrt{c+dx^3}+dx^9\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**6*sqrt(c + d*x**3) + d*x**9*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

3.325 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2c^2}{27d^4\sqrt{c+dx^3}}+\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^4}$$

```
[Out] (2*c^2)/(27*d^4*Sqrt[c + d*x^3]) - (4*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*x^
3)^(3/2))/(9*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81
*d^4)
```

Rubi [A] time = 0.10356, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 87, 43, 63, 206}

$$\frac{2c^2}{27d^4\sqrt{c+dx^3}}+\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(27*d^4*Sqrt[c + d*x^3]) - (4*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*x^
3)^(3/2))/(9*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81
*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{c^2}{9d^3(c+dx)^{3/2}} - \frac{7c}{d^3\sqrt{c+dx}} - \frac{x}{d^2\sqrt{c+dx}} + \frac{512c^2}{9d^3(8c-dx)\sqrt{c+dx}}\right) dx, x, \ldots\\
&= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{\left(512c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^3} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+\ldots}} \ldots\right)}{3d\ldots}\\
&= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{\left(1024c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^4} - \frac{\text{Subst}\left(\int \left(\ldots\right.\right.}{\ldots}\\
&= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2\left(c+dx^3\right)^{3/2}}{9d^4} + \frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}
\end{aligned}
$$

Mathematica [C] time = 0.0447, size = 66, normalized size = 0.73

$$-\frac{2\left(512c^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-456c^2+60cdx^3+3d^2x^6\right)}{27d^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(-456*c^2 + 60*c*d*x^3 + 3*d^2*x^6 + 512*c^2*Hypergeometric2F1[-1/2, 1,
 1/2, (c + d*x^3)/(9*c)]))/(27*d^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.053, size = 560, normalized size = 6.2

$$-\frac{1}{d}\left(-\frac{2\,c^2}{3\,d^3}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\,x^3}{9\,d^2}\sqrt{dx^3+c}-\frac{10\,c}{9\,d^3}\sqrt{dx^3+c}\right)-8\,\frac{c}{d^2}\left(2/3\,\frac{c}{d^2}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}+2/3\,\frac{\sqrt{dx^3+c}}{d^2}\right)+\frac{128\,c^2}{3\,d^4}\frac{1}{\sqrt{dx^3+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d*(-2/3/d^3*c^2/((x^3+1/d*c)*d)^(1/2)+2/9/d^2*x^3*(d*x^3+c)^(1/2)-10/9*c
*(d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/3/d^2*c/((x^3+1/d*c)*d)^(1/2)+2/3*(d*x^3+c
)^(1/2)/d^2)+128/3*c^2/d^4/(d*x^3+c)^(1/2)-512*c^3/d^3*(2/27/d/c/((x^3+1/d*
c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(
-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3
)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2
*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.584, size = 436, normalized size = 4.84

$$\left[\frac{2\left(256\left(cdx^3+c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\left(3d^2x^6+60cdx^3+56c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3+cd^4\right)},-\frac{2\left(512\left(cdx^3+c^2\right)\sqrt{-c}\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(256*(c*d*x^3 + c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c)
)/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c))
/(d^5*x^3 + c*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.53924, size = 111, normalized size = 1.23

$$-\frac{1024\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^4}+\frac{2\,c^2}{27\sqrt{dx^3+c}d^4}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^8+18\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1024/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/27*c^2
/(sqrt(d*x^3 + c)*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*sqrt(d*x^3 + c)*c*
d^8)/d^12
```

3.326 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] (-2*c)/(27*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(3*d^3) + (128*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rubi [A] time = 0.0732029, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 87, 63, 206}

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(27*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(3*d^3) + (128*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{c}{9d^2(c+dx)^{3/2}}-\frac{1}{d^2\sqrt{c+dx}}+\frac{64c}{9d^2(8c-dx)\sqrt{c+dx}}\right)dx,x,x^3\right)\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{(64c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27d^2}\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{(128c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^3}\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}
\end{aligned}
$$

Mathematica [C] time = 0.0327667, size = 53, normalized size = 0.75

$$
-\frac{2\left(64c\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-54c+9dx^3\right)}{27d^3\sqrt{c+dx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(-54*c + 9*d*x^3 + 64*c*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*
c)]))/(27*d^3*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 501, normalized size = 7.1

$$-\frac{1}{d^2}\left(d\left(\frac{2c}{3d^2}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2}{3d^2}\sqrt{dx^3+c}\right)-\frac{16c}{3d}\frac{1}{\sqrt{dx^3+c}}\right)-64\frac{c^2}{d^2}\left(\frac{2}{27cd}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{\frac{i}{243}\sqrt{2}}{d^3c^2}\sum_{\_alpha=RootOf\left(\_Z^3d-8c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d^2*(d*(2/3/d^2*c/((x^3+1/d*c)*d)^(1/2)+2/3*(d*x^3+c)^(1/2)/d^2)-16/3*c/
d/(d*x^3+c)^(1/2))-64*c^2/d^2*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c
^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*
c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2
*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)
^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/1
8/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54468, size = 377, normalized size = 5.31

$$\left[\frac{2\left(32\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\left(9dx^3+10c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3+cd^3\right)},-\frac{2\left(64\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\left(\right.\right.}{81\left(d^4x^3+cd^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(32*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3), -
2/81*(64*(d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(9
*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12141, size = 78, normalized size = 1.1

$$-\frac{128\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^3}-\frac{2\sqrt{dx^3+c}}{3\,d^3}-\frac{2\,c}{27\sqrt{dx^3+c}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -128/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/3*sqrt(d*
x^3 + c)/d^3 - 2/27*c/(sqrt(d*x^3 + c)*d^3)
```

3.327 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

```
[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
Sqrt[c]*d^2)
```

Rubi [A] time = 0.0457229, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 78, 63, 206}

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
Sqrt[c]*d^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{8\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d}\\ &= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^2}\\ &= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}\end{aligned}$$

Mathematica [A] time = 0.030077, size = 49, normalized size = 0.94

$$\frac{2\left(\frac{3}{\sqrt{c+dx^3}} + \frac{8\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{81d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/
(81*d^2)
```

Maple [C] time = 0.01, size = 456, normalized size = 8.8

$$\frac{2}{3\,d^{2}}\frac{1}{\sqrt{dx^{3}+c}}-8\,\frac{c}{d}\left(\frac{2}{27\,cd}\frac{1}{\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{\frac{i}{243}\sqrt{2}}{d^{3}c^{2}}\sum_{\_alpha=RootOf\left(\_Z^{3}d-8\,c\right)}\frac{\sqrt[3]{-d^{2}c}\left(i\sqrt[3]{-d^{2}c}\_alpha\,\sqrt{3}d-i\left(-d^{2}c\right)^{2/3}\sqrt{3}+2\right.}{\sqrt{dx^{3}+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3/d^2/(d*x^3+c)^(1/2)-8/d*c*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c
^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*
c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2
*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)
^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/1
8/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54801, size = 344, normalized size = 6.62

$$\left[\frac{2\left(4\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\sqrt{dx^3+c}c\right)}{81\left(cd^3x^3+c^2d^2\right)},-\frac{2\left(8\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-3\sqrt{dx^3+c}c\right)}{81\left(cd^3x^3+c^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(4*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 3*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d*x
^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)
*c)/(c*d^3*x^3 + c^2*d^2)]
```

Sympy [A] time = 19.0056, size = 58, normalized size = 1.12

$$\begin{cases} \frac{2\left(\frac{1}{27d\sqrt{c+dx^3}}-\frac{8\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81d\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Piecewise((2*(1/(27*d*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-
c)))/(81*d*sqrt(-c)))/d, Ne(d, 0)), (x**6/(48*c**(5/2)), True))
```

Giac [A] time = 1.12979, size = 63, normalized size = 1.21

$$-\frac{2\left(\frac{8\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{3}{\sqrt{dx^3+c}d}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*(8*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3/(sqrt(d*x^3
+ c)*d))/d
```

3.328 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

```
[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
c^(3/2)*d)
```

Rubi [A] time = 0.0486025, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
c^(3/2)*d)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27c}\\
&= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27cd}\\
&= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}
\end{aligned}$$

Mathematica [C] time = 0.0117527, size = 43, normalized size = 0.78

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(27*c*d*Sqrt[c + d*
x^3])
```

Maple [C] time = 0.008, size = 435, normalized size = 7.9

$$-\frac{2}{27cd}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{\frac{i}{243}\sqrt{2}}{d^3c^2}\sum_{\_alpha=RootOf\left(\_Z^3d-8c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/27/d/c/((x^3+1/d*c)*d)^(1/2)-1/243*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55829, size = 339, normalized size = 6.16

$$\left[\frac{\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\sqrt{dx^3+c}c}{81\left(c^2d^2x^3+c^3d\right)},-\frac{2\left(\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\sqrt{dx^3+c}c\right)}{81\left(c^2d^2x^3+c^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/81*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(
d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3 +
 c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*c)/
(c^2*d^2*x^3 + c^3*d)]
```

Sympy [A] time = 18.4083, size = 51, normalized size = 0.93

$$-\frac{2}{27cd\sqrt{c+dx^3}}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81cd\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(27*c*d*sqrt(c + d*x**3)) - 2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(81*c*
d*sqrt(-c))
```

Giac [A] time = 1.12314, size = 65, normalized size = 1.18

$$-\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd}-\frac{2}{27\sqrt{dx^3+c}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 2/27/(sqrt(d*x^
3 + c)*c*d)
```

3.329 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=76

$$\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

```
[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5
/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))
```

Rubi [A] time = 0.0702243, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 85, 156, 63, 208, 206}

$$\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5
/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd+d^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27c^2d}$$
$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2}$$
$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{108c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12c^2d}$$
$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

Mathematica [C] time = 0.0206862, size = 63, normalized size = 0.83

$$\frac{9\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right) - {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{108c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] + 9*Hypergeometric2F1[
-1/2, 1, 1/2, 1 + (d*x^3)/c])/(108*c^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.027, size = 485, normalized size = 6.4

$$-\frac{d}{8\,c}\left(\frac{2}{27\,cd}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}} + \frac{\frac{i}{243}\sqrt{2}}{d^3c^2}\sum_{\_alpha=RootOf\left(d\_Z^3-8\,c\right)} \sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/8*d/c*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-d^2*
c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^
2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-
d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2
*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2
*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c
*(2/3/c/((x^3+1/d*c)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)
```

Fricas [A] time = 1.53056, size = 517, normalized size = 6.8

$$\left[\frac{\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+48\,\sqrt{dx^3+c}c}{648\left(c^3dx^3+c^4\right)}, \frac{27\left(dx^3+c\right)\sqrt{-c}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt
(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), 1/324*(27*(d*x^3
 + c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d*x^3 + c)*sqrt(-c)*ar
```

```
ctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c
^4)]
```

Sympy [A] time = 19.0613, size = 78, normalized size = 1.03

$$\frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{324c^2\sqrt{-c}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2/(27*c**2*sqrt(c + d*x**3)) - atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(324*c**
2*sqrt(-c)) + atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c**2*sqrt(-c))
```

Giac [A] time = 1.10815, size = 92, normalized size = 1.21

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c^2} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}c^2} + \frac{2}{27\sqrt{dx^3+c}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sqr
t(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)
```

3.330 $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

```
[Out] (-25*d)/(216*c^3*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*Sqrt[c + d*x^3]) + (d*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(96*c^(7/2))
```

Rubi [A] time = 0.0950825, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-25*d)/(216*c^3*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*Sqrt[c + d*x^3]) + (d*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(96*c^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4 \left(8c-dx^3\right) \left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{11cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{\frac{99c^2d^2}{2}-\frac{25}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{108c^4d}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{(11d)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^3} + \frac{d^2\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{11\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c^3} + \frac{d\operatorname{Subs}}{}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Mathematica [C] time = 0.0288534, size = 77, normalized size = 0.77

$$\frac{-dx^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right) - 99dx^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right) - 36c}{864c^3x^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-36*c - d*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] - 99*d*x^
3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(864*c^3*x^3*Sqrt[c + d*x
^3])
```

Maple [C] time = 0.024, size = 549, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/64*d^2/c^2*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((
-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/
2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha
*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-
(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+
1/8/c*(-2/3*d/c^2/((x^3+1/d*c)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/64*d/c^2*(2/3/c/((x^3+1/d*c)*d)^(1/2
)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)
```

Fricas [A] time = 1.37141, size = 629, normalized size = 6.29

$$\left[\frac{\left(d^2x^6+cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+297\left(d^2x^6+cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\left(25\,cdx^3+9\,c^2\right)\sqrt{dx^3+c}}{5184\left(c^4dx^6+c^5x^3\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5184*((d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 2*sq
rt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c)
)/(c^4*d*x^6 + c^5*x^3), -1/2592*(297*(d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d
*x^3 + c)*sqrt(-c)/c) + 12*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6
 + c^5*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1405, size = 128, normalized size = 1.28

$$-\frac{1}{2592}\,d\left(\frac{297\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{12\left(25\,dx^3+9\,c\right)}{\left(\left(dx^3+c\right)^{\frac{3}{2}}-\sqrt{dx^3+c}c\right)c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2592*d*(297*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + arctan(1/3
*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 12*(25*d*x^3 + 9*c)/(((d*x^3 +
c)^(3/2) - sqrt(d*x^3 + c)*c)*c^3))
```

3.331 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{245d^2}{1728c^4\sqrt{c+dx^3}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

```
[Out] (245*d^2)/(1728*c^4*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*Sqrt[c + d*x^3]) + (3*
d)/(64*c^3*x^3*Sqrt[c + d*x^3]) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2)
)
```

Rubi [A] time = 0.123816, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{245d^2}{1728c^4\sqrt{c+dx^3}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (245*d^2)/(1728*c^4*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*Sqrt[c + d*x^3]) + (3*
d)/(64*c^3*x^3*Sqrt[c + d*x^3]) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2)
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$=-\frac{1}{48c^2x^6\sqrt{c+dx^3}}-\frac{\text{Subst}\left(\int \frac{18cd-\frac{5d^2x}{2}}{x^2(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{48c^2}$$

$$=-\frac{1}{48c^2x^6\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}+\frac{\text{Subst}\left(\int \frac{218c^2d^2-27cd^3x}{x(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{384c^4}$$

$$=\frac{245d^2}{1728c^4\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}+\frac{\text{Subst}\left(\int \frac{981c^3d^3-\frac{245}{2}c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx,\right.}{1728c^6d}$$

$$=\frac{245d^2}{1728c^4\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}+\frac{\left(109d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{1536c^4}$$

$$=\frac{245d^2}{1728c^4\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}+\frac{(109d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x\right.}{768c^4}$$

$$=\frac{245d^2}{1728c^4\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2}{}$$

Mathematica [C] time = 0.0355619, size = 91, normalized size = 0.71

$$\frac{-d^2x^6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+981d^2x^6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+36c\left(9dx^3-4c\right)}{6912c^4x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (36*c*(-4*c + 9*d*x^3) - d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3
)/(9*c)] + 981*d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(691
2*c^4*x^6*Sqrt[c + d*x^3])
```

Maple [C] time = 0.029, size = 636, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/512*d^3/c^3*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum(
(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3))
)/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1
/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d
-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*
c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
+1/8/c*(2/3*d^2/c^3/((x^3+1/d*c)*d)^(1/2)-1/6*(d*x^3+c)^(1/2)/c^2/x^6+7/12*
d*(d*x^3+c)^(1/2)/c^3/x^3-5/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2))
+1/64*d/c^2*(-2/3*d/c^2/((x^3+1/d*c)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d
*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/512*d^2/c^3*(2/3/c/((x^3+1/d*c
)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)
```

Fricas [A] time = 1.41284, size = 695, normalized size = 5.43

$$\left[\frac{\left(d^{3}x^{9}+cd^{2}x^{6}\right)\sqrt{c}\log\left(\frac{dx^{3}+6\sqrt{dx^{3}+c}\sqrt{c}+10c}{dx^{3}-8c}\right)+2943\left(d^{3}x^{9}+cd^{2}x^{6}\right)\sqrt{c}\log\left(\frac{dx^{3}-2\sqrt{dx^{3}+c}\sqrt{c}+2c}{x^{3}}\right)+24\left(245cd^{2}x^{6}+81c^{2}dx^{3}-\right.}{41472\left(c^{5}dx^{9}+c^{6}x^{6}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/41472*((d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3
- 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x^3
- 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), 1/20736*(2943*(d^3*x^9 +
c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d^3*x^9 + c*d^2*x
^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(245*c*d^2*x^6 + 8
1*c^2*d*x^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13101, size = 146, normalized size = 1.14

$$\frac{1}{20736}d^{2}\left(\frac{2943\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^{4}}-\frac{\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}c^{4}}+\frac{1536}{\sqrt{dx^{3}+c}c^{4}}+\frac{108\left(13\left(dx^{3}+c\right)^{\frac{3}{2}}-17\sqrt{dx^{3}+c}c\right)}{c^{4}d^{2}x^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/20736*d^2*(2943*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 1536/(sqrt(d*x^3 + c)*c^4) +
 108*(13*(d*x^3 + c)^(3/2) - 17*sqrt(d*x^3 + c)*c)/(c^4*d^2*x^6))
```

3.332 $\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=629

$$-\frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\ldots}}{9\ 3^{3/4}d^{8}\ldots}$$

```
[Out] (2*x^2)/(27*d^2*Sqrt[c + d*x^3]) - (56*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3
) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) + (32*c^(1/6)*ArcTan
h[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d^(8/3)) - (32*
c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) + (28*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (56*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^
(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.724423, antiderivative size = 629, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {470, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\ldots}}{9\ 3^{3/4}d^{8/3}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*d^2*Sqrt[c + d*x^3]) - (56*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3
) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) + (32*c^(1/6)*ArcTan
h[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d^(8/3)) - (32*
c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) + (28*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (56*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^
(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^7}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \frac{x\left(16c^2-14cdx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27cd^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{14cx}{\sqrt{c+dx^3}} - \frac{96c^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27cd^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{28\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27d^2} + \frac{(64c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{9d^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{16\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27d^3} - \frac{28\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{27d^{7/3}} + \frac{\left(16\sqrt[3]{c}\right)\int \frac{1+\cdots}{\left(2-\frac{\sqrt[3]{d}\cdots}{\sqrt[3]{c}}\right)\cdots}}{27d^{7/3}\cdots}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right)\cdots}}}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{\cdots}\right)\cdots\right)\cdots}}}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} + \frac{32\sqrt[6]{c}\tan\cdots}{\cdots}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} + \frac{32\sqrt[6]{c}\tan\cdots}{\cdots}$$

Mathematica [C] time = 0.0771293, size = 127, normalized size = 0.2

$$\frac{x^2\left(7dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20c\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+20c\right)}{270cd^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(20*c - 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 7*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(270*c*d^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.048, size = 1810, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d^2*(d*(-2/3/d*x^2/((x^3+1/d*c)*d)^(1/2)-8/9*I/d^2*3^(1/2)*(-d^2*c)^(1/3
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2))))+8*c*(2/3/c*x^2/((x^3+1/d*c)*d)^(1/2)+2/9*I/c*3^(1/2)/
d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))))-64*c^2/d^2*(-2/27/c^2*x^2/((x^3+1/d*c
)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^
2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
```

```
^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^
2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/243*I/c
^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3
))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2
*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^7}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^7/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.333 $\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=635

$$\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}+\frac{4\tanh^{-1}}{81c}$$

```
[Out] (-2*x^2)/(27*c*d*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(27*c*d^(5/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*c^(5/6)*d^(5/3)) + (4*ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(81*c^(5/6)*d^(5/3)) - (4*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[
3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*
3^(3/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x
)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.730387, antiderivative size = 635, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {471, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}+\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}-\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*x^2)/(27*c*d*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(27*c*d^(5/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*c^(5/6)*d^(5/3)) + (4*ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(81*c^(5/6)*d^(5/3)) - (4*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[
3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*
3^(3/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x
)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(16c-\frac{dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \left(\frac{x}{2\sqrt{c+dx^3}} + \frac{12cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{8\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} - \frac{2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{27cd^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{27cd^{4/3}} + \frac{2\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}}{27c^{2/3}d^{4/3}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{9\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\cdots}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} + \frac{4\tanh^{-1}\left(\cdots\right)}{81c^{5/6}\cdots}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} + \frac{4\tanh^{-1}\left(\cdots\right)}{81c^{5/6}\cdots}$$

Mathematica [C] time = 0.0794521, size = 126, normalized size = 0.2

$$-\frac{x^2\left(dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80c\right)}{1080c^2d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(x^2*(80*c - 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
-((d*x^3)/c), (d*x^3)/(8*c)]))/(1080*c^2*d*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 1346, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d*(2/3/c*x^2/((x^3+1/d*c)*d)^(1/2)+2/9*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2))))-8/d*c*(-2/27/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/
2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha
*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(
1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*
```

```
d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2
*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^4}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^4/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.334 $\int \frac{x}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=632

$$-\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-$$

```
[Out] (2*x^2)/(27*c^2*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(27*c^2*d^(2/3)*((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)) - ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/
3)*x))/Sqrt[c + d*x^3]]/(54*Sqrt[3]*c^(11/6)*d^(2/3)) + ArcTanh[(c^(1/3) +
d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(162*c^(11/6)*d^(2/3)) - ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])]/(162*c^(11/6)*d^(2/3)) + (Sqrt[2 - Sqrt[3]]*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4
)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(5/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rubi [A] time = 0.709147, antiderivative size = 632, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {472, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}}-\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*c^2*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(27*c^2*d^(2/3)*((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)) - ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/
3)*x))/Sqrt[c + d*x^3]]/(54*Sqrt[3]*c^(11/6)*d^(2/3)) + ArcTanh[(c^(1/3) +
d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(162*c^(11/6)*d^(2/3)) - ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])]/(162*c^(11/6)*d^(2/3)) + (Sqrt[2 - Sqrt[3]]*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4
)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(5/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \frac{x\left(\frac{5cd}{2}-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{dx}{2\sqrt{c+dx^3}} - \frac{3cdx}{2(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27c^2d}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27c^2} + \frac{\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9c}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{108c^2d} - \frac{\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{27c^2\sqrt[3]{d}} + \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{108c^{5/3}\sqrt[3]{d}} -$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right.}}}{9\,3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{}}{\left((1+\right.}}}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\right.}{3\sqrt[6]{c}}\right.}{162c^{11/6}d}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\right.}{3\sqrt[6]{c}}\right.}{162c^{11/6}d}$$

Mathematica [C] time = 0.0696329, size = 124, normalized size = 0.2

$$\frac{x^2\left(2dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-25c\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+160c\right)}{2160c^3\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(160*c - 25*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 2*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), (d*x^3)/(8*c)]))/(2160*c^3*Sqrt[c + d*x^3])
```

Maple [C] time = 0.009, size = 875, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/27/c^2*x^2/((x^3+1/d*c)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}x}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.335 $\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=653

$$\frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{}}{\left((1+\sqrt{}\right.}}}{144\ 3^{3/4}c^{8/3}}$$

```
[Out] 2/(27*c^2*x*Sqrt[c + d*x^3]) - (43*Sqrt[c + d*x^3])/(216*c^3*x) + (43*d^(1/
3)*Sqrt[c + d*x^3])/(216*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqr
t[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c
 + d*x^3])])/(1296*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(1296*c^(17/6)) - (43*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(8/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3]) + (43*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3
)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.82577, antiderivative size = 653, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\right.}}}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\right.}{\left(\left(1+\sqrt{3}\right)\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x*Sqrt[c + d*x^3]) - (43*Sqrt[c + d*x^3])/(216*c^3*x) + (43*d^(1/
3)*Sqrt[c + d*x^3])/(216*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqr
t[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c
 + d*x^3])])/(1296*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(1296*c^(17/6)) - (43*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(8/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3]) + (43*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3
)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{43cd}{2}+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \frac{x\left(\frac{175c^2d^2}{2}-\frac{43}{4}cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{108c^4d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \left(\frac{43cd^2x}{4\sqrt{c+dx^3}} + \frac{3c^2d^2x}{2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{108c^4d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{(43d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^3} + \frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{72c^2}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^3} + \frac{\left(43d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}}{432c^3}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\right.}{}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{}\right.}{\sqrt{c+dx^3}}\right.}{432\sqrt{3}c^{17/6}}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{}\right.}{\sqrt{c+dx^3}}\right.}{432\sqrt{3}c^{17/6}}$$

Mathematica [C] time = 0.0814089, size = 140, normalized size = 0.21

$$\frac{-43d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+875cdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(27c+43dx^3\right)}{17280c^4x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(27*c + 43*d*x^3) + 875*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/
2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 43*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17280*c^4*x*Sqrt[c
+ d*x^3])
```

Maple [C] time = 0.023, size = 1361, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-5/9*I/c^2
*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/8*d/c*(-2/27/c^2*x^2/((x^3+1/
d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(
-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)
```

```
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(
-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(
1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/243*
I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(
1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alph
a^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^{11}-6\,cd^2x^8-15\,c^2dx^5-8\,c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^11 - 6*c*d^2*x^8 - 15*c^2*d*x^5 - 8*c^3*x^
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

3.336 $\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=675

$$-\frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\cdots$$

```
[Out] 2/(27*c^2*x^4*Sqrt[c + d*x^3]) - (91*Sqrt[c + d*x^3])/(864*c^3*x^4) + (113*
d*Sqrt[c + d*x^3])/(432*c^4*x) - (113*d^(4/3)*Sqrt[c + d*x^3])/(432*c^4*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3456*Sqrt[3]*c^(23/6)) + (d^(4/3)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(10368*c^(23/6)) -
 (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(23/6)) + (113*Sqr
t[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (113*d^(4/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2
]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.920685, antiderivative size = 675, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}c^{23/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}}-\frac{113d^{4/3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^4*Sqrt[c + d*x^3]) - (91*Sqrt[c + d*x^3])/(864*c^3*x^4) + (113*
d*Sqrt[c + d*x^3])/(432*c^4*x) - (113*d^(4/3)*Sqrt[c + d*x^3])/(432*c^4*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3456*Sqrt[3]*c^(23/6)) + (d^(4/3)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(10368*c^(23/6)) -
 (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(23/6)) + (113*Sqr
t[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (113*d^(4/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2
]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{91cd}{2}+\frac{11d^2x^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{\int \frac{-904c^2d^2+\frac{455}{4}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{432c^4d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \frac{x\left(3610c^3d^3-452c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{3456c^6d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \left(\frac{452c^2d^3x}{\sqrt{c+dx^3}} - \frac{6c^3d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{3456c^6d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\left(113d^2\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{864c^4} + \frac{d^2\int \frac{x}{\left(8c-dx^3\right)\cdots}}{576c\cdots}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{d\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6912c^4} - \frac{(113\cdots}{\cdots}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{113\cdots}{\cdots}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\cdots}{\cdots}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\cdots}{\cdots}$$

Mathematica [C] time = 0.0869576, size = 153, normalized size = 0.23

$$\frac{452d^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-9025cd^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+160c\left(-27c^2+135cdx^3+226\right.}{138240c^5x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (160*c*(-27*c^2 + 135*c*d*x^3 + 226*d^2*x^6) - 9025*c*d^2*x^6*Sqrt[1 + (d*x
^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 452*d^3*x^
9*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(138240*c^5*x^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.026, size = 1864, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64*d/c^2*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-5/9*
I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/64*d^2/c^2*(-2/27/c^2*x^
2/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/
```

```
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*
(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*
(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/
3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^
2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(-1/4*(d*x^3+c)^(1/2)/c^2/x
^4+13/8*d*(d*x^3+c)^(1/2)/c^3/x+2/3*d^2/c^3*x^2/((x^3+1/d*c)*d)^(1/2)+55/72
*I*d/c^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

3.337 $\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=699

$$\frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-9$$

```
[Out] 2/(27*c^2*x^7*Sqrt[c + d*x^3]) - (139*Sqrt[c + d*x^3])/(1512*c^3*x^7) + (60
95*d*Sqrt[c + d*x^3])/(48384*c^4*x^4) - (953*d^2*Sqrt[c + d*x^3])/(3024*c^5
*x) + (953*d^(7/3)*Sqrt[c + d*x^3])/(3024*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
d*x^3]])/(27648*Sqrt[3]*c^(29/6)) + (d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^
2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(82944*c^(29/6)) - (d^(7/3)*ArcTanh[Sqrt[c
+ d*x^3]/(3*Sqrt[c])])/(82944*c^(29/6)) - (953*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2016*3^(
3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (953*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1512*Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3])
```

Rubi [A] time = 1.09814, antiderivative size = 699, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{953d^2\sqrt{c+dx^3}}{3024c^5x}-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}}-\frac{d^{7/3}\tanh^{-1}\left(\ldots\right.}{82944c^{29}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^7*Sqrt[c + d*x^3]) - (139*Sqrt[c + d*x^3])/(1512*c^3*x^7) + (60
95*d*Sqrt[c + d*x^3])/(48384*c^4*x^4) - (953*d^2*Sqrt[c + d*x^3])/(3024*c^5
*x) + (953*d^(7/3)*Sqrt[c + d*x^3])/(3024*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
d*x^3]])/(27648*Sqrt[3]*c^(29/6)) + (d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^
2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(82944*c^(29/6)) - (d^(7/3)*ArcTanh[Sqrt[c
+ d*x^3]/(3*Sqrt[c])])/(82944*c^(29/6)) - (953*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2016*3^(
3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (953*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1512*Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
```

```
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{139cd}{2}+\frac{17d^2x^3}{2}}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{\int \frac{-\frac{6095}{2}c^2d^2+\frac{1529}{4}cd^3x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{756c^4d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{\int \frac{-60992c^3d^3+\frac{30475}{4}c^2d^4x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24192c^6d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{\int \frac{x\left(244010c^4d^4-\right.}{\left(8c-dx^3\right)}}{19353}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{\int \left(\frac{30496c^3d^4x}{\sqrt{c+dx^3}}\right.}{19}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{\left(953d^3\right)\int \frac{}{\sqrt{c+}}}{6048c^5}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} - \frac{d^2\int \frac{2\sqrt[3]{c}d^{2/3}-}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\right.}}{552}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}}{3024c^5\left(\left(1+\right.\right.}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}}{3024c^5\left(\left(1+\right.\right.}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}}{3024c^5\left(\left(1+\right.\right.}$$

Mathematica [C] time = 0.105152, size = 167, normalized size = 0.24

$$\frac{610025cd^3x^9\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(953d^4x^{12}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5c\left(-1647c^2dx^3+864\right.\right.}{7741440c^6x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (610025*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] - 32*(5*c*(864*c^3 - 1647*c^2*d*x^3 + 9153*c*d^2*x^6 + 1
5248*d^3*x^9) + 953*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(7741440*c^6*x^7*Sqrt[c + d*x^3])
```

Maple [C] time = 0.026, size = 2389, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/512*d^2/c^3*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-5
/9*I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/512*d^3/c^3*(-2/27/c^
2*x^2/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
```

```
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)
^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/c
^2/x^7+25/56*d*(d*x^3+c)^(1/2)/c^3/x^4-237/112*d^2*(d*x^3+c)^(1/2)/c^4/x-2/
3*d^3/c^4*x^2/((x^3+1/d*c)*d)^(1/2)-935/1008*I*d^2/c^4*3^(1/2)*(-d^2*c)^(1/
3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2))))+1/64*d/c^2*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*d*(d*x
^3+c)^(1/2)/c^3/x+2/3*d^2/c^3*x^2/((x^3+1/d*c)*d)^(1/2)+55/72*I*d/c^3*3^(1/
2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

3.338 $\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0606034, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.167748, size = 233, normalized size = 3.53

$$\frac{x\left(\frac{64c\left(\frac{256c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-1\right)}{d}+x^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{864c^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] + (64*c*(-1 + (256*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/d)
)/(864*c^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.036, size = 1038, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d*(2/3/c*x/((x^3+1/d*c)*d)^(1/2)-2/9*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-8/d*c*(-2/27/c^2*x/((x^3+1/d*c)*d)^(1/2)+
2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+
1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-
d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)
+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*
c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}x^3}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^3/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.339 $\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0298604, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c^2*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}} = \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.128031, size = 230, normalized size = 3.59

$$\frac{x\left(64\left(\frac{176F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+\frac{1}{c^2}\right)-\frac{dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}\right)}{864\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)])/c^3) + 64*(c^(-2) + (176*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*
x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]
))))))/(864*Sqrt[c + d*x^3])
```

Maple [C] time = 0.005, size = 721, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/27/c^2*x/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
```

```
d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-d
^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)
*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-
d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

3.340 $\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(16*c^2*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0643902, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(16*c^2*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(8c-dx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$=-\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.147944, size = 248, normalized size = 3.76

$$\frac{64c\left(-\frac{7360c^2dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-27c-59dx^3\right)+59d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2}\right.}{27648c^4x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (59*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)] + 64*c*(-27*c - 59*d*x^3 - (7360*c^2*d*x^3*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/
2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c)
, (d*x^3)/(8*c)])))))/(27648*c^4*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.024, size = 1053, normalized size = 16.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-2/3*d/c^2*x/((x^3+1/d*c)*d)^(1/2)-1/2/c^2*(d*x^3+c)^(1/2)/x^2+7/18*
I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(
1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-1/8*d/
c*(-2/27/c^2*x/((x^3+1/d*c)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2
*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(
1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*
d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2
*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^{12}-6\,cd^2x^9-15\,c^2dx^6-8\,c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^12 - 6*c*d^2*x^9 - 15*c^2*d*x^6 - 8*c^3*x^
3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

3.341 $\int \frac{1}{x^6 (8c-dx^3) (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 3/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(40*c^2*x^5*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0637066, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 3/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(40*c^2*x^5*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6\left(8c-dx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.212539, size = 261, normalized size = 3.95

$$\frac{64c\left(\frac{382528c^2d^2x^6F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-432c^2+1269cdx^3+2981d^2x^6\right)-2981d^3x}{1105920c^5x^5\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2981*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 64*c*(-432*c^2 + 1269*c*d*x^3 + 2981*d^2*x^6 + (382528*c^
2*d^2*x^6*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c -
d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*
x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4
/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(1105920*c^5*x^5*Sqrt[c
+ d*x^3])
```

Maple [C] time = 0.028, size = 1402, normalized size = 21.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64*d/c^2*(-2/3*d/c^2*x/((x^3+1/d*c)*d)^(1/2)-1/2/c^2*(d*x^3+c)^(1/2)/x^2+
7/18*I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-1
/64*d^2/c^2*(-2/27/c^2*x/((x^3+1/d*c)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-d^2*c
)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(
1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(
1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c
)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/
3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*
(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c)))+1/8/c*(2/3*d^2/c^3*x/((x^3+1/d*c)*d)^(1/2)-1/5/c^2*(d*x^3+c)^(1
/2)/x^5+17/20/c^3*d*(d*x^3+c)^(1/2)/x^2-91/180*I*d/c^3*3^(1/2)*(-d^2*c)^(1/
3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

3.342 $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=737

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\cdots}}{b^{2/3}\sqrt{\cdots}}$$

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3
)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a +
 b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.293567, antiderivative size = 737, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3
)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a +
 b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1\cdots\right.\right.}{\cdots}\right.}{\sqrt{2}b^{2/3}}$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+b\cdots}}{\sqrt{2}3^{3/4}\sqrt{a}}\right.}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

Mathematica [C] time = 0.0847857, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a + 6*Sqrt[3]*a))])/((20 + 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [C] time = 0.166, size = 977, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/
_alpha*(2*3^(1/2)+3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(-a*b^2)^
(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*
(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*(I*3^(1/
2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(-
3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(-a*b^2)^(2/
3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-6*
b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a*b^2)^(1/3)*_
alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(2*
```

```
I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-4*I*(-a
*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b+2*I*(-
a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*b-3*a*b)/a,
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{bx^3 + 2\,a\left(3\,\sqrt{3} + 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a + bx^3}}{10a + 6\sqrt{3}a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5+3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.343 $\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=757

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\ldots}}{b^{2/3}\ldots}$$

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*
x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a
 - b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.287517, antiderivative size = 757, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\ldots}}{b^{2/3}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*
x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a
 - b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a-bx^3}}\,dx$$

$$= \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right.}{\sqrt{2}}\right)}{2\sqrt{2}b^{2/3}}$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

Mathematica [C] time = 0.0856198, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a + 6*Sqrt[3]*a)])/((20 + 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [C] time = 0.261, size = 924, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)+3)*
(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(
a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a
*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/
3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2
)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3
)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3
)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1
```

```
/2)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(a*b^2)^(2/
3)*_alpha+I*3^(1/2)*a*b-2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(a*b^2)^(2
/3)*_alpha-2*I*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*a*3^(1/2)-
10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-bx^3+ax}}{bx^3-2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{a-bx^3}}{-6\sqrt{3}a-10a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5+3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(a - b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.344 $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=774

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\cdots}}{b^{2/3}\sqrt{\cdots}}$$

```
[Out] (-2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(S
qrt[2]*Sqrt[-a + b*x^3])]])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*
x^3])]])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])]])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.280537, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 304, 219, 1879, 488}

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{-\frac{\sqrt[3]{\cdots}}{\left(\left(1-\cdots\right.\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (-2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(S
qrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*
x^3])])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx + \int \frac{x}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\tan$$

$$= -\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}(}{}\right.}{\sqrt{2}b^{2}}$$

Mathematica [C] time = 0.0638605, size = 87, normalized size = 0.11

$$-\frac{x^2\sqrt{bx^3-a}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{1-\frac{bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a
 + 6*Sqrt[3]*a)])/(4*(5 + 3*Sqrt[3])*a*Sqrt[1 - (b*x^3)/a])
```

Maple [C] time = 0.068, size = 926, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)+3)*(
a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a
*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*
b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3
)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*
b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*
_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+
6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2
```

```
)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(a*b^2)^(2/3)
*_alpha+I*3^(1/2)*a*b-2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(a*b^2)^(2/3
)*_alpha-2*I*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*a*3^(1/2)-10
*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 - a}x}{bx^3 - 2a(3\sqrt{3} + 5)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-a + bx^3}}{-6\sqrt{3}a - 10a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3-a)**(1/2)/(b*x**3-2*a*(5+3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(-a + b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.345 $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=768

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\cdots}}{b^{2/3}\sqrt{\cdots}}$$

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/
(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^
3])])/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.283459, antiderivative size = 768, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {489, 304, 219, 1879, 488}

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\cdots}{\left(\left(1-\cdots\right.\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/
(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^
3])])/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx + \int \frac{x}{\sqrt{-a-bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\tan \ldots}{\ldots}$$

$$= -\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}(1 \ldots}{\ldots}\right)}{2\sqrt{2}b \ldots}$$

Mathematica [C] time = 0.0542953, size = 90, normalized size = 0.12

$$-\frac{x^2\sqrt{-a-bx^3}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{\frac{a+bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/
(10*a + 6*Sqrt[3]*a))])/(4*(5 + 3*Sqrt[3])*a*Sqrt[(a + b*x^3)/a])
```

Maple [C] time = 0.066, size = 983, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1
/_alpha*(2*3^(1/2)+3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(-a*b^2)
^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3
*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*
(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(-a*b^2)^(
2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-
6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a*b^2)^(1/3)
*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(
```

```
2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-4*I*(
-a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b+2*I*
(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*b-3*a*b)/
a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-bx^3 - ax}}{bx^3 + 2\,a\left(3\,\sqrt{3} + 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{-a - bx^3}}{10a + 6\sqrt{3}a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(-b*x**3-2*a*(5+3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(-a - b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.346 $\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=738

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\cdots}}{b^{2/3}\cdots}$$

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(S
qrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]
)/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.250974, antiderivative size = 738, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{bx}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(S
qrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]
)/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}-\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{3^{3/4}\sqrt[6]{a}\tan}{}$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}-\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3}}{\sqrt{2}}\right)}{2\sqrt{2}b^{2/3}}$$

Mathematica [C] time = 0.0904819, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20 - 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [C] time = 0.145, size = 977, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/
_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(-a*b^2)^
(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*
(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*(I*3^(1/
2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(3
*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(-a*b^2)^(2/3
)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+6*b
^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-3*(-a*b^2)^(1/3)*_a
lpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(2*I
```

```
*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+4*I*(-a*
b^2)^(1/3)*_alpha^2*b-2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-2*I*(-a
*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-3*a*b)/a,(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{bx^3 - 2\,a\left(3\,\sqrt{3} - 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a + bx^3}}{-6\sqrt{3}a + 10a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.347 $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=758

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\cdots}}{b^{2/3}\cdots}$$

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3]
)])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rubi [A] time = 0.236822, antiderivative size = 758, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])]])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3]
)])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])]])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}+\frac{3^{3/4}\sqrt[6]{a}\tan}{}$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\right.\right.}{\sqrt{2}}\right.}{\sqrt{2}b^{2/3}}$$

Mathematica [C] time = 0.0779448, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20 - 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [C] time = 0.134, size = 924, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*
(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(
a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a
*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/
3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/
2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/
3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b+6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/
3)+6*I*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(
```

```
1/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1/3)*_alpha^2*b-2*3^(1/2)*(a*b^2)^(2
/3)*_alpha+I*3^(1/2)*a*b+2*I*(a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(a*b^2)^(
2/3)*_alpha+2*I*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)
-10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-bx^3+ax}}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{a-bx^3}}{-10a+6\sqrt{3}a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(a - b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.348 $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=774

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\cdots}}{b^{2/3}\sqrt{-\cdots}}$$

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sq
rt[-a + b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2
/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rubi [A] time = 0.236778, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {489, 304, 219, 1879, 488}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\cdots\right)}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a + b*x^3])]])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])]])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sq
rt[-a + b*x^3])]])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2
/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx-\int \frac{x}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\cdots}{\cdots}\right)}{2\sqrt{2}b^{2\cdots}}$$

$$= \frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+b\cdots}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

Mathematica [C] time = 0.0493932, size = 89, normalized size = 0.11

$$-\frac{x^2\sqrt{bx^3-a}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a-10a}\right)}{4\left(3\sqrt{3}-5\right)a\sqrt{\frac{a-bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, -((b*x^3)/(-1
0*a + 6*Sqrt[3]*a))]])/(4*(-5 + 3*Sqrt[3])*a*Sqrt[(a - b*x^3)/a])
```

Maple [C] time = 0.067, size = 926, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)
^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*
(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(
a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a
*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/
3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2
)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3
)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b+6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3
)+6*I*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1
```

```
/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1/3)*_alpha^2*b-2*3^(1/2)*(a*b^2)^(2/
3)*_alpha+I*3^(1/2)*a*b+2*I*(a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(a*b^2)^(2
/3)*_alpha+2*I*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)-
10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{bx^3 - a}x}{bx^3 + 2a\left(3\sqrt{3} - 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
```

```
[Out] -integrate(sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{-a + bx^3}}{-10a + 6\sqrt{3}a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3-a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(-a + b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.349 $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=768

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),4\sqrt{3}-7\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\ldots}}{b^{2/3}\sqrt{-\ldots}}$$

```
[Out] (2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(
1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt
[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rubi [A] time = 0.230564, antiderivative size = 768, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {489, 304, 219, 1879, 488}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\ldots}}{b^{2/3}\sqrt{-\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(
1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt
[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx - \int \frac{x}{\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\cdots\right.}{\sqrt{2}b^{2\ldots}}$$

$$= \frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-b\cdots}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

Mathematica [C] time = 0.0709835, size = 89, normalized size = 0.12

$$-\frac{x^2\sqrt{-a-bx^3}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{4\left(3\sqrt{3}-5\right)a\sqrt{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/
(10*a - 6*Sqrt[3]*a))])/(4*(-5 + 3*Sqrt[3])*a*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.065, size = 983, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/
_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(-a*b^2)^
(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*
(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*(I*3^(1/
2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*(
3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(-a*b^2)^(2/
3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+6*
b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-3*(-a*b^2)^(1/3)*_
alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(2*
```

```
I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+4*I*(-a
*b^2)^(1/3)*_alpha^2*b-2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-2*I*(-
a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-3*a*b)/a,
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^3 - ax}}{bx^3 - 2a\left(3\sqrt{3} - 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-a - bx^3}}{-6\sqrt{3}a + 10a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(-a - b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.350 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=318

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-$$

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/
6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[
3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0638311, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```


```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/
6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[
3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```


Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2-$$

Mathematica [C] time = 0.0674687, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Maple [C] time = 0.065, size = 538, normalized size = 1.7

$$\frac{-\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3+6a\sqrt{3}+10a\right)}\frac{1}{\_alpha}\sqrt[3]{-ab^2}\sqrt{\frac{i}{2}b\left(2x+\frac{1}{b}\left(-i\sqrt{3}\sqrt[3]{-ab^2}+\sqrt[3]{-ab^2}\right)\right)\frac{1}{\sqrt[3]{-ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{-ab^2}\right)\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(
1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^
2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*
x+1/b*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*
I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/
3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-
a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*
_alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1
/2)*a*b+2*I*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I
*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3 + 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^3}\left(10a+6\sqrt{3}a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5+3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.351 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=324

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-$$

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 -
Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt
[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1
+ Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0696075, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 -
Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt
[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1
+ Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\right.}{}$$

Mathematica [C] time = 0.0732454, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Maple [C] time = 0.063, size = 509, normalized size = 1.6

$$\frac{\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3-6a\sqrt{3}-10a\right)}\frac{1}{\_alpha}\sqrt[3]{ab^2}\sqrt{-\frac{i}{2}b\left(2x+\frac{1}{b}\left(i\sqrt{3}\sqrt[3]{ab^2}+\sqrt[3]{ab^2}\right)\right)\frac{1}{\sqrt[3]{ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{ab^2}\right)\left(-3\sqrt[3]{ab^2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/
2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/
3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I
*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2
)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2
/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b
^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alph
a*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/
2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3
)*_alpha^2*b+2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-2*I*(a*b^2)^(2/3)
*_alpha-2*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha-2*I*a*b+3*a*b)/a,(-I*3^(1/2)/b
*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
,_alpha=RootOf(b*_Z^3-6*a*3^(1/2)-10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-6\sqrt{3}\,a\sqrt{a-bx^3}-10a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5+3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.352 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=328

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0667759, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```


```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```


Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}\right.}{\sqrt{2}\sqrt{-a+bx^3}}\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.0707273, size = 85, normalized size = 0.26

$$-\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))
```

Maple [C] time = 0.06, size = 510, normalized size = 1.6

$$\frac{-\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3-6a\sqrt{3}-10a\right)}\frac{1}{\_alpha}\sqrt[3]{ab^2}\sqrt{-\frac{i}{2}b\left(2x+\frac{1}{b}\left(i\sqrt{3}\sqrt[3]{ab^2}+\sqrt[3]{ab^2}\right)\right)\frac{1}{\sqrt[3]{ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{ab^2}\right)\left(-3\sqrt[3]{ab^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2
/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b
^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alph
a*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/
2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3
)*_alpha^2*b+2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-2*I*(a*b^2)^(2/3)
*_alpha-2*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha-2*I*a*b+3*a*b)/a,(-I*3^(1/2)/b
*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
,_alpha=RootOf(b*_Z^3-6*a*3^(1/2)-10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-6\sqrt{3}a-10a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3-2*a*(5+3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.353 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=330

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x
))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0666485, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```


```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x
))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```


Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\sqrt{2}\sqrt{-a-bx^3}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.0671752, size = 87, normalized size = 0.26

$$-\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))
```

Maple [C] time = 0.059, size = 541, normalized size = 1.6

$$\frac{\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3+6a\sqrt{3}+10a\right)}\frac{1}{\_alpha}\sqrt[3]{-ab^2}\sqrt{\frac{i}{2}b\left(2x+\frac{1}{b}\left(-i\sqrt{3}\sqrt[3]{-ab^2}+\sqrt[3]{-ab^2}\right)\right)\frac{1}{\sqrt[3]{-ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{-ab^2}\right)\left(-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(1
/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x
^3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*
I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/
3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-
a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*
_alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1
/2)*a*b+2*I*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I
*a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(bx^3 + 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{10a\sqrt{-a-bx^3}+6\sqrt{3}a\sqrt{-a-bx^3}+bx^3\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3-2*a*(5+3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x**
3*sqrt(-a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.354 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=310

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\right.}{\sqrt{2}\sqrt{a+bx^3}}\right.}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*
x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sq
rt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0509157, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\right.}{\sqrt{2}\sqrt{a+bx^3}}\right.}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*
x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sq
rt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.0903689, size = 83, normalized size = 0.27

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Maple [C] time = 0.063, size = 538, normalized size = 1.7

$$\frac{\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3-6a\sqrt{3}+10a\right)}\frac{1}{\_alpha}\sqrt[3]{-ab^2}\sqrt{\frac{i}{2}b\left(2x+\frac{1}{b}\left(-i\sqrt{3}\sqrt[3]{-ab^2}+\sqrt[3]{-ab^2}\right)\right)\frac{1}{\sqrt[3]{-ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{-ab^2}\right)\left(-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(1
/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*
(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)
*_alpha*b+6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-3*(-a*
b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_a
lpha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2
)*a*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a
*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} - 5\right)\right)\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.355 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\right.}{\sqrt{2}\sqrt{a-bx^3}}\right.}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sq
rt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0544255, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\right.}{\sqrt{2}\sqrt{a-bx^3}}\right.}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sq
rt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.0728146, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Maple [C] time = 0.063, size = 509, normalized size = 1.6

$$\frac{-\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3+6a\sqrt{3}-10a\right)}\frac{1}{\_alpha}\sqrt[3]{ab^2}\sqrt{-\frac{i}{2}b\left(2x+\frac{1}{b}\left(i\sqrt{3}\sqrt[3]{ab^2}+\sqrt[3]{ab^2}\right)\right)\frac{1}{\sqrt[3]{ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{ab^2}\right)\left(-3\sqrt[3]{ab^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/
2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^
(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b+6
*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_al
pha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(
1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1
/3)*_alpha^2*b-2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b+2*I*(a*b^2)^(2/
3)*_alpha+2*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+3*a*b)/a,(-I*3^(1/2)
/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2
)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)-10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-10a\sqrt{a-bx^3}+6\sqrt{3}a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

3.356 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=320

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\ldots$$

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 +
 Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0532115, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {488}

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```


```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 +
 Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```


Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{(2}{}$$

Mathematica [C] time = 0.0658154, size = 84, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3])
```

Maple [C] time = 0.06, size = 510, normalized size = 1.6

$$\frac{-\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3+6a\sqrt{3}-10a\right)}\frac{1}{\_alpha}\sqrt[3]{ab^2}\sqrt{-\frac{i}{2}b\left(2x+\frac{1}{b}\left(i\sqrt{3}\sqrt[3]{ab^2}+\sqrt[3]{ab^2}\right)\right)\frac{1}{\sqrt[3]{ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{ab^2}\right)\left(-3\sqrt[3]{ab^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(
2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b+6*
b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alp
ha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1
/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1/
3)*_alpha^2*b-2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b+2*I*(a*b^2)^(2/3
)*_alpha+2*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+3*a*b)/a,(-I*3^(1/2)/
b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)
),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)-10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(bx^3 + 2a\left(3\sqrt{3} - 5\right)\right)\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)
```

Fricas [B] time = 90.6997, size = 12766, normalized size = 39.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(1/3*(3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3
```

- sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 - 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - sqrt(3)*(b*x^4 - a*x) + 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 - a*x)) - 1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(1/3*(3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 - 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - sqrt(3)*(b*x^4 - a*x) - 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 23

40)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3
 + 8*a^4) - 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896
*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x
^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/
6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - s
qrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2)
)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 -
 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*
a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4
))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*
a^4)))/(b*x^4 - a*x)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^
4))^(1/6)*log((b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*x^3 +
64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*a^6*b^
4*x^4 + 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^7 + 56
1*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) +
6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5
*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16
*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 64*sqrt(3)*(a*b^3
*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944
)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2
131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 23
40)/(a^5*b^4))^(5/6) + 4*sqrt(1/6)*(168*a^3*b^5*x^9 + 1845*a^4*b^4*x^6 + 13
68*a^5*b^3*x^3 + 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 + 1065*a^4*b^4*x^6 +
 792*a^5*b^3*x^3 + 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3
(1/1944)^(1/6)(5*a*b^4*x^10 + 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 + 64*a^4*
b*x - 3*sqrt(3)*(a*b^4*x^10 + 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqr
t(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6
+ 320*a^3*b*x^3 + 64*a^4)) - 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(1/6)*log((b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*x
^3 + 64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*a
^6*b^4*x^4 + 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^7
 + 561*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/
3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 6
4*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5
 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 64*sqrt(3)*(
a*b^3*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) - 2*sqrt(b*x^3 - a)*(1944*(1
/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(
3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3)
 + 2340)/(a^5*b^4))^(5/6) + 4*sqrt(1/6)*(168*a^3*b^5*x^9 + 1845*a^4*b^4*x^6
 + 1368*a^5*b^3*x^3 + 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 + 1065*a^4*b^4*
x^6 + 792*a^5*b^3*x^3 + 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)
) + 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 + 64
*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(135
1*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2
*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 234

```
0)/(a^5*b^4))^(1/6)*log((b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a
^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 -
 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^
5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b
^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a
^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32
*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(b*x^3
- a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^
4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 511
2*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 +
2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2
340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*
b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x
^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40
*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + 1/24*(1/1944)^(1/
6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 - 100*a*b^3*x^9 +
 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^
10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(22
3*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1
351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498
*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11
 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3
+ 8*a^4) - 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*
a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^
5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6
)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sq
rt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))
*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 -
12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a
^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4)
)^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a
^4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-10a\sqrt{-a+bx^3}+6\sqrt{3}a\sqrt{-a+bx^3}+bx^3\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x*
*3*sqrt(-a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.357 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=322

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\cdots$$

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rubi [A] time = 0.0535043, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {488}

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```


```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```


Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - -$$

Mathematica [C] time = 0.0706998, size = 86, normalized size = 0.27

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])
```

Maple [C] time = 0.063, size = 541, normalized size = 1.7

$$\frac{\frac{i}{27}\sqrt{2}}{ab^3}\sum_{\_alpha=RootOf\left(b\_Z^3-6a\sqrt{3}+10a\right)}\frac{1}{\_alpha}\sqrt[3]{-ab^2}\sqrt{\frac{i}{2}b\left(2x+\frac{1}{b}\left(-i\sqrt{3}\sqrt[3]{-ab^2}+\sqrt[3]{-ab^2}\right)\right)\frac{1}{\sqrt[3]{-ab^2}}}\sqrt{b\left(x-\frac{1}{b}\sqrt[3]{-ab^2}\right)\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*(-I*3^(1
/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x
^3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I
*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3
)*_alpha*b+6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-3*(-a
*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_
alpha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/
2)*a*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*
a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*a*3^(1/2)+10*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} - 5\right)\right)\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 - a)), x)
```

Fricas [B] time = 79.134, size = 12785, normalized size = 39.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(-1/3*(3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b
```

^3 - sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 + 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + sqrt(3)*(b*x^4 + a*x) + 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(-1/3*(3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 + 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + sqrt(3)*(b*x^4 + a*x) - 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(

3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a

```
^4)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log(-(b
^4*x^12 - 68*a*b^3*x^9 + 168*a^2*b^2*x^6 + 544*a^3*b*x^3 + 64*a^4 + 6*(1/9)
^(2/3)*(2799*a^4*b^6*x^10 - 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 - 1440*a^7
*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 - 834*a^5*b^5*x^7 + 561*a^6*b^4*x^4 -
104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(2
6*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqr
t(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(
-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 64*sqrt(3)*(a*b^3*x^9 + 3*a^2*b^2
*x^6 + 3*a^3*b*x^3 + a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a
^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8
 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^
(5/6) - 4*sqrt(1/6)*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b^3*x^3
- 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5*b^3*x^
3 - 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3*(1/1944)^(1/6)
*(5*a*b^4*x^10 - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3*sqrt(3)
*(a*b^4*x^10 - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3
 + 64*a^4)) - 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*
log(-(b^4*x^12 - 68*a*b^3*x^9 + 168*a^2*b^2*x^6 + 544*a^3*b*x^3 + 64*a^4 +
6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 - 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 - 1
440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 - 834*a^5*b^5*x^7 + 561*a^6*b^4
*x^4 - 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(
1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2
- 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*
x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 64*sqrt(3)*(a*b^3*x^9 + 3*
a^2*b^2*x^6 + 3*a^3*b*x^3 + a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*
(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*
b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) - 4*sqrt(1/6)*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b
^3*x^3 - 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5
*b^3*x^3 - 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3*(1/1944
)^(1/6)*(5*a*b^4*x^10 - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3*
sqrt(3)*(a*b^4*x^10 - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqrt(3) + 2
340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^
3*b*x^3 + 64*a^4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.358 $\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=125

$$\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2a^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}$$

```
[Out] (2*a^2*Sqrt[c + d*x^3])/(3*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(9*b^2*
d^2) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rubi [A] time = 0.129485, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2a^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*a^2*Sqrt[c + d*x^3])/(3*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(9*b^2*
d^2) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt{c+dx}}{b^2d}+\frac{a^2\sqrt{c+dx}}{b^2(a+bx)}+\frac{(c+dx)^{3/2}}{bd}\right)dx,x,x^3\right)\\
&= -\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)}{3b^2}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{\left(a^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x\right.}{3b^3}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{\left(2a^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{\right.}{3b^3d}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}-\frac{2a^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.265231, size = 121, normalized size = 0.97

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd\left(c+dx^3\right)+b^2\left(-2c^2+cdx^3+3d^2x^6\right)\right)}{45b^3d^2}-\frac{2a^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x^
3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Maple [C] time = 0.054, size = 514, normalized size = 4.1

$$\frac{1}{b^2}\left(b\left(\frac{2x^6}{15}\sqrt{dx^3+c}+\frac{2cx^3}{45d}\sqrt{dx^3+c}-\frac{4c^2}{45d^2}\sqrt{dx^3+c}\right)-\frac{2a}{9d}\left(dx^3+c\right)^{\frac{3}{2}}\right)+\frac{a^2}{b^2}\left(\frac{2}{3b}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{bd^2}\sum_{\_alpha=RootOf(b\_Z}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 1/b^2*(b*(2/15*x^6*(d*x^3+c)^(1/2)+2/45/d*c*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d
*x^3+c)^(1/2)/d^2)-2/9*a/d*(d*x^3+c)^(3/2))+a^2/b^2*(2/3*(d*x^3+c)^(1/2)/b+
1/3*I/b/d^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(
-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2
-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36972, size = 603, normalized size = 4.82

$$\left[\frac{15\,a^2d^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+\left(b^2cd-5\,abd^2\right)x^3\right)\sqrt{dx^3+c}}{45\,b^3d^2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*c^
2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*s
qrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*c*d +
 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d^2)]
```

Sympy [A] time = 50.9361, size = 128, normalized size = 1.02

$$\frac{2\left(\frac{a^2d^3\sqrt{c+dx^3}}{3b^3}-\frac{a^2d^3(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}\left(-ad^2-bcd\right)}{9b^2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(a**2*d**3*sqrt(c + d*x**3)/(3*b**3) - a**2*d**3*(a*d - b*c)*atan(sqrt(c
+ d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + d*(c + d*x**3
)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b**2))/d**3
```

Giac [A] time = 1.13744, size = 188, normalized size = 1.5

$$\frac{2\left(a^{2}bc-a^{3}d\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\sqrt{-b^{2}c+abd}\,b^{3}}+\frac{2\left(3\left(dx^{3}+c\right)^{\frac{5}{2}}b^{4}d^{8}-5\left(dx^{3}+c\right)^{\frac{3}{2}}b^{4}cd^{8}-5\left(dx^{3}+c\right)^{\frac{3}{2}}ab^{3}d^{9}+15\sqrt{dx^{3}+c}\,a^{2}b^{2}d^{10}\right)}{45\,b^{5}d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^8 - 5*(d*x^3 + c)^(3
/2)*b^4*c*d^8 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^3 + c)*a^2*b^2*
d^10)/(b^5*d^10)
```

3.359 $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=93

$$-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}$$

```
[Out] (-2*a*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b*d) + (2*a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Rubi [A] time = 0.0780381, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (-2*a*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b*d) + (2*a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)\\
&= \frac{2\left(c+dx^3\right)^{3/2}}{9bd}-\frac{a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)}{3b}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}-\frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}-\frac{(2a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}+\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0728928, size = 88, normalized size = 0.95

$$\frac{2\sqrt{c+dx^3}\left(b\left(c+dx^3\right)-3ad\right)}{9b^2d}+\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-3*a*d + b*(c + d*x^3)))/(9*b^2*d) + (2*a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Maple [C] time = 0.008, size = 458, normalized size = 4.9

$$\frac{2}{9\,bd}\left(dx^3+c\right)^{\frac{3}{2}}-\frac{a}{b}\left(\frac{2}{3\,b}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{bd^2}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d-a/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum(
(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3))
)/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1
/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d
-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*
c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33092, size = 427, normalized size = 4.59

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(bdx^3+bc-3\,ad\right)\sqrt{dx^3+c}}{9\,b^2d},\ \frac{2\left(3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)\right.}{9\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
 c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b*d*x^3 + b*c - 3*a*d)*sqrt(d*
x^3 + c))/(b^2*d), 2/9*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*
b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^3 +
c))/(b^2*d)]
```

Sympy [A] time = 15.5837, size = 95, normalized size = 1.02

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{3}{2}}}{9b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**3)**(3
```

```
/2)/(9*b))/d**2
```

Giac [A] time = 1.10186, size = 130, normalized size = 1.4

$$-\frac{2\left(\frac{3\left(abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}-\frac{\left(dx^3+c\right)^{\frac{3}{2}}b^2-3\sqrt{dx^3+c}abd}{b^3}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/9*(3*(a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/
(sqrt(-b^2*c + a*b*d)*b^2) - ((d*x^3 + c)^(3/2)*b^2 - 3*sqrt(d*x^3 + c)*a*b
*d)/b^3)/d
```

3.360 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2))
```

Rubi [A] time = 0.0585363, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}\\
&= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(2(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}\\
&= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0303183, size = 70, normalized size = 1.

$$
\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b} - \frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] ((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[b*c - a*d]])/b^(3/2))/3
```

Maple [C] time = 0.006, size = 434, normalized size = 6.2

$$\frac{2}{3b}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{bd^2}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x
-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c
)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)
/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29459, size = 331, normalized size = 4.73

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}}{3\,b},-\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^3+c}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c))/b, -2/3*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^3 + c))/b]
```

Sympy [A] time = 5.5318, size = 68, normalized size = 0.97

$$\frac{2\left(\frac{d\sqrt{c+dx^3}}{3b}-\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d
 - b*c)/b))/(3*b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A] time = 1.10274, size = 89, normalized size = 1.27

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b}+\frac{2\sqrt{dx^3+c}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b
```

3.361 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

```
[Out] (-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a) + (2*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b])
```

Rubi [A] time = 0.0744782, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a) + (2*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^3\right)\\
&= \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\\
&= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad}\\
&= -\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.0433693, size = 81, normalized size = 0.95

$$
\frac{2\left(\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (2*(-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*a)
```

Maple [C] time = 0.01, size = 476, normalized size = 5.6

$$-\frac{b}{a}\left(\frac{2}{3b}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{bd^2}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a),x)
```

```
[Out] -b/a*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-
3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(2/3*(d*x^3+
c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)
```

Fricas [A] time = 1.46332, size = 851, normalized size = 10.01

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,a},\frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{c}\log\left(\frac{dx^3}{\cdots}\right.}{3\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*
sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
c)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
3 + c)*sqrt(-c)/c))/a]
```

Sympy [A] time = 10.4533, size = 85, normalized size = 1.

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/b)))/d
```

Giac [A] time = 1.1257, size = 117, normalized size = 1.38

$$-\frac{2}{3}\,d\left(\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*d*((b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a*d) - c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.362 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=115

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}-\frac{\sqrt{c+dx^3}}{3ax^3}$$

```
[Out] -Sqrt[c + d*x^3]/(3*a*x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(3*a^2*Sqrt[c]) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2)
```

Rubi [A] time = 0.124872, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}-\frac{\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*a*x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(3*a^2*Sqrt[c]) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
```

```
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\\
&= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(b(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2}-\frac{(2bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2}\\
&= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(2b(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d}-\frac{(2bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x\right.}{3a^2d}\\
&= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}
\end{aligned}
$$

Mathematica [A] time = 0.120621, size = 107, normalized size = 0.93

$$\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}-2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)-\frac{a\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^3])/x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*a^2)
```

Maple [C] time = 0.01, size = 518, normalized size = 4.5

$$\frac{b^2}{a^2}\left(\frac{2}{3b}\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{bd^2}\sum_{\_alpha=RootOf\left(b\_Z^3+a\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x)
```

```
[Out] b^2/a^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*
c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(-1/3*(d*
x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-1/a^2*b*(2
/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{(bx^3 + a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)
```

Fricas [A] time = 1.54522, size = 1160, normalized size = 10.09

$$\left[\frac{2\sqrt{b^2c - abd}\,cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right) - (2\,bc - ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - 2\,\sqrt{dx^3 + c}ac}{6\,a^2cx^3}, \frac{4\,\sqrt{-b^2c + \cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3
 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c
*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), -1/3*(
(2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sqrt(-b^2*c
 + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(d*
x^3 + c)*a*c)/(a^2*c*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^3}}{x^4\left(a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)
```

Giac [A] time = 1.10951, size = 163, normalized size = 1.42

$$\frac{1}{3}\,d^2\left(\frac{2\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2d^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,d^2}-\frac{\sqrt{dx^3+c}}{ad^2x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*d^2*(2*(b^2*c - a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
sqrt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-
c))/(a^2*sqrt(-c)*d^2) - sqrt(d*x^3 + c)/(a*d^2*x^3))
```

3.363 $\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0532768, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.414009, size = 241, normalized size = 3.77

$$\frac{x\left(8\left(\frac{8a^2c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+c+dx^3\right)+\frac{x^3\sqrt{\frac{dx^3}{c}+1}(3bc-5ad)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3}\right.}{a}\right.}{20b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x*(((3*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)])/a + 8*(c + d*x^3 + (8*a^2*c^2*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))))/(20*b*Sqrt[c + d*x^3])
```

Maple [C] time = 0.029, size = 1012, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 1/b*(2/5*x*(d*x^3+c)^(1/2)-2/5*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2))-a/b*(-2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/
3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/
(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3
)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x^3}{bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{c + dx^3}}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x^3}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

3.364 $\int \frac{x\sqrt{c+dx^3}}{a+bx^3} dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0373927, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [A] time = 0.0274026, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{\frac{c+dx^3}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)
])/(2*a*Sqrt[(c + d*x^3)/c])
```

Maple [C] time = 0.03, size = 857, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] -2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
```

```
/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/b/d^2*2^(1/2)*sum(
1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/
3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)
*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2
*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+cx}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+cx}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

3.365 $\int \frac{\sqrt{c+dx^3}}{a+bx^3} dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0258554, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.151536, size = 161, normalized size = 2.73

$$\frac{8acx\sqrt{c+dx^3}F_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)\left(3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-2bcF_1\left(\frac{4}{3};-\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)+8acF_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (8*a*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3
)/a)])/((a + b*x^3)*(8*a*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(-2*b*c*AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] time = 0.006, size = 705, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] -2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
```

```
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*
I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(
1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alph
a^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

3.366 $\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rubi [A] time = 0.0520184, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2\left(a+bx^3\right)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.100876, size = 139, normalized size = 2.24

$$\frac{2bdx^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5x^3\sqrt{\frac{dx^3}{c}+1}(3ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20a\left(c+dx^3\right)}{20a^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + 3*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*x*Sqrt[c +
 d*x^3])
```

Maple [C] time = 0.01, size = 1314, normalized size = 21.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x)
```

```
[Out] 1/a*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-
d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
```

```
*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-
d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-b/a*(-
2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/b/d^2*2^(1/2)*sum(1
/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c
)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3
)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(
1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/
3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*
_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*
I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/
2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{(bx^3 + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

3.367 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)])
/(2*a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0518778, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)])
/(2*a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3\left(a+bx^3\right)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.246733, size = 335, normalized size = 5.23

$$\frac{\frac{a\left(32ac\left(2ac-adx^3+6bcx^3+2bdx^6\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}-bdx^6\sqrt{\frac{dx^3}{c}+1}}{16a^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (a*(32*a*c*(2*a*c + 6*b*c*x^3 - a*d*x^3 + 2*b*d*x^6)*AppellF1
[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(a + b*x^3)*(c + d*
x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*
c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*App
ellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2
, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.008, size = 1010, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x)
```

```
[Out] 1/a*(-1/2/x^2*(d*x^3+c)^(1/2)-1/2*I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)))-b/a*(-2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-
d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)
*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^3}}{x^3 \left(a + bx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

3.368 $\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=154

$$\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}+\frac{2a^2\sqrt{c+dx^3}(bc-ad)}{3b^4}-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}-\frac{2\left(c+dx^3\right)^{5/2}(ad+bc)}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/}}{21bd^2}$$

```
[Out] (2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^4) + (2*a^2*(c + d*x^3)^(3/2))/(9*
b^3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(15*b^2*d^2) + (2*(c + d*x^3)^(7/2
))/(21*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rubi [A] time = 0.15477, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}+\frac{2a^2\sqrt{c+dx^3}(bc-ad)}{3b^4}-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}-\frac{2\left(c+dx^3\right)^{5/2}(ad+bc)}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/}}{21bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```



```
[Out] (2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^4) + (2*a^2*(c + d*x^3)^(3/2))/(9*
b^3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(15*b^2*d^2) + (2*(c + d*x^3)^(7/2
))/(21*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/(3*b^(9/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{(-bc-ad)(c+dx)^{3/2}}{b^2d} + \frac{a^2(c+dx)^{3/2}}{b^2(a+bx)} + \frac{(c+dx)^{5/2}}{bd}\right)dx, x, x^3\right)$$
$$= -\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/2}}{21bd^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{3b^2}$$
$$= \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/2}}{21bd^2} + \frac{\left(a^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x\right.}{3b^3}$$
$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/2}}{21bd^2} + \frac{\left(a^2(bc-\right.}{}$$
$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/2}}{21bd^2} + \frac{\left(2a^2(bc-\right.}{}$$
$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2} + \frac{2\left(c+dx^3\right)^{7/2}}{21bd^2} - \frac{2a^2(bc-}{}$$

Mathematica [A] time = 0.151854, size = 145, normalized size = 0.94

$$\frac{2\left(105a^2(bc-ad)\left(\frac{\sqrt{c+dx^3}}{b} - \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right) + 35a^2\left(c+dx^3\right)^{3/2} - \frac{21b\left(c+dx^3\right)^{5/2}(ad+bc)}{d^2} + \frac{15b^2\left(c+dx^3\right)^{7/2}}{d^2}\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(35*a^2*(c + d*x^3)^(3/2) - (21*b*(b*c + a*d)*(c + d*x^3)^(5/2))/d^2 + (
15*b^2*(c + d*x^3)^(7/2))/d^2 + 105*a^2*(b*c - a*d)*(Sqrt[c + d*x^3]/b - (S
qrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))
))/(315*b^3)
```

Maple [C] time = 0.033, size = 605, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 1/b^2*(b*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105/d*c
^2*x^3*(d*x^3+c)^(1/2)-4/105/d^2*c^3*(d*x^3+c)^(1/2))-2/15*a/d*(d*x^3+c)^(5
/2))+a^2/b^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c
)/d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/
(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2
*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1
/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)
^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(
1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3
)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(
2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(
1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/
3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Ro
otOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.71122, size = 883, normalized size = 5.73

$$\left[-\frac{105\left(a^2bcd^2 - a^3d^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) - 2\left(15\,b^3d^3x^9 + 3\left(8\,b^3cd^2 - 7\,ab^2d^3\right)x^6 - 6\,b^3c^3 - 21\,ab^2c^2 \cdots}{315\,b^4d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b
*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*b^
3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2*d
+ 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^2*b*
d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d^3)*s
qrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a
*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21
*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^
2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13043, size = 261, normalized size = 1.69

$$\frac{2\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{3\,\sqrt{-b^{2}c+abd}b^{4}}+\frac{2\left(15\left(dx^{3}+c\right)^{\frac{7}{2}}b^{6}d^{12}-21\left(dx^{3}+c\right)^{\frac{5}{2}}b^{6}cd^{12}-21\left(dx^{3}+c\right)^{\frac{5}{2}}ab^{5}d^{13}+\right.}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 2/315*(15*(d*x^3 + c)^(7/2)*b^6*
d^12 - 21*(d*x^3 + c)^(5/2)*b^6*c*d^12 - 21*(d*x^3 + c)^(5/2)*a*b^5*d^13 +
35*(d*x^3 + c)^(3/2)*a^2*b^4*d^14 + 105*sqrt(d*x^3 + c)*a^2*b^4*c*d^14 - 10
5*sqrt(d*x^3 + c)*a^3*b^3*d^15)/(b^7*d^14)
```

3.369 $\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=120

$$-\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2}-\frac{2a\sqrt{c+dx^3}\,(bc-ad)}{3b^3}+\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd}$$

```
[Out] (-2*a*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^3) - (2*a*(c + d*x^3)^(3/2))/(9*b^2
) + (2*(c + d*x^3)^(5/2))/(15*b*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rubi [A] time = 0.103848, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2}-\frac{2a\sqrt{c+dx^3}\,(bc-ad)}{3b^3}+\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (-2*a*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^3) - (2*a*(c + d*x^3)^(3/2))/(9*b^2
) + (2*(c + d*x^3)^(5/2))/(15*b*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx = \frac{1}{3} \text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx} \, dx, x, x^3\right)$$
$$= \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{a\, \text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx} \, dx, x, x^3\right)}{3b}$$
$$= -\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{(a(bc-ad))\, \text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx} \, dx, x, x^3\right)}{3b^2}$$
$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{\left(a(bc-ad)^2\right) \text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} \, dx$$
$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{\left(2a(bc-ad)^2\right) \text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} \, dx,\right.}{3b^3d}$$
$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} + \frac{2a(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Mathematica [A] time = 0.110194, size = 111, normalized size = 0.92

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd\left(4c+dx^3\right)+3b^2\left(c+dx^3\right)^2\right)}{45b^3d} + \frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^3
)))/(45*b^3*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*b^(7/2))
```

Maple [C] time = 0.009, size = 531, normalized size = 4.4

$$\frac{2}{15\,bd}\left(dx^3+c\right)^{\frac{5}{2}} - \frac{a}{b}\left(\frac{2\,dx^3}{9\,b}\sqrt{dx^3+c} + \frac{2}{3\,d}\left(-\frac{d\,(ad-2\,bc)}{b^2} - \frac{2\,cd}{3\,b}\right)\sqrt{dx^3+c} + \frac{\frac{i}{3}\sqrt{2}}{b^2d^2}\sum_{\_alpha=RootOf\left(b\_Z^3+a\right)} \frac{-a^2d^2+2\,abc}{ad-bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b/d-a/b*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*
c)/b^2-2/3*d/b*c)/d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a
*b*c*d-b^2*c^2)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))
/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d
^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97544, size = 639, normalized size = 5.32

$$\left[-\frac{15\left(abcd-a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(3\,b^2d^2x^6+3\,b^2c^2-20\,abcd+15\,a^2d^2+\left(6\,b^2cd-5\,abd^2\right.\right.}{45\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a
*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(3*b^2*d^2*x
```

```
^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqr
t(d*x^3 + c))/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b)*ar
ctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (3*b^2*d^2*x^6
+ 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d
*x^3 + c))/(b^3*d)]
```

Sympy [A] time = 94.5664, size = 116, normalized size = 0.97

$$-\frac{2a\left(c+dx^3\right)^{\frac{3}{2}}}{9b^2}-\frac{2a\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{2\left(c+dx^3\right)^{\frac{5}{2}}}{15bd}+\frac{\sqrt{c+dx^3}\left(2a^2d-2abc\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] -2*a*(c + d*x**3)**(3/2)/(9*b**2) - 2*a*(a*d - b*c)**2*atan(sqrt(c + d*x**3
)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**(5/2)
/(15*b*d) + sqrt(c + d*x**3)*(2*a**2*d - 2*a*b*c)/(3*b**3)
```

Giac [A] time = 1.11057, size = 204, normalized size = 1.7

$$-\frac{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^4-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^5-15\,\sqrt{dx^3+c}ab^3cd^5+15\,\sqrt{d\right.}{45\,b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^4
 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*sqrt
(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

3.370 $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=96

$$\frac{2\sqrt{c+dx^3}(bc-ad)}{3b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2\left(c+dx^3\right)^{3/2}}{9b}$$

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b) - (2*
(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^
(5/2))
```

Rubi [A] time = 0.0807584, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}(bc-ad)}{3b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2\left(c+dx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b) - (2*
(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^
(5/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx &= \frac{1}{3} \operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx} \, dx, x, x^3\right) \\
&= \frac{2 \left(c+dx^3\right)^{3/2}}{9b} + \frac{(bc-ad) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx} \, dx, x, x^3\right)}{3b} \\
&= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2 \left(c+dx^3\right)^{3/2}}{9b} + \frac{(bc-ad)^2 \operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} \, dx, x, x^3\right)}{3b^2} \\
&= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2 \left(c+dx^3\right)^{3/2}}{9b} + \frac{\left(2(bc-ad)^2\right) \operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} \, dx, x, \sqrt{c+dx^3}\right)}{3b^2 d} \\
&= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2 \left(c+dx^3\right)^{3/2}}{9b} - \frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0708423, size = 85, normalized size = 0.89

$$\frac{2\sqrt{c+dx^3}\left(-3ad+4bc+bdx^3\right)}{9b^2} - \frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) - (2*(b*c - a*d)^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Maple [C] time = 0.006, size = 507, normalized size = 5.3

$$\frac{2\,dx^3}{9\,b}\sqrt{dx^3+c}+\frac{2}{3\,d}\left(-\frac{d\,(ad-2\,bc)}{b^2}-\frac{2\,cd}{3\,b}\right)\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{b^2d^2}\sum_{\_\text{alpha}=\text{RootOf}\left(b\_Z^3+a\right)}\frac{-a^2d^2+2\,abcd-b^2c^2}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c)/d*(d*x^3+c)^
(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-d^
2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-
d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*
(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c
)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d
^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(
1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d
^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88092, size = 446, normalized size = 4.65

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(bdx^3+4\,bc-3\,ad\right)\sqrt{dx^3+c}}{9\,b^2},-\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqr
t(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3*a
*d)*sqrt(d*x^3 + c))/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-
sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c - 3*
a*d)*sqrt(d*x^3 + c))/b^2]
```

Sympy [A] time = 33.8455, size = 90, normalized size = 0.94

$$\frac{2\left(c+dx^3\right)^{\frac{3}{2}}}{9b}+\frac{\sqrt{c+dx^3}\,(-2ad+2bc)}{3b^2}+\frac{2\,(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-2*a*d + 2*b*c)/(3*b**2) +
2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a
*d - b*c)/b))
```

Giac [A] time = 1.10498, size = 153, normalized size = 1.59

$$\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^2+3\,\sqrt{dx^3+c}b^2c-3\,\sqrt{dx^3+c}abd\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(d*
x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3
```

3.371 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$

Optimal. Leaf size=104

$$\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

```
[Out] (2*d*Sqrt[c + d*x^3])/(3*b) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
(3*a) + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a
*d]])/(3*a*b^(3/2))
```

Rubi [A] time = 0.112071, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 84, 156, 63, 208}

$$\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*d*Sqrt[c + d*x^3])/(3*b) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
(3*a) + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a
*d]])/(3*a*b^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
```

```
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx, x, x^3\right)$$

$$= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{\text{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}$$

$$= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ab}$$

$$= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c}\right.}{3abd}$$

$$= \frac{2d\sqrt{c+dx^3}}{3b} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}$$

Mathematica [A] time = 0.0732751, size = 105, normalized size = 1.01

$$\frac{2\left(a\sqrt{b}d\sqrt{c+dx^3}+(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)-b^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3ab^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*(a*Sqrt[b]*d*Sqrt[c + d*x^3] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]] + (b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*
d]]))/(3*a*b^(3/2))
```

Maple [C] time = 0.009, size = 565, normalized size = 5.4

$$-\frac{b}{a}\left(\frac{2\,dx^3}{9\,b}\sqrt{dx^3+c}+\frac{2}{3\,d}\left(-\frac{d\,(ad-2\,bc)}{b^2}-\frac{2\,cd}{3\,b}\right)\sqrt{dx^3+c}+\frac{\frac{i}{3}\sqrt{2}}{b^2d^2}\sum_{\_alpha=RootOf(b\_Z^3+a)}\frac{-a^2d^2+2\,abcd-b^2c^2}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a),x)
```

```
[Out] -b/a*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c)/d*(d*x
^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c
)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3
)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^
(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha
*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^
2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b+a)))+1/a*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*a
rctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)
```

Fricas [A] time = 2.10785, size = 1080, normalized size = 10.38

$$\left[\frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3ab}, \frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c+}}{x^3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(b*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/(a*b), 1/3*(b*c^
(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c
)*a*d + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)))/(a*b), 1/3*(2*b*sqrt(-c)*c*arctan(sqrt(d*x^3 +
 c)*sqrt(-c)/c) + 2*sqrt(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3
 + a)))/(a*b), 2/3*(b*sqrt(-c)*c*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(
d*x^3 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b
*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [A] time = 30.248, size = 102, normalized size = 0.98

$$\frac{2d\sqrt{c+dx^3}}{3b}+\frac{2c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}-\frac{2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)
```

```
[Out] 2*d*sqrt(c + d*x**3)/(3*b) + 2*c**2*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sq
rt(-c)) - 2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*
b**2*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.13406, size = 159, normalized size = 1.53

$$\frac{2}{3}\,d\left(\frac{c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}+\frac{\sqrt{dx^3+c}}{b}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}abd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*d*(c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*d) + sqrt(d*x^3 + c
)/b - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b*d))
```

3.372 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx$

Optimal. Leaf size=116

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}+\frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

```
[Out] -(c*Sqrt[c + d*x^3])/(3*a*x^3) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(3*a^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b])
```

Rubi [A] time = 0.145402, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}+\frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] -(c*Sqrt[c + d*x^3])/(3*a*x^3) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(3*a^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2} + \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x\right.}{3a^2}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} + \frac{\left(2(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3a^2d}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} + \frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

Mathematica [A] time = 0.106637, size = 108, normalized size = 0.93

$$\frac{-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} + \sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{ac\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^3])/x^3) + Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]] - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/Sqrt[b])/(3*a^2)
```

Maple [C] time = 0.01, size = 620, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x)
```

```
[Out] b^2/a^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c)/d*(
d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-
b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(
1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I
*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3
)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_al
pha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(
-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*b+a))+1/a*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d
*arctanh((d*x^3+c)^(1/2)/c^(1/2)))-1/a^2*b*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c
*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)
```

Fricas [A] time = 2.14663, size = 1220, normalized size = 10.52

$$\left[-\frac{2\,(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+(2\,bc-3\,ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+2\,\sqrt{dx^3+c}ac\quad 4\,(b}{6\,a^2x^3},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2
*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*sqrt
(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 +
 c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sq
rt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(c)
*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c) + (b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(-c)*x
^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 + c)*a*c)/(a^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{x^4\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)
```

Giac [A] time = 1.12009, size = 182, normalized size = 1.57

$$\frac{1}{3}\,d^2\left(\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{\left(2\,bc^2-3\,acd\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}-\frac{\sqrt{dx^3+c}c}{ad^2x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*d^2*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c^2 - 3*a*c*d)*arctan(
sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2) - sqrt(d*x^3 + c)*c/(a*d^2*x^3
))
```

3.373 $\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=65

$$\frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0561818, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3 \left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.456073, size = 280, normalized size = 4.31

$$\frac{x\left(\frac{x^3\sqrt{\frac{dx^3}{c}+1}\left(55a^2d^2-88abcd+27b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-\frac{64a^2c^2(11ad-14bc)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\right.\right.}\right.}{220b^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x*(8*(c + d*x^3)*(14*b*c - 11*a*d + 5*b*d*x^3) + ((27*b^2*c^2 - 88*a*b*c*d
 + 55*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)])/a - (64*a^2*c^2*(-14*b*c + 11*a*d)*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7
/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/
c), -((b*x^3)/a)])))))/(220*b^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.036, size = 1101, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 1/b*(2/11*x^4*d*(d*x^3+c)^(1/2)+28/55*c*x*(d*x^3+c)^(1/2)-18/55*I*c^2*3^(1/
2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-a/b*(2/5*d/b*x*
(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*d/b*c)*3^(1/2)/d*(-d^2*c)^(1/
3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*
(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/
3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^
2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d
-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(c + dx^3\right)^{\frac{3}{2}}}{a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}} x^3}{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

3.374 $\int \frac{x(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=65

$$\frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0399799, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx=\frac{\left(c\sqrt{c+dx^3}\right)\int\frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$=\frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.152379, size = 149, normalized size = 2.29

$$\frac{x^2\left(2dx^3\sqrt{\frac{dx^3}{c}+1}(10bc-7ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5c\sqrt{\frac{dx^3}{c}+1}(7bc-4ad)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+20ad\left(c+d\right.\right.}{70ab\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x^2*(20*a*d*(c + d*x^3) + 5*c*(7*b*c - 4*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1
[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(10*b*c - 7*a*d)*x^3*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))
/(70*a*b*Sqrt[c + d*x^3])
```

Maple [C] time = 0.027, size = 930, normalized size = 14.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/7*d/b*x^2*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-4/7*d/b*c)*3^(1/2)/d*
(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
```

```
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b
*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(
1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alph
a^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(c+dx^3\right)^{\frac{3}{2}}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

3.375 $\int \frac{\left(c+dx^3\right)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=60

$$\frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0275824, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.32665, size = 351, normalized size = 5.85

$$\frac{x\left(\frac{8\left(3dx^3\left(a+bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-4ac\left(2ad^2x^3+b\left(5c^2+2cdx^3+2d^2x^6\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+\frac{dx^3\sqrt{\cdots}}{\cdots}\right.}{20b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (x*((d*(8*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-4*a*c*(2*a*d^2*x^3 + b*(5*c^2 + 2*c*d*
x^3 + 2*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
3*d*x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)
/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(20*b
*Sqrt[c + d*x^3])
```

Maple [C] time = 0.004, size = 776, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/5*d/b*x*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*d/b*c)*3^(1/2)/d*(-
d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1/2)*s
um((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*
(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3
*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

3.376 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=63

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rubi [A] time = 0.0578695, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.120766, size = 148, normalized size = 2.35

$$\frac{2dx^6\sqrt{\frac{dx^3}{c}+1}(2ad+bc)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5cx^3\sqrt{\frac{dx^3}{c}+1}(5ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20ac\left(c+dx^3\right)}{20a^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*c*(c + d*x^3) + 5*c*(-2*b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(b*c + 2*a*d)*x^6*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(2
0*a^2*x*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 1404, normalized size = 22.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x)
```

```
[Out] 1/a*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/2)*(-d^2*c
)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
```

```
d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2))))-b/a*(2/7*d/b*x^2*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d
-2*b*c)/b^2-4/7*d/b*c)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d
^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*
(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(
1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d
^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/
3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/b^
2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-d^2*c)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(
2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^3\right)^{\frac{3}{2}}}{x^2\left(a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}}}{\left(bx^3 + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

3.377 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} dx$

Optimal. Leaf size=65

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0577781, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3\left(a+bx^3\right)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.334687, size = 343, normalized size = 5.28

$$-\frac{dx^6\sqrt{\frac{dx^3}{c}+1}(bc-4ad)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8ac\left(3x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-4ac(2\ldots}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\ldots}}{16a^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -(d*(b*c - 4*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x
^3)/c), -((b*x^3)/a)] + (8*a*c*(-4*a*c*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3 + 2*b
*d*x^6)*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a +
 b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a
+ b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3
*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*x^2*Sqrt[c
 + d*x^3])
```

Maple [C] time = 0.02, size = 1096, normalized size = 16.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x)
```

```
[Out] 1/a*(-1/2*c*(d*x^3+c)^(1/2)/x^2+2/5*d*x*(d*x^3+c)^(1/2)-9/10*I*c*3^(1/2)*(-
d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-b/a*(2/5*d/b*x*(d*x^3+
c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*d/b*c)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*
b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c
)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_a
lpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(
I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^3\right)^{\frac{3}{2}}}{x^3 \left(a + bx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**3*(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}}}{\left(bx^3 + a\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

3.378 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=104

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}$$

```
[Out] (-2*(b*c + a*d)*Sqrt[c + d*x^3])/(3*b^2*d^2) + (2*(c + d*x^3)^(3/2))/(9*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)
*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.104588, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```


```
[Out] (-2*(b*c + a*d)*Sqrt[c + d*x^3])/(3*b^2*d^2) + (2*(c + d*x^3)^(3/2))/(9*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)
*Sqrt[b*c - a*d])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^3\right)\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.14478, size = 91, normalized size = 0.88

$$\frac{2\sqrt{c+dx^3}\left(-3ad-2bc+bdx^3\right)}{9b^2d^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

[Out] (2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) - (2*a^2*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])

Maple [C] time = 0.035, size = 488, normalized size = 4.7

$$\frac{1}{b^2}\left(b\left(\frac{2x^3}{9d}\sqrt{dx^3+c}-\frac{4c}{9d^2}\sqrt{dx^3+c}\right)-\frac{2a}{3d}\sqrt{dx^3+c}\right)-\frac{\frac{i}{3}a^2\sqrt{2}}{b^2d^2}\sum_{\_alpha=\mathrm{RootOf}(b\_Z^3+a)}\frac{1}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x)

[Out] 1/b^2*(b*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)-2/3*a/d*(d*x
^3+c)^(1/2))-1/3*I*a^2/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*
c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.84879, size = 593, normalized size = 5.7

$$\left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^3\right)\sqrt{dx^3+c}}{9\left(b^4cd^2-ab^3d^3\right)},\ \frac{2\left(3\sqrt{-\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x
^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^
3), 2/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
+ a*b*d)/(b*d*x^3 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d
 - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.11457, size = 143, normalized size = 1.38

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^3+c}b^2cd^4-3\,\sqrt{dx^3+c}abd^5\right)}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^3 + c)*b^2*c*d^4 - 3*s
qrt(d*x^3 + c)*a*b*d^5)/(b^3*d^6)
```

3.379 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=74

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

```
[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0697095, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}\\ &= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}\\ &= \frac{2\sqrt{c+dx^3}}{3bd} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0701442, size = 74, normalized size = 1.

$$\frac{2\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\sqrt{b}\sqrt{c+dx^3}}{d}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*((Sqrt[b]*Sqrt[c + d*x^3])/d + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*b^(3/2))
```

Maple [C] time = 0.01, size = 448, normalized size = 6.1

$$\frac{2}{3bd}\sqrt{dx^3+c}+\frac{\frac{i}{3}a\sqrt{2}}{bd^2}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{1}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{\cdot}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b/d+1/3*I*a/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1
/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/
3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70904, size = 437, normalized size = 5.91

$$\left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\sqrt{dx^3+c}\left(b^2c-abd\right)}{3\left(b^3cd-ab^2d^2\right)},-\frac{2\left(\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)\right.}{3\left(b^3cd-ab^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c
)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -2/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^3 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.09414, size = 86, normalized size = 1.16

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^3+c}}{b}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^3 + c)/b)/d
```

3.380 $\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

Rubi [A] time = 0.0495318, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0146385, size = 51, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

Maple [C] time = 0.006, size = 426, normalized size = 8.4

$$\frac{-\frac{i}{3}\sqrt{2}}{d^2}\sum_{\_alpha=RootOf\left(b\_Z^3+a\right)}\frac{1}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*
c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_
alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54466, size = 288, normalized size = 5.65

$$\left[\frac{\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)}{3\,\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)}{3\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b
*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] time = 7.7967, size = 39, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.11201, size = 54, normalized size = 1.06

$$\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.381 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

```
[Out] (-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0758255, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\\
&= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.0777833, size = 81, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*(-(ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*a)
```

Maple [C] time = 0.009, size = 453, normalized size = 5.3

$$\frac{\frac{i}{3}b\sqrt{2}}{ad^2}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{1}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I*b/a/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d
^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+
2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*
c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-2/3*arctanh((d*x^3+c)^(1/2)/c^(
1/2))/a/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

Fricas [A] time = 1.68685, size = 948, normalized size = 11.15

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,ac},\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+}{3\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-
c)/c))/(a*c), 2/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A] time = 14.1218, size = 70, normalized size = 0.82

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + 2
*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c))
```

Giac [A] time = 1.12294, size = 107, normalized size = 1.26

$$-\frac{2}{3}\,d\left(\frac{b\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ad}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*d*(b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*a*d) - arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.382 $\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=117

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

```
[Out] -Sqrt[c + d*x^3]/(3*a*c*x^3) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a^2*c^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*
c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.118177, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*a*c*x^3) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a^2*c^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*
c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2c}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx\right.}{3a^2cd}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.112794, size = 151, normalized size = 1.29

$$\frac{2b^{3/2}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(ad-bc)}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*a*c*x^3) + (2*b*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^
2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*c^(3/2)) + (2*b^(3/2
)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^
2*(-(b*c) + a*d))
```

Maple [C] time = 0.01, size = 498, normalized size = 4.3

$$\frac{-\frac{i}{3}b^2\sqrt{2}}{a^2d^2}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{1}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-3\sqrt[3]{-d^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I*b^2/a^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/
d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(
2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a
*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-1/3*(d*x^3+c)^(1/2)/
c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))+2/3/a^2*b*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)\sqrt{dx^3+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)
```

Fricas [A] time = 1.80769, size = 1261, normalized size = 10.78

$$\left[\frac{2\,bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+(2\,bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx^3+c}ac}{6\,a^2c^2x^3},-\frac{4\,bc^2x\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (2*b*c + a*d)*sqr
t(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3
+ c)*a*c)/(a^2*c^2*x^3), -1/6*(4*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*c)/(a^2*c^2*x^3), 1/3*(b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d
*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^3 + a)) - (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) -
 sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3), -1/3*(2*b*c^2*x^3*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^
3 + c)*a*c)/(a^2*c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.11741, size = 159, normalized size = 1.36

$$\frac{1}{3}\,d^2\left(\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}cd^2}-\frac{\sqrt{dx^3+c}}{acd^2x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*d^2*(2*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*a^2*d^2) - (2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqr
t(-c)*c*d^2) - sqrt(d*x^3 + c)/(a*c*d^2*x^3))
```

3.383 $\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0512265, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

Mathematica [A] time = 0.0292805, size = 65, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{4a\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]

[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])/(4*a*Sqrt[c + d*x^3])
```

Maple [C] time = 0.03, size = 719, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x)

[Out] -2/3*I/b*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
```

```
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/
3*I*a/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)
^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/
(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3 + a\right)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.384 $\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}} dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0370371, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

Mathematica [A] time = 0.027467, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)
/a)])/(2*a*Sqrt[c + d*x^3])
```

Maple [C] time = 0.026, size = 429, normalized size = 6.7

$$\frac{-\frac{i}{3}\sqrt{2}}{d^2}\sum_{\_alpha=RootOf(b\_Z^3+a)}\frac{1}{\_alpha\,(ad-bc)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d
*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2
```

```
/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d
^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*
d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^3 + a)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.385 $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0263589, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.0279172, size = 161, normalized size = 2.73

$$-\frac{8acxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\sqrt{c+dx^3}\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x
^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] time = 0.004, size = 429, normalized size = 7.3

$$\frac{-\frac{i}{3}\sqrt{2}}{d^2}\sum_{\_alpha=RootOf(b\_Z^3+a)}\frac{1}{\_alpha^2(ad-bc)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\sqrt[3]{-d^2c}\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1
/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(
```

```
1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^
(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(
a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.386 $\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x*Sqrt[c + d*x^3]))
```

Rubi [A] time = 0.0534893, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x*Sqrt[c + d*x^3]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.10002, size = 141, normalized size = 2.27

$$\frac{2bdx^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5x^3\sqrt{\frac{dx^3}{c}+1}(ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20a\left(c+dx^3\right)}{20a^2cx\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]*A
ppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*c*x*Sqrt[c +
 d*x^3])
```

Maple [C] time = 0.009, size = 890, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((
x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
```

```
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))
)+1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(
x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*
c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)\sqrt{dx^3+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

3.387 $\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0526242, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.262377, size = 339, normalized size = 5.3

$$\frac{\frac{8a\left(3x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-4ac\left(2ac+3adx^3+6bcx^3+2bdx^6\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{\left(a+bx^3\right)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}-bdx^6\sqrt{\frac{dx^3}{c}+1}}{16a^2cx^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (8*a*(-4*a*c*(2*a*c + 6*b*c*x^3 + 3*a*d*x^3 + 2*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(c +
 d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d
*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*
a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*A
ppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3
/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*c*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.009, size = 738, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*
(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3))
)/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1
/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d
-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*
c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)\sqrt{dx^3+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

3.388 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

```
[Out] (2*c^2)/(3*d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(3*b*d^2)
 - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b
*c - a*d)^(3/2))
```

Rubi [A] time = 0.120525, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 87, 63, 208}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(3*d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(3*b*d^2)
 - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b
*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{c^2}{d(-bc+ad)(c+dx)^{3/2}}+\frac{1}{bd\sqrt{c+dx}}+\frac{a^2}{b(bc-ad)(a+bx)\sqrt{c+dx}}\right)dx, x, x^3\right)\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b(bc-ad)}\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}+\frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd(bc-ad)}\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0687899, size = 100, normalized size = 0.93

$$
\frac{2\left(-a^2d^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+a^2d^2+abd\left(c+dx^3\right)+b^2(-c)\left(2c+dx^3\right)\right)}{3b^2d^2\sqrt{c+dx^3}(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(a^2*d^2 + a*b*d*(c + d*x^3) - b^2*c*(2*c + d*x^3) - a^2*d^2*Hypergeomet
ric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]))/(3*b^2*d^2*(-(b*c) + a*
d)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.037, size = 527, normalized size = 4.9

$$\frac{1}{b^2}\left(b\left(\frac{2c}{3d^2}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2}{3d^2}\sqrt{dx^3+c}\right)+\frac{2a}{3d}\frac{1}{\sqrt{dx^3+c}}\right)+\frac{a^2}{b^2}\left(-\frac{2}{3ad-3bc}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{\frac{i}{3}b\sqrt{2}}{d^2}\sum_{\_alpha=RootOf\left(b\_Z^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b^2*(b*(2/3/d^2*c/((x^3+1/d*c)*d)^(1/2)+2/3*(d*x^3+c)^(1/2)/d^2)+2/3*a/d/
(d*x^3+c)^(1/2))+a^2/b^2*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^2*b*
2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*
c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_
alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46543, size = 896, normalized size = 8.37

$$\left[-\frac{\left(a^2d^3x^3+a^2cd^2\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^3-3\,ab^2c^2d+a^2bcd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2\right.\right.}{3\left(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+\left(b^4c^2d^3-2\,ab^3cd^4+a^2b^2d^5\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c -
 a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3 -
 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3)
*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4*c^2
*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^2)*sq
rt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b
*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^
2 + a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b
^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [A] time = 1.13845, size = 139, normalized size = 1.3

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^2c-abd\right)\sqrt{-b^2c+abd}}+\frac{2\,c^2}{3\left(bcd^2-ad^3\right)\sqrt{dx^3+c}}+\frac{2\,\sqrt{dx^3+c}}{3\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqr
t(-b^2*c + a*b*d)) + 2/3*c^2/((b*c*d^2 - a*d^3)*sqrt(d*x^3 + c)) + 2/3*sqrt
(d*x^3 + c)/(b*d^2)
```

3.389 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

```
[Out] (-2*c)/(3*d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0736587, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(3*d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}\\
&= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}\\
&= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0857927, size = 88, normalized size = 1.07

$$
\frac{2\left(\frac{c(ad-bc)}{d\sqrt{c+dx^3}}+\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}\right)}{3(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((c*(-(b*c) + a*d))/(d*Sqrt[c + d*x^3]) + (a*Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*(b*c - a*d)^2)
```

Maple [C] time = 0.007, size = 487, normalized size = 5.9

$$-\frac{2}{3\,bd}\frac{1}{\sqrt{dx^3+c}}-\frac{a}{b}\left(-\frac{2}{3\,ad-3\,bc}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{\frac{i}{3}b\sqrt{2}}{d^2}\sum_{\_alpha=RootOf\left(b\_Z^3+a\right)}\frac{1}{\left(-ad+bc\right)\left(ad-bc\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/b/d/(d*x^3+c)^(1/2)-a/b*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^
2*b*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-
d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)
+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*
c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b
*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09247, size = 682, normalized size = 8.32

$$\left[-\frac{\left(ad^2x^3+acd\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(b^2c^2-abcd\right)\sqrt{dx^3+c}}{3\left(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+\left(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4\right)x^3\right)},-\frac{2\left(\left(ad^2x^3+acd\right)\sqrt{-b^2c+ab\right.}{3\left(b^3c^3d-2\,ab^2c^2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a*d^2*x^3 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c*d
)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^
2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (b^
2*c^2 - a*b*c*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^
3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [A] time = 1.12362, size = 105, normalized size = 1.28

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{c}{\sqrt{dx^3+c}(bc-ad)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*(b*c - a*d)) + c/(sqrt(d*x^3 + c)*(b*c - a*d)))/d
```

3.390 $\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2}{3\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

```
[Out] 2/(3*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0660656, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{2}{3\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*(b*c - a*d)^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}}+\frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3(bc-ad)}\\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}}+\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3d(bc-ad)}\\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0126225, size = 52, normalized size = 0.68

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)}{3\sqrt{c+dx^3}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)])/(3*(-(b*c
) + a*d)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.005, size = 463, normalized size = 6.

$$-\frac{2}{3ad-3bc}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{\frac{i}{3}b\sqrt{2}}{d^2}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{1}{(-ad+bc)(ad-bc)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^2*b*2^(1/2)*sum(1/(-a*d+b*c)/(
a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/
3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)
*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2
*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82387, size = 494, normalized size = 6.42

$$\left[-\frac{\left(dx^3+c\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}}{3\left(\left(bcd-ad^2\right)x^3+bc^2-acd\right)},-\frac{2\left(\left(dx^3+c\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)\right.}{3\left(\left(bcd-ad^2\right)x^3+bc^2-acd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c
))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), -2/3*((d*x^3 + c)*sqrt(-b/(b*c -
a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b
*c)) - sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]
```

Sympy [A] time = 16.4193, size = 66, normalized size = 0.86

$$-\frac{2}{3\sqrt{c+dx^3}\,(ad-bc)}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(3*sqrt(c + d*x**3)*(a*d - b*c)) - 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b
*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c))
```

Giac [A] time = 1.11195, size = 99, normalized size = 1.29

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}(bc-ad)}+\frac{2}{3\,\sqrt{dx^3+c}(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))
```

3.391 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}-\frac{2d}{3c\sqrt{c+dx^3}(bc-ad)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a*c^(3/2)) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*a*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.112687, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 85, 156, 63, 208}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}-\frac{2d}{3c\sqrt{c+dx^3}(bc-ad)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a*c^(3/2)) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*a*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
```

```
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3c(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3 \right.}{3a(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3acd} - \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} \right.}{3ad(bc-}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.0291704, size = 89, normalized size = 0.78

$$-\frac{2\left(bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(dx^3+c)}{bc-ad}\right)+(ad-bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)\right)}{3ac\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(b*c*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)] + (-(
b*c) + a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(3*a*c*(b*c -
a*d)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.022, size = 512, normalized size = 4.5

$$-\frac{b}{a}\left(-\frac{2}{3\,ad-3\,bc}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{\frac{i}{3}b\sqrt{2}}{d^2}\sum_{\_alpha=\mathrm{RootOf}(b\_Z^3+a)}\frac{1}{(-ad+bc)(ad-bc)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -b/a*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^2*b*2^(1/2)*sum(1/(-a*d+
b*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*
c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2
*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)
^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*
b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*b+a))+1/a*(2/3/c/((x^3+1/d*c)*d)^(1/2)-2/3*arctanh((d*x^3+c
)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)
```

Fricas [B] time = 2.10288, size = 1663, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d*x^
3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*
d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*(b*c^2*d*x^3 + b*c
^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c -
 a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log
((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*
b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*((b*c*d - a*
d^2)*x^3 + b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (b*
c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a*b*c^4 - a^2*c^
3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -2/3*(sqrt(d*x^3 + c)*a*c*d - (b*c^2*
d*x^3 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqr
t(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*b*c^4 - a^2*c^3*d + (a*b*c^
3*d - a^2*c^2*d^2)*x^3)]
```

Sympy [A] time = 23.871, size = 104, normalized size = 0.91

$$\frac{2d}{3c\sqrt{c+dx^3}\,(ad-bc)}+\frac{2b\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2*d/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + 2*b*atan(sqrt(c + d*x**3)/sqrt((a*
d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + 2*atan(sqrt(c + d*x**3
)/sqrt(-c))/(3*a*c*sqrt(-c))
```

Giac [A] time = 1.12172, size = 158, normalized size = 1.39

$$-\frac{2}{3}\left(\frac{b^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(abcd-a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{1}{\sqrt{dx^3+c}\left(bc^2-acd\right)}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}cd}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c*d - a^2*d^
2)*sqrt(-b^2*c + a*b*d)) + 1/(sqrt(d*x^3 + c)*(b*c^2 - a*c*d)) - arctan(sqr
t(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*c*d))*d
```

3.392 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}(bc-ad)}-\frac{1}{3acx^3\sqrt{c+dx^3}}$$

```
[Out] -(d*(b*c - 3*a*d))/(3*a*c^2*(b*c - a*d)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*Sqr
t[c + d*x^3]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c
^(5/2)) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3
*a^2*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.222325, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 152, 156, 63, 208}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}(bc-ad)}-\frac{1}{3acx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d*(b*c - 3*a*d))/(3*a*c^2*(b*c - a*d)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*Sqr
t[c + d*x^3]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c
^(5/2)) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3
*a^2*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad)-\frac{1}{4}bd(bc-3ad)x}{x(a+bx)\sqrt{c+dx}}\right.}{3ac^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)} - \frac{(2bc}{}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right.}{3a^2d(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{2b^{5/2}\tan}{3a^2(}$$

Mathematica [C] time = 0.0453529, size = 117, normalized size = 0.74

$$\frac{2b^2c^2x^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+(ad-bc)\left(x^3(3ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+ac\right)}{3a^2c^2x^3\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b^2*c^2*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]
 + (-(b*c) + a*d)*(a*c + (2*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/2, 1, 1/2
, 1 + (d*x^3)/c]))/(3*a^2*c^2*(b*c - a*d)*x^3*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 575, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] b^2/a^2*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^2*b*2^(1/2)*sum(1/(-a
*d+b*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d
^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2
*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2
*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1
/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*b+a)))+1/a*(-2/3*d/c^2/((x^3+1/d*c)*d)^(1/2)-1/3*(d*x^3+c
)^(1/2)/c^2/x^3+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/a^2*b*(2/3/c/
((x^3+1/d*c)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)
```

Fricas [B] time = 6.0326, size = 2290, normalized size = 14.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2
*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)
) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3
*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3)
 + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))
/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/6*(4*(
b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^
2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(c)*log
((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^2*c^2*d +
(a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2
)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*(((2*b^2*c^2*d + a*b*c*d^2 - 3*a
^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d
))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c -
 a*d)))/(b*x^3 + a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3
)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*
d)*x^3), -1/3*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(
-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + ((2*b^
2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b*c^3 - a^2*c^2*d +
 (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^
2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(a + bx^3\right) \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [A] time = 1.16323, size = 248, normalized size = 1.57

$$\frac{1}{3}\left(\frac{2\,b^{3}\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\left(a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-b^{2}c+abd}}-\frac{\left(dx^{3}+c\right)bc-3\left(dx^{3}+c\right)ad+2\,acd}{\left(abc^{3}d-a^{2}c^{2}d^{2}\right)\left(\left(dx^{3}+c\right)^{\frac{3}{2}}-\sqrt{dx^{3}+c}c\right)}-\frac{\left(2\,bc+3\,ad\right)\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{a^{2}\sqrt{-c}c^{2}d^{2}}\right)d^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c*d^2 - a
^3*d^3)*sqrt(-b^2*c + a*b*d)) - ((d*x^3 + c)*b*c - 3*(d*x^3 + c)*a*d + 2*a*
c*d)/((a*b*c^3*d - a^2*c^2*d^2)*((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)) -
(2*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2*d^2))*d^
2
```

3.393 $\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0570646, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*c*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.169167, size = 231, normalized size = 3.45

$$\frac{x\left(-\frac{64a^2cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}-\frac{bx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-8\right)}{12\sqrt{c+dx^3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-8 - (b*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])/a - (64*a^2*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((
b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(
12*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.03, size = 1069, normalized size = 16.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b*(2/3/c*x/((x^3+1/d*c)*d)^(1/2)-2/9*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-a/b*(2/3*d/c*x/(a*d-b*c)/((x^3+1/d*c)*d)^(
1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2
*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2))+1/3*I/d^2*b*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_a
lpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(bx^3 + a)(dx^3 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a + bx^3\right)\left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3 + a\right)\left(dx^3 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.394 $\int \frac{x}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}} \, dx$

Optimal. Leaf size=67

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0412375, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*c*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{\left(a+bx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.115881, size = 142, normalized size = 2.12

$$\frac{x^2\left(2bdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5\sqrt{\frac{dx^3}{c}+1}(ad+3bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20ad\right)}{30ac\sqrt{c+dx^3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(-20*a*d + 5*(3*b*c + a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a*c*(b*c - a*d)*Sqrt[c +
d*x^3])
```

Maple [C] time = 0.03, size = 907, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3*d/c*x^2/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-d^2
*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
```

```
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^2*b*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(
-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/
2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha
*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-
(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b
+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^3 + a)(dx^3 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.395 $\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0288594, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*c*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(a+bx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.318952, size = 338, normalized size = 5.45

$$\frac{x\left(\frac{bdx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a(ad-bc)}+\frac{32ac\left(3ad-3bc+2bdx^3\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24dx^3\left(a+bx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\right.\right.}{\left(a+bx^3\right)(bc-ad)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}\right)}{12c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/(a*(-(b*c) + a*d)) + (32*a*c*(-3*b*c + 3*a*d + 2*b*d*x^3)*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*d*x^3*(a + b*x^3)*
(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF
1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/((b*c - a*d)*(a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(12*c*Sqrt[c + d*x^3])
```

Maple [C] time = 0.005, size = 753, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3*d/c*x/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-d^2*c
)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
```

```
d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/d^2*b*2^(1/2)*sum(1/(a
*d-b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-
d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^
2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

3.396 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*x*Sqrt[c + d*x^3]))
```

Rubi [A] time = 0.0589631, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*x*Sqrt[c + d*x^3]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2\left(a+bx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.196562, size = 193, normalized size = 2.97

$$\frac{-5x^3\sqrt{\frac{dx^3}{c}+1}\left(5a^2d^2-3abcd+6b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{\frac{dx^3}{c}+1}(3bc-5ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+}{60a^2c^2x\sqrt{c+dx^3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (20*a*(-3*b*c*(c + d*x^3) + a*d*(3*c + 5*d*x^3)) - 5*(6*b^2*c^2 - 3*a*b*c*d
 + 5*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), -((b*x^3)/a)] + 2*b*d*(3*b*c - 5*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^2*c^2*(b*c - a*d)*x*Sq
rt[c + d*x^3])
```

Maple [C] time = 0.008, size = 1392, normalized size = 21.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-5/9*I/c^2*3
^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
```

```
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-b/a*(2/3*d/c*x^2/(a*d-b*c)/((x^3+
1/d*c)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((
x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))
+1/3*I/d^2*b*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x
-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c
)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)
/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + bx^3\right) \left(c + dx^3\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + a\right)\left(dx^3 + c\right)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

3.397 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*c*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0572465, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*c*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(a+bx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.578751, size = 425, normalized size = 6.34

$$\frac{\frac{8a\left(3x^3\left(a^2d\left(3c+7dx^3\right)+ab\left(7d^2x^6-3c^2\right)-3b^2cx^3\left(c+dx^3\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-4ac\left(3a^2d\left(2c+7dx^3\right)+ab\left(-6c^2-3cdx^3+14d^2x^6\right.\right.\right.}{\left(a+bx^3\right)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}}{48a^2c^2x^2\sqrt{c+dx^3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b*d*(3*b*c - 7*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (8*a*(-4*a*c*(-6*b^2*c*x^3*(3*c + d*x^3) + 3*a^2
*d*(2*c + 7*d*x^3) + a*b*(-6*c^2 - 3*c*d*x^3 + 14*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(-3*b^2*c*x^3*(c + d*x^3) +
 a^2*d*(3*c + 7*d*x^3) + a*b*(-3*c^2 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2
, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x
^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x
^3)/a)]))))/(48*a^2*c^2*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.008, size = 1084, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a*(-2/3*d/c^2*x/((x^3+1/d*c)*d)^(1/2)-1/2/c^2*(d*x^3+c)^(1/2)/x^2+7/18*I/
c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/
3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-b/a*(2/3
*d/c*x/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-d^2*c)^(
1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/d^2*b*2^(1/2)*sum(1/(a*d-
b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)
+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2
*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^
2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c
)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2
*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*
c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

3.398 $\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4}+\frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

```
[Out] (7*x^6*Sqrt[c + d*x^3])/(15*d^2) + (x^9*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)
) + (2*c*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/(15*d^4) - (3968*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)
```

Rubi [A] time = 0.0935964, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 97, 153, 147, 63, 206}

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4}+\frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*x^6*Sqrt[c + d*x^3])/(15*d^2) + (x^9*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)
) + (2*c*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/(15*d^4) - (3968*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
```

```
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\mathrm{Subst}\left(\int \frac{x^2\left(3c+\frac{7dx}{2}\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2\,\mathrm{Subst}\left(\int \frac{x\left(-56c^2d-\frac{141}{2}cd^2x\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{15d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4} - \frac{\left(1984c^3\right)\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4} - \frac{\left(3968c^3\right)\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4} - \frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

Mathematica [A] time = 0.0669554, size = 101, normalized size = 0.86

$$\frac{6\sqrt{c+dx^3}\left(770c^2dx^3-9168c^3+19cd^2x^6+d^3x^9\right)+19840c^{5/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{45d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (6*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9) + 1
9840*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4*(-
8*c + d*x^3))
```

Maple [C] time = 0.039, size = 952, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^3*(d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45/d*c*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d
*x^3+c)^(1/2)/d^2)+32/9*c/d*(d*x^3+c)^(3/2))+192*c^2/d^3*(2/3*(d*x^3+c)^(1/
2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3)
)/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*
d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d-8*c)))+512*c^3/d^3*(-1/3/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)
+1/54*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))
/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d
^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0851, size = 502, normalized size = 4.29

$$\left[\frac{2\left(4960\left(c^2dx^3-8c^3\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(d^3x^9+19cd^2x^6+770c^2dx^3-9168c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^5x^3-8cd^4\right)}, \frac{2\left(9920\left(c^2d\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 - 91
68*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*c^3
)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^3*x^9 + 19*c*d^2*x
^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12137, size = 149, normalized size = 1.27

$$\frac{3968\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^4}-\frac{512\sqrt{dx^3+c}c^3}{3\left(dx^3-8c\right)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16}+25\left(dx^3+c\right)^{\frac{3}{2}}cd^{16}+960\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqrt
(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(d*
x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20
```

3.399 $\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

```
[Out] (352*c*Sqrt[c + d*x^3])/(27*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (64*c*(c
 + d*x^3)^(3/2))/(27*d^3*(8*c - d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/(9*d^3)
```

Rubi [A] time = 0.0758963, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 89, 80, 50, 63, 206}

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (352*c*Sqrt[c + d*x^3])/(27*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (64*c*(c
 + d*x^3)^(3/2))/(27*d^3*(8*c - d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(104c^2d+9cd^2x\right)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}$$

$$= \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{(176c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{27d^2}$$

$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(176c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(352c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}$$

$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

Mathematica [A] time = 0.0496656, size = 90, normalized size = 0.88

$$\frac{2\sqrt{c+dx^3}\left(-488c^2+41cdx^3+d^2x^6\right)+352c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6) + 352*c^(3/2)*(8*c - d
*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3*(-8*c + d*x^3))
```

Maple [C] time = 0.014, size = 892, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+16*c/d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*s
um((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3
^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alph
a*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d
^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*
d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c
)))+64*c^2/d^2*(-1/3/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum
((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(
1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*
d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2
*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09995, size = 433, normalized size = 4.25

$$\left[\frac{2\left(88\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(d^2x^6+41cdx^3-488c^2\right)\sqrt{dx^3+c}\right)}{9\left(d^4x^3-8cd^3\right)}, \frac{2\left(176\left(cdx^3-8c^2\right)\sqrt{-c}\arctan\left(\cdots\right.\right.}{9\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(
d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x
^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^
4*x^3 - 8*c*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12101, size = 126, normalized size = 1.24

$$\frac{352\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^3}-\frac{64\sqrt{dx^3+c}c^2}{3\left(dx^3-8c\right)d^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6+48\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(d
*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(d*
x^3 + c)*c*d^6)/d^9
```

3.400 $\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}+\frac{26\sqrt{c+dx^3}}{27d^2}-\frac{26\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

```
[Out] (26*Sqrt[c + d*x^3])/(27*d^2) + (8*(c + d*x^3)^(3/2))/(27*d^2*(8*c - d*x^3)
) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)
```

Rubi [A] time = 0.0585338, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 50, 63, 206}

$$\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}+\frac{26\sqrt{c+dx^3}}{27d^2}-\frac{26\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (26*Sqrt[c + d*x^3])/(27*d^2) + (8*(c + d*x^3)^(3/2))/(27*d^2*(8*c - d*x^3)
) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(8c-dx)^2}\,dx,x,x^3\right)$$
$$= \frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)} - \frac{13\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{27d}$$
$$= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)} - \frac{(13c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d}$$
$$= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)} - \frac{(26c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^2}$$
$$= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)} - \frac{26\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Mathematica [A] time = 0.0402187, size = 79, normalized size = 0.96

$$\frac{6\sqrt{c+dx^3}\left(dx^3-12c\right)+26\sqrt{c}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (6*(-12*c + d*x^3)*Sqrt[c + d*x^3] + 26*Sqrt[c]*(8*c - d*x^3)*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))
```

Maple [C] time = 0.01, size = 874, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d
*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^
2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*
d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c
*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+8/d*c*(-1/3/d*(d*x^3+c)^(
1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d
*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2
/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d
^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82981, size = 378, normalized size = 4.61

$$\left[\frac{13\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+6\sqrt{dx^3+c}\left(dx^3-12c\right)}{9\left(d^3x^3-8cd^2\right)}, \frac{2\left(13\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+\cdots}\right.}{9\left(d^3x^3-8cd^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2),
2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*s
qrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [A] time = 1.11929, size = 100, normalized size = 1.22

$$\frac{2\left(\frac{13\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{3\sqrt{dx^3+c}}{d}-\frac{12\sqrt{dx^3+c}c}{\left(dx^3-8\,c\right)d}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 2/9*(13*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
+ c)/d - 12*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d))/d
```

3.401 $\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[Out] Sqrt[c + d*x^3]/(3*d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(9*Sqrt[c]*d)
```

Rubi [A] time = 0.0493253, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 47, 63, 206}

$$\frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```



```
[Out] Sqrt[c + d*x^3]/(3*d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(9*Sqrt[c]*d)
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(8c-dx)^2}\,dx,x,x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d}\\
&= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}
\end{aligned}$$

Mathematica [A] time = 0.0553801, size = 61, normalized size = 0.95

$$\frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] ((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/S
qrt[c])/(9*d)
```

Maple [C] time = 0.007, size = 439, normalized size = 6.9

$$-\frac{1}{3\,d\left(dx^3-8\,c\right)}\sqrt{dx^3+c}+\frac{\frac{i}{54}\sqrt{2}}{d^3c}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3d-8\,c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x-\frac{1}{d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/3/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80881, size = 342, normalized size = 5.34

$$\left[\frac{\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{18\,\left(cd^2x^3-8\,c^2d\right)},\ \frac{\left(dx^3-8\,c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-3\,\sqrt{dx^3+c}c}{9\,\left(cd^2x^3-8\,c^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3 -
 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)*c
)/(c*d^2*x^3 - 8*c^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [A] time = 1.0921, size = 72, normalized size = 1.12

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d}-\frac{\sqrt{dx^3+c}}{3\,\left(dx^3-8\,c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c)
/((d*x^3 - 8*c)*d)
```

3.402 $\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=88

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

```
[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))
```

Rubi [A] time = 0.0723515, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
```

```
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{192c}$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{96c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96cd}$$

$$= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

Mathematica [A] time = 0.0480696, size = 102, normalized size = 1.16

$$\frac{12\sqrt{c}\sqrt{c+dx^3} + 5\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 3\left(dx^3-8c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (12*Sqrt[c]*Sqrt[c + d*x^3] + 5*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])] + 3*(-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(288*c^(3/2)*(
8*c - d*x^3))
```

Maple [C] time = 0.015, size = 912, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/64*d/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8*d/c*(-1/3/d*(
d*x^3+c)^(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(2/3*(d*x^3+c
)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)
```

Fricas [A] time = 1.88867, size = 536, normalized size = 6.09

$$\left[\frac{5\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-24\sqrt{dx^3+c}c}{576\left(c^2dx^3-8c^3\right)}, \frac{3\left(dx^3-8c\right)\sqrt{-c}\operatorname{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)
*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), 1/288*(3*
(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 5*(d*x^3 - 8*c)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*c)/(c
^2*d*x^3 - 8*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{x\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)
```

Giac [A] time = 1.10394, size = 107, normalized size = 1.22

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c}-\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c}-\frac{\sqrt{dx^3+c}}{24\left(dx^3-8c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c)
```

3.403 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)^2}\,dx$

Optimal. Leaf size=124

$$\frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)}+\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}-\frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)}$$

```
[Out] (d*Sqrt[c + d*x^3])/(96*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c*x^3*(8*c
 - d*x^3)) + (7*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(128*c^(5/2))
```

Rubi [A] time = 0.101621, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d\sqrt{c+dx^3}}{96c^2(8c-dx^3)}+\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}-\frac{\sqrt{c+dx^3}}{24cx^3(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (d*Sqrt[c + d*x^3])/(96*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c*x^3*(8*c
 - d*x^3)) + (7*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(128*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
```

```
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)^2}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{\operatorname{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}-\frac{\operatorname{Subst}\left(\int \frac{-54c^2d^2-9cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{1728c^3d}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{d\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{256c^2}+\frac{\left(7d^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{768c^2}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{128c^2}+\frac{(7d)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx\right.}{384c^2}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}$$

Mathematica [A] time = 0.136018, size = 97, normalized size = 0.78

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(4c-dx^3\right)}{dx^6-8cx^3}+7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{1152c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(115
2*c^(5/2))
```

Maple [C] time = 0.013, size = 957, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/256*d^2/c^3*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64*d^2/c^2*(
-1/3/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)
*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/3
*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/256/
c^3*d*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)
```

Fricas [A] time = 1.95708, size = 636, normalized size = 5.13

$$\left[\frac{7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\left(cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{2304\left(c^3dx^6-8\,c^4x^3\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 -
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^3*d*x^6 - 8*c^4*x^3), 1/1152*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan
(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*
sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^3*d*
x^6 - 8*c^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14145, size = 150, normalized size = 1.21

$$\frac{1}{1152}\,d\left(\frac{9\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{7\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{12\left(\left(dx^3+c\right)^{\frac{3}{2}}-5\,\sqrt{dx^3+c}c\right)}{\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/1152*d*(9*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 7*arctan(1/3*
sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 12*((d*x^3 + c)^(3/2) - 5*sqrt(d
*x^3 + c)*c)/((((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2))
```

3.404 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=164

$$\frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}+\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}$$

```
[Out] (5*d^2*Sqrt[c + d*x^3])/(1536*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c*x^
6*(8*c - d*x^3)) - (7*d*Sqrt[c + d*x^3])/(384*c^2*x^3*(8*c - d*x^3)) + (23*
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(2048*c^(7/2))
```

Rubi [A] time = 0.131336, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}+\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```


```
[Out] (5*d^2*Sqrt[c + d*x^3])/(1536*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c*x^
6*(8*c - d*x^3)) - (7*d*Sqrt[c + d*x^3])/(384*c^2*x^3*(8*c - d*x^3)) + (23*
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(2048*c^(7/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{7cd+\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-6c^2d^2-\frac{21}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{54c^3d^3+45c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27648c^5d}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{4096c^3} + \frac{(2}{}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{2048c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2\tanh^{-1}}{2048}$$

Mathematica [A] time = 0.171532, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(32c^2+28cdx^3-5d^2x^6\right)}{dx^9-8cx^6}+23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18432c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))
```

Maple [C] time = 0.015, size = 1020, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x)
```

```
[Out] -3/4096*d^3/c^4*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/
3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/
(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3
)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/512*d^3/c^3
*(-1/3/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-d^2*c)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1
/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1
/6*(d*x^3+c)^(1/2)/x^6-1/12*d*(d*x^3+c)^(1/2)/c/x^3+1/12*d^2*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/c^(3/2))+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+3/4096/c^4*d^2*(2/3*(d*x^3+c)^(1/2)-
2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2 x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A] time = 2.03926, size = 706, normalized size = 4.3

$$\left[\frac{23\left(d^3x^9 - 8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 9\left(d^3x^9 - 8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - 24\left(5\,cd^2x^6 - 28\,c^2dx^3 -\right.}{36864\left(c^4dx^9 - 8\,c^5x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d*x^
3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), 1/18432*(9*(d^3*x^9 -
 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 23*(d^3*x^9 - 8
*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(5*c*d^2*x
^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11387, size = 140, normalized size = 0.85

$$\frac{1}{18432}\,d^2\left(\frac{9\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3}-\frac{23\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^3}-\frac{12\,\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)c^3}-\frac{48\,\left(dx^3+c\right)^{\frac{3}{2}}}{c^3d^2x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/18432*d^2*(9*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 12*sqrt(d*x^3 + c)/((d*x^3 -
 8*c)*c^3) - 48*(d*x^3 + c)^(3/2)/(c^3*d^2*x^6))
```

3.405 $\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=663

$$\frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}-$$

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d^2) + (746*c*Sqrt[c + d*x^3])/(21*d^(8/3)*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^5*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3
)) + (76*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/(3*Sqrt[3]*d^(8/3)) - (76*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3
*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) + (76*c^(7/6)*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])])/(9*d^(8/3)) - (373*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*d^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (746*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.851378, antiderivative size = 663, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {467, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}-\frac{76c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{76c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}+\frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{21\sqrt[4]{3}d^{8/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d^2) + (746*c*Sqrt[c + d*x^3])/(21*d^(8/3)*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^5*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3
)) + (76*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/(3*Sqrt[3]*d^(8/3)) - (76*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3
*c^(1/6)*Sqrt[c + d*x^3])]])/(9*d^(8/3)) + (76*c^(7/6)*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])]])/(9*d^(8/3)) - (373*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]]])/(7*3^(3/4)*d^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (746*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]]])/(21*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x^4\left(5c+\frac{13dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{3d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{2\int \frac{x\left(104c^2d+\frac{373}{2}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{21d^3}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{2\int \left(-\frac{373cdx}{2\sqrt{c+dx^3}} + \frac{1596c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{21d^3}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{(373c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{21d^2} - \frac{\left(152c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{(38c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3} + \frac{(373c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{21d^{7/3}} - \frac{(38\cdots}{\cdots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}\cdots}{\left(\left(1\cdots\right.\right.}}}{7\,3^{3/4}d^{8/3}\cdots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} - \frac{76c\cdots}{\cdots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} - \frac{76c\cdots}{\cdots}$$

Mathematica [C] time = 0.152907, size = 176, normalized size = 0.27

$$-\frac{373dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+520cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80\left(52c^2x^2\right.}{840d^2\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] -(80*(52*c^2*x^2 + 49*c*d*x^5 - 3*d^2*x^8) + 520*c*x^2*(-8*c + d*x^3)*Sqrt[
1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 37
3*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*
x^3)/c), (d*x^3)/(8*c)])/(840*d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.034, size = 2198, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^2*(2/7*x^2*(d*x^3+c)^(1/2)-2/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2))))+16*c/d^2*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*
(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
```

```
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*
(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I
/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))
/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d
^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c))+64*c^2/d^2*(-1/24/c*x^2*(d*x^3+c)^(1/2)/(d*x^3-8
*c)-1/72*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3
))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1
/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3
)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^
2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d
^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*
c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1
/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x^7}{\left(dx^3 - 8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}\,x^7}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

3.406 $\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=641

$$\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\ldots}{\left(\left(1+\ldots\right.\right.}}}{2\,3^{3/4}d^{5/3}\ldots}$$

```
[Out] (7*Sqrt[c + d*x^3])/(3*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*
Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) + (5*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (5*c^(1/6)*Ar
cTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(5/3)) + (
5*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(5/3)) - (7*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(2*3^(3/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (7*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^(5
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.731008, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {467, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\ldots\right.}}}{2\,3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\ldots\right.}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*Sqrt[c + d*x^3])/(3*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*
Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) + (5*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (5*c^(1/6)*Ar
cTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(9*d^(5/3)) + (
5*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(9*d^(5/3)) - (7*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(2*3^(3/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (7*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^(5
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x\left(2c+\frac{7dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{3d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \left(-\frac{7x}{2\sqrt{c+dx^3}} + \frac{30cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{3d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{7\int \frac{x}{\sqrt{c+dx^3}}\,dx}{6d} - \frac{(10c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6d^2} + \frac{7\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{6d^{4/3}} - \frac{\left(5\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{6d^{4/3}} + \cdots$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\cdots\right.}{2\,3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c\cdots}}$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{5\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2\cdots}{3\sqrt[6]{c}\sqrt{c+dx^3}\cdots}\right.}{9d^{5/3}}$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{5\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2\cdots}{3\sqrt[6]{c}\sqrt{c+dx^3}\cdots}\right.}{9d^{5/3}}$$

Mathematica [C] time = 0.138832, size = 167, normalized size = 0.26

$$\frac{7dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+10cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(c+dx^3\right)}{240cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(240*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 1740, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*s
um(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^
(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1
/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d
*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))+8/d*c*(-1/24/c*x^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/72*I/c*3^(1/2
)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1
```

```
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-
d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/
(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2
)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2
*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(
-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}x^4}{\left(dx^3-8c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}x^4}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

3.407 $\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=644

$$\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{3\sqrt[6]{c}\sqrt{c+\cdots}}\right.}{144c^{5/6}d^{2/3}}\cdots$$

```
[Out] Sqrt[c + d*x^3]/(24*c*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(24*c*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]]/(48*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/3)
 + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(144*c^(5/6)*d^(2/3)) + ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(144*c^(5/6)*d^(2/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3
/4)*c^(2/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*c^(2/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rubi [A] time = 0.723417, antiderivative size = 644, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {469, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}}+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\cdots}\right.\right.}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+\cdots}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(24*c*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]]/(48*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/3)
 + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(144*c^(5/6)*d^(2/3)) + ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(144*c^(5/6)*d^(2/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3
/4)*c^(2/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*c^(2/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-c+\frac{dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{3cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{1}{8}\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{48c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96cd} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{48c\sqrt[3]{d}} - \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{96c^{2/3}\sqrt[3]{d}} + \frac{\sqrt{2-\sqrt{3}}}{24\sqrt{2}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\right.\right.}{16\ 3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}} - -$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}} + -$$

Mathematica [C] time = 0.0864138, size = 164, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5cx^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(c+dx^3\right)}{1920c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 5*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqrt[
1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(19
20*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.008, size = 882, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/24/c*x^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/72*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha
-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}\,x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x}{d^2x^6-16\,cdx^3+64\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}\,x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

3.408 $\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=665

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}}}{32\ 3^{3/4}c^{5/3}\sqrt{\frac{}{\left(\left(1+\right.\right.}}}$$

```
[Out] -Sqrt[c + d*x^3]/(48*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(48*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x*(8*c - d*x^3)) - (d^(1/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(48*Sqr
t[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c
 + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(144*c^(11/6)) - (Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*3^(3/4)*c^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(24*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.82412, antiderivative size = 665, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {469, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E}{32\ 3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(48*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x*(8*c - d*x^3)) - (d^(1/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(48*Sqr
t[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c
 + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(144*c^(11/6)) - (Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*3^(3/4)*c^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(24*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}-\frac{\int \frac{-4c-\frac{5dx^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$=-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}+\frac{\int \frac{x\left(40c^2d-2cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{192c^3}$$

$$=-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}+\frac{\int \left(\frac{2cdx}{\sqrt{c+dx^3}}+\frac{24c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{192c^3}$$

$$=-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}+\frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{96c^2}+\frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{8c}$$

$$=-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}-\frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96c^2}+\frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{96c^2}+\frac{d^{2/3}\int \frac{\ldots}{\left(2-\ldots\right.}}{9\ldots}$$

$$=-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}-\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\ldots}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}{32\,3^{3/4}c^{5/3}\sqrt{\ldots}}$$

$$=-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\tan\ldots}{\ldots}$$

$$=-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\tan\ldots}{\ldots}$$

Mathematica [C] time = 0.10809, size = 179, normalized size = 0.27

$$\frac{d^2x^6\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+50cdx^3\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(6c^2+5cd\right.}{3840c^3\sqrt{c+dx^3}\left(8cx-dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] (-80*c*(6*c^2 + 5*c*d*x^3 - d^2*x^6) + 50*c*d*x^3*(8*c - d*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d^2*x^6*
(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(3840*c^3*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] time = 0.013, size = 2193, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/64/c^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1
/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1
/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*
c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1
/64*d/c^2*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)
^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/
```

```
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d^3*2^(
1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d
^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2
*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2
*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-
1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/
3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))+1/8*d/c*(-1/24/c*x^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/72*I/
c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_
alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+
I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_a
lpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*
(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}}{d^2x^8 - 16\,cdx^5 + 64\,c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^8 - 16*c*d*x^5 + 64*c^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^3}}{x^2\left(-8c + dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(-8*c + d*x**3)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

3.409 $\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=687

$$\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{17d^{4/3}}{\cdots}$$

```
[Out] (-7*Sqrt[c + d*x^3])/(768*c^2*x^4) - (d*Sqrt[c + d*x^3])/(96*c^3*x) + (d^(4
/3)*Sqrt[c + d*x^3])/(96*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c
+ d*x^3]/(24*c*x^4*(8*c - d*x^3)) - (17*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(17/6)) + (17*d^(4/3)
*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(9216*c^(17/
6)) - (17*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(17/6)) - (
Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(64*3^(3/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(48*Sqrt[2]*3^
(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.944179, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {469, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}}+\frac{17d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}-\frac{17d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] (-7*Sqrt[c + d*x^3])/(768*c^2*x^4) - (d*Sqrt[c + d*x^3])/(96*c^3*x) + (d^(4
/3)*Sqrt[c + d*x^3])/(96*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c
+ d*x^3]/(24*c*x^4*(8*c - d*x^3)) - (17*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(17/6)) + (17*d^(4/3)
*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(17/
6)) - (17*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(17/6)) - (
Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(64*3^(3/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(48*Sqrt[2]*3^
(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

<u>Rule 469</u>

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

<u>Rule 583</u>

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

<u>Rule 584</u>

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \frac{-7c-\frac{11dx^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} + \frac{\int \frac{64c^2d+\frac{35}{2}cd^2x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{768c^3}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-460c^3d^2+32c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{6144c^5}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \left(-\frac{32c^2d^2x}{\sqrt{c+dx^3}} - \frac{204c^3d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{6144c^5}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{192c^3} + \frac{\left(17d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{512c^2}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{(17d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6144c^3} + \frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\cdots}{\sqrt{c\cdots}}\cdots}{192c\cdots}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}\cdots\right.}{\cdots}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{3072\sqrt{3}c\cdots}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{3072\sqrt{3}c\cdots}$$

Mathematica [C] time = 0.13435, size = 199, normalized size = 0.29

$$-\frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{7680c^4\sqrt{c+dx^3}}+\frac{115d^2x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{24576c^3\sqrt{c+dx^3}}+\sqrt{c+dx^3}\left(-\frac{d^2x^2}{1536c^3\left(dx^3-8c\right)}-\frac{5d}{512c^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/(256*c^2*x^4) - (5*d)/(512*c^3*x) - (d^2*x^2)/(1536*c^3
*(-8*c + d*x^3))) + (115*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(24576*c^3*Sqrt[c + d*x^3]) - (d^3*x^5*S
qrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(7680*c^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 2671, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/256/c^3*d*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((
x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))
)-1/256*d^2/c^3*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-
d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
```

```
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-
d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/d
^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(
-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2
-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*
c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*d-8*c)))+1/64*d^2/c^2*(-1/24/c*x^2*(d*x^3+c)^(1/2)/(d*x^3-8
*c)-1/72*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3
))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1
/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3
)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^
2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d
^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*
c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1
/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c)))+1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d*x^3+c)^(1/2)/
c/x-1/8*I*d/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
```

```
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^{11}-16\,cdx^8+64\,c^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^11 - 16*c*d*x^8 + 64*c^2*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)^2 x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

3.410 $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=711

$$\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^2\sqrt{c}}{537}$$

```
[Out] (-5*Sqrt[c + d*x^3])/(672*c^2*x^7) - (53*d*Sqrt[c + d*x^3])/(21504*c^3*x^4)
 - (d^2*Sqrt[c + d*x^3])/(5376*c^4*x) + (d^(7/3)*Sqrt[c + d*x^3])/(5376*c^4
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x^7*(8*c - d*
x^3)) - (13*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(12288*Sqrt[3]*c^(23/6)) + (13*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)
*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(36864*c^(23/6)) - (13*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(36864*c^(23/6)) - (Sqrt[2 - Sqrt[3]]*d^(7/3)
*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3584*
3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2688*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rubi [A] time = 1.07148, antiderivative size = 711, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {469, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^2\sqrt{c+dx^3}}{5376c^4x}-\frac{13d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}}+\frac{13d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}}-\frac{13d^{7/3}\tanh}{36864}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^3])/(672*c^2*x^7) - (53*d*Sqrt[c + d*x^3])/(21504*c^3*x^4)
 - (d^2*Sqrt[c + d*x^3])/(5376*c^4*x) + (d^(7/3)*Sqrt[c + d*x^3])/(5376*c^4
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x^7*(8*c - d*
x^3)) - (13*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(12288*Sqrt[3]*c^(23/6)) + (13*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)
*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(36864*c^(23/6)) - (13*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(36864*c^(23/6)) - (Sqrt[2 - Sqrt[3]]*d^(7/3)
*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3584*
3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2688*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
```

```
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-10c-\frac{17dx^3}{2}}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{106c^2d+55cd^2x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1344c^3}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-64c^3d^2-265c^2d^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{43008c^5}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{x\left(2440c^4d^3-32c^3d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{344064c^7}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \left(\frac{32c^3d^3x}{\sqrt{c+dx^3}} + \frac{2184c^4d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{344064c^7}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{d^3\int \frac{x}{\sqrt{c+dx^3}}\,dx}{10752c^4} + \frac{\left(13d^3\right)\int \frac{}{\left(8c-d\right.}}{2048}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\left(13d^2\right)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,d}{24576c^4}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} -$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} -$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} -$$

Mathematica [C] time = 0.14255, size = 209, normalized size = 0.29

$$\frac{1525cd^3x^9\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\left(d^4x^{12}\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+20c\left(24\right.\right.}{3440640c^5x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (1525*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(384*c^4 + 648*c^3*d*x^3 + 243*c^2
*d^2*x^6 - 25*c*d^3*x^9 - 4*d^4*x^12) + d^4*x^12*(8*c - d*x^3)*Sqrt[1 + (d*
x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3440640*
c^5*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 3169, normalized size = 4.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x)
```

```
[Out] 3/4096/c^4*d^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*
(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
))))-3/4096*d^3/c^4*(-2/3*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/
d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
```

```
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/
d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3
*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3
))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2
*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/7*(d*x^3+c)^(1/2)/x^7-3/56*d*(d*x
^3+c)^(1/2)/c/x^4+15/112*d^2*(d*x^3+c)^(1/2)/c^2/x+5/112*I*d^2/c^2*3^(1/2)*
(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))+1/512*d^3/c^3*(-1/24/c*x^2*(d*x^3+c)^(1/
2)/(d*x^3-8*c)-1/72*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-
d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-
d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/216*I
/d^3/c*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3
))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2
*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
```

```
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d
*x^3+c)^(1/2)/c/x-1/8*I*d/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*
(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*
(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)^2 x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

3.411 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=134

$$\frac{1664c^3\sqrt{c+dx^3}}{d^4}-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

```
[Out] (1664*c^3*Sqrt[c + d*x^3])/d^4 + (3*x^6*(c + d*x^3)^(3/2))/(7*d^2) + (x^9*(
c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (2*c*(c + d*x^3)^(3/2)*(694*c + 51*
d*x^3))/(21*d^4) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rubi [A] time = 0.110103, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 97, 153, 147, 50, 63, 206}

$$\frac{1664c^3\sqrt{c+dx^3}}{d^4}-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (1664*c^3*Sqrt[c + d*x^3])/d^4 + (3*x^6*(c + d*x^3)^(3/2))/(7*d^2) + (x^9*(
c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (2*c*(c + d*x^3)^(3/2)*(694*c + 51*
d*x^3))/(21*d^4) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx=\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{(8c-dx)^2}\,dx,x,x^3\right)$$

$$=\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{\mathrm{Subst}\left(\int \frac{x^2\sqrt{c+dx}\left(3c+\frac{9dx}{2}\right)}{8c-dx}\,dx,x,x^3\right)}{3d}$$

$$=\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{2\,\mathrm{Subst}\left(\int \frac{x\sqrt{c+dx}\left(-72c^2d-\frac{255}{2}cd^2x\right)}{8c-dx}\,dx,x,x^3\right)}{21d^3}$$

$$=\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}-\frac{\left(832c^3\right)\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\ldots\right.}{d^3}$$

$$=\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}-\frac{(748\ldots}{}$$

$$=\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}-\frac{(149\ldots}{}$$

$$=\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}-\frac{4992\ldots}{}$$

Mathematica [A] time = 0.0690827, size = 111, normalized size = 0.83

$$\frac{2\left(\sqrt{c+dx^3}\left(301c^2d^2x^6+12206c^3dx^3-145328c^4+16cd^3x^9+d^4x^{12}\right)+52416c^{7/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{21d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*(Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c
*d^3*x^9 + d^4*x^12) + 52416*c^(7/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])]))/(21*d^4*(-8*c + d*x^3))
```

Maple [C] time = 0.039, size = 998, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^3*(d*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105/d*c
^2*x^3*(d*x^3+c)^(1/2)-4/105/d^2*c^3*(d*x^3+c)^(1/2))+32/15*c/d*(d*x^3+c)^(
5/2))+192*c^2/d^3*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3
*c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*
c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2
*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)
^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/1
8/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c)))+512*c^3/d^3*(-3/d*c*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+
c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)
^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_al
pha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89089, size = 562, normalized size = 4.19

$$\left[\frac{2\left(26208\left(c^{3}dx^{3}-8\,c^{4}\right)\sqrt{c}\log\left(\frac{dx^{3}-6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+\left(d^{4}x^{12}+16\,cd^{3}x^{9}+301\,c^{2}d^{2}x^{6}+12206\,c^{3}dx^{3}-145328\,c^{4}\right)\sqrt{dx^{3}}\right.}{21\left(d^{5}x^{3}-8\,cd^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 +
12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21*(5
2416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) +
(d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)*
sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12225, size = 171, normalized size = 1.28

$$\frac{4992\,c^{4}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^{4}}-\frac{1536\,\sqrt{dx^{3}+c}c^{4}}{\left(dx^{3}-8\,c\right)d^{4}}+\frac{2\left(\left(dx^{3}+c\right)^{\frac{7}{2}}d^{24}+21\left(dx^{3}+c\right)^{\frac{5}{2}}cd^{24}+448\left(dx^{3}+c\right)^{\frac{3}{2}}c^{2}d^{24}+15680\,\sqrt{d}\right.}{21\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d*
x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x^3
 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 + c)
*c^3*d^24)/d^28
```

3.412 $\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal. Leaf size=119

$$\frac{160c^2\sqrt{c+dx^3}}{d^3} - \frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

```
[Out] (160*c^2*Sqrt[c + d*x^3])/d^3 + (160*c*(c + d*x^3)^(3/2))/(27*d^3) + (2*(c
+ d*x^3)^(5/2))/(15*d^3) + (64*c*(c + d*x^3)^(5/2))/(27*d^3*(8*c - d*x^3))
- (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rubi [A] time = 0.0936134, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 89, 80, 50, 63, 206}

$$\frac{160c^2\sqrt{c+dx^3}}{d^3} - \frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (160*c^2*Sqrt[c + d*x^3])/d^3 + (160*c*(c + d*x^3)^(3/2))/(27*d^3) + (2*(c
+ d*x^3)^(5/2))/(15*d^3) + (64*c*(c + d*x^3)^(5/2))/(27*d^3*(8*c - d*x^3))
- (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(168c^2d+9cd^2x\right)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}$$

$$= \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{(80c)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d^2}$$

$$= \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(80c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d^2}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(720c^3\right)\text{Subst}\left(\int \frac{}{(8c-}\right.}{d^2}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(1440c^3\right)\text{Subst}\left(\int \frac{}{9c}\right.}{d^3}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{3\sqrt{c}}\right.}{d^3}$$

Mathematica [A] time = 0.0753497, size = 102, normalized size = 0.86

$$\frac{2\sqrt{c+dx^3}\left(2515c^2dx^3-29944c^3+62cd^2x^6+3d^3x^9\right)+21600c^{5/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{45d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9)
 + 21600*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^
3*(-8*c + d*x^3))
```

Maple [C] time = 0.014, size = 920, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/d^3+16*c/d^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)
^(1/2)/d+3*I/d^3*c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(
1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alph
a^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))+64*c^2/d^2*(-3/d*c*(d*x^3+c)^(1/2)/(d*x^3-8
*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*
(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2
*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*
d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88515, size = 509, normalized size = 4.28

$$\left[\frac{2\left(5400\left(c^2dx^3-8c^3\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(3d^3x^9+62cd^2x^6+2515c^2dx^3-29944c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^4x^3-8cd^3\right)}, \frac{2\left(10800\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 - 2
9944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 - 8*
c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^3*x^9 + 62*c*d^
2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12936, size = 150, normalized size = 1.26

$$\frac{480\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3}-\frac{192\sqrt{dx^3+c}c^3}{\left(dx^3-8c\right)d^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}+80\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+3120\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x^
3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x^3
 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15
```

3.413 $\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}+\frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)}+\frac{14\left(c+dx^3\right)^{3/2}}{27d^2}+\frac{14c\sqrt{c+dx^3}}{d^2}$$

```
[Out] (14*c*Sqrt[c + d*x^3])/d^2 + (14*(c + d*x^3)^(3/2))/(27*d^2) + (8*(c + d*x^
3)^(5/2))/(27*d^2*(8*c - d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^2
```

Rubi [A] time = 0.0748169, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 50, 63, 206}

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}+\frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)}+\frac{14\left(c+dx^3\right)^{3/2}}{27d^2}+\frac{14c\sqrt{c+dx^3}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```


```
[Out] (14*c*Sqrt[c + d*x^3])/d^2 + (14*(c + d*x^3)^(3/2))/(27*d^2) + (8*(c + d*x^
3)^(5/2))/(27*d^2*(8*c - d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^2
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2} \, dx = \frac{1}{3} \operatorname{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(8c-dx)^2} \, dx, x, x^3\right)$$

$$= \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{7 \operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx} \, dx, x, x^3\right)}{9d}$$

$$= \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{(7c) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx} \, dx, x, x^3\right)}{d}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{\left(63c^2\right) \operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}} \, dx, x, x^3\right)}{d}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{\left(126c^2\right) \operatorname{Subst}\left(\int \frac{1}{9c-x^2} \, dx, x, \sqrt{c+dx^3}\right)}{d^2}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{42c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Mathematica [A] time = 0.0520667, size = 90, normalized size = 0.93

$$\frac{2\sqrt{c+dx^3}\left(-524c^2+44cdx^3+d^2x^6\right)+378c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6) + 378*c^(3/2)*(8*c - d
*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))
```

Maple [C] time = 0.012, size = 902, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3*c*2^(1/2)*sum
((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(
1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*
d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2
*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)+8/d*c*(-3/d*c*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3
*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)
^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(
1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/
d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90032, size = 437, normalized size = 4.51

$$\left[\frac{189\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2\left(d^2x^6+44cdx^3-524c^2\right)\sqrt{dx^3+c}}{9\left(d^3x^3-8cd^2\right)}, \frac{2\left(189\left(cdx^3-8c^2\right)\sqrt{-c}\arctan\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(189*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c)
)/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c))/
(d^3*x^3 - 8*c*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12488, size = 126, normalized size = 1.3

$$\frac{42\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2}-\frac{24\sqrt{dx^3+c}c^2}{\left(dx^3-8c\right)d^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4+51\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 42*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 24*sqrt(d*x^3
+ c)*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*sqrt(d*x^3 +
 c)*c*d^4)/d^6
```

3.414 $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=77

$$\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{\sqrt{c+dx^3}}{d}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] Sqrt[c + d*x^3]/d + (c + d*x^3)^(3/2)/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rubi [A] time = 0.0600984, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {444, 47, 50, 63, 206}

$$\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{\sqrt{c+dx^3}}{d}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/d + (c + d*x^3)^(3/2)/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(8c-dx)^2}\,dx,x,x^3\right)$$
$$= \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)$$
$$= \frac{\sqrt{c+dx^3}}{d} + \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{1}{2}(9c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$= \frac{\sqrt{c+dx^3}}{d} + \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{(9c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d}$$
$$= \frac{\sqrt{c+dx^3}}{d} + \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Mathematica [C] time = 0.0148847, size = 43, normalized size = 0.56

$$\frac{2\left(c+dx^3\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{dx^3+c}{9c}\right)}{1215c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (c + d*x^3)/(9*c)])/(12
15*c^2*d)
```

Maple [C] time = 0.007, size = 451, normalized size = 5.9

$$-3\,\frac{c\sqrt{dx^3+c}}{d\left(dx^3-8\,c\right)} + \frac{2}{3\,d}\sqrt{dx^3+c} + \frac{\frac{i}{2}\sqrt{2}}{d^3}\sum_{\_alpha=RootOf\left(\_Z^3d-8\,c\right)} \sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -3/d*c*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*
sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1
/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*
3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)
+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alp
ha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-
d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c
*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*
c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72932, size = 373, normalized size = 4.84

$$\left[\frac{9\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2\left(2dx^3-25c\right)\sqrt{dx^3+c}}{6\left(d^2x^3-8cd\right)}, \frac{9\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(2dx^3-2\right.}{3\left(d^2x^3-8cd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d), 1
/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*d*
x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09158, size = 93, normalized size = 1.21

$$\frac{3\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{2\sqrt{dx^3+c}}{3\,d}-\frac{3\sqrt{dx^3+c}c}{\left(dx^3-8\,c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 2/3*sqrt(d*x^3 + c)
/d - 3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d)
```

3.415 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} \, dx$

Optimal. Leaf size=85

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Rubi [A] time = 0.0790129, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 98, 156, 63, 208, 206}

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)^2}\,dx,x,x^3\right)\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{\text{Subst}\left(\int \frac{-c^2d+\frac{7}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24cd}\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{1}{192}\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)-\frac{1}{64}(9d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{9}{32}\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{96d}\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0451007, size = 100, normalized size = 1.18

$$
\frac{36\sqrt{c}\sqrt{c+dx^3}+\left(9dx^3-72c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+\left(dx^3-8c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}\left(8c-dx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (36*Sqrt[c]*Sqrt[c + d*x^3] + (-72*c + 9*d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])] + (-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c]*(8
*c - d*x^3))
```

Maple [C] time = 0.013, size = 956, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/64*d/c^2*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3*c*2^(
1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/
3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)
*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2
*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*d-8*c))+1/8*d/c*(-3/d*c*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+c)^(1/2)/d
+1/2*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c
)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-
3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-
(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c))+1/64/c^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)
^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)
```

Fricas [A] time = 1.63902, size = 524, normalized size = 6.16

$$\left[\frac{9\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-72\sqrt{dx^3+c}c}{192\left(cdx^3-8c^2\right)}, \frac{\left(dx^3-8c\right)\sqrt{-c}\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/192*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + (d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*s
qrt(c) + 2*c)/x^3) - 72*sqrt(d*x^3 + c)*c)/(c*d*x^3 - 8*c^2), 1/96*((d*x^3
- 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 9*(d*x^3 - 8*c)*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 36*sqrt(d*x^3 + c)*c)/(c*d*x^3
- 8*c^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12132, size = 95, normalized size = 1.12

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}}+\frac{3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}}-\frac{3\sqrt{dx^3+c}}{8\left(dx^3-8c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) + 3/32*arctan(1/3*sqrt(d*x^3
 + c)/sqrt(-c))/sqrt(-c) - 3/8*sqrt(d*x^3 + c)/(d*x^3 - 8*c)
```

3.416 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=121

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}$$

```
[Out] (5*d*Sqrt[c + d*x^3])/(96*c*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*x^3*(8*c -
 d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(3/2))
```

Rubi [A] time = 0.103458, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(96*c*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*x^3*(8*c -
 d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)^2}\,dx,x,x^3\right)$$

$$=-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}-\frac{\mathrm{Subst}\left(\int \frac{-14c^2d-\frac{19}{2}cd^2x}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}$$

$$=\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{\mathrm{Subst}\left(\int \frac{126c^3d^2+45c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{1728c^3d}$$

$$=\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{(7d)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{768c}+\frac{\left(9d^2\right)\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{256c}$$

$$=\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{7\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{384c}+\frac{(9d)\,\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx,\right.}{128c}$$

$$=\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

Mathematica [A] time = 0.132811, size = 97, normalized size = 0.8

$$\frac{\frac{4\sqrt{c}\sqrt{c+dx^3}\left(4c-5dx^3\right)}{dx^6-8cx^3}+9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*(4*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(38
4*c^(3/2))
```

Maple [C] time = 0.015, size = 1014, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/256*d^2/c^3*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3*c*
2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^
(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1
/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d
*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))+1/64*d^2/c^2*(-3/d*c*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+c)
^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(
1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alph
a^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*
(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/256/c^3*d*(2/
9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)
^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)
```

Fricas [A] time = 1.64422, size = 636, normalized size = 5.26

$$\left[\frac{9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\left(5\,cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{768\left(c^2dx^6-8\,c^3x^3\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 - 2
*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^2*d*x^6 - 8*c^3*x^3), 1/384*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(
sqrt(d*x^3 + c)*sqrt(-c)/c) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) - 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d*
x^6 - 8*c^3*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1618, size = 153, normalized size = 1.26

$$\frac{1}{384}\,d\left(\frac{7\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{9\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{4\left(5\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c\right)}{\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/384*d*(7*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9*arctan(1/3*sqr
t(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 4*(5*(d*x^3 + c)^(3/2) - 9*sqrt(d*x^3
 + c)*c)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c))
```

3.417 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx$

Optimal. Leaf size=161

$$\frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)}+\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}-\frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

```
[Out] (7*d^2*Sqrt[c + d*x^3])/(512*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*x^6*(
8*c - d*x^3)) - (23*d*Sqrt[c + d*x^3])/(384*c*x^3*(8*c - d*x^3)) + (15*d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(2048*c^(5/2))
```

Rubi [A] time = 0.133409, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)}+\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}-\frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```


```
[Out] (7*d^2*Sqrt[c + d*x^3])/(512*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*x^6*(
8*c - d*x^3)) - (23*d*Sqrt[c + d*x^3])/(384*c*x^3*(8*c - d*x^3)) + (15*d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(2048*c^(5/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)^2}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-23c^2d-\frac{37}{2}cd^2x}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{102c^3d^2+\frac{69}{2}c^2d^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{384c^3}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-918c^4d^3-189c^3d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27648c^5d}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\left(17d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{4096c^2} + \cdots$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{(17d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{2048c^2} \cdots$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2\tanh^{-1}\left(\cdots\right.}{2048c^{5\cdots}}$$

Mathematica [A] time = 0.156764, size = 112, normalized size = 0.7

$$\frac{\frac{4\sqrt{c}\sqrt{c+dx^3}\left(32c^2+92cdx^3-21d^2x^6\right)}{dx^9-8cx^6}+45d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-51d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sqrt
```

```
[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))
```

Maple [C] time = 0.016, size = 1075, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x)
```

```
[Out] -3/4096*d^3/c^4*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*c*(d*x^3+c)^(1/2)/d+3*I/d^3*c
*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)
^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(
1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/
d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))+1/512*d^3/c^3*(-3/d*c*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+
c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)
^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_al
pha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/6*c*(d*x^3+c)^(1/2)/x^6-5/12
*d*(d*x^3+c)^(1/2)/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/
256/c^3*d*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arcta
nh((d*x^3+c)^(1/2)/c^(1/2)))+3/4096/c^4*d^2*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*
c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A] time = 1.63929, size = 707, normalized size = 4.39

$$\left[\frac{45\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+51\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\left(21\,cd^2x^6-92\,c^2dx^3\right.}{12288\left(c^3dx^9-8\,c^4x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d
*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*d*x
^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), 1/6144*(51*(d^3*x^9
- 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 45*(d^3*x^9 -
8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(21*c*d^2*
x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12683, size = 161, normalized size = 1.

$$\frac{1}{6144}\,d^2\left(\frac{51\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{45\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{36\,\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)c^2}-\frac{16\left(3\left(dx^3+c\right)^{\frac{3}{2}}-2\,\sqrt{dx^3+c}c\right)}{c^2d^2x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/6144*d^2*(51*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 45*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 36*sqrt(d*x^3 + c)/((d*x^3 -
 8*c)*c^2) - 16*(3*(d*x^3 + c)^(3/2) - 2*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^6))
```

3.418 $\int \frac{x^7\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=681

$$\frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[Out] (103*c*x^2*Sqrt[c + d*x^3])/(13*d^2) + (19*x^5*Sqrt[c + d*x^3])/(39*d) + (5
906*c^2*Sqrt[c + d*x^3])/(13*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 (x^5*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (108*Sqrt[3]*c^(13/6)*ArcTan
[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) - (108*c
^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/
3) + (108*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) - (2953*3^
(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(13*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (5906*Sqrt[2]*c^(7
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(13
*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.939014, antiderivative size = 681, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {467, 581, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{108\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{108c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{108c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+}}{3\sqrt{}}\right.}{d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (103*c*x^2*Sqrt[c + d*x^3])/(13*d^2) + (19*x^5*Sqrt[c + d*x^3])/(39*d) + (5
906*c^2*Sqrt[c + d*x^3])/(13*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 (x^5*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (108*Sqrt[3]*c^(13/6)*ArcTan
[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) - (108*c
^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/
3) + (108*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) - (2953*3^
(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(13*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (5906*Sqrt[2]*c^(7
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(13
*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
```

```
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
```

x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[8*b*c + a*d, 0]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rule 2145

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{x^7\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x^4\sqrt{c+dx^3}\left(5c+\frac{19dx^3}{2}\right)}{8c-dx^3}\,dx}{3d}$$

$$= \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\int \frac{x^4\left(-\frac{825c^2d}{2}-\frac{2163}{4}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{39d^2}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{4\int \frac{x\left(-8652c^3d^2-\frac{62013}{4}c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{4\int \left(\frac{62013c^2d^2x}{4\sqrt{c+dx^3}} - \frac{132678c^3d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{\left(2953c^2\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{13d^2} - \frac{\left(1944c^3\right)\int \frac{}{(8c-dx}}{d^2}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{\left(162c^2\right)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^3} + \frac{(295}{}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{2953\sqrt[4]{3}\sqrt{}}{}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{108\sqrt{3}c^{13}}{}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{108\sqrt{3}c^{13}}{}$$

Mathematica [C] time = 0.157897, size = 191, normalized size = 0.28

$$\frac{4120c^2x^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+2953cdx^5\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80x^2\left(-3\right.}{520d^2\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (80*x^2*(-412*c^3 - 388*c^2*d*x^3 + 25*c*d^2*x^6 + d^3*x^9) + 4120*c^2*x^2*
(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 2953*c*d*x^5*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(520*d^2*(-8*c + d*x^3)*Sqrt[
c + d*x^3])
```

Maple [C] time = 0.035, size = 2223, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^2*(2/13*x^5*d*(d*x^3+c)^(1/2)+32/91*c*x^2*(d*x^3+c)^(1/2)-18/91*I*c^2*3
^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))+16*c/d^2*(2/7*x^2*(d*x^3+c)^(1/
2)-44/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
```

```
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*s
um(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^
(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1
/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d
*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))+64*c^2/d^2*(-3/8*x^2/(d*x^3-8*c)*(d*x^3+c)^(1/2)-19/24*I*3^(1/
2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2
*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(
-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^
2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^
2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^7}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^7}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

3.419 $\int \frac{x^4\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=657

$$\frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d) + (265*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)
) + (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(5/3) - (9*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6
)*Sqrt[c + d*x^3])])/d^(5/3) + (9*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/d^(5/3) - (265*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (265*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.805319, antiderivative size = 657, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {467, 581, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}+\frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}\cdots}{7\sqrt[4]{3}d^{5/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d) + (265*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)
) + (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(5/3) - (9*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6
)*Sqrt[c + d*x^3])])/d^(5/3) + (9*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/d^(5/3) - (265*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (265*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^2 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x\sqrt{c+dx^3}\left(2c+\frac{13dx^3}{2}\right)}{8c-dx^3}\,dx}{3d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\int \frac{x\left(-111c^2d-\frac{795}{4}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{21d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\int \left(\frac{795cdx}{4\sqrt{c+dx^3}} - \frac{1701c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{21d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{(265c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{14d} - \frac{\left(162c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{(27c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2d^2} + \frac{(265c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{14d^{4/3}} -$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{14d^5}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

Mathematica [C] time = 0.160137, size = 176, normalized size = 0.27

$$-\frac{53dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+74cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+16x^2\left(37c^2+\right.}{112d\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] -(16*x^2*(37*c^2 + 35*c*d*x^3 - 2*d^2*x^6) + 74*c*x^2*(-8*c + d*x^3)*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 53*
d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^
3)/c), (d*x^3)/(8*c)])/(112*d*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 1747, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d*(2/7*x^2*(d*x^3+c)^(1/2)-44/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/
2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))
^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d
^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+
2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
```

```
^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^
(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+8/d*c*(-3/8*x^2/(d*x^3-8*c)*(d*x^3+c
)^(1/2)-19/24*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(
1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/
2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)
+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2
*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^
2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c
)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/
18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha
+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^4}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^7+cx^4\right)\sqrt{dx^3+c}}{d^2x^6-16\,cdx^3+64\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^7 + c*x^4)*sqrt(d*x^3 + c)/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(dx^3+c\right)^{\frac{3}{2}}x^4}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

3.420 $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=638

$$\frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{16d^{2/3}\sqrt{\cdots}}$$

```
[Out] (19*Sqrt[c + d*x^3])/(8*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*x
^2*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(16*d^(2/3)) - (9*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(16*d^(2/3))
+ (9*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(16*d^(2/3)) - (19*3^(1/
4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(16*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/
4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.718172, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {468, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{16d^{2/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (19*Sqrt[c + d*x^3])/(8*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*x
^2*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(16*d^(2/3)) - (9*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*d^(2/3))
+ (9*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(16*d^(2/3)) - (19*3^(1/
4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(16*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/
4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{\int \frac{x\left(-15c^2d-\frac{57}{2}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{\int \left(\frac{57cdx}{2\sqrt{c+dx^3}} - \frac{243c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{24cd}$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{19}{16}\int \frac{x}{\sqrt{c+dx^3}}\,dx - \frac{1}{8}(81c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{27\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{32d} + \frac{19\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{16\sqrt[3]{d}} - \frac{\left(27\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{32\sqrt[3]{d}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\right.}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}} - \frac{9\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\right.}{3\sqrt[6]{c}\sqrt{}}\right.}{16d^{2/3}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}} - \frac{9\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\right.}{3\sqrt[6]{c}\sqrt{}}\right.}{16d^{2/3}}$$

Mathematica [C] time = 0.182889, size = 141, normalized size = 0.22

$$\frac{x^2\left(-\frac{19dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c} - 25\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + \frac{240\left(c+dx^3\right)}{8c-dx^3}\right)}{640\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (x^2*((240*(c + d*x^3))/(8*c - d*x^3) - 25*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - (19*d*x^3*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c))/(640*Sqrt[c +
 d*x^3])
```

Maple [C] time = 0.007, size = 873, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -3/8*x^2/(d*x^3-8*c)*(d*x^3+c)^(1/2)-19/24*I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2
*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(
I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)
^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^4+cx\right)\sqrt{dx^3+c}}{d^2x^6-16\,cdx^3+64\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x)*sqrt(d*x^3 + c)/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

3.421 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=522

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\ldots}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}{32c^{2/3}\sqrt{\ldots}}$$

```
[Out] -Sqrt[c + d*x^3]/(16*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(16*c*((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x*(8*c - d*x^3)) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*Sqrt[2]*3^(1/4)*c^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.256064, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {468, 21, 325, 303, 218, 1877}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)\ldots}}}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\ldots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -Sqrt[c + d*x^3]/(16*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(16*c*((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x*(8*c - d*x^3)) - (3^(1/4)*
```

```
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*Sqrt[2]*3^(1/4)*c^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx &= \frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}+\frac{\int \frac{12c^2d-\frac{3}{2}cd^2x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}\\
&= \frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}+\frac{1}{16}\int \frac{1}{x^2\sqrt{c+dx^3}}\,dx\\
&= -\frac{\sqrt{c+dx^3}}{16cx}+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}+\frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c}\\
&= -\frac{\sqrt{c+dx^3}}{16cx}+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}+\frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c}+\frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{16c^{2/3}}\\
&= -\frac{\sqrt{c+dx^3}}{16cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{32c^{2/3}\sqrt{\frac{\sqrt[3]{\cdots}}{\left(\left(1+\cdots\right.\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.618073, size = 242, normalized size = 0.46

$$\frac{\left(2c-dx^3\right)\sqrt{c+dx^3}}{16cx\left(dx^3-8c\right)}-\frac{\sqrt[6]{-1}\sqrt[3]{-d}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}-1\right)}\sqrt{\frac{(-d)^{2/3}x^2}{c^{2/3}}+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+1}\left(\sqrt[3]{-1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right),\sqrt[3]{-\cdots}\right)\right)}{16\sqrt[4]{3}\sqrt[3]{c}\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] ((2*c - d*x^3)*Sqrt[c + d*x^3])/(16*c*x*(-8*c + d*x^3)) - ((-1)^(1/6)*(-d)^
(1/3)*Sqrt[(-1)^(5/6)*(-1 + ((-d)^(1/3)*x)/c^(1/3))]*Sqrt[1 + ((-d)^(1/3)*x
)/c^(1/3) + ((-d)^(2/3)*x^2)/c^(2/3)]*((-I)*Sqrt[3]*EllipticE[ArcSin[Sqrt[-
(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/3)*E
llipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)
^(1/3)]))/(16*3^(1/4)*c^(1/3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 2217, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/64/c^2*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/2)*(-
d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/64*d/c^2*(2/7*x^2*(d*x^3+c)^(1/2)-44/7*I
*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d
```

```
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alp
ha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3
^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alph
a*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d
^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*
d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c
)))+1/8*d/c*(-3/8*x^2/(d*x^3-8*c)*(d*x^3+c)^(1/2)-19/24*I*3^(1/2)/d*(-d^2*c
)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_
alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}}}{\left(dx^3 - 8\,c\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^3 + c\right)^{\frac{3}{2}}}{d^2x^8 - 16\,cdx^5 + 64\,c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^(3/2)/(d^2*x^8 - 16*c*d*x^5 + 64*c^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

3.422 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx$

Optimal. Leaf size=684

$$\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9\sqrt{3}d^{\cdots}}{\cdots}$$

```
[Out] (-13*Sqrt[c + d*x^3])/(256*c*x^4) - (d*Sqrt[c + d*x^3])/(32*c^2*x) + (d^(4/
3)*Sqrt[c + d*x^3])/(32*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[
c + d*x^3])/(8*x^4*(8*c - d*x^3)) - (9*Sqrt[3]*d^(4/3)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(4/3)*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)
) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3^(
1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(64*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[2]*3^(1/
4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.940406, antiderivative size = 684, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {468, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}+\frac{d^{4}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] (-13*Sqrt[c + d*x^3])/(256*c*x^4) - (d*Sqrt[c + d*x^3])/(32*c^2*x) + (d^(4/
3)*Sqrt[c + d*x^3])/(32*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[
c + d*x^3])/(8*x^4*(8*c - d*x^3)) - (9*Sqrt[3]*d^(4/3)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(4/3)*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(1024*c^(11/6)
) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(1024*c^(11/6)) - (3^(
1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(64*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[2]*3^(1/
4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{\int \frac{39c^2d+\frac{51}{2}cd^2x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{\int \frac{-192c^3d^2-\frac{195}{2}c^2d^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{768c^3d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{\int \frac{x\left(1740c^4d^3-96c^3d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{6144c^5d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{\int \left(\frac{96c^3d^3x}{\sqrt{c+dx^3}} + \frac{972c^4d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{6144c^5d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{64c^2} + \frac{\left(81d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{512c}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{(27d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^2} + \frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt{c+\cdots}}}{64c^2}\cdots$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\cdots\right.}{\cdots}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{1024c^{11\cdots}}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{1024c^{11\cdots}}$$

Mathematica [C] time = 0.134494, size = 199, normalized size = 0.29

$$-\frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{2560c^3\sqrt{c+dx^3}}+\frac{145d^2x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{8192c^2\sqrt{c+dx^3}}+\sqrt{c+dx^3}\left(-\frac{3d^2x^2}{512c^2\left(dx^3-8c\right)}-\frac{13d}{512c^2x}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/(256*c*x^4) - (13*d)/(512*c^2*x) - (3*d^2*x^2)/(512*c^2
*(-8*c + d*x^3))) + (145*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1,
5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(8192*c^2*Sqrt[c + d*x^3]) - (d^3*x^5*Sq
rt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/
(2560*c^3*Sqrt[c + d*x^3])
```

Maple [C] time = 0.015, size = 2690, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/256/c^3*d*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/2)
*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/256*d^2/c^3*(2/7*x^2*(d*x^3+c)^(1/2)-
44/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
```

```
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*sum(
1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/
3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)
*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2
*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*d-8*c)))+1/64*d^2/c^2*(-3/8*x^2/(d*x^3-8*c)*(d*x^3+c)^(1/2)-19/24*I*3^(1/2
)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2*
c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^
2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-
d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2
*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2
*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/
c^2*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d*3^(1/2)*(-
d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
```

```
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{d^2x^{11}-16\,cdx^8+64\,c^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^(3/2)/(d^2*x^11 - 16*c*d*x^8 + 64*c^2*x^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

3.423 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx$

Optimal. Leaf size=708

$$\frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-19d$$

```
[Out] (-11*Sqrt[c + d*x^3])/(224*c*x^7) - (83*d*Sqrt[c + d*x^3])/(7168*c^2*x^4) -
 (19*d^2*Sqrt[c + d*x^3])/(1792*c^3*x) + (19*d^(7/3)*Sqrt[c + d*x^3])/(1792
*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x^7*(8*c
 - d*x^3)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*
x))/Sqrt[c + d*x^3]])/(4096*c^(17/6)) + (9*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/
3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(4096*c^(17/6)) - (9*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4096*c^(17/6)) - (19*3^(1/4)*Sqrt[2 - Sqrt[3
]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3584*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*Sqrt[2]*3^(1/4)*c^(8/3)*Sqr
t[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3])
```

Rubi [A] time = 1.07565, antiderivative size = 708, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {468, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}-\frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}}-\frac{9d^{7/3}\tan}{409}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-11*Sqrt[c + d*x^3])/(224*c*x^7) - (83*d*Sqrt[c + d*x^3])/(7168*c^2*x^4) -
 (19*d^2*Sqrt[c + d*x^3])/(1792*c^3*x) + (19*d^(7/3)*Sqrt[c + d*x^3])/(1792
*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x^7*(8*c
 - d*x^3)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*
x))/Sqrt[c + d*x^3]])/(4096*c^(17/6)) + (9*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/
3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(4096*c^(17/6)) - (9*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4096*c^(17/6)) - (19*3^(1/4)*Sqrt[2 - Sqrt[3
]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3584*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*Sqrt[2]*3^(1/4)*c^(8/3)*Sqr
t[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
```

```
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\int \frac{66c^2d+\frac{105}{2}cd^2x^3}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \frac{-498c^3d^2-363c^2d^3x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1344c^3d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\int \frac{3648c^4d^3+1245c^3d^4x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{43008c^5d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-28200c^5d^4+1824c^4d^5x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{344064c^7d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \left(-\frac{1824c^4d^4x}{\sqrt{c+dx^3}} - \frac{13608c^5d^4x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{344064c^7d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\left(19d^3\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{3584c^3} + \frac{\left(81d^3\right)\int}{}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\left(27d^2\right)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}}{8192c^3}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}$$

Mathematica [C] time = 0.150801, size = 212, normalized size = 0.3

$$-\frac{19d^4x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{143360c^4\sqrt{c+dx^3}}+\frac{1175d^3x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{229376c^3\sqrt{c+dx^3}}+\sqrt{c+dx^3}\left(-\frac{3d^3x^2}{4096c^3\left(dx^3-8c\right)}-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/(448*c*x^7) - (41*d)/(7168*c^2*x^4) - (283*d^2)/(28672*
c^3*x) - (3*d^3*x^2)/(4096*c^3*(-8*c + d*x^3))) + (1175*d^3*x^2*Sqrt[(c + d
*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(229376*c
^3*Sqrt[c + d*x^3]) - (19*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1,
 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(143360*c^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.014, size = 3186, normalized size = 4.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x)
```

```
[Out] 3/4096/c^4*d^2*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1
/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3
^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-3/4096*d^3/c^4*(2/7*x^2*(d*x^3+c)^(1
/2)-44/7*I*c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3
))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
```

```
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3*I*c/d^3*2^(1/2)*
sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)
^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(
1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/
d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))+1/64/c^2*(-1/7*c*(d*x^3+c)^(1/2)/x^7-17/56*d*(d*x^3+c)^(1/2)/
x^4-27/112*d^2/c*(d*x^3+c)^(1/2)/x-9/112*I*d^2/c*3^(1/2)*(-d^2*c)^(1/3)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2
*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(
I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2))))+1/512*d^3/c^3*(-3/8*x^2/(d*x^3-8*c)*(d*x^3+c)^(1/2)-19/24*
I*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alph
a*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3
)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^
(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha
```

```
*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^
2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)
))+1/256/c^3*d*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d
*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

3.424 $\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=95

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}+\frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4}$$

```
[Out] (8*x^6*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (2*Sqrt[c + d*x^3]*(170*c
+ 7*d*x^3))/(27*d^4) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(81*d^4)
```

Rubi [A] time = 0.0727786, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 98, 147, 63, 206}

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}+\frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```


```
[Out] (8*x^6*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (2*Sqrt[c + d*x^3]*(170*c
+ 7*d*x^3))/(27*d^4) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(81*d^4)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x\left(16c^2+21cdx\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^2}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(1472c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^3}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(2944c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^4}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

Mathematica [A] time = 0.0493686, size = 91, normalized size = 0.96

$$\frac{6\sqrt{c+dx^3}\left(-1360c^2+114cdx^3+3d^2x^6\right)+2944c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (6*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6) + 2944*c^(3/2)*(8*
c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4*(-8*c + d*x^3))
```

Maple [C] time = 0.036, size = 916, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/d^3*(d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)+32/3*c*(d*x^
3+c)^(1/2)/d)+64/9*I*c/d^6*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-
d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d
*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)
+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*
c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+512*c^3/d^3*(-1/27/d/c*(d*x^3+c)^(1/
2)/(d*x^3-8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/
d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(
2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82425, size = 455, normalized size = 4.79

$$\left[\frac{2\left(736\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(3d^2x^6+114cdx^3-1360c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3-8cd^4\right)}, \frac{2\left(1472\left(cdx^3-8c^2\right)\sqrt{-c}\,\mathrm{ar}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(736*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt(d*x^3
 + c))/(d^5*x^3 - 8*c*d^4), 2/81*(1472*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/
3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt
(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1555, size = 126, normalized size = 1.33

$$\frac{2944\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^4}-\frac{512\sqrt{dx^3+c}c^2}{27\left(dx^3-8\,c\right)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^8+45\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2944/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/27*sq
rt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^4) + 2/9*((d*x^3 + c)^(3/2)*d^8 + 45*sqr
t(d*x^3 + c)*c*d^8)/d^12
```

3.425 $\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)}+\frac{2\sqrt{c+dx^3}}{3d^3}-\frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) + (64*c*Sqrt[c + d*x^3])/(27*d^3*(8*c - d*x^3))
 - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rubi [A] time = 0.0636044, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 89, 80, 63, 206}

$$\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)}+\frac{2\sqrt{c+dx^3}}{3d^3}-\frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) + (64*c*Sqrt[c + d*x^3])/(27*d^3*(8*c - d*x^3))
 - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{40c^2d+9cd^2x}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^3}$$
$$= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{(112c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^2}$$
$$= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{(224c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^3}$$
$$= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Mathematica [A] time = 0.041675, size = 82, normalized size = 0.99

$$-\frac{64c\sqrt{c+dx^3}}{27d^3\left(dx^3-8c\right)} + \frac{2\sqrt{c+dx^3}}{3d^3} - \frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) - (64*c*Sqrt[c + d*x^3])/(27*d^3*(-8*c + d*x^3)
) - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Maple [C] time = 0.013, size = 874, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^3+16/27*I/d^5*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*
(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2
*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*
d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+64*c^2/d^2*(-1/27/d/c*(d*x^
3+c)^(1/2)/(d*x^3-8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*
(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3
*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78132, size = 390, normalized size = 4.7

$$\left[\frac{2\left(56\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(9dx^3-104c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3-8cd^3\right)}, \frac{2\left(112\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\right.}{81\left(d^4x^3-8cd^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^
3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)
 + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.10507, size = 93, normalized size = 1.12

$$\frac{224\, c \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}d^3} + \frac{2\,\sqrt{dx^3 + c}}{3\,d^3} - \frac{64\,\sqrt{dx^3 + c}c}{27\left(dx^3 - 8\,c\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 224/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) + 2/3*sqrt(d*x
^3 + c)/d^3 - 64/27*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^3)
```

3.426 $\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

```
[Out] (8*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Rubi [A] time = 0.0506441, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 78, 63, 206}

$$\frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{5\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27d}\\
&= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^2}\\
&= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}
\end{aligned}
$$

Mathematica [A] time = 0.04724, size = 63, normalized size = 0.98

$$
-\frac{8\sqrt{c+dx^3}}{27d^2\left(dx^3-8c\right)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Maple [C] time = 0.012, size = 861, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/27*I/d^4/c*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/
(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^
2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))+8/d*c*(-1/27/d/c*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/486
*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c
)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-
3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-
(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79474, size = 356, normalized size = 5.56

$$\left[\frac{5\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-24\sqrt{dx^3+c}c}{81\left(cd^3x^3-8c^2d^2\right)},\frac{2\left(5\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-12\sqrt{dx^3+c}c\right)}{81\left(cd^3x^3-8c^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(5*
(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x
^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.10975, size = 78, normalized size = 1.22

$$\frac{2\left(\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{12\sqrt{dx^3+c}}{\left(dx^3-8c\right)d}\right)}{81d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3 +
 c)/((d*x^3 - 8*c)*d))/d
```

3.427 $\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}$$

```
[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)]/(81*c^(3/2)*d)
```

Rubi [A] time = 0.0493981, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)]/(81*c^(3/2)*d)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\ &= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}+\frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{54c}\\ &= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}+\frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27cd}\\ &= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}\end{aligned}$$

Mathematica [A] time = 0.0294008, size = 64, normalized size = 0.96

$$\frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3}+\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])])/(81*c^(3/2)*d)
```

Maple [C] time = 0.007, size = 442, normalized size = 6.6

$$-\frac{1}{27\,cd\left(dx^3-8\,c\right)}\sqrt{dx^3+c}-\frac{\frac{i}{486}\sqrt{2}}{d^3c^2}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^3d-8\,c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27/d/c*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2
*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82914, size = 351, normalized size = 5.24

$$\left[\frac{\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{162\,\left(c^2d^2x^3-8\,c^3d\right)},-\frac{\left(dx^3-8\,c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,\sqrt{dx^3+c}c}{81\,\left(c^2d^2x^3-8\,c^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((d*
x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 +
 c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.10711, size = 80, normalized size = 1.19

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}cd}-\frac{\sqrt{dx^3+c}}{27\,\left(dx^3-8\,c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^3
 + c)/((d*x^3 - 8*c)*c*d)
```

3.428 $\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

```
[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))
```

Rubi [A] time = 0.075345, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 103, 156, 63, 208, 206}

$$\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2d}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2} + \frac{(13d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^2}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{864c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right.}{96c^2d}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

Mathematica [A] time = 0.0731727, size = 83, normalized size = 0.94

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + 13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 27\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))
```

Maple [C] time = 0.014, size = 880, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/1728*I/d^2/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*
(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1
/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d
*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha
^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*
3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*
(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*d-8*c))+1/8*d/c*(-1/27/d/c*(d*x^3+c)^(1/2)/(d*x^3-8*c
)-1/486*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*
(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1
/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d
*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha
^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*
3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*
(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*d-8*c)))-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)
```

Fricas [A] time = 1.85705, size = 544, normalized size = 6.18

$$\left[\frac{13\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+27\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-24\sqrt{dx^3+c}c}{5184\left(c^3dx^3-8c^4\right)}, \frac{27\left(dx^3-8c\right)\sqrt{-c \ldots}}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5184*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
 c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), 1/2592
*(27*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 13*(d*x^3
- 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)
*c)/(c^3*d*x^3 - 8*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.12233, size = 107, normalized size = 1.22

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^2}-\frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}c^2}-\frac{\sqrt{dx^3+c}}{216\left(dx^3-8c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 13/2592*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/216*sqrt(d*x^3 + c)/((d*x^3 - 8
*c)*c^2)
```

3.429 $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=124

$$\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

```
[Out] (5*d*Sqrt[c + d*x^3])/(864*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c^2*x^3
*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2
)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(7/2))
```

Rubi [A] time = 0.10125, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```


```
[Out] (5*d*Sqrt[c + d*x^3])/(864*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c^2*x^3
*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2
)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(7/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{2cd-\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{-18c^2d^2+5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^4d}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c^3} + \frac{\left(11d^2\right)\text{Subst}\left(\right.}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c^3} + \frac{(11d)\,\text{Sub}}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

Mathematica [A] time = 0.133074, size = 97, normalized size = 0.78

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(36c-5dx^3\right)}{dx^6-8cx^3} + 11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 27d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{10368c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(10368*c^(7/2))
```

Maple [C] time = 0.014, size = 926, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/6912*I/d/c^4*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3
))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2
*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c))+1/64*d^2/c^2*(-1/27/d/c*(d*x^3+c)^(1/2)/(d*x^3-
8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)
^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_al
pha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*
d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/384*d*arctanh((d*x^3+c)^(1/2)
/c^(1/2))/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3 + c}\left(dx^3 - 8\,c\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^4), x)
```

Fricas [A] time = 1.78352, size = 653, normalized size = 5.27

$$\left[\frac{11\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\left(5\,cdx^3-36\,c^2\right)\sqrt{dx^3}}{20736\left(c^4dx^6-8\,c^5x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*s
qrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3
 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(d*x
^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(27*(d^2*x^6 - 8*c*d*x^3)*sqrt(-
c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 11*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3
+ c))/(c^4*d*x^6 - 8*c^5*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13434, size = 153, normalized size = 1.23

$$-\frac{1}{10368}\,d\left(\frac{27\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{11\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^3}+\frac{12\left(5\left(dx^3+c\right)^{\frac{3}{2}}-41\,\sqrt{dx^3+c}c\right)}{\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10368*d*(27*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 11*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 12*(5*(d*x^3 + c)^(3/2) - 41
*sqrt(d*x^3 + c)*c)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^3))
```

3.430 $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=164

$$-\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)}+\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)}$$

```
[Out] (-35*d^2*Sqrt[c + d*x^3])/(13824*c^4*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c
^2*x^6*(8*c - d*x^3)) + (3*d*Sqrt[c + d*x^3])/(128*c^3*x^3*(8*c - d*x^3)) +
 (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))
```

Rubi [A] time = 0.128709, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$-\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)}+\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-35*d^2*Sqrt[c + d*x^3])/(13824*c^4*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c
^2*x^6*(8*c - d*x^3)) + (3*d*Sqrt[c + d*x^3])/(128*c^3*x^3*(8*c - d*x^3)) +
 (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{9cd-\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{38c^2d^2-\frac{27}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^4}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-342c^3d^3+3}{x(8c-dx)\sqrt{}}\right.}{27648c}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{\left(19d^2\right)\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{12288c}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{(19d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+}\right.}{614}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right.}{165888c^{9/2}}$$

Mathematica [A] time = 0.180005, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(288c^2-324cdx^3+35d^2x^6\right)}{dx^9-8cx^6}+31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-513d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{165888c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x^
6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))
```

Maple [C] time = 0.013, size = 989, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/36864*I/c^5*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3)
)/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*
d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d-8*c))+1/512*d^3/c^3*(-1/27/d/c*(d*x^3+c)^(1/2)/(d*x^3-
8*c)-1/486*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)
^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_al
pha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*
d*(d*x^3+c)^(1/2)/c^2/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))
+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1
/2))/c^(3/2))-1/2048*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A] time = 1.73586, size = 722, normalized size = 4.4

$$\left[\frac{31\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+513\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+24\left(35\,cd^2x^6-324\,c\right.}{331776\left(c^5dx^9-8\,c^6x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log(
(d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*c^2
*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6*x^6), 1/165888*(513*(
d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 31*(d^
3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(
35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6
*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09662, size = 158, normalized size = 0.96

$$\frac{1}{165888}\,d^2\left(\frac{513\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^4}-\frac{31\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^4}-\frac{12\,\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)c^4}+\frac{432\left(\left(dx^3+c\right)^{\frac{3}{2}}-2\,\sqrt{dx^3+c}c\right)}{c^4d^2x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/165888*d^2*(513*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 12*sqrt(d*x^3 + c)/((d*x^
3 - 8*c)*c^4) + 432*((d*x^3 + c)^(3/2) - 2*sqrt(d*x^3 + c)*c)/(c^4*d^2*x^6)
)
```

3.431 $\int \frac{x^7}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=641

$$\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{9\;3^{3/4}d^{8\cdots}}$$

```
[Out] (62*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (8*
x^2*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (44*c^(1/6)*ArcTan[(Sqrt[3]*c
^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) - (44*
c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d
^(8/3)) + (44*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) -
(31*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (62*Sqrt[2]*c
^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c
^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(27*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.711494, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {470, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\right)\cdots\right.}}}{9\;3^{3/4}d^{8/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (62*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (8*
x^2*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (44*c^(1/6)*ArcTan[(Sqrt[3]*c
^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) - (44*
c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d
^(8/3)) + (44*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) -
(31*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (62*Sqrt[2]*c
^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c
^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(27*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

<u>Rule 470</u>

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

<u>Rule 584</u>

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

<u>Rule 303</u>

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

<u>Rule 218</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\int \frac{x\left(16c^2+31cdx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\int \left(-\frac{31cx}{\sqrt{c+dx^3}} + \frac{264c^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{31\int \frac{x}{\sqrt{c+dx^3}}dx}{27d^2} - \frac{(88c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{9d^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{22\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{27d^3} + \frac{31\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{27d^{7/3}} - \frac{\left(22\sqrt[3]{c}\right)\int \cdots}{27d\cdots}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{9\,3^{3/4}d^{8/3}\sqrt{\frac{\cdots}{\left(\left(1+\cdots\right.\right.}}}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{44\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} - \frac{44\sqrt[6]{c}\,\mathrm{t}\cdots}{\cdots}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{44\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} - \frac{44\sqrt[6]{c}\,\mathrm{t}\cdots}{\cdots}$$

Mathematica [C] time = 0.146808, size = 167, normalized size = 0.26

$$\frac{31dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+40cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+320cx^2\left(c+dx\cdots\right.}{1080cd^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (320*c*x^2*(c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 31*d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(1080*c*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.035, size = 1737, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/d^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+16/27*I/d^5*2^(1/2)*su
m(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(
1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c
)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/
3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*
(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c))+64*c^2/d^2*(-1/216/c^2*x^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/648*I/c
^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellipt
```

```
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum
(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2
*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1
/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)
^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(
1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3
)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(
2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(
1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**7/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.432 $\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=647

$$\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\left(\ldots\right.}{3\ldots}\right.}{81c^{5/6\ldots}}$$

```
[Out] Sqrt[c + d*x^3]/(27*c*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(27*c*d*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
 d^(1/3)*x))/Sqrt[c + d*x^3]]/(27*Sqrt[3]*c^(5/6)*d^(5/3)) - ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(81*c^(5/6)*d^(5/3)) + ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3
/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rubi [A] time = 0.714585, antiderivative size = 647, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {471, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+(1\ldots}{\sqrt[3]{d}x+(1\ldots}\right.\right.}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(27*c*d*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
 d^(1/3)*x))/Sqrt[c + d*x^3]]/(27*Sqrt[3]*c^(5/6)*d^(5/3)) - ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(81*c^(5/6)*d^(5/3)) + ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3
/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{\int \frac{x\left(2c+\frac{dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{6cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{2\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{54cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{54cd^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{54cd^{4/3}} - \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{54c^{2/3}d^{4/3}} \ldots$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2} \ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d} \ldots\right.}}}{18\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\ldots}} \ldots$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} - \frac{\tanh^{-1}\left(\ldots\right)}{81c^{5/6} \ldots} \ldots$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} - \frac{\tanh^{-1}\left(\ldots\right)}{81c^{5/6} \ldots} \ldots$$

Mathematica [C] time = 0.0833419, size = 166, normalized size = 0.26

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+10cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(c+dx^3\right)}{2160c^2d\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
2160*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 1304, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/27*I/d^4/c*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c
)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_a
lpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d-8*c))+8/d*c*(-1/216/c^2*x^2*(d*x^3+c)^(1/2)/(d*
x^3-8*c)-1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2
*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/
3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2
*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-7/1944*I/c
^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-
d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3
))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(
```

```
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2
*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^
(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}x^4}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**4/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.433 $\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=644

$$\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{7\tan}{4}$$

```
[Out] Sqrt[c + d*x^3]/(216*c^2*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^
2*Sqrt[c + d*x^3])/(216*c^2*(8*c - d*x^3)) - (7*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqrt[3]*c^(11/6)*d^(2/3)) + (7*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1296*c^(11/6)
*d^(2/3)) - (7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1296*c^(11/6)*d^(2/3)
) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(
1/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.699168, antiderivative size = 644, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {472, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}}+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^
2*Sqrt[c + d*x^3])/(216*c^2*(8*c - d*x^3)) - (7*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqrt[3]*c^(11/6)*d^(2/3)) + (7*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(1296*c^(11/6)
*d^(2/3)) - (7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1296*c^(11/6)*d^(2/3)
) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(
1/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\int \frac{x\left(25cd-\frac{d^2x^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{21cdx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^2} + \frac{7\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{72c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^2d} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{432c^2\sqrt[3]{d}} + \frac{7\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\cdots}{864c^{5/3}\sqrt[3]{d}}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\cdots\right.}}}{144\ 3^{3/4}c^{5/3}d^{2/3}\cdots}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}} + \frac{7\tanh\cdots}{129\cdots}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}} + \frac{7\tanh\cdots}{129\cdots}$$

Mathematica [C] time = 0.07337, size = 164, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+125cx^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(c+dx^3\right)\cdots}{17280c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 125*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
17280*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.009, size = 882, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/216/c^2*x^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(
1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1
/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}x}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.434 $\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=665

$$\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{288\ 3^{3/4}c^{8/3}\cdots}$$

```
[Out] (-7*Sqrt[c + d*x^3])/(432*c^3*x) + (7*d^(1/3)*Sqrt[c + d*x^3])/(432*c^3*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*c^2*x*(8*c - d*x^3
)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3
]])/(216*Sqrt[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(648*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(648*c^(17/6)) - (7*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (7*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.828396, antiderivative size = 665, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\cdots}}{288\ 3^{3/4}c^{8/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-7*Sqrt[c + d*x^3])/(432*c^3*x) + (7*d^(1/3)*Sqrt[c + d*x^3])/(432*c^3*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*c^2*x*(8*c - d*x^3
)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3
]])/(216*Sqrt[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(648*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(648*c^(17/6)) - (7*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (7*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} + \frac{\int \frac{28cd+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-160c^2d^2+14cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1728c^4d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\int \left(-\frac{14cd^2x}{\sqrt{c+dx^3}} - \frac{48c^2d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{1728c^4d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} + \frac{(7d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{864c^3} + \frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{36c^2}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{432c^3} + \frac{\left(7d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\cdots}{\sqrt{c+dx^3}}\cdots}{864c^3}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}\cdots\right.}{\cdots}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{\cdots}\right.}{\sqrt{c+\cdots}}\cdots\right.}{216\sqrt{3}c^{17/6}\cdots}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{\cdots}\right.}{\sqrt{c+\cdots}}\cdots\right.}{216\sqrt{3}c^{17/6}\cdots}$$

Mathematica [C] time = 0.122201, size = 180, normalized size = 0.27

$$\frac{7d^2x^6\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+200cdx^3\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(54c^2+4\right.}{34560c^4\sqrt{c+dx^3}\left(8cx-dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(54*c^2 + 47*c*d*x^3 - 7*d^2*x^6) + 200*c*d*x^3*(8*c - d*x^3)*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d
^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/(34560*c^4*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] time = 0.014, size = 1761, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64/c^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2))))-1/1728*I/d^2/c^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)
^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
```

```
)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8*d/c*(-1/216/c^2*x^2*(d*x^3
+c)^(1/2)/(d*x^3-8*c)-1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*
(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/
3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^
2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^{11}-15\,cd^2x^8+48\,c^2dx^5+64\,c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^3*x^11 - 15*c*d^2*x^8 + 48*c^2*d*x^5 + 64*c^3*x
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

3.435 $\int \frac{1}{x^5(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=687

$$-\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\ldots$$

```
[Out] (-31*Sqrt[c + d*x^3])/(6912*c^3*x^4) + (5*d*Sqrt[c + d*x^3])/(864*c^4*x) -
(5*d^(4/3)*Sqrt[c + d*x^3])/(864*c^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 Sqrt[c + d*x^3]/(216*c^2*x^4*(8*c - d*x^3)) - (25*d^(4/3)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27648*Sqrt[3]*c^(23/6)) +
 (25*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/
(82944*c^(23/6)) - (25*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(82944
*c^(23/6)) + (5*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(576*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(
1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
- (5*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(432*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.927277, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{25d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{23/6}}+\frac{25d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}}-\frac{25d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-31*Sqrt[c + d*x^3])/(6912*c^3*x^4) + (5*d*Sqrt[c + d*x^3])/(864*c^4*x) -
(5*d^(4/3)*Sqrt[c + d*x^3])/(864*c^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 Sqrt[c + d*x^3]/(216*c^2*x^4*(8*c - d*x^3)) - (25*d^(4/3)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27648*Sqrt[3]*c^(23/6)) +
 (25*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/
(82944*c^(23/6)) - (25*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(82944
*c^(23/6)) + (5*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(576*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(
1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
- (5*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(432*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}+\frac{\int \frac{31cd+\frac{11d^2x^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}-\frac{\int \frac{320c^2d^2-\frac{155}{2}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{6912c^4d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}+\frac{\int \frac{x\left(-980c^3d^3+160c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{55296c^6d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}+\frac{\int \left(-\frac{160c^2d^3x}{\sqrt{c+dx^3}}+\frac{300c^3d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{55296c^6d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}-\frac{\left(5d^2\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1728c^4}+\frac{\left(25d^2\right)\int \frac{}{(8c}}{46}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}-\frac{(25d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{55296c^4}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}+\frac{5}{}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}-\frac{25}{}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}-\frac{25}{}$$

Mathematica [C] time = 0.137667, size = 196, normalized size = 0.29

$$\frac{245cd^2x^6\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16\left(d^3x^9\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+2c\left(-135c\right.\right.}{221184c^5x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (245*c*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(2*c*(216*c^3 - 135*c^2*d*x^3 - 311*c*d
^2*x^6 + 40*d^3*x^9) + d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(221184*c^5*x^4*(8*c - d*x
^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 2240, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/256/c^3*d*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/
2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))
^(1/2))))-1/6912*I/d/c^4*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x
-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c
)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1
```

```
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64*d^2/c^2*(-1/216/c^2*x^2*
(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/
2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))
^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(
x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*
c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d
)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-1/4*(d*x^3+c)^(1
/2)/c/x^4+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I*d/c^2*3^(1/2)*(-d^2*c)^(1/3)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\,(dx^3-8\,c)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\,(dx^3-8\,c)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

3.436 $\int \frac{1}{x^8(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=711

$$\frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-$$

```
[Out] (-17*Sqrt[c + d*x^3])/(6048*c^3*x^7) + (391*d*Sqrt[c + d*x^3])/(193536*c^4*
x^4) - (289*d^2*Sqrt[c + d*x^3])/(48384*c^5*x) + (289*d^(7/3)*Sqrt[c + d*x^
3])/(48384*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*
c^2*x^7*(8*c - d*x^3)) - (17*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(110592*Sqrt[3]*c^(29/6)) + (17*d^(7/3)*ArcTanh[
(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(331776*c^(29/6)) - (
17*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(331776*c^(29/6)) - (289*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(32256*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (289*d^(7/3)*
(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
+ d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(24192*
Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 1.04372, antiderivative size = 711, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{289d^2\sqrt{c+dx^3}}{48384c^5x}-\frac{17d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}}+\frac{17d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}}-\frac{17d^{7/3}\ \text{t}}{33}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-17*Sqrt[c + d*x^3])/(6048*c^3*x^7) + (391*d*Sqrt[c + d*x^3])/(193536*c^4*
x^4) - (289*d^2*Sqrt[c + d*x^3])/(48384*c^5*x) + (289*d^(7/3)*Sqrt[c + d*x^
3])/(48384*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*
c^2*x^7*(8*c - d*x^3)) - (17*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(110592*Sqrt[3]*c^(29/6)) + (17*d^(7/3)*ArcTanh[
(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(331776*c^(29/6)) - (
17*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(331776*c^(29/6)) - (289*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(32256*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (289*d^(7/3)*
(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
+ d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(24192*
Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
```

```
+ b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} + \frac{\int \frac{34cd+\frac{17d^2x^3}{2}}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \frac{\int \frac{782c^2d^2-187cd^3x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{12096c^4d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} + \frac{\int \frac{18496c^3d^3-1955c^2d^4x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{387072c^6d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-76840c^4d^4+\cdots\right.}{\left(8c-dx^3\right)\cdots}}{30965\cdots}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \frac{\int \left(-\frac{9248c^3d^4x}{\sqrt{c+dx^3}}\cdots\right.}{309\cdots}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} + \frac{\left(289d^3\right)\int \frac{\cdots}{\sqrt{c\cdots}}}{96768c^5\cdots}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \frac{\left(17d^2\right)\int \frac{2\sqrt[3]{\cdots}}{\left(4+\frac{2}{\cdots}\right.\cdots}}{22\cdots}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\cdots}{21\cdots}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\cdots}{21\cdots}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\cdots}{21\cdots}$$

Mathematica [C] time = 0.152896, size = 212, normalized size = 0.3

$$-\frac{289d^4x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3870720c^6\sqrt{c+dx^3}}+\frac{9605d^3x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{6193152c^5\sqrt{c+dx^3}}+\sqrt{c+dx^3}\left(-\frac{d^3x^2}{110592c^5\left(dx^3-8c\right)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/(448*c^3*x^7) + (15*d)/(7168*c^4*x^4) - (171*d^2)/(2867
2*c^5*x) - (d^3*x^2)/(110592*c^5*(-8*c + d*x^3))) + (9605*d^3*x^2*Sqrt[(c +
 d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(619315
2*c^5*Sqrt[c + d*x^3]) - (289*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2
, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(3870720*c^6*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 2738, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 3/4096/c^4*d^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^
(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
)))^(1/2))))-1/36864*I/c^5*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*
(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2
*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi
```

```
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*
d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-1/7*(d*x^3+c)^(1
/2)/c/x^7+11/56*d*(d*x^3+c)^(1/2)/c^2/x^4-55/112*d^2*(d*x^3+c)^(1/2)/c^3/x-
55/336*I*d^2/c^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1
/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))+1/512*d^3/c^3*(
-1/216/c^2*x^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(
1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1
/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*
(-1/4*(d*x^3+c)^(1/2)/c/x^4+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I*d/c^2*3^(1/2
)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(
```

```
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3 + c}\left(dx^3 - 8\,c\right)^2 x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

3.437 $\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 1/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0544029, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 1/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^7\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.298717, size = 239, normalized size = 3.62

$$\frac{x\left(\frac{256\left(-\frac{32c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+c+dx^3\right)}{8c-dx^3}-\frac{23dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}\right)}{864d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((-23*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/c + (256*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(864*d^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.035, size = 1431, normalized size = 21.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/d^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+16
/27*I/d^5*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)
^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_al
pha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d-8*c))+64*c^2/d^2*(-1/216/c^2*x*(d*x^3+c)^(1/2)/(
d*x^3-8*c)+1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d
^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*
(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*
(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3
*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{dx^3 + c}\left(dx^3 - 8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3 + c}x^6}{d^3x^9 - 15\,cd^2x^6 + 48\,c^2dx^3 + 64\,c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^6/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-8c + dx^3\right)^2\sqrt{c + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**6/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{dx^3+c}\left(dx^3-8c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.438 $\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0549255, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.168172, size = 237, normalized size = 3.59

$$\frac{x\left(x^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{64c\left(-\frac{32c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+c+dx^3\right)}{d\left(dx^3-8c\right)}\right)}{1728c^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] - (64*c*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(d*(-8*c + d
*x^3))))/(1728*c^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.008, size = 1150, normalized size = 17.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/27*I/d^4/c*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2
*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*
_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+8/d*c*(-1/216/c^2*x*(d*x^3+c)^(1/2)/(d*
x^3-8*c)+1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2
*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-
I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d
*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^
2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*
d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c
*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3 + c}x^3}{d^3x^9 - 15\,cd^2x^6 + 48\,c^2dx^3 + 64\,c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{dx^3 + c}\left(dx^3 - 8\,c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.439 $\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0283876, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^2*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.159929, size = 237, normalized size = 3.7

$$\frac{x\left(\frac{dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{64\left(\frac{832F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+\frac{c+dx^3}{c^2}\right)}{8c-dx^3}\right)}{13824\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (64*((c + d*x^3)/c^2 + (832*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(13824*Sqrt[c + d*x^3])
```

Maple [C] time = 0.006, size = 728, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/216/c^2*x*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/648*I/c^2*3^(1/2)/d*(-d^2*c)^(1/
3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
```

```
^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(1/_al
pha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+
I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_a
lpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*
(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

3.440 $\int \frac{1}{x^3(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(128*c^2*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0551217, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(128*c^2*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.172358, size = 266, normalized size = 4.03

$$\frac{\frac{29d^2x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^4}-\frac{4096dxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-\frac{64\left(c+dx^3\right)\left(29d\cdots\right.}{c^3x^2\left(dx^3\cdots\right.}}{221184\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3)*(-216*c + 29*d*x^3))/(c^3*x^2*(-8*c + d*x^3)) + (29*d^2*x
^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*
c)])/c^4 - (4096*d*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(c*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(221184*Sqrt
[c + d*x^3])
```

Maple [C] time = 0.013, size = 1455, normalized size = 22.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64/c^2*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-1/1728*I/d^2/c^3*2^(1/2)*sum(1/_alpha^2*(-
d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/
(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2
)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2
*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(
-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/
8*d/c*(-1/216/c^2*x*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/648*I/c^2*3^(1/2)/d*(-d^2
*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*su
m(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)
^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(
1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/
d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^{12}-15\,cd^2x^9+48\,c^2dx^6+64\,c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^3*x^12 - 15*c*d^2*x^9 + 48*c^2*d*x^6 + 64*c^3*x
^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

3.441 $\int \frac{1}{x^6(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 1/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(320*c^2*x^5*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0564458, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 1/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(320*c^2*x^5*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.201282, size = 279, normalized size = 4.23

$$\frac{-\frac{119d^3x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^5}+\frac{1404928d^2xF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^2\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+\frac{64\left(c+dx^3\right)\left(8\cdots\right.}{\cdots}}{2211840\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((64*(c + d*x^3)*(864*c^2 - 1080*c*d*x^3 + 119*d^2*x^6))/(c^4*x^5*(-8*c + d
*x^3)) - (119*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/c^5 + (1404928*d^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((
d*x^3)/c), (d*x^3)/(8*c)])/(c^2*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d
*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)]))))/(2211840*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 1782, normalized size = 27.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

[Out] 1/256/c^3*d*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
3^(1/2)/d(-d^2*c)^(1/3)))^(1/2)))-1/6912*I/d/c^4*2^(1/2)*sum(1/_alpha^2*(
-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/
2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha
*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-
(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1
/64*d^2/c^2*(-1/216/c^2*x*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/648*I/c^2*3^(1/2)/d
*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1
/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-
d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^
2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/5/c/x^5*(d*x^3+c)^(1/2)+7/20*d/c^2/x^2*(d*
x^3+c)^(1/2)-7/60*I*d/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-
d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
(-I(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

3.442 $\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

```
[Out] (8*x^6)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*(38*c + 39*d*x^3))/(81*
d^4*Sqrt[c + d*x^3]) - (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
243*d^4)
```

Rubi [A] time = 0.0757577, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 98, 146, 63, 206}

$$\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (8*x^6)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*(38*c + 39*d*x^3))/(81*
d^4*Sqrt[c + d*x^3]) - (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
243*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3}{(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$=\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{\mathrm{Subst}\left(\int \frac{x\left(16c^2+13cdx\right)}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{27cd^2}$$

$$=\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{(320c)\,\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{81d^3}$$

$$=\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{(640c)\,\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{81d^4}$$

$$=\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

Mathematica [C] time = 0.0368726, size = 88, normalized size = 0.93

$$\frac{6\left(752c^2-198cdx^3+9d^2x^6\right)-640c\left(8c-dx^3\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{81d^4\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (6*(752*c^2 - 198*c*d*x^3 + 9*d^2*x^6) - 640*c*(8*c - d*x^3)*Hypergeometric
2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(81*d^4*(-8*c + d*x^3)*Sqrt[c + d*x^3
])
```

Maple [C] time = 0.046, size = 970, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d^3*(d*(2/3/d^2*c/((x^3+1/d*c)*d)^(1/2)+2/3*(d*x^3+c)^(1/2)/d^2)-32/3*c/d
/(d*x^3+c)^(1/2))+192*c^2/d^3*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c
^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*
c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2
*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)
^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/1
8/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c))+512*c^3/d^3*(-2/243/d/c^2/((x^3+1/d*c)*d)^(1/2)-1/243/d/c^2
*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/1458*I/d^3/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_
alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86252, size = 532, normalized size = 5.6

$$\left[\frac{2\left(160\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(27\,d^2x^6-274\,cdx^3-304\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(d^6x^6-7\,cd^5x^3-8\,c^2d^4\right)},\frac{2\left(320\left(d^2x^6-7\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(160*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*c^
2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^3 - 8*c^2*d^4), 2/243*(320*(d^2*x^
6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*
(27*d^2*x^6 - 274*c*d*x^3 - 304*c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^
3 - 8*c^2*d^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15573, size = 119, normalized size = 1.25

$$\frac{640\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^4}+\frac{2\sqrt{dx^3+c}}{3\,d^4}-\frac{2\left(85\left(dx^3+c\right)c+3\,c^2\right)}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 640/243*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/3*sqrt(d*
x^3 + c)/d^4 - 2/81*(85*(d*x^3 + c)*c + 3*c^2)/(((d*x^3 + c)^(3/2) - 9*sqrt
(d*x^3 + c)*c)*d^4)
```

3.443 $\int \frac{x^8}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

```
[Out] -22/(81*d^3*Sqrt[c + d*x^3]) + (64*c)/(27*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]
) - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*Sqrt[c]*d^3)
```

Rubi [A] time = 0.0651967, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 89, 78, 63, 206}

$$\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -22/(81*d^3*Sqrt[c + d*x^3]) + (64*c)/(27*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]
) - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*Sqrt[c]*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-24c^2d+9cd^2x}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{27cd^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{16\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{81d^2}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{32\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{81d^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

Mathematica [A] time = 0.0880438, size = 71, normalized size = 0.86

$$\frac{2\left(\frac{3\left(8c+11dx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)
```

Maple [C] time = 0.016, size = 926, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/d^3/(d*x^3+c)^(1/2)+16*c/d^2*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d
^3/c^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-
d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^
2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*d-8*c))+64*c^2/d^2*(-2/243/d/c^2/((x^3+1/d*c)*d)^(1/2)-1/243/d/
c^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/1458*I/d^3/c^3*2^(1/2)*sum((-d^2*c)^(1/3)
*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/
3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(
1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/
(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*
3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/
2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3
)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13018, size = 495, normalized size = 5.96

$$\left[\frac{2\left(8\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(11\,cdx^3+8\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}, \frac{2\left(16\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\,\mathrm{arc}\right.}{243\left(cd^5x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 +
 c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 + c)
)/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^3 -
 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*(11*c*d*x^3 + 8
*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17336, size = 90, normalized size = 1.08

$$\frac{32\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^3} - \frac{2\left(11\,dx^3+8\,c\right)}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 32/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/81*(11*d*x^3
 + 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d^3)
```

3.444 $\int \frac{x^5}{\left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}} \, dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2}{81cd^2\sqrt{c+dx^3}}$$

```
[Out] -2/(81*c*d^2*Sqrt[c + d*x^3]) + 8/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(3/2)*d^2)
```

Rubi [A] time = 0.0622635, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 51, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2}{81cd^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(81*c*d^2*Sqrt[c + d*x^3]) + 8/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(3/2)*d^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{9d}$$
$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{81cd}$$
$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81cd^2}$$
$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

Mathematica [C] time = 0.0236642, size = 68, normalized size = 0.8

$$-\frac{2\left(\left(dx^3-8c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+12c\right)}{81cd^2\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(12*c + (-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c
)]))/(81*c*d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 908, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-d^2*c)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(
2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+8/d*c*(-2/
243/d/c^2/((x^3+1/d*c)*d)^(1/2)-1/243/d/c^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/1
458*I/d^3/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))
/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d
^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07353, size = 487, normalized size = 5.73

$$\left[\frac{\left(d^{2} x^{6}-7 c d x^{3}-8 c^{2}\right) \sqrt{c} \log \left(\frac{d x^{3}+6 \sqrt{d x^{3}+c} \sqrt{c}+10 c}{d x^{3}-8 c}\right)-6\left(c d x^{3}+4 c^{2}\right) \sqrt{d x^{3}+c}}{243\left(c^{2} d^{4} x^{6}-7 c^{3} d^{3} x^{3}-8 c^{4} d^{2}\right)},-\frac{2\left(\left(d^{2} x^{6}-7 c d x^{3}-8 c^{2}\right) \sqrt{-c} \arctan \left(\frac{\sqrt{}}{}\right.\right.}{243\left(c^{2} d^{4} x^{6}-7 c^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^
2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^
2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12575, size = 103, normalized size = 1.21

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}cd}+\frac{3\left(dx^3+4c\right)}{\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)cd}\right)}{243d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/243*(arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) + 3*(d*x^3 + 4*
c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c*d))/d
```

3.445 $\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=88

$$-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}+\frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[Out] -1/(81*c^2*d*Sqrt[c + d*x^3]) + 1/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)
```

Rubi [A] time = 0.0638284, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}+\frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/(81*c^2*d*Sqrt[c + d*x^3]) + 1/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{18c}\\
&= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{162c^2}\\
&= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81c^2d}\\
&= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}
\end{aligned}$$

Mathematica [C] time = 0.0120503, size = 43, normalized size = 0.49

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{243c^2d\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (c + d*x^3)/(9*c)])/(243*c^2*d*Sqrt[c +
 d*x^3])
```

Maple [C] time = 0.007, size = 463, normalized size = 5.3

$$-\frac{2}{243\,dc^2}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{1}{243\,dc^2\left(dx^3-8\,c\right)}\sqrt{dx^3+c}-\frac{\frac{i}{1458}\sqrt{2}}{d^3c^3}\sum_{\_alpha=RootOf\left(\_Z^3d-8\,c\right)}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/243/d/c^2/((x^3+1/d*c)*d)^(1/2)-1/243/d/c^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-
1/1458*I/d^3/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/
3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^
2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93065, size = 482, normalized size = 5.48

$$\left[\frac{\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\left(cdx^3-5\,c^2\right)\sqrt{dx^3+c}}{486\left(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d\right)},-\frac{\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}}{3\,c}\right.}{243\left(c^3d^3x^6-7\,c^4d^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(c^
3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 - 5*c^2)*sqrt
(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1286, size = 97, normalized size = 1.1

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{243\,\sqrt{-c}c^2d}-\frac{dx^3-5\,c}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3
- 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)
```

3.446 $\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}}+\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

```
[Out] 5/(648*c^3*Sqrt[c + d*x^3]) + 1/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(7776*c^(7/2)) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(96*c^(7/2))
```

Rubi [A] time = 0.0949663, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$\frac{1}{216c^2(8c-dx^3)\sqrt{c+dx^3}}+\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*Sqrt[c + d*x^3]) + 1/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(7776*c^(7/2)) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(96*c^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{216c^2d}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-\frac{81}{2}c^2d^2+\frac{15}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{972c^4d^2}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^3} + \frac{(7d)\,\text{Sub}}{}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{2592c^3} + \frac{\text{Su}}{}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Mathematica [C] time = 0.036877, size = 97, normalized size = 0.92

$$\frac{\left(7dx^3-56c\right)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+27\left(8c-dx^3\right)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+12c}{2592c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (12*c + (-56*c + 7*d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)
] + 27*(8*c - d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(2592*
c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 953, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/64*d/c^2*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum((-d
^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)
*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-
d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/
8*d/c*(-2/243/d/c^2/((x^3+1/d*c)*d)^(1/2)-1/243/d/c^2*(d*x^3+c)^(1/2)/(d*x^
3-8*c)-1/1458*I/d^3/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2
*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*
_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(2/3/c/((x^3+1/d*c)*d)^(1/2)-
2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

Fricas [A] time = 1.68887, size = 724, normalized size = 6.83

$$\left[\frac{7\left(d^{2}x^{6}-7\,cdx^{3}-8\,c^{2}\right)\sqrt{c}\log\left(\frac{dx^{3}+6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+81\left(d^{2}x^{6}-7\,cdx^{3}-8\,c^{2}\right)\sqrt{c}\log\left(\frac{dx^{3}-2\,\sqrt{dx^{3}+c}\sqrt{c}+2\,c}{x^{3}}\right)+24\left(5\,cdx^{3}-4\right.}{15552\left(c^{4}d^{2}x^{6}-7\,c^{5}dx^{3}-8\,c^{6}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/15552*(7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqr
t(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3 - 4
3*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 - 8*c^6), 1/7776*(81*(d^
2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*
(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/
c) + 12*(5*c*d*x^3 - 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 -
8*c^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10591, size = 126, normalized size = 1.19

$$\frac{\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}c^{3}}-\frac{7\,\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{7776\,\sqrt{-c}c^{3}}+\frac{5\,dx^{3}-43\,c}{648\left(\left(dx^{3}+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^{3}+c}c\right)c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 7/7776*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/648*(5*d*x^3 - 43*c)/(((d*x^3 +
c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^3)
```

3.447 $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{35d}{2592c^4\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

```
[Out] (-35*d)/(2592*c^4*Sqrt[c + d*x^3]) + (5*d)/(864*c^3*(8*c - d*x^3)*Sqrt[c +
d*x^3]) - 1/(24*c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*d*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(384*c^(9/2))
```

Rubi [A] time = 0.124815, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{35d}{2592c^4\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```


```
[Out] (-35*d)/(2592*c^4*Sqrt[c + d*x^3]) + (5*d)/(864*c^3*(8*c - d*x^3)*Sqrt[c +
d*x^3]) - 1/(24*c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*d*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(384*c^(9/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{10cd-\frac{5d^2x}{2}}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2}$$

$$= \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{-90c^2d^2+15cd^3x}{x(8c-dx)(c+dx)^{3/2}}\, d\right.}{1728c^4d}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Sub}}{}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{(5d)}{}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\,\text{Su}}{}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{5d\,\text{t}}{}$$

Mathematica [C] time = 0.0462997, size = 117, normalized size = 0.82

$$\frac{5dx^3\left(dx^3-8c\right)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+135dx^3\left(dx^3-8c\right)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+12c\left(5dx^3-36c\right)}{10368c^4x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (12*c*(-36*c + 5*d*x^3) + 5*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1,
 1/2, (c + d*x^3)/(9*c)] + 135*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + (d*x^3)/c])/(10368*c^4*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 1019, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/256*d^2/c^3*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum(
(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3))
)/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1
/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d
^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d
-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*
c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
+1/64*d^2/c^2*(-2/243/d/c^2/((x^3+1/d*c)*d)^(1/2)-1/243/d/c^2*(d*x^3+c)^(1/
2)/(d*x^3-8*c)-1/1458*I/d^3/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1
/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^
(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(
```

```
-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-2/3*d/c^2/((x^3+1/d
*c)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(5/2))+1/256/c^3*d*(2/3/c/((x^3+1/d*c)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)
```

Fricas [A] time = 1.67074, size = 837, normalized size = 5.85

$$\left[\frac{5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+405\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24}{62208\left(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 6*sq
rt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6 -
 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) -
24*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 -
 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x
^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 5*(d^3*x^9 - 7*c*d^2*x^6
- 8*c^2*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(35*c*d
^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7*c^6*d*x
^6 - 8*c^7*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10383, size = 171, normalized size = 1.2

$$-\frac{1}{31104}\,d\left(\frac{405\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^4}+\frac{5\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}c^4}+\frac{12\left(35\left(dx^3+c\right)^2-335\left(dx^3+c\right)c+192\,c^2\right)}{\left(\left(dx^3+c\right)^{\frac{5}{2}}-10\left(dx^3+c\right)^{\frac{3}{2}}c+9\,\sqrt{dx^3+c}c^2\right)c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/31104*d*(405*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 5*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 12*(35*(d*x^3 + c)^2 - 335*(
d*x^3 + c)*c + 192*c^2)/(((d*x^3 + c)^(5/2) - 10*(d*x^3 + c)^(3/2)*c + 9*sq
rt(d*x^3 + c)*c^2)*c^4))
```

3.448 $\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{665d^2}{41472c^5\sqrt{c+dx^3}}+\frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}}-\frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}+\frac{17d}{384c^3x^3\left(8c-dx\right.}$$

```
[Out] (665*d^2)/(41472*c^5*Sqrt[c + d*x^3]) - (71*d^2)/(13824*c^4*(8*c - d*x^3)*S
qrt[c + d*x^3]) - 1/(48*c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (17*d)/(38
4*c^3*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (13*d^2*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(497664*c^(11/2)) - (33*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(2048*c^(11/2))
```

Rubi [A] time = 0.162809, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$-\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{665d^2}{41472c^5\sqrt{c+dx^3}}+\frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}}-\frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}+\frac{17d}{384c^3x^3\left(8c-dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (665*d^2)/(41472*c^5*Sqrt[c + d*x^3]) - (71*d^2)/(13824*c^4*(8*c - d*x^3)*S
qrt[c + d*x^3]) - 1/(48*c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (17*d)/(38
4*c^3*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (13*d^2*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(497664*c^(11/2)) - (33*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(2048*c^(11/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{17cd-\frac{7d^2x}{2}}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{198c^2d^2-\frac{85}{2}c\cdots}{x(8c-dx)^2(c+d\cdots}\cdots\right)}{384c^4}$$

$$= -\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{\cdots}}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\cdots}{38\cdots}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\cdots}{38\cdots}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\cdots}{38\cdots}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\cdots}{38\cdots}$$

Mathematica [C] time = 0.0543834, size = 135, normalized size = 0.73

$$\frac{13d^2x^6\left(dx^3-8c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-3\left(4c\left(288c^2-612cdx^3+71d^2x^6\right)+891d^2x^6\left(dx^3-8c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)\right)}{165888c^5x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (13*d^2*x^6*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c
)] - 3*(4*c*(288*c^2 - 612*c*d*x^3 + 71*d^2*x^6) + 891*d^2*x^6*(-8*c + d*x^
3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(165888*c^5*x^6*(8*c -
d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.015, size = 1106, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -3/4096*d^3/c^4*(2/27/d/c/((x^3+1/d*c)*d)^(1/2)+1/243*I/d^3/c^2*2^(1/2)*sum
((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(
1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*
d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2
*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)+1/512*d^3/c^3*(-2/243/d/c^2/((x^3+1/d*c)*d)^(1/2)-1/243/d/c^2*(d*x^3+c)^(
1/2)/(d*x^3-8*c)-1/1458*I/d^3/c^3*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x
-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c
)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
```

```
2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(2/3*d^2/c^3/((x^3+
1/d*c)*d)^(1/2)-1/6*(d*x^3+c)^(1/2)/c^2/x^6+7/12*d*(d*x^3+c)^(1/2)/c^3/x^3-
5/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2))+1/256/c^3*d*(-2/3*d/c^2/(
(x^3+1/d*c)*d)^(1/2)-1/3*(d*x^3+c)^(1/2)/c^2/x^3+d*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/c^(5/2))+3/4096/c^4*d^2*(2/3/c/((x^3+1/d*c)*d)^(1/2)-2/3*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)
```

Fricas [A] time = 1.66096, size = 919, normalized size = 4.97

$$\left[\frac{13\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+8019\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{995328\left(c^6d^2x^{12}-7\,c^7dx^9-8\,c^8x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/995328*(13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 +
 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^12 - 7*c*d^
3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d*x^3 + 864*c^4)*s
qrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6), 1/497664*(8019*(d
^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt
(-c)/c) - 13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(1/3*s
```

```
qrt(d*x^3 + c)*sqrt(-c)/c) + 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^
3*d*x^3 + 864*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1429, size = 180, normalized size = 0.97

$$\frac{1}{497664}d^2\left(\frac{8019\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^5}-\frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}c^5}+\frac{12\left(341\,dx^3-2731\,c\right)}{\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)c^5}+\frac{1296\left(3\left(dx^3+c\right)^{\frac{3}{2}}-4\sqrt{d\right.}{c^5d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/497664*d^2*(8019*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13*arc
tan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 12*(341*d*x^3 - 2731*c)/
(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) + 1296*(3*(d*x^3 + c)^(3/2)
 - 4*sqrt(d*x^3 + c)*c)/(c^5*d^2*x^6))
```

3.449 $\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=668

$$\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}}-\frac{4\tanh^{-1}}{243c}$$

```
[Out] (-2*x^2)/(81*c*d^2*Sqrt[c + d*x^3]) + (8*x^2)/(27*d^2*(8*c - d*x^3)*Sqrt[c
+ d*x^3]) + (2*Sqrt[c + d*x^3])/(81*c*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) + (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
])/(81*Sqrt[3]*c^(5/6)*d^(8/3)) - (4*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(
1/6)*Sqrt[c + d*x^3])]])/(243*c^(5/6)*d^(8/3)) + (4*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])]])/(243*c^(5/6)*d^(8/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]]])/(27*3^(3/4)*c^(2/3)*d^(8/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]]])/(81*3^(1/4)*c^(2/3)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.801042, antiderivative size = 668, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {470, 579, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}}-\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}}+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*x^2)/(81*c*d^2*Sqrt[c + d*x^3]) + (8*x^2)/(27*d^2*(8*c - d*x^3)*Sqrt[c
+ d*x^3]) + (2*Sqrt[c + d*x^3])/(81*c*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) + (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
])/(81*Sqrt[3]*c^(5/6)*d^(8/3)) - (4*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(
1/6)*Sqrt[c + d*x^3])]])/(243*c^(5/6)*d^(8/3)) + (4*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(243*c^(5/6)*d^(8/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*c^(2/3)*d^(8/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(81*3^(1/4)*c^(2/3)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(16c^2+7cdx^3\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{27cd^2}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(-72c^3d-\frac{9}{2}c^2d^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{729c^3d^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \left(\frac{9c^2dx}{2\sqrt{c+dx^3}} - \frac{108c^3dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{729c^3d^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{8\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27d^2} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{81cd^2}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{81cd^3} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\sqrt{c+\ldots}}}{81c\ldots}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{2-\ldots}}{\ldots}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{4\tan^{-1}\ldots}{81\ldots}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{4\tan^{-1}\ldots}{81\ldots}$$

Mathematica [C] time = 0.105254, size = 168, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+40cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(4c+dx^3\right)}{3240c^2d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(4*c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(3240*c^2*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.04, size = 2255, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d^2*(2/3/c*x^2/((x^3+1/d*c)*d)^(1/2)+2/9*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2
*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(
I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2))))+16*c/d^2*(-2/27/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3
^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
```

```
/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_a
lpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(
1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I
*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3
)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_al
pha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(
-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8
*c)))+64*c^2/d^2*(2/243*x^2/c^3/((x^3+1/d*c)*d)^(1/2)-1/1944*x^2/c^3*(d*x^3
+c)^(1/2)/(d*x^3-8*c)+5/1944*I/c^3*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*
(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)))-5/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d
*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2
/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d
^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}x^7}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^7/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.450 $\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=671

$$\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\text{tan}$$

```
[Out] -x^2/(81*c^2*d*Sqrt[c + d*x^3]) + x^2/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]
) + Sqrt[c + d*x^3]/(81*c^2*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) -
ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(81*Sqrt[3]
*c^(11/6)*d^(5/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])]/(243*c^(11/6)*d^(5/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c
^(11/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)], -7 - 4*Sqrt[3]])/(54*3^(3/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(81*3^(1/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.800403, antiderivative size = 671, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {471, 579, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}}+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{81}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -x^2/(81*c^2*d*Sqrt[c + d*x^3]) + x^2/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]
) + Sqrt[c + d*x^3]/(81*c^2*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) -
ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(81*Sqrt[3]
*c^(11/6)*d^(5/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])]/(243*c^(11/6)*d^(5/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c
^(11/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)], -7 - 4*Sqrt[3]])/(54*3^(3/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(81*3^(1/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(2c-\frac{5dx^3}{2}\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{27cd}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(45c^2d-\frac{9}{4}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{729c^3d^2}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \left(\frac{9cdx}{4\sqrt{c+dx^3}} + \frac{27c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{729c^3d^2}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{162c^2d} + \frac{2\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{162c^2d^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt{c+\cdots}}}{162c\cdots}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{2-\cdots}}{\cdots}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{\cdots}}{8\cdots}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{\cdots}}{8\cdots}$$

Mathematica [C] time = 0.100183, size = 169, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+50cx^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(dx^3-5c\right)}{6480c^3d\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(-5*c + d*x^3) + 50*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(6480*c^3*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.012, size = 1788, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d*(-2/27/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
```

```
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_
alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+8/d*c*(2/243*x^2
/c^3/((x^3+1/d*c)*d)^(1/2)-1/1944*x^2/c^3*(d*x^3+c)^(1/2)/(d*x^3-8*c)+5/194
4*I/c^3*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-5/5832*I/c^3/d^3*2^(1/2
)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*
c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d
^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2
*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)
^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/1
8/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}x^4}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.451 $\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=665

$$-\frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\ldots$$

```
[Out] (5*x^2)/(648*c^3*Sqrt[c + d*x^3]) + x^2/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (5*Sqrt[c + d*x^3])/(648*c^3*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)) - (5*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(1296*Sqrt[3]*c^(17/6)*d^(2/3)) + (5*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(3888*c^(17/6)*d^(2/3)) - (5*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])])/(3888*c^(17/6)*d^(2/3)) + (5*Sqrt[2 - Sqrt[3]]*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(432*3^(3/4)*c^(8/
3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/4)*c^(8/3)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3])
```

Rubi [A] time = 0.798692, antiderivative size = 665, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {472, 579, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}}+\frac{5\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\left(\sqrt[3]{c}+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (5*x^2)/(648*c^3*Sqrt[c + d*x^3]) + x^2/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (5*Sqrt[c + d*x^3])/(648*c^3*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)) - (5*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(1296*Sqrt[3]*c^(17/6)*d^(2/3)) + (5*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])]])/(3888*c^(17/6)*d^(2/3)) - (5*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])]])/(3888*c^(17/6)*d^(2/3)) + (5*Sqrt[2 - Sqrt[3]]*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(432*3^(3/4)*c^(8/
3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/4)*c^(8/3)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{x\left(25cd+\frac{5d^2x^3}{2}\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(\frac{45c^2d^2}{2}-\frac{45}{4}cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \left(\frac{45cd^2x}{4\sqrt{c+dx^3}} - \frac{135c^2d^2x}{2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{2916c^4d^2}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1296c^3} + \frac{5\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2592c^3d} - \frac{5\int \frac{(1-\sqrt{}}{\sqrt{}}}{129}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{5\sqrt{2}}{}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{5\,\mathrm{ta}}{}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{5\,\mathrm{ta}}{}$$

Mathematica [C] time = 0.112377, size = 167, normalized size = 0.25

$$\frac{dx^5\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+16cx^2\left(43c-5dx^3\right)}{10368c^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (16*c*x^2*(43*c - 5*d*x^3) + 5*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*App
ellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(8*c - d*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(10368*c^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 903, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/243*x^2/c^3/((x^3+1/d*c)*d)^(1/2)-1/1944*x^2/c^3*(d*x^3+c)^(1/2)/(d*x^3-8
*c)+5/1944*I/c^3*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^
(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x
+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^
(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-5/5832*I/c^3/d
^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(
-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2
-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
```

```
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*
c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 112*
c^3*d*x^3 + 64*c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.452 $\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=686

$$\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left((1+\cdots\right.}}}{864\,3^{3/4}c^{11/3}\cdots}$$

```
[Out] 5/(648*c^3*x*Sqrt[c + d*x^3]) + 1/(216*c^2*x*(8*c - d*x^3)*Sqrt[c + d*x^3])
 - (31*Sqrt[c + d*x^3])/(1296*c^4*x) + (31*d^(1/3)*Sqrt[c + d*x^3])/(1296*c
^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1296*Sqrt[3]*c^(23/6)) + (d^(1/3)
*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3888*c^(23/
6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3888*c^(23/6)) - (31*
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(864*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (31*d^(1/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(648*Sqrt
[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.922697, antiderivative size = 686, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {472, 579, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}}{864\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\cdots}{\left((1+\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x*Sqrt[c + d*x^3]) + 1/(216*c^2*x*(8*c - d*x^3)*Sqrt[c + d*x^3])
 - (31*Sqrt[c + d*x^3])/(1296*c^4*x) + (31*d^(1/3)*Sqrt[c + d*x^3])/(1296*c
^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1296*Sqrt[3]*c^(23/6)) + (d^(1/3)
*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3888*c^(23/
6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3888*c^(23/6)) - (31*
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(864*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (31*d^(1/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(648*Sqrt
[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

<u>Rule 472</u>

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

<u>Rule 579</u>

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

<u>Rule 583</u>

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
```

2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[8*b*c + a*d, 0]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rule 2145

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{28cd+\frac{11d^2x^3}{2}}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{-558c^2d^2+\frac{225}{4}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \frac{x\left(2340c^3d^3-279c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}}{23328c^6d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \left(\frac{279c^2d^3x}{\sqrt{c+dx^3}} + \frac{108c}{\left(8c-dx^3\right)}\right.}{23328c^6d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{(31d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{2592c^4} +$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+d}}}{2592c^4}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{d}\sqrt{c+dx}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{d}\sqrt{c+dx}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{d}\sqrt{c+dx}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\right.}$$

Mathematica [C] time = 0.12759, size = 180, normalized size = 0.26

$$\frac{31d^2x^6\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+650cdx^3\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(162c^2\right.}{103680c^5\sqrt{c+dx^3}\left(8cx-dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(162*c^2 + 227*c*d*x^3 - 31*d^2*x^6) + 650*c*d*x^3*(8*c - d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] +
31*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -(
(d*x^3)/c), (d*x^3)/(8*c)])/(103680*c^5*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] time = 0.014, size = 2269, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-5/9*I/
c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/64*d/c^2*(-2/27/c^2*x^2/((
x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x
-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*
```

```
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+
1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^
2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2
*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(
1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8*d/c*(2/243*x^2/c^3/((x^3+1/d*c)*d
)^(1/2)-1/1944*x^2/c^3*(d*x^3+c)^(1/2)/(d*x^3-8*c)+5/1944*I/c^3*3^(1/2)/d*(
-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-5/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-d^
2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-
d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*
(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c
)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^
(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d
^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(
1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d
^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{14}-14\,cd^3x^{11}+33\,c^2d^2x^8+112\,c^3dx^5+64\,c^4x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^14 - 14*c*d^3*x^11 + 33*c^2*d^2*x^8 + 112*c
^3*d*x^5 + 64*c^4*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

3.453 $\int \frac{1}{x^5\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=708

$$-\frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-$$

```
[Out] 5/(648*c^3*x^4*Sqrt[c + d*x^3]) + 1/(216*c^2*x^4*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (253*Sqrt[c + d*x^3])/(20736*c^4*x^4) + (77*d*Sqrt[c + d*x^3])/(2592
*c^5*x) - (77*d^(4/3)*Sqrt[c + d*x^3])/(2592*c^5*((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)) - (11*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqr
t[c + d*x^3]])/(82944*Sqrt[3]*c^(29/6)) + (11*d^(4/3)*ArcTanh[(c^(1/3) + d^
(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(248832*c^(29/6)) - (11*d^(4/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(248832*c^(29/6)) + (77*Sqrt[2 - Sqrt[3
]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(1728*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (77*d^(4/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1296*Sqrt[2]*3^(1/4)*c
^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [A] time = 1.03154, antiderivative size = 708, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {472, 579, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{11d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}}+\frac{11d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}}-\frac{11d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^4*Sqrt[c + d*x^3]) + 1/(216*c^2*x^4*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (253*Sqrt[c + d*x^3])/(20736*c^4*x^4) + (77*d*Sqrt[c + d*x^3])/(2592
*c^5*x) - (77*d^(4/3)*Sqrt[c + d*x^3])/(2592*c^5*((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)) - (11*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqr
t[c + d*x^3]])/(82944*Sqrt[3]*c^(29/6)) + (11*d^(4/3)*ArcTanh[(c^(1/3) + d^
(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(248832*c^(29/6)) - (11*d^(4/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(248832*c^(29/6)) + (77*Sqrt[2 - Sqrt[3
]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(1728*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (77*d^(4/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1296*Sqrt[2]*3^(1/4)*c
^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
```

```
x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5 \left(8c - dx^3\right)^2 \left(c + dx^3\right)^{3/2}} \, dx = \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} + \frac{\int \frac{31cd + \frac{17d^2x^3}{2}}{x^5\left(8c - dx^3\right)\left(c + dx^3\right)^{3/2}} \, dx}{216c^2 d}$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{\int \frac{-\frac{2277}{2}c^2d^2 + \frac{495}{4}cd^3x^3}{x^5\left(8c - dx^3\right)\sqrt{c + dx^3}} \, dx}{2916c^4 d^2}$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{\int \frac{-22176c^3d^3 + \frac{11385}{4}}{x^2\left(8c - dx^3\right)\sqrt{c +}}}{93312c^6 d}$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c + dx^3}}{2592c^5x} -$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c + dx^3}}{2592c^5x} -$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c + dx^3}}{2592c^5x} -$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c + dx^3}}{2592c^5x} -$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c + dx^3}}{2592c^5x} -$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c + dx^3}}{2592c^5x} -$$

$$= \frac{5}{648c^3x^4\sqrt{c + dx^3}} + \frac{1}{216c^2x^4 \left(8c - dx^3\right) \sqrt{c + dx^3}} - \frac{253\sqrt{c + dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c + dx^3}}{2592c^5x} -$$

Mathematica [C] time = 0.149872, size = 198, normalized size = 0.28

$$\frac{-16\left(77d^3x^9\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+10c\left(-2997c^2dx^3+648c^3-4565cd^2x^6+616d^3x^9\right)\right)-24475}{3317760c^6x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-24475*c*d^2*x^6*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(10*c*(648*c^3 - 2997*c^2*d*x^3 - 456
5*c*d^2*x^6 + 616*d^3*x^9) + 77*d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3317760*c^6*x^4*
(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.014, size = 2774, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/256/c^3*d*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-5/9
*I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/256*d^2/c^3*(-2/27/c^2*
x^2/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*
```

I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/
d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(
2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64*d^2/c^2*(2/243*x^2/c^3/((x
^3+1/d*c)*d)^(1/2)-1/1944*x^2/c^3*(d*x^3+c)^(1/2)/(d*x^3-8*c)+5/1944*I/c^3*
3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-5/5832*I/c^3/d^3*2^(1/2)*sum(1/
_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)
+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_
alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I
*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))+1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*d*(d*x^3+c)^(1/2)/c^3/x
+2/3*d^2/c^3*x^2/((x^3+1/d*c)*d)^(1/2)+55/72*I*d/c^3*3^(1/2)*(-d^2*c)^(1/3)
(I(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
c)^(1/3))^(1/2)((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2

```
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

3.454 $\int \frac{1}{x^8(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=732

$$\frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[Out] 5/(648*c^3*x^7*Sqrt[c + d*x^3]) + 1/(216*c^2*x^7*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (191*Sqrt[c + d*x^3])/(18144*c^4*x^7) + (8257*d*Sqrt[c + d*x^3])/(58
0608*c^5*x^4) - (5179*d^2*Sqrt[c + d*x^3])/(145152*c^6*x) + (5179*d^(7/3)*S
qrt[c + d*x^3])/(145152*c^6*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*d^(7/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(331776
*Sqrt[3]*c^(35/6)) + (7*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*
Sqrt[c + d*x^3])])/(995328*c^(35/6)) - (7*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(995328*c^(35/6)) - (5179*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(96768*3^(3/4)*c^
(17/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (5179*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(72576*Sqrt[2]*3^(1/4)*c^(17/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
 d*x^3])
```

Rubi [A] time = 1.17801, antiderivative size = 732, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {472, 579, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5179d^2\sqrt{c+dx^3}}{145152c^6x}-\frac{7d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}}+\frac{7d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}}-\frac{7d^{7/3}\ t}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^7*Sqrt[c + d*x^3]) + 1/(216*c^2*x^7*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (191*Sqrt[c + d*x^3])/(18144*c^4*x^7) + (8257*d*Sqrt[c + d*x^3])/(58
0608*c^5*x^4) - (5179*d^2*Sqrt[c + d*x^3])/(145152*c^6*x) + (5179*d^(7/3)*S
qrt[c + d*x^3])/(145152*c^6*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*d^(7/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(331776
*Sqrt[3]*c^(35/6)) + (7*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*
Sqrt[c + d*x^3])])/(995328*c^(35/6)) - (7*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(995328*c^(35/6)) - (5179*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(96768*3^(3/4)*c^
(17/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (5179*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(72576*Sqrt[2]*3^(1/4)*c^(17/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
 d*x^3])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
```

```
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
```

```
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{34cd+\frac{23d^2x^3}{2}}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{-1719c^2d^2+\frac{765}{4}cd^3x^3}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{\int \frac{-74313c^3d^3+\frac{18909}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+}}}{163296c^6d}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} -$$

Mathematica [C] time = 0.165154, size = 210, normalized size = 0.29

$$\frac{829375cd^3x^9\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\left(5179d^4x^{12}\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\right.}{92897280c^7x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (829375*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(10368*c^4 - 18792*c^3*d*x^3 + 1
01817*c^2*d^2*x^6 + 153269*c*d^3*x^9 - 20716*d^4*x^12) + 5179*d^4*x^12*(8*c
 - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x
^3)/(8*c)]))/(92897280*c^7*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.014, size = 3299, normalized size = 4.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 3/4096/c^4*d^2*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-
5/9*I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-3/4096*d^3/c^4*(-2/27/
c^2*x^2/((x^3+1/d*c)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+
```

```
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^
(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(
x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*
c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d
)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/7*(d*x^3+c)^(1
/2)/c^2/x^7+25/56*d*(d*x^3+c)^(1/2)/c^3/x^4-237/112*d^2*(d*x^3+c)^(1/2)/c^4
/x-2/3*d^3/c^4*x^2/((x^3+1/d*c)*d)^(1/2)-935/1008*I*d^2/c^4*3^(1/2)*(-d^2*c
)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2))))+1/512*d^3/c^3*(2/243*x^2/c^3/((x^3+1/d*c)*d)^(1
/2)-1/1944*x^2/c^3*(d*x^3+c)^(1/2)/(d*x^3-8*c)+5/1944*I/c^3*3^(1/2)/d*(-d^2
*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2)))-5/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-d^2*c)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*
c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^
2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1
```

```
/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c
)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256/c
^3*d*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*d*(d*x^3+c)^(1/2)/c^3/x+2/3*d^2/c^3
*x^2/((x^3+1/d*c)*d)^(1/2)+55/72*I*d/c^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1
/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1
/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
)^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3
)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2
)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

3.455 $\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=256

$$\frac{2x(4c+dx^3)}{81cd^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}cd^{7/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] (2*x*(4*c + d*x^3))/(81*c*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) - (2*Sqrt[2 +
Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(81*3^(1/4)*c*d^(7/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rubi [C] time = 0.0600647, antiderivative size = 66, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{7}{3};2,\frac{3}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 3/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^3*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^7\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{7}{3};2,\frac{3}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

Mathematica [C] time = 0.519133, size = 189, normalized size = 0.74

$$-\frac{6\sqrt[3]{-d}x\left(4c+dx^3\right)+2i3^{3/4}\sqrt[3]{c}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-d}x-\sqrt[3]{c}\right)}{\sqrt[3]{c}}}\sqrt{\frac{(-d)^{2/3}x^2}{c^{2/3}}+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+1}\left(dx^3-8c\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right),\sqrt[3]{-1}\right)}{243c(-d)^{7/3}\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(6*(-d)^(1/3)*x*(4*c + d*x^3) + (2*I)*3^(3/4)*c^(1/3)*Sqrt[((-1)^(5/6)*(-c
^(1/3) + (-d)^(1/3)*x))/c^(1/3)]*Sqrt[1 + ((-d)^(1/3)*x)/c^(1/3) + ((-d)^(2
/3)*x^2)/c^(2/3)]*(-8*c + d*x^3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d
)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)])/(243*c*(-d)^(7/3)*(-8*c + d*x^3)
*Sqrt[c + d*x^3])
```

Maple [C] time = 0.039, size = 1791, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)

[Out] 1/d^2*(2/3/c*x/((x^3+1/d*c)*d)^(1/2)-2/9*I/c*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+16/d^2*c*(-2/27/c^2*x/((x^3+1/d*c)*d)^(1
/2)+2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1
/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/
d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(
2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+64*c^2/d^2*(2/243/c^3*x/((x^3+1
/d*c)*d)^(1/2)-1/1944/c^3*x*(d*x^3+c)^(1/2)/(d*x^3-8*c)-5/1944*I/c^3*3^(1/2
)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))-1/972*I/c^3/d^3*2
^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)
^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3
*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(
-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(

```
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}x^6}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^6/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.456 $\int \frac{x^3}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=66

$$\frac{x^4 \sqrt{\frac{dx^3}{c}+1} F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3 \sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^3*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.058012, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4 \sqrt{\frac{dx^3}{c}+1} F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3 \sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^3*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.259336, size = 242, normalized size = 3.67

$$\frac{x\left(\frac{3x^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{192\left(\frac{160F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+\frac{dx^3-5c}{c^2}\right)}{d(8c-dx^3)}\right)}{15552\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((3*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (192*((-5*c + d*x^3)/c^2 + (160*AppellF1[1/3, 1/2, 1, 4/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))
/(d*(8*c - d*x^3))))/(15552*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 1478, normalized size = 22.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d*(-2/27/c^2*x/((x^3+1/d*c)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1/3)*
(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*
c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha
^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3
^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+
(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alph
a*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d
^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*
d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c
)))+8/d*c*(2/243/c^3*x/((x^3+1/d*c)*d)^(1/2)-1/1944/c^3*x*(d*x^3+c)^(1/2)/(
d*x^3-8*c)-5/1944*I/c^3*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-
d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c
)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(dx^3 + c\right)^{\frac{3}{2}}\left(dx^3 - 8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^3 + c}x^3}{d^4x^{12} - 14\,cd^3x^9 + 33\,c^2d^2x^6 + 112\,c^3dx^3 + 64\,c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(dx^3 + c\right)^{\frac{3}{2}}\left(dx^3 - 8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.457 $\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^3*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0294427, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^3*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\sqrt{1+\frac{dx^3}{c}}\int\frac{1}{\left(8c-dx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$=\frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.227945, size = 253, normalized size = 3.95

$$\frac{x\left(192c\left(\frac{1216c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+\frac{5dx^3-43c}{dx^3-8c}\right)-15dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\right.\right.}{124416c^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-15*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 192*c*((-43*c + 5*d*x^3)/(-8*c + d*x^3) + (1216*c^2*Appell
F1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/
3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))))))/(124416*c^4*Sqrt[c + d*x^3])
```

Maple [C] time = 0.004, size = 747, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/243/c^3*x/((x^3+1/d*c)*d)^(1/2)-1/1944/c^3*x*(d*x^3+c)^(1/2)/(d*x^3-8*c)-
5/1944*I/c^3*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3
))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2
```

```
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*
(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/
3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^
2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 112*c^
3*d*x^3 + 64*c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

3.458 $\int \frac{1}{x^3(8c-dx^3)^2(c+dx^3)^{3/2}} dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(128*c^3*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0574086, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(128*c^3*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.207029, size = 259, normalized size = 3.92

$$\frac{\frac{64c\left(-\frac{19648c^2dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}-648c^2-1249cdx^3+167d^2x^6\right)}{8c-dx^3}+167d^2x^6\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{663552c^5x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (167*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + (64*c*(-648*c^2 - 1249*c*d*x^3 + 167*d^2*x^6 - (19648*c^2*d
*x^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF
1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1
/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((
d*x^3)/c), (d*x^3)/(8*c)]))))/(8*c - d*x^3))/(663552*c^5*x^2*Sqrt[c + d*x^3
])
```

Maple [C] time = 0.013, size = 1805, normalized size = 27.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*(-2/3*d/c^2*x/((x^3+1/d*c)*d)^(1/2)-1/2/c^2*(d*x^3+c)^(1/2)/x^2+7/
18*I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))-1/6
4*d/c^2*(-2/27/c^2*x/((x^3+1/d*c)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-d^2*c)^(1
/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_a
lpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)
+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_
alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I
*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))+1/8*d/c*(2/243/c^3*x/((x^3+1/d*c)*d)^(1/2)-1/1944/c^3*x*(d*x^3+c)^(
1/2)/(d*x^3-8*c)-5/1944*I/c^3*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-
1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))
^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_
alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{15}-14\,cd^3x^{12}+33\,c^2d^2x^9+112\,c^3dx^6+64\,c^4x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^15 - 14*c*d^3*x^12 + 33*c^2*d^2*x^9 + 112*c
^3*d*x^6 + 64*c^4*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

3.459 $\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 3/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(320*c^3*x^5*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0614282, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 3/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(320*c^3*x^5*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.196494, size = 283, normalized size = 4.29

$$\frac{-\frac{2027d^3x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^6}+\frac{16789504d^2xF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+\frac{64(-7128\ldots}{\ldots}}{6635520\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((64*(2592*c^3 - 7128*c^2*d*x^3 - 15373*c*d^2*x^6 + 2027*d^3*x^9))/(c^5*x^5
*(-8*c + d*x^3)) - (2027*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^6 + (16789504*d^2*x*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(c^3*(8*c - d*x^3)*(32*c*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2,
2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^
3)/c), (d*x^3)/(8*c)]))))/(6635520*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 2156, normalized size = 32.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/256/c^3*d*(-2/3*d/c^2*x/((x^3+1/d*c)*d)^(1/2)-1/2/c^2*(d*x^3+c)^(1/2)/x^2
+7/18*I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-
1/256*d^2/c^3*(-2/27/c^2*x/((x^3+1/d*c)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-d^2
*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/243*I/c^2/d^3*2^(1/2)*su
m(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)
^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2
*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(
1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(
1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/
d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))+1/64*d^2/c^2*(2/243/c^3*x/((x^3+1/d*c)*d)^(1/2)-1/1944/c^3*x*
(d*x^3+c)^(1/2)/(d*x^3-8*c)-5/1944*I/c^3*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-d^2*
c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^
2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-
d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2
*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2*c)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2
*c)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/
```

```
c^2*(2/3*d^2/c^3*x/((x^3+1/d*c)*d)^(1/2)-1/5/c^2*(d*x^3+c)^(1/2)/x^5+17/20/
c^3*d*(d*x^3+c)^(1/2)/x^2-91/180*I*d/c^3*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

3.460 $\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}-\frac{a\sqrt{c+dx^3}(4bc-5ad)}{3b^3(bc-ad)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

```
[Out] -(a*(4*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(
3/2))/(9*b^2*d) - (a^2*(c + d*x^3)^(3/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) +
 (a*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*
b^(7/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.192206, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}-\frac{a\sqrt{c+dx^3}(4bc-5ad)}{3b^3(bc-ad)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -(a*(4*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(
3/2))/(9*b^2*d) - (a^2*(c + d*x^3)^(3/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) +
 (a*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*
b^(7/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(a+bx)^2}\,dx,x,x^3\right)$$

$$= -\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(-\frac{1}{2}a(2bc-3ad)+b(bc-ad)x\right)}{a+bx}\,dx,x,x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{}{(a+bx)}\right.}{6b^3}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+}\right.}{3b^3d}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right.}{3b^{7/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.24721, size = 147, normalized size = 0.91

$$\frac{-\frac{a^2\left(c+dx^3\right)^{3/2}}{a+bx^3} + \frac{a(5ad-4bc)\left(\sqrt{b}\sqrt{c+dx^3}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}} + \frac{2\left(c+dx^3\right)^{3/2}(bc-ad)}{3d}}{3b^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*(b*c - a*d)*(c + d*x^3)^(3/2))/(3*d) - (a^2*(c + d*x^3)^(3/2))/(a + b*x
^3) + (a*(-4*b*c + 5*a*d)*(Sqrt[b]*Sqrt[c + d*x^3] - Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]))/b^(3/2))/(3*b^2*(b*c - a*d))
```

Maple [C] time = 0.037, size = 917, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b^2/d-2*a/b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2
)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+
I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_a
lpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*
(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf
(_Z^3*b+a)))+a^2/b^2*(-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/d/b*2^(1/2)*su
m(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c
)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*
c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^
(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b
/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9075, size = 975, normalized size = 6.06

$$\left[-\frac{3\left(4\,a^{2}bcd-5\,a^{3}d^{2}+\left(4\,ab^{2}cd-5\,a^{2}bd^{2}\right)x^{3}\right)\sqrt{b^{2}c-abd}\log\left(\frac{bdx^{3}+2\,bc-ad-2\,\sqrt{dx^{3}+c}\sqrt{b^{2}c-abd}}{bx^{3}+a}\right)-2\left(2\left(b^{4}cd-ab^{3}d^{2}\right)x^{6}+2\right.}{18\left(ab^{5}cd-a^{2}b^{4}d^{2}+\left(b^{6}cd-ab^{5}d^{2}\right)x^{3}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(
b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^
2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*s
qrt(d*x^3 + c))/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3), -1/9
*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*b*d^2 +
2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c*d
- a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11787, size = 184, normalized size = 1.14

$$-\frac{\sqrt{dx^{3}+c}a^{2}d}{3\left(\left(dx^{3}+c\right)b-bc+ad\right)b^{3}}-\frac{\left(4\,abc-5\,a^{2}d\right)\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{3\,\sqrt{-b^{2}c+abd}b^{3}}+\frac{2\left(\left(dx^{3}+c\right)^{\frac{3}{2}}b^{4}d^{2}-6\,\sqrt{dx^{3}+c}ab^{3}d^{3}\right)}{9\,b^{6}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/((((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*c
 - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^3)/
(b^6*d^3)
```

3.461 $\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{c+dx^3}(2bc-3ad)}{3b^2(bc-ad)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}+\frac{a\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(3/2
))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.1095, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{c+dx^3}(2bc-3ad)}{3b^2(bc-ad)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}+\frac{a\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(3/2
))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)$$
$$= \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$
$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b^2}$$
$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d}$$
$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.13852, size = 117, normalized size = 0.86

$$\frac{\frac{(2bc-3ad)\left(\sqrt{b}\sqrt{c+dx^3}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}} + \frac{a\left(c+dx^3\right)^{3/2}}{a+bx^3}}{3b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((a*(c + d*x^3)^(3/2))/(a + b*x^3) + ((2*b*c - 3*a*d)*(Sqrt[b]*Sqrt[c + d*x
^3] - Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]))/
b^(3/2))/(3*b*(b*c - a*d))
```

Maple [C] time = 0.01, size = 897, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*
(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3
*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(-1/3*(d*x^3+
c)^(1/2)/b/(b*x^3+a)-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*
c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alp
ha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84243, size = 707, normalized size = 5.2

$$\left[-\frac{\left(\left(2\,b^2c-3\,abd\right)x^3+2\,abc-3\,a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(3\,ab^2c-3\,a^2bd+2\left(b^3c-ab^2d\right)x^3\right.}{6\left(ab^4c-a^2b^3d+\left(b^5c-ab^4d\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*lo
g((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 +
a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c))/
(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d)*x^
3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^
2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d
)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [A] time = 1.10802, size = 150, normalized size = 1.1

$$\frac{\frac{\sqrt{dx^3+c}ad^2}{\left(\left(dx^3+c\right)b-bc+ad\right)b^2}+\frac{2\sqrt{dx^3+c}d}{b^2}+\frac{\left(2\,bcd-3\,ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(d*x^3 + c)*a*d^2/(((d*x^3 + c)*b - b*c + a*d)*b^2) + 2*sqrt(d*x^3
 + c)*d/b^2 + (2*b*c*d - 3*a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*
b*d))/(sqrt(-b^2*c + a*b*d)*b^2))/d
```

3.462 $\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

```
[Out] -Sqrt[c + d*x^3]/(3*b*(a + b*x^3)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0651737, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 47, 63, 208}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*b*(a + b*x^3)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}+\frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b}\\
&= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}+\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b}\\
&= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.0828787, size = 80, normalized size = 1.

$$
\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{ad-bc}}\right)}{3b^{3/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [C] time = 0.007, size = 453, normalized size = 5.7

$$-\frac{1}{3\,b\left(bx^3+a\right)}\sqrt{dx^3+c}-\frac{\frac{i}{6}\sqrt{2}}{bd}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{1}{ad-bc}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d^2c}}}\sqrt{d\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*
c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^
2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1
/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c
)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65338, size = 533, normalized size = 6.66

$$\left[\frac{\left(bdx^3+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{6\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x^3\right)},\ \frac{\left(bdx^3+ad\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{}}{}\right.}{3\left(ab^3c-a^2b^2d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sq
rt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 + a*
d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^
3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c
- a*b^3*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [A] time = 1.10827, size = 107, normalized size = 1.34

$$\frac{1}{3}\,d\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^3+c}}{\left(\left(dx^3+c\right)b-bc+ad\right)b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d*(arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b) - sqrt(d*x^3 + c)/(((d*x^3 + c)*b - b*c + a*d)*b))
```

3.463 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=121

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)}$$

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt
[c]])/(3*a^2) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(3*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.117871, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt
[c]])/(3*a^2) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(3*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
```

```
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+d\right.}{3a^2d}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.212566, size = 112, normalized size = 0.93

$$\frac{\frac{a\sqrt{c+dx^3}}{a+bx^3} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}} - 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3])/(a + b*x^3) - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]] + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(Sq
rt[b]*Sqrt[b*c - a*d]))/(3*a^2)
```

Maple [C] time = 0.012, size = 934, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2
*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(
1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2
*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/
2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_a
lpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_al
pha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-b/a*(-1/3*(d
*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3
))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2
)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)
*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(2
/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)
```

Fricas [B] time = 1.76131, size = 1800, normalized size = 14.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))
- 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3
)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b^2*c - a
^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3
), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2
*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b
*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4
*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c
 - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/
(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*a
rctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)
```

Giac [A] time = 1.12058, size = 170, normalized size = 1.4

$$\frac{1}{3}\,d^2\left(\frac{\sqrt{dx^3+c}}{\left(\left(dx^3+c\right)b-bc+ad\right)ad}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d^2*(sqrt(d*x^3 + c)/(((d*x^3 + c)*b - b*c + a*d)*a*d) - (2*b*c - a*d)*
arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*d^
2) + 2*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2))
```

3.464 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

```
[Out] (-2*b*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*x^3*(a +
b*x^3)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*Sqrt[c])
- (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d
]])/(3*a^3*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.217879, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 99, 151, 156, 63, 208}

$$-\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-2*b*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*x^3*(a +
b*x^3)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*Sqrt[c])
- (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d
]])/(3*a^3*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6a^3} - \frac{(4bc-ad)\,\text{Su}}{}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^3d} - \frac{(4bc-a}{}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d}}{\sqrt{bc-ad}}\right.}{3a^3\sqrt{bc-ad}}$$

Mathematica [A] time = 0.228915, size = 190, normalized size = 1.18

$$\frac{\sqrt{c}\left(a\left(a+2bx^3\right)\sqrt{c+dx^3}(bc-ad)+\sqrt{b}x^3\left(a+bx^3\right)(4bc-3ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)-x^3\left(a+bx^3\right)\left(a^2d^2-5a\right.}{3a^3\sqrt{c}x^3\left(a+bx^3\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*x^3*(a + b*x^3)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]]) + Sqrt[c]*(a*(b*c - a*d)*(a + 2*b*x^3)*Sqrt[c + d*x^3] + Sqrt[b
]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*x^3*(a + b*x^3)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]]))/(3*a^3*Sqrt[c]*(-(b*c) + a*d)*x^3*(a + b*x^3))
```

Maple [C] time = 0.013, size = 978, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 2/a^3*b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^
2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_a
lpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-1/3
*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+b^2/a^
2*(-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-d^2
*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(
-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^
2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^
2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
)-2/a^3*b*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)^2 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)
```

Fricas [A] time = 1.80802, size = 1858, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(
b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt
(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*
x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*
c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*
c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + (
(4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(
a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c^2 - 3*a*b*c*
d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a))
 + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/3
*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c
*x^6 + a^4*c*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1641, size = 258, normalized size = 1.6

$$-\frac{1}{3}\,d^{3}\left(\frac{2\left(dx^{3}+c\right)^{\frac{3}{2}}b-2\,\sqrt{dx^{3}+c}bc+\sqrt{dx^{3}+c}ad}{\left(\left(dx^{3}+c\right)^{2}b-2\left(dx^{3}+c\right)bc+bc^{2}+\left(dx^{3}+c\right)ad-acd\right)a^{2}d^{2}}-\frac{\left(4\,b^{2}c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}a^{3}d^{3}}+\frac{\left(4\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{a^{3}\sqrt{-c}d^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*d^3*((2*(d*x^3 + c)^(3/2)*b - 2*sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*
a*d)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*
d)*a^2*d^2) - (4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*
b*d))/(sqrt(-b^2*c + a*b*d)*a^3*d^3) + (4*b*c - a*d)*arctan(sqrt(d*x^3 + c)
/sqrt(-c))/(a^3*sqrt(-c)*d^3))
```

3.465 $\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0513995, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.214935, size = 235, normalized size = 3.67

$$\frac{x\left(\frac{8\left(\frac{8ac^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}-c-dx^3\right)}{a+bx^3}+\frac{5dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}\right)}{24b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x*((5*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/a + (8*(-c - d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d
*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a
+ b*x^3)))/(24*b*Sqrt[c + d*x^3])
```

Maple [C] time = 0.037, size = 1468, normalized size = 22.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)

[Out] 1/b*(-2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2
*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*
_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c)
,(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(1/3/a*x*(d*x^3+c)^(1/2)/(b*
x^3+a)-1/9*I/a/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1
/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1
/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/
3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x^3}{(bx^3 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{c + dx^3}}{(a + bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}x^3}{(bx^3 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

3.466 $\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0407256, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{x\sqrt{1+\frac{dx^3}{c}}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.103338, size = 153, normalized size = 2.39

$$\frac{-dx^5\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5cx^2\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+10ax^2\left(c+dx^3\right)}{30a^2\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (10*a*x^2*(c + d*x^3) + 5*c*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*x^5*(a + b*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(30*a^2*(a
 + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.034, size = 908, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+
1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/
```

```
3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)
^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3
^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_alpha/(a*d-b*c)*(-d^2
*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(
-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^
2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^
2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + cx}}{\left(bx^3 + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}x}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

3.467 $\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0271805, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.200597, size = 232, normalized size = 3.93

$$\frac{x\left(\frac{dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a^2}+\frac{8\left(\frac{16c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+\frac{c+dx^3}{a}\right)}{a+bx^3}\right)}{24\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a^2 + (8*((c + d*x^3)/a + (16*c^2*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
-((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(
a + b*x^3)))/(24*Sqrt[c + d*x^3])
```

Maple [C] time = 0.005, size = 753, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alph
a^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c
)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*
c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^
(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b
/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^3}}{(a + bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{(bx^3 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

3.468 $\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rubi [A] time = 0.0544932, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.136067, size = 172, normalized size = 2.77

$$\frac{8bdx^6\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5x^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}(9ad-8bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20a}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(3*a + 4*b*x^3)*(c + d*x^3) + 5*(-8*b*c + 9*a*d)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 8
*b*d*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^
3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 2227, normalized size = 35.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*
```

```
(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c
)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*
(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^
(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/a^
2*b*(-2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/b/d^2*2^(1/2)
*sum(1/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c
)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*
c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^
(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b
/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a)))-b/a*(1/3/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(
1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/
(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b
*c)/_alpha/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*
(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
```

```
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-
d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_
alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

3.469 $\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)])
/(2*a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0512235, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)])
/(2*a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.293319, size = 338, normalized size = 5.28

$$\frac{\frac{a\left(32ac\left(6ac-3adx^3+30bcx^3+10bdx^6\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(3a+5bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}-5bdx^6\sqrt{\vphantom{x}}}{48a^3x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-5*b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + (a*(32*a*c*(6*a*c + 30*b*c*x^3 - 3*a*d*x^3 + 10*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(3*a + 5*b*x^3)
*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)]
+ a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3
)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2
*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[
4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^
3])
```

Maple [C] time = 0.01, size = 1768, normalized size = 27.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-1/2/x^2*(d*x^3+c)^(1/2)-1/2*I*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-
d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2
)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3
))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)))-1/a^2*b*(-2/3*I/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/
2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3)
)^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c
)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-d^2*c)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2
*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)
^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-b/
a*(1/3/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/b*3^(1/2)*(-d^2*c)^(1/3)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_a
lpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3
)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^
2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d
^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*
c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/
2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2
```

```
*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

3.470 $\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2\left(a+bx^3\right)(bc-ad)}-\frac{a\left(c+dx^3\right)^{3/2}(4bc-7ad)}{9b^3(bc-ad)}-\frac{a\sqrt{c+dx^3}(4bc-7ad)}{3b^4}+\frac{a(4bc-7ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

```
[Out] -(a*(4*b*c - 7*a*d)*Sqrt[c + d*x^3])/(3*b^4) - (a*(4*b*c - 7*a*d)*(c + d*x^
3)^(3/2))/(9*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(5/2))/(15*b^2*d) - (a^2*(c
+ d*x^3)^(5/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rubi [A] time = 0.237007, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2\left(a+bx^3\right)(bc-ad)}-\frac{a\left(c+dx^3\right)^{3/2}(4bc-7ad)}{9b^3(bc-ad)}-\frac{a\sqrt{c+dx^3}(4bc-7ad)}{3b^4}+\frac{a(4bc-7ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(4*b*c - 7*a*d)*Sqrt[c + d*x^3])/(3*b^4) - (a*(4*b*c - 7*a*d)*(c + d*x^
3)^(3/2))/(9*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(5/2))/(15*b^2*d) - (a^2*(c
+ d*x^3)^(5/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(-\frac{1}{2}a(2bc-5ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-7ad))\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-7ad))\,\text{Subst}\left(\int \cdots\right.}{6b^3 \cdots}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \cdots$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \cdots$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \cdots$$

Mathematica [A] time = 0.162128, size = 162, normalized size = 0.86

$$\frac{\sqrt{c+dx^3}\left(5a^2bd\left(14dx^3-19c\right)+105a^3d^2+2ab^2\left(3c^2-34cdx^3-7d^2x^6\right)+6b^3x^3\left(c+dx^3\right)^2\right)}{45b^4d\left(a+bx^3\right)} + \frac{a(4bc-7ad)\sqrt{bc-ad}\,\text{ta}\cdots}{3b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c
+ 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b*x
^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Maple [C] time = 0.039, size = 1003, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b^2/d-2*a/b^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*
d-2*b*c)/b^2-2/3*d/b*c)/d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^
(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alp
ha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+a^2/b^2*(1/3*(a*d-b*c)/b^2*(d*x^3+c)
^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-d^2*c)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*
c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^
2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1
/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c
)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18314, size = 944, normalized size = 4.99

$$\left[-\frac{15\left(4\,a^2bcd-7\,a^3d^2+\left(4\,ab^2cd-7\,a^2bd^2\right)x^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(6\,b^3d^2x^9+2\left(6\,b^3cd-7\,ab^2d\right.\right.}{90\left(b^5dx^3+ab^4d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt
((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c
- a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^
6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d
+ 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*(4*a^
2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a*d)/b)
*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^3*d^2*x
^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3
*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^5*d*x^3 + a*b^4*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15928, size = 285, normalized size = 1.51

$$-\frac{\left(4\,ab^2c^2-11\,a^2bcd+7\,a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^4}-\frac{\sqrt{dx^3+c}a^2bcd-\sqrt{dx^3+c}a^3d^2}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^4}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^8d^4-10\left(dx^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt
(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b*c
*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(3*(
d*x^3 + c)^(5/2)*b^8*d^4 - 10*(d*x^3 + c)^(3/2)*a*b^7*d^5 - 30*sqrt(d*x^3 +
 c)*a*b^7*c*d^5 + 45*sqrt(d*x^3 + c)*a^2*b^6*d^6)/(b^10*d^5)
```

3.471 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=163

$$\frac{\left(c+dx^3\right)^{3/2}(2bc-5ad)}{9b^2(bc-ad)}+\frac{\sqrt{c+dx^3}(2bc-5ad)}{3b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{a\left(c+dx^3\right)^{5/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3) + ((2*b*c - 5*a*d)*(c + d*x^3)^(3
/2))/(9*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(5/2))/(3*b*(b*c - a*d)*(a + b*x^
3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(3*b^(7/2))
```

Rubi [A] time = 0.136803, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(c+dx^3\right)^{3/2}(2bc-5ad)}{9b^2(bc-ad)}+\frac{\sqrt{c+dx^3}(2bc-5ad)}{3b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{a\left(c+dx^3\right)^{5/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3) + ((2*b*c - 5*a*d)*(c + d*x^3)^(3
/2))/(9*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(5/2))/(3*b*(b*c - a*d)*(a + b*x^
3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(3*b^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-5ad)\,\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-5ad)\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b^2}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{((2bc-5ad)(bc-ad))}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{((2bc-5ad)(bc-ad))}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-5ad)\sqrt{bc-ad}\,\mathrm{t}}{3b^{7/2}}$$

Mathematica [A] time = 0.101708, size = 125, normalized size = 0.77

$$\frac{\sqrt{c+dx^3}\left(-15a^2d+ab\left(11c-10dx^3\right)+2b^2x^3\left(4c+dx^3\right)\right)}{9b^3\left(a+bx^3\right)} - \frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x^
3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Maple [C] time = 0.009, size = 983, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c)/d*(d*x^
3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)
*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(
1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-
d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*
d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2
*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*b+a)))-a/b*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)^(1
/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^
(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alp
ha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6323, size = 679, normalized size = 4.17

$$\left[-\frac{3\left(\left(2\,b^2c-5\,abd\right)x^3+2\,abc-5\,a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(2\,b^2dx^6+2\left(4\,b^2c-5\,abd\right)x^3+11\,abc\right.}{18\left(b^4x^3+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b)
*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x
^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^2*d
)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*
a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c
- a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*
c - 15*a^2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11715, size = 234, normalized size = 1.44

$$\frac{\left(2\,b^2c^2-7\,abcd+5\,a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3}+\frac{\sqrt{dx^3+c}abcd-\sqrt{dx^3+c}a^2d^2}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^4+3\,\sqrt{dx^3+c}b^4c-6\,\sqrt{\right.}}{9\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - sqr
t(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 + c)^
(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6
```

3.472 $\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal. Leaf size=94

$$-\frac{d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx^3)^{3/2}}{3b(a+bx^3)}+\frac{d\sqrt{c+dx^3}}{b^2}$$

```
[Out] (d*Sqrt[c + d*x^3])/b^2 - (c + d*x^3)^(3/2)/(3*b*(a + b*x^3)) - (d*Sqrt[b*c
 - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rubi [A] time = 0.0812484, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 47, 50, 63, 208}

$$-\frac{d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx^3)^{3/2}}{3b(a+bx^3)}+\frac{d\sqrt{c+dx^3}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (d*Sqrt[c + d*x^3])/b^2 - (c + d*x^3)^(3/2)/(3*b*(a + b*x^3)) - (d*Sqrt[b*c
 - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2} \, dx = \frac{1}{3} \operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2} \, dx, x, x^3\right)$$
$$= -\frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{d \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx} \, dx, x, x^3\right)}{2b}$$
$$= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{(d(bc-ad)) \operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} \, dx, x, x^3\right)}{2b^2}$$
$$= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{(bc-ad) \operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} \, dx, x, \sqrt{c+dx^3}\right)}{b^2}$$
$$= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} - \frac{d\sqrt{bc-ad} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.0226425, size = 54, normalized size = 0.57

$$\frac{2d\left(c+dx^3\right)^{5/2} {}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b\left(dx^3+c\right)}{ad-bc}\right)}{15(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (2*d*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x^3))/(-(
b*c) + a*d))])/(15*(-(b*c) + a*d)^2)
```

Maple [C] time = 0.005, size = 466, normalized size = 5.

$$\frac{ad-bc}{3b^2\left(bx^3+a\right)}\sqrt{dx^3+c} + \frac{2d}{3b^2}\sqrt{dx^3+c} + \frac{\frac{i}{2}\sqrt{2}}{db^2} \sum_{\_alpha=RootOf\left(b\_Z^3+a\right)} \sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\frac{1}{\sqrt[3]{-d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)^(1/2)/b^2+1/2*I
/d/b^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-
d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^
2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57281, size = 493, normalized size = 5.24

$$\left[\frac{3\left(bdx^3+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(2\,bdx^3-bc+3\,ad\right)\sqrt{dx^3+c}}{6\left(b^3x^3+ab^2\right)},-\frac{3\left(bdx^3+ad\right)\sqrt{-\frac{bc-ad}{b}}\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c +
```

```
3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(-(b
*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
(2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14556, size = 161, normalized size = 1.71

$$\frac{1}{3}d\left(\frac{3\left(bc-ad\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}+\frac{2\sqrt{dx^3+c}}{b^2}-\frac{\sqrt{dx^3+c}bc-\sqrt{dx^3+c}ad}{\left(\left(dx^3+c\right)b-bc+ad\right)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d*(3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 2*sqrt(d*x^3 + c)/b^2 - (sqrt(d*x^3 + c)*b*c - sqrt(d
*x^3 + c)*a*d)/(((d*x^3 + c)*b - b*c + a*d)*b^2))
```

3.473 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}(bc-ad)}{3ab\left(a+bx^3\right)}$$

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) - (2*c^(3/2)*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(3*a^2) + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*b^(3/2))
```

Rubi [A] time = 0.13963, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}(bc-ad)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) - (2*c^(3/2)*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(3*a^2) + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*b^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ab}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{((bc-ad)(2bc+ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx\right.}{6a^2b}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{((bc-ad)(2bc+ad))\,\text{Subst}\left(\int \right.}{3a^2bd}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{bc-ad}(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

Mathematica [A] time = 0.163404, size = 122, normalized size = 0.93

$$\frac{\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + \frac{a\sqrt{c+dx^3}(bc-ad)}{b\left(a+bx^3\right)} - 2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) - 2*c^(3/2)*ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))/(3*a^2)
```

Maple [C] time = 0.011, size = 1036, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c)/d*
(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d
-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^
(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+
I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_a
lpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*
(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf
(_Z^3*b+a)))-b/a*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+
c)^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^
2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2
*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c
),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(2/9*d*x^3*(d*x^3+c)^(1/2
)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)
```

Fricas [A] time = 1.81942, size = 1485, normalized size = 11.34

$$\left[\frac{\left(\left(2\,b^2c+abd\right)x^3+2\,abc+a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(b^2cx^3+abc\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+\cdots}{6\left(a^2b^2x^3+a^3b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b*
d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a))
 + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/3*((
(2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*sqrt(c
)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 + c)*(a*b
*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arc
tan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)
*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt(
(b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*
x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a
*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + 2*(b^2
*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 +
c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15097, size = 223, normalized size = 1.7

$$\frac{1}{3}d^2\left(\frac{2c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}+\frac{\sqrt{dx^3+c}bc-\sqrt{dx^3+c}ad}{\left(\left(dx^3+c\right)b-bc+ad\right)abd}-\frac{\left(2b^2c^2-abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2bd^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d^2*(2*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*d^2) + (sqrt(
d*x^3 + c)*b*c - sqrt(d*x^3 + c)*a*d)/((((d*x^3 + c)*b - b*c + a*d)*a*b*d) -
 (2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b
*d))/(sqrt(-b^2*c + a*b*d)*a^2*b*d^2))
```

3.474 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{\sqrt{c+dx^3}(2bc-ad)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}-\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - (c*Sqrt[c + d*x^3])/
(3*a*x^3*(a + b*x^3)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/(3*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b])
```

Rubi [A] time = 0.257946, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 98, 151, 156, 63, 208}

$$-\frac{\sqrt{c+dx^3}(2bc-ad)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}-\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - (c*Sqrt[c + d*x^3])/
(3*a*x^3*(a + b*x^3)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/(3*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3} + \frac{((bc-ad)(4}{}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d} + \frac{((bc}{}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad)\,\text{t}}{3a^3\sqrt{}}$$

Mathematica [A] time = 0.219075, size = 142, normalized size = 0.84

$$\frac{\frac{a\sqrt{c+dx^3}\left(-ac+adx^3-2bcx^3\right)}{x^3(a+bx^3)} + \sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + Sqr
t[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]] - (Sqrt[b*c - a*d]*(4
*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b])/(3
*a^3)
```

Maple [C] time = 0.011, size = 1093, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 2/a^3*b^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c)/d
*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*
d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)
+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_
alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I
*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*b+a)))+1/a^2*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/
2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+b^2/a^2*(1/3*(a*d-b*c)/b^2*(d*x^3+c)
^(1/2)/(b*x^3+a)+2/3*d*(d*x^3+c)^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-d^2*c)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*
c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^
2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1
/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c
)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2
/a^3*b*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh
((d*x^3+c)^(1/2)/c^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)
```

Fricas [A] time = 1.84515, size = 1782, normalized size = 10.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*
log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^
3 + a)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 +
a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x
^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*
sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3
*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2
*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1
/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3
)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt
((b*c - a*d)/b))/(b*x^3 + a)) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^
3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^
2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/
b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(
-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*a*b*c - a^2*d)*x^3 + a^2*c)*sq
rt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15051, size = 300, normalized size = 1.76

$$-\frac{1}{3}\,d^{3}\left(\frac{2\left(dx^{3}+c\right)^{\frac{3}{2}}bc-2\sqrt{dx^{3}+c}bc^{2}-\left(dx^{3}+c\right)^{\frac{3}{2}}ad+2\sqrt{dx^{3}+c}acd}{\left(\left(dx^{3}+c\right)^{2}b-2\left(dx^{3}+c\right)bc+bc^{2}+\left(dx^{3}+c\right)ad-acd\right)a^{2}d^{2}}-\frac{\left(4\,b^{2}c^{2}-5\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}a^{3}d^{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*d^3*((2*(d*x^3 + c)^(3/2)*b*c - 2*sqrt(d*x^3 + c)*b*c^2 - (d*x^3 + c)^
(3/2)*a*d + 2*sqrt(d*x^3 + c)*a*c*d)/((((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c
+ b*c^2 + (d*x^3 + c)*a*d - a*c*d)*a^2*d^2) - (4*b^2*c^2 - 5*a*b*c*d + a^2*
d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a
^3*d^3) + (4*b*c^2 - 3*a*c*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c
)*d^3))
```

3.475 $\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=65

$$\frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.056264, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.422904, size = 338, normalized size = 5.2

$$\frac{x^4\left(\frac{d\sqrt{\frac{dx^3}{c}+1}(43bc-55ad)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-8acd\left(11ad+b\left(c+6dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3\left(c+dx^3\right)\left(-11ad+5bc-6bdx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c}\right.\right.\right.}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},\right.\right.}\right.}{120b^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^4*((d*(43*b*c - 55*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-8*a*c*d*(11*a*d + b*(c + 6*d*x^3))*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*(c + d*x^3)*(5*b*c
- 11*a*d - 6*b*d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((
a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*
AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(120*b^2*Sqrt[c
 + d*x^3])
```

Maple [C] time = 0.039, size = 1587, normalized size = 24.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)

[Out] 1/b*(2/5*d/b*x*(d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*d/b*c)*3^(1/2)
/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1
/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2
*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(
1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3))
)^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2
*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/
2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_a
lpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_al
pha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(-1/3*(a
*d-b*c)/a/b*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*d-b*c)/
a)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a
/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c)*(-d^2
c)^(1/3)(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(
-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^
2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^
2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

3.476 $\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal. Leaf size=65

$$\frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0405539, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```


```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= \frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.181383, size = 177, normalized size = 2.72

$$\frac{x^2\left(-dx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}(bc-7ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5c\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}(2ad+bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\right.\right.}{30a^2b\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^2*(-10*a*(-(b*c) + a*d)*(c + d*x^3) + 5*c*(b*c + 2*a*d)*(a + b*x^3)*Sqrt
[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*
(b*c - 7*a*d)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a^2*b*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.035, size = 955, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2+1/6/b^2*d*(
a*d-b*c)/a)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3)
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c
)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/18*I/a/b^2/d^2*2^
(1/2)*sum((7*a^2*d^2-5*a*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-d^2*c)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3)
)^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-
d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*
_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

3.477 $\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=60

$$\frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0267157, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.316535, size = 339, normalized size = 5.65

$$\frac{x\left(\frac{a\left(24x^3\left(c+dx^3\right)(bc-ad)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-64ac\left(bc\left(3c+dx^3\right)-ad^2x^3\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+dx^3\sqrt{\frac{dx^3}{c}+1}(\right.}{24a^2b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (x*(d*(b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)] + (a*(-64*a*c*(-(a*d^2*x^3) + b*c*(3*c + d*x^3))*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*(b*c - a*d)*x^3*(
c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] +
a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(24*a^2*b*Sqrt[c + d*x^3])
```

Maple [C] time = 0.005, size = 801, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*
d-b*c)/a)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/
(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1
/18*I/a/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c
)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3
)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^
(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha
*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^
2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

[Out] Exception raised: UnboundLocalError

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

3.478 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rubi [A] time = 0.0569014, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)^2}\,dx=\frac{\left(c\sqrt{c+dx^3}\right)\int\frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$=-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] time = 0.177493, size = 190, normalized size = 3.02

$$\frac{2dx^6\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}(4bc-ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5cx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}(11ad-8bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\right.}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(c + d*x^3)*(3*a*c + 4*b*c*x^3 - a*d*x^3) + 5*c*(-8*b*c + 11*a*d)*x^
3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
-((b*x^3)/a)] + 2*d*(4*b*c - a*d)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sq
rt[c + d*x^3])
```

Maple [C] time = 0.014, size = 2364, normalized size = 37.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x)
```

[Out] 1/a^2*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/2)*(-d^2
c)^(1/3)(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/
2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/
2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I
(x+1/2/d(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(
1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2))))-1/a^2*b*(2/7*d/b*x^2*(d*x^3+c)^(1/2)-2/3*I*(-
d*(a*d-2*b*c)/b^2-4/7*d/b*c)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1
/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^
(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d
/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1
/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*
c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/
3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-d^2
c)^(1/3)(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d
^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(
-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)
^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^
2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^
2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
)-b/a*(-1/3*(a*d-b*c)/a/b*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2+1/6/
b^2*d*(a*d-b*c)/a)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c
)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*
(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)
/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(

```
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/18*I/a/b^2
/d^2*2^(1/2)*sum((7*a^2*d^2-5*a*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-d^2*c)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2
*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)
^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

3.479 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rubi [A] time = 0.0553509, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```



```
[Out] -(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*x^2*Sqrt[1 + (d*x^3)/c])
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^3\right)^{3/2}}{x^3\left(a+bx^3\right)^2}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.376524, size = 370, normalized size = 5.69

$$\frac{\frac{8a\left(4ac\left(a\left(6c^2-15cdx^3-4d^2x^6\right)+10bcx^3\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(c+dx^3\right)\left(3ac-2adx^3+5bcx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\cdots\right.\right.\right.}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}}{48a^3x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-(d*(5*b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)]) + (8*a*(4*a*c*(10*b*c*x^3*(3*c + d*x^3) + a*(6*c^
2 - 15*c*d*x^3 - 4*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] - 3*x^3*(c + d*x^3)*(3*a*c + 5*b*c*x^3 - 2*a*d*x^3)*(2*b*c*AppellF
1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 1902, normalized size = 29.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-1/2*c*(d*x^3+c)^(1/2)/x^2+2/5*d*x*(d*x^3+c)^(1/2)-9/10*I*c*3^(1/2)*
(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(
-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-1/a^2*b*(2/5*d/b*x*(
d*x^3+c)^(1/2)-2/3*I*(-d*(a*d-2*b*c)/b^2-2/5*d/b*c)*3^(1/2)/d*(-d^2*c)^(1/3
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^
2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(
-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3
)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2
*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-
b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-b/a*(-1/3*(a*d-b*c)/a/b*x*(d
*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*d-b*c)/a)*3^(1/2)/d*(-d
^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+
1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a/b^2/d^2*2^(1/2)
*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/
2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
```

```
*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3
))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alph
a-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3 + c)^{\frac{3}{2}}}{(bx^3 + a)^2 x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

3.480 $\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^3}}{3b^2(a+bx^3)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}+\frac{2\sqrt{c+dx^3}}{3b^2d}$$

```
[Out] (2*Sqrt[c + d*x^3])/(3*b^2*d) - (a^2*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)*(a
 + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*b^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.143646, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^3}}{3b^2(a+bx^3)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}+\frac{2\sqrt{c+dx^3}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b^2*d) - (a^2*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)*(a
 + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$=-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2(bc-ad)}$$
$$=\frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}-\frac{(a(4bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b^2(bc-ad)}$$
$$=\frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}-\frac{(a(4bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d(bc-ad)}$$
$$=\frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.251641, size = 107, normalized size = 0.87

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}\left(\frac{a^2}{\left(a+bx^3\right)(ad-bc)}+\frac{2}{d}\right)}{b^2}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((Sqrt[c + d*x^3]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^3))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/3
```

Maple [C] time = 0.037, size = 911, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b^2/d+2/3*I*a/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c
)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2
*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/
3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)
^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)
^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+a^2
/b^2*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*
c)^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(
1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I
*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3
)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_al
pha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(
-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73021, size = 983, normalized size = 7.99

$$\left[\frac{\left(4\,a^2bcd-3\,a^3d^2+\left(4\,ab^2cd-3\,a^2bd^2\right)x^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2\right.}{6\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2*
c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*
d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3), -1
/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11259, size = 181, normalized size = 1.47

$$-\frac{\sqrt{dx^3+c}a^2d}{3\left(b^3c-ab^2d\right)\left(\left(dx^3+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{2\,\sqrt{dx^3+c}}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^3 + c)*b - b*c + a*d))
- 1/3*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 2/3*sqrt(d*x^3 + c)/(b^2*d)
```

3.481 $\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^3}}{3b(a+bx^3)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0844353, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^3}}{3b(a+bx^3)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```


```
[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}+\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}+\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0916238, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{\left(a+bx^3\right)(bc-ad)}+\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*b^(
3/2))
```

Maple [C] time = 0.01, size = 892, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^
2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2
*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c
),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(1/3/(a*d-b*c)*(d*x^3+c)^(1/
2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(
x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*
c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63511, size = 722, normalized size = 7.29

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^3+2\,abc-a^2d\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{6\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^3\right)},\frac{\left(\left(2\,b^2c-abd\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) +
 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.10433, size = 157, normalized size = 1.59

$$\frac{\frac{\sqrt{dx^3+c}ad^2}{(b^2c-abd)((dx^3+c)b-bc+ad)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.482 $\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^3]/(3*(b*c - a*d)*(a + b*x^3)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0726292, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*(b*c - a*d)*(a + b*x^3)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6(bc-ad)}\\ &= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3(bc-ad)}\\ &= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0846254, size = 85, normalized size = 0.98

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]/((-(b*c) + a*d)*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3
```

Maple [C] time = 0.007, size = 457, normalized size = 5.3

$$\frac{1}{(3\,ad-3\,bc)\left(bx^3+a\right)}\sqrt{dx^3+c}-\frac{\frac{i}{6}\sqrt{2}}{d}\sum_{\_alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{1}{(ad-bc)^2}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2\,x+\frac{1}{d}\left(-i\sqrt{3}\sqrt[3]{-d^2c}+\sqrt[3]{-d^2c}\right)\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(
-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))
/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/
2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha
*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-
(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c
)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*
(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*
(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b
+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59947, size = 639, normalized size = 7.34

$$\left[-\frac{\left(bdx^3+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{6\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^3\right)},-\frac{\left(bdx^3+ad\right)\sqrt{-b^2c+abd}\arctan\left(\right.}{3\left(ab^3c^2-2\,a^2b^2cd+a^3bd\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^3)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.12037, size = 124, normalized size = 1.43

$$-\frac{1}{3}\,d\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{\sqrt{dx^3+c}}{\left(\left(dx^3+c\right)b-bc+ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d*(arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) + sqrt(d*x^3 + c)/(((d*x^3 + c)*b - b*c + a*d)*(b*c - a*d)))
```

3.483 $\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(bc-ad)}$$

```
[Out] (b*Sqrt[c + d*x^3])/(3*a*(b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.13866, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```


```
[Out] (b*Sqrt[c + d*x^3])/(3*a*(b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(3/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{6a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{}{a-} \right.}{3a^2d(bc-}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.233109, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^3}}{\left(a+bx^3\right)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*a^2)
```

Maple [C] time = 0.01, size = 915, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a^2*b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(
-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3
)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2
*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-
b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-b/a*(1/3/(a*d-b*c)*(d*x^3+c)^
(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-d^2*c)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(
d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2
)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d
^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*
c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-2/3*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/a^2/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)
```

Fricas [A] time = 1.96192, size = 1831, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c
)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3
 + b*c)) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqr
t(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^3), 1/6*(2*sqrt(d*x^3 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^3 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c^2 - 3*a^2*
c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c
 - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c)*
a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.69251, size = 207, normalized size = 1.57

$$-\frac{1}{3}\,d^{2}\left(\frac{\left(2\,b^{2}c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\left(a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-b^{2}c+abd}}-\frac{\sqrt{dx^{3}+c}b}{\left(abcd-a^{2}d^{2}\right)\left(\left(dx^{3}+c\right)b-bc+ad\right)}-\frac{2\,\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{a^{2}\sqrt{-c}d^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d^2*((2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d)
)/((a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x^3 + c)*b/((a*b*
c*d - a^2*d^2)*((d*x^3 + c)*b - b*c + a*d)) - 2*arctan(sqrt(d*x^3 + c)/sqrt
(-c))/(a^2*sqrt(-c)*d^2))
```

3.484 $\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b\sqrt{c+dx^3}(2bc-ad)}{3a^2c\left(a+bx^3\right)(bc-ad)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}$$

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*c*(b*c - a*d)*(a + b*x^3)) - Sqrt
[c + d*x^3]/(3*a*c*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(3*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.242982, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b\sqrt{c+dx^3}(2bc-ad)}{3a^2c\left(a+bx^3\right)(bc-ad)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*c*(b*c - a*d)*(a + b*x^3)) - Sqrt
[c + d*x^3]/(3*a*c*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(3*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x,\right.}{3a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{\left(b^2(4bc-5ad)\right)\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x\right.}{6a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{\left(b^2(4bc-5ad)\right)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \right.}{3a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b^{3/2}(4bc-5}{3a}$$

Mathematica [A] time = 0.545622, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^3}\left(a^2d+ab\left(dx^3-c\right)-2b^2cx^3\right)}{x^3\left(a+bx^3\right)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(a^2*d - 2*b^2*c*x^3 + a*b*(-c + d*x^3)))/((b*c - a*d)*
x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(3*a^3*c)
```

Maple [C] time = 0.013, size = 961, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/a^3*b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/
d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(
2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*
d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a
*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*
(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-1/3*(d*x^3+c)^(1/2
)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))+b^2/a^2*(1/3/(a*d-b
*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-d^2*c)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)
))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(
2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))+4/3/
a^3*b*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^4), x)
```

Fricas [A] time = 2.32533, size = 2504, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)
*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 +
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a
^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*l
og((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arcta
n(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/3*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13201, size = 362, normalized size = 1.96

$$\frac{1}{3}d^3\left(\frac{\left(4b^3c-5ab^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)^{\frac{3}{2}}b^2c-2\sqrt{dx^3+c}b^2c^2-\left(dx^3+c\right)^{\frac{3}{2}}abd+2\sqrt{dx^3+c}abcd-\sqrt{dx}}{\left(a^2bc^2d^2-a^3cd^3\right)\left(\left(dx^3+c\right)^2b-2\left(dx^3+c\right)bc+bc^2+\left(dx^3+c\right)ad-a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*d^3*((4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d
))/((a^3*b*c*d^3 - a^4*d^4)*sqrt(-b^2*c + a*b*d)) - (2*(d*x^3 + c)^(3/2)*b^
2*c - 2*sqrt(d*x^3 + c)*b^2*c^2 - (d*x^3 + c)^(3/2)*a*b*d + 2*sqrt(d*x^3 +
c)*a*b*c*d - sqrt(d*x^3 + c)*a^2*d^2)/((a^2*b*c^2*d^2 - a^3*c*d^3)*((d*x^3
+ c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)) - (4*b*c +
 a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c*d^3))
```

3.485 $\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0523328, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$=\frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.211872, size = 238, normalized size = 3.72

$$\frac{x\left(\frac{8\left(\frac{8ac^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}+c+dx^3\right)}{a+bx^3}+\frac{dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}\right)}{24\sqrt{c+dx^3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a + (8*(c + d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^
3)/c), -((b*x^3)/a)])/(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a +
b*x^3)))/(24*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.036, size = 1207, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x
-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c
)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)
/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(-1/3*b/(a*d-b*c)/a
*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/(a*d-b*c)/a*3^(1/2)*(-d^2*c)^(1/3)*(I*(x
+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1
/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a/d^2*2^(1/2)*sum((-7*a*d+4*b*c)/(
a*d-b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(
1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(
-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d
^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
,1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_a
lpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.486 $\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0393523, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{x}{\left(a+bx^3\right)^2 \sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.153541, size = 172, normalized size = 2.69

$$\frac{-bdx^5\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5x^2\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}(bc-3ad)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+10ab}{30a^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (10*a*b*x^2*(c + d*x^3) + 5*(b*c - 3*a*d)*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/
c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - b*d*x^5*(a + b*
x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)
/a)])/(30*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.034, size = 923, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*b/(a*d-b*c)/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/a*3^(1/2)*
(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/
```

```
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2
*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*a*d+2*b*c)/(
a*d-b*c)^2/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/
3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d
^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2
*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2
*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1
/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^
2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.487 $\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0278168, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.234979, size = 392, normalized size = 6.64

$$\frac{3bx^4\left(dx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8a\left(c+dx^3\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};\cdots\right)\right)}{24a^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\cdots\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]*(8*a*(3*b*
c - 3*a*d + b*d*x^3) + b*d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3
, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + 3*b*x^4*(8*a*(c + d*x^3) + d*
x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/(24*a^2*
(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))
```

Maple [C] time = 0.004, size = 769, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*b/(a*d-b*c)/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/(a*d-b*c)/a*3^(1/2)*(-
d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a/d^2*2^(1/2)*su
m((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d
^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+
2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*
c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2
*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2 \sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

3.488 $\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x*Sqrt[c + d*x^3]))
```

Rubi [A] time = 0.0544206, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x*Sqrt[c + d*x^3]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.232227, size = 226, normalized size = 3.65

$$\frac{-5x^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(3a^2d^2-15abcd+8b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+20a\left(c+dx^3\right)\left(3a^2d-3ab\left(c-dx^3\right)-4b^2\right.}{60a^3cx\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (20*a*(c + d*x^3)*(3*a^2*d - 4*b^2*c*x^3 - 3*a*b*(c - d*x^3)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^6*(a
+ b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*
x^3)/a)])/(60*a^3*c*(b*c - a*d)*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 1818, normalized size = 29.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a^2*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
```

```
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d
*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/
2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(
-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)
)))+1/3*I/a^2*b/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*
(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*
(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/
2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-
d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^
2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3
*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^
(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-b/a*(-1/3*b/(a*d-b
*c)/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/a*3^(1/2)*(-d^2*c)^(1/3
)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^
2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1
/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3)))^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*a*d+2*b*c)/(a*d-b*c)^2/_a
lpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(
1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I
*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3
)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_al
pha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(
-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-
3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

3.489 $\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0522244, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3 \left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} \\
&= -\frac{\sqrt{1+\frac{dx^3}{c}}\, F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}
\end{aligned}
$$

Mathematica [B] time = 0.548454, size = 411, normalized size = 6.42

$$
\frac{\frac{a\left(32ac\left(3a^2d\left(2c+3dx^3\right)+3ab\left(-2c^2+7cdx^3+2d^2x^6\right)-10b^2cx^3\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+24x^3\left(c+dx^3\right)\left(-3a^2d+3ab\left(c-dx^3\right)+5b^2cx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\cdots\right.\right.\right.}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}}{48a^3cx^2\sqrt{c+dx^3}(ad-bc)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*(5*b*c - 3*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (a*(32*a*c*(-10*b^2*c*x^3*(3*c + d*x^3) + 3*a^2*
d*(2*c + 3*d*x^3) + 3*a*b*(-2*c^2 + 7*c*d*x^3 + 2*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*x^3*(c + d*x^3)*(-3*a^2*d + 5*
b^2*c*x^3 + 3*a*b*(c - d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/
c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3
*c*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 1512, normalized size = 23.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a^2*(-1/2/c/x^2*(d*x^3+c)^(1/2)+1/6*I/c*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/
d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^
(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*
c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/3*I/a^2*b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b
*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1
/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*
3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)
+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_
alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alp
ha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*
I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-
d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c
*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3
/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_
Z^3*b+a))-b/a*(-1/3*b/(a*d-b*c)/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/(a*d-b*
c)/a*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a
/d^2*2^(1/2)*sum((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2
)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha
-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

3.490 $\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2d^2+2b^2c^2}{3b^2d\sqrt{c+dx^3}(bc-ad)^2}-\frac{a^2}{3b^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

```
[Out] -(2*b^2*c^2 + a^2*d^2)/(3*b^2*d*(b*c - a*d)^2*Sqrt[c + d*x^3]) - a^2/(3*b^2
*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.19494, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 78, 63, 208}

$$-\frac{a^2d^2+2b^2c^2}{3b^2d\sqrt{c+dx^3}(bc-ad)^2}-\frac{a^2}{3b^2\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```



```
[Out] -(2*b^2*c^2 + a^2*d^2)/(3*b^2*d*(b*c - a*d)^2*Sqrt[c + d*x^3]) - a^2/(3*b^2
*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(5/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(2bc+ad)+b(bc-ad)x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\operatorname{Subst}\left(\int \frac{}{(a\cdots}\right.}{6b(bc-ad\cdots}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\operatorname{Subst}\left(\int \frac{}{a-\cdots}\right.}{3bd(bc-\cdots}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c\cdots}}{\sqrt{bc\cdots}}\right.}{3b^{3/2}(bc-ad)^{5/2}}$$

Mathematica [A] time = 0.278391, size = 134, normalized size = 0.9

$$\frac{\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{\sqrt{b}\left(a^2d\left(c+dx^3\right)+2abc^2+2b^2c^2x^3\right)}{d\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)^2}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d)
^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2))/(3*b^(3/2))
```

Maple [C] time = 0.053, size = 978, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/b^2/d/(d*x^3+c)^(1/2)-2*a/b^2*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/
3*I/d^2*b*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-
1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)
^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/
(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/
d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))+a^2/b^2*(-2/3*d/(a*d-b*
c)^2/((x^3+1/d*c)*d)^(1/2)-1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)+1/2*
I/d*b*2^(1/2)*sum(1/(a*d-b*c)^3*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2
)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^
(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1
/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alp
ha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^
(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*
3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89976, size = 1470, normalized size = 9.87

$$\left[-\frac{\left(\left(4\,ab^2cd^2-a^2bd^3\right)x^6+4\,a^2bc^2d-a^3cd^2+\left(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right.}{6\left(ab^5c^4d-3\,a^2b^4c^3d^2+3\,a^3b^3c^2d^3-a^4b^2cd^4+\left(b^6c^3d^2-3\,ab^5c^2d^3+3\,a^2b^4cd^4-a^3b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3
+ 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(2*
a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d + a^2*
b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*c^3*d
^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d^3 + 3
*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2*a^3*b^
3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2
*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d
 + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^
4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*
d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2
*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13833, size = 263, normalized size = 1.77

$$-\frac{\left(4\,abc-a^{2}d\right)\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{3\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)\sqrt{-b^{2}c+abd}}-\frac{2\left(dx^{3}+c\right)b^{2}c^{2}-2\,b^{2}c^{3}+2\,abc^{2}d+\left(dx^{3}+c\right)a^{2}d^{2}}{3\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)\left(\left(dx^{3}+c\right)^{\frac{3}{2}}b-\sqrt{dx^{3}+c}bc+\sqrt{dx^{3}+c}ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)*
b^2*c^2 - 2*b^2*c^3 + 2*a*b*c^2*d + (d*x^3 + c)*a^2*d^2)/((b^3*c^2*d - 2*a*
b^2*c*d^2 + a^2*b*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*
x^3 + c)*a*d))
```

3.491 $\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{a}{3b\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

```
[Out] (2*b*c + a*d)/(3*b*(b*c - a*d)^2*Sqrt[c + d*x^3]) + a/(3*b*(b*c - a*d)*(a +
 b*x^3)*Sqrt[c + d*x^3]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.114101, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{3b\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```


```
[Out] (2*b*c + a*d)/(3*b*(b*c - a*d)^2*Sqrt[c + d*x^3]) + a/(3*b*(b*c - a*d)*(a +
 b*x^3)*Sqrt[c + d*x^3]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(5/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+d}}\right.}{6(bc-ad)^2}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3d(bc-ad)^2}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.0286215, size = 91, normalized size = 0.68

$$\frac{\left(a+bx^3\right)(ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+a(bc-ad)}{3b\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (a*(b*c - a*d) + (2*b*c + a*d)*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2,
(b*(c + d*x^3))/(b*c - a*d)])/(3*b*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3
])
```

Maple [C] time = 0.01, size = 958, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^2*b*2^(1/2)*sum(1/(-a*d+b
*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c
)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^
2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*
c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^
(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^
(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b
/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)
^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a)))-a/b*(-2/3*d/(a*d-b*c)^2/((x^3+1/d*c)*d)^(1/2)-1/3*b/(a
*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)+1/2*I/d*b*2^(1/2)*sum(1/(a*d-b*c)^3*(-d
^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(
-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)
*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3
^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-
d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/
2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^
(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-
d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-
d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a
)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79297, size = 1278, normalized size = 9.54

$$\left[\frac{\left(\left(2\,b^2cd + abd^2\right)x^6 + 2\,abc^2 + a^2cd + \left(2\,b^2c^2 + 3\,abcd + a^2d^2\right)x^3\right)\sqrt{b^2c - abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right) + 2\left(3\,ab\right.}{6\left(ab^4c^4 - 3\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 - a^4bcd^3 + \left(b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4\right)x^6 + \left(b^5c^4 - \right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b
*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqr
t(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*b*c
*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c^4 -
 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*b^4*c
^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*d + 2*
a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^
2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*a
rctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c^2
- 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(
a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4
*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14581, size = 244, normalized size = 1.82

$$\frac{\frac{\left(2\,bcd+ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{2\left(dx^3+c\right)bcd-2\,bc^2d+\left(dx^3+c\right)ad^2+2\,acd^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3 + c)*b*c*d -
2*b*c^2*d + (d*x^3 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d)))/d
```

3.492 $\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{d}{\sqrt{c+dx^3}(bc-ad)^2}-\frac{1}{3\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] -(d/((b*c - a*d)^2*Sqrt[c + d*x^3])) - 1/(3*(b*c - a*d)*(a + b*x^3)*Sqrt[c
+ d*x^3]) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(b*c - a*d)^(5/2)
```

Rubi [A] time = 0.0913992, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$-\frac{d}{\sqrt{c+dx^3}(bc-ad)^2}-\frac{1}{3\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d/((b*c - a*d)^2*Sqrt[c + d*x^3])) - 1/(3*(b*c - a*d)*(a + b*x^3)*Sqrt[c
+ d*x^3]) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(b*c - a*d)^(5/2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{2(bc-ad)}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(bd)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, \right.}{2(bc-ad)^2}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+\right.}{(bc-ad)^2}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.0170034, size = 54, normalized size = 0.5

$$-\frac{2d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b\left(dx^3+c\right)}{ad-bc}\right)}{3\sqrt{c+dx^3}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x^3))/(-(b*c) + a*d))])/(
3*(-(b*c) + a*d)^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.007, size = 485, normalized size = 4.5

$$-\frac{2d}{3(ad-bc)^2}\frac{1}{\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{b}{3(ad-bc)^2(bx^3+a)}\sqrt{dx^3+c}+\frac{\frac{i}{2}b\sqrt{2}}{d}\sum_{\_alpha=RootOf(b\_Z^3+a)}\frac{1}{(ad-bc)^3}\sqrt[3]{-d^2c}\sqrt{\frac{i}{2}d\left(2x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3*d/(a*d-b*c)^2/((x^3+1/d*c)*d)^(1/2)-1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/
(b*x^3+a)+1/2*I/d*b*2^(1/2)*sum(1/(a*d-b*c)^3*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x
-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c
)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)
/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)
/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84924, size = 923, normalized size = 8.55

$$\left[\frac{3\left(bd^2x^6+\left(bcd+ad^2\right)x^3+acd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\left(3\,bdx^3+bc+2\,ad\right)\sqrt{dx^3+c}}{6\left(\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^6+ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^3\right)},\ \frac{3\left(bd^2\right.}{3\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(b/(b*c - a*d))*log((
b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/
(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), 1/3*(3*(b*d^2*x^6 + (b*c
*d + a*d^2)*x^3 + a*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c
- a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (3*b*d*x^3 + b*c + 2*a*d)*sq
rt(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10527, size = 203, normalized size = 1.88

$$-\frac{1}{3}\,d\left(\frac{3\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\left(dx^3+c\right)b-2\,bc+2\,ad}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*d*(3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (3*(d*x^3 + c)*b - 2*b*c + 2*a*d)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c
 + sqrt(d*x^3 + c)*a*d))
```

3.493 $\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b}{3a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}(bc-ad)^2}$$

```
[Out] (d*(b*c + 2*a*d))/(3*a*c*(b*c - a*d)^2*Sqrt[c + d*x^3]) + b/(3*a*(b*c - a*d
)*(a + b*x^3)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^
2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(3*a^2*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.222598, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b}{3a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)}+\frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(3*a*c*(b*c - a*d)^2*Sqrt[c + d*x^3]) + b/(3*a*(b*c - a*d
)*(a + b*x^3)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^
2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(3*a^2*(b*c - a*d)^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)^2-\frac{1}{4}bd(}{x(a+bx)\sqrt{c+}}\right.}{3ac(bc-ad}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2c}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \right.}{3a^2cd}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} + \frac{b^{3/2}(}{}$$

Mathematica [C] time = 0.104378, size = 123, normalized size = 0.72

$$\frac{\frac{b(2bc-5ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)}{a(ad-bc)} + \left(\frac{2b}{a}-\frac{2d}{c}\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right) + \frac{b}{a+bx^3}}{3a\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b/(a + b*x^3) + (b*(2*b*c - 5*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c +
 d*x^3))/(b*c - a*d)])/(a*(-(b*c) + a*d)) + ((2*b)/a - (2*d)/c)*Hypergeomet
ric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(3*a*(b*c - a*d)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.01, size = 1002, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a^2*b*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^2*b*2^(1/2)*sum(1/(-
a*d+b*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1
/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-
d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(
-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^
2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),
1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_al
pha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d
^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*b+a))-b/a*(-2/3*d/(a*d-b*c)^2/((x^3+1/d*c)*d)^(1/2)-1/3
*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)+1/2*I/d*b*2^(1/2)*sum(1/(a*d-b*c)^
3*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3
)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^
(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(
-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha
*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*
3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^
2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2
/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b+a)))+1/a^2*(2/3/c/((x^3+1/d*c)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(
1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)
```

Fricas [B] time = 3.26953, size = 3657, normalized size = 21.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log
((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*
b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))
/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^
3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
 a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*
b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b
*d*x^3 + b*c)) + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 -
 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + (a*b^2*
c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^
3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^
2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5
*c^2*d^3)*x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (2*a*b^2*c^4 - 5
*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c
^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d
 - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a*b
^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3
*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b
^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^3 + b*c)) + 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^
3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b^2*c^3 + 2*a^3*c*
d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*
```

```
d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12252, size = 319, normalized size = 1.85

$$-\frac{1}{3}\,d^2\left(\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2b^2c^2d^2-2\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{\left(dx^3+c\right)b^2c+2\left(dx^3+c\right)abd-2\,abcd+2\,a^2d^2}{\left(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*d^2*((2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*
d))/((a^2*b^2*c^2*d^2 - 2*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)) - ((
d*x^3 + c)*b^2*c + 2*(d*x^3 + c)*a*b*d - 2*a*b*c*d + 2*a^2*d^2)/((a*b^2*c^3
*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*
c + sqrt(d*x^3 + c)*a*d)) - 2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c
)*c*d^2))
```

3.494 $\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{3a^2c^2\sqrt{c+dx^3}(bc-ad)^2}-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}-\frac{b(2bc-a}{3a^2c\left(a+bx^3\right)\sqrt{c+}}$$

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*Sqrt[c +
d*x^3]) - (b*(2*b*c - a*d))/(3*a^2*c*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3
]) - 1/(3*a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.356918, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{3a^2c^2\sqrt{c+dx^3}(bc-ad)^2}-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}-\frac{b(2bc-a}{3a^2c\left(a+bx^3\right)\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*Sqrt[c +
d*x^3]) - (b*(2*b*c - a*d))/(3*a^2*c*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3
]) - 1/(3*a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4}{x(}\right.}{3a}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{}}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{}}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{}}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{}}$$

Mathematica [C] time = 0.11314, size = 189, normalized size = 0.78

$$\frac{b^2c^2x^3\left(a+bx^3\right)(4bc-7ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right) - (ad-bc)\left(x^3\left(a+bx^3\right)\left(3a^2d^2+abcd-4b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+\right.\right.}{3a^3c^2x^3\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b^2*c^2*(4*b*c - 7*a*d)*x^3*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (b
*(c + d*x^3))/(b*c - a*d)] - (-(b*c) + a*d)*(a*c*(a^2*d - 2*b^2*c*x^3 + a*b
*(-c + d*x^3)) + (-4*b^2*c^2 + a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Hyperge
ometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(3*a^3*c^2*(b*c - a*d)^2*x^3*(a +
 b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.013, size = 1067, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/a^3*b^2*(-2/3/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-1/3*I/d^2*b*2^(1/2)*sum(1/(
-a*d+b*c)/(a*d-b*c)*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(
1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(
-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2
)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*
(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d
^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d
^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)
,1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_a
lpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-
d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*b+a))+1/a^2*(-2/3*d/c^2/((x^3+1/d*c)*d)^(1/2)-1/3*(d*x
^3+c)^(1/2)/c^2/x^3+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+b^2/a^2*(-2
/3*d/(a*d-b*c)^2/((x^3+1/d*c)*d)^(1/2)-1/3*b/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b
*x^3+a)+1/2*I/d*b*2^(1/2)*sum(1/(a*d-b*c)^3*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1
/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^
(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)
*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(
a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d
*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2/a^3*b*(2/3/c/((x^3+1/d
*c)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)
```

Fricas [B] time = 5.04311, size = 4741, normalized size = 19.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(b/(b*c -
 a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b
*c - a*d)))/(b*x^3 + a)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^
3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b
*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2
+ 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x
^3) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^
2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*
d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a
^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2
*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*
b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (
4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6
+ (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt
(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2
*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c
*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3
)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9
```

```
 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b
^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2*((4*b^4*c^3*d - 5*a*b^3
*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d -
7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2
*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c
)*sqrt(-c)/c) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3
*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt
(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*
sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^
2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*
c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((
a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4
*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*
d + a^6*c^4*d^2)*x^3), -1/3*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*
c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4
*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(
b*c - a*d))/(b*d*x^3 + b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*
c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a
^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*
d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*
b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4
*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c
^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14504, size = 510, normalized size = 2.12

$$\frac{1}{3}\,d^{3}\left(\frac{\left(4\,b^{4}c-7\,ab^{3}d\right)\arctan\left(\frac{\sqrt{dx^{3}+c}b}{\sqrt{-b^{2}c+abd}}\right)}{\left(a^{3}b^{2}c^{2}d^{3}-2\,a^{4}bcd^{4}+a^{5}d^{5}\right)\sqrt{-b^{2}c+abd}}-\frac{2\left(dx^{3}+c\right)^{2}b^{3}c^{2}-2\left(dx^{3}+c\right)b^{3}c^{3}-2\left(dx^{3}+c\right)^{2}ab^{2}cd+3\left(dx^{3}+c\right)a}{\left(a^{2}b^{2}c^{4}d^{2}-2\,a^{3}bc^{3}d^{3}+a^{4}c^{2}d^{4}\right)\left(\left(dx^{3}+c\right)^{\frac{5}{2}}b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*d^3*((4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d
))/((a^3*b^2*c^2*d^3 - 2*a^4*b*c*d^4 + a^5*d^5)*sqrt(-b^2*c + a*b*d)) - (2*
(d*x^3 + c)^2*b^3*c^2 - 2*(d*x^3 + c)*b^3*c^3 - 2*(d*x^3 + c)^2*a*b^2*c*d +
 3*(d*x^3 + c)*a*b^2*c^2*d + 3*(d*x^3 + c)^2*a^2*b*d^2 - 7*(d*x^3 + c)*a^2*
b*c*d^2 + 2*a^2*b*c^2*d^2 + 3*(d*x^3 + c)*a^3*d^3 - 2*a^3*c*d^3)/((a^2*b^2*
c^4*d^2 - 2*a^3*b*c^3*d^3 + a^4*c^2*d^4)*((d*x^3 + c)^(5/2)*b - 2*(d*x^3 +
c)^(3/2)*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^(3/2)*a*d - sqrt(d*x^3 +
 c)*a*c*d)) - (4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c
)*c^2*d^3))
```

3.495 $\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0569109, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*c*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.287747, size = 381, normalized size = 5.69

$$-\frac{x^4\left(\left(3bdx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8a\left(2ad+b\left(c+3dx^3\right)\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+ad\right.\right.}{8a\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)^2\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(x^4*(-8*a*b*c*d*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]*(8
*a + (a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)]) + (8*a*(2*a*d + b*(c + 3*d*x^3)) + 3*b*d*x^3*(a + b*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])
*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Appell
F1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/(8*a*(b*c - a*d)^2*(a +
 b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] time = 0.045, size = 1593, normalized size = 23.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b*(2/3*d/c*x/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-
d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(
1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)
+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/d^2*b*2^(1/2)*sum
(1/(a*d-b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*
c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(
-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^
(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2
-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/
d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(2/3*d^2/c*x/(a*d-b*c)^2/((x^3+1/d*c)*d)
^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(1/3*d^2/c/(
a*d-b*c)^2+1/6*b*d/(a*d-b*c)^2/a)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^
2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*
((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))
^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(
-d^2*c)^(1/3)))^(1/2))+1/18*I/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^
3/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^
2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(
1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c
)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/
3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*
(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1
/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.496 $\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=67

$$\frac{x^2 \sqrt{\frac{dx^3}{c}+1} F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2 c\sqrt{c+dx^3}}$$

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0425051, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2 \sqrt{\frac{dx^3}{c}+1} F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2 c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```



```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*c*Sqrt[c + d*x^3])
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.257256, size = 216, normalized size = 3.22

$$-\frac{x^2\left(5\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(a^2d^2+6abcd-b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-10a\left(2a^2d^2+2abd^2x^3+b^2c\left(c+dx^3\right)\right)+bdx\right.}{30a^2c\left(a+bx^3\right)\sqrt{c+dx^3}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(x^2*(-10*a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)) + 5*(-(b^2*c^2
) + 6*a*b*c*d + a^2*d^2)*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2,
 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + b*d*(b*c + 2*a*d)*x^3*(a + b*x^3)*Sq
rt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))/
(30*a^2*c*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.039, size = 986, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3*d^2/c*x^2/(a*d-b*c)^2/((x^3+1/d*c)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x^2*(
d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(-1/3*d^2/c/(a*d-b*c)^2-1/6*b*d/(a*d-b*c)^2/
a)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^
2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d
```

```
*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c
)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^
(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^
(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/18*I/a/d^2*b*2^(1/2)*sum((
11*a*d-2*b*c)/(a*d-b*c)^3/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/
2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)
^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+
1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_al
pha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)
^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I
*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^2 \left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.497 $\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}} dx$

Optimal. Leaf size=62

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*c*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0275229, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*c*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.524862, size = 381, normalized size = 6.15

$$\frac{x\left(\frac{a\left(64ac\left(3a^2d^2+2abd\left(dx^3-3c\right)+b^2c\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(2a^2d^2+2abd^2x^3+b^2c\left(c+dx^3\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\right.\right.}{\left(a+bx^3\right)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right.}{24a^2c\sqrt{c+dx^3}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(b*d*(b*c + 2*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (a*(64*a*c*(3*a^2*d^2 + 2*a*b*d*(-3*c + d*x^3)
+ b^2*c*(3*c + d*x^3))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a
)] - 24*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3))*(2*b*c*AppellF1
[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])))))/(24*a^2*c*(b*c - a*d)^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.004, size = 830, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3*d^2/c*x/(a*d-b*c)^2/((x^3+1/d*c)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x*(d*x^
3+c)^(1/2)/(b*x^3+a)-2/3*I*(1/3*d^2/c/(a*d-b*c)^2+1/6*b*d/(a*d-b*c)^2/a)*3^
(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^
(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^
2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/
3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-
d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(
-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/18*I/a/d^2*
b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(
d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2
)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d
^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3
^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*
c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2 \left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

3.498 $\int \frac{1}{x^2(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*c*x*Sqrt[c + d*x^3]))
```

Rubi [A] time = 0.0583374, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*c*x*Sqrt[c + d*x^3]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.376118, size = 308, normalized size = 4.74

$$\frac{2bdx^6\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(5a^2d^2-6abcd+4b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5x^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(-6a^2bcd^2+5a^3d^3\right.}{60a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-20*a*(4*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(3*c + 5*d*x^3) + 3*a*b^2*c*(c^
2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 5*d^2*x^6)) + 5*(-
8*b^3*c^3 + 21*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 5*a^3*d^3)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2
*b*d*(4*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c
]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*c^2*(b*c
- a*d)^2*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 2383, normalized size = 36.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a^2*(-(d*x^3+c)^(1/2)/c^2/x-2/3*d/c^2*x^2/((x^3+1/d*c)*d)^(1/2)-5/9*I/c^2
*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d
^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+
c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(
1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1
/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1
/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3
)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))))-1/a^2*b*(2/3*d/c*x^2/(a*d-b*c)/
((x^3+1/d*c)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*
(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1
/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*
3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/
3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/
2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))
^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)
/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(
1/2)))+1/3*I/d^2*b*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-d^2*c)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)
*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-
3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3
^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))-b/a*(2/3*d^2/c*x
^2/(a*d-b*c)^2/((x^3+1/d*c)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x^2*(d*x^3+c)^(1
/2)/(b*x^3+a)-2/3*I*(-1/3*d^2/c/(a*d-b*c)^2-1/6*b*d/(a*d-b*c)^2/a)*3^(1/2)/
d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))
*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(
1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*((-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(
-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2
*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/d*(-d^2*c)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d
```

```
^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I
*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))+1/18*I/a/d^2*b*2^(1/2)*sum((11*a*d-2*b*
c)/(a*d-b*c)^3/_alpha*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)
^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3
*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(
-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*
(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

3.499 $\int \frac{1}{x^3(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*c*x^2*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.055826, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*c*x^2*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.914784, size = 515, normalized size = 7.69

$$\frac{a\left(32ac\left(2a^2bd\left(-6c^2-6cdx^3+7d^2x^6\right)+3a^3d^2\left(2c+7dx^3\right)+3ab^2c\left(2c^2-13cdx^3-4d^2x^6\right)+10b^3c^2x^3\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(a^2bd\left(-6c^2-3cdx^3+7d^2x^6\right.\right.\right.}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-(b*d*(5*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1
[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + (a*(32*a*c*(10*b^3*c^2*x^
3*(3*c + d*x^3) + 3*a^3*d^2*(2*c + 7*d*x^3) + 3*a*b^2*c*(2*c^2 - 13*c*d*x^3
 - 4*d^2*x^6) + 2*a^2*b*d*(-6*c^2 - 6*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(5*b^3*c^2*x^3*(c + d*x^3)
 + a^3*d^2*(3*c + 7*d*x^3) + 3*a*b^2*c*(c^2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*
d*(-6*c^2 - 3*c*d*x^3 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(4
8*a^3*c^2*(b*c - a*d)^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] time = 0.011, size = 1919, normalized size = 28.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a^2*(-2/3*d/c^2*x/((x^3+1/d*c)*d)^(1/2)-1/2/c^2*(d*x^3+c)^(1/2)/x^2+7/18*
I/c^2*3^(1/2)*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/
d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*
c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2
)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(
1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)))-1/a^2*
b*(2/3*d/c*x/(a*d-b*c)/((x^3+1/d*c)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-d^
2*c)^(1/3)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/
2)*d/(-d^2*c)^(1/3))^(1/2)*((x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1
/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
)*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2
*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2))+1/3*I/d^2*b*2^(1/2)*sum(1
/(a*d-b*c)^2/_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)
^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3
*(-d^2*c)^(1/3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-d^2*c)^(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-d^2*c)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(
-d^2*c)^(1/3)*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/
2),1/2*b/d*(2*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*
(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*b+a)))-b/a*(2/3*d^2/c*x/(a*d-b*c)^2/((x^3+1/d*c)*d)^(
1/2)+1/3*b^2/(a*d-b*c)^2/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(1/3*d^2/c/(a*
d-b*c)^2+1/6*b*d/(a*d-b*c)^2/a)*3^(1/2)/d*(-d^2*c)^(1/3)*(I*(x+1/2/d*(-d^2*
c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2)*((
x-1/d*(-d^2*c)^(1/3))/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2
)*d/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-d^2*c)^(1/3)-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(
1/2),(I*3^(1/2)/d*(-d^2*c)^(1/3)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d
^2*c)^(1/3)))^(1/2))+1/18*I/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/
_alpha^2*(-d^2*c)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-d^2*c)^(1/3)+(-d^2*
c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)*(d*(x-1/d*(-d^2*c)^(1/3))/(-3*(-d^2*c)^(1/
3)+I*3^(1/2)*(-d^2*c)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-d^2*c)^
(1/3)+(-d^2*c)^(1/3)))/(-d^2*c)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-d^2*c)^(1
```

```
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2*c)^(2/3)+2*_alpha^2*d^2-(-d^2*c)^(1/3)
*_alpha*d-(-d^2*c)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2*c)^(1/3)
-1/2*I*3^(1/2)/d*(-d^2*c)^(1/3))*3^(1/2)*d/(-d^2*c)^(1/3))^(1/2),1/2*b/d*(2
*I*(-d^2*c)^(1/3)*3^(1/2)*_alpha^2*d-I*(-d^2*c)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-d^2*c)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-d^2*c)^(1/3
)/(-3/2/d*(-d^2*c)^(1/3)+1/2*I*3^(1/2)/d*(-d^2*c)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

3.500 $\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=134

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+23))\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(7/2))/(b*e*(23 + 2*m)) - (a^2*(2*a*B*(1 + m
) - A*b*(23 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/2, (
1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(23 + 2*m)*Sqrt[1 + (b*x^3
)/a])
```

Rubi [A] time = 0.0791409, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+23))\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```


```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(7/2))/(b*e*(23 + 2*m)) - (a^2*(2*a*B*(1 + m
) - A*b*(23 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/2, (
1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(23 + 2*m)*Sqrt[1 + (b*x^3
)/a])
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (ex)^m \left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\int (ex)^m\left(a+bx^3\right)^{5/2}dx}{b\left(\frac{23}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(a^2\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex)^m\left(1+\frac{bx^3}{a}\right.}{b\left(\frac{23}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{a^2(2aB(1+m)-Ab(23+2m))(ex)^{1+m}\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{5}{2},\right.}{be(1+m)(23+2m)\sqrt{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.111362, size = 113, normalized size = 0.84

$$\frac{a^2x\sqrt{a+bx^3}(ex)^m\left(A(m+4)\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(-\frac{5}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (a^2*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-5/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-5/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int (ex)^m \left(bx^3+a\right)^{\frac{5}{2}} \left(Bx^3+A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}} (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2x^9 + \left(2\,Bab + Ab^2\right)x^6 + \left(Ba^2 + 2\,Aab\right)x^3 + Aa^2\right)\sqrt{bx^3+a}\,(ex)^m\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

3.501 $\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx$

Optimal. Leaf size=132

$$\frac{2B\left(a+bx^3\right)^{5/2}(ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+17))\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(5/2))/(b*e*(17 + 2*m)) - (a*(2*a*B*(1 + m)
- A*b*(17 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(17 + 2*m)*Sqrt[1 + (b*x^3)/
a])
```

Rubi [A] time = 0.0743111, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\left(a+bx^3\right)^{5/2}(ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+17))\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(5/2))/(b*e*(17 + 2*m)) - (a*(2*a*B*(1 + m)
- A*b*(17 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(17 + 2*m)*Sqrt[1 + (b*x^3)/
a])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right) \int (ex)^m \left(a+bx^3\right)^{3/2} dx}{b\left(\frac{17}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(a\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right)\sqrt{a+bx^3}\right) \int (ex)^m \left(1+\frac{bx^3}{a}\right.}{b\left(\frac{17}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{a(2aB(1+m)-Ab(17+2m))(ex)^{1+m}\sqrt{a+bx^3}\, {}_2F_1\left(-\frac{3}{2}, \frac{1-}{}\right.}{be(1+m)(17+2m)\sqrt{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.0792829, size = 111, normalized size = 0.84

$$\frac{ax\sqrt{a+bx^3}(ex)^m \left(A(m+4)\, {}_2F_1\left(-\frac{3}{2}, \frac{m+1}{3}; \frac{m+4}{3}; -\frac{bx^3}{a}\right) + B(m+1)x^3\, {}_2F_1\left(-\frac{3}{2}, \frac{m+4}{3}; \frac{m+7}{3}; -\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-3/2, (1 + m)/3,
(4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-3/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (ex)^m \left(bx^3+a\right)^{\frac{3}{2}} \left(Bx^3+A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}} (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}\,(ex)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] time = 56.0889, size = 252, normalized size = 1.91

$$\frac{Aa^{\frac{3}{2}}e^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{A\sqrt{a}be^m x^4 x^m \Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}e^m x^4 x^m \Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{3\Gamma\left(\frac{m}{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + A*sqrt(a)*b*e**m*x**4*
x**m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + B*a**(3/2)*e**m*x**4*x**m*gamma(m/3 + 4/
3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(m/3 + 7/3)) + B*sqrt(a)*b*e**m*x**7*x**m*gamma(m/3 + 7/3)*hyper((-1/2, m/
3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

3.502 $\int (ex)^m \sqrt{a+bx^3}\left(A+Bx^3\right) dx$

Optimal. Leaf size=131

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(3/2))/(b*e*(11 + 2*m)) - ((2*a*B*(1 + m) -
A*b*(11 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 +
m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(11 + 2*m)*Sqrt[1 + (b*x^3)/a]
)
```

Rubi [A] time = 0.073593, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```


```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(3/2))/(b*e*(11 + 2*m)) - ((2*a*B*(1 + m) -
A*b*(11 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 +
m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(11 + 2*m)*Sqrt[1 + (b*x^3)/a]
)
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)dx &= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)}-\frac{\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\int (ex)^m\sqrt{a+bx^3}\,dx}{b\left(\frac{11}{2}+m\right)}\\ &= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)}-\frac{\left(\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex)^m\sqrt{1+\frac{bx^3}{a}}\,dx}{b\left(\frac{11}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}\\ &= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)}-\frac{(2aB(1+m)-Ab(11+2m))(ex)^{1+m}\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\right.}{be(1+m)(11+2m)\sqrt{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.070676, size = 110, normalized size = 0.84

$$\frac{x\sqrt{a+bx^3}(ex)^m\left(A(m+4)\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-1/2, (1 + m)/3, (4
 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-1/2, (4 + m)/3, (
7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (ex)^m \sqrt{bx^3+a}\left(Bx^3+A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] time = 8.68436, size = 122, normalized size = 0.93

$$\frac{A\sqrt{a}e^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{B\sqrt{a}e^m x^4 x^m \Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

```
[Out] A*sqrt(a)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*sqrt(a)*e**m*x**4*x**
m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(
I*pi)/a)/(3*gamma(m/3 + 7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

3.503 $\int \frac{(ex)^m\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=131

$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

```
[Out] (2*B*(e*x)^(1 + m)*Sqrt[a + b*x^3])/(b*e*(5 + 2*m)) - ((2*a*B*(1 + m) - A*b
*(5 + 2*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m
)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(5 + 2*m)*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0708553, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```



```
[Out] (2*B*(e*x)^(1 + m)*Sqrt[a + b*x^3])/(b*e*(5 + 2*m)) - ((2*a*B*(1 + m) - A*b
*(5 + 2*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m
)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(5 + 2*m)*Sqrt[a + b*x^3])
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)}-\frac{\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{b\left(\frac{5}{2}+m\right)}\\ &= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)}-\frac{\left(\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}\,dx}{b\left(\frac{5}{2}+m\right)\sqrt{a+bx^3}}\\ &= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)}-\frac{(2aB(1+m)-Ab(5+2m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(5+2m)\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.0766648, size = 110, normalized size = 0.84

$$\frac{x\sqrt{\frac{bx^3}{a}+1}(ex)^m\left(A(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[1/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[1/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int (ex)^m\left(Bx^3+A\right)\frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^m}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)(ex)^m}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Sympy [C] time = 5.49894, size = 119, normalized size = 0.91

$$\frac{Ae^m x x^m \Gamma\left(\frac{m}{3} + \frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{3} + \frac{1}{3} \\ \frac{m}{3} + \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)} + \frac{Be^m x^4 x^m \Gamma\left(\frac{m}{3} + \frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{3} + \frac{4}{3} \\ \frac{m}{3} + \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{m}{3} + \frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3
*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3)) + B*e**m*x**4*x**m*gamma(m
/3 + 4/3)*hyper((1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*sqrt(a)*gamma(m/3 + 7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^m}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

3.504 $\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+A(b-2bm))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*a*B*(1 + m) +
 A*(b - 2*b*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1
 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0771552, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 365, 364}

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+A(b-2bm))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*a*B*(1 + m) +
 A*(b - 2*b*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1
 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}}+\frac{(2aB(1+m)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{3ab}\\ &= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}}+\frac{\left((2aB(1+m)+A(b-2bm))\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}\,dx}{3ab\sqrt{a+bx^3}}\\ &= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}}+\frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3abe(1+m)\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.0848411, size = 113, normalized size = 0.85

$$\frac{x\sqrt{\frac{bx^3}{a}+1}(ex)^m\left(A(m+4)\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(\frac{3}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{a(m+1)(m+4)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[3/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[3/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/(a*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int (ex)^m \left(Bx^3+A\right) \left(bx^3+a\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)(ex)^m}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^m}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^m}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

3.505 $\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((A*b*(7 - 2*m)
 + 2*a*B*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2,
(1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(9*a^2*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0789417, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 365, 364}

$$\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((A*b*(7 - 2*m)
 + 2*a*B*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2,
(1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(9*a^2*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^m \left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{2(Ab-aB)(ex)^{1+m}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\right)\int \frac{(ex)^m}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}\\
&= \frac{2(Ab-aB)(ex)^{1+m}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\left(1+\frac{bx^3}{a}\right)^{3/2}}\,dx}{9a^2b\sqrt{a+bx^3}}\\
&= \frac{2(Ab-aB)(ex)^{1+m}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{(Ab(7-2m)+2aB(1+m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{9a^2be(1+m)\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.0890926, size = 113, normalized size = 0.85

$$\frac{x\sqrt{\frac{bx^3}{a}+1}(ex)^m\left(A(m+4)\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(\frac{5}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{a^2(m+1)(m+4)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[5/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[5/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/(a^2*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (ex)^{m} \left(B x^{3}+A\right) \left(b x^{3}+a\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B x^{3}+A\right) (ex)^{m}}{\left(b x^{3}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B x^{3}+A\right) \sqrt{b x^{3}+a}\,(ex)^{m}}{b^{3} x^{9}+3\,a b^{2} x^{6}+3\,a^{2} b x^{3}+a^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2
*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^m}{\left(bx^3 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

3.506 $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*b^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0933625, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*b^(3/2)*d^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)}{6bd}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.180121, size = 123, normalized size = 1.4

$$
\frac{b\sqrt{d}\sqrt{a+bx^3}\left(c+dx^3\right)-\sqrt{bc-ad}(ad+bc)\sqrt{\frac{b\left(c+dx^3\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{3b^2d^{3/2}\sqrt{c+dx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^3]*(c + d*x^3) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^3))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^3])/Sqrt[b*c - a*
d]])/(3*b^2*d^(3/2)*Sqrt[c + d*x^3])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int x^5 \frac{1}{\sqrt{bx^3+a}} \frac{1}{\sqrt{dx^3+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40686, size = 597, normalized size = 6.78

$$\left[\frac{4\sqrt{bx^3+a}\sqrt{dx^3+c}\,bd + (bc+ad)\sqrt{bd}\log\left(8\,b^2d^2x^6 + b^2c^2 + 6\,abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^3 - 4\left(2\,bdx^3 + bc + ad\right)\right.}{12\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 - 4
*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d)))/(b^2*d
^2), 1/6*(2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*ar
ctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)
/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a + bx^3}\sqrt{c + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.17714, size = 140, normalized size = 1.59

$$\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right|\right)}{\sqrt{bd}d}+\frac{\sqrt{bx^3+a}\sqrt{b^2c+\left(bx^3+a\right)bd-abd}}{bd}}{3\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*c + a*d)*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b*x^3 + a)*sqrt(b^2*c + (b*x^3 + a)
*b*d - a*b*d)/(b*d))/abs(b)
```

3.507 $\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Rubi [A] time = 0.0594079, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.0652979, size = 85, normalized size = 1.77

$$\frac{2\sqrt{c+dx^3}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^3\right)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^3])/Sqrt[b*c - a*d]])/(3*S
qrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^3))/(b*c - a*d)])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int x^{2}\frac{1}{\sqrt{b x^{3}+a}}\frac{1}{\sqrt{d x^{3}+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.29587, size = 440, normalized size = 9.17

$$\left[\frac{\sqrt{bd}\log\left(8\,b^{2}d^{2}x^{6}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\left(b^{2}cd+abd^{2}\right)x^{3}+4\left(2\,bdx^{3}+bc+ad\right)\sqrt{bx^{3}+a}\sqrt{dx^{3}+c}\sqrt{bd}\right)}{6\,bd},-\frac{\sqrt{-bd}\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqr
t(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x^3))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [A] time = 1.14752, size = 73, normalized size = 1.52

$$-\frac{2\,b\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right|\right)}{3\,\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d -
a*b*d)))/(sqrt(b*d)*abs(b))
```

3.508 $\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Rubi [A] time = 0.0482415, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {446, 93, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^3\right)\\ &= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.015774, size = 48, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49494, size = 455, normalized size = 9.48

$$\left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^3-4\left((bc+ad)x^3+2\,ac\right)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)}{6\,ac}, \frac{\sqrt{-ac}\arctan\left(\frac{\left((bc+ad)x^3+2\,ac\right)\sqrt{bx^3+a}\sqrt{dx^3+}}{2\left(abcdx^6+a^2c^2+\left(abc^2+a^2cd\right)\right.}\right.}{3\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x
^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^3 +
 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [B] time = 1.12382, size = 120, normalized size = 2.5

$$-\frac{2\sqrt{bd}b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{3\sqrt{-abcd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*
d)*abs(b))
```

3.509 $\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=91

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

```
[Out] -(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Rubi [A] time = 0.0801235, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 96, 93, 208}

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)}{6ac}\\ &= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3ac}\\ &= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0500886, size = 91, normalized size = 1.

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}} - \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87254, size = 636, normalized size = 6.99

$$\left[\frac{\sqrt{ac}(bc+ad)x^3\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^6+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^3+4\left((bc+ad)x^3+2\,ac\right)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)-4\sqrt{bx^3+a}\sqrt{dx^3+c}ac}{12\,a^2c^2x^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a*c)*(b*c + a*d)*x^3*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 + 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b
*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6) - 4*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*a*c)/(a^2*c^2*x^3), -1/6*(sqrt(-a*c)*(b*c + a*d)*x^3*arctan(1/2*((b*c +
a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 +
```

```
 a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3)) + 2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*a*
c)/(a^2*c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [B] time = 1.31371, size = 558, normalized size = 6.13

$$\frac{\sqrt{bd}b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd}-\frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^3+a\right)bd-\cdots}\right.\right.}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^3+a}\sqrt{bd}\cdots\right.\right.}\right.}{3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^3 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^3 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

3.510 $\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^5\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] (x^5*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0973033, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^5\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$
$$= \frac{x^5\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [A] time = 0.0474621, size = 90, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^4\frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}x^4}{bdx^6+(bc+ad)x^3+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^4/(b*d*x^6 + (b*c + a*d)*x^3 + a
*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.511 $\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^4\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] (x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.101098, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [A] time = 0.0447409, size = 90, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^3\frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}{bdx^6+(bc+ad)x^3+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3/(b*d*x^6 + (b*c + a*d)*x^3 + a
*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.512 $\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0706296, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\\
&= \frac{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}
\end{aligned}
$$

Mathematica [A] time = 0.0430736, size = 90, normalized size = 1.02

$$
\frac{x^2\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$
\int x\frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}x}{bdx^6+(bc+ad)x^3+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x/(b*d*x^6 + (b*c + a*d)*x^3 + a*c
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.513 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.046574, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.230123, size = 170, normalized size = 2.05

$$-\frac{8acxF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}\left(3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a
 + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((
d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}}{bdx^6+(bc+ad)x^3+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^6 + (b*c + a*d)*x^3 + a*c),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

3.514 $\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1/2, 1/2, 2/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]))
```

Rubi [A] time = 0.097289, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1/2, 1/2, 2/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$

$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.156673, size = 189, normalized size = 2.2

$$\frac{8bdx^6\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5x^3\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}(ad+bc)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-20\left(a+bx\right.}{20acx\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*(a + b*x^3)*(c + d*x^3) + 5*(b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 8*
b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3
, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c*x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + a}\sqrt{dx^3 + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}\sqrt{dx^3 + c}}{bdx^8 + (bc + ad)x^5 + acx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^8 + (b*c + a*d)*x^5 + a*c*x
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^3}\sqrt{c + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

3.515 $\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] -(Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1/2, 1/2, 1/3, -((
b*x^3)/a), -((d*x^3)/c)])/(2*x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] time = 0.0957892, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1/2, 1/2, 1/3, -((
b*x^3)/a), -((d*x^3)/c)])/(2*x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}} \\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}} \\
&= -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}
\end{aligned}
$$

Mathematica [B] time = 0.253591, size = 365, normalized size = 4.15

$$
\frac{bdx^6\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4\left(3x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(2ac+3 \cdots\right.\right.}{8acF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\cdots\right.\right.}}{8acx^2\sqrt{a+bx^3}\sqrt{c+dx^3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*(-4*a*c*(2*a*c + 3*b*c*x^3 + 3*a*d*x^3
+ 2*b*d*x^6)*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x
^3*(a + b*x^3)*(c + d*x^3)*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^
3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*Appel
lF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(8*a*c*x^2*Sqrt[a +
b*x^3]*Sqrt[c + d*x^3])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$
\int \frac{1}{x^3}\frac{1}{\sqrt{bx^3+a}}\frac{1}{\sqrt{dx^3+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}}{bdx^9+(bc+ad)x^6+acx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^9 + (b*c + a*d)*x^6 + a*c*x
^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

3.516 $\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=161

$$-\frac{a^2e^{7/2}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}\left(a+b\right.}{9be}$$

```
[Out] (a*(2*A*b - a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b^2) + ((2*A*b - a*B)
*(e*x)^(9/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/
(9*b*e) - (a^2*(2*A*b - a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)
*Sqrt[a + b*x^3])])/(24*b^(5/2))
```

Rubi [A] time = 0.112619, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$-\frac{a^2e^{7/2}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}\left(a+b\right.}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (a*(2*A*b - a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b^2) + ((2*A*b - a*B)
*(e*x)^(9/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/
(9*b*e) - (a^2*(2*A*b - a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)
*Sqrt[a + b*x^3])])/(24*b^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(-9Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\sqrt{a+bx^3}\,dx}{9b}$$
$$= \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} + \frac{(a(2Ab-aB))\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}dx}{8b}$$
$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$
$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

Mathematica [A] time = 0.207192, size = 145, normalized size = 0.9

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(-3a^2B+2ab\left(3A+Bx^3\right)+4b^2x^3\left(3A+2Bx^3\right)\right)+3a^{3/2}(aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{72b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-3*a^2
*B + 2*a*b*(3*A + B*x^3) + 4*b^2*x^3*(3*A + 2*B*x^3)) + 3*a^(3/2)*(-2*A*b +
 a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*
x^3)/a])
```

Maple [C] time = 0.388, size = 7293, normalized size = 45.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)
```

Fricas [A] time = 4.42509, size = 656, normalized size = 4.07

$$\left[-\frac{3\left(Ba^3 - 2\,Aa^2b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e + 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4\left(8\,Bb^2e^3x^7 + 2\left(Bab + 6\,A\right.\right.}{288\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(
8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)*
sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/144*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(-e/b
)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*
(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)
```

```
*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)
```

3.517 $\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=324

$$-\frac{3^{3/4}a^{5/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-7aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3ae^2\sqrt{ex}\sqrt{\ldots}}{}$$

```
[Out] (3*a*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b^2) + ((16*A*b -
 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(80*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^
(3/2))/(8*b*e) - (3^(3/4)*a^(5/3)*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b^2*Sqrt[(b
^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Rubi [A] time = 0.319787, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 279, 321, 329, 225}

$$-\frac{3^{3/4}a^{5/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3ae^2\sqrt{ex}\sqrt{a+bx^3}(1\ldots}{320b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (3*a*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b^2) + ((16*A*b -
 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(80*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^
(3/2))/(8*b*e) - (3^(3/4)*a^(5/3)*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b^2*Sqrt[(b
^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt
```

```
[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be} - \frac{\left(-8Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\sqrt{a+bx^3}\,dx}{8b}$$

$$= \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be} + \frac{(3a(16Ab-7aB))\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{160b}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

Mathematica [C] time = 0.134845, size = 112, normalized size = 0.35

$$\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(a(7aB-16Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(7aB-16Ab-10bBx^3\right)\right)}{80b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(-16*A*b
+ 7*a*B - 10*b*B*x^3)) + a*(-16*A*b + 7*a*B)*Hypergeometric2F1[-1/2, 1/6, 7
/6, -((b*x^3)/a)]))/(80*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.068, size = 4175, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] 1/320*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^3/(-a*b^2)^(1/3)*(84*I*B*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/
2))*(-a*b^2)^(1/3)*3^(1/2)*x*a^3*b*e-21*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*
a^2*b+48*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(
-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a*b^2-192*I*A*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a
*b^2)^(1/3)*3^(1/2)*x*a^2*b^2*e+40*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^6*b
^3+96*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1
)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a^2*b*e-96*A*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x
^2*a^2*b^3*e-42*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I
*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a^3*e+42*B*(-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^
```

```
(1/2))*x^2*a^3*b^2*e-120*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^6*b^3+192*A*(-a*b^2)^(1
/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3
^(1/2)-3))^(1/2))*x*a^2*b^2*e-42*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^3*b^2*e-84
*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3
^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x*a^3*b*e+64*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(
1/2)*x^3*b^3-96*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3
^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+
I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*a^2*b*e+42*B*(-a*b^2)^(2/3)*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/
2)-3))^(1/2))*a^3*e-192*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^3*b^3-36*B*(1/b^2*e*x*(-
b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e
*x)^(1/2)*x^3*a*b^2+96*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)
+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2*b^3*e+12*I*B*(1/b^
2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
```

```
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1
/2)*((b*x^3+a)*e*x)^(1/2)*x^3*a*b^2-144*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a*b^2+63*B
*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3
)*((b*x^3+a)*e*x)^(1/2)*a^2*b)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e
*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Be^2x^5 + Ae^2x^2\right)\sqrt{bx^3 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

3.518 $\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=581

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\cdots$$

```
[Out] ((14*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*b*e) + (3*(1 + Sqrt[3])*
a*(14*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(5/3)*(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)) + (B*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*b*e) - (3*3^(
1/4)*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(112*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(
3/4)*(1 - Sqrt[3])*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(5/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rubi [A] time = 0.62115, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 308, 225, 1881}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}a^{4/3}e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((14*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*b*e) + (3*(1 + Sqrt[3])*
a*(14*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(5/3)*(a^(1/3) + (1
```

```
+ Sqrt[3])*b^(1/3)*x)) + (B*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*b*e) - (3*3^(
1/4)*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(112*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(
3/4)*(1 - Sqrt[3])*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(5/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{\left(-7Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{7b}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab-5aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,d}{112b}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab-5aB))\,\mathrm{Subst}\left(\int\right.}{56be}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{(3a(14Ab-5aB))\,\mathrm{Subst}\left(\int\right.}{112}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{3\left(1+\sqrt{3}\right)a(14Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{B(ex)^{5/2}}{}$$

Mathematica [C] time = 0.0973849, size = 94, normalized size = 0.16

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left((14Ab-5aB)\,{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+5B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)\right)}{35b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (14*A
*b - 5*a*B)*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)]))/(35*b*Sqrt[1
 + (b*x^3)/a])
```

Maple [C] time = 0.066, size = 5358, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bex^4+Aex\right)\sqrt{bx^3+a}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] time = 70.2171, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)}+\frac{B\sqrt{a}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(11/6)) + B*sqrt(a)*e**(3/2)*x**(11/2)*gamma(11/6
)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

3.519 $\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=121

$$\frac{a\sqrt{e}(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

```
[Out] ((4*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(3/2)*(a +
b*x^3)^(3/2))/(6*b*e) + (a*(4*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/
2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(3/2))
```

Rubi [A] time = 0.0893504, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a\sqrt{e}(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```


```
[Out] ((4*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(3/2)*(a +
b*x^3)^(3/2))/(6*b*e) + (a*(4*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/
2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(3/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)dx=\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}-\frac{\left(-6Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{6b}$$

$$=\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}+\frac{(a(4Ab-aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{8b}$$

$$=\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}+\frac{(a(4Ab-aB))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,d\right.}{4be}$$

$$=\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}+\frac{(a(4Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,d\right.}{12be}$$

$$=\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}+\frac{(a(4Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,\right.}{12be}$$

$$=\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}+\frac{a(4Ab-aB)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right.}{12b^{3/2}}$$

Mathematica [A] time = 0.124822, size = 119, normalized size = 0.98

$$\frac{\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(B\left(a+2bx^3\right)+4Ab\right)-\sqrt{a}(aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{12b^{3/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(4*A*b + B*
(a + 2*b*x^3)) - Sqrt[a]*(-4*A*b + a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])
)/(12*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.056, size = 6858, normalized size = 56.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Fricas [A] time = 4.40025, size = 501, normalized size = 4.14

$$\left[-\frac{\left(Ba^2 - 4\,Aab\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4\left(2\,Bbx^4 + (Ba + 4\,Ab)x\right)\sqrt{bx}}{48\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((B*a^2 - 4*A*a*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
- 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*x^4
 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/24*((B*a^2 - 4*A*a*b)*s
qrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*
e)) + 2*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]
```

Sympy [A] time = 13.3376, size = 201, normalized size = 1.66

$$\frac{A\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{3\sqrt{b}}+\frac{Ba^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}{12be\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}}-\frac{Ba^2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{12b^{\frac{3}{2}}}+\frac{Bb\,(ex)^{\frac{15}{2}}}{6\sqrt{a}e^7\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*a**(3/2)*(e*x)**(3/2)/(12
*b*e*sqrt(1 + b*x**3/a)) + B*sqrt(a)*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**3/a
)) - B*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(12*b**(
3/2)) + B*b*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.520 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=286

$$\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+bx^3}(}{20b}$$

```
[Out] ((10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b*e) + (B*Sqrt[e*x]*(a + b*x
^3)^(3/2))/(5*b*e) + (3^(3/4)*a^(2/3)*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*b*e*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rubi [A] time = 0.229096, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 225}

$$\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+bx^3}(10Ab-}{20be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] ((10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b*e) + (B*Sqrt[e*x]*(a + b*x
^3)^(3/2))/(5*b*e) + (3^(3/4)*a^(2/3)*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*b*e*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} - \frac{\left(-5Ab+\frac{aB}{2}\right)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{40b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x\right.}{20be}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} + \frac{3^{3/4}a^{2/3}(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\ }}{40be\sqrt{\frac{}{(\ }}}$$

Mathematica [C] time = 0.0600449, size = 93, normalized size = 0.33

$$\frac{x\sqrt{a+bx^3}\left((10Ab-aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)+B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)\right)}{5b\sqrt{ex}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (10*A*b - a*B)*Hype
rgeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*b*Sqrt[e*x]*Sqrt[1 + (b*x^
3)/a])
```

Maple [C] time = 0.053, size = 3721, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x)
```

[Out] -1/20*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(60*I*A*(-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(
2/3)*3^(1/2)*a*b*e-3*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a*b-4*I*B*(1/b^2*e*
x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*
((b*x^3+a)*e*x)^(1/2)*x^3*b^2+60*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a*b^3*e+12*I
B(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^
(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a^2*b*e-60*A*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2*a*
b^3*e-10*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(
-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b^2+6*B*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b
^2*e+120*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+
1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a*b^2*e-12*B*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1

/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^
2)^(1/3)*x*a^2*b*e-6*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2*b^2*e-60*A*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
3^(1/2)(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1
/2))*(-a*b^2)^(2/3)*a*b*e+6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1
/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3
^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*e+12*B*(1/b^
2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*
x^3+a)*e*x)^(1/2)*x^3*b^2-120*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*
3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a*
b^2*e-6*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)
+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a^2*e+30*A*(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x
)^(1/2)*b^2+9*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a*b)/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/
2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}\sqrt{ex}}{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

Sympy [C] time = 6.83115, size = 97, normalized size = 0.34

$$\frac{A\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{B\sqrt{a}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(e)*gamma(7/6)) + B*sqrt(a)*x**(7/2)*gamma(7/6)*hyper((-1/2,
7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

3.521 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=580

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}(aB+8Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{bx}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{bx}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt[3]{\cdots}}{\cdots}$$

```
[Out] ((8*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*a*e^4) + (3*(1 + Sqrt[3])*(8
*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(a*e*Sqrt[e*x]) - (3*3^(1/4)*a^(1
/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(8*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sq
rt[3])*a^(1/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.591689, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 308, 225, 1881}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}(aB+8Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{bx}+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{bx}+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{bx}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt[3]{a}\sqrt{ex}\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] ((8*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*a*e^4) + (3*(1 + Sqrt[3])*(8
*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[
```

```
3])*b^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(a*e*Sqrt[e*x]) - (3*3^(1/4)*a^(1
/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(8*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sq
rt[3])*a^(1/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(8Ab+aB)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{8e^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right.}{4e^4}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} - \frac{(3(8Ab+aB))\,\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\right.}{8b^{2/3}e^4}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} + \frac{3\left(1+\sqrt{3}\right)(8Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} - \frac{3\sqrt[4]{}}{}$$

Mathematica [C] time = 0.0564763, size = 98, normalized size = 0.17

$$-\frac{4x^4\sqrt{a+bx^3}\left(-\frac{aB}{2}-4Ab\right){}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5a(ex)^{3/2}\sqrt{\frac{bx^3}{a}+1}}-\frac{2Ax\left(a+bx^3\right)^{3/2}}{a(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (-2*A*x*(a + b*x^3)^(3/2))/(a*(e*x)^(3/2)) - (4*(-4*A*b - (a*B)/2)*x^4*Sqrt
[a + b*x^3]*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)])/(5*a*(e*x)^(3
/2)*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.062, size = 5736, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}\sqrt{ex}}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] time = 6.57141, size = 100, normalized size = 0.17

$$\frac{A\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{B\sqrt{a}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(3/2),x)
```

```
[Out] A*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a)
/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + B*sqrt(a)*x**(5/2)*gamma(5/6)*hyper((-1/
2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

3.522 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+2Ab)}{3ae^4}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

```
[Out] ((2*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*e^4) - (2*A*(a + b*x^3)^(3
/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^
(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rubi [A] time = 0.0844829, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+2Ab)}{3ae^4}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((2*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*e^4) - (2*A*(a + b*x^3)^(3
/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^
(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2e^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

Mathematica [A] time = 0.187416, size = 87, normalized size = 0.74

$$\frac{x\sqrt{a+bx^3}\left(\frac{x^{3/2}(aB+2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{bx^3}{a}+1}} - 2A + Bx^3\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(-2*A + B*x^3 + ((2*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*
x^(3/2))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x^3)/a])))/(3*(e*x)^(5/2))
```

Maple [C] time = 0.059, size = 6668, normalized size = 56.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

Fricas [A] time = 4.32924, size = 486, normalized size = 4.12

$$\left[\frac{(Ba + 2\,Ab)\sqrt{be}x^2 \log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,bx^4 + ax\right)\sqrt{bx^3 + a}\sqrt{be}\sqrt{ex}\right) + 4\left(Bbx^3 - 2\,Ab\right)\sqrt{bx^3 + a}\sqrt{ex}}{12\,be^3x^2}, -\frac{(B}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((B*a + 2*A*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e -
 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(B*b*x^3 - 2*A*
b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b*e)*x
^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(
B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [A] time = 22.1409, size = 160, normalized size = 1.36

$$-\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}}-\frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] -2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2)*
sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) +
B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

3.523 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=283

$$\frac{3^{3/4}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+bx^3}(5aB}{10ae^4}$$

```
[Out] ((4*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(10*a*e^4) - (2*A*(a + b*x^3)^(
3/2))/(5*a*e*(e*x)^(5/2)) + (3^(3/4)*(4*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(20*a^(1/3)*e^4*Sq
rt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.227216, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 225}

$$\frac{3^{3/4}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+bx^3}(5aB+4Ab)}{10ae^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] ((4*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(10*a*e^4) - (2*A*(a + b*x^3)^(
3/2))/(5*a*e*(e*x)^(5/2)) + (3^(3/4)*(4*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(20*a^(1/3)*e^4*Sq
rt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}} + \frac{(4Ab+5aB)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5ae^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}} + \frac{(3(4Ab+5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{20e^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}} + \frac{(3(4Ab+5aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right.}{10e^4}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}} + \frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\sqrt[3]{a}+(1+\cdots\right.}}}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x(\cdots}{\left(\sqrt[3]{a}+(\cdots\right.}}}$$

Mathematica [C] time = 0.0555128, size = 97, normalized size = 0.34

$$\frac{2x\sqrt{a+bx^3}\left(x^3(5aB+4Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-A\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\right)}{5a(ex)^{7/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]) + (4*A*b + 5*a*B
)*x^3*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sq
rt[1 + (b*x^3)/a])
```

Maple [C] time = 0.056, size = 3512, normalized size = 12.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x)
```

```
[Out] -1/10*(b*x^3+a)^(1/2)/x^2/(-a*b^2)^(1/3)/b*(24*I*A*(-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(
2/3)*3^(1/2)*x^3*b*e-5*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^3*b-48*I*A*(-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3
))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x^4*b^2*e-24*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*
3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^5*b^3*e+48*
A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(
1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^4*b^2*e-24*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*
x^3*b*e+30*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1
/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^5*a*b^2*e-12*A*(1/b^2*e*x*(-b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2
)*b+4*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*
b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b-60*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
```

```
llipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1
/3)*3^(1/2)*x^4*a*b*e+24*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/
2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^
(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^5*b^3*e-30*B*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1
/2))*x^5*a*b^2*e+60*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/
(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^4*a*b*e-30*B*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1
/2))*(-a*b^2)^(2/3)*x^3*a*e+30*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I
*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-
1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x^3
*a*e+15*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*
b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^3*b)/e^3/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/
2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\sqrt{ex}}{e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

3.524 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=564

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] (-2*(2*A*b + 7*a*B)*Sqrt[a + b*x^3])/(7*a*Sqrt[x]) + (3*(1 + Sqrt[3])*b^(1/
3)*(2*A*b + 7*a*B)*Sqrt[x]*Sqrt[a + b*x^3])/(7*a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(7*a*x^(7/2)) - (3*3^(1/4)*b^(1/3)*(2*
A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a
^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(7*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sqrt[3])*b^(1
/3)*(2*A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(14*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.529974, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {453, 277, 329, 308, 225, 1881}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{x}\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

```
[Out] (-2*(2*A*b + 7*a*B)*Sqrt[a + b*x^3])/(7*a*Sqrt[x]) + (3*(1 + Sqrt[3])*b^(1/
3)*(2*A*b + 7*a*B)*Sqrt[x]*Sqrt[a + b*x^3])/(7*a*(a^(1/3) + (1 + Sqrt[3])*b
```

```
^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(7*a*x^(7/2)) - (3*3^(1/4)*b^(1/3)*(2*
A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a
^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(7*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sqrt[3])*b^(1
/3)*(2*A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(14*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{9/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}} - \frac{\left(2\left(-Ab-\frac{7aB}{2}\right)\right)\int\frac{\sqrt{a+bx^3}}{x^{3/2}}\,dx}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}} + \frac{(3b(2Ab+7aB))\int\frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}} + \frac{(6b(2Ab+7aB))\operatorname{Subst}\left(\int\frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}} - \frac{\left(3\sqrt[3]{b}(2Ab+7aB)\right)\operatorname{Subst}\left(\int\frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}}{\sqrt{a+bx^6}}\right.}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} + \frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}} - \frac{3}{}$$

Mathematica [C] time = 0.0926879, size = 81, normalized size = 0.14

$$\frac{2\sqrt{a+bx^3}\left(-\frac{x^3(7aB+2Ab)\,_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-A\left(a+bx^3\right)\right)}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((2*A*b + 7*a*B)*x^3*Hypergeometric2
F1[-1/2, -1/6, 5/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(7*a*x^(7/2))
```

Maple [C] time = 0.106, size = 5911, normalized size = 10.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Sympy [C] time = 112.877, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\Gamma\left(-\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{6},-\frac{1}{2}\\-\frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)}+\frac{B\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{x}\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(9/2),x)
```

```
[Out] A*sqrt(a)*gamma(-7/6)*hyper((-7/6, -1/2), (-1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*x**(7/2)*gamma(-1/6)) + B*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6
,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(x)*gamma(5/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

3.525 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}+\frac{2}{3}\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

```
[Out] (-2*B*Sqrt[a + b*x^3])/(3*x^(3/2)) - (2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2))
+ (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/3
```

Rubi [A] time = 0.0458653, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {451, 277, 329, 275, 217, 206}

$$-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}+\frac{2}{3}\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (-2*B*Sqrt[a + b*x^3])/(3*x^(3/2)) - (2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2))
+ (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/3
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}\,dx &= -\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + B\int \frac{\sqrt{a+bx^3}}{x^{5/2}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + (bB)\int \frac{\sqrt{x}}{\sqrt{a+bx^3}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + (2bB)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{3/2}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{a+bx^3}}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.163341, size = 87, normalized size = 1.1

$$
\frac{2\sqrt{a+bx^3}\left(\frac{3\sqrt{a}\sqrt{b}B\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \frac{a\left(A+3Bx^3\right)+Abx^3}{x^{9/2}}\right)}{9a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((A*b*x^3 + a*(A + 3*B*x^3))/x^(9/2)) + (3*Sqrt[a]*Sqr
t[b]*B*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[1 + (b*x^3)/a]))/(9*a)
```

Maple [C] time = 0.066, size = 3759, normalized size = 47.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x)
```

```
[Out] -2/9*(b*x^3+a)^(1/2)/x^(9/2)/b*(3*I*B*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*x)^(1/2)*3^(1/2)*x^3*a*b-18*I*B*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I
*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x^5*a-36*I*B*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x^6*a*b+18*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)
*3^(1/2)*x^5*a+I*A*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)*((b*x^3+a)*x)^(1/2)*3^(1/2)*x^3*b^2-18*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^7*a*b^2+I*A*(1/
b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*x)^(1/2)
*3^(1/2)*a*b+18*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*
3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^7*a*b^2+36*
B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(
1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^6*a*b-36*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
```

```
cPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3
^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2
)-3))^(1/2))*(-a*b^2)^(1/3)*x^6*a*b-18*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF(
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)
+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5*a
+18*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5*a+18*I
*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^
(1/2)-3))^(1/2))*3^(1/2)*x^7*a*b^2-9*B*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*x)^(1/2)*x^3*a*b+36*I*B*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2
)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x^6*a*b-3*A*(1/b^2*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*x)^(1/2)*x^3*b^2-1
8*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^7*a*b^2-3*A*(1/
b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*x)^(1/2)
*a*b)/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3)))^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18257, size = 437, normalized size = 5.53

$$\left[\frac{3\,Ba\sqrt{b}x^5\log\left(-8\,b^2x^6-8\,abx^3-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\sqrt{b}\sqrt{x}-a^2\right)-4\left((3\,Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}\sqrt{x}}{18\,ax^5},-\frac{3\,Ba\sqrt{-b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqr
t(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^
3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 + a)*
sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 +
a)*sqrt(x))/(a*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19035, size = 147, normalized size = 1.86

$$-\frac{2\,Bb\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}+\frac{2\left(3\,Bab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,Ba\sqrt{-b}\sqrt{b}+A\sqrt{-b}b^{\frac{3}{2}}\right)}{9\,a\sqrt{-b}}-\frac{2\left(3\,Ba^3\sqrt{b+\frac{a}{x^3}}+Aa^2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -2/3*B*b*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/9*(3*B*a*b*arctan(sq
rt(b)/sqrt(-b)) + 3*B*a*sqrt(-b)*sqrt(b) + A*sqrt(-b)*b^(3/2))/(a*sqrt(-b))
 - 2/9*(3*B*a^3*sqrt(b + a/x^3) + A*a^2*(b + a/x^3)^(3/2))/a^3
```

3.526 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}} dx$

Optimal. Leaf size=269

$$-\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-11aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(2Ab}{55ax^{5/2}}$$

```
[Out] (2*(2*A*b - 11*a*B)*Sqrt[a + b*x^3])/(55*a*x^(5/2)) - (2*A*(a + b*x^3)^(3/2
))/(11*a*x^(11/2)) - (3^(3/4)*b*(2*A*b - 11*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(55*a^(4/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rubi [A] time = 0.199148, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {453, 277, 329, 225}

$$-\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-11aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(2Ab-11aB)}{55ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

```
[Out] (2*(2*A*b - 11*a*B)*Sqrt[a + b*x^3])/(55*a*x^(5/2)) - (2*A*(a + b*x^3)^(3/2
))/(11*a*x^(11/2)) - (3^(3/4)*b*(2*A*b - 11*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(55*a^(4/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{13/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{\left(2\left(Ab-\frac{11aB}{2}\right)\right)\int \frac{\sqrt{a+bx^3}}{x^{7/2}}\,dx}{11a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{(3b(2Ab-11aB))\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{(6b(2Ab-11aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{\cdots}}{\left(\sqrt[3]{a}+(1+\cdots\right.}}}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}\cdots\right.}{\left(\sqrt[3]{a}+(1+\cdots\right.}}}$$

Mathematica [C] time = 0.0866127, size = 80, normalized size = 0.3

$$\frac{2\sqrt{a+bx^3}\left(\frac{x^3(2Ab-11aB)\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-5A\left(a+bx^3\right)\right)}{55ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + ((2*A*b - 11*a*B)*x^3*Hypergeometric
2F1[-5/6, -1/2, 1/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(55*a*x^(11/2))
```

Maple [C] time = 0.068, size = 3690, normalized size = 13.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x)
```

```
[Out] 2/55*(b*x^3+a)^(1/2)*(-66*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
```

```
/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^8*a*b^2-24*I*A*(-a*b^
2)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^
(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^7*b^2+132*I*B*(-a*b^2)^(1/3)*3^(1/2)*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3)
)^(1/2))*x^7*a*b-5*I*A*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*(1/b^2*x*
(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a+12*I*A*(-a*b^2)^(2/3)*3
^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(
I*3^(1/2)-3))^(1/2))*x^6*b-12*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^
(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I
*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^8*b^3-3*I*A*((b*x^3+a)*x)^(
1/2)*(-a*b^2)^(1/3)*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3)))^(1/2)*x^3*b+66*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))
/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^8*a*b^2+24*A*(-a*b^2)^(1/3)*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(
1/2))*x^7*b^2-132*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I
*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
```

```
-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-
1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^7*a*b-12*A*(-a*b^2)^(2/3
)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(
1/2)-3))^(1/2))*x^6*b+66*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^6*a-11*I*B*((b*x^
3+a)*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*x^3*a+12*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^8*b^3+9*A*((b*x^3
+a)*x)^(1/2)*(-a*b^2)^(1/3)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3)))^(1/2)*x^3*b+33*B*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(1/3)*(1/b^2*x*(-b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*a-66*I*B*(-a*b^2)^(2/3)*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^
(1/2)-3))^(1/2))*x^6*a+15*A*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(1/3)*(1/b^2*x*(-b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a)/x^(11/2)/(-a*b^2)^(1/3)/(
(b*x^3+a)*x)^(1/2)/a/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^{\frac{13}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

3.527 $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=201

$$\frac{a^2e^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-3aB)}{192b^2}-\frac{a^3e^{7/2}(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72be}+\frac{a(ex)^{9/2}\cdots}{}$$

```
[Out] (a^2*(8*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b^2) + (a*(8*A*b
 - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(96*b*e) + ((8*A*b - 3*a*B)*(e*x)^(9
/2)*(a + b*x^3)^(3/2))/(72*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e
) - (a^3*(8*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqr
t[a + b*x^3])])/(192*b^(5/2))
```

Rubi [A] time = 0.138526, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^2e^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-3aB)}{192b^2}-\frac{a^3e^{7/2}(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72be}+\frac{a(ex)^{9/2}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b^2) + (a*(8*A*b
 - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(96*b*e) + ((8*A*b - 3*a*B)*(e*x)^(9
/2)*(a + b*x^3)^(3/2))/(72*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e
) - (a^3*(8*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqr
t[a + b*x^3])])/(192*b^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{\left(-12Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\,dx}{12b}$$

$$= \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} + \frac{(a(8Ab-3aB))\int (ex)^{7/}}{16b}$$

$$= \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+b\right.}{12be}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)}{}$$

Mathematica [A] time = 0.257227, size = 167, normalized size = 0.83

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(6a^2b\left(4A+Bx^3\right)-9a^3B+8ab^2x^3\left(14A+9Bx^3\right)+16b^3x^6\left(4A+3Bx^3\right)\right)+3a^{5/2}(3aB\right.}{576b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-9*a^3
*B + 6*a^2*b*(4*A + B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A
 + 9*B*x^3)) + 3*a^(5/2)*(-8*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]
]))/(576*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.065, size = 7705, normalized size = 38.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)
```

Fricas [A] time = 4.37852, size = 791, normalized size = 3.94

$$\left[-\frac{3\left(3\,Ba^4-8\,Aa^3b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(48\,Bb^3e^3x^{10}+8\left(9\,Bab\right.\right.}{2304\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/2304*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*
x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) -
4*(48*B*b^3*e^3*x^10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*
A*a*b^2)*e^3*x^4 - 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x)
)/b^2, -1/1152*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3
```

```
+ a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(48*B*b^3*e^3*x^10 + 8
*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*A*a*b^2)*e^3*x^4 - 3*(3*
B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)
```

3.528 $\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=364

$$-\frac{9\ 3^{3/4} a^{8/3} e^2 \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right) \sqrt[3]{b} x\right)^2}} (22 A b-7 a B) \text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right) \sqrt[3]{b} x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right) \sqrt[3]{b} x}\right),\frac{1}{4} \left(2+\sqrt{3}\right)\right)}{14080 b^2 \sqrt{\frac{\sqrt[3]{b} x \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right) \sqrt[3]{b} x\right)^2}} \sqrt{a+b x^3}}+\frac{27 a^2 e^2 \sqrt{}$$

```
[Out] (27*a^2*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(7040*b^2) + (9*a*(
22*A*b - 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1760*b*e) + ((22*A*b - 7*a*B)
*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(176*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(5/2
))/(11*b*e) - (9*3^(3/4)*a^(8/3)*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(14080*b^2*Sqrt[(
b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rubi [A] time = 0.309358, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 279, 321, 329, 225}

$$\frac{27 a^2 e^2 \sqrt{ex} \sqrt{a+b x^3} (22 A b-7 a B)}{7040 b^2}-\frac{9\ 3^{3/4} a^{8/3} e^2 \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right) \sqrt[3]{b} x\right)^2}} (22 A b-7 a B) F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right) \sqrt[3]{b} x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right) \sqrt[3]{b} x+\sqrt[3]{a}}\right)\right.}{14080 b^2 \sqrt{\frac{\sqrt[3]{b} x \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right) \sqrt[3]{b} x\right)^2}} \sqrt{a+b x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```



```
[Out] (27*a^2*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(7040*b^2) + (9*a*(
22*A*b - 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1760*b*e) + ((22*A*b - 7*a*B)
*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(176*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(5/2
))/(11*b*e) - (9*3^(3/4)*a^(8/3)*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x
```

```
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(14080*b^2*Sqrt[(
b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{\left(-11Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}dx}{11b}$$

$$= \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} + \frac{(9a(22Ab-7aB))\int (ex)}{352b}$$

$$= \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a\right.}{11be}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7}{}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7}{}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7}{}$$

Mathematica [C] time = 0.155794, size = 116, normalized size = 0.32

$$\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(a^2(7aB-22Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)^2\sqrt{\frac{bx^3}{a}+1}\left(7aB-22Ab-16bBx^3\right)\right)}{176b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-22*A*
b + 7*a*B - 16*b*B*x^3)) + a^2*(-22*A*b + 7*a*B)*Hypergeometric2F1[-3/2, 1/
6, 7/6, -((b*x^3)/a)]))/(176*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.059, size = 4619, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] 1/7040*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/(-a*b^2)^(1/3)/b^3*(378*B*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/
2))*x^2*a^4*b^2*e-1188*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^3*e-1188*A*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(
-a*b^2)^(2/3)*a^3*b*e-2376*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(
1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*
3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a^3*b
^2*e+756*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2
)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a^4*b*e-1782*A*(1/b^2*e
*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3
+a)*e*x)^(1/2)*a^2*b^2+567*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a^3*b-1920*B*(1/b^2*e*x
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a
)*e*x)^(1/2)*x^9*b^4-2640*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^6*b^4-3000*B*(1/b^2*e*
x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*
```

3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+
a)*e*x)^(1/2)*x^6*a*b^3-5016*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^3*a*b^3-324*B*(1/b^
2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*
x^3+a)*e*x)^(1/2)*x^3*a^2*b^2-756*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^4*b*e+
2376*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(
I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^3*b^2*e-378*I*B*(-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b
^2)^(2/3)*3^(1/2)*a^4*e+594*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a^2*b^2-189*
I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(
1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a^3*b+640*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(
1/2)*x^9*b^4+880*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^6*b^4+1000*I*B*(1/b^2
*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/
2)*((b*x^3+a)*e*x)^(1/2)*x^6*a*b^3+1672*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*
x^3*a*b^3+108*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1
/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^3*a^2*b^2+1188*I*A*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*

```
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3)
)^(1/2))*3^(1/2)*x^2*a^3*b^3*e-378*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^4*b^2*e+
1188*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)
/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a^3*b*e+378*B*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(
-a*b^2)^(2/3)*a^4*e)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbe^2x^8 + (Ba + Ab)e^2x^5 + Aae^2x^2\right)\sqrt{bx^3 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*e^2*x^8 + (B*a + A*b)*e^2*x^5 + A*a*e^2*x^2)*sqrt(b*x^3 + a)*
sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

3.529 $\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=621

$$-\frac{9\, 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\ldots$$

```
[Out] (9*a*(4*A*b - a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*b*e) + (27*(1 + Sqrt[3
])*a^2*(4*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(5/3)*(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)) + ((4*A*b - a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(2
8*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*b*e) - (27*3^(1/4)*a^(7/3)*(
4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCo
s[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)],
 (2 + Sqrt[3])/4])/(448*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(
1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt[
3])*a^(7/3)*(4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(896*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(
1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.660539, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 308, 225, 1881}

$$\frac{27\left(1+\sqrt{3}\right)a^2e\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{9\, 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\ldots\right.}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (9*a*(4*A*b - a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*b*e) + (27*(1 + Sqrt[3
])*a^2*(4*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(5/3)*(a^(1/3) + (
```

```
1 + Sqrt[3])*b^(1/3)*x)) + ((4*A*b - a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(2
8*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*b*e) - (27*3^(1/4)*a^(7/3)*(
4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCo
s[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)],
 (2 + Sqrt[3])/4])/(448*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(
1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt[
3])*a^(7/3)*(4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(896*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(
1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} - \frac{\left(-10Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}dx}{10b}$$

$$= \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} + \frac{(9a(4Ab-aB))\int (ex)^{3/2}\cdots}{56b}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+b\cdots\right.}{10be}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+b\cdots\right.}{10be}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+b\cdots\right.}{10be}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{27\left(1+\sqrt{3}\right)a^2(4Ab-aB)e\sqrt{ex}\sqrt{a+bx^3}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{(4A\cdots}{}$$

Mathematica [C] time = 0.115566, size = 96, normalized size = 0.15

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(a(4Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)^2\right)}{10b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(4*
A*b - a*B)*Hypergeometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)]))/(10*b*Sqrt[1
+ (b*x^3)/a])
```

Maple [C] time = 0.062, size = 5790, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bbex^7 + (Ba + Ab)ex^4 + Aaex\right)\sqrt{bx^3 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*e*x^7 + (B*a + A*b)*e*x^4 + A*a*e*x)*sqrt(b*x^3 + a)*sqrt(e*x
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

3.530 $\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=161

$$\frac{a^2\sqrt{e}(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a+bx^3\right.}{9be}$$

```
[Out] (a*(6*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b*e) + ((6*A*b - a*B)*(e*
x)^(3/2)*(a + b*x^3)^(3/2))/(36*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9
*b*e) + (a^2*(6*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*S
qrt[a + b*x^3])])/(24*b^(3/2))
```

Rubi [A] time = 0.113435, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a^2\sqrt{e}(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a+bx^3\right.}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a*(6*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b*e) + ((6*A*b - a*B)*(e*
x)^(3/2)*(a + b*x^3)^(3/2))/(36*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9
*b*e) + (a^2*(6*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*S
qrt[a + b*x^3])])/(24*b^(3/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} - \frac{\left(-9Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{3/2}dx}{9b}$$

$$= \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{(a(6Ab-aB))\int \sqrt{ex}\sqrt{a+b}}{8b}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

Mathematica [A] time = 0.171383, size = 143, normalized size = 0.89

$$\frac{\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(3a^2B+2ab\left(15A+7Bx^3\right)+4b^2x^3\left(3A+2Bx^3\right)\right)-3a^{3/2}(aB-6Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{72b^{3/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(3*a^2*B +
4*b^2*x^3*(3*A + 2*B*x^3) + 2*a*b*(15*A + 7*B*x^3)) - 3*a^(3/2)*(-6*A*b + a
*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^
3)/a])
```

Maple [C] time = 0.047, size = 7290, normalized size = 45.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*sqrt(e*x), x)
```

Fricas [A] time = 4.28633, size = 614, normalized size = 3.81

$$\left[-\frac{3\left(Ba^3 - 6\,Aa^2b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4\left(8\,Bb^2x^7 + 2\left(7\,Bab + 6\,A\right.\right.}{288\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*B*
b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 +
a)*sqrt(e*x))/b, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^
3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(8*B*b^2*x^7 + 2*(7*
B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b
```

]

Sympy [B] time = 37.9075, size = 335, normalized size = 2.08

$$\frac{Aa^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}\,\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}{12e\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}b\,(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}}+\frac{Aa^2\sqrt{e}\,\mathrm{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{4\sqrt{b}}+\frac{Ab^2\,(ex)^{\frac{15}{2}}}{6\sqrt{a}e^7\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^{\frac{5}{2}}\,(ex)^{\frac{3}{2}}}{24be\sqrt{1+\frac{bx^3}{a}}}+\frac{17B}{72e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(3/2)*(e*x)**(3/2)/
(12*e*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**
3/a)) + A*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(4*sq
rt(b)) + A*b**2*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a)) + B*a**(5
/2)*(e*x)**(3/2)/(24*b*e*sqrt(1 + b*x**3/a)) + 17*B*a**(3/2)*(e*x)**(9/2)/(
72*e**4*sqrt(1 + b*x**3/a)) + 11*B*sqrt(a)*b*(e*x)**(15/2)/(36*e**7*sqrt(1
+ b*x**3/a)) - B*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2))
)/(24*b**(3/2)) + B*b**2*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.531 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=324

$$\frac{9\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\left(a+bx\right.}{}$$

```
[Out] (9*a*(16*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b*e) + ((16*A*b - a*B)*
Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(5/2))/(8*
b*e) + (9*3^(3/4)*a^(5/3)*(16*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b*e*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.25567, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 225}

$$\frac{9\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}(16\ldots}{80be}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (9*a*(16*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b*e) + ((16*A*b - a*B)*
Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(5/2))/(8*
b*e) + (9*3^(3/4)*a^(5/3)*(16*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b*e*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} - \frac{\left(-8Ab+\frac{aB}{2}\right)\int \frac{\left(a+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx}{8b}$$

$$= \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{(9a(16Ab-aB))\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{160b}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{(2}{}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{(2}{}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{9}{}$$

Mathematica [C] time = 0.0713055, size = 96, normalized size = 0.3

$$\frac{x\sqrt{a+bx^3}\left(a(16Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)+B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)^2\right)}{8b\sqrt{ex}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(16*A*b - a*B)*
Hypergeometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)]))/(8*b*Sqrt[e*x]*Sqrt[1 + (
b*x^3)/a])
```

Maple [C] time = 0.042, size = 4173, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x)

[Out] -1/320*(b*x^3+a)^(1/2)*x/(-a*b^2)^(1/3)/b^2*(-208*I*A*(1/b^2*e*x*(-b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e
*x)^(1/2)*a*b^2-40*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^6*b^3-76*I*B*(1/b^2
*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/
2)*((b*x^3+a)*e*x)^(1/2)*x^3*a*b^2+864*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(
1/2)*a^2*b*e-54*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I
*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^3*b^2*e+108*I*B*(-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2
))*(-a*b^2)^(1/3)*3^(1/2)*x*a^3*b*e-864*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^3*e-1728*
I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3
^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a^2*b^2*e+54*B*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2
```

*a^3*b^2*e+120*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^6*b^3+1728*A*(-a*b^2)^(1/3)*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3)
)^(1/2))*x*a^2*b^2*e-64*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^3*b^3-108*B*(-
a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2
)+1)/(I*3^(1/2)-3))^(1/2))*x*a^3*b*e-27*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*
a^2*b-864*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)
+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*a^2*b*e+54*B*(-a*b^2)^(2/3)*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))
^(1/2))*a^3*e+192*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^
(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^3*b^3+228*B*(1/b^2*e*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(
1/2)*x^3*a*b^2-54*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/
(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a^3*e+864*I*A*(-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-

```
a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)
-3))^(1/2))*3^(1/2)*x^2*a^2*b^3*e+624*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a*b^2+81*B*(
1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*
((b*x^3+a)*e*x)^(1/2)*a^2*b)/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3
)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}\sqrt{ex}}{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x),
 x)
```

Sympy [C] time = 25.8476, size = 199, normalized size = 0.61

$$\frac{Aa^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{A\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)}+\frac{B\sqrt{a}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right)2}{3\sqrt{e}\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + A*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-1/
2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*a**
(3/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(e)*gamma(13/6)) + B*sqrt(a)*b*x**(13/2)*gamma(13/6)*hyper((-1/
2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

3.532 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=614

$$-\frac{9\,3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+14Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\cdots}{\cdots}$$

```
[Out] (9*(14*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*e^4) + (27*(1 + Sqrt[3])
*a*(14*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(2/3)*e^2*(a^(1/3) + (1
 + Sqrt[3])*b^(1/3)*x)) + ((14*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7
*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(a*e*Sqrt[e*x]) - (27*3^(1/4)*a^(4/3)*(14
*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(
a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
 + Sqrt[3])/4])/(112*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt
[3])*a^(4/3)*(14*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.644303, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 308, 225, 1881}

$$-\frac{9\,3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+14Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}a^{4/3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (9*(14*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*e^4) + (27*(1 + Sqrt[3])
*a*(14*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(2/3)*e^2*(a^(1/3) + (1
 + Sqrt[3])*b^(1/3)*x)) + ((14*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7
*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(a*e*Sqrt[e*x]) - (27*3^(1/4)*a^(4/3)*(14
*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(
a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
 + Sqrt[3])/4])/(112*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt
[3])*a^(4/3)*(14*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
```

```
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}} + \frac{(14Ab+aB)\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\,dx}{ae^3}$$

$$= \frac{(14Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}} + \frac{(9(14Ab+aB))\int (ex)^{3/2}\sqrt{a+bx^3}\,d}{14e^3}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}} + \frac{(2}{}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}} + \frac{(2}{}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}} - \frac{(2}{}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{27\left(1+\sqrt{3}\right)a(14Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{112b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{(14Ab+aB}{}$$

Mathematica [C] time = 0.0736531, size = 84, normalized size = 0.14

$$\frac{2x\sqrt{a+bx^3}\left(\frac{x^3(aB+14Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \frac{5A\left(a+bx^3\right)^2}{a}\right)}{5(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + ((14*A*b + a*B)*x^3*Hypergeo
metric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(3
/2))
```

Maple [C] time = 0.052, size = 6142, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bbx^6 + (Ba + Ab)x^3 + Aa)\sqrt{bx^3 + a}\sqrt{ex}}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x
^2), x)
```

Sympy [C] time = 31.626, size = 202, normalized size = 0.33

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)}+\frac{A\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{B\sqrt{a}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right)2}{3e^{\frac{3}{2}}\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(3/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + A*sqrt(a)*b*x**(5/2)*gamma(5/6)*hyper((
-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B
*a**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B*sqrt(a)*b*x**(11/2)*gamma(11/6)*hyper
((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

3.533 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}} dx$

Optimal. Leaf size=152

$$\frac{(ex)^{3/2}(a+bx^3)^{3/2}(aB+4Ab)}{6ae^4}+\frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+4Ab)}{4e^4}+\frac{a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

```
[Out] ((4*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(4*e^4) + ((4*A*b + a*B)*(e*x)^
(3/2)*(a + b*x^3)^(3/2))/(6*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(
3/2)) + (a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*
x^3])])/(4*Sqrt[b]*e^(5/2))
```

Rubi [A] time = 0.104042, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}(a+bx^3)^{3/2}(aB+4Ab)}{6ae^4}+\frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+4Ab)}{4e^4}+\frac{a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}}-\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((4*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(4*e^4) + ((4*A*b + a*B)*(e*x)^
(3/2)*(a + b*x^3)^(3/2))/(6*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(
3/2)) + (a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*
x^3])])/(4*Sqrt[b]*e^(5/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(4Ab+aB)\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\,dx}{ae^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3(4Ab+aB))\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{4e^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3a(4A}{}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3a(4A}{}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(a(4Ab}{}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(a(4Ab}{}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{a(4Ab}{}$$

Mathematica [A] time = 0.145976, size = 126, normalized size = 0.83

$$\frac{x\sqrt{a+bx^3}\left(\sqrt{b}\sqrt{\frac{bx^3}{a}+1}\left(-8aA+5aBx^3+4Abx^3+2bBx^6\right)+3\sqrt{a}x^{3/2}(aB+4Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{12\sqrt{b}(ex)^{5/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(Sqrt[b]*Sqrt[1 + (b*x^3)/a]*(-8*a*A + 4*A*b*x^3 + 5*a*B
*x^3 + 2*b*B*x^6) + 3*Sqrt[a]*(4*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2
))/Sqrt[a]]))/(12*Sqrt[b]*(e*x)^(5/2)*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.049, size = 7108, normalized size = 46.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

Fricas [A] time = 4.1967, size = 590, normalized size = 3.88

$$\left[\frac{3\left(Ba^2 + 4\,Aab\right)\sqrt{be}x^2\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,bx^4 + ax\right)\sqrt{bx^3 + a}\sqrt{be}\sqrt{ex}\right) + 4\left(2\,Bb^2x^6 + \left(5\,Bab + 4\,Ab^2\right)\right.}{48\,be^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(2*B*b^2*
x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*
x^2), -1/24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sq
rt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(2*B*b^2*x^6 + (5*B*a*b + 4*A*b
```

```
^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [B] time = 62.0412, size = 289, normalized size = 1.9

$$-\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}-\frac{2A\sqrt{a}bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B}{4e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3/
2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*sqr
t(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2) + B
*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2)/(1
2*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt(1 +
b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/2)) +
B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

3.534 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=314

$$\frac{9\,3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\left(a+bx^3\right.}{5}$$

```
[Out] (9*(2*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*e^4) + ((2*A*b + a*B)*Sqrt[
e*x]*(a + b*x^3)^(3/2))/(5*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(5*a*e*(e*x)^(5
/2)) + (9*3^(3/4)*a^(2/3)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*e^4*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.246942, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 225}

$$\frac{9\,3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\left(a+bx^3\right)^{3/2}(aB}{5ae^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```


```
[Out] (9*(2*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*e^4) + ((2*A*b + a*B)*Sqrt[
e*x]*(a + b*x^3)^(3/2))/(5*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(5*a*e*(e*x)^(5
/2)) + (9*3^(3/4)*a^(2/3)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*e^4*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = -\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{(2Ab+aB)\int \frac{(a+bx^3)^{3/2}}{\sqrt{ex}}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{(9(2Ab+aB))\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{10e^3}$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{(27a(2A}{}$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{(27a(2A}{}$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{9\ 3^{3/4}a^{2}}{}$$

Mathematica [C] time = 0.0705467, size = 85, normalized size = 0.27

$$\frac{2x\sqrt{a+bx^3}\left(\frac{5x^3(aB+2Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \frac{A(a+bx^3)^2}{a}\right)}{5(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((A*(a + b*x^3)^2)/a) + (5*(2*A*b + a*B)*x^3*Hyperge
ometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(7
/2))
```

Maple [C] time = 0.046, size = 3966, normalized size = 12.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*(-108*x^5*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*
3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*a*b^3*e-54*x^5*B*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/
2))*a^2*b^2*e-54*x^3*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))
/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*e+8*I*A*(1/b^2*e*x*
(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((
b*x^3+a)*e*x)^(1/2)*a*b-4*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^6*b^2-13*I*x
^3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^
(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a*b+54*I*x^3*B*(-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(
2/3)*3^(1/2)*a^2*e+39*x^3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a*b+30*x^3*A*(1/b^2*e*x*
(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)
*e*x)^(1/2)*b^2+12*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^6*b^2-24*A*(1/b^2*e*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(
```

1/2)*a*b+216*x^4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a*b^2*e+108*x^4*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*a^2*b*e-108*x^3*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a*b*e-216*I*x^4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*a*b^2*e+54*I*x^5*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a^2*b^2*e+108*I*x^3*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a*b*e-108*I*x^4*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*a^2*b*e-10*I*x^3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b^2+108*I*x^5*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)

```
-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*b^3*e)/x
^2/(-a*b^2)^(1/3)/b/e^3/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3)/(1/
b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}\sqrt{ex}}{e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x
^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

3.535 $\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=241

$$\frac{a^3e^2(ex)^{3/2}\sqrt{a+bx^3}(10Ab-3aB)}{384b^2}-\frac{a^4e^{7/2}(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}+\frac{a^2(ex)^{9/2}\sqrt{a+bx^3}(10Ab-3aB)}{192be}+\frac{(ex)^{9/2}\,(\ldots}{\ldots}$$

```
[Out] (a^3*(10*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(384*b^2) + (a^2*(10
*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(192*b*e) + (a*(10*A*b - 3*a*B)*
(e*x)^(9/2)*(a + b*x^3)^(3/2))/(144*b*e) + ((10*A*b - 3*a*B)*(e*x)^(9/2)*(a
 + b*x^3)^(5/2))/(120*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) - (
a^4*(10*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ b*x^3])])/(384*b^(5/2))
```

Rubi [A] time = 0.158972, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^3e^2(ex)^{3/2}\sqrt{a+bx^3}(10Ab-3aB)}{384b^2}-\frac{a^4e^{7/2}(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}+\frac{a^2(ex)^{9/2}\sqrt{a+bx^3}(10Ab-3aB)}{192be}+\frac{(ex)^{9/2}\,(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```


```
[Out] (a^3*(10*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(384*b^2) + (a^2*(10
*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(192*b*e) + (a*(10*A*b - 3*a*B)*
(e*x)^(9/2)*(a + b*x^3)^(3/2))/(144*b*e) + ((10*A*b - 3*a*B)*(e*x)^(9/2)*(a
 + b*x^3)^(5/2))/(120*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) - (
a^4*(10*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ b*x^3])])/(384*b^(5/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be} - \frac{\left(-15Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}dx}{15b}$$

$$= \frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be} + \frac{(a(10Ab-3aB))\int (ex)^{7/}}{16b}$$

$$= \frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be} + \frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be} + \frac{B(ex)^{9/2}\left(a\right.}{15}$$

$$= \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be} + \frac{(10Ab-3a}{}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3}{}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3}{}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3}{}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3}{}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3}{}$$

Mathematica [A] time = 0.293084, size = 188, normalized size = 0.78

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(4a^2b^2x^3\left(295A+186Bx^3\right)+30a^3b\left(5A+Bx^3\right)-45a^4B+16ab^3x^6\left(85A+63Bx^3\right)+9\right.\right.}{5760b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-45*a^
4*B + 30*a^3*b*(5*A + B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(8
```

```
5*A + 63*B*x^3) + 4*a^2*b^2*x^3*(295*A + 186*B*x^3)) + 15*a^(7/2)*(-10*A*b
+ 3*a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(5760*b^(5/2)*Sqrt[x]*Sqrt[1
+ (b*x^3)/a])
```

Maple [C] time = 0.063, size = 8117, normalized size = 33.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)
```

Fricas [A] time = 4.58304, size = 940, normalized size = 3.9

$$\left[-\frac{15\left(3\,Ba^5 - 10\,Aa^4b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e + 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4\left(384\,Bb^4e^3x^{13} + 48\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b
*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b))
 - 4*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B*a^
2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*
(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/11520*(15*
(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*
x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 +
10*A*b^4)*e^3*x^10 + 8*(93*B*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b
 + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a
)*sqrt(e*x))/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)
```

3.536 $\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx$

Optimal. Leaf size=404

$$-\frac{27\,3^{3/4}a^{11/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{11264b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{81a^3e^2\cdots}{}$$

```
[Out] (81*a^3*(4*A*b - a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(5632*b^2) + (27*a^2*(
4*A*b - a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1408*b*e) + (15*a*(4*A*b - a*B)*
(e*x)^(7/2)*(a + b*x^3)^(3/2))/(704*b*e) + ((4*A*b - a*B)*(e*x)^(7/2)*(a +
b*x^3)^(5/2))/(44*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(7/2))/(14*b*e) - (27*3
^(3/4)*a^(11/3)*(4*A*b - a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(11264*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.34375, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 279, 321, 329, 225}

$$\frac{81a^3e^2\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{5632b^2}-\frac{27\,3^{3/4}a^{11/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)\cdots\right)}{11264b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```


```
[Out] (81*a^3*(4*A*b - a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(5632*b^2) + (27*a^2*(
4*A*b - a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1408*b*e) + (15*a*(4*A*b - a*B)*
(e*x)^(7/2)*(a + b*x^3)^(3/2))/(704*b*e) + ((4*A*b - a*B)*(e*x)^(7/2)*(a +
b*x^3)^(5/2))/(44*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(7/2))/(14*b*e) - (27*3
^(3/4)*a^(11/3)*(4*A*b - a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
```

```
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(11264*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx=\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}-\frac{\left(-14Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}dx}{14b}$$

$$=\frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}+\frac{(15a(4Ab-aB))\int (ex)^{5/2}}{88b}$$

$$=\frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be}+\frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be}+\frac{B(ex)^{7/2}\left(a\right.}{14b}$$

$$=\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be}+\frac{(4Ab-a}{}$$

$$=\frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2}+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{15a(4Ab-}{}$$

$$=\frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2}+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{15a(4Ab-}{}$$

$$=\frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2}+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{15a(4Ab-}{}$$

Mathematica [C] time = 0.178099, size = 116, normalized size = 0.29

$$\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(7a^3(aB-4Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)^3\sqrt{\frac{bx^3}{a}+1}\left(7aB-28Ab-22bBx^3\right)\right)}{308b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^3*Sqrt[1 + (b*x^3)/a]*(-28*A*
b + 7*a*B - 22*b*B*x^3)) + 7*a^3*(-4*A*b + a*B)*Hypergeometric2F1[-5/2, 1/6
, 7/6, -((b*x^3)/a)]))/(308*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.059, size = 5063, normalized size = 12.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2e^2x^{11} + \left(2\,Bab + Ab^2\right)e^2x^8 + \left(Ba^2 + 2\,Aab\right)e^2x^5 + Aa^2e^2x^2\right)\sqrt{bx^3 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*e^2*x^11 + (2*B*a*b + A*b^2)*e^2*x^8 + (B*a^2 + 2*A*a*b)*e^
2*x^5 + A*a^2*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

3.537 $\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=661

$$-\frac{27\, 3^{3/4} \left(1-\sqrt{3}\right) a^{10/3} e \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} (26Ab-5aB) \text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{23296 b^{5/3} \sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} \sqrt{a+bx^3}}$$

```
[Out] (27*a^2*(26*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(5824*b*e) + (81*(1 +
 Sqrt[3])*a^3*(26*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(11648*b^(5/3)*
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*a*(26*A*b - 5*a*B)*(e*x)^(5/2)*(a
 + b*x^3)^(3/2))/(728*b*e) + ((26*A*b - 5*a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2
))/(260*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(7/2))/(13*b*e) - (81*3^(1/4)*a^(
10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(11648*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/
4)*(1 - Sqrt[3])*a^(10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(23296*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rubi [A] time = 0.707938, antiderivative size = 661, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 308, 225, 1881}

$$\frac{81\left(1+\sqrt{3}\right) a^3 e \sqrt{ex} \sqrt{a+bx^3} (26Ab-5aB)}{11648 b^{5/3} \left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{27\, 3^{3/4} \left(1-\sqrt{3}\right) a^{10/3} e \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} (26Ab-5aB) F\left(\right.}{23296 b^{5/3} \sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (27*a^2*(26*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(5824*b*e) + (81*(1 +
 Sqrt[3])*a^3*(26*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(11648*b^(5/3)*
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*a*(26*A*b - 5*a*B)*(e*x)^(5/2)*(a
 + b*x^3)^(3/2))/(728*b*e) + ((26*A*b - 5*a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2
))/(260*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(7/2))/(13*b*e) - (81*3^(1/4)*a^(
10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(11648*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/
4)*(1 - Sqrt[3])*a^(10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(23296*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
```

```
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}-\frac{\left(-13Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\,dx}{13b}$$
$$=\frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}+\frac{(3a(26Ab-5aB))\int (e}{104}$$
$$=\frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be}+\frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be}+\frac{B(ex)^{5/2}}{}$$
$$=\frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}+\frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be}+\frac{(26A}{}$$
$$=\frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}+\frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be}+\frac{(26A}{}$$
$$=\frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}+\frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be}+\frac{(26A}{}$$
$$=\frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}+\frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

Mathematica [C] time = 0.133452, size = 99, normalized size = 0.15

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(a^2(26Ab-5aB)\,{}_2F_1\left(-\frac{5}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+5B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)^3\right)}{65b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)^3*Sqrt[1 + (b*x^3)/a] + a^2
*(26*A*b - 5*a*B)*Hypergeometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)]))/(65*b*
Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.061, size = 6202, normalized size = 9.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^2ex^{10} + \left(2\,Bab + Ab^2\right)ex^7 + \left(Ba^2 + 2\,Aab\right)ex^4 + Aa^2ex\right)\sqrt{bx^3 + a}\sqrt{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*e*x^10 + (2*B*a*b + A*b^2)*e*x^7 + (B*a^2 + 2*A*a*b)*e*x^4
+ A*a^2*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

3.538 $\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=201

$$\frac{5a^3\sqrt{e}(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/2}\left(a+\right.}{}$$

```
[Out] (5*a^2*(8*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b*e) + (5*a*(8*A*b -
 a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(288*b*e) + ((8*A*b - a*B)*(e*x)^(3/2)
*(a + b*x^3)^(5/2))/(72*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) +
 (5*a^3*(8*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
 + b*x^3])])/(192*b^(3/2))
```

Rubi [A] time = 0.127124, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{5a^3\sqrt{e}(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/2}\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```



```
[Out] (5*a^2*(8*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b*e) + (5*a*(8*A*b -
 a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(288*b*e) + ((8*A*b - a*B)*(e*x)^(3/2)
*(a + b*x^3)^(5/2))/(72*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) +
 (5*a^3*(8*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
 + b*x^3])])/(192*b^(3/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} - \frac{\left(-12Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\,dx}{12b}$$
$$= \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} + \frac{(5a(8Ab-aB))\int \sqrt{ex}\left(a+\right.}{48b}$$
$$= \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex)^{3/2}\left(a+bx\right.}{12be}$$
$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)}{72}$$
$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)}{72}$$
$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)}{72}$$
$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)}{72}$$
$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)}{72}$$

Mathematica [A] time = 0.376751, size = 146, normalized size = 0.73

$$\frac{x\sqrt{ex}\sqrt{a+bx^3}\left(\frac{(8Ab-aB)\left(\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(33a^2+26abx^3+8b^2x^6\right)+15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{48\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}}+B\left(a+bx^3\right)^3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 + ((8*A*b - a*B)*(Sqrt[b]*x^(
3/2)*Sqrt[1 + (b*x^3)/a]*(33*a^2 + 26*a*b*x^3 + 8*b^2*x^6) + 15*a^(5/2)*Arc
Sinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(48*Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]))
```

```
)/(12*b)
```

Maple [C] time = 0.049, size = 7702, normalized size = 38.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*sqrt(e*x), x)
```

Fricas [A] time = 4.31232, size = 738, normalized size = 3.67

$$\left[-\frac{15\left(Ba^4 - 8\,Aa^3b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4\left(48\,Bb^3x^{10} + 8\left(17\,Bab^2\right.\right.}{2304\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2304*(15*(B*a^4 - 8*A*a^3*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
 a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(48
```

```
*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2)*x
^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/1152*(15*(
B*a^4 - 8*A*a^3*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-
e/b)/(2*b*e*x^3 + a*e)) + 2*(48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 +
 2*(59*B*a^2*b + 104*A*a*b^2)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3
+ a)*sqrt(e*x))/b]
```

Sympy [B] time = 111.188, size = 413, normalized size = 2.05

$$\frac{Aa^{\frac{5}{2}}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa^{\frac{5}{2}}(ex)^{\frac{3}{2}}}{8e\sqrt{1+\frac{bx^3}{a}}}+\frac{35Aa^{\frac{3}{2}}b(ex)^{\frac{9}{2}}}{72e^4\sqrt{1+\frac{bx^3}{a}}}+\frac{17A\sqrt{a}b^2(ex)^{\frac{15}{2}}}{36e^7\sqrt{1+\frac{bx^3}{a}}}+\frac{5Aa^3\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{24\sqrt{b}}+\frac{Ab^3(ex)^{\frac{21}{2}}}{9\sqrt{a}e^{10}\sqrt{1+\frac{bx^3}{a}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(5/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(5/2)*(e*x)**(3/2)/
(8*e*sqrt(1 + b*x**3/a)) + 35*A*a**(3/2)*b*(e*x)**(9/2)/(72*e**4*sqrt(1 + b
*x**3/a)) + 17*A*sqrt(a)*b**2*(e*x)**(15/2)/(36*e**7*sqrt(1 + b*x**3/a)) +
5*A*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(24*sqrt(b)
) + A*b**3*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a)) + 5*B*a**(7/2
)*(e*x)**(3/2)/(192*b*e*sqrt(1 + b*x**3/a)) + 133*B*a**(5/2)*(e*x)**(9/2)/(
576*e**4*sqrt(1 + b*x**3/a)) + 127*B*a**(3/2)*b*(e*x)**(15/2)/(288*e**7*sqr
t(1 + b*x**3/a)) + 23*B*sqrt(a)*b**2*(e*x)**(21/2)/(72*e**10*sqrt(1 + b*x**
3/a)) - 5*B*a**4*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(19
2*b**(3/2)) + B*b**3*(e*x)**(27/2)/(12*sqrt(a)*e**13*sqrt(1 + b*x**3/a))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.539 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=364

$$\frac{27\,3^{3/4}a^{8/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(22Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27a^2\sqrt{ex}\sqrt{}}{}$$

```
[Out] (27*a^2*(22*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(1408*b*e) + (3*a*(22*A*b
 - a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(352*b*e) + ((22*A*b - a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(176*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(7/2))/(11*b*e) + (
27*3^(3/4)*a^(8/3)*(22*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2816*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) +
b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.283724, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 225}

$$\frac{27\,3^{3/4}a^{8/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(22Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27a^2\sqrt{ex}\sqrt{a+bx^3}(}{1408be}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```


```
[Out] (27*a^2*(22*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(1408*b*e) + (3*a*(22*A*b
 - a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(352*b*e) + ((22*A*b - a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(176*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(7/2))/(11*b*e) + (
27*3^(3/4)*a^(8/3)*(22*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
```

```
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2816*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) +
b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be} - \frac{\left(-11Ab+\frac{aB}{2}\right)\int \frac{\left(a+bx^3\right)^{5/2}}{\sqrt{ex}}\,dx}{11b}$$

$$= \frac{(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{176be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be} + \frac{(15a(22Ab-aB))\int \frac{\left(a+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx}{352b}$$

$$= \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{176be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be} +$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}\,(}{176be}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}\,(}{176be}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}\,(}{176be}$$

Mathematica [C] time = 0.0637703, size = 84, normalized size = 0.23

$$\frac{x\sqrt{a+bx^3}\left(B\left(a+bx^3\right)^3 - \frac{a^2(aB-22Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}\right)}{11b\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 - (a^2*(-22*A*b + a*B)*Hypergeometric2F
1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b*Sqrt[e*x])
```

Maple [C] time = 0.045, size = 4617, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x)
```

```
[Out] -1/1408*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(162*B*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2*a^4*b
^2*e-3564*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)
+1)/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^3*e-3564*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)
*a^3*b*e+3102*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a^2*b^2+243*B*(1/b^2*e*x*(-b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)
*a^3*b+384*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(
-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^9*b^4+528*A*(1/b^2*e*x*(-b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^
6*b^4+324*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/
2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a^4*b*e-7128*I*A*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a^3*b^2*e-376*I*B*(1/b^2*e*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
```

2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x
)^(1/2)*x^6*a*b^3-356*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^3*a^2*b^2-616*I*
A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/
3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^3*a*b^3+1128*B*(1/b^2*e*x*(-b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^
6*a*b^3+1848*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^3*a*b^3+1068*B*(1/b^2*e*x*(-b*x+(-a
b^2)^(1/3))(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/
2)*x^3*a^2*b^2-324*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(
I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^4*b*e+7128*A*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(
1/2))*(-a*b^2)^(1/3)*x*a^3*b^2*e+3564*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1
/2)*a^3*b*e-162*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I
*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^4*b^2*e+3564*I*A*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/
2))*3^(1/2)*x^2*a^3*b^3*e-162*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*
3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-

```
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a^4*
e+162*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/
(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^4*e-1034*I*A*(1/b^2*e*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x
)^(1/2)*a^2*b^2-81*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a^3*b-128*I*B*(1/b^2*
e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2
)*((b*x^3+a)*e*x)^(1/2)*x^9*b^4-176*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^6*
b^4)/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2x^9+\left(2\,Bab+Ab^2\right)x^6+\left(Ba^2+2\,Aab\right)x^3+Aa^2\right)\sqrt{bx^3+a}\sqrt{ex}}{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

Sympy [C] time = 133.694, size = 308, normalized size = 0.85

$$\frac{Aa^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\\frac{7}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\\frac{13}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)}+\frac{A\sqrt{a}b^{2}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\\frac{19}{6}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{19}{6}\right)}+\frac{Ba^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(5/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + 2*A*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((
-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + A*
sqrt(a)*b**2*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*a**(5/2)*x**(7/2)*gamma(7/6)*hyp
er((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
+ 2*B*a**(3/2)*b*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*sqrt(a)*b**2*x**(19/2)*gamma
(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*ga
mma(25/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^{3}+A\right)\left(bx^{3}+a\right)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

3.540 $\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=650

$$-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+20Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+$$

```
[Out] (27*a*(20*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*e^4) + (81*(1 + Sqrt
[3])*a^2*(20*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(2/3)*e^2*(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*(20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(
3/2))/(28*e^4) + ((20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*a*e^4)
- (2*A*(a + b*x^3)^(7/2))/(a*e*Sqrt[e*x]) - (81*3^(1/4)*a^(7/3)*(20*A*b + a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(448*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/4)*(1 - Sqrt[3])*a^
(7/3)*(20*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3
)*x)], (2 + Sqrt[3])/4])/(896*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.698379, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 308, 225, 1881}

$$\frac{81\left(1+\sqrt{3}\right)a^2\sqrt{ex}\sqrt{a+bx^3}(aB+20Ab)}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+20Ab)F\left(\cos\right.}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (27*a*(20*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*e^4) + (81*(1 + Sqrt
[3])*a^2*(20*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(2/3)*e^2*(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*(20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(
3/2))/(28*e^4) + ((20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*a*e^4)
- (2*A*(a + b*x^3)^(7/2))/(a*e*Sqrt[e*x]) - (81*3^(1/4)*a^(7/3)*(20*A*b + a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(448*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/4)*(1 - Sqrt[3])*a^
(7/3)*(20*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3
)*x)], (2 + Sqrt[3])/4])/(896*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
```

```
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} + \frac{(20Ab+aB)\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\,dx}{ae^3}$$

$$= \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} + \frac{(3(20Ab+aB))\int (ex)^{3/2}\left(a+bx^3\right)^{3/}}{4e^3}$$

$$= \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} +$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5}}{10ae}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5}}{10ae}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5}}{10ae}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)} + 3(20Ab +$$

Mathematica [C] time = 0.0439877, size = 87, normalized size = 0.13

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a^2x^3(aB+20Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - 5A\left(a+bx^3\right)^3\right)}{5a(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3)^3 + (a^2*(20*A*b + a*B)*x^3*Hypergeo
metric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)^
(3/2))
```

Maple [C] time = 0.056, size = 6530, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2x^9 + \left(2\,Bab + Ab^2\right)x^6 + \left(Ba^2 + 2\,Aab\right)x^3 + Aa^2\right)\sqrt{bx^3 + a}\sqrt{ex}}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] time = 159.126, size = 311, normalized size = 0.48

$$\frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{A\sqrt{a}b^2x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}+\frac{Ba^{\frac{5}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(5/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + 2*A*a**(3/2)*b*x**(5/2)*gamma(5/6)*hype
r((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
+ A*sqrt(a)*b**2*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*
exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*a**(5/2)*x**(5/2)*gamma(5/6
)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(1
1/6)) + 2*B*a**(3/2)*b*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b
*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*sqrt(a)*b**2*x**(17/2
)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**
(3/2)*gamma(23/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

3.541 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(aB+6Ab)}{9ae^4}+\frac{5(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+6Ab)}{36e^4}+\frac{5a(ex)^{3/2}\sqrt{a}}{}$$

```
[Out] (5*a*(6*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*e^4) + (5*(6*A*b + a*B)
*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*e^4) + ((6*A*b + a*B)*(e*x)^(3/2)*(a +
b*x^3)^(5/2))/(9*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + (5*
a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])
/(24*Sqrt[b]*e^(5/2))
```

Rubi [A] time = 0.128335, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(aB+6Ab)}{9ae^4}+\frac{5(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+6Ab)}{36e^4}+\frac{5a(ex)^{3/2}\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (5*a*(6*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*e^4) + (5*(6*A*b + a*B)
*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*e^4) + ((6*A*b + a*B)*(e*x)^(3/2)*(a +
b*x^3)^(5/2))/(9*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + (5*
a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])
/(24*Sqrt[b]*e^(5/2))
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}} + \frac{(6Ab+aB)\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\,dx}{ae^3}$$

$$= \frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5(6Ab+aB))\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\,d}{6e^3}$$

$$= \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}} + \cdots$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}\left(\cdots\right.}{9ae^4}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}\left(\cdots\right.}{9ae^4}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}\left(\cdots\right.}{9ae^4}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}\left(\cdots\right.}{9ae^4}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}\left(\cdots\right.}{9ae^4}$$

Mathematica [A] time = 0.193192, size = 150, normalized size = 0.8

$$\frac{x\sqrt{a+bx^3}\left(\sqrt{b}\sqrt{\frac{bx^3}{a}+1}\left(a^2\left(33Bx^3-48A\right)+a\left(54Abx^3+26bBx^6\right)+4b^2x^6\left(3A+2Bx^3\right)\right)+15a^{3/2}x^{3/2}(aB+6Ab)\,\mathrm{sinh}\cdots\right.}{72\sqrt{b}(ex)^{5/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(Sqrt[b]*Sqrt[1 + (b*x^3)/a]*(4*b^2*x^6*(3*A + 2*B*x^3)
+ a^2*(-48*A + 33*B*x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^(3/2)*(6*A*b
 + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*Sqrt[b]*(e*x)^(5/2
```

```
)*Sqrt[1 + (b*x^3)/a])
```

Maple [C] time = 0.053, size = 7544, normalized size = 40.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

Fricas [A] time = 4.33935, size = 714, normalized size = 3.8

$$\left[\frac{15\left(Ba^3 + 6\,Aa^2b\right)\sqrt{be}x^2 \log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,bx^4 + ax\right)\sqrt{bx^3 + a}\sqrt{be}\sqrt{ex}\right) + 4\left(8\,Bb^3x^9 + 2\left(13\,Bab^2 + 6\right.\right.}{288\,be^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(8*B*
```

```
b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*
a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/144*(15*(B*a^3 + 6*A
*a^2*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b
*e*x^3 + a*e)) - 2*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b
 + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

3.542 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=352

$$\frac{27\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+16Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\left(a+b\right.}{}$$

```
[Out] (27*a*(16*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*e^4) + (3*(16*A*b +
5*a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*e^4) + ((16*A*b + 5*a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(40*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(5*a*e*(e*x)^(5/2))
 + (27*3^(3/4)*a^(5/3)*(16*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*e^4*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.286675, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 225}

$$\frac{27\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+16Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\left(a+bx^3\right)^{5/2}(5a}{40ae^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (27*a*(16*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*e^4) + (3*(16*A*b +
5*a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*e^4) + ((16*A*b + 5*a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(40*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(5*a*e*(e*x)^(5/2))
 + (27*3^(3/4)*a^(5/3)*(16*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
```

```
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*e^4*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}} + \frac{(16Ab+5aB)\int \frac{\left(a+bx^3\right)^{5/2}}{\sqrt{ex}}\,dx}{5ae^3}$$

$$= \frac{(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{40ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}} + \frac{(3(16Ab+5aB))\int \frac{\left(a+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx}{16e^3}$$

$$= \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{40ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}} + \frac{(2}{}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}\,(}{40ae^4}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}\,(}{40ae^4}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}\,(}{40ae^4}$$

Mathematica [C] time = 0.0466687, size = 88, normalized size = 0.25

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a^2x^3(5aB+16Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - A\left(a+bx^3\right)^3\right)}{5a(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)^3) + (a^2*(16*A*b + 5*a*B)*x^3*Hyperg
eometric2F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)
^(7/2))
```

Maple [C] time = 0.049, size = 4422, normalized size = 12.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x)
```

```
[Out] -1/320*(b*x^3+a)^(1/2)*(-2592*a^2*x^5*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*b^3*e-810*a^3*x^5*B*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2
)-3))^(1/2))*b^2*e-810*a^3*x^3*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3
^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+
I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*e-368*I*a*x^3
*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1
/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b^2-40*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)
*x^9*b^3-140*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^6*a*b^2-1620*I*a^3*x^4*B*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/
2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*b*e+2592*I*a^2*x^3*A*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a
```

```
*b^2)^(2/3)*3^(1/2)*b*e+810*I*a^3*x^5*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*b^2*e+1104*a*
x^3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)
^(1/3)*((b*x^3+a)*e*x)^(1/2)*b^2+705*a^2*x^3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*b+420
*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1
/3)*((b*x^3+a)*e*x)^(1/2)*x^6*a*b^2+120*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^9*b^3+19
2*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(
1/3)*((b*x^3+a)*e*x)^(1/2)*x^6*b^3-384*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a^2*b+810*I
*a^3*x^3*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+
1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*e-64*I*A*(1/b^2*e*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a
)*e*x)^(1/2)*x^6*b^3+2592*I*a^2*x^5*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*b^3*e+128*I*A*(
1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*
3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a^2*b-5184*I*a^2*x^4*A*(-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)
^(1/3)*3^(1/2)*b^2*e-235*I*a^2*x^3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
```

```
(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b+5184*
a^2*x^4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1
)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*b^2*e+1620*a^3*x^4*B*(-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(
-a*b^2)^(1/3)*b*e-2592*a^2*x^3*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3
^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+
I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*b*e)/x^2/b/(-
a*b^2)^(1/3)/e^3/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bb^2x^9 + \left(2\,Bab + Ab^2\right)x^6 + \left(Ba^2 + 2\,Aab\right)x^3 + Aa^2\right)\sqrt{bx^3 + a}\sqrt{ex}}{e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

3.543 $\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=121

$$\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}(4Ab-3aB)}{12b^2}-\frac{ae^{7/2}(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

```
[Out] ((4*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b^2) + (B*(e*x)^(9/2)
*Sqrt[a + b*x^3])/(6*b*e) - (a*(4*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(5/2))
```

Rubi [A] time = 0.0860604, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 321, 329, 275, 217, 206}

$$\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}(4Ab-3aB)}{12b^2}-\frac{ae^{7/2}(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((4*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b^2) + (B*(e*x)^(9/2)
*Sqrt[a + b*x^3])/(6*b*e) - (a*(4*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(-6Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{6b}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{8b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,\right.}{4b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,\right.}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x\right.}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{a(4Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}$$

Mathematica [A] time = 0.121261, size = 97, normalized size = 0.8

$$\frac{e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}\left(-3aB+4Ab+2bBx^3\right)+a(3aB-4Ab)\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)\right)}{12b^{5/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3)
 + a*(-4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]]))/(12*b^(5
/2)*Sqrt[x])
```

Maple [C] time = 0.059, size = 6861, normalized size = 56.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{7}{2}}}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/sqrt(b*x^3 + a), x)
```

Fricas [A] time = 4.16013, size = 551, normalized size = 4.55

$$\left[-\frac{\left(3\,Ba^2-4\,Aab\right)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(2\,Bbe^3x^4-(3\,Ba-4\,Ab)e\right.}{48\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B
*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/24*(
(3*B*a^2 - 4*A*a*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*s
qrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sq
rt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.544 $\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=286

$$-\frac{a^{2/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-7aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e^2\sqrt{ex}\sqrt{a+bx}}{20}$$

```
[Out] ((10*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b^2) + (B*(e*x)^(7/2)*
Sqrt[a + b*x^3])/(5*b*e) - (a^(2/3)*(10*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^
2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.22163, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 321, 329, 225}

$$-\frac{a^{2/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e^2\sqrt{ex}\sqrt{a+bx^3}(10Ab-}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```



```
[Out] ((10*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b^2) + (B*(e*x)^(7/2)*
Sqrt[a + b*x^3])/(5*b*e) - (a^(2/3)*(10*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^
2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(-5Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(a(10Ab-7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{40b^2}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(a(10Ab-7aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x\right.}{20b^2}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{a^{2/3}(10Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}}{\left(\sqrt[3]{\right.}}}}{40\sqrt[4]{3}b^2\sqrt{\frac{}{\left(\sqrt[3]{\right.}}}}$$

Mathematica [C] time = 0.100398, size = 98, normalized size = 0.34

$$\frac{e^2\sqrt{ex}\left(a\sqrt{\frac{bx^3}{a}+1}(7aB-10Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\left(7aB-10Ab-4bBx^3\right)\right)}{20b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (e^2*Sqrt[e*x]*(-((a + b*x^3)*(-10*A*b + 7*a*B - 4*b*B*x^3)) + a*(-10*A*b +
 7*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])
)/(20*b^2*Sqrt[a + b*x^3])
```

Maple [C] time = 0.053, size = 3723, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/20*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^3/(-a*b^2)^(1/3)*(28*I*B*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/
2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*(-a*b^2)^(1/3)*3^(1/2)*x*a^2*b*e-40*I*A*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x
+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(
1/2)*x*a*b^2*e+4*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^3*b^2-14*I*B*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^
(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*3^(1/2)*x^2*a^2*b^2*e+10*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b^2-20*A*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/
2)-3))^(1/2))*x^2*a*b^3*e+20*I*A*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*a*b*e
+14*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I
*3^(1/2)-3))^(1/2))*x^2*a^2*b^2*e+40*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a*b^2
```

*e-28*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
3^(1/2)(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/
(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^2*b*e+20*I*A*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/
2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*3^(1/2)
*x^2*a*b^3*e-20*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3
^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a*b*e+14*B*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a
*b^2)^(2/3)*a^2*e-12*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*x^3*b^2-14*I*B*((I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-
3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
(-a*b^2)^(2/3)*3^(1/2)*a^2*e-7*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*a*b-30*A*
(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)
*((b*x^3+a)*e*x)^(1/2)*b^2+21*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*a*b)/((b*x^3+a)*e*x)
^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Be^2x^5 + Ae^2x^2)\sqrt{ex}}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Sympy [C] time = 169.649, size = 94, normalized size = 0.33

$$\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{13}{6}\right)} + \frac{Be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(13/6)) + B*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((
1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(19/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

3.545 $\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=543

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt[3]{a}\ldots}{\ldots}$$

```
[Out] (B*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*b*e) + ((1 + Sqrt[3])*(8*A*b - 5*a*B)*e*
Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x))
- (3^(1/4)*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (
(1 - Sqrt[3])*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rubi [A] time = 0.500764, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*b*e) + ((1 + Sqrt[3])*(8*A*b - 5*a*B)*e*
Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x))
```

```
- (3^(1/4)*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (
(1 - Sqrt[3])*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} - \frac{\left(-4Ab+\frac{5aB}{2}\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{4b}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{(8Ab-5aB)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{4be}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} - \frac{(8Ab-5aB)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{8b^{5/3}e} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}(8\ldots\right.}{\ldots}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{\left(1+\sqrt{3}\right)(8Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\ldots\right.}{8b\ldots}$$

Mathematica [C] time = 0.0894605, size = 80, normalized size = 0.15

$$\frac{x(ex)^{3/2}\left(\sqrt{\frac{bx^3}{a}+1}(8Ab-5aB)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+5B\left(a+bx^3\right)\right)}{20b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x*(e*x)^(3/2)*(5*B*(a + b*x^3) + (8*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(20*b*Sqrt[a + b*x^3])
```

Maple [C] time = 0.056, size = 4914, normalized size = 9.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/4*e*(e*x)^(1/2)*(b*x^3+a)^(1/2)*(15*B*x^3*a*b^2*e-24*A*(-a*b^2)^(1/3)*x^2
*b^2*e-24*A*(-a*b^2)^(2/3)*x*b*e+15*B*(-a*b^2)^(2/3)*x*a*e-3*B*(1/b^2*e*x*(
-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^2*
b^2+8*I*A*3^(1/2)*x^3*b^3*e+15*B*(-a*b^2)^(1/3)*x^2*a*b*e-24*A*x^3*b^3*e+8*
I*A*(-a*b^2)^(1/3)*3^(1/2)*x^2*b^2*e+8*I*A*(-a*b^2)^(2/3)*3^(1/2)*x*b*e-5*I
*B*(-a*b^2)^(2/3)*3^(1/2)*x*a*e+I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3)))^(1/2)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*x^2*b^2-5*I*B*3^(1/2)*x
^3*a*b^2*e+8*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^
(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x^2*b^2*e-16*I*A*(-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3
))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x*b*e+10*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*
3^(1/2)*x*a*e-30*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*
3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*a*e-10*B*(-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
```

)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-
a*b^2)^(1/3)*x^2*a*b*e+15*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2
)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(
1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^2*a*b*e-8*I*A*(-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)
-3))^(1/2))*3^(1/2)*a*b^2*e+5*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*
3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a^2*b*e-16*A*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))
^(1/2))*a*b^2*e+24*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(
I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*a*b^2*e+10*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*
3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*a^2*b*e-15*B*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/
2)-3))^(1/2))*a^2*b*e-5*I*B*(-a*b^2)^(1/3)*3^(1/2)*x^2*a*b*e+16*A*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
3^(1/2)(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^

```
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1
/2))*(-a*b^2)^(1/3)*x^2*b^2*e-24*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I
*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-
1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^2*b^2*e-3
2*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3
^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*b*e+48*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
cE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*
b*e+20*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)
/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*a*e-5*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1
/3)*3^(1/2)*x^2*a*b*e)/b^3/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(
-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bex^4 + Aex\right)\sqrt{ex}}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Sympy [C] time = 30.2011, size = 94, normalized size = 0.17

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{6}\right)} + \frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{6}\\\frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((
1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(17/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{3}{2}}}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

3.546 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{e}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

```
[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(S
qrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rubi [A] time = 0.0673085, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 329, 275, 217, 206}

$$\frac{\sqrt{e}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```


```
[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(S
qrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}-\frac{\left(-3Ab+\frac{3aB}{2}\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{3b}\\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{be}\\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3be}\\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be}\\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0424473, size = 78, normalized size = 0.94

$$
\frac{\sqrt{ex}\left((2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)+\sqrt{b}Bx^{3/2}\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (Sqrt[e*x]*(Sqrt[b]*B*x^(3/2)*Sqrt[a + b*x^3] + (2*A*b - a*B)*ArcTanh[(Sqrt
[b]*x^(3/2))/Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

Maple [C] time = 0.044, size = 6424, normalized size = 77.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)\sqrt{ex}}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Fricas [A] time = 4.2287, size = 417, normalized size = 5.02

$$\left[\frac{4\sqrt{bx^3 + a}\sqrt{ex}Bx - (Ba - 2\,Ab)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)}{12\,b}, \frac{2\sqrt{bx^3 + a}\sqrt{ex}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(e*x)*B*x - (B*a - 2*A*b)*sqrt(e/b)*log(-8*b^2
*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e
*x)*sqrt(e/b)))/b, 1/6*(2*sqrt(b*x^3 + a)*sqrt(e*x)*B*x + (B*a - 2*A*b)*sqr
t(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)
))/b]
```

Sympy [A] time = 6.05434, size = 107, normalized size = 1.29

$$\frac{2A\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{3\sqrt{b}}+\frac{B\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3be}-\frac{Ba\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*
sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*b*e) - B*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*b**(3/2))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.547 $\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=249

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

```
[Out] (B*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b*e) + ((4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*a^(1/
3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.182986, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {459, 329, 225}

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```


```
[Out] (B*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b*e) + ((4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*a^(1/
3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx = \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} - \frac{\left(-2Ab+\frac{aB}{2}\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{2b}$$

$$= \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{2be}$$

$$= \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\right.\right.}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0562964, size = 80, normalized size = 0.32

$$\frac{x\sqrt{\frac{bx^3}{a}+1}(4Ab-aB)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+Bx\left(a+bx^3\right)}{2b\sqrt{ex}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

```
[Out] (B*x*(a + b*x^3) + (4*A*b - a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/
6, 1/2, 7/6, -((b*x^3)/a)])/(2*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Maple [C] time = 0.042, size = 3275, normalized size = 13.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(8*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*
b^3*e-2*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)
+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a*b^2*e-16*I*A*(-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)
^(1/3)*3^(1/2)*x*b^2*e+4*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/
2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^
(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a*b*e+8
*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
```

-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*b*e-8*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2*b^3*e-2*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a*e+2*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2*e+16*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*b^2*e-4*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a*b*e-8*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*b*e+2*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a*e-I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b+3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x)^(1/2)*b)/

```
(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(
1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}}{bex^4 + aex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e*x^4 + a*e*x), x)
```

Sympy [C] time = 3.38823, size = 94, normalized size = 0.38

$$\frac{A\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/
6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*sqrt(e)*gamma(13/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

3.548 $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=542

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{ex}\left(\sqrt[3]{a}\cdots\right.}{\cdots}$$

```
[Out] (-2*A*Sqrt[a + b*x^3])/(a*e*Sqrt[e*x]) + ((1 + Sqrt[3])*(2*A*b + a*B)*Sqrt[
e*x]*Sqrt[a + b*x^3])/(a*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) -
 (3^(1/4)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellipt
icE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1
- Sqrt[3])*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rubi [A] time = 0.504986, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {453, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(a*e*Sqrt[e*x]) + ((1 + Sqrt[3])*(2*A*b + a*B)*Sqrt[
e*x]*Sqrt[a + b*x^3])/(a*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) -
```

```
(3^(1/4)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellipt
icE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1
- Sqrt[3])*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{(2Ab+aB)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{ae^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{(2(2Ab+aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{ae^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} - \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{ab^{2/3}e^4} - \frac{\left(\left(1-\sqrt{3}\right)(2Ab+aB)\right.}{\ldots}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\ldots}{\left(\sqrt[3]{a}+\ldots\right.}}}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\ldots}{\left(\ldots\right.}}}$$

Mathematica [C] time = 0.0418165, size = 83, normalized size = 0.15

$$\frac{x\left(2x^3\sqrt{\frac{bx^3}{a}+1}(aB+2Ab)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)-10A\left(a+bx^3\right)\right)}{5a(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (x*(-10*A*(a + b*x^3) + 2*(2*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(5*a*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Maple [C] time = 0.051, size = 5385, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}}{be^2x^5 + ae^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e^2*x^5 + a*e^2*x^2), x)
```

Sympy [C] time = 5.23486, size = 97, normalized size = 0.18

$$\frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{1}{2}\\\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (
11/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(3/2)*gamma(11/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

3.549 $\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

```
[Out] (-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(3
/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rubi [A] time = 0.0570957, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {451, 329, 275, 217, 206}

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(3
/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{e^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{(2B)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{2B\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

Mathematica [A] time = 0.038327, size = 65, normalized size = 0.87

$$\frac{2x\left(\frac{Bx^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^3}}{a}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[
a + b*x^3]])/Sqrt[b]))/(3*(e*x)^(5/2))
```

Maple [C] time = 0.042, size = 3397, normalized size = 45.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a)^(1/2)/x/b^2*(6*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*
3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^4*a*b^2*e-6*I*B*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^
(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^4*a*b^2*e-12*I*B*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3)
)^(1/2))*3^(1/2)*(-a*b^2)^(1/3)*x^3*a*b*e+12*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
```

b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-
1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3
^(1/2)-3))^(1/2))*3^(1/2)*(-a*b^2)^(1/3)*x^3*a*b*e+6*I*B*(-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1
/2)*(-a*b^2)^(2/3)*x^2*a*e-6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
b^2)^(1/3)))^(1/2)((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(
1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*
3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^4*a*b^2*e-6*I*B*(-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3
^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*(-a*b^2)^(2/3)*x^2*a*e+6*B*(-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/
(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^4*a*b^2*e+12*B*(-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)
^(1/3)*x^3*a*b*e-12*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^
(1/3)*x^3*a*b*e-6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I
*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^2*a*e+6*B*(-(I*3^(1/2)-3

```
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(
1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^2*a*e+I*A*(1/b^2*e*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*((b*x^3+a)*e*x)^(1/2)*b^
2-3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*((b*x^3+
a)*e*x)^(1/2)*b^2)/e^2/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/2)/a/(I*3^(1/2)-3)/(1
/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40648, size = 425, normalized size = 5.67

$$\left[\frac{\sqrt{be}Bax^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\sqrt{be}\sqrt{ex}\right)-4\,\sqrt{bx^3+a}\sqrt{ex}Ab}{6\,abe^3x^2},-\frac{\sqrt{-be}Bax^2\arctan\left(\frac{2\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b*e)*B*a*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4
 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) - 4*sqrt(b*x^3 + a)*sqrt(e*x)*
A*b)/(a*b*e^3*x^2), -1/3*(sqrt(-b*e)*B*a*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(
-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) + 2*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)/(a*
```

```
b*e^3*x^2)]
```

Sympy [A] time = 29.5105, size = 60, normalized size = 0.8

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}}+\frac{2B\,\text{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3/
2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Giac [A] time = 1.24993, size = 146, normalized size = 1.95

$$-\frac{2}{3}\left(\frac{B\arctan\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{\sqrt{-be}}\right)e^{(-1)}}{\sqrt{-be}}+\frac{\sqrt{be+\frac{ae}{x^3}}Ae^{(-2)}}{a}-\frac{\left(Ba\arctan\left(\frac{\sqrt{b}e^{\frac{1}{2}}}{\sqrt{-be}}\right)e+\sqrt{-be}A\sqrt{b}e^{\frac{1}{2}}\right)e^{(-2)}}{\sqrt{-be}a}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*arctan(sqrt(b*e + a*e/x^3)/sqrt(-b*e))*e^(-1)/sqrt(-b*e) + sqrt(b*e
 + a*e/x^3)*A*e^(-2)/a - (B*a*arctan(sqrt(b)*e^(1/2)/sqrt(-b*e))*e + sqrt(-
b*e)*A*sqrt(b)*e^(1/2))*e^(-2)/(sqrt(-b*e)*a))*e^(-1)
```

3.550 $\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

```
[Out] (-2*A*Sqrt[a + b*x^3])/(5*a*e*(e*x)^(5/2)) - ((2*A*b - 5*a*B)*Sqrt[e*x]*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b
^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4
)*a^(4/3)*e^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.183095, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 329, 225}

$$-\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```


```
[Out] (-2*A*Sqrt[a + b*x^3])/(5*a*e*(e*x)^(5/2)) - ((2*A*b - 5*a*B)*Sqrt[e*x]*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b
^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4
)*a^(4/3)*e^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{5ae^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2(2Ab-5aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{5ae^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(\right.\right.}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.0460112, size = 82, normalized size = 0.33

$$-\frac{2x\left(x^3\sqrt{\frac{bx^3}{a}+1}(2Ab-5aB)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+A\left(a+bx^3\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*x*(A*(a + b*x^3) + (2*A*b - 5*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^3])
```

Maple [C] time = 0.043, size = 3303, normalized size = 13.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/5*(b*x^3+a)^(1/2)*(-4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/
2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^5*b^3*e+10*B*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*x^5*a*
b^2*e+10*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+
1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3*a*e+20*I*x^4*B*(-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
```

)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-
a*b^2)^(1/3)*3^(1/2)*a*b*e+4*I*x^3*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*b
*e-8*I*x^4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2
)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*b^2*e+3*A*(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*((b*x^3+a)*e*x
)^(1/2)*b-10*I*x^3*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(
I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*a*e+4*I*x^5*A*(-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-
3))^(1/2))*3^(1/2)*b^3*e-10*I*x^5*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*b^2*e+8*A*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(1/3)*x^4*b^2*e-20*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^4*a*b
*e-4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*

```
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(
I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3*b*e-I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(
1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*e*x)^(1
/2)*b)/x^2/(-a*b^2)^(1/3)/b/a/e^3/(e*x)^(1/2)/((b*x^3+a)*e*x)^(1/2)/(I*3^(1
/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}}{be^4x^7 + ae^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e^4*x^7 + a*e^4*x^4), x)
```

Sympy [C] time = 176.107, size = 97, normalized size = 0.39

$$\frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (
7/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(7/2)*gamma(7/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

3.551 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}}+\frac{e^{7/2}(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

```
[Out] -((2*A*b - 3*a*B)*e^2*(e*x)^(3/2))/(3*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(9/2)
)/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)
^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rubi [A] time = 0.0864786, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 288, 329, 275, 217, 206}

$$-\frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}}+\frac{e^{7/2}(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -((2*A*b - 3*a*B)*e^2*(e*x)^(3/2))/(3*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(9/2)
)/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)
^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} - \frac{\left(-3Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{(2Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Mathematica [A] time = 0.163377, size = 109, normalized size = 0.91

$$\frac{e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\left(3aB-2Ab+bBx^3\right)-\sqrt{a}\sqrt{\frac{bx^3}{a}+1}(3aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{x}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3) - Sqrt[a]*(-2*A*
b + 3*a*B)*Sqrt[1 + (b*x^3)/a]*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(3*b^(5
/2)*Sqrt[x]*Sqrt[a + b*x^3])
```

Maple [C] time = 0.065, size = 7016, normalized size = 58.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(ex)^{\frac{7}{2}}}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(3/2), x)
```

Fricas [A] time = 4.33937, size = 667, normalized size = 5.56

$$\left[-\frac{\left(\left(3\,Bab - 2\,Ab^2\right)e^3x^3 + \left(3\,Ba^2 - 2\,Aab\right)e^3\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4}{12\left(b^3x^3 + ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(e/b)*l
og(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 +
a)*sqrt(e*x)*sqrt(e/b)) - 4*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b*x^
3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2), 1/6*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3
*B*a^2 - 2*A*a*b)*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sq
rt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.552 $\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-7aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{e^2\sqrt{ex}(4Ab-7aB)}{6b^2\sqrt{a+bx^3}}$$

```
[Out] -((4*A*b - 7*a*B)*e^2*Sqrt[e*x])/(6*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(7/2))/
(2*b*e*Sqrt[a + b*x^3]) + ((4*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(12*3^(1/4)*a^(1/3)*b^2*
Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.221321, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 288, 329, 225}

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{e^2\sqrt{ex}(4Ab-7aB)}{6b^2\sqrt{a+bx^3}}+\frac{B(e}{2be\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```


```
[Out] -((4*A*b - 7*a*B)*e^2*Sqrt[e*x])/(6*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(7/2))/
(2*b*e*Sqrt[a + b*x^3]) + ((4*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(12*3^(1/4)*a^(1/3)*b^2*
Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^
```

```
2]*Sqrt[a + b*x^3])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} - \frac{\left(-2Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{2b}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{\left((4Ab-7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{12b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{\left((4Ab-7aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{6b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\right.}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}}$$

Mathematica [C] time = 0.111221, size = 87, normalized size = 0.3

$$\frac{e^2\sqrt{ex}\left(\sqrt{\frac{bx^3}{a}+1}(4Ab-7aB)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+7aB-4Ab+3bBx^3\right)}{6b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (e^2*Sqrt[e*x]*(-4*A*b + 7*a*B + 3*b*B*x^3 + (4*A*b - 7*a*B)*Sqrt[1 + (b*x^
3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(6*b^2*Sqrt[a + b*x^
3])
```

Maple [C] time = 0.059, size = 3760, normalized size = 13.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] -1/6*e^2/x*(e*x)^(1/2)*(4*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x*b^2+28*I*B*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a
)*e*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x*a*b-14*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e
*x)^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*a-16*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/
2)*(-a*b^2)^(1/3)*3^(1/2)*x*b^2+8*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(
-a*b^2)^(2/3)*3^(1/2)*b-8*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2
)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(
1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*x^2*b^3-3*I
*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1
/3)*3^(1/2)*x^4*b^2+14*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*x^2*a*b^2+8*I*
A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(
```

1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*3^(1/2)*x^2*b^3+16*A*(-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((
b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*x*b^2-28*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)
^(1/2)*(-a*b^2)^(1/3)*x*a*b-8*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^
(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I
*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2
)^(2/3)*b+14*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1
/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*a+9*B*(1/
b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x^
4*b^2-7*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-
a*b^2)^(1/3)*3^(1/2)*x*a*b-14*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*
3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*3^(1/
2)*x^2*a*b^2-12*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1
/2)*(-a*b^2)^(1/3)*x*b^2+21*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x*a*b)/(b*x^3+a)^(1/2)/b^3/(-a*b^2)^(1/3)/(I
*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{5}{2}}}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Be^2x^5 + Ae^2x^2\right)\sqrt{bx^3 + a}\sqrt{ex}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b
*x^3 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{5}{2}}}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

3.553 $\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e\sqrt{ex}\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*Sqrt[a + b*x^3]) - ((1 + Sqrt[3])*(2*A
*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) + ((2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(b^(
1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + ((1 - Sqrt[3])*(2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(6*3^(1/4)*a^(2/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.502503, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 329, 308, 225, 1881}

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*Sqrt[a + b*x^3]) - ((1 + Sqrt[3])*(2*A
*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(a^(1/3) + (1 + Sqrt[
```

```
3])*b^(1/3)*x)) + ((2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(b^(
1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + ((1 - Sqrt[3])*(2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(6*3^(1/4)*a^(2/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(-Ab+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{(2(2Ab-5aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab-5aB)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3ab^{5/3}e} + \frac{\left(\left(1-\sqrt{3}\right)(2Ab-\ldots\right.}{\ldots}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{\left(1+\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\ldots}}{3^{3/4}a^{2/3}b^{5\ldots}}$$

Mathematica [C] time = 0.105907, size = 77, normalized size = 0.14

$$\frac{x(ex)^{3/2}\left(\sqrt{\frac{bx^3}{a}+1}(2Ab-5aB)\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{bx^3}{a}\right)+5aB\right)}{5ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(e*x)^(3/2)*(5*a*B + (2*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a*b*Sqrt[a + b*x^3])
```

Maple [C] time = 0.065, size = 5392, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{3}{2}}}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bex^4 + Aex\right)\sqrt{bx^3 + a}\sqrt{ex}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b*x^3 +
 a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{3}{2}}}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

3.554 $\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}} dx$

Optimal. Leaf size=85

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTan
h[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rubi [A] time = 0.0623942, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {452, 329, 275, 217, 206}

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTan
h[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0872688, size = 93, normalized size = 1.09

$$\frac{2\sqrt{ex}\left(a^{3/2}B\sqrt{\frac{bx^3}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}(Ab-aB)\right)}{3ab^{3/2}\sqrt{x}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[e*x]*(Sqrt[b]*(A*b - a*B)*x^(3/2) + a^(3/2)*B*Sqrt[1 + (b*x^3)/a]*A
rcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(3*a*b^(3/2)*Sqrt[x]*Sqrt[a + b*x^3])
```

Maple [C] time = 0.04, size = 3654, normalized size = 43.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3*(e*x)^(1/2)/(b*x^3+a)^(1/2)/b^3*(-6*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(
1/2)*3^(1/2)*x^2*a*b^2+12*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1
/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3
^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^
(1/3)*3^(1/2)*x*a*b-12*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)
+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2
)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*
x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x*a*b+6*I*B*(-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
```

```
(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/
(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*a-6*I*B*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/
2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*a+6*B*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/
2))*((b*x^3+a)*e*x)^(1/2)*x^2*a*b^2+I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*x^2*b^3-6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I
*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1
/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*x^2*a*b^2-12*B*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a
)*e*x)^(1/2)*(-a*b^2)^(1/3)*x*a*b+12*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*3^(1/2
))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))
^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*x*a*b-I*B*(1/b^2*e*x*(-b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*x^2*a*b^2+6*B*(-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2)
)*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*a-6*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
```

```
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),(-1+I*
3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/
2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*a+6*I*B*(-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/
2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*3^(1/2)*x^2*a*b^2-3*A*(1/
b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^2*b^3+3*B*(1/b^
2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^2*a*b^2)/x/(I*3^(
1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\sqrt{ex}}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(3/2), x)
```

Fricas [A] time = 2.52996, size = 510, normalized size = 6.

$$\left[-\frac{4\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex}\,x-\left(Babx^3+Ba^2\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)}{6\left(ab^2x^3+a^2b\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x - (B*a*b*x^3 + B*a^2)*sqrt
(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b
*x^3 + a)*sqrt(e*x)*sqrt(e/b)))/(a*b^2*x^3 + a^2*b), -1/3*(2*sqrt(b*x^3 + a
)*(B*a - A*b)*sqrt(e*x)*x + (B*a*b*x^3 + B*a^2)*sqrt(-e/b)*arctan(2*sqrt(b*
x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)))/(a*b^2*x^3 + a^2*b)]
```

Sympy [A] time = 53.1422, size = 95, normalized size = 1.12

$$\frac{2A\sqrt{e}x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+B\left(\frac{2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{2\sqrt{e}x^{\frac{3}{2}}}{3\sqrt{a}b\sqrt{1+\frac{bx^3}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*sqr
t(1 + b*x**3/a)))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.555 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=258

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*A*b + a*B)*Sqrt[e
*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqr
t[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3
*3^(1/4)*a^(4/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.188183, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {457, 329, 225}

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```



```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*A*b + a*B)*Sqrt[e
*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqr
t[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3
*3^(1/4)*a^(4/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2(2Ab+aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2\right.\right.}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.056167, size = 79, normalized size = 0.31

$$\frac{2x\left(\sqrt{\frac{bx^3}{a}+1}(aB+2Ab)\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-aB+Ab\right)}{3ab\sqrt{ex}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(A*b - a*B + (2*A*b + a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6,
1/2, 7/6, -((b*x^3)/a)]))/(3*a*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Maple [C] time = 0.044, size = 3565, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] -2/3/(b*x^3+a)^(1/2)/b^2/(-a*b^2)^(1/3)/a*(2*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e
*x)^(1/2)*3^(1/2)*x^2*a*b^2+4*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*
3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*3^(1/
2)*x^2*b^3-I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
*(-a*b^2)^(1/3)*3^(1/2)*x*b^2+4*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a
```

*b^2)^(2/3)*3^(1/2)*b-4*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2
)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(
1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1
/3)*3^(1/2)*x*a*b-4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/
(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*x^2*b^3+2*I*B*(-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-
3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*a-2*B*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))
*((b*x^3+a)*e*x)^(1/2)*x^2*a*b^2+8*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-
a*b^2)^(1/3)*x*b^2+4*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))
/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*x
*a*b-4*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)
/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*b-2*B*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
3^(1/2)(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^

```
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1
/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*a+I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(
1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x*a*b-8*I*A*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))
^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x*b^2+3*A*(1/b^2*e*x*(
-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x*b^2-3*B*(
1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*
x*a*b)/(e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}}{b^2ex^7 + 2\,abex^4 + a^2ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*e*x^7 + 2*a*b*e*x^4 + a
^2*e*x), x)
```

Sympy [C] time = 142.286, size = 94, normalized size = 0.36

$$\frac{A\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3
*a**(3/2)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((7/6, 3/2), (13
/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*sqrt(e)*gamma(13/6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

3.556 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=585

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(1+\sqrt{3}\right)}{3a^2b^{2/3}e^2}$$

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^3]) - (2*(4*A*b - a*B)*(e*x)^(5/2))/(3*a
^2*e^4*Sqrt[a + b*x^3]) + (2*(1 + Sqrt[3])*(4*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^3])/(3*a^2*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (2*(4*A*
b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(
1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 +
Sqrt[3])/4])/(3^(3/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1 - Sqrt[
3])*(4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.570769, antiderivative size = 585, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 329, 308, 225, 1881}

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)\right.}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^3]) - (2*(4*A*b - a*B)*(e*x)^(5/2))/(3*a
^2*e^4*Sqrt[a + b*x^3]) + (2*(1 + Sqrt[3])*(4*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
```

```
b*x^3])/(3*a^2*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (2*(4*A*
b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(
1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 +
Sqrt[3])/4])/(3^(3/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1 - Sqrt[
3])*(4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{(4Ab-aB)\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(2(4Ab-aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(4(4Ab-aB))\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3a^2e^4}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} - \frac{(2(4Ab-aB))\,\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x,\right.}{3a^2b^{2/3}e^4}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2(4Ab-a}{}$$

Mathematica [C] time = 0.0420644, size = 77, normalized size = 0.13

$$\frac{x\left(2x^3\sqrt{\frac{bx^3}{a}+1}(aB-4Ab)\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{bx^3}{a}\right)-10aA\right)}{5a^2(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-10*a*A + 2*(-4*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/
6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a^2*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Maple [C] time = 0.052, size = 5563, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}\sqrt{ex}}{b^2e^2x^8+2\,abe^2x^5+a^2e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*e^2*x^8 + 2*a*b*e^2*x^5
 + a^2*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

3.557 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))/
(3*a^2*e^4*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.029186, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))/
(3*a^2*e^4*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}} - \frac{(2Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{ae^3}$$
$$= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}} - \frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.0206157, size = 45, normalized size = 0.67

$$\frac{x\left(-2aA+2aBx^3-4Abx^3\right)}{3a^2(ex)^{5/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-2*a*A - 4*A*b*x^3 + 2*a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])
```

Maple [A] time = 0.006, size = 39, normalized size = 0.6

$$-\frac{2\,x\left(2\,Ax^3b-Bax^3+Aa\right)}{3\,a^2}\frac{1}{\sqrt{bx^3+a}}(ex)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/(e*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

Fricas [A] time = 1.31067, size = 117, normalized size = 1.75

$$\frac{2\left((Ba-2\,Ab)x^3-Aa\right)\sqrt{bx^3+a}\sqrt{ex}}{3\left(a^2be^3x^5+a^3e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b*e^3*x^5 + a^
3*e^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

3.558 $\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{ex}(8Ab-5aB)}{15a^2e^4\sqrt{a+bx^3}}$$

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x])/
(15*a^2*e^4*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(15*3^(1/4)*a^(7/3)*e
^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.222652, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 290, 329, 225}

$$-\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{ex}(8Ab-5aB)}{15a^2e^4\sqrt{a+bx^3}}-\frac{}{5ae(ex}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x])/
(15*a^2*e^4*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(15*3^(1/4)*a^(7/3)*e
^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{(8Ab-5aB)\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx}{5ae^3}$$

$$=-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}}-\frac{(2(8Ab-5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{15a^2e^3}$$

$$=-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}}-\frac{(4(8Ab-5aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{15a^2e^4}$$

$$=-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^2\cdots}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\cdots\right.}}\cdots}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{bx}\left(\sqrt[3]{a}\cdots\right.}{\left(\sqrt[3]{a}+\left(1+\cdots\right.\right.}}\cdots}$$

Mathematica [C] time = 0.0495363, size = 95, normalized size = 0.34

$$\frac{x\left(4x^3\sqrt{\frac{bx^3}{a}+1}(5aB-8Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-2\left(3aA-5aBx^3+8Abx^3\right)\right)}{15a^2(ex)^{7/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-2*(3*a*A + 8*A*b*x^3 - 5*a*B*x^3) + 4*(-8*A*b + 5*a*B)*x^3*Sqrt[1 + (b
*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(15*a^2*(e*x)^(7/
2)*Sqrt[a + b*x^3])
```

Maple [C] time = 0.046, size = 3783, normalized size = 13.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/15/x^2*(-20*I*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3
^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*3^(1/2)*x^4*a*b^2+5*I
*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1
/3)*3^(1/2)*x^3*a*b+32*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)
+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/
3)*3^(1/2)*x^2*b-64*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2)
)/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*
3^(1/2)*x^3*b^2+32*I*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))
/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*3^(1/2)*x^4*b^3-
32*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*
3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*x^4*b^3+40*I*B*(-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x
^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x^3*a*b+20*B*(-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+
a)*e*x)^(1/2)*x^4*a*b^2+64*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/
```

```
2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^
(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(
1/3)*x^3*b^2-40*B*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3
^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(1/3)*x^3*a*
b-32*A*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(
I*3^(1/2)-3))^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*x^2*b+20*B*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))
^(1/2))*((b*x^3+a)*e*x)^(1/2)*(-a*b^2)^(2/3)*x^2*a-8*I*A*(1/b^2*e*x*(-b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x^3*b^2-3
*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^
(1/3)*3^(1/2)*a*b+24*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
))^(1/2)*(-a*b^2)^(1/3)*x^3*b^2-15*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x^3*a*b-20*I*B*(-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(I*3^(1/2)+1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(-1+I*3^(1/2))/(I*3^(1/2)+1)/(I*3^(1/2)-3))^(1/2))*((b*x^3+
a)*e*x)^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*x^2*a+9*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a*b)/(b*x^3+a)^(1/2)/(-a*b^
2)^(1/3)/b/a^2/e^3/(e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}}{b^2e^4x^{10} + 2\,abe^4x^7 + a^2e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*e^4*x^10 + 2*a*b*e^4*x^
7 + a^2*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

3.559 $\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}} dx$

Optimal. Leaf size=114

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*B*e^2*(e*x)^(3
/2))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(
e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rubi [A] time = 0.0750009, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {452, 288, 329, 275, 217, 206}

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*B*e^2*(e*x)^(3
/2))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(
e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{B\int \frac{(ex)^{7/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{b}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{\left(Be^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{\left(2Be^2\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{\left(2Be^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{\left(2Be^2\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

<u>Mathematica</u> [A] time = 0.273049, size = 119, normalized size = 1.04

$$\frac{2e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\left(-3a^2B-4abBx^3+Ab^2x^3\right)+3a^{3/2}B\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{9ab^{5/2}\sqrt{x}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3) + 3*
a^(3/2)*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]
]))/(9*a*b^(5/2)*Sqrt[x]*(a + b*x^3)^(3/2))
```

Maple [C] time = 0.076, size = 7081, normalized size = 62.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{7}{2}}}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(5/2), x)
```

Fricas [A] time = 2.52442, size = 726, normalized size = 6.37

$$\left[\frac{3\left(Bab^2e^3x^6 + 2\,Ba^2be^3x^3 + Ba^3e^3\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3 + a}\sqrt{ex}\sqrt{\frac{e}{b}}\right) - 4\left(\left(4\,Bab - \right.\right.}{18\left(ab^4x^6 + 2\,a^2b^3x^3 + a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(e/b)*log(-8
*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sq
rt(e*x)*sqrt(e/b)) - 4*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x
^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2), -1/9*(3*(B*a*b^2*
e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a
```

```
)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*((4*B*a*b - A*b^2)*e^3*x^
4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 +
a^3*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.560 $\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=299

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2e^2\sqrt{ex}(7aB+2Ab}{27ab^2\sqrt{a+bx^3}}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(7/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*(2*A*b + 7*a*B
)*e^2*Sqrt[e*x])/(27*a*b^2*Sqrt[a + b*x^3]) + ((2*A*b + 7*a*B)*e^2*Sqrt[e*x
]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[
3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*
3^(1/4)*a^(4/3)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.228818, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 288, 329, 225}

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2e^2\sqrt{ex}(7aB+2Ab)}{27ab^2\sqrt{a+bx^3}}+\frac{2(ex}{9ab}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(7/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*(2*A*b + 7*a*B
)*e^2*Sqrt[e*x])/(27*a*b^2*Sqrt[a + b*x^3]) + ((2*A*b + 7*a*B)*e^2*Sqrt[e*x
]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[
3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*
3^(1/4)*a^(4/3)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
```

```
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(Ab+\frac{7aB}{2}\right)\right)\int \frac{(ex)^{5/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{\left((2Ab+7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{\left(2(2Ab+7aB)e^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{27ab^2}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}}}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\right.}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right.\right.}}}$$

Mathematica [C] time = 0.148311, size = 108, normalized size = 0.36

$$\frac{2e^2\sqrt{ex}\left(-7a^2B+\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(7aB+2Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-2ab\left(A+5Bx^3\right)+Ab^2x^3\right)}{27ab^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^2*Sqrt[e*x]*(-7*a^2*B + A*b^2*x^3 - 2*a*b*(A + 5*B*x^3) + (2*A*b + 7*a
*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x
^3)/a)]))/(27*a*b^2*(a + b*x^3)^(3/2))
```

Maple [C] time = 0.078, size = 7083, normalized size = 23.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(ex)^{\frac{5}{2}}}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Be^2x^5 + Ae^2x^2)\sqrt{bx^3 + a}\sqrt{ex}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*x^9 + 3*a*b
^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(ex)^{\frac{5}{2}}}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

3.561 $\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=596

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\left(1+\sqrt{3}\right)e}{27a^2b^{5/3}}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(4*A*b + 5*a*B
)*(e*x)^(5/2))/(27*a^2*b*e*Sqrt[a + b*x^3]) - (2*(1 + Sqrt[3])*(4*A*b + 5*a
*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) + (2*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(9*3^(3/4)*a^(5/3)*b^(5/3)*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + ((1 - Sqrt[3])*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(5/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.572681, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 290, 329, 308, 225, 1881}

$$-\frac{2\left(1+\sqrt{3}\right)e\sqrt{ex}\sqrt{a+bx^3}(5aB+4Ab)}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(4*A*b + 5*a*B
)*(e*x)^(5/2))/(27*a^2*b*e*Sqrt[a + b*x^3]) - (2*(1 + Sqrt[3])*(4*A*b + 5*a
*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) + (2*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(9*3^(3/4)*a^(5/3)*b^(5/3)*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + ((1 - Sqrt[3])*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(5/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
```

```
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}dx = \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(2Ab+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(2(4Ab+5aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}dx}{27a^2b}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(4(4Ab+5aB))\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}dx, x, \sqrt{ex}\right)}{27a^2be}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(4Ab+5aB))\,\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}dx, \right.}{27a^2b^{5/3}e}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{2\left(1+\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{2(4Ab}{}$$

<u>Mathematica</u> [C] time = 0.117283, size = 86, normalized size = 0.14

$$\frac{x(ex)^{3/2}\left(\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(5aB+4Ab)\,{}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{11}{6};-\frac{bx^3}{a}\right)-5a^2B\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(e*x)^(3/2)*(-5*a^2*B + (4*A*b + 5*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/
2))
```

Maple [C] time = 0.096, size = 10786, normalized size = 18.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^3 + A)(ex)^{\frac{3}{2}}}{(bx^3 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(Bex^4 + Aex)\sqrt{bx^3 + a}\sqrt{ex}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*x^9 + 3*a*b^2*x^6
 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{3}{2}}}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

3.562 $\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)*
(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0311775, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 264}

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)*
(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{3/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(3Ab+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$
$$= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.0371331, size = 44, normalized size = 0.56

$$\frac{2x\sqrt{ex}\left(3aA+aBx^3+2Abx^3\right)}{9a^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))
```

Maple [A] time = 0.006, size = 39, normalized size = 0.5

$$\frac{2\,x\left(2\,Ax^3b+Bax^3+3\,Aa\right)}{9\,a^2}\sqrt{ex}\left(bx^3+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] 2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^3+A\right)\sqrt{ex}}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(5/2), x)
```

Fricas [A] time = 1.36708, size = 128, normalized size = 1.62

$$\frac{2\left((Ba+2\,Ab)x^4+3\,Aax\right)\sqrt{bx^3+a}\sqrt{ex}}{9\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b^2*x^6 +
2*a^3*b*x^3 + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47911, size = 86, normalized size = 1.09

$$\frac{2\,x^{\frac{3}{2}}\left(\frac{3\,Ae^5}{a}+\frac{\left(Ba^5b^5e^{21}+2\,Aa^4b^6e^{21}\right)x^3e^{(-16)}}{a^6b^5}\right)e^{\frac{3}{2}}}{9\left(bx^3e^4+ae^4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*x^(3/2)*(3*A*e^5/a + (B*a^5*b^5*e^21 + 2*A*a^4*b^6*e^21)*x^3*e^(-16)/(a
^6*b^5))*e^(3/2)/(b*x^3*e^4 + a*e^4)^(3/2)
```

3.563 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=297

$$\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(aB+8Ab)}{27a^2be\sqrt{a+bx^3}}+\frac{2\cdots}{9a\cdots}$$

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(8*A*b + a*B)*Sq
rt[e*x])/(27*a^2*b*e*Sqrt[a + b*x^3]) + (2*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^
(7/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.222083, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 290, 329, 225}

$$\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(aB+8Ab)}{27a^2be\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(Ab-\cdots}{9abe\left(a+b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```


```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(8*A*b + a*B)*Sq
rt[e*x])/(27*a^2*b*e*Sqrt[a + b*x^3]) + (2*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^
(7/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(4Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(8Ab+aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(4(8Ab+aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{27a^2be}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos\right.}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{\ }}$$

Mathematica [C] time = 0.093222, size = 107, normalized size = 0.36

$$\frac{2x\left(-2a^2B+2\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(aB+8Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+ab\left(11A+Bx^3\right)+8Ab^2x^3\right)}{27a^2b\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-2*a^2*B + 8*A*b^2*x^3 + a*b*(11*A + B*x^3) + 2*(8*A*b + a*B)*(a + b*
x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(
27*a^2*b*Sqrt[e*x]*(a + b*x^3)^(3/2))
```

Maple [C] time = 0.059, size = 7077, normalized size = 23.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} \sqrt{ex}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(Bx^3 + A)\sqrt{bx^3 + a}\sqrt{ex}}{b^3ex^{10} + 3\,ab^2ex^7 + 3\,a^2bex^4 + a^3ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*e*x^10 + 3*a*b^2*e*x^7
+ 3*a^2*b*e*x^4 + a^3*e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2)/(e*x)**(1/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} \sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

3.564 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=624

$$-\frac{4\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{8\left(1+\sqrt{\ldots}\right.}{27a^3b^{2\ldots}}$$

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^3)^(3/2)) - (2*(10*A*b - a*B)*(e*x)^(5/2))/(
9*a^2*e^4*(a + b*x^3)^(3/2)) - (8*(10*A*b - a*B)*(e*x)^(5/2))/(27*a^3*e^4*S
qrt[a + b*x^3]) + (8*(1 + Sqrt[3])*(10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(27*a^3*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (8*(10*A*b - a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(9*3^(3/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x
))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*(1 - Sqrt[3
])*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.6386, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 329, 308, 225, 1881}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(10Ab-aB)}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{4\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x\ldots}\right.\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^3)^(3/2)) - (2*(10*A*b - a*B)*(e*x)^(5/2))/(
9*a^2*e^4*(a + b*x^3)^(3/2)) - (8*(10*A*b - a*B)*(e*x)^(5/2))/(27*a^3*e^4*S
qrt[a + b*x^3]) + (8*(1 + Sqrt[3])*(10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(27*a^3*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (8*(10*A*b - a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(9*3^(3/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x
))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*(1 - Sqrt[3
])*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
```

```
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{(10Ab-aB)\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{5/2}}\,dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(4(10Ab-aB))\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{(8(10Ab-aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx}}}{27a^3e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{(16(10Ab-aB))\,\mathrm{Subst}}{27a}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} - \frac{(8(10Ab-aB))\,\mathrm{Subst}\left(\right.}{2}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{8\left(1+\sqrt{3}\right)(10Ab-aB)}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1-\right.\right.}$$

Mathematica [C] time = 0.0423889, size = 85, normalized size = 0.14

$$\frac{2x\left(x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(aB-10Ab)\,{}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{11}{6};-\frac{bx^3}{a}\right)-5a^2A\right)}{5a^3(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-5*a^2*A + (-10*A*b + a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(5*a^3*(e*x)^(3/2)*(a + b*x^3)^
(3/2))
```

Maple [C] time = 0.085, size = 10961, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}}{b^3e^2x^{11} + 3\,ab^2e^2x^8 + 3\,a^2be^2x^5 + a^3e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*e^2*x^11 + 3*a*b^2*e^2*
x^8 + 3*a^2*b*e^2*x^5 + a^3*e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

3.565 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - (2*(4*A*b - a*B)*(e*x)^(3/2)
)/(9*a^2*e^4*(a + b*x^3)^(3/2)) - (4*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^3*e^4*
Sqrt[a + b*x^3])
```

Rubi [A] time = 0.0464514, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - (2*(4*A*b - a*B)*(e*x)^(3/2)
)/(9*a^2*e^4*(a + b*x^3)^(3/2)) - (4*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^3*e^4*
Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx &= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{(4Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{5/2}}\,dx}{ae^3}\\ &= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{(2(4Ab-aB))\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{3a^2e^3}\\ &= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.0367259, size = 65, normalized size = 0.62

$$\frac{x\left(-6a^2\left(A-Bx^3\right)+4abx^3\left(Bx^3-6A\right)-16Ab^2x^6\right)}{9a^3(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (x*(-16*A*b^2*x^6 - 6*a^2*(A - B*x^3) + 4*a*b*x^3*(-6*A + B*x^3)))/(9*a^3*(
e*x)^(5/2)*(a + b*x^3)^(3/2))
```

Maple [A] time = 0.006, size = 62, normalized size = 0.6

$$-\frac{2\,x\left(8\,Ab^2x^6-2\,Bx^6ab+12\,aAbx^3-3\,Bx^3a^2+3\,Aa^2\right)}{9\,a^3}\left(bx^3+a\right)^{-\frac{3}{2}}(ex)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9*x*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(b*x^3+a)
^(3/2)/a^3/(e*x)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

Fricas [A] time = 1.32226, size = 193, normalized size = 1.86

$$\frac{2\left(2\left(Bab - 4\,Ab^2\right)x^6 + 3\left(Ba^2 - 4\,Aab\right)x^3 - 3\,Aa^2\right)\sqrt{bx^3 + a}\sqrt{ex}}{9\left(a^3b^2e^3x^8 + 2\,a^4be^3x^5 + a^5e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x^
3 + a)*sqrt(e*x)/(a^3*b^2*e^3*x^8 + 2*a^4*b*e^3*x^5 + a^5*e^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

3.566 $\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{16\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{16\sqrt{ex}(14Ab-5}{135a^3e^4\sqrt{a+b}}$$

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*(a + b*x^3)^(3/2)) - (2*(14*A*b - 5*a*B)*Sqrt[e*x
])/(45*a^2*e^4*(a + b*x^3)^(3/2)) - (16*(14*A*b - 5*a*B)*Sqrt[e*x])/(135*a^
3*e^4*Sqrt[a + b*x^3]) - (16*(14*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(135*3^(1/4)*a^(10/3)*e^4
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rubi [A] time = 0.257692, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 290, 329, 225}

$$-\frac{16\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{16\sqrt{ex}(14Ab-5aB)}{135a^3e^4\sqrt{a+bx^3}}-\frac{2}{45}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```


```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*(a + b*x^3)^(3/2)) - (2*(14*A*b - 5*a*B)*Sqrt[e*x
])/(45*a^2*e^4*(a + b*x^3)^(3/2)) - (16*(14*A*b - 5*a*B)*Sqrt[e*x])/(135*a^
3*e^4*Sqrt[a + b*x^3]) - (16*(14*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(135*3^(1/4)*a^(10/3)*e^4
```

```
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{(14Ab-5aB)\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx}{5ae^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(8(14Ab-5aB))\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx}{45a^2e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{(16(14Ab-5aB))\int}{135a^3e}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{(32(14Ab-5aB))\,S}{}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{16(14Ab-5aB)\sqrt{e}}{}$$

Mathematica [C] time = 0.0750694, size = 121, normalized size = 0.38

$$\frac{x\left(a^2\left(110Bx^3-54A\right)+32x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}(5aB-14Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+a\left(80bBx^6-308Abx^3\right)-224Ab^2x^6\right)}{135a^3(ex)^{7/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (x*(-224*A*b^2*x^6 + a^2*(-54*A + 110*B*x^3) + a*(-308*A*b*x^3 + 80*b*B*x^6
) + 32*(-14*A*b + 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(135*a^3*(e*x)^(7/2)*(a + b*x^3)^(3/2))
```

Maple [C] time = 0.066, size = 7299, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}\sqrt{ex}}{b^3e^4x^{13} + 3\,ab^2e^4x^{10} + 3\,a^2be^4x^7 + a^3e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*e^4*x^13 + 3*a*b^2*e^4*
x^10 + 3*a^2*b*e^4*x^7 + a^3*e^4*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

3.567 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=220

$$-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}+\frac{\sqrt[3]{2}a^{10/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}+$$

```
[Out] -((a^3*(a + b*x^3)^(1/3))/(b^4*d)) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) + (a
*(a + b*x^3)^(7/3))/(7*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) + (2^(1/3)*a^
(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(10/3)*Log[a - b*x^3])/(3*2^(2/3)*b^4*d) - (a^(10/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4*d)
```

Rubi [A] time = 0.235602, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}+\frac{\sqrt[3]{2}a^{10/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^3*(a + b*x^3)^(1/3))/(b^4*d)) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) + (a
*(a + b*x^3)^(7/3))/(7*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) + (2^(1/3)*a^
(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(10/3)*Log[a - b*x^3])/(3*2^(2/3)*b^4*d) - (a^(10/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2\sqrt[3]{a+bx}}{b^3d} + \frac{a(a+bx)^{4/3}}{b^3d} - \frac{(a+bx)^{7/3}}{b^3d} + \frac{a^3\sqrt[3]{a+bx}}{b^3(ad-bdx)}\right)dx, x, x^3\right)$$

$$= -\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{a^3\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{\left(2a^4\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,d\right.}{3b^3}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d} + \frac{a^{10/3}\operatorname{Subst}}{}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d} - \frac{a^{10/3}\log\left(\sqrt[3]{}\right.}{}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{\sqrt[3]{2}a^{10/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^4d} + \frac{a}{}$$

Mathematica [A] time = 0.196546, size = 230, normalized size = 1.05

$$-\frac{111a^2bx^3\sqrt[3]{a+bx^3} + 507a^3\sqrt[3]{a+bx^3} + 140\sqrt[3]{2}a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) - 70\sqrt[3]{2}a^{10/3}\log\left(2^{2/3}a^{2/3} + \sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}\right.}{420b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(507*a^3*(a + b*x^3)^(1/3) + 111*a^2*b*x^3*(a + b*x^3)^(1/3) + 66*a*b^2*x^
6*(a + b*x^3)^(1/3) + 42*b^3*x^9*(a + b*x^3)^(1/3) - 140*2^(1/3)*Sqrt[3]*a^
(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 140*2^(1
/3)*a^(10/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 70*2^(1/3)*a^(10/3)
*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)])/(420*b^4*d)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34805, size = 520, normalized size = 2.36

$$-\frac{140\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+70\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)\right.}{420b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(2^
(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(1/3))
/(b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.568 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=174

$$-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}+\frac{\sqrt[3]{2}a^{7/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}$$

```
[Out] -((a^2*(a + b*x^3)^(1/3))/(b^3*d)) - (a + b*x^3)^(7/3)/(7*b^3*d) + (2^(1/3)
*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(7/3)*Log[a - b*x^3])/(3*2^(2/3)*b^3*d) - (a^(7/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3*d)
```

Rubi [A] time = 0.17849, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}+\frac{\sqrt[3]{2}a^{7/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^2*(a + b*x^3)^(1/3))/(b^3*d)) - (a + b*x^3)^(7/3)/(7*b^3*d) + (2^(1/3)
*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(7/3)*Log[a - b*x^3])/(3*2^(2/3)*b^3*d) - (a^(7/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{(a+bx)^{4/3}}{b^2d} + \frac{a^2\sqrt[3]{a+bx}}{b^2(ad-bdx)}\right)dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{a^2\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^2}$$
$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b^2}$$
$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d} + \frac{a^{7/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d} + \frac{a^{8/3}\ \mathrm{S}}{}$$
$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d} - \frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d} - \frac{\left(\sqrt[3]{2}a^{7/3}\right)\operatorname{Subst}}{}$$
$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{\sqrt[3]{2}a^{7/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^3d} + \frac{a^{7/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^3d} - \frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}\right.}{2^{2/3}b^3}$$

Mathematica [A] time = 0.124325, size = 207, normalized size = 1.19

$$\frac{1}{3}\left(-\frac{3a^2\sqrt[3]{a+bx^3}}{b^3d} - \frac{\frac{2\sqrt[3]{2}a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{bd} - \frac{\sqrt[3]{2}a^{7/3}\left(\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\right)}{bd}}{2b^2} - \frac{3\left(a+\right.}{7b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-3*a^2*(a + b*x^3)^(1/3))/(b^3*d) - (3*(a + b*x^3)^(7/3))/(7*b^3*d) - ((2
*2^(1/3)*a^(7/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(b*d) - (2^(1/3)
*a^(7/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a
^(1/3))] + Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)]))/(b*d))/(2*b^2))/3
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^8}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33823, size = 483, normalized size = 2.78

$$-\frac{14\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+7\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-}{42b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/42*(14*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^2*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3
 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 7*2^(1/3)*(-a)^(1/3)*a^2*log(2^(2/
3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) -
 14*2^(1/3)*(-a)^(1/3)*a^2*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*
(b^2*x^6 + 2*a*b*x^3 + 8*a^2)*(b*x^3 + a)^(1/3))/(b^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^8 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.569 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=172

$$\frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}+\frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}$$

```
[Out] -((a*(a + b*x^3)^(1/3))/(b^2*d)) - (a + b*x^3)^(4/3)/(4*b^2*d) + (2^(1/3)*a
^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(4/3)*Log[a - b*x^3])/(3*2^(2/3)*b^2*d) - (a^(4/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^2*d)
```

Rubi [A] time = 0.148568, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 80, 50, 57, 617, 204, 31}

$$\frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}+\frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(b^2*d)) - (a + b*x^3)^(4/3)/(4*b^2*d) + (2^(1/3)*a
^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(4/3)*Log[a - b*x^3])/(3*2^(2/3)*b^2*d) - (a^(4/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^2*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d} + \frac{a^{4/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} + \frac{a^{5/3}\operatorname{Sul}}{}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} - \frac{\left(\sqrt[3]{2}a^{4/3}\right)\operatorname{Subst}\Big(}{}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d} - \frac{a^{4/3}\log\Big(\sqrt[3]{2}\sqrt[3]{a}-}{2^{2/3}b^2d}$$

Mathematica [A] time = 0.0967885, size = 186, normalized size = 1.08

$$-\frac{4\sqrt[3]{2}a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) - 2\sqrt[3]{2}a^{4/3}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right) - 4\sqrt[3]{2}\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt[3]*a^
(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*2^(1/3)
*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 2*2^(1/3)*a^(4/3)*Log[2
^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(1
2*b^2*d)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{x^5}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.26959, size = 451, normalized size = 2.62

$$-\frac{4\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-4}{12b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*2^(1/3)*(-a)^(1/3)*a*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 +
a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2*2^(1/3)*(-a)^(1/3)*a*log(2^(2/3)*(-
a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 4*2^
(1/3)*(-a)^(1/3)*a*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 3*(b*x^3 +
 5*a)*(b*x^3 + a)^(1/3))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^5 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.570 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd}$$

```
[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(1/3)*Log[a - b*x^3]
)/(3*2^(2/3)*b*d) - (a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
2/3)*b*d)
```

Rubi [A] time = 0.118523, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {444, 50, 57, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(1/3)*Log[a - b*x^3]
)/(3*2^(2/3)*b*d) - (a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
2/3)*b*d)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{1}{3}(2a)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd} + \frac{\sqrt[3]{a}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} + \frac{a^{2/3}\operatorname{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{}}\right.}{\sqrt[3]{}}$$
$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} - \frac{\left(\sqrt[3]{2}\sqrt[3]{a}\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.}{bd}$$
$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

Mathematica [A] time = 0.047603, size = 167, normalized size = 1.11

$$\frac{\sqrt[3]{2}\sqrt[3]{a}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-6\sqrt[3]{a+bx^3}-2\sqrt[3]{2}\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{a}\tan}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a +
 b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[2^(1/3)*a^(1/3) -
(a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*b*d)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2629, size = 417, normalized size = 2.78

$$-\frac{2\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}(-}{6bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^
(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2/3
) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3)*(
-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(1/3)
)/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{x^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.571 $\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=214

$$\frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d}-$$

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*d)) + (2^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(2/3)*d) - Log[x]/(2*a^(2/3)*d) + Log[a - b*x^3]/(3*2^(2/
3)*a^(2/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*d) - Log[2^(1/3
)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)*d)
```

Rubi [A] time = 0.173803, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {446, 83, 57, 617, 204, 31}

$$\frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*d)) + (2^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(2/3)*d) - Log[x]/(2*a^(2/3)*d) + Log[a - b*x^3]/(3*2^(2/
3)*a^(2/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*d) - Log[2^(1/3
)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(ad-bdx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right) + \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d} + \frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{a^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d} - \frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}$$

Mathematica [A] time = 0.0979623, size = 233, normalized size = 1.09

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-\sqrt[3]{2}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+}{6a^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)
*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[
a^(1/3) - (a + b*x^3)^(1/3)] + 2*2^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^
(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 2^(
1/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)])/(6*a^(2/3)*d)
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-bdx^3+ad\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x), x)
```

Fricas [A] time = 1.3914, size = 809, normalized size = 3.78

$$-\frac{2\sqrt{3}2^{\frac{1}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+2^{\frac{1}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*a^2*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/3)*(b*x^
3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 2^(1/3)*a^2*(-1/a^2)^(1/3)*l
og(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(1/3)
+ (b*x^3 + a)^(2/3)) - 2*2^(1/3)*a^2*(-1/a^2)^(1/3)*log(2^(1/3)*a*(-1/a^2)^
(1/3) + (b*x^3 + a)^(1/3)) + 2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(a^2)^(1/6)
*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + (
a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2
)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax+bx^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.572 $\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=268

$$-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}+\frac{b\sqrt[3]{a+bx^3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+\cdots}}{\sqrt{\cdots}}\right.}{3\sqrt{3}a^{5/3}\cdots}$$

```
[Out] (b*(a + b*x^3)^(1/3))/(3*a^2*d) - (a + b*x^3)^(4/3)/(3*a^2*d*x^3) - (4*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*d) + (2^(1/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(5/3)*d) - (2*b*Log[x])/(3*a^(5/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(2/3)*a^(5/3)*d) + (2*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(5/3)*d)
```

Rubi [A] time = 0.247268, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 156, 50, 57, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}+\frac{b\sqrt[3]{a+bx^3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+\cdots}}{\sqrt{\cdots}}\right.}{3\sqrt{3}a^{5/3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/(3*a^2*d) - (a + b*x^3)^(4/3)/(3*a^2*d*x^3) - (4*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*d) + (2^(1/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(5/3)*d) - (2*b*Log[x])/(3*a^(5/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(2/3)*a^(5/3)*d) + (2*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(5/3)*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd+\frac{1}{3}b^2dx\right)}{x(ad-bdx)}\,dx, x, x^3\right)}{3a^2d}$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} + \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3a^2} + \frac{(4b)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9a^2d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\right.}{9ad}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{5/3}d} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{5/3}d} + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d} - \frac{b\log\left(\sqrt[3]{2}\right.}{2}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}d} + \frac{\sqrt[3]{2}b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\,\mathrm{l}}{3}$$

Mathematica [A] time = 0.108945, size = 280, normalized size = 1.04

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^3} + 4bx^3\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right) - 3\sqrt[3]{2}bx^3\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] -(6*a^(2/3)*(a + b*x^3)^(1/3) + 8*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^
(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*(a
+ b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 8*b*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)
] + 6*2^(1/3)*b*x^3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] + 4*b*x^3*Log[
a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 3*2^(1/3)*b*x^3*
Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/(18*a^(5/3)*d*x^3)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-bdx^3+ad\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)
```

Fricas [A] time = 1.39113, size = 905, normalized size = 3.38

$$-\frac{6\sqrt{3}2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+3\cdot 2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 3*2^(1/3)*a^2*b*x^3*(
-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*a*
(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 6*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*lo
g(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 8*sqrt(3)*(a^2)^(1/6)*a*b
*x^3*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^
(1/3)*(a^2)^(2/3))/a^2) + 4*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a + (a^
2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 8*(a^2)^(2/3)*b*x^3*log((b*x^
3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2)/(a^3*d*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^4+bx^7}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.573 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

Optimal. Leaf size=283

$$\frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}-\frac{11b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}d}+\frac{\sqrt[3]{2}b^2\tan^{-1}\left(\frac{2^2}{}\right.}{\sqrt{3}a^8}$$

```
[Out] (-2*b*(a + b*x^3)^(1/3))/(9*a^2*d*x^3) - (a + b*x^3)^(4/3)/(6*a^2*d*x^6) -
(11*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(8/3)*d) + (2^(1/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*d) - (11*b^2*Log[x])/(18*a^(8/3)*d) + (
b^2*Log[a - b*x^3])/(3*2^(2/3)*a^(8/3)*d) + (11*b^2*Log[a^(1/3) - (a + b*x^
3)^(1/3)])/(18*a^(8/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/
(2^(2/3)*a^(8/3)*d)
```

Rubi [A] time = 0.264317, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 149, 156, 57, 617, 204, 31}

$$\frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}-\frac{11b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}d}+\frac{\sqrt[3]{2}b^2\tan^{-1}\left(\frac{2^2}{}\right.}{\sqrt{3}a^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```


```
[Out] (-2*b*(a + b*x^3)^(1/3))/(9*a^2*d*x^3) - (a + b*x^3)^(4/3)/(6*a^2*d*x^6) -
(11*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(8/3)*d) + (2^(1/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*d) - (11*b^2*Log[x])/(18*a^(8/3)*d) + (
b^2*Log[a - b*x^3])/(3*2^(2/3)*a^(8/3)*d) + (11*b^2*Log[a^(1/3) - (a + b*x^
3)^(1/3)])/(18*a^(8/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/
(2^(2/3)*a^(8/3)*d)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(ad-bdx)}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd-\frac{2}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx,x,x^3\right)}{6a^2d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{22}{9}a^2b^2d^2-\frac{14}{9}ab^3d^2x}{x(a+bx)^{2/3}(ad-bdx)}\,dx,x,x^3\right)}{6a^3d^2}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx,x,x^3\right)}{3a^2} + \frac{\left(11b^2\right)\text{Subst}\left(\int \frac{1}{x(a+bx}\right.}{27a^2d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d} - \frac{\left(11b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a}\right.}{18a^{8/3}d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d} + \frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d} - \frac{b^2\,l}{}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{8/3}d} + \frac{\sqrt[3]{2}b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{8/3}d} - \frac{11b^2\log(x)}{18a^{8/3}d}$$

Mathematica [A] time = 0.128286, size = 314, normalized size = 1.11

$$-\frac{11b^2x^6\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-9\sqrt[3]{2}b^2x^6\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+21a^{2/3}bx^3\sqrt[3]{a}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] -(9*a^(5/3)*(a + b*x^3)^(1/3) + 21*a^(2/3)*b*x^3*(a + b*x^3)^(1/3) + 22*Sqr
t[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*2^(1/
3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]
] - 22*b^2*x^6*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 18*2^(1/3)*b^2*x^6*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] + 11*b^2*x^6*Log[a^(2/3) + a^(1/3)*(a +
b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*b^2*x^6*Log[2^(2/3)*a^(2/3) +
 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a^(8/3)*d*x^6)
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^{7} \left(-b d \,x^{3}+a d \right)}\sqrt[3]{b \,x^{3}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b x^{3}+a\right)^{\frac{1}{3}}}{\left(b d x^{3}-a d\right) x^{7}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^7), x)
```

Fricas [A] time = 1.38086, size = 952, normalized size = 3.36

$$-\frac{18\sqrt{3}2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+9\cdot 2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(
2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 9*2^(1/3)*a^2*b^2*
x^6*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/
3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 18*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^
(1/3)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 22*sqrt(3)*(a^2)^
(1/6)*a*b^2*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + 11*(a^2)^(2/3)*b^2*x^6*log((b*x^3 + a)
^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 22*(a^2)^(2/3)*
b^2*x^6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x^3 + 3*a^3)*(b
*x^3 + a)^(1/3))/(a^4*d*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.574 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=268

$$\frac{a^2 \log \left(ad - bdx^3\right)}{3\ 2^{2/3} b^{8/3} d} + \frac{11 a^2 \log \left(\sqrt[3]{b} x - \sqrt[3]{a + bx^3}\right)}{18 b^{8/3} d} - \frac{a^2 \log \left(\sqrt[3]{2} \sqrt[3]{b} x - \sqrt[3]{a + bx^3}\right)}{2^{2/3} b^{8/3} d} + \frac{11 a^2 \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9 \sqrt{3} b^{8/3} d} - \frac{\sqrt[3]{2} a^2 \tan^{-1}\left(\right.}{\sqrt{3} b^{8}}$$

```
[Out] (-7*a*x^2*(a + b*x^3)^(1/3))/(18*b^2*d) - (x^5*(a + b*x^3)^(1/3))/(6*b*d) +
 (11*a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*
b^(8/3)*d) - (2^(1/3)*a^2*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/
3))/Sqrt[3]])/(Sqrt[3]*b^(8/3)*d) + (a^2*Log[a*d - b*d*x^3])/(3*2^(2/3)*b^(
8/3)*d) + (11*a^2*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^(8/3)*d) - (a^2
*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3)*b^(8/3)*d)
```

Rubi [C] time = 0.0596524, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^8 \sqrt[3]{a + bx^3} F_1\left(\frac{8}{3}; -\frac{1}{3}, 1; \frac{11}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{8ad \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), (b*x^3)/a
])/(8*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^7 \sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^8 \sqrt[3]{a+bx^3} F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.191141, size = 177, normalized size = 0.66

$$\frac{22abx^5\left(1-\frac{b^2x^6}{a^2}\right)^{2/3} F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5x^2\left(\left(1-\frac{bx^3}{a}\right)^{2/3}\left(7a^2+10abx^3+3b^2x^6\right)-7a^2\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\right.\right.}{90b^2d\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (22*a*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/
a), (b*x^3)/a] - 5*x^2*((1 - (b*x^3)/a)^(2/3)*(7*a^2 + 10*a*b*x^3 + 3*b^2*x
^6) - 7*a^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^
3)/(a - b*x^3)]))/(90*b^2*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{x^7}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^7}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

Fricas [A] time = 1.42261, size = 980, normalized size = 3.66

$$-\frac{18\sqrt{3}2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-18\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+9\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 18*2^(1/3)*a^2*b^2*(
-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 9*2
^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)
*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 22*sqrt(3
)*a^2*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 22*a^2*(b^2)^(2/3)*log(-((b^
2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 11*a^2*(b^2)^(2/3)*log(((b^2)^(1/3)*
b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*
b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^7 \sqrt[3]{a+bx^3}}{-a+bx^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}} x^7}{bdx^3-ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

3.575 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$

Optimal. Leaf size=233

$$\frac{a \log \left(ad-bdx^3\right)}{3\ 2^{2/3} b^{5/3} d}+\frac{2 a \log \left(\sqrt[3]{b} x-\sqrt[3]{a+bx^3}\right)}{3 b^{5/3} d}-\frac{a \log \left(\sqrt[3]{2} \sqrt[3]{b} x-\sqrt[3]{a+bx^3}\right)}{2^{2/3} b^{5/3} d}+\frac{4 a \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3 \sqrt{3} b^{5/3} d}-\frac{\sqrt[3]{2} a \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right.}{\sqrt{3} b^{5/3} d}$$

```
[Out] -(x^2*(a + b*x^3)^(1/3))/(3*b*d) + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^
3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d) - (2^(1/3)*a*ArcTan[(1 + (2*2^(1/
3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(5/3)*d) + (a*Log[a*d
 - b*d*x^3])/(3*2^(2/3)*b^(5/3)*d) + (2*a*Log[b^(1/3)*x - (a + b*x^3)^(1/3)
])/(3*b^(5/3)*d) - (a*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3)*
b^(5/3)*d)
```

Rubi [C] time = 0.0625171, antiderivative size = 66, normalized size of antiderivative = 0.28, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^5 \sqrt[3]{a+bx^3} F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]
)/(5*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x^5\sqrt[3]{a+bx^3}F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.149134, size = 160, normalized size = 0.69

$$\frac{4bx^5\left(1-\frac{b^2x^6}{a^2}\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5x^2\left(\left(a+bx^3\right)\left(1-\frac{bx^3}{a}\right)^{2/3}-a\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)\right)}{15bd\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (4*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a),
 (b*x^3)/a] - 5*x^2*((a + b*x^3)*(1 - (b*x^3)/a)^(2/3) - a*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a - b*x^3)]))/(15*b*d*(a
 + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^4}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

Fricas [A] time = 1.45501, size = 933, normalized size = 4.

$$-\frac{6\sqrt{3}2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-6\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+3\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(
b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 6*2^(1/3)*a*b^2*(-1/b^2
)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 3*2^(1/3)
*a*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3
+ a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a)^(1/
3)*b^2*x^2 + 8*sqrt(3)*a*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x
+ 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 8*a*(b^2)
^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 4*a*(b^2)^(2/3)*log(
((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)
/x^2))/(b^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^4 \sqrt[3]{a+bx^3}}{-a+bx^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{bdx^3-ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

3.576 $\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=201

$$\frac{\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}-\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}$$

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d) -
 (2^(1/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(S
qrt[3]*b^(2/3)*d) + Log[a*d - b*d*x^3]/(3*2^(2/3)*b^(2/3)*d) + Log[b^(1/3)*
x - (a + b*x^3)^(1/3)]/(2*b^(2/3)*d) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^
(1/3)]/(2^(2/3)*b^(2/3)*d)
```

Rubi [C] time = 0.042767, antiderivative size = 66, normalized size of antiderivative = 0.33, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a]
)/(2*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.0380564, size = 63, normalized size = 0.31

$$\frac{x^2\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3
)/a])/(2*d*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

[Out] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)

Fricas [B] time = 1.3942, size = 869, normalized size = 4.32

$$-\frac{2\sqrt{3}2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-2\cdot 2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}b^2\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*2^(1/3)*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 2*2^(1/3)*b^2*(-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 2^(1/3)*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 2*sqrt(3)*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3 + a\right)^{\frac{1}{3}} x}{bdx^3 - ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

3.577 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt[3]{a+bx^3}}{adx}+\frac{\sqrt[3]{b}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}ad}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}-\frac{\sqrt[3]{2}\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}ad}$$

```
[Out] -((a + b*x^3)^(1/3)/(a*d*x)) - (2^(1/3)*b^(1/3)*ArcTan[(1 + (2*2^(1/3)*b^(1
/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*d) + (b^(1/3)*Log[a*d - b*d*
x^3])/(3*2^(2/3)*a*d) - (b^(1/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]
)/(2^(2/3)*a*d)
```

Rubi [C] time = 0.0700063, antiderivative size = 77, normalized size of antiderivative = 0.49, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1-\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)}{adx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 - (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, (-2*b*x^3)/(a - b*x^3)])/(a*d*x*(1 + (b*x^3)/a)^(1/3)))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1-\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)}{adx\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.0166263, size = 45, normalized size = 0.29

$$-\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{2bx^3}{bx^3+a}\right)}{adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1, 2/3, (2*b*x^3)/(a + b*x^3)]
)/(a*d*x))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-bdx^3+ad\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

3.578 $\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=183

$$\frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}-\frac{\sqrt[3]{2}b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^2d}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

```
[Out] -(a + b*x^3)^(1/3)/(4*a*d*x^4) - (5*b*(a + b*x^3)^(1/3))/(4*a^2*d*x) - (2^(
1/3)*b^(4/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])
/(Sqrt[3]*a^2*d) + (b^(4/3)*Log[a*d - b*d*x^3])/(3*2^(2/3)*a^2*d) - (b^(4/3
)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3)*a^2*d)
```

Rubi [C] time = 0.422017, antiderivative size = 117, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{a^2-bx^3\left(a+3bx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+3bx^3\left(a-bx^3\right)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+4abx^3+3b^2x^6}{4a^2dx^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -(a^2 + 4*a*b*x^3 + 3*b^2*x^6 - b*x^3*(a + 3*b*x^3)*Hypergeometric2F1[2/3,
1, 5/3, (2*b*x^3)/(a + b*x^3)] + 3*b*x^3*(a - b*x^3)*Hypergeometric2F1[2/3,
 2, 5/3, (2*b*x^3)/(a + b*x^3)])/(4*a^2*d*x^4*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{a^2+4abx^3+3b^2x^6-bx^3\left(a+3bx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)+3bx^3\left(a-bx^3\right)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)}{4a^2dx^4\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 4.85741, size = 112, normalized size = 0.61

$$-\frac{\left(a+bx^3\right)^{4/3}\left(\left(a^2-4abx^3+3b^2x^6\right)\,{}_2F_1\left(1,1;\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)+9bx^3\left(a+bx^3\right)\,{}_2F_1\left(2,2;\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)\right)}{4a^2dx^4\left(a-bx^3\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(4/3)*((a^2 - 4*a*b*x^3 + 3*b^2*x^6)*Hypergeometric2F1[1, 1,
2/3, (-2*b*x^3)/(a - b*x^3)] + 9*b*x^3*(a + b*x^3)*Hypergeometric2F1[2, 2,
5/3, (-2*b*x^3)/(a - b*x^3)]))/(4*a^2*d*x^4*(a - b*x^3)^2)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(-bdx^3+ad\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{\left(bdx^3 - ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^5+bx^8}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

3.579 $\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=210

$$-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}+\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}-\frac{\sqrt[3]{2}b^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^3d}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{\sqrt[3]{a+}}{7ad}$$

```
[Out] -(a + b*x^3)^(1/3)/(7*a*d*x^7) - (2*b*(a + b*x^3)^(1/3))/(7*a^2*d*x^4) - (8
*b^2*(a + b*x^3)^(1/3))/(7*a^3*d*x) - (2^(1/3)*b^(7/3)*ArcTan[(1 + (2*2^(1/
3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^3*d) + (b^(7/3)*Log[a
*d - b*d*x^3])/(3*2^(2/3)*a^3*d) - (b^(7/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*
x^3)^(1/3)])/(2^(2/3)*a^3*d)
```

Rubi [C] time = 19.7802, antiderivative size = 244, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-9bx^3\left(a-bx^3\right)^2\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-2bx^3\left(2a^2+3abx^3+9b^2x^6\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+15a^2bx^3\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)}{28a^3dx^7\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] -(4*a^3 + 10*a^2*b*x^3 + 24*a*b^2*x^6 + 18*b^3*x^9 - 2*b*x^3*(2*a^2 + 3*a*b
*x^3 + 9*b^2*x^6)*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] + 1
5*a^2*b*x^3*Hypergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 12*a*b^
2*x^6*Hypergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 27*b^3*x^9*Hy
pergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 9*b*x^3*(a - b*x^3)^2
*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, (2*b*x^3)/(a + b*x^3)])/(28*a^3*d
*x^7*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{4a^3+10a^2bx^3+24ab^2x^6+18b^3x^9-2bx^3\left(2a^2+3abx^3+9b^2x^6\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)+15a^2bx}{}$$

Mathematica [C] time = 5.10644, size = 135, normalized size = 0.64

$$\frac{7b^3x^9\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)-\left(1-\frac{bx^3}{a}\right)^{2/3}\left(3a^2bx^3+a^3+10ab^2x^6+8b^3x^9\right)}{7a^3dx^7\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] (-((1 - (b*x^3)/a)^(2/3)*(a^3 + 3*a^2*b*x^3 + 10*a*b^2*x^6 + 8*b^3*x^9)) +
7*b^3*x^9*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)
/(a - b*x^3)])/(7*a^3*d*x^7*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^8\left(-bdx^3+ad\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{\left(bdx^3 - ad\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

3.580 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=237

$$-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}+\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}-\frac{\sqrt[3]{2}b^{10/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^4d}$$

```
[Out] -(a + b*x^3)^(1/3)/(10*a*d*x^10) - (11*b*(a + b*x^3)^(1/3))/(70*a^2*d*x^7)
- (37*b^2*(a + b*x^3)^(1/3))/(140*a^3*d*x^4) - (169*b^3*(a + b*x^3)^(1/3))/
(140*a^4*d*x) - (2^(1/3)*b^(10/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^4*d) + (b^(10/3)*Log[a*d - b*d*x^3])/(3*2^
(2/3)*a^4*d) - (b^(10/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/
3)*a^4*d)
```

Rubi [C] time = 29.209, antiderivative size = 423, normalized size of antiderivative = 1.78, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-54bx^3\left(a-bx^3\right)^2\left(2a+3bx^3\right)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+27bx^3\left(a-bx^3\right)^3\,{}_4F_3\left(\frac{2}{3},2,2,2;1,1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-36a^2b^2x^6\,{}_2F_1\left(\frac{2}{3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] -(28*a^4 + 64*a^3*b*x^3 + 90*a^2*b^2*x^6 + 216*a*b^3*x^9 + 162*b^4*x^12 - 2
8*a^3*b*x^3*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 36*a^2*
b^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 54*a*b^3*x^
9*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 162*b^4*x^12*Hype
rgeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] + 117*a^3*b*x^3*Hypergeom
etric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 99*a^2*b^2*x^6*Hypergeometri
c2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 81*a*b^3*x^9*Hypergeometric2F1[2
/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 297*b^4*x^12*Hypergeometric2F1[2/3, 2,
 5/3, (2*b*x^3)/(a + b*x^3)] - 54*b*x^3*(a - b*x^3)^2*(2*a + 3*b*x^3)*Hyper
geometricPFQ[{2/3, 2, 2}, {1, 5/3}, (2*b*x^3)/(a + b*x^3)] + 27*b*x^3*(a -
b*x^3)^3*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, (2*b*x^3)/(a + b*x^
```

```
3)])/(280*a^4*d*x^10*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{28a^4+64a^3bx^3+90a^2b^2x^6+216ab^3x^9+162b^4x^{12}-28a^3bx^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)-36a^2b^2x^6}{}$$

Mathematica [C] time = 5.15406, size = 130, normalized size = 0.55

$$\frac{-59a^2b^2x^6-36a^3bx^3-14a^4+\frac{140b^4x^{12}\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)}{\left(1-\frac{bx^3}{a}\right)^{2/3}}-206ab^3x^9-169b^4x^{12}}{140a^4dx^{10}\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] (-14*a^4 - 36*a^3*b*x^3 - 59*a^2*b^2*x^6 - 206*a*b^3*x^9 - 169*b^4*x^12 + (
140*b^4*x^12*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x
^3)/(a - b*x^3)])/(1 - (b*x^3)/a)^(2/3))/(140*a^4*d*x^10*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^{11}\left(-bd x^{3}+a d\right)}\sqrt[3]{b x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b x^{3}+a\right)^{\frac{1}{3}}}{\left(b d x^{3}-a d\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

3.581 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=521

$$-\frac{2a^2x\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\left(a+bx^3\right)^{2/3}}-\frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}+\frac{a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}b^{7/3}d}-\frac{\sqrt[3]{2}a^{5/3}\log}{}$$

```
[Out] (-3*a*x*(a + b*x^3)^(1/3))/(5*b^2*d) - (x^4*(a + b*x^3)^(1/3))/(5*b*d) - (2
^(1/3)*a^(5/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)*d) - (a^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3)
 + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*b^(7/3)*d) - (
2*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
])/(5*b^2*d*(a + b*x^3)^(2/3)) - (a^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3)*
x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(7/3)*d) + (a^(5/3)*Log[1 + (2^(2/3)*(a
^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(7/3)*d) - (2^(1/3)*a^(5/3)*Log[1 + (2^(1/
3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(7/3)*d) + (a^(5/3)*Log[
2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) +
 b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*b^(7/3)*d)
```

Rubi [C] time = 0.0625733, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^7\sqrt[3]{a+bx^3}F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), (b*x^3)/a
])/(7*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^6\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x^7\sqrt[3]{a+bx^3}F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.332316, size = 234, normalized size = 0.45

$$\frac{\frac{48a^4xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}+7abx^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-4(}{20b^2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-4*(a + b*x^3)*(3*a*x + b*x^4) + 7*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[
4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^4*x*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/
3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)
/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))
```

```
/(20*b^2*d*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^6}{-bdx^3 + ad} \sqrt[3]{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}} x^6}{bdx^3 - ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^6 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**6*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}} x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

3.582 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=494

$$-\frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}+\frac{a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}b^{4/3}d}-\frac{\sqrt[3]{2}a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{4/3}d}+\frac{a^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{l}\right.}{\left(a+bx^3\right)}\right.}{}$$

```
[Out] -(x*(a + b*x^3)^(1/3))/(2*b*d) - (2^(1/3)*a^(2/3)*ArcTan[(1 - (2*2^(1/3)*(a
^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*d) - (a^
(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3
]])/(2^(2/3)*Sqrt[3]*b^(4/3)*d) - (a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric
2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*d*(a + b*x^3)^(2/3)) - (a^(2/3)*Log[
2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(4/3)*d) +
 (a^(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^
(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(4/3)*d) - (2
^(1/3)*a^(2/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/
(3*b^(4/3)*d) + (a^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3
)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*b^
(4/3)*d)
```

Rubi [C] time = 0.0623486, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]
)/(4*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^3\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.194387, size = 225, normalized size = 0.46

$$\frac{x\left(\frac{4\left(\frac{4a^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}-a-bx^3\right)}{b}+3x^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x*(3*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b
*x^3)/a] + (4*(-a - b*x^3 + (4*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
 (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*
```

```
x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*
AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/b))/(8*d*(a + b*x^
3)^(2/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^3}{-bdx^3 + ad}\sqrt[3]{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}} x^3}{bdx^3 - ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^3 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

3.583 $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=416

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right.}{6\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$

```
[Out] -((2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*a^(1/3)*b^(1/3)*d)) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b
^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)*d)
- Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)
*b^(1/3)*d) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) -
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1
/3)*d) - (2^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)
])/(3*a^(1/3)*b^(1/3)*d) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x
^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a
^(1/3)*b^(1/3)*d)
```

Rubi [C] time = 0.0311958, antiderivative size = 61, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/
(a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.145371, size = 154, normalized size = 0.37

$$\frac{4ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{d\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/(d*(a - b*x^3)*(4*a*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + AppellF
1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{-bdx^3+ad}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

3.584 $\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=496

$$-\frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d}+\frac{b^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{4/3}d}-\frac{\sqrt[3]{2}b^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{4/3}d}+\frac{b^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\left(a+bx^3\right.}\right.}{}$$

```
[Out] -(a + b*x^3)^(1/3)/(2*a*d*x^2) - (2^(1/3)*b^(2/3)*ArcTan[(1 - (2*2^(1/3)*(a
^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(4/3)*d) - (b^
(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3
]])/(2^(2/3)*Sqrt[3]*a^(4/3)*d) + (b*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric
2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*d*(a + b*x^3)^(2/3)) - (b^(2/3)*Log[
2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(4/3)*d) +
 (b^(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^
(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(4/3)*d) - (2
^(1/3)*b^(2/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/
(3*a^(4/3)*d) + (b^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3
)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*a^
(4/3)*d)
```

Rubi [C] time = 0.0638079, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2adx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), (b*x^3)/a])/
(2*a*d*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2adx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.137502, size = 231, normalized size = 0.47

$$\frac{\frac{48a^3bx^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}+b^2x^6\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-4a\left(\cdots\right.}{8a^2dx^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] (-4*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*b*x^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*
x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)
/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))/(8*a^2*d*x^
```

```
2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^3(-bdx^3 + ad)} \sqrt[3]{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(bdx^3 - ad)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^3+bx^6}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**3 + b*x**6), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

3.585 $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=523

$$\frac{2b^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\left(a+bx^3\right)^{2/3}}-\frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}a^{7/3}d}+\frac{b^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{7/3}d}-\frac{\sqrt[3]{2}b^{5/3}\log}{}$$

```
[Out] -(a + b*x^3)^(1/3)/(5*a*d*x^5) - (3*b*(a + b*x^3)^(1/3))/(5*a^2*d*x^2) - (2
^(1/3)*b^(5/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*a^(7/3)*d) - (b^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3)
 + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a^(7/3)*d) + (
2*b^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
])/(5*a^2*d*(a + b*x^3)^(2/3)) - (b^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3)*
x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(7/3)*d) + (b^(5/3)*Log[1 + (2^(2/3)*(a
^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(7/3)*d) - (2^(1/3)*b^(5/3)*Log[1 + (2^(1/
3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^(7/3)*d) + (b^(5/3)*Log[
2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) +
 b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*a^(7/3)*d)
```

Rubi [C] time = 0.067572, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5adx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), (b*x^3)/a])
/(5*a*d*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5adx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.17375, size = 243, normalized size = 0.46

$$\frac{\frac{3b^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}-\frac{4\left(a^2+4abx^3+3b^2x^6\right)}{a^2x^5}+\frac{112b^2xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right.}}{20d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-4*(a^2 + 4*a*b*x^3 + 3*b^2*x^6))/(a^2*x^5) + (3*b^3*x^4*(1 + (b*x^3)/a)^
(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/a^3 + (112*b^2*x
*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2
```

```
/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a]))))/(20*d*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{x^{6} \left(-bd\,x^{3}+ad\right)}\sqrt[3]{b\,x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(bdx^{3}-ad\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^6+bx^9}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**6/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**6 + b*x**9), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

3.586 $\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=223

$$-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}$$

```
[Out] -(a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) + (a
*(a + b*x^3)^(8/3))/(8*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) - (2^(2/3)*a^
(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(11/3)*Log[a - b*x^3])/(3*2^(1/3)*b^4*d) - (a^(11/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4*d)
```

Rubi [A] time = 0.250387, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) + (a
*(a + b*x^3)^(8/3))/(8*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) - (2^(2/3)*a^
(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(11/3)*Log[a - b*x^3])/(3*2^(1/3)*b^4*d) - (a^(11/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a^2(a+bx)^{2/3}}{b^3d} + \frac{a(a+bx)^{5/3}}{b^3d} - \frac{(a+bx)^{8/3}}{b^3d} + \frac{a^3(a+bx)^{2/3}}{b^3(ad-bdx)}\right)dx, x, x^3\right)$$

$$= -\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d} + \frac{a\left(a+bx^3\right)^{8/3}}{8b^4d} - \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} + \frac{a^3\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d} - \frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d} + \frac{a\left(a+bx^3\right)^{8/3}}{8b^4d} - \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} + \frac{\left(2a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-b\cdots}\right.}{3b^3}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d} - \frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d} + \frac{a\left(a+bx^3\right)^{8/3}}{8b^4d} - \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} + \frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^4d} + \frac{a^{11/3}\cdots}{\cdots}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d} - \frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d} + \frac{a\left(a+bx^3\right)^{8/3}}{8b^4d} - \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} + \frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^4d} - \frac{a^{11/3}\cdots}{\cdots}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d} - \frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d} + \frac{a\left(a+bx^3\right)^{8/3}}{8b^4d} - \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} - \frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{\sqrt{3}b^4d}$$

Mathematica [A] time = 0.252983, size = 163, normalized size = 0.73

$$-\frac{3\left(a+bx^3\right)^{2/3}\left(98a^2bx^3+293a^3+65ab^2x^6+40b^3x^9\right) - 220\ 2^{2/3}a^{11/3}\log\left(a-bx^3\right) + 660\ 2^{2/3}a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+b\cdots}\right.}{1320b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(3*(a + b*x^3)^(2/3)*(293*a^3 + 98*a^2*b*x^3 + 65*a*b^2*x^6 + 40*b^3*x^9)
+ 440*2^(2/3)*Sqrt[3]*a^(11/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1
/3))/Sqrt[3]] - 220*2^(2/3)*a^(11/3)*Log[a - b*x^3] + 660*2^(2/3)*a^(11/3)*
Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(1320*b^4*d)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{-bd x^{3}+a d} \left(b x^{3}+a \right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60753, size = 559, normalized size = 2.51

$$-\frac{440 \cdot 4^{\frac{1}{3}} \sqrt{3} \left(-a^{2}\right)^{\frac{1}{3}} a^{3} \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} \left(b x^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}}-\sqrt{3} a}{3 a}\right) + 220 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a^{3} \log\left(4^{\frac{2}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{2}{3}} + 2 \left(b x^{3}+a\right)^{\frac{2}{3}} a \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*a^
3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^
(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2)^(2
/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b*x^3
+ 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.587 $\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=177

$$-\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}$$

```
[Out] -(a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d) - (2^(2/3)
*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(8/3)*Log[a - b*x^3])/(3*2^(1/3)*b^3*d) - (a^(8/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3*d)
```

Rubi [A] time = 0.199533, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$-\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```


```
[Out] -(a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d) - (2^(2/3)
*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(8/3)*Log[a - b*x^3])/(3*2^(1/3)*b^3*d) - (a^(8/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3*d)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{(a+bx)^{5/3}}{b^2d}+\frac{a^2(a+bx)^{2/3}}{b^2(ad-bdx)}\right)dx,x,x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^2\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)}{3b^2}$$
$$= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx,x,x^3\right)}{3b^2}$$
$$= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d}+\frac{a^{8/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}-$$
$$= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}+\frac{\left(2^{2/3}a^{8/3}\right)}{}$$
$$= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{}\right.}{}$$

Mathematica [A] time = 0.163851, size = 153, normalized size = 0.86

$$\frac{-3\left(\left(a+bx^3\right)^{2/3}\left(5a^2+2abx^3+b^2x^6\right)+4\,2^{2/3}a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)\right)+4\,2^{2/3}a^{8/3}\log\left(a-bx^3\right)-8\,2^{2/3}\sqrt{3}a^{8/3}\,\text{ta}}{24b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3)
)/Sqrt[3]] + 4*2^(2/3)*a^(8/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(5*a^2
 + 2*a*b*x^3 + b^2*x^6) + 4*2^(2/3)*a^(8/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^
3)^(1/3)]))/(24*b^3*d)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^8}{-bdx^3+ad}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.65444, size = 517, normalized size = 2.92

$$-\frac{8 \cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a^2\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+4 \cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a^2\log\left(4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{2}{3}}a-2 \cdot 4\right.}{24\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/24*(8*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^2*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4*4^(1/3)*(-a^2)^(1/3)*a^2*log(
4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*
(-a^2)^(1/3)*a) - 8*4^(1/3)*(-a^2)^(1/3)*a^2*log(-4^(2/3)*(-a^2)^(2/3) + 2*
(b*x^3 + a)^(1/3)*a) + 3*(b^2*x^6 + 2*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/3))/(
b^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.588 $\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=175

$$\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}$$

```
[Out] -(a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) - (2^(2/3)*a
^(5/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(5/3)*Log[a - b*x^3])/(3*2^(1/3)*b^2*d) - (a^(5/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^2*d)
```

Rubi [A] time = 0.156465, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 80, 50, 55, 617, 204, 31}

$$\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) - (2^(2/3)*a
^(5/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(5/3)*Log[a - b*x^3])/(3*2^(1/3)*b^2*d) - (a^(5/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^2*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{a\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^2d} + \frac{a^{5/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d} - \ldots$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d} + \frac{\left(2^{2/3}a^{5/3}\right)\text{Su}\ldots}{\ldots}$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} - \frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^2d} + \frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\ldots\right)}{\sqrt[3]{\ldots}}$$

Mathematica [A] time = 0.129372, size = 143, normalized size = 0.82

$$\frac{5\ 2^{2/3}a^{5/3}\log\left(a-bx^3\right) - 3\left(5\ 2^{2/3}a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + \left(a+bx^3\right)^{2/3}\left(7a+2bx^3\right)\right) - 10\ 2^{2/3}\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+\ldots}}{\sqrt[3]{a}}\ldots}{\sqrt{3}}\right)}{30b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-10*2^(2/3)*Sqrt[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3
))/Sqrt[3]] + 5*2^(2/3)*a^(5/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(7*a
+ 2*b*x^3) + 5*2^(2/3)*a^(5/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(
30*b^2*d)
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^5}{-bdx^3+ad}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84128, size = 493, normalized size = 2.82

$$-\frac{10\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^{2}\right)^{\frac{1}{3}} a \arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+5\cdot 4^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{1}{3}} a \log\left(4^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{2}{3}}+2\left(bx^{3}+a\right)^{\frac{2}{3}}a-2\cdot \cdots\right.}{30\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/30*(10*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3
 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 5*4^(1/3)*(-a^2)^(1/3)*a*log(4^(
2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a
^2)^(1/3)*a) - 10*4^(1/3)*(-a^2)^(1/3)*a*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x
^3 + a)^(1/3)*a) + 3*(2*b*x^3 + 7*a)*(b*x^3 + a)^(2/3))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^{5}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.589 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3} dx$

Optimal. Leaf size=153

$$\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd}-\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

```
[Out] -(a + b*x^3)^(2/3)/(2*b*d) - (2^(2/3)*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(2/3)*Log[a - b*x^3]
)/(3*2^(1/3)*b*d) - (a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
1/3)*b*d)
```

Rubi [A] time = 0.125755, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {444, 50, 55, 617, 204, 31}

$$\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd}-\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```



```
[Out] -(a + b*x^3)^(2/3)/(2*b*d) - (2^(2/3)*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(2/3)*Log[a - b*x^3]
)/(3*2^(1/3)*b*d) - (a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
1/3)*b*d)
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^{2/3}}{2bd} + \frac{1}{3}(2a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^{2/3}}{2bd} + \frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}bd} + \frac{a^{2/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd} - \frac{a\operatorname{Subst}\left(\int \frac{}{2^{2/3}a^{2/}}\right.}{}$$
$$= -\frac{\left(a+bx^3\right)^{2/3}}{2bd} + \frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd} + \frac{\left(2^{2/3}a^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{bd}$$
$$= -\frac{\left(a+bx^3\right)^{2/3}}{2bd} - \frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}bd} + \frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}$$

Mathematica [A] time = 0.0713689, size = 130, normalized size = 0.85

$$\frac{2^{2/3}a^{2/3}\log\left(a-bx^3\right) - 3\left(2^{2/3}a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\left(a+bx^3\right)^{2/3}\right) - 2\,2^{2/3}\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3)
)/Sqrt[3]] + 2^(2/3)*a^(2/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3) + 2^(2/3
)*a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(6*b*d)
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^2}{-bdx^3+ad}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56344, size = 454, normalized size = 2.97

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+4^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{2}{3}}+2\left(bx^{3}+a\right)^{\frac{2}{3}}a-2\cdot 4^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{1}{3}}\cdots\right.}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 + a
)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(1/3
)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(1/
3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^{2}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.590 $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

Optimal. Leaf size=214

$$\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{ad}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{ad}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}}-$$

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
d) - (2^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(1/3)*d) - Log[x]/(2*a^(1/3)*d) + Log[a - b*x^3]/(3*2^(1/3)*
a^(1/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*d) - Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)]/(2^(1/3)*a^(1/3)*d)
```

Rubi [A] time = 0.18207, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {446, 83, 55, 617, 204, 31}

$$\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{ad}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{ad}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{ad}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
d) - (2^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(1/3)*d) - Log[x]/(2*a^(1/3)*d) + Log[a - b*x^3]/(3*2^(1/3)*
a^(1/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*d) - Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)]/(2^(1/3)*a^(1/3)*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(ad-bdx)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx,x,x^3\right) + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx,\right.}{d}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{\sqrt[3]{a}d}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{2^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d} -$$

Mathematica [A] time = 0.0695741, size = 164, normalized size = 0.77

$$\frac{2^{2/3}\log\left(a-bx^3\right)+3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-3\,2^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\right.}{6\sqrt[3]{a}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(2/3)*
Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*Log[x
] + 2^(2/3)*Log[a - b*x^3] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)] - 3*2^(2/3)
*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*d)
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-bdx^3+ad\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)
```

Fricas [A] time = 1.69716, size = 1530, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 +
 a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log(
(2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a -
 a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 4^(1
/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)
*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2
/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a
^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*s
qrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*(-1/a)^(1/
3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3)
+ 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/
3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3
```

```
+ a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a
*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax+bx^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.591 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=269

$$-\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3}+\frac{b\left(a+bx^3\right)^{2/3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d}+\frac{5b\tan^{-1}\left(\frac{2}{}\right.}{3\sqrt{3}a}$$

```
[Out] (b*(a + b*x^3)^(2/3))/(3*a^2*d) - (a + b*x^3)^(5/3)/(3*a^2*d*x^3) + (5*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)
*d) - (2^(2/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(4/3)*d) - (5*b*Log[x])/(6*a^(4/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(1/3)*a^(4/3)*d) + (5*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(4/3)*d)
```

Rubi [A] time = 0.263398, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 156, 50, 55, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3}+\frac{b\left(a+bx^3\right)^{2/3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d}+\frac{5b\tan^{-1}\left(\frac{2}{}\right.}{3\sqrt{3}a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(2/3))/(3*a^2*d) - (a + b*x^3)^(5/3)/(3*a^2*d*x^3) + (5*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)
*d) - (2^(2/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(4/3)*d) - (5*b*Log[x])/(6*a^(4/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(1/3)*a^(4/3)*d) + (5*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(4/3)*d)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(ad-bdx)}dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd+\frac{2}{3}b^2dx\right)}{x(ad-bdx)}dx, x, x^3\right)}{3a^2d}$$
$$= -\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{b^2\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}dx, x, x^3\right)}{3a^2} + \frac{(5b)\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}dx, x, x^3\right)}{9a^2d}$$
$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}dx, x, x^3\right)}{3a} + \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\right.}{9ad}$$
$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{4/3}d} - \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}dx, x, \sqrt[3]{a+bx}\right.}{6a^{4/3}d}$$
$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} - \frac{b\log\left(\right.}{}$$
$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{5b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{4/3}d} - \frac{2^{2/3}b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} - \frac{5b\log(x)}{6a^{4/3}d} +$$

Mathematica [A] time = 0.0839781, size = 213, normalized size = 0.79

$$\frac{10\sqrt{3}bx^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 3\left(2\sqrt[3]{a}\left(a+bx^3\right)^{2/3} - 2^{2/3}bx^3\log\left(a-bx^3\right) - 5bx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + 3\ 2^{2/3}bx^3\log\left(\right.\right.}{18a^{4/3}dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*(
2*a^(1/3)*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 5*b*x^3*Log[x] - 2^(2/3)*b*x^3*Log[a
 - b*x^3] - 5*b*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 3*2^(2/3)*b*x^3*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(18*a^(4/3)*d*x^3)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-bd x^3+ad\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)
```

Fricas [A] time = 1.69358, size = 1727, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 15*sqrt(1/3)*a*b*x^3*sqrt(-1
/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3
+ a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*
a)/x^3) + 3*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-
1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*
b*x^3*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + 5*a
^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) -
 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)
/(a^2*d*x^3), -1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 3*4^(1/3)*a*b*x
^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*
(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(-4
^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 30*sqrt(1/3)*a^(2/3)*b*x^3*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 5*a^(2/3)*b*x^3*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 10*a^(2/3)*b
*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)/(a^2*d*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^4+bx^7}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.592 $\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx$

Optimal. Leaf size=284

$$\frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d}-\frac{2^{2/3}b^2\tan^{-1}\left(\cdots\right.}{\sqrt{3}a\cdots}$$

```
[Out] (-5*b*(a + b*x^3)^(2/3))/(18*a^2*d*x^3) - (a + b*x^3)^(5/3)/(6*a^2*d*x^6) +
 (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt
[3]*a^(7/3)*d) - (2^(2/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*d) - (7*b^2*Log[x])/(9*a^(7/3)*d) + (b
^2*Log[a - b*x^3])/(3*2^(1/3)*a^(7/3)*d) + (7*b^2*Log[a^(1/3) - (a + b*x^3)
^(1/3)])/(9*a^(7/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^
(1/3)*a^(7/3)*d)
```

Rubi [A] time = 0.279263, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 149, 156, 55, 617, 204, 31}

$$\frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d}-\frac{2^{2/3}b^2\tan^{-1}\left(\cdots\right.}{\sqrt{3}a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```


```
[Out] (-5*b*(a + b*x^3)^(2/3))/(18*a^2*d*x^3) - (a + b*x^3)^(5/3)/(6*a^2*d*x^6) +
 (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt
[3]*a^(7/3)*d) - (2^(2/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*d) - (7*b^2*Log[x])/(9*a^(7/3)*d) + (b
^2*Log[a - b*x^3])/(3*2^(1/3)*a^(7/3)*d) + (7*b^2*Log[a^(1/3) - (a + b*x^3)
^(1/3)])/(9*a^(7/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^
(1/3)*a^(7/3)*d)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(ad-bdx)}dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd-\frac{1}{3}b^2dx\right)}{x^2(ad-bdx)}dx,x,x^3\right)}{6a^2d}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{28}{9}a^2b^2d^2-\frac{8}{9}ab^3d^2x}{x\sqrt[3]{a+bx}(ad-bdx)}dx,x,x^3\right)}{6a^3d^2}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}dx,x,x^3\right)}{3a^2} + \frac{\left(14b^2\right)\text{Subst}\left(\int \cdots\right.}{27a^2 \cdots}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{7/3}d} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}dx,x,\cdots\right.}{9a^{7/3}d}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d} - \cdots$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{7/3}d} - \frac{2^{2/3}b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d} - \frac{7b^2\log\cdots}{9a^{7/3}\cdots}$$

Mathematica [A] time = 0.14117, size = 247, normalized size = 0.87

$$\frac{28\sqrt{3}b^2x^6\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\left(3a^{4/3}\left(a+bx^3\right)^{2/3}-3\ 2^{2/3}b^2x^6\log\left(a-bx^3\right)-14b^2x^6\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+9\ 2^{2/3}b^2x^6\right.}{54a^{7/3}dx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (28*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3
*(3*a^(4/3)*(a + b*x^3)^(2/3) + 8*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 6*2^(2/
3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]
] + 14*b^2*x^6*Log[x] - 3*2^(2/3)*b^2*x^6*Log[a - b*x^3] - 14*b^2*x^6*Log[a
^(1/3) - (a + b*x^3)^(1/3)] + 9*2^(2/3)*b^2*x^6*Log[2^(1/3)*a^(1/3) - (a +
b*x^3)^(1/3)]))/(54*a^(7/3)*d*x^6)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^7\left(-bdx^3+ad\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)
```

Fricas [A] time = 1.72148, size = 1816, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3
)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*sq
rt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b
*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3)
 + 3*a)/x^3) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/
3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 18*4^
(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1
/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(8*a*
b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6), -1/54*(18*4^(1/3)*sqrt(3)*a*
b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1
/3) - 1/3*sqrt(3)) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 +
a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) -
 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 +
 a)^(1/3)) - 84*sqrt(1/3)*a^(2/3)*b^2*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^
3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3)
- a^(1/3)) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.593 $\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=264

$$\frac{a^2\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}b^{7/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{7/3}d}+\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{9b^{7/3}d}-\frac{14a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}d}+\frac{2^{2/3}a^2\tan^{-1}}{\sqrt{3}b}$$

```
[Out] (-4*a*x*(a + b*x^3)^(2/3))/(9*b^2*d) - (x^4*(a + b*x^3)^(2/3))/(6*b*d) - (1
4*a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(
7/3)*d) + (2^(2/3)*a^2*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*b^(7/3)*d) + (a^2*Log[a*d - b*d*x^3])/(3*2^(1/3)*b^(7/3
)*d) - (a^2*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(1/3)*b^(7/3)*d)
 + (7*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(7/3)*d)
```

Rubi [C] time = 0.0632764, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), (b*x^3)/a
])/(7*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.263719, size = 244, normalized size = 0.92

$$\frac{2\,2^{2/3}a^2\sqrt[3]{a+bx^3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)+21ab^{4/3}x^4\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3},\right.}{54b^{7/3}d\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-3*b^(1/3)*(a + b*x^3)*(8*a*x + 3*b*x^4) + 21*a*b^(4/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + 2*2^(2/3)*a^2
*(a + b*x^3)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)
^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1
 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3
)^(1/3)]))/(54*b^(7/3)*d*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^6}{-bdx^3+ad}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

Fricas [A] time = 1.69421, size = 1875, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 42*sqrt(1/3)*a^2*b*sqrt(-1
/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4
/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
b^(2/3)) + 2*a) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2
/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*4^(1/3
)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^
3 + a)^(2/3))/x^2) - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
 + 14*a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d), -1/54
*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*lo
g(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-
1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b
```

```
*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 84*sqrt(1/3)*a^2*b^(2/3)*arct
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 28*a^2*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 14*a^2*b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8
*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^6\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

3.594 $\int \frac{x^3 \left(a+bx^3\right)^{2/3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=229

$$\frac{a \log \left(ad-bdx^3\right)}{3 \sqrt[3]{2} b^{4/3} d}-\frac{a \log \left(\sqrt[3]{2} \sqrt[3]{b} x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2} b^{4/3} d}+\frac{5 a \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b} x\right)}{6 b^{4/3} d}-\frac{5 a \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3 \sqrt{3} b^{4/3} d}+\frac{2^{2/3} a \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3} b^{4/3} d}$$

```
[Out] -(x*(a + b*x^3)^(2/3))/(3*b*d) - (5*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)
^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d) + (2^(2/3)*a*ArcTan[(1 + (2*2^(1/3)
*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*d) + (a*Log[a*d -
 b*d*x^3])/(3*2^(1/3)*b^(4/3)*d) - (a*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(
1/3)])/(2^(1/3)*b^(4/3)*d) + (5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6
*b^(4/3)*d)
```

Rubi [C] time = 0.0634338, antiderivative size = 66, normalized size of antiderivative = 0.29, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^4 \left(a+bx^3\right)^{2/3} F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]
)/(4*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3 (a + bx^3)^{2/3}}{ad - bdx^3} dx = \frac{(a + bx^3)^{2/3} \int \frac{x^3 \left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^4 (a + bx^3)^{2/3} F_1\left(\frac{4}{3}; -\frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{4ad\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.227412, size = 216, normalized size = 0.94

$$\frac{\frac{15x^4 \sqrt[3]{\frac{bx^3}{a}+1} F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{2^{2/3} a \left(\log\left(\frac{2^{2/3} b^{2/3} x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}} + 1\right) - 2\log\left(1 - \frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}} + 1}{\sqrt{3}}\right)\right)}{b^{4/3}} - \frac{12x(a+bx^3)^{2/3}}{b}}{36d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-12*x*(a + b*x^3)^(2/3))/b + (15*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3,
1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/(a + b*x^3)^(1/3) + (2^(2/3)*a*(2*Sq
rt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log
[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)
/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/b^(4/3))/(36*
d)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^3}{-bd x^3+ad}\left(b x^3+a\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b x^3+a\right)^{\frac{2}{3}}x^3}{bd x^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

Fricas [A] time = 1.69662, size = 1786, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)
*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^(2
/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x
^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/
3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*
(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-
1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/
3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
+ a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
```

```
 + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1
/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3)
)/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3
 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(
1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^
2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/
3))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^3\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

3.595 $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=200

$$\frac{\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\sqrt[3]{b}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}+\frac{2^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}$$

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d)
) + (2^(2/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])
/(Sqrt[3]*b^(1/3)*d) + Log[a*d - b*d*x^3]/(3*2^(1/3)*b^(1/3)*d) - Log[2^(1/
3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2^(1/3)*b^(1/3)*d) + Log[-(b^(1/3)*x) +
(a + b*x^3)^(1/3)]/(2*b^(1/3)*d)
```

Rubi [C] time = 0.0328836, antiderivative size = 61, normalized size of antiderivative = 0.3, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/
(a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.154119, size = 156, normalized size = 0.78

$$\frac{4ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{d\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/(d*(a - b*x^3)*(4*a*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + 2*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{-bdx^3+ad}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

Fricas [A] time = 1.64314, size = 1659, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 3*sqrt(1/3)*b*sqrt(-1/b^(2/3))*l
og(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (
b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) +
2*a) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 +
 a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) -
 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 2
*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(2*4^
(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-1/b)^(1/3))/x) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*
(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(
1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b
*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2
*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
```

```
+ a)^(2/3))/x^2))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

3.596 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$

Optimal. Leaf size=157

$$\frac{b^{2/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}ad}+\frac{2^{2/3}b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}ad}-\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}$$

```
[Out] -(a + b*x^3)^(2/3)/(2*a*d*x^2) + (2^(2/3)*b^(2/3)*ArcTan[(1 + (2*2^(1/3)*b^
(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*d) + (b^(2/3)*Log[a*d - b*
d*x^3])/(3*2^(1/3)*a*d) - (b^(2/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3
)])/(2^(1/3)*a*d)
```

Rubi [C] time = 0.0707691, antiderivative size = 79, normalized size of antiderivative = 0.5, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)}{2adx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3
, (-2*b*x^3)/(a - b*x^3)])/(2*a*d*x^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)}{2adx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.0161884, size = 47, normalized size = 0.3

$$-\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};\frac{2bx^3}{bx^3+a}\right)}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 1, 1/3, (2*b*x^3)/(a + b*x^3)])
/(2*a*d*x^2)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-bdx^3+ad\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^3+bx^6}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(bdx^3 - ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

3.597 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

Optimal. Leaf size=182

$$\frac{b^{5/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}a^2d}-\frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^2d}+\frac{2^{2/3}b^{5/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^2d}-\frac{7b\left(a+bx^3\right)^{2/3}}{10a^2dx^2}-\frac{\left(a+bx^3\right)^{2/3}}{5adx^5}$$

```
[Out] -(a + b*x^3)^(2/3)/(5*a*d*x^5) - (7*b*(a + b*x^3)^(2/3))/(10*a^2*d*x^2) + (
2^(2/3)*b^(5/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3
]])/(Sqrt[3]*a^2*d) + (b^(5/3)*Log[a*d - b*d*x^3])/(3*2^(1/3)*a^2*d) - (b^(
5/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(1/3)*a^2*d)
```

Rubi [C] time = 0.429248, antiderivative size = 121, normalized size of antiderivative = 0.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{2a^2-4bx^3\left(2a+3bx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+12bx^3\left(a-bx^3\right)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+5abx^3+3b^2x^6}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -(2*a^2 + 5*a*b*x^3 + 3*b^2*x^6 - 4*b*x^3*(2*a + 3*b*x^3)*Hypergeometric2F1
[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*b*x^3*(a - b*x^3)*Hypergeometric2
F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)])/(10*a^2*d*x^5*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{2a^2+5abx^3+3b^2x^6-4bx^3\left(2a+3bx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)+12bx^3\left(a-bx^3\right)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right.}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

Mathematica [C] time = 4.66952, size = 124, normalized size = 0.68

$$\frac{\text{Gamma}\left(-\frac{2}{3}\right)\left(a+bx^3\right)^{5/3}\left(\left(2a^2-5abx^3+3b^2x^6\right)\,{}_2F_1\left(1,1;\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)+18bx^3\left(a+bx^3\right)\,{}_2F_1\left(2,2;\frac{4}{3};-\frac{2bx^3}{a-bx^3}\right)\right)}{15a^2dx^5\text{Gamma}\left(\frac{1}{3}\right)\left(a-bx^3\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(5/3)*Gamma[-2/3]*((2*a^2 - 5*a*b*x^3 + 3*b^2*x^6)*Hypergeomet
ric2F1[1, 1, 1/3, (-2*b*x^3)/(a - b*x^3)] + 18*b*x^3*(a + b*x^3)*Hypergeome
tric2F1[2, 2, 4/3, (-2*b*x^3)/(a - b*x^3)]))/(15*a^2*d*x^5*(a - b*x^3)^2*Ga
mma[1/3])
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^6\left(-bdx^3+ad\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^6+bx^9}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

3.598 $\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx$

Optimal. Leaf size=209

$$-\frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2}+\frac{b^{8/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^3d}-\frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d}+\frac{2^{2/3}b^{8/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^3d}-\frac{b(a+bx^3)^{2/3}}{4a^2dx^5}$$

```
[Out] -(a + b*x^3)^(2/3)/(8*a*d*x^8) - (b*(a + b*x^3)^(2/3))/(4*a^2*d*x^5) - (5*b
^2*(a + b*x^3)^(2/3))/(8*a^3*d*x^2) + (2^(2/3)*b^(8/3)*ArcTan[(1 + (2*2^(1/
3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^3*d) + (b^(8/3)*Log[a
*d - b*d*x^3])/(3*2^(1/3)*a^3*d) - (b^(8/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*
x^3)^(1/3)])/(2^(1/3)*a^3*d)
```

Rubi [C] time = 10.6764, antiderivative size = 244, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-18bx^3(a-bx^3)^2\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-4bx^3(5a^2+6abx^3+9b^2x^6)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+42a^2bx^3\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)}{40a^3dx^8\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] -(5*a^3 + 11*a^2*b*x^3 + 15*a*b^2*x^6 + 9*b^3*x^9 - 4*b*x^3*(5*a^2 + 6*a*b*
x^3 + 9*b^2*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 42
*a^2*b*x^3*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*a*b^2
*x^6*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b^3*x^9*Hyp
ergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 18*b*x^3*(a - b*x^3)^2
*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)])/(40*a^3*d
*x^8*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{5a^3+11a^2bx^3+15ab^2x^6+9b^3x^9-4bx^3\left(5a^2+6abx^3+9b^2x^6\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)+42a^2bx^3\,{}_2F\ldots}{\ldots}$$

Mathematica [A] time = 5.21285, size = 179, normalized size = 0.86

$$\frac{4\,2^{2/3}b^{8/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)-\frac{3\left(a+bx^3\right)^{2/3}\left(a^2+2abx^3+5b^2x^6\right)}{x^8}}{24a^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 4*2^(2/3)*b^(8/
3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]]
 - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(24*a
^3*d)
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^{9} \left(-b d \,x^{3}+a d \right)} \left(b \,x^{3}+a \right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(bdx^3 - ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

3.599 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx$

Optimal. Leaf size=236

$$-\frac{293b^3\left(a+bx^3\right)^{2/3}}{440a^4dx^2}-\frac{49b^2\left(a+bx^3\right)^{2/3}}{220a^3dx^5}+\frac{b^{11/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}a^4d}-\frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^4d}+\frac{2^{2/3}b^{11/3}\tan^{-1}\left(\cdots\right.}{\sqrt{3}a^4d}$$

```
[Out] -(a + b*x^3)^(2/3)/(11*a*d*x^11) - (13*b*(a + b*x^3)^(2/3))/(88*a^2*d*x^8)
- (49*b^2*(a + b*x^3)^(2/3))/(220*a^3*d*x^5) - (293*b^3*(a + b*x^3)^(2/3))/
(440*a^4*d*x^2) + (2^(2/3)*b^(11/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a +
b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^4*d) + (b^(11/3)*Log[a*d - b*d*x^3])/(3*
2^(1/3)*a^4*d) - (b^(11/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(
1/3)*a^4*d)
```

Rubi [C] time = 17.6006, antiderivative size = 391, normalized size of antiderivative = 1.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-54bx^3\left(a-bx^3\right)^2\left(5a+6bx^3\right){}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+54bx^3\left(a-bx^3\right)^3{}_4F_3\left(\frac{1}{3},2,2,2;1,1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-180a^2b^2x^6\,{}_2F_1\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] -(40*a^4 + 85*a^3*b*x^3 + 99*a^2*b^2*x^6 + 135*a*b^3*x^9 + 81*b^4*x^12 - 16
0*a^3*b*x^3*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 180*a^2
*b^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 216*a*b^3*
x^9*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 324*b^4*x^12*Hy
pergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 396*a^3*b*x^3*Hyperge
ometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 198*a^2*b^2*x^6*Hypergeome
tric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 594*b^4*x^12*Hypergeometric2F
1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b*x^3*(a - b*x^3)^2*(5*a + 6*b*x
^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)] + 54*b*
x^3*(a - b*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2}, {1, 1, 4/3}, (2*b*x^3)/
(a + b*x^3)])/(440*a^4*d*x^11*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40a^4+85a^3bx^3+99a^2b^2x^6+135ab^3x^9+81b^4x^{12}-160a^3bx^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)-180a^2b^2x^6\,{}_2}{}$$

Mathematica [A] time = 5.22345, size = 196, normalized size = 0.83

$$\frac{\frac{2^{2/3}b^{11/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{bx}}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{a^4}-\frac{3\left(a+bx^3\right)^{2/3}\left(65a^2bx^3+40a^3+98ab^2x^6+293b^3x^9\right)}{220a^4x^{11}}}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9)
)/(220*a^4*x^11) + (2^(2/3)*b^(11/3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1
/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x
```

```
^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(
1/3)*x)/(b + a*x^3)^(1/3)]))/a^4)/(6*d)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{x^{12}\left(-bd\,x^{3}+ad\right)}\left(b\,x^{3}+a\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{\left(bdx^{3}-ad\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

3.600 $\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=512

$$-\frac{19a^2x^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}}+\frac{a^{7/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}b^{8/3}d}-\frac{2^{2/3}a^{7/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{8/3}d}-\frac{a^{7/3}}{}$$

```
[Out] (-9*a*x^2*(a + b*x^3)^(2/3))/(28*b^2*d) - (x^5*(a + b*x^3)^(2/3))/(7*b*d) +
 (2^(2/3)*a^(7/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)
^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(8/3)*d) + (a^(7/3)*ArcTan[(1 + (2^(1/3)*(a^(1
/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*b^(8/3)*d)
- (19*a^2*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x
^3)/a)])/(28*b^2*d*(a + b*x^3)^(1/3)) + (a^(7/3)*Log[((a^(1/3) - b^(1/3)*x)
^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(8/3)*d) + (a^(7/3)*Log[1 + (2^(
2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/
3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*b^(8/3)*d) - (2^(2/3)*a^(7/3)*Log[1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(8/3)*d) - (a^(7/
3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^
3)^(1/3))/a^(1/3)])/(2*2^(1/3)*b^(8/3)*d)
```

Rubi [C] time = 0.0658789, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^8\left(a+bx^3\right)^{2/3}F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), (b*x^3)/a
])/(8*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^8\left(a+bx^3\right)^{2/3}F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.104658, size = 147, normalized size = 0.29

$$\frac{45a^2x^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5\left(9a^2x^2+13abx^5+4b^2x^8\right)+38abx^5\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{140b^2d\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-5*(9*a^2*x^2 + 13*a*b*x^5 + 4*b^2*x^8) + 45*a^2*x^2*(1 + (b*x^3)/a)^(1/3)
*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] + 38*a*b*x^5*(1 + (b*x
^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(140*b^2*
d*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^{7}}{-b d x^{3}+a d}\left(b x^{3}+a\right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b x^{3}+a\right)^{\frac{2}{3}} x^{7}}{b d x^{3}-a d}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^{7}\left(a+b x^{3}\right)^{\frac{2}{3}}}{-a+b x^{3}}\,d x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**7*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

3.601 $\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=485

$$\frac{a^{4/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}b^{5/3}d}-\frac{2^{2/3}a^{4/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{5/3}d}-\frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}+ 2^{2/}$$

```
[Out] -(x^2*(a + b*x^3)^(2/3))/(4*b*d) + (2^(2/3)*a^(4/3)*ArcTan[(1 - (2*2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(5/3)*d) + (
a^(4/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(1/3)*Sqrt[3]*b^(5/3)*d) - (3*a*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeo
metric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(4*b*d*(a + b*x^3)^(1/3)) + (a^(4/3
)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(5/3
)*d) + (a^(4/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3)
 - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*b^(5/3)*d
) - (2^(2/3)*a^(4/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1
/3)])/(3*b^(5/3)*d) - (a^(4/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3)
- (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*b^(5/3)*d)
```

Rubi [C] time = 0.0655671, antiderivative size = 66, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^5\left(a+bx^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]
)/(5*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
```

```
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^5\left(a+bx^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.0824392, size = 127, normalized size = 0.26

$$\frac{x^2\left(6bx^3\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+5a\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5\left(a+bx^3\right)\right)}{20bd\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(-5*(a + b*x^3) + 5*a*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3,
 -((b*x^3)/a), (b*x^3)/a] + 6*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3
, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(20*b*d*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^4}{-bdx^3+ad}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x^4\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

[Out] -Integral(x**4*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] integrate(-(b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)

3.602 $\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3} dx$

Optimal. Leaf size=457

$$\frac{\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}b^{2/3}d}-\frac{2^{2/3}\sqrt[3]{a}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}d}-\frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d}+ 2^{2/3}$$

```
[Out] (2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d) + (a^(1/3)*ArcTan[(1 + (2^(1/3)*(a^(1/
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*b^(2/3)*d) -
 (x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])
/(2*d*(a + b*x^3)^(1/3)) + (a^(1/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) +
 b^(1/3)*x))/a])/(6*2^(1/3)*b^(2/3)*d) + (a^(1/3)*Log[1 + (2^(2/3)*(a^(1/3)
 + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b
*x^3)^(1/3)])/(3*2^(1/3)*b^(2/3)*d) - (2^(2/3)*a^(1/3)*Log[1 + (2^(1/3)*(a^
(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(2/3)*d) - (a^(1/3)*Log[(b^(1/
3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(
1/3)]])/(2*2^(1/3)*b^(2/3)*d)
```

Rubi [C] time = 0.0465118, antiderivative size = 66, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a]
)/(2*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
```

```
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.0392283, size = 63, normalized size = 0.14

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^3
)/a])/(2*d*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{-bdx^3+ad}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

3.603 $\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx$

Optimal. Leaf size=483

$$\frac{\sqrt[3]{b}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}a^{2/3}d}-\frac{2^{2/3}\sqrt[3]{b}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{2/3}d}-\frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}+\frac{2^{2/3}\cdots}{\cdots}$$

```
[Out] -((a + b*x^3)^(2/3)/(a*d*x)) + (2^(2/3)*b^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(2/3)*d) + (b^(1
/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]
)/(2^(1/3)*Sqrt[3]*a^(2/3)*d) + (b*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric
2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*a*d*(a + b*x^3)^(1/3)) + (b^(1/3)*Log[
((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*a^(2/3)*d) +
 (b^(1/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^
(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*a^(2/3)*d) - (2
^(2/3)*b^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/
(3*a^(2/3)*d) - (b^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(
2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*a^(2/3)*d)
```

Rubi [C] time = 0.069205, antiderivative size = 64, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{(a+bx^3)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{adx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), (b*x^3)/a])
/(a*d*x*(1 + (b*x^3)/a)^(2/3)))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(ad-bdx^3\right)}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{adx\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.0959109, size = 136, normalized size = 0.28

$$\frac{15abx^3\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2\left(b^2x^6\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+5a\left(a+bx^3\right)\right)}{10a^2dx\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] (15*a*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
(b*x^3)/a] - 2*(5*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/
3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(10*a^2*d*x*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(-b d \,x^{3}+a d \right)} \left(b \,x^{3}+a \right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b x^{3}+a\right)^{\frac{2}{3}}}{\left(b d x^{3}-a d\right) x^{2}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\left(a+b x^{3}\right)^{\frac{2}{3}}}{-a x^{2}+b x^{5}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**2 + b*x**5), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

3.604 $\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx$

Optimal. Leaf size=512

$$\frac{3b^2x^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}}+\frac{b^{4/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}a^{5/3}d}-\frac{2^{2/3}b^{4/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{5/3}d}-\frac{b^{4/3}\,\mathrm{lo}\cdots}{\cdots}$$

```
[Out] -(a + b*x^3)^(2/3)/(4*a*d*x^4) - (3*b*(a + b*x^3)^(2/3))/(2*a^2*d*x) + (2^(
2/3)*b^(4/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3
))/Sqrt[3]])/(Sqrt[3]*a^(5/3)*d) + (b^(4/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) +
 b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a^(5/3)*d) + (3*
b^2*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)
])/(4*a^2*d*(a + b*x^3)^(1/3)) + (b^(4/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(
1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*a^(5/3)*d) + (b^(4/3)*Log[1 + (2^(2/3)*(a
^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3)])/(3*2^(1/3)*a^(5/3)*d) - (2^(2/3)*b^(4/3)*Log[1 + (2^(1/
3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^(5/3)*d) - (b^(4/3)*Log[
(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3
))/a^(1/3)])/(2*2^(1/3)*a^(5/3)*d)
```

Rubi [C] time = 0.0689181, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{(a+bx^3)^{2/3}F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4adx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), (b*x^3)/a])
/(4*a*d*x^4*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(ad-bdx^3\right)}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4adx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.0810564, size = 148, normalized size = 0.29

$$\frac{-5a\left(a^2+7abx^3+6b^2x^6\right)-6b^3x^9\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+35ab^2x^6\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{20a^3dx^4\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*a*(a^2 + 7*a*b*x^3 + 6*b^2*x^6) + 35*a*b^2*x^6*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] - 6*b^3*x^9*(1 + (b*x^3)/
a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(20*a^3*d*x^4
*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^{5} \left(-b d x^{3}+a d \right)} \left(b x^{3}+a \right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b x^{3}+a\right)^{\frac{2}{3}}}{\left(b d x^{3}-a d\right) x^{5}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^5+bx^8}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**5 + b*x**8), x)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

3.605 $\int \frac{x^{14}}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=127

$$\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] (2*(1 - x^3)^(5/3))/5 - (1 - x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + ArcTan[(1
 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^
(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0934224, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*(1 - x^3)^(5/3))/5 - (1 - x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + ArcTan[(1
 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^
(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-2(1-x)^{2/3}+2(1-x)^{5/3}-(1-x)^{8/3}+\frac{1}{\sqrt[3]{1-x}(1+x)}\right)dx, x, x^3\right)$$
$$= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\right.$$
$$= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}(}{}$$
$$= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}\right.}{}$$

Mathematica [A] time = 0.100535, size = 113, normalized size = 0.89

$$\frac{1}{660}\left(3\left(1-x^3\right)^{2/3}\left(-20x^9+5x^6-38x^3+53\right)-55\ 2^{2/3}\log\left(x^3+1\right)+165\ 2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+110\ 2^{2/3}\sqrt{3}\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9) + 110*2^(2/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 55*2^(2/3)*Log[1 + x^3] + 165*
2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/660
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x^{14}}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^14/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [A] time = 1.46089, size = 161, normalized size = 1.27

$$\frac{1}{11}\left(-x^3+1\right)^{\frac{11}{3}}-\frac{1}{4}\left(-x^3+1\right)^{\frac{8}{3}}+\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/11*(-x^3 + 1)^(11/3) - 1/4*(-x^3 + 1)^(8/3) + 1/6*sqrt(3)*2^(2/3)*arctan(
1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 2/5*(-x^3 + 1)^(5/3)
- 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) +
 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A] time = 2.23223, size = 359, normalized size = 2.83

$$-\frac{1}{220}\left(20\,x^9-5\,x^6+38\,x^3-53\right)\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6
)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12
*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2
^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.606 $\int \frac{x^{11}}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=128

$$-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(2/3)/2 + (1 - x^3)^(5/3)/5 - (1 - x^3)^(8/3)/8 - ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)
) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0879538, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```


```
[Out] -(1 - x^3)^(2/3)/2 + (1 - x^3)^(5/3)/5 - (1 - x^3)^(8/3)/8 - ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)
) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3} + (1-x)^{5/3} - \frac{1}{\sqrt[3]{1-x}(1+x)}\right) dx, x, x^3\right)$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx\right.$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\right.}{2}$$

Mathematica [A] time = 0.0508381, size = 128, normalized size = 1.

$$\frac{1}{120}\left(-15\left(1-x^3\right)^{2/3}x^6 + 6\left(1-x^3\right)^{2/3}x^3 - 51\left(1-x^3\right)^{2/3} + 10\ 2^{2/3}\log\left(x^3+1\right) - 30\ 2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) - 20\ 2^{2/3}\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-51*(1 - x^3)^(2/3) + 6*x^3*(1 - x^3)^(2/3) - 15*x^6*(1 - x^3)^(2/3) - 20*
2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*
Log[1 + x^3] - 30*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/120
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^11/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [A] time = 1.4344, size = 161, normalized size = 1.26

$$-\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}}-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^3 + 1)^(8/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Fricas [A] time = 2.39276, size = 448, normalized size = 3.5

$$-\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/40*(5*x^6
- 2*x^3 + 17)*(-x^3 + 1)^(2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.607 $\int \frac{x^8}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=97

$$\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] (1 - x^3)^(5/3)/5 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
1/3))
```

Rubi [A] time = 0.0770901, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```


```
[Out] (1 - x^3)^(5/3)/5 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
1/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-(1-x)^{2/3}+\frac{1}{\sqrt[3]{1-x}(1+x)}\right)dx, x, x^3\right) \\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\right.}{2\sqrt[3]{}} \\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} \\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0553751, size = 96, normalized size = 0.99

$$
\frac{1}{60}\left(12\left(1-x^3\right)^{5/3}-5\ 2^{2/3}\log\left(x^3+1\right)+15\ 2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+10\ 2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 5*2^(2/3)*Log[1 + x^3] + 15*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(
1/3)])/60
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$
\int \frac{x^8}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^8/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [A] time = 1.41949, size = 131, normalized size = 1.35

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A] time = 2.13027, size = 328, normalized size = 3.38

$$-\frac{1}{5}\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(s
qrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) +
2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (
-x^3 + 1)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.608 $\int \frac{x^5}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=98

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(2/3)/2 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)
*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^
(1/3))
```

Rubi [A] time = 0.0637919, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 55, 617, 204, 31}

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/2 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)
*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^
(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)+\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\right.}{2\sqrt[3]{}}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0313085, size = 95, normalized size = 0.97

$$
\frac{1}{12}\left(-6\left(1-x^3\right)^{2/3}+2^{2/3}\log\left(x^3+1\right)-3\ 2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-2\ 2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
])/12
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$
\int \frac{x^5}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [A] time = 1.44807, size = 131, normalized size = 1.34

$$-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Fricas [A] time = 1.78259, size = 419, normalized size = 4.28

$$-\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 +
1)^(2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.609 $\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=82

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0588563, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right)\\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}\\
&= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.0176259, size = 73, normalized size = 0.89

$$\frac{-\log\left(x^3+1\right)+3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6*2^(1/3))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [A] time = 1.44403, size = 116, normalized size = 1.41

$$\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A] time = 1.71607, size = 282, normalized size = 3.44

$$\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
```

```
1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.610 $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=137

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 -
x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(1/3)
) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0910683, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 55, 618, 204, 31, 617}

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 -
x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(1/3)
) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
p}, x] && !IntegerQ[p]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x,\right.$$
$$= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{\sqrt[3]{2}}$$
$$= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - $$

Mathematica [A] time = 0.0389278, size = 133, normalized size = 0.97

$$\frac{1}{12}\left(2^{2/3}\log\left(x^3+1\right) + 6\log\left(1-\sqrt[3]{1-x^3}\right) - 3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + 4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right) - 2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcT
an[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 6*Log[x] + 2^(2/3)*Log[1 + x^3]
 + 6*Log[1 - (1 - x^3)^(1/3)] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/1
2
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^3+1\right)}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)
```

Fricas [C] time = 11.5527, size = 1335, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3
*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))
- 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log(3/8
*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)
^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)
)*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/
3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3)*
(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/3*log(-1/24*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3))^3 + (-x^3 + 1)^(1/3) - 4/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1
)^(1/3) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.611 $\int \frac{1}{x^4 \sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=157

$$-\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+$$

```
[Out] -(1 - x^3)^(2/3)/(3*x^3) - (2*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[x]/3 - Log[1 + x^3]/(6*2^(1/3)) - Log[1 - (1 - x^3)^(1/3)]/3 + Log[2^(
1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.103411, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(3*x^3) - (2*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[x]/3 - Log[1 + x^3]/(6*2^(1/3)) - Log[1 - (1 - x^3)^(1/3)]/3 + Log[2^(
1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x^2(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{\frac{2}{3}-\frac{x}{3}}{\sqrt[3]{1-x}x(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} - \frac{2}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{}{1+\cdots}\right.$$
$$= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{2}{3}\,\text{Subs}\cdots$$
$$= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} - \frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x)}{3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1\cdots\right.$$

Mathematica [A] time = 0.148786, size = 153, normalized size = 0.97

$$\frac{1}{36}\left(3\left(-\frac{4\left(1-x^3\right)^{2/3}}{x^3} - 2^{2/3}\log\left(x^3+1\right) - 4\log\left(1-\sqrt[3]{1-x^3}\right) + 3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + 2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}\cdots}{\sqrt{3}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 3*((-4*(1 - x^3)^(2/3
))/x^3 + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] +
4*Log[x] - 2^(2/3)*Log[1 + x^3] - 4*Log[1 - (1 - x^3)^(1/3)] + 3*2^(2/3)*Lo
g[2^(1/3) - (1 - x^3)^(1/3)]))/36
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(x^3+1\right)}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)
```

Fricas [A] time = 1.7055, size = 541, normalized size = 3.45

$$6\sqrt{6}2^{\frac{1}{6}}x^3\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}x^3\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+6\cdot 2^{\frac{2}{3}}x^3\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*(6*sqrt(6)*2^(1/6)*x^3*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)
*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*x^3*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3)
+ (-x^3 + 1)^(2/3)) + 6*2^(2/3)*x^3*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 8*sq
rt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 4*x^3*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*x^3*log((-x^3 + 1)^(1/3) - 1) -
12*(-x^3 + 1)^(2/3))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt[3]{-(x-1)\left(x^2+x+1\right)(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.612 $\int \frac{x^6}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{3}\left(1-x^3\right)^{2/3}x-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(x*(1 - x^3)^(2/3))/3 + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3
*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sq
rt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*
2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/3
```

Rubi [A] time = 0.149213, antiderivative size = 226, normalized size of antiderivative = 1.47, number of steps used = 15, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {494, 470, 522, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{9}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{2}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(x*(1 - x^3)^(2/3))/3 + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3
*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sq
rt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/9 - (2*Log[1 + x/
(1 - x^3)^(1/3)])/9 - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(
1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/
3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
```

```
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^6}{\left(1+x^3\right)^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1-x^3}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{9}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{9}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-}\right.$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{1}{9}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}\right.}{6\sqrt[3]{2}}$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{9}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) -$$

Mathematica [C] time = 0.11141, size = 144, normalized size = 0.94

$$\frac{1}{36}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right) - 12\left(1-x^3\right)^{2/3}x + 2^{2/3}\left(-\log\left(\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right) + 2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right) + 2\sqrt{3}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-12*x*(1 - x^3)^(2/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^(2
/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - Log[
1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] + 2*Log[
1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/36
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^6}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [A] time = 1.80758, size = 572, normalized size = 3.71

$$-\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x-\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x-2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(2/3)*x - 1/6*sqrt(6)*2^(1/6)*arctan(-1/6*2^(1/6)*(sqrt(6)*
2^(1/3)*x - 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) + 1/6*2^(2/3)*log((2^(1/3)*x + (
-x^3 + 1)^(1/3))/x) - 1/12*2^(2/3)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1
/3)*x + (-x^3 + 1)^(2/3))/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqr
t(3)*(-x^3 + 1)^(1/3))/x) - 2/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/9*log((x^
```

```
2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.613 $\int \frac{x^3}{\sqrt[3]{1-x^3}\left(1+x^3\right)} \, dx$

Optimal. Leaf size=135

$$\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/
3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + Log[x + (1 - x^3)^(
1/3)]/2
```

Rubi [A] time = 0.104934, antiderivative size = 207, normalized size of antiderivative = 1.53, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {494, 481, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\cdots}{\cdots}\right)}{\sqrt{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^2/(1 - x^3)
^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3 + Log[1 + (2^(
2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[
1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^3}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{2}\,\text{Subst}$$

$$= -\frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log}{}$$

$$= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-}}\right.$$

Mathematica [C] time = 0.0180336, size = 26, normalized size = 0.19

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])/4
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x^3}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [C] time = 11.7116, size = 1392, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-1
)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-
1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
```

```
))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(
1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)*(-1)^
(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(
-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(
1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(1/3)*x*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/3*sqrt(3)*a
rctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*log(1/24*(x*(I
*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/3))/x) - 1/6*
log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.614 $\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.057282, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x\right.}{6\sqrt[3]{2}}$$
$$= -\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$
$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

Mathematica [A] time = 0.0663128, size = 112, normalized size = 1.27

$$\frac{-\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)+2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + (
2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^
(1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] time = 21.9704, size = 660, normalized size = 7.5

$$-\frac{1}{18}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-16x^5+\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.615 $\int \frac{1}{x^3 \sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal. Leaf size=105

$$-\frac{(1-x^3)^{2/3}}{2x^2}+\frac{\log(x^3+1)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(2/3)/(2*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0715571, antiderivative size = 139, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$-\frac{(1-x^3)^{2/3}}{2x^2}+\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(2*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1+x^3}{x^3\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x\right.$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2}{}\right.}{\sqrt[3]{2}}$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

Mathematica [C] time = 0.360042, size = 82, normalized size = 0.78

$$-\frac{\left(-6x^6+4x^3+2\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2x^3}{x^3-1}\right)-3x^3\left(x^3+1\right)\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{2x^3}{x^3-1}\right)}{4x^2\left(1-x^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((2 + 4*x^3 - 6*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] -
3*x^3*(1 + x^3)*Hypergeometric2F1[4/3, 2, 7/3, (2*x^3)/(-1 + x^3)])/(4*x^2*
(1 - x^3)^(4/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(x^3+1\right)}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

Fricas [B] time = 22.0639, size = 841, normalized size = 8.01

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^2\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^2*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/
3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3)*
(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^
6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1/3)*x^
2*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x^3
```

```
+ 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*x^2*log(-(3*2^
(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6
- 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 18*
(-x^3 + 1)^(2/3))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

3.616 $\int \frac{1}{x^6\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=124

$$\frac{(1-x^3)^{2/3}}{5x^2}-\frac{(1-x^3)^{2/3}}{5x^5}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(2/3)/(5*x^5) + (1 - x^3)^(2/3)/(5*x^2) - ArcTan[(1 - (2*2^(1/3)
*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3))
+ Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rubi [A] time = 0.0882245, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{(1-x^3)^{5/3}}{5x^5}-\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(5/3)/(5*x^5) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Subst}\left(\int \frac{(1+x^3)^2}{x^6(1+2x^3)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{x^6}+\frac{1}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{(1-x^3)^{5/3}}{5x^5} + \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{(1-x^3)^{5/3}}{5x^5} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \right.$$
$$= -\frac{(1-x^3)^{5/3}}{5x^5} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}}{\sqrt[3]{1-}}\right.}{\sqrt[3]{2}}$$
$$= -\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

Mathematica [A] time = 5.13696, size = 123, normalized size = 0.99

$$\frac{-\log\left(\frac{2^{2/3}x^2}{(x^3-1)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)}{6\sqrt[3]{2}} - \frac{(1-x^3)^{5/3}}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(5/3)/(5*x^5) + (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)
^(1/3))/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1
 + x^3)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/(6*2^(1/3))
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{1}{x^{6}\left(x^{3}+1\right)} \frac{1}{\sqrt[3]{-x^{3}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

Fricas [B] time = 22.0348, size = 726, normalized size = 5.85

$$-\frac{10\sqrt{6}2^{\frac{1}{6}}x^{5}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^{9}-111x^{6}+33x^{3}-1\right)+12\sqrt{6}\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}\right)-10\cdot 2^{\frac{2}{3}}x^{5}\log\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/180*(10*sqrt(6)*2^(1/6)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7
 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3
 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105
*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2
^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*x^5*log((3*
```

```
2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(
2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(x^3 - 1)*(-x^3 + 1)
^(2/3))/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}} x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

3.617 $\int \frac{1}{x^9 \sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal. Leaf size=141

$$-\frac{17(1-x^3)^{2/3}}{40x^2}+\frac{(1-x^3)^{2/3}}{20x^5}-\frac{(1-x^3)^{2/3}}{8x^8}+\frac{\log(x^3+1)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(2/3)/(8*x^8) + (1 - x^3)^(2/3)/(20*x^5) - (17*(1 - x^3)^(2/3))/
(40*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqr
t[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2
^(1/3))
```

Rubi [A] time = 0.0920679, antiderivative size = 175, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{(1-x^3)^{8/3}}{8x^8}-\frac{(1-x^3)^{5/3}}{5x^5}-\frac{(1-x^3)^{2/3}}{2x^2}+\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(2*x^2) - (1 - x^3)^(5/3)/(5*x^5) - (1 - x^3)^(8/3)/(8*x^8
) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^
(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{\left(1+x^3\right)^3}{x^9\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{x^9}+\frac{1}{x^6}+\frac{1}{x^3}+\frac{1}{-1-2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\text{Subst}\left(\int \frac{1}{-1-2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{3}\text{Subs}$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}-\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\right.$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-$$

Mathematica [A] time = 5.09531, size = 133, normalized size = 0.94

$$\frac{\log\left(\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}-\frac{\left(1-x^3\right)^{2/3}\left(17x^6-2x^3+5\right)}{40x^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(2/3)*(5 - 2*x^3 + 17*x^6))/(40*x^8) + (-2*Sqrt[3]*ArcTan[(-1 +
 (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Log[1 + (2^(2/3)*x^2)/(-1 + x^3
)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^
(1/3)])/(6*2^(1/3))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^9\left(x^3+1\right)}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

Fricas [B] time = 21.6347, size = 876, normalized size = 6.21

$$-\frac{20\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^8\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(20*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^8*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(
2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3
)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*
```

```
x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 20*2^(2/3)*(-1)^(1/3)
*x^8*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x
^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 10*2^(2/3)*(-1)^(1/3)*x^8*log(
-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(1
9*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
 + 9*(17*x^6 - 2*x^3 + 5)*(-x^3 + 1)^(2/3))/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^9 \sqrt[3]{-(x-1)(x^2+x+1)}(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

3.618 $\int \frac{x^7}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=271

$$-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{1}{4}\left(1-x^3\right)^{2/3}x^2+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}$$

```
[Out] -(x^2*(1 - x^3)^(2/3))/4 + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3]
)/4 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 0.0167184, antiderivative size = 26, normalized size of antiderivative = 0.1, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^8*AppellF1[8/3, 1/3, 1, 11/3, x^3, -x^3])/8
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0273191, size = 40, normalized size = 0.15

$$\frac{1}{4}x^2\left(F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)-\left(1-x^3\right)^{2/3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(-(1 - x^3)^(2/3) + AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]))/4
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^7}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^7}{x^6-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)*x^7/(x^6 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.619 $\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=254

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-\ldots}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3
])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sq
rt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x
)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)
^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 0.0162367, antiderivative size = 26, normalized size of antiderivative = 0.1, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0176386, size = 26, normalized size = 0.1

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x^4}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^4}{x^6-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)*x^4/(x^6 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.620 $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal. Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+$$

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[
3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 0.0125221, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)} dx = \frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0133988, size = 26, normalized size = 0.11

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{x^3+1}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Fricas [B] time = 16.7047, size = 1049, normalized size = 4.5

$$-\frac{1}{36}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+12\sqrt{6}(-1)^{\frac{1}{3}}\left(x^{16}-33x^{13}+110\right.\right.}{6\left(x^{18}-102x^{15}+447x^{12}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{-(x-1)\left(x^{2}+x+1\right)}(x+1)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

3.621 $\int \frac{1}{x^2 \sqrt[3]{1-x^3}\left(1+x^3\right)} dx$

Optimal. Leaf size=270

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\ldots$$

```
[Out] -((1 - x^3)^(2/3)/x) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqr
t[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sq
rt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2
- Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^
3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)]/(4*2^(1/3))
```

Rubi [C] time = 0.0172514, antiderivative size = 24, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(AppellF1[-1/3, 1/3, 1, 2/3, x^3, -x^3]/x)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = -\frac{F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Mathematica [C] time = 0.0475008, size = 67, normalized size = 0.25

$$-\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(2/3)/x) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - (x^5*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(x^3+1\right)}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^8-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)/(x^8 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

3.622 $\int \frac{1}{x^5 \sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal. Leaf size=289

$$\frac{1}{4}x^2 {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-x^3)^{2/3}}{2x}-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\right.}{4\sqrt[3]{2}}$$

```
[Out] -(1 - x^3)^(2/3)/(4*x^4) + (1 - x^3)^(2/3)/(2*x) + ArcTan[(1 - (2*2^(1/3)*(
1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + (x^2*Hypergeometri
c2F1[1/3, 2/3, 5/3, x^3])/4 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 +
 (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(
6*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-
1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 0.0181233, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -AppellF1[-4/3, 1/3, 1, -1/3, x^3, -x^3]/(4*x^4)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = -\frac{F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Mathematica [C] time = 0.0395003, size = 76, normalized size = 0.26

$$\frac{2x^9F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)+15x^6F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+5\left(1-x^3\right)^{2/3}\left(2x^3-1\right)}{20x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (5*(1 - x^3)^(2/3)*(-1 + 2*x^3) + 15*x^6*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] + 2*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/(20*x^4)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(x^3+1\right)}\frac{1}{\sqrt[3]{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^{11}-x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)/(x^11 - x^5), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt[3]{-(x-1)\left(x^2+x+1\right)}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

3.623 $\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=125

$$-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(1/3) + (1 - x^3)^(4/3)/4 - (1 - x^3)^(7/3)/7 + ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3))
- Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0949, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 57, 617, 204, 31}

$$-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + (1 - x^3)^(4/3)/4 - (1 - x^3)^(7/3)/7 + ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3))
- Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{1}{(1-x)^{2/3}} - \sqrt[3]{1-x} + (1-x)^{4/3} - \frac{1}{(1-x)^{2/3}(1+x)}\right)dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} - \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \cdots$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\cdots\right.}{2\ 2^{2/3}}$$

Mathematica [A] time = 0.067436, size = 151, normalized size = 1.21

$$\frac{1}{84}\left(-12\sqrt[3]{1-x^3}x^6 + 3\sqrt[3]{1-x^3}x^3 - 75\sqrt[3]{1-x^3} - 14\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + 7\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 14\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-75*(1 - x^3)^(1/3) + 3*x^3*(1 - x^3)^(1/3) - 12*x^6*(1 - x^3)^(1/3) + 14*
2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 14*2^(1/3)*
Log[2^(1/3) - (1 - x^3)^(1/3)] + 7*2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3)
+ (1 - x^3)^(2/3)])/84
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^11/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [A] time = 1.43818, size = 161, normalized size = 1.29

$$-\frac{1}{7}\left(-x^3+1\right)^{\frac{7}{3}}+\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/7*(-x^3 + 1)^(7/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Fricas [A] time = 1.52061, size = 463, normalized size = 3.7

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/28*(4*x^6 - x^3 + 25)*(-x^3 + 1)^(1/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.624 $\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=98

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] (1 - x^3)^(4/3)/4 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
2/3))
```

Rubi [A] time = 0.0749959, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 57, 617, 204, 31}

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```


```
[Out] (1 - x^3)^(4/3)/4 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\sqrt[3]{1-x}+\frac{1}{(1-x)^{2/3}(1+x)}\right)dx, x, x^3\right)$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x,\right.}{2\sqrt[3]{2}}$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right.}{2^{2/3}}$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] time = 0.036311, size = 135, normalized size = 1.38

$$\frac{1}{12}\left(-3\sqrt[3]{1-x^3}x^3+3\sqrt[3]{1-x^3}+2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}}{\ }\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(1/3) - 3*x^3*(1 - x^3)^(1/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
] - 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^8}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^8/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [A] time = 1.41675, size = 131, normalized size = 1.34

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A] time = 1.53393, size = 346, normalized size = 3.53

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1
/4*(x^3 - 1)*(-x^3 + 1)^(1/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.625 $\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=95

$$-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(1/3) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*S
qrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2
/3))
```

Rubi [A] time = 0.0617366, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 57, 617, 204, 31}

$$-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*S
qrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2
/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1}\right.}{2\sqrt[3]{2}}$$
$$= -\sqrt[3]{1-x^3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}$$
$$= -\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] time = 0.0284433, size = 118, normalized size = 1.24

$$\frac{1}{12}\left(-12\sqrt[3]{1-x^3} - 2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + \sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3
) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \frac{x^5}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [A] time = 1.43109, size = 131, normalized size = 1.38

$$\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Fricas [A] time = 1.54081, size = 431, normalized size = 4.54

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 (-x^3 + 1)^(1/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.626 $\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal. Leaf size=83

$$-\frac{\log(x^3+1)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) - Log[1
+ x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0555175, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 57, 617, 204, 31}

$$-\frac{\log(x^3+1)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```


```
[Out] -(ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) - Log[1
+ x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0229493, size = 94, normalized size = 1.13

$$
-\frac{-2\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[2^(1/3) -
 (1 - x^3)^(1/3)] + Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/(6*
2^(2/3))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{x^2}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [A] time = 1.42407, size = 116, normalized size = 1.4

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Fricas [A] time = 1.51327, size = 301, normalized size = 3.63

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.627 $\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=137

$$\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] -(ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1
 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(2
/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/
3))
```

Rubi [A] time = 0.0896032, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 57, 618, 204, 31, 617}

$$\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1
 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(2
/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/
3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
```

```
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{2/3}x(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) - \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\,2^{2/3}} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+x+x^2}\,dx\right.$$
$$= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\,2^{2/3}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}} - \frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{}$$
$$= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\,2^{2/3}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

Mathematica [A] time = 0.03803, size = 179, normalized size = 1.31

$$\frac{1}{12}\left(4\log\left(1-\sqrt[3]{1-x^3}\right) - 2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + \sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) - 2\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{1-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[1 - (1 - x^3)^(1/3)] - 2
*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)
^(1/3) + (1 - x^3)^(2/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/1
2
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^3+1\right)}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)
```

Fricas [A] time = 1.5104, size = 605, normalized size = 4.42

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\Bigl(\frac{1}{6}\cdot 4^{\frac{1}{6}}\Bigl(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\,(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\Bigr)\Bigr)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\Bigl(-4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\,(-x^3+1)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}\,(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((
-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.628 $\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=158

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(}{6}$$

```
[Out] -(1 - x^3)^(1/3)/(3*x^3) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[
3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log
[x]/6 - Log[1 + x^3]/(6*2^(2/3)) - Log[1 - (1 - x^3)^(1/3)]/6 + Log[2^(1/3)
 - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.102788, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 57, 618, 204, 31, 617}

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3)/(3*x^3) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[
3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log
[x]/6 - Log[1 + x^3]/(6*2^(2/3)) - Log[1 - (1 - x^3)^(1/3)]/6 + Log[2^(1/3)
 - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x^2(1+x)}\,dx,x,x^3\right)$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{1}{3}\,\text{Subst}\left(\int \frac{\frac{1}{3}-\frac{2x}{3}}{(1-x)^{2/3}x(1+x)}\,dx,x,x^3\right)$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx,x,x^3\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,x,x^3\right)$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\log(x)}{6}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^3}\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{}{1+}\right.$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\log(x)}{6}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{1}{3}\,\text{Subs}$$
$$= -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x)}{6}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\right.$$

Mathematica [A] time = 0.066116, size = 196, normalized size = 1.24

$$\frac{1}{36}\left(-\frac{12\sqrt[3]{1-x^3}}{x^3}-4\log\left(1-\sqrt[3]{1-x^3}\right)+6\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-3\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)+2\log\left(\left(1-\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[
3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 4*L
og[1 - (1 - x^3)^(1/3)] + 6*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] - 3*2^(1
/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + 2*Log[1 + (1 - x^3
)^(1/3) + (1 - x^3)^(2/3)])/36
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(x^3+1\right)}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)
```

Fricas [A] time = 1.59326, size = 560, normalized size = 3.54

$$-\frac{12\cdot 4^{\frac{1}{6}}\sqrt{3}x^3\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)+3\cdot 4^{\frac{2}{3}}x^3\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)-6\cdot 4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(12*4^(1/6)*sqrt(3)*x^3*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1
)^(1/3) + 4^(1/3)*sqrt(3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3) +
2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 + 1)
^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3))
- 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3 + 1)
^(1/3) - 1) + 24*(-x^3 + 1)^(1/3))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.629 $\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=160

$$-\frac{1}{3}\sqrt[3]{1-x^3}x^2+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(x^2*(1 - x^3)^(1/3))/3 + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*S
qrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt
[3]) + Log[1 + x^3]/(6*2^(2/3)) + Log[-x - (1 - x^3)^(1/3)]/6 - Log[-(2^(1/
3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.162947, antiderivative size = 228, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {494, 470, 584, 634, 618, 204, 628, 292, 31, 617}

$$-\frac{1}{3}\sqrt[3]{1-x^3}x^2-\frac{1}{18}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+\right.}{3\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(x^2*(1 - x^3)^(1/3))/3 + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*S
qrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt
[3]) - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/18 + Log[1 + x/(1 -
 x^3)^(1/3)]/9 + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x
^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^7}{\left(1+x^3\right)^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{x\left(2+x^3\right)}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int\left(\frac{1}{3(1+x)} + \frac{-1-x}{3\left(1-x+x^2\right)} + \frac{3x}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{-1-x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \text{Subst}\left(\int\right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{6}\text{Subst}\left(\right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right.$$
$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) +$$

Mathematica [C] time = 0.0550037, size = 78, normalized size = 0.49

$$\frac{1}{15}x^2\left(x^3\left(-F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};x^3,-x^3\right)\right) + \frac{5\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{2x^3}{x^3+1}\right)}{\left(x^3+1\right)^{2/3}} - 5\sqrt[3]{1-x^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(-5*(1 - x^3)^(1/3) - x^3*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3] + (5*H
ypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)])/(1 + x^3)^(2/3)))/15
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{x^{7}}{x^{3}+1}\left(-x^{3}+1\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{7}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [A] time = 1.58954, size = 698, normalized size = 4.36

$$-\frac{1}{3}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}+\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}x\right)}{6\,x}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\,x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(1/3)*x^2 + 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/
6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 4^(1/3)*sqrt(3)*x)/x) + 1
/12*4^(2/3)*(-1)^(1/3)*log(-(4^(2/3)*(-1)^(1/3)*x - 2*(-x^3 + 1)^(1/3))/x)
- 1/24*4^(2/3)*(-1)^(1/3)*log((2*4^(1/3)*(-1)^(2/3)*x^2 + 4^(2/3)*(-1)^(1/3
```

```
)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-1/3*(
sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^(1/3))
/x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.630 $\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=139

$$-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/
3)) - Log[-x - (1 - x^3)^(1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*
2^(2/3))
```

Rubi [A] time = 0.103182, antiderivative size = 207, normalized size of antiderivative = 1.49, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {494, 481, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{6}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\cdots}{\cdots}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^2/(1 - x^3)
^(2/3) - x/(1 - x^3)^(1/3)]/6 - Log[1 + x/(1 - x^3)^(1/3)]/3 - Log[1 + (2^(
2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[
1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\text{Subst}}{\ }$$

$$= -\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{2}\text{Su}$$

$$= \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{lo}}{\ }$$

$$= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log\left(1+\frac{\ }{\sqrt[3]{1}}\right.$$

Mathematica [C] time = 0.0338653, size = 26, normalized size = 0.19

$$\frac{1}{5}x^5 F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3])/5
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x^4}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [A] time = 1.54412, size = 555, normalized size = 3.99

$$\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}x-4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\,x}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}\left(-x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(4^(1/3)*sqrt(3)*x - 4^(2/3)*sqrt(3
)*(-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/
x) - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3
 + 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 +
1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1
)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.631 $\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=88

$$\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
Log[1 + x^3]/(6*2^(2/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0611541, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= -\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}$$

$$= \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

Mathematica [C] time = 0.0066669, size = 38, normalized size = 0.43

$$\frac{x^2\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{2x^3}{x^3+1}\right)}{2\left(x^3+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^2*Hypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)])/(2*(1 + x^3)^(2/3
))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [B] time = 13.8144, size = 797, normalized size = 9.06

$$-\frac{1}{18}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(19\,x^8-16\,x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-12\sqrt{3}\,(-1)^{\frac{1}{3}}\left(5\,x^7+4\,x^4-x\right)\left(-x^3+1\right)\right.}{6\left(109\,x^9-105\,x^6+3\,x^3+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1
)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/36*4^(2/3)*(-1)^(1/3)*log(-
(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 + 1) -
 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/72*4^(2/3)*(-1)^(1/3)*log((6*4^(1/3)*
(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6 - 16*x
^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.632 $\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[-(2^(1/3)*x) - (1 - x^3)
^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0718635, antiderivative size = 137, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
(2^(2/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1
- x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3)
)
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1+x^3}{x^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2\cdots}\right.}{2\sqrt[3]{\cdots}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\cdots}{\sqrt[3]{\cdots}}\right.}{2^{2/3}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

Mathematica [C] time = 0.370987, size = 81, normalized size = 0.79

$$-\frac{5\left(-3x^6+x^3+2\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2x^3}{x^3-1}\right)-12x^3\left(x^3+1\right){}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{2x^3}{x^3-1}\right)}{10x\left(1-x^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(5*(2 + x^3 - 3*x^6)*Hypergeometric2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3)] -
12*x^3*(1 + x^3)*Hypergeometric2F1[5/3, 2, 8/3, (2*x^3)/(-1 + x^3)])/(10*x*
(1 - x^3)^(5/3))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(x^{3}+1\right)}\left(-x^{3}+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

Fricas [B] time = 13.694, size = 694, normalized size = 6.74

$$\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}x\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+12\,\sqrt{3}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\left(71\,x^{9}-111\,x^{6}+33\,x^{3}-1\right)\right)}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right)+2\cdot 4^{\frac{2}{3}}x\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(-x\cdots\right.}{\cdots}\right.}{72\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(4*4^(1/6)*sqrt(3)*x*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(19*x^8 - 1
6*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*sqrt(3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(
2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6
+ 3*x^3 + 1)) + 2*4^(2/3)*x*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 +
 1)^(2/3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x*log((6*4^(1/3)*(5*x
```

```
^4 - x)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 72*(-x^3 + 1)^(1/3))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

3.633 $\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt[3]{1-x^3}}{4x}-\frac{\sqrt[3]{1-x^3}}{4x^4}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(1 - x^3)^(1/3)/(4*x^4) + (1 - x^3)^(1/3)/(4*x) - ArcTan[(1 - (2*2^(1/3)*x
)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3)) -
Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rubi [A] time = 0.0844884, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{4/3}}{4x^4}+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(4/3)/(4*x^4) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(2/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{\left(1+x^3\right)^2}{x^5\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{x^5}+\frac{x}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} + \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$
$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+\cdots}\right.}{2\cdots}$$
$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\cdots\right.}{2^{2/3}}$$
$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

Mathematica [C] time = 8.51815, size = 145, normalized size = 1.17

$$-\frac{81\left(x^3+1\right)^2 x^3\text{HypergeometricPFQ}\left(\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{x^3-1}\right)+216\left(x^9+x^6\right)\text{HypergeometricPFQ}\left(\left\{\frac{2}{3},2,2\right\},\left\{\cdots\right.\right.}{60x^4\left(1-x^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(5*(-1 - 9*x^3 + x^6 + 9*x^9 + (4 - 13*x^3 - 20*x^6 + 9*x^9)*Hypergeometri
c2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3)]) + 216*(x^6 + x^9)*HypergeometricPFQ[
{2/3, 2, 2}, {1, 8/3}, (2*x^3)/(-1 + x^3)] + 81*x^3*(1 + x^3)^2*Hypergeomet
ricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, (2*x^3)/(-1 + x^3)])/(60*x^4*(1 - x^3)^
(5/3))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^{5}\left(x^{3}+1\right)}\left(-x^{3}+1\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

Fricas [B] time = 13.0231, size = 856, normalized size = 6.9

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^{4}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-12\sqrt{3}(-1)^{\frac{1}{3}}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\left(71\,x^{9}-111\,x^{6}+33\,x^{3}-1\right)\right)}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(
1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x
```

```
^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x
^4*log(-(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^
3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^4*log((6*4
^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6
 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18
*(x^3 - 1)*(-x^3 + 1)^(1/3))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}} (x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

3.634 $\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=291

$$-\frac{1}{2}\sqrt[3]{1-x^3}x+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}$$

```
[Out] -(x*(1 - x^3)^(1/3))/2 + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/S
qrt[3]]/(2^(2/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(6*2^
(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1
 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*
2^(2/3)) - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1
 - x^3)^(1/3)]/(12*2^(2/3))
```

Rubi [C] time = 0.0189698, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^7*AppellF1[7/3, 2/3, 1, 10/3, x^3, -x^3])/7
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.118928, size = 115, normalized size = 0.4

$$\frac{1}{2}x\sqrt[3]{1-x^3}\left(-\frac{4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x
^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3,
 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/2
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^6}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [A] time = 17.7998, size = 949, normalized size = 3.26

$$\frac{1}{36}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-48\sqrt{3}\left(x^{14}-2\,x^{11}-6\,x^{8}-2\right.}{6\left(x^{18}-102\,x^{15}+447\,x^{12}-628\,x^{9}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(x^16 - 33*x^13
 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*sqrt(3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*
x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x
^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3 + 1)^
(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^3 + 1
))/(x^6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 + x^2)*(
-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10
- 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)
) - 1/2*(-x^3 + 1)^(1/3)*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{6}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.635 $\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal. Leaf size=294

$$\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right.}{12\ 2^{2/3}}$$

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3
])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sq
rt[3]) + (x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 - Log[2^(2/3) - (1 - x
)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3
) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*(1 -
x))/(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3
) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rubi [C] time = 0.0185345, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0327678, size = 26, normalized size = 0.09

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^3}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}x^3}{x^6-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(1/3)*x^3/(x^6 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.636 $\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=293

$$\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+\right.}{12\ 2^{2/3}}$$

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[
3]) + (x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 + Log[2^(2/3) - (1 - x)/(
1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) -
 (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*(1 - x))
/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) +
 (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rubi [C] time = 0.0088603, antiderivative size = 21, normalized size of antiderivative = 0.07, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.118103, size = 111, normalized size = 0.38

$$-\frac{4xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{(1-x^3)^{2/3}(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3])/((1 - x^3)^(2/3)*(1 + x^3)*(-4
*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, 2/3, 2, 7/3,
x^3, -x^3] - 2*AppellF1[4/3, 5/3, 1, 7/3, x^3, -x^3])))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+1}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^6-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(1/3)/(x^6 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

3.637 $\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=294

$$-\frac{\sqrt[3]{1-x^3}}{2x^2}-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\log\left(\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}-\ldots$$

```
[Out] -(1 - x^3)^(1/3)/(2*x^2) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) - Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(6*
2^(2/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/
(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(
3*2^(2/3)) + Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/
(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rubi [C] time = 0.0188149, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};x^3,-x^3\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -AppellF1[-2/3, 2/3, 1, 1/3, x^3, -x^3]/(2*x^2)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx = -\frac{F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};x^3,-x^3\right)}{2x^2}$$

Mathematica [C] time = 0.0868362, size = 120, normalized size = 0.41

$$\frac{\sqrt[3]{1-x^3}\left(\frac{4x^3F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-1\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 +
 x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/
3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/(2*x^2)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(x^3+1\right)}\left(-x^3+1\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

Fricas [A] time = 19.6917, size = 1095, normalized size = 3.72

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^2\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x\right)\left(-x^3+1\right)^{\frac{1}{3}}+48\sqrt{3}(-1)^{\frac{1}{3}}\left(x^{14}-2\,x^{11}-6\,x^{8}-2\,x^{5}+x^{2}\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\cdots\right.}{6\left(x^{18}-102\,x^{15}+447\,x^{12}-628\,x^{9}+447\,x^{6}-102\,x^{3}+1\right)}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/144*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^2*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1
)^(1/3) + 48*sqrt(3)*(-1)^(1/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3
 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^
6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3
+ 1)) + 4^(2/3)*(-1)^(1/3)*x^2*log((24*4^(1/3)*(-1)^(2/3)*(x^8 - 4*x^5 + x^
2)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 +
 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x
^6 + 4*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x^2*log(-(12*(-x^3 + 1)^(2/3)*x^2 +
 3*4^(2/3)*(-1)^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(-1)^(2/3)*(x^6
+ 2*x^3 + 1))/(x^6 + 2*x^3 + 1)) + 72*(-x^3 + 1)^(1/3))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

3.638 $\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} dx$

Optimal. Leaf size=141

$$\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)
^(8/3)/8 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]
) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [A] time = 0.113576, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 87, 43, 627, 51, 55, 617, 204, 31}

$$\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)
^(8/3)/8 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]
) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^4}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}}-\frac{x^2}{\sqrt[3]{1-x}}+\frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\right)dx, x, x^3\right)$$
$$= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}}\,dx, x, x^3\right)+\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\,dx, x, x^3\right)$$
$$= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}}-2(1-x)^{2/3}+(1-x)^{5/3}\right)dx, x, x^3\right)+\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{}{(1-}\right.$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right.$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{}}\right.$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}$$

Mathematica [C] time = 0.0234512, size = 53, normalized size = 0.38

$$\frac{20\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-5x^9-x^6-23x^3+29}{40\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (29 - 23*x^3 - x^6 - 5*x^9 + 20*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2
])/(40*(1 - x^3)^(1/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^{14}}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^14/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [A] time = 1.42169, size = 173, normalized size = 1.23

$$\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}}+\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/8*(-x^3 + 1)^(8/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^(1/
3) + (-x^3 + 1)^(1/3)) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Fricas [A] time = 1.78337, size = 406, normalized size = 2.88

$$\frac{10\sqrt{6}2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+10}{120\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/120*(10*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x
^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x
^3 + 1)^(1/3)) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.639 $\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=130

$$-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2
^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [A] time = 0.101246, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 87, 43, 783, 78, 55, 617, 204, 31}

$$-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2
^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{x}{\sqrt[3]{1-x}}-\frac{x}{\sqrt[3]{1-x}\left(-1+x^2\right)}\right)dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}}\,dx, x, x^3\right)\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\left(-1+x^2\right)}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int\left(\frac{1}{\sqrt[3]{1-x}}-(1-x)^{2/3}\right)dx, x, x^3\right)\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(-1-x)(1-x)^{4/3}}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt[3]{1-x}}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\right.}{4\sqrt[3]{}}$$

Mathematica [C] time = 0.0167046, size = 48, normalized size = 0.37

$$\frac{-5\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-2x^6-x^3+13}{10\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

[Out] (13 - x^3 - 2*x^6 - 5*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(10*(1 - x^3)^(1/3))

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(-x^3+1)^(4/3)/(x^3+1),x)

[Out] int(x^11/(-x^3+1)^(4/3)/(x^3+1),x)

Maxima [A] time = 1.43204, size = 161, normalized size = 1.24

$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")

[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)

Fricas [A] time = 1.7085, size = 500, normalized size = 3.85

$$-\frac{10\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+5\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)\right.}{120\left(x^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/120*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(
6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 5*2^(2/3)*(-1)^(1/3)*(
x^3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x
^3 + 1)^(2/3)) - 10*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) +
(-x^3 + 1)^(1/3)) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.640 $\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=115

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [A] time = 0.0901354, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 87, 627, 51, 55, 617, 204, 31}

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}}+\frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\right)dx, x, x^3\right)$$
$$= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\,dx, x, x^3\right)$$
$$= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Su}}{}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\right.}{2\sqrt[3]{2}}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [C] time = 0.0126791, size = 41, normalized size = 0.36

$$\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-x^3+1}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] (1 - x^3 + Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(2*(1 - x^3)^(1/3)
)
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \frac{x^8}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^8/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [A] time = 1.47437, size = 146, normalized size = 1.27

$$\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/3)
+ 1/2/(-x^3 + 1)^(1/3)
```

Fricas [A] time = 1.8143, size = 377, normalized size = 3.28

$$\frac{2\sqrt{6}2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) + 12*(x^3 - 2)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.641 $\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*(1 - x^3)^(1/3)) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rubi [A] time = 0.0683699, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\right.}{4\sqrt[3]{}}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] time = 0.072203, size = 95, normalized size = 0.95

$$\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}} + 2^{2/3}\log\left(x^3+1\right) - 3\ 2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) - 2\ 2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
])/24
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{x^5}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [A] time = 1.44085, size = 131, normalized size = 1.31

$$-\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

Fricas [B] time = 1.81803, size = 468, normalized size = 4.68

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\right.}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.642 $\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rubi [A] time = 0.0683153, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {444, 51, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```


```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx\right.}{4\sqrt[3]{2}}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [C] time = 0.0071791, size = 34, normalized size = 0.34

$$\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2]/(2*(1 - x^3)^(1/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{x^2}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [A] time = 1.43897, size = 131, normalized size = 1.31

$$\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

Fricas [A] time = 1.7583, size = 363, normalized size = 3.63

$$\frac{2\sqrt{6}2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.643 $\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=154

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\log$$

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - A
rcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/2
 + Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (
1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [A] time = 0.110702, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 85, 156, 55, 618, 204, 31, 617}

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\log$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - A
rcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/2
 + Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (
1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
```

```
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6}\,\text{Subst}\left(\int \frac{2+x}{\sqrt[3]{1-x}x(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\,\text{Su}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}}{}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1\right.$$

Mathematica [C] time = 0.0150435, size = 54, normalized size = 0.35

$$\frac{2\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-x^3\right) - {}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2] + 2*Hypergeometric2F1[-1/3,
1, 2/3, 1 - x^3])/(2*(1 - x^3)^(1/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)x}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)
```

Fricas [A] time = 1.80998, size = 701, normalized size = 4.55

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) - 8*sqrt(3)*(x^3 - 1)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/
3*sqrt(3)) + 4*(x^3 - 1)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*(
x^3 - 1)*log((-x^3 + 1)^(1/3) - 1) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.644 $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)} dx$

Optimal. Leaf size=175

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{5}{6\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}}{}\right.}{2\sqrt[3]{2}}$$

```
[Out] 5/(6*(1 - x^3)^(1/3)) - 1/(3*x^3*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)
^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]
]/(2*2^(1/3)*Sqrt[3]) - Log[x]/6 - Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 -
 x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [A] time = 0.115659, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 156, 51, 55, 618, 204, 31, 617}

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{5}{6\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}}{}\right.}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 5/(6*(1 - x^3)^(1/3)) - 1/(3*x^3*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)
^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]
]/(2*2^(1/3)*Sqrt[3]) - Log[x]/6 - Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 -
 x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{4/3}x^2(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{-\frac{1}{3}-\frac{4x}{3}}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{4/3}x}\,dx, x, x^3\right) + \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3$$
$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right) + \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,d\right.$$
$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{6}$$
$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-\right.}{4\sqrt[3]{2}}$$
$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6} - \frac{\log\left(1+x\right.}{12\sqrt[3]{2}}$$

Mathematica [C] time = 0.0209776, size = 64, normalized size = 0.37

$$\frac{3x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)+2x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-x^3\right)-2}{6x^3\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-2 + 3*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2] + 2*x^3*Hypergeome
tric2F1[-1/3, 1, 2/3, 1 - x^3])/(6*x^3*(1 - x^3)^(1/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{3}+1\right) x^{4}} \left(-x^{3}+1\right)^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)
```

Fricas [A] time = 1.83717, size = 633, normalized size = 3.62

$$6\,\sqrt{6}2^{\frac{1}{6}}\left(x^{6}-x^{3}\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\,\sqrt{6}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}\left(x^{6}-x^{3}\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(x^6 - x^3)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*(x^6 - x^3)*log(2^(2/3) + 2^(1/3)*(
-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*(x^6 - x^3)*log(-2^(1/3) +
(-x^3 + 1)^(1/3)) + 8*sqrt(3)*(x^6 - x^3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/
3) + 1/3*sqrt(3)) - 4*(x^6 - x^3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
 1) + 8*(x^6 - x^3)*log((-x^3 + 1)^(1/3) - 1) - 12*(5*x^3 - 2)*(-x^3 + 1)^(
2/3))/(x^6 - x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.645 $\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=174

$$\frac{x^4}{2\sqrt[3]{1-x^3}}+\frac{5}{6}\left(1-x^3\right)^{2/3}x+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{1}{6}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}}{2}$$

```
[Out] x^4/(2*(1 - x^3)^(1/3)) + (5*x*(1 - x^3)^(2/3))/6 + ArcTan[(1 - (2*x)/(1 -
x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3
))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[-(2^(1/3)
*x) - (1 - x^3)^(1/3)]/(4*2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/6
```

Rubi [C] time = 0.0178236, antiderivative size = 26, normalized size of antiderivative = 0.15, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{10}x^{10}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^10*AppellF1[10/3, 4/3, 1, 13/3, x^3, -x^3])/10
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{10}x^{10}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.172993, size = 152, normalized size = 0.87

$$\frac{1}{72}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)-\frac{12\left(2x^3-5\right)x}{\sqrt[3]{1-x^3}}-5\,2^{2/3}\left(-\log\left(\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*x*(-5 + 2*x^3))/(1 - x^3)^(1/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^
3, -x^3] - 5*2^(2/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3)
)/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3
)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/72
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^9}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] time = 1.93425, size = 776, normalized size = 4.46

$$6\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x+2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)+6\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^
(1/3)*x + 2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3))/x) + 6*2^(2/3)*(-1)^(1/3)*
(x^3 - 1)*log((2^(1/3)*(-1)^(2/3)*x + (-x^3 + 1)^(1/3))/x) - 3*2^(2/3)*(-1)
^(1/3)*(x^3 - 1)*log(-(2^(2/3)*(-1)^(1/3)*x^2 + 2^(1/3)*(-1)^(2/3)*(-x^3 +
1)^(1/3)*x - (-x^3 + 1)^(2/3))/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt
(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1
/3))/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2
) + 12*(2*x^4 - 5*x)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**9/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.646 $\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal. Leaf size=153

$$\frac{x}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right) + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] x/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
 - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3])
- Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3
)) - Log[x + (1 - x^3)^(1/3)]/2
```

Rubi [C] time = 0.0180263, antiderivative size = 26, normalized size of antiderivative = 0.17, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^7*AppellF1[7/3, 4/3, 1, 10/3, x^3, -x^3])/7
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.138084, size = 142, normalized size = 0.93

$$\frac{1}{24}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)+\frac{12x}{\sqrt[3]{1-x^3}}+2^{2/3}\left(\log\left(\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^(
2/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Lo
g[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Lo
g[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/24
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^6}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] time = 1.85947, size = 655, normalized size = 4.28

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x-2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)-2\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)}{x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(-1/6*2^(1/6)*(sqrt(6)*2^(1/3)*x -
 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^
3 + 1)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^
(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)
*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3)
)/x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) +
 12*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.647 $\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] x/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 0.018378, antiderivative size = 38, normalized size of antiderivative = 0.36, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{x^3+1}\right)}{4\left(x^3+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)])/(4*(1 + x^3)^(4/3
))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{1+x^3}\right)}{4\left(1+x^3\right)^{4/3}}$$

Mathematica [C] time = 0.0092013, size = 38, normalized size = 0.36

$$\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{x^3+1}\right)}{4\left(x^3+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)])/(4*(1 + x^3)^(4/3
))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{x^3}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] time = 21.7687, size = 876, normalized size = 8.26

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)
*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^
(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 -
111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1
/3)*(x^3 - 1)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)
^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*(x
^3 - 1)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(
-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 +
 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.648 $\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] x/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [A] time = 0.0762032, antiderivative size = 140, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/
(1 - x^3)^(1/3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$$
$$= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\right.$$
$$= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}}{\sqrt[3]{1-}}\right.}{2\sqrt[3]{2}}$$
$$= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

Mathematica [C] time = 0.0419575, size = 100, normalized size = 0.94

$$\frac{-7\left(3x^3+4\right)\left(x^3-1\right)^2 {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2x^3}{x^3-1}\right)-12\left(x^9+x^6\right) {}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+7\left(3x^3+4\right)\left(x^3-1\right)^2}{14x^2\left(1-x^3\right)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (7*(-1 + x^3)^2*(4 + 3*x^3) - 7*(-1 + x^3)^2*(4 + 3*x^3)*Hypergeometric2F1[
1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] - 12*(x^6 + x^9)*Hypergeometric2F1[2, 7/3,
 10/3, (2*x^3)/(-1 + x^3)])/(14*x^2*(1 - x^3)^(7/3))
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [B] time = 21.7424, size = 741, normalized size = 6.99

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-2\cdot 2^{\frac{2}{3}}\left(x^3-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 -
 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^(1/3
)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^3
 - 1)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^
3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(-x^3 +
 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.649 $\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(1-x^3\right)^{2/3}}{x^2}+\frac{1}{2x^2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*x^2*(1 - x^3)^(1/3)) - (1 - x^3)^(2/3)/x^2 + ArcTan[(1 - (2*2^(1/3)*x)
/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3))
- Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 8.12141, antiderivative size = 204, normalized size of antiderivative = 1.65, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{-18\left(x^3+1\right)^2x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-54x^{12}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-84x^9\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-30x^6\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\right.}{14x^5\left(1-x^3\right)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(14 + 56*x^3 - 91*x^6 - 42*x^9 + 63*x^12 - 7*(1 - x^3)^2*(2 + 12*x^3 + 9*x
^6)*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 30*x^6*Hypergeomet
ric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 84*x^9*Hypergeometric2F1[2, 7/3,
 10/3, (-2*x^3)/(1 - x^3)] - 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^
3)/(1 - x^3)] - 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}
, (-2*x^3)/(1 - x^3)])/(14*x^5*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{14+56x^3-91x^6-42x^9+63x^{12}-7\left(1-x^3\right)^2\left(2+12x^3+9x^6\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)-3}{}$$

Mathematica [C] time = 2.11795, size = 192, normalized size = 1.55

$$\frac{18\left(x^3+1\right)^2x^6\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{2x^3}{x^3-1}\right)+54x^{12}{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+84x^9{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+}{14x^5\left(1-x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-14 - 56*x^3 + 91*x^6 + 42*x^9 - 63*x^12 + 7*(-1 + x^3)^2*(2 + 12*x^3 + 9*
x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] + 30*x^6*Hypergeome
tric2F1[2, 7/3, 10/3, (2*x^3)/(-1 + x^3)] + 84*x^9*Hypergeometric2F1[2, 7/3
, 10/3, (2*x^3)/(-1 + x^3)] + 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (2*x^
3)/(-1 + x^3)] + 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3
}, (2*x^3)/(-1 + x^3)])/(14*x^5*(1 - x^3)^(7/3))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(x^3+1\right)}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

Fricas [B] time = 23.2538, size = 900, normalized size = 7.26

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^5-x^2\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^5 - x^2)*arctan(1/6*2^(1/6)*(6*sqrt(
6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1
)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9
- 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^
(1/3)*(x^5 - x^2)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*
(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3
)*(x^5 - x^2)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(
1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/
(x^6 + 2*x^3 + 1)) + 36*(2*x^3 - 1)*(-x^3 + 1)^(2/3))/(x^5 - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

3.650 $\int \frac{1}{x^6\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=144

$$-\frac{4\left(1-x^3\right)^{2/3}}{5x^2}-\frac{7\left(1-x^3\right)^{2/3}}{10x^5}+\frac{1}{2x^5\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*x^5*(1 - x^3)^(1/3)) - (7*(1 - x^3)^(2/3))/(10*x^5) - (4*(1 - x^3)^(2/
3))/(5*x^2) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3
)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)
]/(4*2^(1/3))
```

Rubi [C] time = 8.31832, antiderivative size = 397, normalized size of antiderivative = 2.76, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{54\left(x^3+1\right)^2\left(6x^3+1\right)x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+54\left(x^3+1\right)^3x^6\,{}_4F_3\left(2,2,2,\frac{7}{3};1,1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+567x^{15}\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(28 - 182*x^3 - 476*x^6 + 819*x^9 + 378*x^12 - 567*x^15 - 28*Hypergeometri
c2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 182*x^3*Hypergeometric2F1[1/3, 1, 4
/3, (-2*x^3)/(1 - x^3)] + 476*x^6*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(
1 - x^3)] - 819*x^9*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 37
8*x^12*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 567*x^15*Hyperg
eometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 36*x^6*Hypergeometric2F1[2,
7/3, 10/3, (-2*x^3)/(1 - x^3)] + 342*x^9*Hypergeometric2F1[2, 7/3, 10/3, (-
2*x^3)/(1 - x^3)] + 972*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 -
x^3)] + 594*x^15*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 54*x
^6*(1 + x^3)^2*(1 + 6*x^3)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (-2*x^
3)/(1 - x^3)] + 54*x^6*(1 + x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1,
 10/3}, (-2*x^3)/(1 - x^3)])/(70*x^8*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{28-182x^3-476x^6+819x^9+378x^{12}-567x^{15}-28\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)+182x^3\,{}_2F_1\left(\frac{1}{3}\right.}{}$$

Mathematica [C] time = 5.01472, size = 373, normalized size = 2.59

$$\frac{-54\left(x^3+1\right)^2\left(6x^3+1\right)x^6\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{2x^3}{x^3-1}\right)-54\left(x^3+1\right)^3x^6\text{HypergeometricPFQ}\left(\left\{2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-28 + 182*x^3 + 476*x^6 - 819*x^9 - 378*x^12 + 567*x^15 + 28*Hypergeometri
c2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] - 182*x^3*Hypergeometric2F1[1/3, 1, 4
/3, (2*x^3)/(-1 + x^3)] - 476*x^6*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-
1 + x^3)] + 819*x^9*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] + 37
8*x^12*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] - 567*x^15*Hyperg
eometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] + 36*x^6*Hypergeometric2F1[2,
7/3, 10/3, (2*x^3)/(-1 + x^3)] - 342*x^9*Hypergeometric2F1[2, 7/3, 10/3, (2
*x^3)/(-1 + x^3)] - 972*x^12*Hypergeometric2F1[2, 7/3, 10/3, (2*x^3)/(-1 +
x^3)] - 594*x^15*Hypergeometric2F1[2, 7/3, 10/3, (2*x^3)/(-1 + x^3)] - 54*x
^6*(1 + x^3)^2*(1 + 6*x^3)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (2*x^3
)/(-1 + x^3)] - 54*x^6*(1 + x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1,
 10/3}, (2*x^3)/(-1 + x^3)])/(70*x^8*(1 - x^3)^(7/3))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)x^6}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

Fricas [B] time = 22.1061, size = 780, normalized size = 5.42

$$-\frac{10\sqrt{6}2^{\frac{1}{6}}\left(x^8-x^5\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-10\cdot 2^{\frac{2}{3}}\left(x^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(x^8 - x^5)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3
)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6
+ 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x
^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^3 +
1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(
2/3)*(x^8 - x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*
x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) +
 36*(8*x^6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

3.651 $\int \frac{1}{x^9(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=162

$$-\frac{49(1-x^3)^{2/3}}{40x^2}-\frac{13(1-x^3)^{2/3}}{20x^5}-\frac{5(1-x^3)^{2/3}}{8x^8}+\frac{1}{2x^8\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/(2*x^8*(1 - x^3)^(1/3)) - (5*(1 - x^3)^(2/3))/(8*x^8) - (13*(1 - x^3)^(2/
3))/(20*x^5) - (49*(1 - x^3)^(2/3))/(40*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1
 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - L
og[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rubi [C] time = 10.5945, antiderivative size = 643, normalized size of antiderivative = 3.97, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{-81x^{18}\,{}_5F_4\left(2,2,2,2,\frac{7}{3};1,1,1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-324x^{15}\,{}_5F_4\left(2,2,2,2,\frac{7}{3};1,1,1,\frac{10}{3}\right.\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(70 - 308*x^3 + 1162*x^6 + 2856*x^9 - 4914*x^12 - 2268*x^15 + 3402*x^18 -
70*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 308*x^3*Hypergeomet
ric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 1162*x^6*Hypergeometric2F1[1/3, 1
, 4/3, (-2*x^3)/(1 - x^3)] - 2856*x^9*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^
3)/(1 - x^3)] + 4914*x^12*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)
] + 2268*x^15*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 3402*x^1
8*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 66*x^6*Hypergeometri
c2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 312*x^9*Hypergeometric2F1[2, 7/3,
10/3, (-2*x^3)/(1 - x^3)] - 2268*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x
^3)/(1 - x^3)] - 6696*x^15*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^
3)] - 4050*x^18*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 27*x^
6*(1 + x^3)^2*(7 - 18*x^3 - 105*x^6)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/
3}, (-2*x^3)/(1 - x^3)] + 54*x^6*(1 - 15*x^3)*(1 + x^3)^3*HypergeometricPFQ
[{2, 2, 2, 7/3}, {1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 81*x^6*HypergeometricP
```

```
FQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 324*x^9*Hyperg
eometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 486*x
^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3
)] - 324*x^15*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3
)/(1 - x^3)] - 81*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}
, (-2*x^3)/(1 - x^3)])/(280*x^11*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{70-308x^3+1162x^6+2856x^9-4914x^{12}-2268x^{15}+3402x^{18}-70\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right.}{}$$

Mathematica [A] time = 5.16572, size = 136, normalized size = 0.84

$$\frac{1}{120}\left(5\ 2^{2/3}\left(\log\left(\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)\right)-\frac{3\left(-49x^9+23x^6+x^3\right.}{x^8\sqrt[3]{1-x^3}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 5*2^(2/3)*(-2*Sqr
t[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Log[1 + (2^(2/
3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/
3)*x)/(-1 + x^3)^(1/3)]))/120
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)x^9}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

Fricas [B] time = 22.2233, size = 933, normalized size = 5.76

$$-\frac{10\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^{11}-x^8)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-\cdots}{6(109x^9-105x^6+3x^3+1)}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^11 - x^8)*arctan(1/6*2^(1/6)*(6*sq
rt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*
(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x
^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(
-1)^(1/3)*(x^11 - x^8)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(
2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(-
1)^(1/3)*(x^11 - x^8)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/
3) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)
^(1/3))/(x^6 + 2*x^3 + 1)) + 9*(49*x^9 - 23*x^6 - x^3 - 5)*(-x^3 + 1)^(2/3)
```

```
)/(x^11 - x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^9\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

3.652 $\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=292

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}\left(1-x^3\right)^{2/3}x^2-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-\right.}{8\sqrt[3]{}}$$

```
[Out] x^5/(2*(1 - x^3)^(1/3)) + (3*x^2*(1 - x^3)^(2/3))/4 - ArcTan[(1 - (2*2^(1/3
)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(
1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeo
metric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) - Lo
g[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) +
 Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rubi [C] time = 0.0188917, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{11}x^{11}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^11*AppellF1[11/3, 4/3, 1, 14/3, x^3, -x^3])/11
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{10}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{11}x^{11}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0729359, size = 71, normalized size = 0.24

$$\frac{1}{20}x^2\left(-4x^3F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-15F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-\frac{5\left(x^3-3\right)}{\sqrt[3]{1-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*((-5*(-3 + x^3))/(1 - x^3)^(1/3) - 15*AppellF1[2/3, 1/3, 1, 5/3, x^3,
-x^3] - 4*x^3*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/20
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{x^{10}}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^{10}}{x^9-x^6-x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x^10/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**10/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.653 $\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=274

$$-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+$$

```
[Out] x^2/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (3*x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x
^3])/4 + Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/
(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 +
 (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 -
 x^3)^(1/3)]/(8*2^(1/3))
```

Rubi [C] time = 0.0183977, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^8*AppellF1[8/3, 4/3, 1, 11/3, x^3, -x^3])/8
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0635399, size = 66, normalized size = 0.24

$$\frac{1}{10}x^2\left(-3x^3F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-5F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{5}{\sqrt[3]{1-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - 3*x^3*A
ppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^7}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^7}{x^9-x^6-x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x^7/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.654 $\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=274

$$-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}-$$

```
[Out] x^2/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3
])/4 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1
 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) + Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(8*2^(1/3))
```

Rubi [C] time = 0.0178863, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 4/3, 1, 8/3, x^3, -x^3])/5
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0557329, size = 66, normalized size = 0.24

$$\frac{1}{10}x^2\left(x^3\left(-F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)\right)-5F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{5}{\sqrt[3]{1-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - x^3*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^4}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^4}{x^9-x^6-x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x^4/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.655 $\int \frac{x}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} dx$

Optimal. Leaf size=274

$$-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+ \cdots$$

```
[Out] x^2/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3
])/4 + Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(8*2^(1/3))
```

Rubi [C] time = 0.0130731, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 4/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.0320443, size = 45, normalized size = 0.16

$$\frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{10}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) - (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x}{x^3+1}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}x}{x^{9}-x^{6}-x^{3}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

3.656 $\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=292

$$-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{3\left(1-x^3\right)^{2/3}}{2x}+\frac{1}{2\sqrt[3]{1-x^3}x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-}\right.}{8\sqrt[3]{2}}$$

```
[Out] 1/(2*x*(1 - x^3)^(1/3)) - (3*(1 - x^3)^(2/3))/(2*x) - ArcTan[(1 - (2*2^(1/3
)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(
1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (3*x^2*Hyperg
eometric2F1[1/3, 2/3, 5/3, x^3])/4 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) -
Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(
1/3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3))
 + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rubi [C] time = 0.0190919, antiderivative size = 24, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(AppellF1[-1/3, 4/3, 1, 2/3, x^3, -x^3]/x)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2 \left(1-x^3\right)^{4/3} \left(1+x^3\right)} \, dx = -\frac{F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Mathematica [C] time = 0.0588202, size = 76, normalized size = 0.26

$$-\frac{3}{10}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{3x^3-2}{2\sqrt[3]{1-x^3}x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-2 + 3*x^3)/(2*x*(1 - x^3)^(1/3)) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] - (3*x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)x^2}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^{11}-x^8-x^5+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^11 - x^8 - x^5 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

3.657 $\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=308

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x}-\frac{3\left(1-x^3\right)^{2/3}}{4x^4}+\frac{1}{2\sqrt[3]{1-x^3}x^4}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-$$

```
[Out] 1/(2*x^4*(1 - x^3)^(1/3)) - (3*(1 - x^3)^(2/3))/(4*x^4) - (1 - x^3)^(2/3)/x
 + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqr
t[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*
Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 +
 x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rubi [C] time = 0.0196007, antiderivative size = 26, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -AppellF1[-4/3, 4/3, 1, -1/3, x^3, -x^3]/(4*x^4)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5 \left(1-x^3\right)^{4/3} \left(1+x^3\right)}\,dx = -\frac{F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Mathematica [C] time = 0.0811564, size = 79, normalized size = 0.26

$$-\frac{4x^9F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)+5x^6F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{5\left(-4x^6+x^3+1\right)}{\sqrt[3]{1-x^3}}}{20x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -((5*(1 + x^3 - 4*x^6))/(1 - x^3)^(1/3) + 5*x^6*AppellF1[2/3, 1/3, 1, 5/3,
x^3, -x^3] + 4*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/(20*x^4)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)x^5}\left(-x^3+1\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^{14}-x^{11}-x^8+x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] integral((-x^3 + 1)^(2/3)/(x^14 - x^11 - x^8 + x^5), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(-x**3+1)**(4/3)/(x**3+1),x)

[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)

3.658 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=264

$$\frac{\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{4b^3d^3}-\frac{\left(a+bx^3\right)^{7/3}(2ad+bc)}{7b^3d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}-\frac{c^3\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{13/3}}+$$

```
[Out] -((c^3*(a + b*x^3)^(1/3))/d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)
^(4/3))/(4*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2) + (a +
b*x^3)^(10/3)/(10*b^3*d) - (c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^3*(b*
c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rubi [A] time = 0.386141, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{4b^3d^3}-\frac{\left(a+bx^3\right)^{7/3}(2ad+bc)}{7b^3d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}-\frac{c^3\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{13/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```



```
[Out] -((c^3*(a + b*x^3)^(1/3))/d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)
^(4/3))/(4*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2) + (a +
b*x^3)^(10/3)/(10*b^3*d) - (c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^3*(b*
c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{4/3}}{b^2d^2} + \frac{(a+bx)^{7/3}}{b^2d} - \frac{c^3\sqrt[3]{a+bx}}{d^3(c+dx)}\right)\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^3d} - \frac{c^3\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx\right.}{3d^3}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^3d} + \frac{\left(c^3(b\right.}{}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^3d} - \frac{c^3\sqrt[3]{b}}{}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^3d} - \frac{c^3\sqrt[3]{b}}{}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^3d} - \frac{c^3\sqrt[3]{b}}{}$$

Mathematica [A] time = 0.534551, size = 270, normalized size = 1.02

$$\frac{\frac{105d\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3} - \frac{60d^2\left(a+bx^3\right)^{7/3}(2ad+bc)}{b^3} + \frac{42d^3\left(a+bx^3\right)^{10/3}}{b^3} - \frac{70c^3\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\,\text{l}\right.}{\sqrt[3]{d}}}{420d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-420*c^3*(a + b*x^3)^(1/3) + (105*d*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x
^3)^(4/3))/b^3 - (60*d^2*(b*c + 2*a*d)*(a + b*x^3)^(7/3))/b^3 + (42*d^3*(a
+ b*x^3)^(10/3))/b^3 - (70*c^3*(b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c - a*d)^
```

```
(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
 a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))/(420*
d^4)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77748, size = 749, normalized size = 2.84

$$-\frac{140\,\sqrt{3}b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\,\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+70\,b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b
*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
```

```
 + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/
3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c - a*d
)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*d^3*x
^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^3)*
(b*x^3 + a)^(1/3))/(b^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [A] time = 1.26662, size = 512, normalized size = 1.94

$$-\frac{\left(b^{34}c^4d^6-ab^{33}c^3d^7\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{34}cd^{10}-ab^{33}d^{11}\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(-b*c*d^2 + a*d^
3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^30*c^3*d^
6 - 35*(b*x^3 + a)^(4/3)*b^29*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^28*c*d^8 - 3
5*(b*x^3 + a)^(4/3)*a*b^28*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^27*d^9 + 40*(b*x
^3 + a)^(7/3)*a*b^27*d^9 - 35*(b*x^3 + a)^(4/3)*a^2*b^27*d^9)/(b^30*d^10)
```

3.659 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=220

$$-\frac{\left(a+bx^3\right)^{4/3}(ad+bc)}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{a+bx^3}}{d^3}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a}\right.}{2d^{10/3}}$$

```
[Out] (c^2*(a + b*x^3)^(1/3))/d^3 - ((b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*d^2) +
 (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c^2
*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rubi [A] time = 0.258599, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{4/3}(ad+bc)}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{a+bx^3}}{d^3}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a}\right.}{2d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(1/3))/d^3 - ((b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*d^2) +
 (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c^2
*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{(-bc-ad)\sqrt[3]{a+bx}}{bd^2}+\frac{(a+bx)^{4/3}}{bd}+\frac{c^2\sqrt[3]{a+bx}}{d^2(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\operatorname{Subst}\left(\int\frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}-\frac{\left(c^2(bc-ad)\right)\operatorname{Subst}\left(\int\frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right.}{3d^3}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{\left(c^2\sqrt[3]{bc-ad}\right)\operatorname{Su}}{}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\right.}{}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}}+\frac{c^2\sqrt[3]{bc-}}{}$$

Mathematica [A] time = 0.33104, size = 230, normalized size = 1.05

$$\frac{-\frac{21d\left(a+bx^3\right)^{4/3}(ad+bc)}{b^2}+\frac{12d^2\left(a+bx^3\right)^{7/3}}{b^2}+\frac{14c^2\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-}\right.}{\sqrt[3]{d}}}{84d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (84*c^2*(a + b*x^3)^(1/3) - (21*d*(b*c + a*d)*(a + b*x^3)^(4/3))/b^2 + (12*
d^2*(a + b*x^3)^(7/3))/b^2 + (14*c^2*(b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1
 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*
```

```
(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))
/(84*d^3)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{x^8}{dx^3 + c} \sqrt[3]{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7464, size = 644, normalized size = 2.93

$$-\frac{28\sqrt{3}b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+14\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
```

```
 + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1
/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*c -
a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b^2*d^
2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^3)*(b*
x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8 \sqrt[3]{a + bx^3}}{c + dx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [A] time = 1.22617, size = 432, normalized size = 1.96

$$\frac{\left(b^{17}c^3d^4 - ab^{16}c^2d^5\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{17}cd^7 - ab^{16}d^8\right)} - \frac{\sqrt{3}\left(-bcd^2 + ad^3\right)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt(3
)*(-b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-
(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28*(28*(b*x^3 + a)^(1/3)*b^14*c^2*d^4 - 7
*(b*x^3 + a)^(4/3)*b^13*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^12*d^6 - 7*(b*x^3 + a
)^(4/3)*a*b^12*d^6)/(b^14*d^7)
```

3.660 $\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=186

$$-\frac{c\sqrt[3]{a+bx^3}}{d^2}-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}+$$

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) - (c*(b*c - a*d)^(
1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(7/3)) + (c*
(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^
(7/3))
```

Rubi [A] time = 0.202219, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$-\frac{c\sqrt[3]{a+bx^3}}{d^2}-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```


```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) - (c*(b*c - a*d)^(
1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(7/3)) + (c*
(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^
(7/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} + \frac{(c(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{\left(c\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+\right.}{2d^{7/3}}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}} -$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-a}}{}$$

Mathematica [A] time = 0.338361, size = 204, normalized size = 1.1

$$-\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{c\sqrt[3]{bc-ad}\left(-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+\right.\right.}{6d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) + (c*(b*c - a*d)^(
1/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)]))/(6*d^(7/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^5}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13028, size = 535, normalized size = 2.88

$$-\frac{4\sqrt{3}bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-a}{d}\right.\right.}{12\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 +
a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*b*
c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a
*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log((b*
x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b*x^3
+ a)^(1/3))/(b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \sqrt[3]{a + bx^3}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [A] time = 1.2089, size = 362, normalized size = 1.95

$$-\frac{\frac{4\left(b^2c^2d^2-abcd^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^4-ad^5}-\frac{4\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{d^3}-\frac{2\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\cdots\right.}{\cdots}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/12*(4*(b^2*c^2*d^2 - a*b*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 +
a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) - 4*sqrt(3)*(-b*c*d^2
 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)
/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 - 2*(-b*c*d^2 + a*d^3)^(1/3)*b*c*log
((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*
d)/d)^(2/3))/d^3 + 3*(4*(b*x^3 + a)^(1/3)*b*c*d^2 - (b*x^3 + a)^(4/3)*d^3)/
d^4)/b
```

3.661 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal. Leaf size=159

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}}+\frac{\sqrt[3]{a+bx^3}}{d}$$

```
[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)) + ((b*c - a*d)^(1/3)
*Log[c + d*x^3])/(6*d^(4/3)) - ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(4/3))
```

Rubi [A] time = 0.154765, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {444, 50, 58, 617, 204, 31}

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}}+\frac{\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)) + ((b*c - a*d)^(1/3)
*Log[c + d*x^3])/(6*d^(4/3)) - ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(4/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \frac{(bc-ad)^{2/3}}{}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\text{Subst}}{}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}} + \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\right.}{2d^{4/3}}$$

Mathematica [A] time = 0.148236, size = 205, normalized size = 1.29

$$\frac{\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2}{6d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3
)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)
*(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x^2}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04317, size = 483, normalized size = 3.04

$$-\frac{2\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(
1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + (-(b*c
- a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^
(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^
(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [A] time = 1.2206, size = 301, normalized size = 1.89

$$\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd-ad^2\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c -
 a*d)/d)^(1/3)))/(b*c*d - a*d^2) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d
)/d)^(1/3))/d^2 + (b*x^3 + a)^(1/3)/d - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/
d)^(2/3))/d^2
```

3.662 $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}}+\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a\right.}}{2c}$$

```
[Out] -((a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(1/3)) - (a^(1/3)*Log[x])/(2*c) - ((b
*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c*d^(1/3)) + (a^(1/3)*Log[a^(1/3) - (a +
 b*x^3)^(1/3)])/(2*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)])/(2*c*d^(1/3))
```

Rubi [A] time = 0.234189, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 83, 57, 617, 204, 31, 58}

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}}+\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a\right.}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```



```
[Out] -((a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(1/3)) - (a^(1/3)*Log[x])/(2*c) - ((b
*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c*d^(1/3)) + (a^(1/3)*Log[a^(1/3) - (a +
 b*x^3)^(1/3)])/(2*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)])/(2*c*d^(1/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(c+dx)}\,dx,x,x^3\right)$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^3\right)}{3c} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3c}$$

$$= -\frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{2/3}\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x}\right.}{2c}$$

$$= -\frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a}\right.}{2c\sqrt[3]{d}}$$

$$= -\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}c} - \frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}}{}$$

Mathematica [A] time = 0.30883, size = 268, normalized size = 1.09

$$\frac{\frac{\sqrt[3]{bc-ad}\left(-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)\right)}{\sqrt[3]{d}} - \sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a}\right.\right.}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] (-(a^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] -
 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/
3) + (a + b*x^3)^(2/3)])) + ((b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))/(6*c)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(dx^3+c\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)
```

Fricas [A] time = 2.23319, size = 714, normalized size = 2.9

$$-\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1
/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(3)
*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a) +
a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + ((b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)
```

```
^(1/3) + ((b*c - a*d)/d)^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)
) - 2*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))
/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)
```

Giac [A] time = 2.52698, size = 447, normalized size = 1.82

$$-\frac{1}{6}\left(\frac{2\,(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^2c^2-abcd}+\frac{2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{bc}+\frac{a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+(b\right.}{bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b
*c - a*d)/d)^(1/3)))/(b^2*c^2 - a*b*c*d) + 2*sqrt(3)*a^(1/3)*arctan(1/3*sqr
t(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(b*c) + a^(1/3)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(b*c) - 2*a^(1/3)*log(abs((
b*x^3 + a)^(1/3) - a^(1/3)))/(b*c) - 2*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d
)/d)^(1/3))/(b*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d))*b
```

3.663 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

Optimal. Leaf size=340

$$\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{(bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2}-\frac{\log(x)(bc-3ad)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}-d^{2/3}\cdots$$

```
[Out] (d*(a + b*x^3)^(1/3))/c^2 + ((b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*a*c^2) - (
a + b*x^3)^(4/3)/(3*a*c*x^3) - ((b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^
3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d
)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]])/(Sqrt[3]*c^2) - ((b*c - 3*a*d)*Log[x])/(6*a^(2/3)*c^2) + (d^(2/3)*(b*c
- a*d)^(1/3)*Log[c + d*x^3])/(6*c^2) + ((b*c - 3*a*d)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(6*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rubi [A] time = 0.383712, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{(bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2}-\frac{\log(x)(bc-3ad)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}-d^{2/3}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(1/3))/c^2 + ((b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*a*c^2) - (
a + b*x^3)^(4/3)/(3*a*c*x^3) - ((b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^
3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d
)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]])/(Sqrt[3]*c^2) - ((b*c - 3*a*d)*Log[x])/(6*a^(2/3)*c^2) + (d^(2/3)*(b*c
- a*d)^(1/3)*Log[c + d*x^3])/(6*c^2) + ((b*c - 3*a*d)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(6*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{1}{3}(-bc+3ad)-\frac{bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{9c^2} - \frac{(d(bc\ldots}{}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c^2} \ldots$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c^2} \ldots$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\,\tan\ldots}{\sqrt{\ldots}}$$

Mathematica [A] time = 0.92963, size = 366, normalized size = 1.08

$$\frac{\frac{(bc-3ad)\left(3\sqrt[3]{a+bx^3}-\frac{1}{2}\sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)}{3c} + \frac{ad^{2/3}\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(b\ldots\right.\right.}{\ldots}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-((a + b*x^3)^(4/3)/x^3) + ((b*c - 3*a*d)*(3*(a + b*x^3)^(1/3) - (a^(1/3)*
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)]))/2))/(3*c) + (a*d^(2/3)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt
```

```
[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d
)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(2*c))/(3*a*c)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(dx^3+c\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)
```

Fricas [A] time = 2.69548, size = 1044, normalized size = 3.07

$$-\frac{6\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\arctan\left(-\frac{2\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right)+3\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*arctan(-1/3*(2*sqrt(3)*(-
b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
- a*d^2)) + 3*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(2/3)*d^2 +
(-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
6*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*
d^3)^(1/3)) + 2*sqrt(3)*(a*b*c - 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-1
/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt
(-(-a^2)^(1/3))/a^2) + (-a^2)^(2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 + a)^(2/3)
*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*(b*c
 - 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)
*a^2*c)/(a^2*c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)
```

Giac [A] time = 2.60832, size = 525, normalized size = 1.54

$$\frac{1}{18}\left(\frac{6\left(bcd-ad^2\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^3c^3-ab^2c^2d}-\frac{6\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{b^2c^2}-\frac{2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/18*(6*(b*c*d - a*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) -
(-(b*c - a*d)/d)^(1/3)))/(b^3*c^3 - a*b^2*c^2*d) - 6*sqrt(3)*(-b*c*d^2 + a*
d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3)
)/(-(b*c - a*d)/d)^(1/3))/(b^2*c^2) - 2*sqrt(3)*(a^(1/3)*b*c - 3*a^(4/3)*d)
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a*b^2*c^2) -
3*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2) + 2*(b*c - 3*a*d)*log(
abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*b^2*c^2) - (a^(1/3)*b*c - 3*a^(4
/3)*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a*b^2*
c^2) - 6*(b*x^3 + a)^(1/3)/(b^2*c*x^3))*b^2
```

3.664 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

Optimal. Leaf size=370

$$-\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}+\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3}+\frac{\log(x)\left(-9a^2d^2+3ab\right.}{18a^{5/3}c^3}$$

```
[Out] ((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(9*a*c^2*x^3) - (a + b*x^3)^(4/3)/(6*a*c*
x^6) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/
(Sqrt[3]*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(5/3)*c^3)
 - (d^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 3*a*b*c
*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(5/3)*c^3) + (d^(5/
3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2
*c^3)
```

Rubi [A] time = 0.488675, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 149, 156, 57, 617, 204, 31, 58}

$$-\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}+\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3}+\frac{\log(x)\left(-9a^2d^2+3ab\right.}{18a^{5/3}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```


```
[Out] ((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(9*a*c^2*x^3) - (a + b*x^3)^(4/3)/(6*a*c*
x^6) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/
(Sqrt[3]*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(5/3)*c^3)
 - (d^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 3*a*b*c
*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(5/3)*c^3) + (d^(5/
3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2
```

```
*c^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(c+dx)}\,dx,x,x^3\right)$$

$$=-\frac{\left(a+bx^3\right)^{4/3}}{6acx^6}-\frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{2}{3}(bc+3ad)+\frac{2bdx}{3}\right)}{x^2(c+dx)}\,dx,x,x^3\right)}{6ac}$$

$$=\frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3}-\frac{\left(a+bx^3\right)^{4/3}}{6acx^6}-\frac{\text{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+3abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-6ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{6ac^2}$$

$$=\frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3}-\frac{\left(a+bx^3\right)^{4/3}}{6acx^6}+\frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3c^3}-\frac{\left(b^2c^2+3ab\right.}{}$$

$$=\frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3}-\frac{\left(a+bx^3\right)^{4/3}}{6acx^6}+\frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^3}$$

$$=\frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3}-\frac{\left(a+bx^3\right)^{4/3}}{6acx^6}+\frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^3}$$

$$=\frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3}-\frac{\left(a+bx^3\right)^{4/3}}{6acx^6}+\frac{\left(b^2c^2+3abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{5/3}c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\,\text{ta}}{\sqrt{}}$$

Mathematica [A] time = 1.35555, size = 411, normalized size = 1.11

$$-\frac{\frac{2\left(-9a^2d^2+3abcd+b^2c^2\right)\left(3\sqrt[3]{a+bx^3}-\frac{1}{2}\sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)}{9ac^2}+\frac{ad^{5/3}\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(4/3)/x^6 - (2*(b*c + 3*a*d)*(a + b*x^3)^(4/3))/(3*a*c*x^3) +
 (2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(3*(a + b*x^3)^(1/3) - (a^(1/3)*(2*Sq
rt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) -
 (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)
```

```
^(2/3)]))/2))/(9*a*c^2) + (a*d^(5/3)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[
3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
 b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a
*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/c^2)/(6*a*c)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^{7}\left(dx^{3}+c\right)}\sqrt[3]{bx^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(dx^{3}+c\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)
```

Fricas [A] time = 11.634, size = 1142, normalized size = 3.09

$$-\frac{18\sqrt{3}\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}a^{3}dx^{6}\arctan\left(-\frac{2\sqrt{3}\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^{2}\right)}{3\left(bcd-ad^{2}\right)}\right)+9\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}a^{3}dx^{6}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}d^{2}-\left(bcd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*arctan(-1/3*(2*sqrt(3)*
(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
 - a*d^2)) + 9*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(2/3)*d^2
- (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) -
18*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a
*d^3)^(1/3)) - 2*sqrt(3)*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2)^(1/6)*
x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(
1/3)*(a^2)^(2/3))/a^2) - (b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*
log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) +
2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(1/3)*a
 - (a^2)^(2/3)) + 3*(3*a^3*c^2 + (a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(
1/3))/(a^3*c^3*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)
```

Giac [A] time = 2.73655, size = 653, normalized size = 1.76

$$-\frac{1}{54}\left(\frac{18\left(bcd^2-ad^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^4c^4-ab^3c^3d}-\frac{18\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right.}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right.}{b^3c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/54*(18*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3
) - (-(b*c - a*d)/d)^(1/3)))/(b^4*c^4 - a*b^3*c^3*d) - 18*sqrt(3)*(-b*c*d^2
 + a*d^3)^(1/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d
)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(b^3*c^3) - 9*(-b*c*d^2 + a*d^3)^(1/3)*d*l
og((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c -
a*d)/d)^(2/3))/(b^3*c^3) - 2*sqrt(3)*(a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9
*a^(7/3)*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(
a^2*b^3*c^3) + 2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3
) - a^(1/3)))/(a^(5/3)*b^3*c^3) - (a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9*a^
(7/3)*d^2)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^
2*b^3*c^3) + 3*((b*x^3 + a)^(4/3)*b*c + 2*(b*x^3 + a)^(1/3)*a*b*c - 6*(b*x^
3 + a)^(4/3)*a*d + 6*(b*x^3 + a)^(1/3)*a^2*d)/(a*b^4*c^2*x^6))*b^3
```

3.665 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=336

$$-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3}-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}d^3}-\frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(c+\right.}{6d^3}$$

```
[Out] -((6*b*c - a*d)*x^2*(a + b*x^3)^(1/3))/(18*b*d^2) + (x^5*(a + b*x^3)^(1/3))
/(6*d) - ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/(a +
b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)*d^3) + (c^(5/3)*(b*c - a*d)^(1/3
)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]
)/(Sqrt[3]*d^3) - (c^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^3) - ((9*
b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^(5
/3)*d^3) + (c^(5/3)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (
a + b*x^3)^(1/3)])/(2*d^3)
```

Rubi [C] time = 0.0544801, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{a+bx^3}F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x^8\sqrt[3]{a+bx^3}F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.285313, size = 226, normalized size = 0.67

$$\frac{5cx^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(6bc-ad)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)+\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}\left(ad-6bc+3bdx^3\right)\right)-2x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+\right.}{90bcd^2\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-2*(-9*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^
3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^
2*((a + b*x^3)*(-6*b*c + a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b*c
- a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*
d)*x^3)/(a*(c + d*x^3))]))/(90*b*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2
/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{x^7}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}} x^7}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

Fricas [A] time = 11.6838, size = 1175, normalized size = 3.5

$$18\sqrt{3}\left(bc^3 - ac^2d\right)^{\frac{1}{3}} b^3c \arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(bc^3-ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right) + 18\left(bc^3 - ac^2d\right)^{\frac{1}{3}} b^3c \log\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}c-\left(bc^3-ac^2d\right)^{\frac{1}{3}}x}{x}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*arctan(-1/3*(sqrt(3)*(b*c^2
- a*c*d)*x + 2*sqrt(3)*(b*c^3 - a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 -
 a*c*d)*x)) + 18*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3 + a)^(1/3)*c - (
b*c^3 - a*c^2*d)^(1/3)*x)/x) - 9*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3
+ a)^(2/3)*c^2 + (b*c^3 - a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)*c*x + (b*c^3 - a
*c^2*d)^(2/3)*x^2)/x^2) + 2*sqrt(3)*(9*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*(
b^2)^(1/6)*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3
)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*(
b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 3*a
*b*c*d - a^2*d^2)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b
```

```
^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d - a
*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^3*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7 \sqrt[3]{a + bx^3}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}} x^7}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

3.666 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal. Leaf size=276

$$\frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^2}+\frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-}}{\sqrt[3]{c}}\right.}{2d^2}$$

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*d) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)*d^2) - (c^(2/3)*(b*c - a*d)^(
1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(Sqrt[3]*d^2) + (c^(2/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^2) + (
(3*b*c - a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(2/3)*d^2) - (c^(2/3
)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*d^2)
```

Rubi [C] time = 0.0540282, antiderivative size = 64, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt[3]{a+bx^3}F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^5\sqrt[3]{a+bx^3}F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.17884, size = 185, normalized size = 0.67

$$\frac{x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(ad-3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}-a\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\right.\right.}{15cd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((-3*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1[5/
3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^2*((a + b*x^3)*(1 + (d*
x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((
-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(15*c*d*(a + b*x^3)^(2/3)*(1 + (d*x^3
)/c)^(2/3))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{x^4}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

Fricas [B] time = 2.59089, size = 1077, normalized size = 3.9

$$6\left(bx^3+a\right)^{\frac{1}{3}}b^2dx^2+6\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{1}{3}}b^2\arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right)+6\left(-bc^3+ac^2d\right)^{\frac{1}{3}}b^2\log\left(\frac{(bx\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*sqrt(3)*(-b*c^3 + a*c^2*d)^(1/3)*b^
2*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 + a*c^2*d)^(2/
3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^(1/3)*b^2
*log(((b*x^3 + a)^(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) - 3*(-b*c^3 + a*
c^2*d)^(1/3)*b^2*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x
^3 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) - 2*sqrt(3)*(3*b^2*c
 - a*b*d)*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqr
t(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) + 2*(-b^2
)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - (-b^
2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^
```

```
(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

3.667 $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d}+\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+\right.}{2d}$$

```
[Out] -((b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d)) + ((b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(1/3)*d) - ((b*c - a*d)^(1/3)*Log[c + d
*x^3])/(6*c^(1/3)*d) - (b^(1/3)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d) +
 ((b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(1/3)*d)
```

Rubi [C] time = 0.0377606, antiderivative size = 64, normalized size of antiderivative = 0.27, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.0305237, size = 65, normalized size = 0.28

$$\frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{a+bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*((a + b*x^3)/a)^(1/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

Fricas [A] time = 1.89708, size = 807, normalized size = 3.45

$$2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)-2\sqrt{3}\,(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3\,bx}\right)+2\,(-b)^{\frac{1}{3}}\log\left(\frac{(-b)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b*c - a*d)/c)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2
*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) - 2*sq
rt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)
^(2/3))/(b*x)) + 2*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2
*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 + a)^(1/3))/x
) - (-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^
3 + a)^(2/3))/x^2) - ((b*c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) +
 (b*x^3 + a)^(1/3)*x*((b*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

3.668 $\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

```
[Out] -((a + b*x^3)^(1/3)/(c*x)) - ((b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)) + ((b*c -
 a*d)^(1/3)*Log[c + d*x^3])/(6*c^(4/3)) - ((b*c - a*d)^(1/3)*Log[((b*c - a*
d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3))
```

Rubi [C] time = 0.0636257, antiderivative size = 87, normalized size of antiderivative = 0.52, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(c*x*(1 + (b*x^3)/a)^(1/3))
)
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1+\frac{dx^3}{c}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.0280787, size = 81, normalized size = 0.48

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x*(1 + (b*x^3)/a)^(1/3)))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(dx^3+c\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

3.669 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt[3]{a+bx^3}(bc-4ad)}{4ac^2x}-\frac{d\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}}$$

```
[Out] -(a + b*x^3)^(1/3)/(4*c*x^4) - ((b*c - 4*a*d)*(a + b*x^3)^(1/3))/(4*a*c^2*x
) + (d*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)) - (d*(b*c - a*d)^(1/3)*Log[c + d
*x^3])/(6*c^(7/3)) + (d*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
 - (a + b*x^3)^(1/3)])/(2*c^(7/3))
```

Rubi [C] time = 0.132877, antiderivative size = 145, normalized size of antiderivative = 0.71, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{x^3\left(c-3dx^3\right)(-(bc-ad))\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+2c\left(a+bx^3\right)\left(c-3dx^3\right)}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(2*c*(a + b*x^3)*(c - 3*d*x^3) - (b*c - a*d)*x^3*(c - 3*d*x^3)*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*(b*c - a*d)*x^3*
(c + d*x^3)*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)])/(8*c^3*x^4*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{2c\left(a+bx^3\right)\left(c-3dx^3\right)-(bc-ad)x^3\left(c-3dx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)+3(bc-ad)x^3\left(c+dx^3\right)\,{}_2F_{}}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 0.398873, size = 146, normalized size = 0.72

$$-\frac{x^3\left(3dx^3-c\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+2c\left(a+bx^3\right)\left(c-3dx^3\right)}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(2*c*(a + b*x^3)*(c - 3*d*x^3) + (b*c - a*d)*x^3*(-c + 3*d*x^3)*Hypergeome
tric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*(b*c - a*d)*x^3
*(c + d*x^3)*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))])/(8*c^3*x^4*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(dx^3+c\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

3.670 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx$

Optimal. Leaf size=258

$$\frac{\sqrt[3]{a+bx^3}\left(-28a^2d^2+7abcd+3b^2c^2\right)}{28a^2c^3x}+\frac{d^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{10/3}}-\frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}-\frac{d^2\sqrt[3]{bc-a}}{}$$

```
[Out] -(a + b*x^3)^(1/3)/(7*c*x^7) - ((b*c - 7*a*d)*(a + b*x^3)^(1/3))/(28*a*c^2*
x^4) + ((3*b^2*c^2 + 7*a*b*c*d - 28*a^2*d^2)*(a + b*x^3)^(1/3))/(28*a^2*c^3
*x) - (d^2*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(
a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(10/3)) + (d^2*(b*c - a*d)^(1/3)*Lo
g[c + d*x^3])/(6*c^(10/3)) - (d^2*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*
x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(10/3))
```

Rubi [C] time = 0.939877, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-2x^3\left(2c^2-3cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-12bc}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2*x^6
+ 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*Hyperg
eometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3*x^3*
Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*a*c^
2*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
 12*b*c^2*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 12*a*c*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hypergeometr
icPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(56*c^4*x^
```

```
7*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{8ac^3+8bc^3x^3-12ac^2dx^3-12bc^2dx^6+36acd^2x^6+36bcd^2x^9-2(bc-ad)x^3\left(2c^2-3cdx^3+9d^2x^6\right.}{}$$

Mathematica [C] time = 1.59537, size = 451, normalized size = 1.75

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\text{HypergeometricPFQ}\left(\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)-2x^3\left(2c^2-3cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2*x^6
+ 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*Hyperg
```

```
eometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3*x^3*
Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*a*c^
2*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
 12*b*c^2*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 12*a*c*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hypergeometr
icPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(56*c^4*x^
7*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{1}{x^8\left(dx^3+c\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

3.671 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

Optimal. Leaf size=318

$$-\frac{\sqrt[3]{a+bx^3}\left(35a^2bcd^2-140a^3d^3+15ab^2c^2d+9b^3c^3\right)}{140a^3c^4x}+\frac{\sqrt[3]{a+bx^3}\left(-35a^2d^2+5abcd+3b^2c^2\right)}{140a^2c^3x^4}-\frac{d^3\sqrt[3]{bc-ad}\log\left(c+dx\right.}{6c^{13/3}}$$

```
[Out] -(a + b*x^3)^(1/3)/(10*c*x^10) - ((b*c - 10*a*d)*(a + b*x^3)^(1/3))/(70*a*c
^2*x^7) + ((3*b^2*c^2 + 5*a*b*c*d - 35*a^2*d^2)*(a + b*x^3)^(1/3))/(140*a^2
*c^3*x^4) - ((9*b^3*c^3 + 15*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 140*a^3*d^3)*(a
 + b*x^3)^(1/3))/(140*a^3*c^4*x) + (d^3*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b
*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(13/3)
) - (d^3*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^(13/3)) + (d^3*(b*c - a*d)^
(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(13/3))
```

Rubi [C] time = 2.64184, antiderivative size = 905, normalized size of antiderivative = 2.85, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-324bcd^3x^{12}-162ad^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}+162bcd^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}-297ad^4\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^1}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -(56*a*c^4 + 56*b*c^4*x^3 - 72*a*c^3*d*x^3 - 72*b*c^3*d*x^6 + 108*a*c^2*d^2
*x^6 + 108*b*c^2*d^2*x^9 - 324*a*c*d^3*x^9 - 324*b*c*d^3*x^12 - 28*b*c^4*x^
3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 28*a*
c^3*d*x^3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 36*b*c^3*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 36*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3
)/(c*(a + b*x^3))] - 54*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] + 54*a*c*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[2
/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeomet
```

```
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 117*b*c^4*x^3*Hype
rgeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 117*a*c^3*d
*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99
*b*c^3*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] + 81*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] - 81*a*c*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[2/3, 2
, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F
1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c - a*d)*x^3*(2*c
 - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*Hypergeometri
cPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560*
c^5*x^10*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{56ac^4+56bc^4x^3-72ac^3dx^3-72bc^3dx^6+108ac^2d^2x^6+108bc^2d^2x^9-324acd^3x^9-324bcd^3x^{12}-}{}$$

Mathematica [C] time = 3.79019, size = 905, normalized size = 2.85

$$-\frac{-324bcd^3x^{12} - 162ad^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12} + 162bcd^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12} - 297ad^4\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^1}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -(56*a*c^4 + 56*b*c^4*x^3 - 72*a*c^3*d*x^3 - 72*b*c^3*d*x^6 + 108*a*c^2*d^2
*x^6 + 108*b*c^2*d^2*x^9 - 324*a*c*d^3*x^9 - 324*b*c*d^3*x^12 - 28*b*c^4*x^
3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 28*a*
c^3*d*x^3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 36*b*c^3*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 36*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3
)/(c*(a + b*x^3))] - 54*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] + 54*a*c*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[2
/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 117*b*c^4*x^3*Hype
rgeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 117*a*c^3*d
*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99
*b*c^3*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] + 81*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] - 81*a*c*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[2/3, 2
, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F
1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 54*(b*c - a*d)*x^3*(c +
 d*x^3)^2*(-2*c + 3*d*x^3)*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*Hypergeometr
icPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560
*c^5*x^10*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^{11}\left(dx^3+c\right)} \sqrt[3]{bx^3+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

3.672 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=64

$$\frac{x^7 \sqrt[3]{a+bx^3} F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0509833, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7 \sqrt[3]{a+bx^3} F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^6\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\
&= \frac{x^7\sqrt[3]{a+bx^3}F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}
$$

Mathematica [B] time = 0.542318, size = 281, normalized size = 4.39

$$
\frac{x\left(-\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(a^2d^2+5abcd-10b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2(ad-5bc)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};\right.\right.}\right.}{40bd^2\left(a+bx^3\right)^{2/3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + a*d + 2*b*d*x^3) - ((-10*b^2*c^2 + 5*a*b*c*d +
a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + a*d)*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3,
 -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x
^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d
*x^3)/c)])))))/(40*b*d^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$
\int \frac{x^6}{dx^3+c}\sqrt[3]{bx^3+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

3.673 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal. Leaf size=64

$$\frac{x^4 \sqrt[3]{a+bx^3} F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0530495, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4 \sqrt[3]{a+bx^3} F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^3\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.280696, size = 240, normalized size = 3.75

$$\frac{x\left(4\left(\frac{4a^2c^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+a+bx^3\right)+\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-2bc)F_1\left(\frac{4}{3};\frac{2}{3},\right.}{c}\right.}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(((-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])/c + 4*(a + b*x^3 + (4*a^2*c^2*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^3}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}} x^3}{dx^3 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

3.674 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.025742, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.0403401, size = 160, normalized size = 2.71

$$\frac{4acx\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(bcF_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

3.675 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0531591, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.243332, size = 327, normalized size = 5.11

$$\frac{\frac{c\left(16ac\left(a\left(c+3dx^3\right)+bdx^6\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}-bdx^6\left(\frac{bx^3}{a}+1\right)^{2/3}}{8c^2x^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (c*(16*a*c*(b*d*x^6 + a*(c + 3*d*x^3))*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)]))))/(8*c^2*x^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(dx^3+c\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**3*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

3.676 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(5*c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0544718, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```


```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(5*c*x^5*(1 + (b*x^3)/a)^(1/3))
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.354018, size = 289, normalized size = 4.52

$$\frac{\frac{16x\left(-10a^2d^2+5abcd+b^2c^2\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}(5ad-bc)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3}-}{40\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + b*c*x^3 - 5*a*d*x^3))/(a*c^2*x^5) + (b*d*(-(b*c)
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/(a*c^3) + (16*(b^2*c^2 + 5*a*b*c*d - 10*a^2*d^2)*x*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^6\left(dx^3+c\right)}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{\left(dx^3 + c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a + bx^3}}{x^6\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**6/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**6*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

3.677 $\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=266

$$\frac{(a+bx^3)^{5/3}(a^2d^2+abcd+b^2c^2)}{5b^3d^3}-\frac{(a+bx^3)^{8/3}(2ad+bc)}{8b^3d^2}+\frac{(a+bx^3)^{11/3}}{11b^3d}-\frac{c^3(a+bx^3)^{2/3}}{2d^4}+\frac{c^3(bc-ad)^{2/3}\log(c+dx^3}{6d^{14/3}}$$

```
[Out] -(c^3*(a + b*x^3)^(2/3))/(2*d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^
3)^(5/3))/(5*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d^2) + (a
+ b*x^3)^(11/3)/(11*b^3*d) - (c^3*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(14/3)) + (c^3*(
b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[
(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3))
```

Rubi [A] time = 0.322075, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$\frac{(a+bx^3)^{5/3}(a^2d^2+abcd+b^2c^2)}{5b^3d^3}-\frac{(a+bx^3)^{8/3}(2ad+bc)}{8b^3d^2}+\frac{(a+bx^3)^{11/3}}{11b^3d}-\frac{c^3(a+bx^3)^{2/3}}{2d^4}+\frac{c^3(bc-ad)^{2/3}\log(c+dx^3}{6d^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```


```
[Out] -(c^3*(a + b*x^3)^(2/3))/(2*d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^
3)^(5/3))/(5*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d^2) + (a
+ b*x^3)^(11/3)/(11*b^3*d) - (c^3*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(14/3)) + (c^3*(
b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[
(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{\left(b^2c^2+abcd+a^2d^2\right)(a+bx)^{2/3}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{5/3}}{b^2d^2} + \frac{(a+bx)^{8/3}}{b^2d} - \frac{c^3(a+b}{d^3(c+}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} - \frac{c^3\,\text{Subst}\left(\int \frac{(a+bx)}{c+dx}\right.}{3d^3}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} +$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} +$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} +$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} -$$

Mathematica [C] time = 0.105893, size = 148, normalized size = 0.56

$$\frac{\left(a+bx^3\right)^{2/3}\left(3a^2bd^2\left(11c-4dx^3\right)+18a^3d^3+2ab^2d\left(44c^2-11cdx^3+5d^2x^6\right)+220b^3c^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+b^3\left(88c^2dx\right.\right.}{440b^3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) + 2*a*b^2*d*(
44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*d^2*
x^6 + 40*d^3*x^9) + 220*b^3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^
3))/(-(b*c) + a*d)]))/(440*b^3*d^4)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{d x^{3}+c}\left(b x^{3}+a\right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88651, size = 1033, normalized size = 3.88

$$-\frac{440\sqrt{3}b^{3}c^{3}\left(-\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}d\left(-\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+220\,b^{3}c^{3}\left(-\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\log\left(\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*a
rctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2 -
2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(
```

```
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2
/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^3*c*d^2
- 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3
*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^3 + a)^(2/
3))/(b^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21092, size = 552, normalized size = 2.08

$$-\frac{\left(b^{37}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ab^{36}c^3d^8\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{37}cd^{11} - ab^{36}d^{12}\right)} - \frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\cdots}{\cdots}\right)}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d)
^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)
*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b
*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3 + a)^
(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/
d^6 - 1/440*(220*(b*x^3 + a)^(2/3)*b^33*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^32
*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^31*c*d^9 - 88*(b*x^3 + a)^(5/3)*a*b^31*c*
d^9 - 40*(b*x^3 + a)^(11/3)*b^30*d^10 + 110*(b*x^3 + a)^(8/3)*a*b^30*d^10 -
 88*(b*x^3 + a)^(5/3)*a^2*b^30*d^10)/(b^33*d^11)
```

3.678 $\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=223

$$-\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2}+\frac{(a+bx^3)^{8/3}}{8b^2d}+\frac{c^2(a+bx^3)^{2/3}}{2d^3}-\frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\right.}{2d^{11/3}}$$

```
[Out] (c^2*(a + b*x^3)^(2/3))/(2*d^3) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^2*d^
2) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^
(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(11/3)) -
(c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(11/3)) + (c^2*(b*c - a*d)^(2/3
)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3))
```

Rubi [A] time = 0.259346, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$-\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2}+\frac{(a+bx^3)^{8/3}}{8b^2d}+\frac{c^2(a+bx^3)^{2/3}}{2d^3}-\frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\right.}{2d^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(2/3))/(2*d^3) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^2*d^
2) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^
(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(11/3)) -
(c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(11/3)) + (c^2*(b*c - a*d)^(2/3
)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8 \left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{2/3}}{bd^2} + \frac{(a+bx)^{5/3}}{bd} + \frac{c^2(a+bx)^{2/3}}{d^2(c+dx)}\right) dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^2d} + \frac{c^2\,\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^2d} - \frac{\left(c^2(bc-ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx,\right.}{3d^3}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}} + \frac{\left(c^2(bc-\right.}{}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}} + \frac{c^2(bc-}{}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^2d} + \frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}} - $$

Mathematica [C] time = 0.0682273, size = 104, normalized size = 0.47

$$\frac{\left(a+bx^3\right)^{2/3}\left(-3a^2d^2-20b^2c^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2abd\left(dx^3-4c\right)+b^2\left(20c^2-8cdx^3+5d^2x^6\right)\right)}{40b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(20*c^2 - 8*c
*d*x^3 + 5*d^2*x^6) - 20*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x
^3))/(-(b*c) + a*d)]))/(40*b^2*d^3)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^8}{dx^3 + c} \left(bx^3 + a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62024, size = 900, normalized size = 4.04

$$40\sqrt{3}b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-20\,b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arcta
n(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*
```

```
d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3
 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*d - 3*a^
2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3))/(b^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8 \left(a + bx^3\right)^{\frac{2}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [A] time = 1.20182, size = 473, normalized size = 2.12

$$\frac{\left(b^{19}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ab^{18}c^2d^6\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{19}cd^8 - ab^{18}d^9\right)} + \frac{\sqrt{3}\left(-bcd^2 + ad^3\right)^{\frac{2}{3}}c^2\arctan\left(\cdots\right.}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)^
(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c^
2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
- a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a)^(2/
3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5
 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^15*c*d^6
 + 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(b^16*d^8
)
```

3.679 $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=188

$$-\frac{c\left(a+bx^3\right)^{2/3}}{2d^2}+\frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}-\frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{bc}}}{\sqrt{3}}\right.}{\sqrt{3}d^{8/3}}$$

```
[Out] -(c*(a + b*x^3)^(2/3))/(2*d^2) + (a + b*x^3)^(5/3)/(5*b*d) - (c*(b*c - a*d)
^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(8/3)) - (
c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(8/3))
```

Rubi [A] time = 0.198692, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 56, 617, 204, 31}

$$-\frac{c\left(a+bx^3\right)^{2/3}}{2d^2}+\frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}-\frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{bc}}}{\sqrt{3}}\right.}{\sqrt{3}d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -(c*(a + b*x^3)^(2/3))/(2*d^2) + (a + b*x^3)^(5/3)/(5*b*d) - (c*(b*c - a*d)
^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(8/3)) - (
c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(8/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{5/3}}{5bd} - \frac{c\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)}{3d}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{(c(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx,x,x^3\right)}{3d^2}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{\left(c(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,d\right.}{2d^{8/3}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{\right.}}{2d^{8/3}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} - \frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} -$$

Mathematica [C] time = 0.0247067, size = 68, normalized size = 0.36

$$\frac{\left(a+bx^3\right)^{2/3}\left(5bc\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2ad-5bc+2bdx^3\right)}{10bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3 + 5*b*c*Hypergeometric2F1[2/
3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]))/(10*b*d^2)
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{x^5}{dx^3+c}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59756, size = 815, normalized size = 4.34

$$-\frac{10\sqrt{3}bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+5\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(
1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2
)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^
2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^
(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3))/(b*d^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [A] time = 1.19439, size = 402, normalized size = 2.14

$$-\frac{\frac{10\left(b^2c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-abcd^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^5-ad^6}+\frac{10\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{d^4}-\frac{5\left(-bcd^2+\cdots\right.}{\cdots}}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/30*(10*(b^2*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b*c*d^4*(-(b*c - a*d)/d)^
(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b*c*d^5 - a*d^6) + 10*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(
1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)
^(1/3))/d^4 - 5*(-b*c*d^2 + a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 3*(5*(b*
x^3 + a)^(2/3)*b*c*d^3 - 2*(b*x^3 + a)^(5/3)*d^4)/d^5)/b
```

3.680 $\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=162

$$-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}}+\frac{\left(a+bx^3\right)}{2d}$$

```
[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*
x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)) - ((b*c - a*d)^(
2/3)*Log[c + d*x^3])/(6*d^(5/3)) + ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(5/3))
```

Rubi [A] time = 0.170246, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {444, 50, 56, 617, 204, 31}

$$-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}}+\frac{\left(a+bx^3\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```


```
[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*
x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)) - ((b*c - a*d)^(
2/3)*Log[c + d*x^3])/(6*d^(5/3)) + ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(5/3))
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2 \left(a+bx^3\right)^{2/3}}{c+dx^3} \, dx = \frac{1}{3} \text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx} \, dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad) \, \text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)} \, dx, x, x^3\right)}{3d}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)^{2/3} \log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3} \, \text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x} \, dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{5/3}} - \cdots$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)^{2/3} \log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}} - \cdots$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} + \frac{(bc-ad)^{2/3} \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}} - \frac{(bc-ad)^{2/3} \log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3} \log\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.0155015, size = 47, normalized size = 0.29

$$-\frac{\left(a+bx^3\right)^{2/3} \left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d(bx^3+a)}{ad-bc}\right)-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-
(b*c) + a*d)]))/(2*d)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^2}{dx^3+c} \left(bx^3+a\right)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.53618, size = 736, normalized size = 4.54

$$2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - sq
rt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/
3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b
*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(b
*x^3 + a)^(2/3))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [B] time = 1.23453, size = 350, normalized size = 2.16

$$\frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd^2-ad^3\right)}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2d}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\right.}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^2
 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (
b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3
```

3.681 $\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx$

Optimal. Leaf size=245

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-\right.}{2cd^{2/3}}$$

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*c) - ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(2/3)) - (a^(2/3)*Log[x])/(2*c) + ((b*c
- a*d)^(2/3)*Log[c + d*x^3])/(6*c*d^(2/3)) + (a^(2/3)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*c*d^(2/3))
```

Rubi [A] time = 0.228895, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 83, 55, 617, 204, 31, 56}

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-\right.}{2cd^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*c) - ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(2/3)) - (a^(2/3)*Log[x])/(2*c) + ((b*c
- a*d)^(2/3)*Log[c + d*x^3])/(6*c*d^(2/3)) + (a^(2/3)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*c*d^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(c+dx)}\,dx,x,x^3\right)$$

$$=\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^3\right)}{3c}+\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx,x,x^3\right)}{3c}$$

$$=-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{a^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2c}+\frac{a\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{\ldots}}\right.}{}$$

$$=-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\ldots\right.}{2cd^{2/3}}$$

$$=\frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}c}-\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}\ldots$$

Mathematica [C] time = 0.0693176, size = 115, normalized size = 0.47

$$\frac{a^{2/3}\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)+3\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (3*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c)
 + a*d)] + a^(2/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/(6*c)
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)
```

Fricas [B] time = 1.76431, size = 1033, normalized size = 4.22

$$-\frac{2\sqrt{3}\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-2\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{1}{3}}}{3\,a}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2
*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) +
 sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt
(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(1/3))/a) + (-(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2
- 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + (a^2)^(1/3)*log((b*x^3 + a)^(2/3)*a +
(a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(-(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) -
```

```
(b*x^3 + a)^(1/3)*(b*c - a*d)) - 2*(a^2)^(1/3)*log((b*x^3 + a)^(1/3)*a - (a
^2)^(2/3)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)
```

Giac [A] time = 2.90919, size = 487, normalized size = 1.99

$$-\frac{1}{6}\left(\frac{2\left(bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^2c^2-abcd}-\frac{2\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{bc}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(2*(b*c*(-(b*c - a*d)/d)^(1/3) - a*d*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2
 - a*b*c*d) - 2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a
^(1/3))/a^(1/3))/(b*c) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
a^(1/3) + a^(2/3))/(b*c) - 2*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/
(b*c) + 2*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a
)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(b*c*d^2) - (-b*c
*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)
/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2))*b
```

3.682 $\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx$

Optimal. Leaf size=347

$$\frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{\left(a+bx^3\right)^{2/3}(2bc-3ad)}{6ac^2}-\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2}+\frac{\sqrt[3]{d}(bc-$$

```
[Out] (d*(a + b*x^3)^(2/3))/(2*c^2) + ((2*b*c - 3*a*d)*(a + b*x^3)^(2/3))/(6*a*c^
2) - (a + b*x^3)^(5/3)/(3*a*c*x^3) + ((2*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b
*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*c^2) - ((2*b*c - 3*a*d)*Log[x])/(6*a^(1/3)*c^2) - (d^(
1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^2) + ((2*b*c - 3*a*d)*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rubi [A] time = 0.389123, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 55, 617, 204, 31, 56}

$$\frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{\left(a+bx^3\right)^{2/3}(2bc-3ad)}{6ac^2}-\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2}+\frac{\sqrt[3]{d}(bc-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```


```
[Out] (d*(a + b*x^3)^(2/3))/(2*c^2) + ((2*b*c - 3*a*d)*(a + b*x^3)^(2/3))/(6*a*c^
2) - (a + b*x^3)^(5/3)/(3*a*c*x^3) + ((2*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b
*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*c^2) - ((2*b*c - 3*a*d)*Log[x])/(6*a^(1/3)*c^2) - (d^(
1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^2) + ((2*b*c - 3*a*d)*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(-2bc+3ad)-\frac{2bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{9c^2} -$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\Big(}{6c^2}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\Big(}{6c^2}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-}{}$$

Mathematica [C] time = 0.0989132, size = 202, normalized size = 0.58

$$\frac{-9\sqrt[3]{a}dx^3\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right) + 2\sqrt{3}x^3(2bc-3ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 6\sqrt[3]{a}c\left(a+bx^3\right)^{2/3} + 6bcx^3\log\Big(\sqrt[3]{a}-}{18\sqrt[3]{a}c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-6*a^(1/3)*c*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(2*b*c - 3*a*d)*x^3*ArcTan[(1 +
 (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 9*a^(1/3)*d*x^3*(a + b*x^3)^(2/3
)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)] - 6*b*c*x^
3*Log[x] + 9*a*d*x^3*Log[x] + 6*b*c*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)] -
```

```
9*a*d*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(1/3)*c^2*x^3)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (dx^3 + c)} (bx^3 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)
```

Fricas [A] time = 2.37889, size = 2473, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3
 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)
^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) -
 6*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt
```

```
(3)*(b*c - a*d) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^
3 + a)^(1/3))/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)
^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(2*b*c - 3*a*d)*(-a
)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1
/3)*a*x^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(2/3)) + 6*(b*x^3 + a)^(2/3)*a*c)/(a*c^2*x^3), 1/18*(6*sqrt(1/3
)*(2*a*b*c - 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 +
a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) - (
2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1
/3) + (-a)^(1/3)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-
(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/
3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) - 6*(b*x^3
+ a)^(2/3)*a*c)/(a*c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^4\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)
```

Giac [A] time = 2.67912, size = 585, normalized size = 1.69

$$\frac{1}{18}\left(\frac{6\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^3c^3-ab^2c^2d}+\frac{2\left(2a^{\frac{1}{3}}bc-3a^{\frac{4}{3}}d\right)\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}\right.\right.}{a^{\frac{2}{3}}b^2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/18*(6*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*
c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^3
*c^3 - a*b^2*c^2*d) + 2*(2*a^(1/3)*b*c - 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(
1/3) - a^(1/3)))/(a^(2/3)*b^2*c^2) + 6*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d
)/d)^(1/3))/(b^2*c^2*d) - 3*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c
^2*d) + 2*sqrt(3)*(2*a^(5/3)*b*c - 3*a^(8/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^
3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*b^2*c^2) - (2*a^(5/3)*b*c - 3*a^(8/3)
*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^2*b^2*c
^2) - 6*(b*x^3 + a)^(2/3)/(b^2*c*x^3))*b^2
```

3.683 $\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx$

Optimal. Leaf size=370

$$-\frac{(-9a^2d^2+6abcd+b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}-\frac{(-9a^2d^2+6abcd+b^2c^2)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}c^3}+\frac{\log(x)\,(-9a^2d^2+6ab}{18a^{4/3}c^3}$$

```
[Out] ((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(18*a*c^2*x^3) - (a + b*x^3)^(5/3)/(6*a*c
*x^6) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^
(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(4/3)*c^3
) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 6*a*b*
c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(4/3)*c^3) - (d^(4
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*c^3)
```

Rubi [A] time = 0.496307, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 149, 156, 55, 617, 204, 31, 56}

$$-\frac{(-9a^2d^2+6abcd+b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}-\frac{(-9a^2d^2+6abcd+b^2c^2)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}c^3}+\frac{\log(x)\,(-9a^2d^2+6ab}{18a^{4/3}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```


```
[Out] ((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(18*a*c^2*x^3) - (a + b*x^3)^(5/3)/(6*a*c
*x^6) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^
(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(4/3)*c^3
) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 6*a*b*
c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(4/3)*c^3) - (d^(4
```

```
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*c^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(bc+6ad)+\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+6abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-3ad)x}{x\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{6ac^2}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c^3} - \frac{\left(b^2c^2+6\right.}{}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+\right.}{6c^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+\right.}{6c^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{4/3}c^3} - \frac{d^{4/3}(bc-ad)}{}$$

Mathematica [C] time = 0.272646, size = 240, normalized size = 0.65

$$\frac{3x^6\log(x)\left(-9a^2d^2+6abcd+b^2c^2\right) - 3x^6\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) - 2\sqrt{3}x^6\left(-9a^2d^2+6abcd+b^2\right.}{54a^{4/3}c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (3*a^(1/3)*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3) - 2*Sqrt[3]
*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^
(1/3))/Sqrt[3]] + 27*a^(4/3)*d^2*x^6*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/
3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2
```

```
*d^2)*x^6*Log[x] - 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*Log[a^(1/3) - (a
 + b*x^3)^(1/3)])/(54*a^(4/3)*c^3*x^6)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^7 \left(dx^3+c\right)} \left(bx^3+a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)
```

Fricas [A] time = 9.31567, size = 2691, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arc
tan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d
^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a
```

```
^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*
d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 -
a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)
^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a
*b*c*d^2 - a^2*d^3)^(2/3)) + 3*sqrt(1/3)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d
^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^
(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/
3)*a^(2/3) + 3*a)/x^3) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a
*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(3*a^2
*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6), -1/54
*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arctan(-1
/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*
a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)
*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(2/3)) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a*b
*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*sqrt(1/3
)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d^2)*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)
^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) + 3*(3*a^2*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)
*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^7\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)
```

Giac [A] time = 2.64391, size = 698, normalized size = 1.89

$$-\frac{1}{54}\left(\frac{18\left(bcd^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^4c^4-ab^3c^3d}+\frac{18\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{}\right.}{b^3c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/54*(18*(b*c*d^2*(-(b*c - a*d)/d)^(1/3) - a*d^3*(-(b*c - a*d)/d)^(1/3))*(
-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/
(b^4*c^4 - a*b^3*c^3*d) + 18*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sq
rt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3)
)/(b^3*c^3) - 9*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^3*c^3) + 2*sqrt
(3)*(a^(2/3)*b^2*c^2 + 6*a^(5/3)*b*c*d - 9*a^(8/3)*d^2)*arctan(1/3*sqrt(3)*
(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*b^3*c^3) + 2*(a^(1/3)*b^2*c^2
 + 6*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(
a^(5/3)*b^3*c^3) - (a^(2/3)*b^2*c^2 + 6*a^(5/3)*b*c*d - 9*a^(8/3)*d^2)*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^2*b^3*c^3) + 3*
(2*(b*x^3 + a)^(5/3)*b*c + (b*x^3 + a)^(2/3)*a*b*c - 6*(b*x^3 + a)^(5/3)*a*
d + 6*(b*x^3 + a)^(2/3)*a^2*d)/(a*b^4*c^2*x^6))*b^3
```

3.684 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=334

$$-\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{4/3}d^3}+\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}d^3}-\frac{c^{4/3}(bc-ad)^{2/3}\log\left(c\right.}{6d^3}$$

```
[Out] -((3*b*c - a*d)*x*(a + b*x^3)^(2/3))/(9*b*d^2) + (x^4*(a + b*x^3)^(2/3))/(6
*d) + ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x
^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(4/3)*d^3) - (c^(4/3)*(b*c - a*d)^(2/3)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
Sqrt[3]*d^3) - (c^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^3) + (c^(4/3
)*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*d^3) - ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3
)^(1/3)])/(18*b^(4/3)*d^3)
```

Rubi [C] time = 0.0558136, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.613674, size = 525, normalized size = 1.57

$$3x^4\sqrt[3]{\frac{bx^3}{a}+1}\sqrt[3]{bc-ad}\left(-a^2d^2-6abcd+9b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2c\left(-a^2\sqrt[3]{c}d\sqrt[3]{a+bx^3}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc}}{\sqrt[3]{ax^3+}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (3*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*x^4*(1 + (b*x^3)/a)^
(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-18*a*b
*c*(b*c - a*d)^(1/3)*x + 6*a^2*d*(b*c - a*d)^(1/3)*x - 18*b^2*c*(b*c - a*d)
^(1/3)*x^4 + 15*a*b*d*(b*c - a*d)^(1/3)*x^4 + 9*b^2*d*(b*c - a*d)^(1/3)*x^7
 - 2*Sqrt[3]*a*c^(1/3)*(-3*b*c + a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c
 - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*(-3*b*
c + a*d)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^
(1/3)] + 3*a*b*c^(4/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x
```

```
^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] -
a^2*c^(1/3)*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b +
a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*b*c*
d^2*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x^6}{dx^3 + c} \left(bx^3 + a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}} x^6}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

Fricas [B] time = 9.23199, size = 2734, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*arctan(-
1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^
```

```
(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d + a^2
*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*x - (b*x
^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(
1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2
+ (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x + (b*x^3 +
a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2
*b*d^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*
sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2
/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d - a^2*d^
2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2) - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(b^2*d^3), -1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2
*c*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/
3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c
- a*d)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x
+ (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*
d^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c
*d - a^2*d^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x
^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2*b*d^2)*arc
tan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3
*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))/(b^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6 \left(a + bx^3\right)^{\frac{2}{3}}}{c + dx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

3.685 $\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal. Leaf size=272

$$\frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^2}-\frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6\sqrt[3]{b}d^2}-\frac{(3bc-2a}{}$$

```
[Out] (x*(a + b*x^3)^(2/3))/(3*d) - ((3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)*d^2) + (c^(1/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(Sqrt[3]*d^2) + (c^(1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^2) - (
c^(1/3)*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(
1/3)])/(2*d^2) + ((3*b*c - 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6
*b^(1/3)*d^2)
```

Rubi [C] time = 0.0591927, antiderivative size = 64, normalized size of antiderivative = 0.24, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^4\left(a+bx^3\right)^{2/3} F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.49689, size = 286, normalized size = 1.05

$$\frac{\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}(2ad-3bc)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{a+bx^3}}+\frac{2\left(-a\sqrt[3]{c}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)+6x\left(a+bx^3\right)^{2/3}\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)-2\sqrt{3}a\sqrt[3]{c}\tan\right.}{\sqrt[3]{bc-ad}}}{36d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*(-3*b*c + 2*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(c*(a + b*x^3)^(1/3)) + (2*(6*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)
*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*Log[c^(1/3) - ((b*c
 - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^
(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)]))/(b*c - a*d)^(1/3))/(36*d)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^3}{d x^3 + c} \left(b x^3 + a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^3 + a\right)^{\frac{2}{3}} x^3}{d x^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

Fricas [B] time = 2.36371, size = 2546, normalized size = 9.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - 2*a*b*d)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d
^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b
*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c -
2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b
```

```
^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*
d))/x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*
a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^
2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2
))/(b*d^2), 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(1/3)*(3*b^2*c - 2*a*b*
d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3
))*sqrt(-(-b)^(1/3)/b)/x) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^
(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2
*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a
)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c
^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/
x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*a*b*
c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^2*c*
d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2))/(
b*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a + bx^3\right)^{\frac{2}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}} x^3}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

3.686 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=233

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d}$$

```
[Out] (b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)
 - ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*d) - ((b*c - a*d)^(2/3)*Log[c + d*x^
3])/(6*c^(2/3)*d) + ((b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(2*c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)
^(1/3)])/(2*d)
```

Rubi [C] time = 0.0273266, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.040759, size = 161, normalized size = 0.69

$$\frac{4acx\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*b*c*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Fricas [B] time = 1.7259, size = 1102, normalized size = 4.73

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}\cdots}{3\,bx}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

3.687 $\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx$

Optimal. Leaf size=169

$$\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}}-\frac{\left(a+bx^3\right)^{2/3}}{2cx^2}$$

```
[Out] -(a + b*x^3)^(2/3)/(2*c*x^2) + ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d
)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)) + ((b*c
 - a*d)^(2/3)*Log[c + d*x^3])/(6*c^(5/3)) - ((b*c - a*d)^(2/3)*Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3))
```

Rubi [C] time = 0.0627971, antiderivative size = 89, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{2cx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3
, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(2*c*x^2*(1 + (b*x^3)/a)^(2/
3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}\left(1+\frac{dx^3}{c}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{2cx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.0338207, size = 83, normalized size = 0.49

$$-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{2cx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(2*c*x^2*(1 + (b*x^3)/a)^(2/3))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3\right)^{\frac{2}{3}}}{x^3\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

3.688 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=206

$$-\frac{\left(a+bx^3\right)^{2/3}(2bc-5ad)}{10ac^2x^2}-\frac{d(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{8/3}}+\frac{d(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}}-\frac{d(bc-ad)^{2/3}\tan^{-1}\left(\cdots\right)}{\sqrt{3}c^{8/3}}$$

```
[Out] -(a + b*x^3)^(2/3)/(5*c*x^5) - ((2*b*c - 5*a*d)*(a + b*x^3)^(2/3))/(10*a*c^
2*x^2) - (d*(b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)) - (d*(b*c - a*d)^(2/3)*Log[
c + d*x^3])/(6*c^(8/3)) + (d*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^
(1/3) - (a + b*x^3)^(1/3)])/(2*c^(8/3))
```

Rubi [C] time = 0.134778, antiderivative size = 148, normalized size of antiderivative = 0.72, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-2x^3\left(2c-3dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+c\left(a+bx^3\right)\left(2c-3dx^3\right)}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -(c*(a + b*x^3)*(2*c - 3*d*x^3) - 2*(b*c - a*d)*x^3*(2*c - 3*d*x^3)*Hyperge
ometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c - a*d)*
x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(10*c^3*x^5*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{c\left(a+bx^3\right)\left(2c-3dx^3\right)-2(bc-ad)x^3\left(2c-3dx^3\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)+6(bc-ad)x^3\left(c+dx^3\right)}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Mathematica [C] time = 0.389426, size = 148, normalized size = 0.72

$$-\frac{2x^3\left(3dx^3-2c\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+c\left(a+bx^3\right)\left(2c-3dx^3\right)}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -(c*(a + b*x^3)*(2*c - 3*d*x^3) + 2*(b*c - a*d)*x^3*(-2*c + 3*d*x^3)*Hyperg
eometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c - a*d)
*x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))])/(10*c^3*x^5*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x^6\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^6\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

3.689 $\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx$

Optimal. Leaf size=257

$$\frac{(a+bx^3)^{2/3}(-20a^2d^2+8abcd+3b^2c^2)}{40a^2c^3x^2}+\frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}}-\frac{d^2(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}}+\frac{d^2(bc$$

```
[Out] -(a + b*x^3)^(2/3)/(8*c*x^8) - ((b*c - 4*a*d)*(a + b*x^3)^(2/3))/(20*a*c^2*
x^5) + ((3*b^2*c^2 + 8*a*b*c*d - 20*a^2*d^2)*(a + b*x^3)^(2/3))/(40*a^2*c^3
*x^2) + (d^2*(b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(11/3)) + (d^2*(b*c - a*d)^(2/3)*
Log[c + d*x^3])/(6*c^(11/3)) - (d^2*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3
)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(11/3))
```

Rubi [C] time = 0.970983, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-9x^3(c+dx^3)^2(bc-ad)\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-2x^3(5c^2-6cdx^3+9d^2x^6)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-6bc^2d\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^6 + 9
*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3*Hype
rgeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c^2*d*
x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6*b
*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[
{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*x^8*(a +
```

```
 b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{5ac^3+5bc^3x^3-6ac^2dx^3-6bc^2dx^6+9acd^2x^6+9bcd^2x^9-2(bc-ad)x^3\left(5c^2-6cdx^3+9d^2x^6\right)\,{}_2\cdots}{\cdots}$$

Mathematica [C] time = 1.62771, size = 451, normalized size = 1.75

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\text{HypergeometricPFQ}\left(\left\{\frac{1}{3},2,2\right\},\left\{1,\frac{4}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)-2x^3\left(5c^2-6cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^6 + 9
*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hypergeome
```

```
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3*Hype
rgeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c^2*d*
x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6*b
*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[
{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*x^8*(a +
 b*x^3)^(1/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^{9} \left(d x^{3}+c \right)} \left(b x^{3}+a \right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{3}+a\right)^{\frac{2}{3}}}{\left(d x^{3}+c\right) x^{9}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3\right)^{\frac{2}{3}}}{x^9 \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

3.690 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)} dx$

Optimal. Leaf size=320

$$-\frac{(a+bx^3)^{2/3}(88a^2bcd^2-220a^3d^3+33ab^2c^2d+18b^3c^3)}{440a^3c^4x^2}+\frac{(a+bx^3)^{2/3}(-44a^2d^2+11abcd+6b^2c^2)}{220a^2c^3x^5}-\frac{d^3(bc-ad)^{2/3}\log}{6c^{14/3}}$$

```
[Out] -(a + b*x^3)^(2/3)/(11*c*x^11) - ((2*b*c - 11*a*d)*(a + b*x^3)^(2/3))/(88*a
*c^2*x^8) + ((6*b^2*c^2 + 11*a*b*c*d - 44*a^2*d^2)*(a + b*x^3)^(2/3))/(220*
a^2*c^3*x^5) - ((18*b^3*c^3 + 33*a*b^2*c^2*d + 88*a^2*b*c*d^2 - 220*a^3*d^3
)*(a + b*x^3)^(2/3))/(440*a^3*c^4*x^2) - (d^3*(b*c - a*d)^(2/3)*ArcTan[(1 +
 (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^
(14/3)) - (d^3*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^(14/3)) + (d^3*(b*c -
 a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(1
4/3))
```

Rubi [C] time = 2.62553, antiderivative size = 819, normalized size of antiderivative = 2.56, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-81bcd^3x^{12}-162ad^4\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}+162bcd^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}-297ad^4\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^2*d^2*
x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^12 - 80*b*c^4*x^3*Hy
pergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*a*c^3*
d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9
0*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1, 4/3,
((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[1/3
, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeometri
```

```
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3*Hyperg
eometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c^3*d*x
^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99*b
*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] - 27*(b*c - a*d)*x^3*(5*c - 6*d*x^3)*(c + d*x
^3)^2*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2},
 {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(440*c^5*x^11*(a + b*x^3)
^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511 (Rubi steps)

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40ac^4+40bc^4x^3-45ac^3dx^3-45bc^3dx^6+54ac^2d^2x^6+54bc^2d^2x^9-81acd^3x^9-81bcd^3x^{12}-80}{}$$

Mathematica [C] time = 3.91969, size = 819, normalized size = 2.56

$$-\frac{-81bcd^3x^{12}-162ad^4\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}+162bcd^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}-297ad^4\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^2*d^2*
x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^12 - 80*b*c^4*x^3*Hy
pergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*a*c^3*
d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9
0*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1, 4/3,
((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[1/3
, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeometri
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3*Hyperg
eometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c^3*d*x
^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99*b
*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^2*(-5*c + 6*
d*x^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2}
, {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(440*c^5*x^11*(a + b*x^3
)^(1/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^{12}\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

3.691 $\int \frac{x^7\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^8\left(a+bx^3\right)^{2/3}F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] (x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] time = 0.0579732, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\left(a+bx^3\right)^{2/3}F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^8\left(a+bx^3\right)^{2/3} F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] time = 0.200559, size = 181, normalized size = 2.83

$$\frac{x^2\left(-2x^3\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2+7abcd-14b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5ac\sqrt[3]{\frac{bx^3}{a}+1}(7bc-2ad)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.}{140bcd^2\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(5*c*(a + b*x^3)*(-7*b*c + 2*a*d + 4*b*d*x^3) + 5*a*c*(7*b*c - 2*a*d)*
(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)
] - 2*(-14*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(140*b*c*d^2*(a + b*x^3
)^(1/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{x^7}{dx^3+c}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^7}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}} x^7}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

3.692 $\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal. Leaf size=64

$$\frac{x^5 \left(a+bx^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] (x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] time = 0.0561503, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5 \left(a+bx^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^5\left(a+bx^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.0949248, size = 141, normalized size = 2.2

$$\frac{2x^5\sqrt[3]{\frac{bx^3}{a}+1}(ad-2bc)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5acx^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(a+bx^3\right)}{20cd\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*(-2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(
1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*c*d*(a + b
*x^3)^(1/3))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^4}{dx^3+c}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}} x^4}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(a + bx^3\right)^{\frac{2}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}} x^4}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

3.693 $\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^2 \left(a+bx^3\right)^{2/3} F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] time = 0.0398454, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2 \left(a+bx^3\right)^{2/3} F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [A] time = 0.0323984, size = 65, normalized size = 1.02

$$\frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{a+bx^3}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*((a + b*x^3)/a)^(2/3))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x}{dx^3+c}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

3.694 $\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx$

Optimal. Leaf size=62

$$-\frac{(a+bx^3)^{2/3} F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] -(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*x*(1 + (b*x^3)/a)^(2/3)))
```

Rubi [A] time = 0.0601531, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{(a+bx^3)^{2/3} F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*x*(1 + (b*x^3)/a)^(2/3)))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] time = 0.0911039, size = 138, normalized size = 2.23

$$\frac{2bdx^6\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5x^3\sqrt[3]{\frac{bx^3}{a}+1}(ad-2bc)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-10c\left(a+bx^3\right)}{10c^2x\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) - 5*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/
3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/
3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*c^2*x*(a + b
*x^3)^(1/3))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**2*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

3.695 $\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{(a+bx^3)^{2/3} F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] -((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(4*c*x^4*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] time = 0.0605682, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{(a+bx^3)^{2/3} F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(4*c*x^4*(1 + (b*x^3)/a)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] time = 0.156636, size = 181, normalized size = 2.83

$$\frac{5x^6\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2-4abcd+b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^9\sqrt[3]{\frac{bx^3}{a}+1}(bc-2ad)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5c\left(\right.}{20ac^3x^4\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] (-5*c*(a + b*x^3)*(2*b*c*x^3 + a*(c - 4*d*x^3)) + 5*(b^2*c^2 - 4*a*b*c*d +
2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a
), -((d*x^3)/c)] + 2*b*d*(b*c - 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c^3*x^4*(a + b*x^3)^(1/
3))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^3)^{\frac{2}{3}}}{x^5\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**5*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

3.696 $\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=251

$$-\frac{(a+bx^3)^{7/3}(ad+bc)}{7b^2d^2}+\frac{(a+bx^3)^{10/3}}{10b^2d}+\frac{c^2(a+bx^3)^{4/3}}{4d^3}-\frac{c^2\sqrt[3]{a+bx^3}(bc-ad)}{d^4}-\frac{c^2(bc-ad)^{4/3}\log(c+dx^3)}{6d^{13/3}}+\frac{c^2(bc-}{}$$

```
[Out] -((c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4) + (c^2*(a + b*x^3)^(4/3))/(4*d^3
) - ((b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (a + b*x^3)^(10/3)/(10*b^
2*d) - (c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[c
 + d*x^3])/(6*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^
(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rubi [A] time = 0.362667, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{(a+bx^3)^{7/3}(ad+bc)}{7b^2d^2}+\frac{(a+bx^3)^{10/3}}{10b^2d}+\frac{c^2(a+bx^3)^{4/3}}{4d^3}-\frac{c^2\sqrt[3]{a+bx^3}(bc-ad)}{d^4}-\frac{c^2(bc-ad)^{4/3}\log(c+dx^3)}{6d^{13/3}}+\frac{c^2(bc-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -((c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4) + (c^2*(a + b*x^3)^(4/3))/(4*d^3
) - ((b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (a + b*x^3)^(10/3)/(10*b^
2*d) - (c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[c
 + d*x^3])/(6*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^
(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8 \left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{4/3}}{bd^2}+\frac{(a+bx)^{7/3}}{bd}+\frac{c^2(a+bx)^{4/3}}{d^2(c+dx)}\right)dx,x,x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{c^2\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)}{3d^2}$$

$$= \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{\left(c^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right.}{3d^3}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{\left(c^2(bc-ad)\right.}{}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/}}{6d}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/}}{6d}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/}}{}$$

Mathematica [A] time = 0.432544, size = 258, normalized size = 1.03

$$\frac{-\frac{60d\left(a+bx^3\right)^{7/3}(ad+bc)}{b^2}+\frac{42d^2\left(a+bx^3\right)^{10/3}}{b^2}-\frac{70c^2(bc-ad)\left(\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+\right.\right.}{d^{4/3}}}{420d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (105*c^2*(a + b*x^3)^(4/3) - (60*d*(b*c + a*d)*(a + b*x^3)^(7/3))/b^2 + (42
*d^2*(a + b*x^3)^(10/3))/b^2 - (70*c^2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(
```

```
1/3) + (b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3
))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^
(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])))/d^(4/3))/(420*d^3)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^8}{dx^3 + c} \left(bx^3 + a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48239, size = 829, normalized size = 3.3

$$\frac{140 \sqrt{3}\left(b^3c^3 - ab^2c^2d\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}} d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} - \sqrt{3}(bc-ad)}{3\,(bc-ad)}\right) + 70\left(b^3c^3 - ab^2c^2d\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left(bx^3 + a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1
/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a
*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*c*d^2
- 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^
3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 + a)^(1/3
))/(b^2*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19153, size = 532, normalized size = 2.12

$$-\frac{\left(b^{24}c^4d^6-2\,ab^{23}c^3d^7+a^2b^{22}c^2d^8\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{23}cd^{10}-ab^{22}d^{11}\right)}+\frac{\sqrt{3}\left(bc^3-ac^2d\right)\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\,\mathrm{arctan}}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)^
(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 - a
*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b
```

```
^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b
^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*a*b^18*d^
9)/(b^20*d^10)
```

3.697 $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=211

$$-\frac{c\left(a+bx^3\right)^{4/3}}{4d^2}+\frac{c\sqrt[3]{a+bx^3}(bc-ad)}{d^3}+\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c(b}{}$$

```
[Out] (c*(b*c - a*d)*(a + b*x^3)^(1/3))/d^3 - (c*(a + b*x^3)^(4/3))/(4*d^2) + (a
+ b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b
*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c*(b*c - a*
d)^(4/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rubi [A] time = 0.244996, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$-\frac{c\left(a+bx^3\right)^{4/3}}{4d^2}+\frac{c\sqrt[3]{a+bx^3}(bc-ad)}{d^3}+\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (c*(b*c - a*d)*(a + b*x^3)^(1/3))/d^3 - (c*(a + b*x^3)^(4/3))/(4*d^2) + (a
+ b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b
*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c*(b*c - a*
d)^(4/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5 \left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{7/3}}{7bd} - \frac{c\,\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{(c(bc-ad))\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} - \frac{\left(c(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x\right.}{3d^3}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}} - \frac{\left(c(bc-ad)^{4/\cdots}\right.}{\cdots}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3}\cdots}{\cdots}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c(bc\cdots}{\cdots}$$

Mathematica [A] time = 0.256264, size = 255, normalized size = 1.21

$$-\frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{c(bc-ad)\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\cdots\right.\right.}{6d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -(c*(a + b*x^3)^(4/3))/(4*d^2) + (a + b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)
*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2
*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/
3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*L
```

```
og[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3
)*(a + b*x^3)^(2/3)]))/(6*d^(10/3))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^5}{dx^3 + c} \left(bx^3 + a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54, size = 683, normalized size = 3.24

$$28\sqrt{3}\left(b^2c^2 - abcd\right)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}} d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}} - \sqrt{3}(bc-ad)}{3\,(bc-ad)}\right) + 14\left(b^2c^2 - abcd\right)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left(bx^3 + a\right)^{\frac{2}{3}} - \left(bx^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*
```

```
c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2
/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 28
*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c -
a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - (
7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3))/(b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \left(a + bx^3\right)^{\frac{4}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [B] time = 1.21291, size = 470, normalized size = 2.23

$$\frac{\left(b^{10}c^3d^4 - 2\,ab^9c^2d^5 + a^2b^8cd^6\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left|\left(bx^3 + a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^9cd^7 - ab^8d^8\right)} - \frac{\sqrt{3}\left(-bcd^2 + ad^3\right)^{\frac{1}{3}}\left(bc^2 - acd\right)\arctan\left(\right.}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3)
*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*d^
8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^
4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3) + (b
*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28
*(28*(b*x^3 + a)^(1/3)*b^8*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^7*c*d^5 - 28*(b*
x^3 + a)^(1/3)*a*b^7*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^6*d^6)/(b^7*d^7)
```

3.698 $\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=187

$$-\frac{\sqrt[3]{a+bx^3}(bc-ad)}{d^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\cdots}{\cdots}}{\cdots}\right)}{\sqrt{3}d^{7/3}}$$

```
[Out] -(((b*c - a*d)*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*d) - ((b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]])/(Sqrt[3]*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(7/3)) +
 ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(7/3))
```

Rubi [A] time = 0.207714, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {444, 50, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}(bc-ad)}{d^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\cdots}{\cdots}}{\cdots}\right)}{\sqrt{3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*d) - ((b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]])/(Sqrt[3]*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(7/3)) +
 ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(7/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2 \left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)$$
$$= \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)}{3d}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3d^2}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\right.}{2d^{7/3}}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{\right.}{2d^{7/3}}$$
$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right.}{6d^{7/3}}$$

<u>Mathematica</u> [A] time = 0.295421, size = 232, normalized size = 1.24

$$\frac{(ad-bc)\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+\right.\right.}{6d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(4/3)/(4*d) + ((-(b*c) + a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*
Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
 a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(6*d^(7/3))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^2}{d x^3 + c} \left(b x^3 + a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57733, size = 572, normalized size = 3.06

$$\frac{4\sqrt{3}(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right.}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(
b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*d)*(
-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(
b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + bx^3\right)^{\frac{4}{3}}}{c + dx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [A] time = 1.24612, size = 401, normalized size = 2.14

$$-\frac{\left(b^2c^2d^2 - 2\,abcd^3 + a^2d^4\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left|\left(bx^3 + a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd^4 - ad^5\right)} + \frac{\sqrt{3}\left(-bcd^2 + ad^3\right)^{\frac{1}{3}}(bc - ad)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}\right.}{3\left(-\frac{bc-}{}\right.}\right.}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs((
b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(3)
*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 +
a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c -
 a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b*c
*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4
```

3.699 $\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=261

$$\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-\right.}{2cd^{4}}$$

```
[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(4/3)) - (a^(4
/3)*Log[x])/(2*c) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c*d^(4/3)) + (a^(
4/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(4/3)*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(4/3))
```

Rubi [A] time = 0.304392, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 84, 156, 57, 617, 204, 31, 58}

$$\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-\right.}{2cd^{4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(4/3)) - (a^(4
/3)*Log[x])/(2*c) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c*d^(4/3)) + (a^(
4/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(4/3)*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(4/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{\text{Subst}\left(\int \frac{a^2d+b(-bc+2ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3cd}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} - \frac{a^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{5/3}}{}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{4/3}\log}{}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}c} + \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}}{6}$$

Mathematica [A] time = 0.539185, size = 331, normalized size = 1.27

$$\frac{a^{4/3}\left(-\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)+\frac{(bc-ad)\left(\sqrt[3]{bc-ad}\log\left(\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (6*a*(a + b*x^3)^(1/3) - a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a
^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]) + ((b*c - a*d)*(6*d^(1/3)*(a
 + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a +
b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c -
a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)
^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^
(2/3)]))/d^(4/3))/(6*c)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)
```

Fricas [A] time = 2.62887, size = 811, normalized size = 3.11

$$-\frac{2\sqrt{3}a^{\frac{4}{3}}d\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{4}{3}}d\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{4}{3}}d\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-2\sqrt{3}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(b*
c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(
(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 6*(b*x^3 + a)^(1
/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3
+ a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c - a*d)*(
(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))/(c*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3\right)^{\frac{4}{3}}}{x\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)
```

Giac [A] time = 2.80749, size = 510, normalized size = 1.95

$$\frac{1}{6}\left(\frac{2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^2c^2d - abcd^2} - \frac{2\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{bc} - \frac{a^{\frac{4}{3}}\log\left(\left(b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^
3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d - a*b*c*d^2) - 2*sqrt(3)
*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(b*c)
- a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(b*c
) + 2*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(b*c) + 6*(b*x^3 + a)^(
1/3)/d - 2*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*
(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(b*c
*d^2) - (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2))*b
```

3.700 $\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)} dx$

Optimal. Leaf size=399

$$\frac{d(a+bx^3)^{4/3}}{4c^2}+\frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2}+\frac{\sqrt[3]{a+bx^3}(4bc-3ad)}{3c^2}-\frac{\sqrt[3]{a+bx^3}(bc-ad)}{c^2}-\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{d}}+$$

```
[Out] ((4*b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*c^2) - ((b*c - a*d)*(a + b*x^3)^(1/3
))/c^2 + (d*(a + b*x^3)^(4/3))/(4*c^2) + ((4*b*c - 3*a*d)*(a + b*x^3)^(4/3)
)/(12*a*c^2) - (a + b*x^3)^(7/3)/(3*a*c*x^3) - (a^(1/3)*(4*b*c - 3*a*d)*Arc
Tan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*c^2) - (
(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(Sqrt[3]*c^2*d^(1/3)) - (a^(1/3)*(4*b*c - 3*a*d)*Log[x])/(6*c
^2) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^2*d^(1/3)) + (a^(1/3)*(4*b*c
- 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c^2) + ((b*c - a*d)^(4/3)*Log
[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*d^(1/3))
```

Rubi [A] time = 0.48352, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{d(a+bx^3)^{4/3}}{4c^2}+\frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2}+\frac{\sqrt[3]{a+bx^3}(4bc-3ad)}{3c^2}-\frac{\sqrt[3]{a+bx^3}(bc-ad)}{c^2}-\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{d}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```



```
[Out] ((4*b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*c^2) - ((b*c - a*d)*(a + b*x^3)^(1/3
))/c^2 + (d*(a + b*x^3)^(4/3))/(4*c^2) + ((4*b*c - 3*a*d)*(a + b*x^3)^(4/3)
)/(12*a*c^2) - (a + b*x^3)^(7/3)/(3*a*c*x^3) - (a^(1/3)*(4*b*c - 3*a*d)*Arc
Tan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*c^2) - (
(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(Sqrt[3]*c^2*d^(1/3)) - (a^(1/3)*(4*b*c - 3*a*d)*Log[x])/(6*c
^2) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^2*d^(1/3)) + (a^(1/3)*(4*b*c
```

```
- 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c^2) + ((b*c - a*d)^(4/3)*Log
[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*d^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{3acx^3} - \frac{\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-4bc+3ad)-\frac{4bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{3acx^3} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(4bc-3ad)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3} + \frac{(4bc-3ad)\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9c^2} -$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3}$$

Mathematica [A] time = 1.21008, size = 389, normalized size = 0.97

$$\frac{\frac{(4bc-3ad)\left(-\frac{1}{2}a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)+3a\sqrt[3]{a+bx^3}+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{3c} + \frac{a\left(3d^{4/3}\left(a+bx^3\right)^{4/3}-2(bc-\right.}{}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-((a + b*x^3)^(7/3)/x^3) + ((4*b*c - 3*a*d)*(3*a*(a + b*x^3)^(1/3) + (3*(a
 + b*x^3)^(4/3))/4 - (a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/
```

```
a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/2))/(3*c) + (a*(3*d^(4/3)*(a +
 b*x^3)^(4/3) - 2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) + (b*c - a*d)^(1
/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)]))))/(4*c*d^(1/3)))/(3*a*c)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

Fricas [A] time = 3.54093, size = 946, normalized size = 2.37

$$6\sqrt{3}(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\sqrt{3}(4\,bc-3\,ad)\,(-a)^{\frac{1}{3}}\,x^3\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(-\cdots}{3\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(
3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c -
 a*d)) + 2*sqrt(3)*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*arctan(1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + (4*b*c - 3*a*d)*(-a)^(1/3)*x^3*
log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 3*(b*c
 - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(4*b*c - 3*a*d)*(-a)
^(1/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*c - a*d)*x^3*(-(b*c -
 a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 +
 a)^(1/3)*a*c)/(c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3\right)^{\frac{4}{3}}}{x^4 \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)
```

Giac [A] time = 2.86263, size = 568, normalized size = 1.42

$$-\frac{1}{18}\left(\frac{6\left(b^2c^2 - 2\,abcd + a^2d^2\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3 + a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^3c^3 - ab^2c^2d} + \frac{2\sqrt{3}\left(4\,a^{\frac{1}{3}}bc - 3\,a^{\frac{4}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right.}{3\,a^{\frac{1}{3}}}\right.}{b^2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/18*(6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*
x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^3*c^3 - a*b^2*c^2*d) + 2*sqrt(
3)*(4*a^(1/3)*b*c - 3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
a^(1/3))/a^(1/3))/(b^2*c^2) + (4*a^(1/3)*b*c - 3*a^(4/3)*d)*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(b^2*c^2) - 6*sqrt(3)*(-b*c*d
^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(b^2*c^2*d) - 3*(-b*c*d^2 + a*
d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a
*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d) - 2*(4*a*b*c - 3*a^2*d)*
log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*b^2*c^2) + 6*(b*x^3 + a)^(1/
3)*a/(b^2*c*x^3))*b^2
```

3.701 $\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx$

Optimal. Leaf size=440

$$\frac{\sqrt[3]{a+bx^3}\left(9a^2d^2-12abcd+2b^2c^2\right)}{9ac^3}+\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3}-\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\tan^{-1}}{9\sqrt{3}a^{2/3}c^3}$$

```
[Out] (d*(b*c - a*d)*(a + b*x^3)^(1/3))/c^3 + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^
2)*(a + b*x^3)^(1/3))/(9*a*c^3) - ((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(18*a*c
^2*x^3) - (a + b*x^3)^(7/3)/(6*a*c*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*
d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3
)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^3) - ((2*b^2*c^2 - 12*a*b*
c*d + 9*a^2*d^2)*Log[x])/(18*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[
c + d*x^3])/(6*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(18*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)
```

Rubi [A] time = 0.622864, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {446, 103, 149, 156, 50, 57, 617, 204, 31, 58}

$$\frac{\sqrt[3]{a+bx^3}\left(9a^2d^2-12abcd+2b^2c^2\right)}{9ac^3}+\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3}-\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\tan^{-1}}{9\sqrt{3}a^{2/3}c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```



```
[Out] (d*(b*c - a*d)*(a + b*x^3)^(1/3))/c^3 + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^
2)*(a + b*x^3)^(1/3))/(9*a*c^3) - ((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(18*a*c
^2*x^3) - (a + b*x^3)^(7/3)/(6*a*c*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*
d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3
)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^3) - ((2*b^2*c^2 - 12*a*b*
```

```
c*d + 9*a^2*d^2)*Log[x])/(18*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[
c + d*x^3])/(6*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(18*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3(c+dx)}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\mathrm{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-bc+6ad)-\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx,x,x^3\right)}{6ac}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{2}{9}\left(2b^2c^2-12abcd+9a^2d^2\right)-\frac{2}{9}bd(2bc-3ad)x\right)}{x(c+dx)}\,dx,x,\right.}{6ac^2}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\mathrm{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)}{3c^3} + \frac{\left(2b^2c^2-12a\right.}{}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/}}{6acx^6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/}}{6acx^6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/}}{6acx^6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/}}{6acx^6}$$

Mathematica [A] time = 0.982329, size = 429, normalized size = 0.98

$$\frac{x^6\left(4\left(9a^2d^2-12abcd+2b^2c^2\right)\left(-\frac{1}{2}a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (-18*a*c^2*(a + b*x^3)^(7/3) - 6*c*(b*c - 6*a*d)*x^3*(a + b*x^3)^(7/3) + x^
6*(4*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(3*a*(a + b*x^3)^(1/3) + (3*(a +
```

```
b*x^3)^(4/3))/4 - (a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)
*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/2) - 9*a^2*d^(2/3)*(3*d^(4/3)*(a
+ b*x^3)^(4/3) - 2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) + (b*c - a*d)^(
1/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
 + b*x^3)^(2/3)])))))/(108*a^2*c^3*x^6)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^7 \left(d x^3+c\right)} \left(b x^3+a\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^3+a\right)^{\frac{4}{3}}}{\left(d x^3+c\right) x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)
```

Fricas [A] time = 12.0079, size = 1204, normalized size = 2.74

$$18 \sqrt{3}\left(a^2bc - a^3d\right)\left(bcd^2 - ad^3\right)^{\frac{1}{3}} x^6 \arctan\left(-\frac{2 \sqrt{3}\left(bcd^2-ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}} - \sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right) - 2\sqrt{3}\left(2\,ab^2c^2 - 12\,a^2bcd + 9\,a^3d^2\right)\left(a^2\right)^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(1/3)*x^6*arctan(-1/3*
(2*sqrt(3)*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d
^2))/(b*c*d - a*d^2)) - 2*sqrt(3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*d^2)*
(a^2)^(1/6)*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(
a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(a^2)^(2/3)) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((
b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1
/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 18*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(3*a^3*c^2
+ (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(1/3))/(a^2*c^3*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.21773, size = 689, normalized size = 1.57

$$\frac{1}{54}\left(\frac{18\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^4c^4-ab^3c^3d}-\frac{18\,\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\cdots}{\cdots}\right)}{b^3c^3}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/54*(18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs
((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^4*c^4 - a*b^3*c^3*d) - 18*
sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 +
 a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(b^3*c^3) - 9*(
-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^3*c^3) - 2*sqrt(3)*(
2*a^(1/3)*b^2*c^2 - 12*a^(4/3)*b*c*d + 9*a^(7/3)*d^2)*arctan(1/3*sqrt(3)*(2
*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a*b^3*c^3) + 2*(2*b^2*c^2 - 12*a*b*
c*d + 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*b^3*c^3) -
(2*a^(1/3)*b^2*c^2 - 12*a^(4/3)*b*c*d + 9*a^(7/3)*d^2)*log((b*x^3 + a)^(2/3
) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a*b^3*c^3) - 3*(7*(b*x^3 + a)^(4/
3)*b*c - 4*(b*x^3 + a)^(1/3)*a*b*c - 6*(b*x^3 + a)^(4/3)*a*d + 6*(b*x^3 + a
)^(1/3)*a^2*d)/(b^4*c^2*x^6))*b^3
```

3.702 $\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=334

$$-\frac{(2a^2d^2-12abcd+9b^2c^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3}-\frac{(2a^2d^2-12abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{2/3}d^3}-\frac{c^{2/3}(bc-ad)^{4/3}\log(}{6d^3}$$

```
[Out] -((6*b*c - 7*a*d)*x^2*(a + b*x^3)^(1/3))/(18*d^2) + (b*x^5*(a + b*x^3)^(1/3
))/(6*d) - ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(2/3)*d^3) + (c^(2/3)*(b*c - a*d)
^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqr
t[3]])/(Sqrt[3]*d^3) - (c^(2/3)*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^3) -
 ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/
(18*b^(2/3)*d^3) + (c^(2/3)*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(
1/3) - (a + b*x^3)^(1/3)])/(2*d^3)
```

Rubi [C] time = 0.0590289, antiderivative size = 65, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{ax^5\sqrt[3]{a+bx^3}F_1\left(\frac{5}{3};-\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -4/3, 1, 8/3, -((b*x^3)/a), -((d*x^3
)/c)])/(5*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^5\sqrt[3]{a+bx^3}F_1\left(\frac{5}{3};-\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.267245, size = 225, normalized size = 0.67

$$\frac{2x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}\left(2a^2d^2-12abcd+9b^2c^2\right)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(6bc-7ad)\,{}_2F_1\left(\frac{2}{3},\right.\right.}{90cd^2\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x
^3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x
^2*((a + b*x^3)*(-6*b*c + 7*a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b
*c - 7*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c)
 + a*d)*x^3)/(a*(c + d*x^3))]))/(90*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)
^(2/3))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{x^4}{dx^3+c}\left(bx^3+a\right)^{\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

Fricas [A] time = 11.9539, size = 1281, normalized size = 3.84

$$2\sqrt{3}\left(9\,b^3c^2-12\,ab^2cd+2\,a^2bd^2\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3\,b^2x}\right)-18\sqrt{3}\left(b^3c-ab^2d\right)\left(-b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(9*b^3*c^2 - 12*a*b^2*c*d + 2*a^2*b*d^2)*sqrt(-(-b^2)^(1/3)
)*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2
)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 18*sqrt(3)*(b^3*c - a*b^2*d)*(-b*c^
3 + a*c^2*d)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c
^3 + a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) - 2*(9*b^2*c^2
```

```
- 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(
1/3)*b)/x) + (9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2)
 - 18*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1/3)*log(((b*x^3 + a)^(1/3)*c +
 (-b*c^3 + a*c^2*d)^(1/3)*x)/x) + 9*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1
/3)*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)
*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d -
 7*a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(a + bx^3\right)^{\frac{4}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}} x^4}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

3.703 $\int \frac{x(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal. Leaf size=277

$$\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d^2}+\frac{\sqrt[3]{b}(3bc-4ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6d^2}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d^2}+\frac{\sqrt[3]{b}(3bc-\cdots}{\cdots}$$

```
[Out] (b*x^2*(a + b*x^3)^(1/3))/(3*d) + (b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(1 + (2*b
^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^2) - ((b*c - a*d)^(4/3)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(Sqrt[3]*c^(1/3)*d^2) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(1/3)*d^2)
 + (b^(1/3)*(3*b*c - 4*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*d^2) - (
(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(
2*c^(1/3)*d^2)
```

Rubi [C] time = 0.0439897, antiderivative size = 65, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{ax^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -4/3, 1, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.215738, size = 198, normalized size = 0.71

$$\frac{2bx^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(4ad-3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5x^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(3ad-2bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)\right.}{30cd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (2*b*(-3*b*c + 4*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Appel
lF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*x^2*(2*b*c*(a + b*x^3
)*(1 + (d*x^3)/c)^(2/3) + a*(-2*b*c + 3*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(30*c*d*(a
 + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x}{dx^3+c}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

Fricas [A] time = 3.50299, size = 968, normalized size = 3.49

$$6\left(bx^3+a\right)^{\frac{1}{3}}bdx^2-6\sqrt{3}(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}(3\,bc-4\,ad)\,(-b)^{\frac{1}{3}}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b*d*x^2 - 6*sqrt(3)*(b*c - a*d)*((b*c - a*d)/c)^(
1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b
*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b*c - 4*a*d)*(-b)^(1/3)
*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^(2/3))/(b*x)) -
 2*(3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 6
*(b*c - a*d)*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 +
 a)^(1/3))/x) + (3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a
)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(b*c - a*d)*((b*c - a*d)
/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) + (b*x^3 + a)^(1/3)*x*((b*c - a*d)
/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

3.704 $\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$

Optimal. Leaf size=254

$$-\frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}-\frac{b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{4/3}d}+\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}d}$$

```
[Out] -((a*(a + b*x^3)^(1/3))/(c*x)) - (b^(4/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d) + ((b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c
- a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)*d)
- ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(4/3)*d) - (b^(4/3)*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/(2*d) + ((b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)
/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3)*d)
```

Rubi [C] time = 0.0590058, antiderivative size = 63, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{1}{3};-\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3)*AppellF1[-1/3, -4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*x*(1 + (b*x^3)/a)^(1/3)))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{1}{3};-\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.323071, size = 161, normalized size = 0.63

$$\frac{2b^2cx^6\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-\frac{5ax^3\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-2bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{\left(\frac{dx^3}{c}+1\right)^{2/3}}-10ac\left(a+bx^3\right)}{10c^2x\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-10*a*c*(a + b*x^3) + 2*b^2*c*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3,
 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] - (5*a*(-2*b*c + a*d)*x^3*(1 + (b*x^3)
/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x
^3))])/(1 + (d*x^3)/c)^(2/3))/(10*c^2*x*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

3.705 $\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=201

$$-\frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{4c^2x}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}}$$

```
[Out] -(a*(a + b*x^3)^(1/3))/(4*c*x^4) - ((5*b*c - 4*a*d)*(a + b*x^3)^(1/3))/(4*c
^2*x) - ((b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
+ b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)) + ((b*c - a*d)^(4/3)*Log[c + d
*x^3])/(6*c^(7/3)) - ((b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)])/(2*c^(7/3))
```

Rubi [C] time = 0.0635623, antiderivative size = 90, normalized size of antiderivative = 0.45, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{4cx^4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/3, -
1/3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(4*c*x^4*(1 + (b*x^3)/a)^
(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^5\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{a\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4cx^4\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.0328093, size = 84, normalized size = 0.42

$$-\frac{a\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{4cx^4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/3, -
1/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(4*c*x^4*(1 + (b*x^3)/a)^(1/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^5\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

3.706 $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx$

Optimal. Leaf size=250

$$-\frac{\sqrt[3]{a+bx^3}\left(28a^2d^2-35abcd+4b^2c^2\right)}{28ac^3x}-\frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{28c^2x^4}-\frac{d(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{10/3}}+\frac{d(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}\right.}{2c^{10/3}}$$

```
[Out] -(a*(a + b*x^3)^(1/3))/(7*c*x^7) - ((8*b*c - 7*a*d)*(a + b*x^3)^(1/3))/(28*
c^2*x^4) - ((4*b^2*c^2 - 35*a*b*c*d + 28*a^2*d^2)*(a + b*x^3)^(1/3))/(28*a*
c^3*x) + (d*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(10/3)) - (d*(b*c - a*d)^(4/3)*Log
[c + d*x^3])/(6*c^(10/3)) + (d*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/
c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(10/3))
```

Rubi [C] time = 0.506586, antiderivative size = 169, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{12cx^3\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx^3\right)+5bcx^3\right)-2x^6(bc-\right.}{28c^4x^7\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] (12*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)*Hypergeometric2F1[-1/3, 2, 2/
3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (4*c - 3*d*x^3)*(c*(a + b*x^3)*(5*b
*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(28*c^4*x^7*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^8\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^8\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{12c(bc-ad)x^3\left(a+bx^3\right)\left(c+dx^3\right){}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)-\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(5bcx^3+a\left(c-\right.\right.\right.}{28c^4x^7\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 0.422977, size = 179, normalized size = 0.72

$$-\frac{a\left(\frac{bx^3}{a}+1\right)\left(12cx^3\left(a+bx^3\right)\left(c+dx^3\right)(ad-bc)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx^3\right)+5bcx^3\right)\right.\right.}{28c^4x^7\left(a+bx^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -(a*(1 + (b*x^3)/a)*(12*c*(-(b*c) + a*d)*x^3*(a + b*x^3)*(c + d*x^3)*Hyperg
eometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + (4*c - 3*d*x
^3)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hype
rgeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])))/(28*c^4*x^
7*(a + b*x^3)^(5/3))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{1}{x^{8} \left(d x^{3}+c \right)} \left(b x^{3}+a \right)^{\frac{4}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{3}+a\right)^{\frac{4}{3}}}{\left(d x^{3}+c\right) x^{8}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}}}{\left(dx^3 + c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

3.707 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=318

$$\frac{\sqrt[3]{a+bx^3}\left(-175a^2bcd^2+140a^3d^3+20ab^2c^2d+6b^3c^3\right)}{140a^2c^4x}-\frac{\sqrt[3]{a+bx^3}\left(35a^2d^2-40abcd+2b^2c^2\right)}{140ac^3x^4}+\frac{d^2(bc-ad)^{4/3}\log\left(c+\right.}{6c^{13/3}}$$

```
[Out] -(a*(a + b*x^3)^(1/3))/(10*c*x^10) - ((11*b*c - 10*a*d)*(a + b*x^3)^(1/3))/
(70*c^2*x^7) - ((2*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*(a + b*x^3)^(1/3))/(1
40*a*c^3*x^4) + ((6*b^3*c^3 + 20*a*b^2*c^2*d - 175*a^2*b*c*d^2 + 140*a^3*d^
3)*(a + b*x^3)^(1/3))/(140*a^2*c^4*x) - (d^2*(b*c - a*d)^(4/3)*ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(
13/3)) + (d^2*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(13/3)) - (d^2*(b*c -
a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(13
/3))
```

Rubi [C] time = 1.80114, antiderivative size = 260, normalized size of antiderivative = 0.82, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{-18cx^3\left(a+bx^3\right)\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(-\frac{1}{3},2,2;\frac{2}{3},1;\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6cx^3\left(a+bx^3\right)\left(11c^2+2cdx^3-9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\right.}{140c^5x^{10}\left(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] (6*c*(b*c - a*d)*x^3*(a + b*x^3)*(11*c^2 + 2*c*d*x^3 - 9*d^2*x^6)*Hypergeom
etric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (14*c^2 - 12*c*
d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c -
a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
]) - 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{-1/3
, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(140*c^5*x^10*(a + b
*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{6c(bc-ad)x^3\left(a+bx^3\right)\left(11c^2+2cdx^3-9d^2x^6\right)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)-\left(14c^2-12cdx^3+9d^2x^6\right)\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.65054, size = 270, normalized size = 0.85

$$-\frac{a\left(\frac{bx^3}{a}+1\right)\left(18cx^3\left(a+bx^3\right)\left(c+dx^3\right)^2(bc-ad)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{3},2,2\right\},\left\{\frac{2}{3},1\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)+6cx^3\left(a+bx^3\right)\left(\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -(a*(1 + (b*x^3)/a)*(6*c*(b*c - a*d)*x^3*(a + b*x^3)*(-11*c^2 - 2*c*d*x^3 +
 9*d^2*x^6)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + (14*c^2 - 12*c*d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c -
4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))]) + 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*Hy
```

```
pergeometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])
)/(140*c^5*x^10*(a + b*x^3)^(5/3))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^{11} \left(d x^{3}+c\right)} \left(b x^{3}+a\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{3}+a\right)^{\frac{4}{3}}}{\left(d x^{3}+c\right) x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

3.708 $\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt[3]{a+bx^3}\left(260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4+78ab^3c^3d+36b^4c^4\right)}{1820a^3c^5x}+\frac{\sqrt[3]{a+bx^3}\left(-520a^2bcd^2+455a^3d^3+26ab^2\right.}{1820a^2c^4x^4}$$

```
[Out] -(a*(a + b*x^3)^(1/3))/(13*c*x^13) - ((14*b*c - 13*a*d)*(a + b*x^3)^(1/3))/
(130*c^2*x^10) - ((4*b^2*c^2 - 143*a*b*c*d + 130*a^2*d^2)*(a + b*x^3)^(1/3)
)/(910*a*c^3*x^7) + ((12*b^3*c^3 + 26*a*b^2*c^2*d - 520*a^2*b*c*d^2 + 455*a
^3*d^3)*(a + b*x^3)^(1/3))/(1820*a^2*c^4*x^4) - ((36*b^4*c^4 + 78*a*b^3*c^3
*d + 260*a^2*b^2*c^2*d^2 - 2275*a^3*b*c*d^3 + 1820*a^4*d^4)*(a + b*x^3)^(1/
3))/(1820*a^3*c^5*x) + (d^3*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1
/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(16/3)) - (d^3*(b*
c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(16/3)) + (d^3*(b*c - a*d)^(4/3)*Log[((
b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(16/3))
```

Rubi [C] time = 4.39057, antiderivative size = 1446, normalized size of antiderivative = 3.69, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] -(140*a^2*c^5 + 840*a*b*c^5*x^3 - 686*a^2*c^4*d*x^3 + 700*b^2*c^5*x^6 - 131
6*a*b*c^4*d*x^6 + 612*a^2*c^3*d^2*x^6 - 630*b^2*c^4*d*x^9 + 1152*a*b*c^3*d^
2*x^9 - 513*a^2*c^2*d^3*x^9 + 540*b^2*c^3*d^2*x^12 - 918*a*b*c^2*d^3*x^12 +
 324*a^2*c*d^4*x^12 - 405*b^2*c^2*d^3*x^15 + 324*a*b*c*d^4*x^15 - 828*a*b*c
^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/
3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*b^2*c^4*d*x^9*Hypergeome
tric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 234*a*b*c^3*d^2*
```

```
x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 32
4*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + 324*b^2*c^3*d^2*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] - 918*a*b*c^2*d^3*x^12*Hypergeometric2F1[-1/3, 2,
2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a^2*c*d^4*x^12*Hypergeometric
2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*b^2*c^2*d^3*x^15
*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a
*b*c*d^4*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] - 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*b^2*c^4*d*x^9*Hypergeometric2F
1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 504*a*b*c^3*d^2*x^9*Hyp
ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*a^2*c^
2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 216*b^2*c^3*d^2*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 432*a*b*c^2*d^3*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 216*a^2*c*d^4*x^12*Hypergeometric2F1[2/3, 1,
 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b^2*c^2*d^3*x^15*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*a*b*c*d^4*x^15
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*a^
2*d^5*x^15*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(7*c - 6*d*x^3)*(c + d*x^3)^2*Hypergeo
metricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*c
*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2, 2
}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(1820*c^6*x^13*(a + b*x
^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{140a^2c^5+840abc^5x^3-686a^2c^4dx^3+700b^2c^5x^6-1316abc^4dx^6+612a^2c^3d^2x^6-630b^2c^4dx^9+}{}$$

Mathematica [C] time = 3.91673, size = 1446, normalized size = 3.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] (-140*a^2*c^5 - 840*a*b*c^5*x^3 + 686*a^2*c^4*d*x^3 - 700*b^2*c^5*x^6 + 131
6*a*b*c^4*d*x^6 - 612*a^2*c^3*d^2*x^6 + 630*b^2*c^4*d*x^9 - 1152*a*b*c^3*d^
2*x^9 + 513*a^2*c^2*d^3*x^9 - 540*b^2*c^3*d^2*x^12 + 918*a*b*c^2*d^3*x^12 -
 324*a^2*c*d^4*x^12 + 405*b^2*c^2*d^3*x^15 - 324*a*b*c*d^4*x^15 + 828*a*b*c
^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] + 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/
3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*b^2*c^4*d*x^9*Hypergeome
tric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 234*a*b*c^3*d^2*
x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 32
4*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] - 324*b^2*c^3*d^2*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] + 918*a*b*c^2*d^3*x^12*Hypergeometric2F1[-1/3, 2,
2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a^2*c*d^4*x^12*Hypergeometric
2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*b^2*c^2*d^3*x^15
*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a
*b*c*d^4*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] + 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5
```

```
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*b^2*c^4*d*x^9*Hypergeometric2F
1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 504*a*b*c^3*d^2*x^9*Hyp
ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*a^2*c^
2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 216*b^2*c^3*d^2*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 432*a*b*c^2*d^3*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] + 216*a^2*c*d^4*x^12*Hypergeometric2F1[2/3, 1,
 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*b^2*c^2*d^3*x^15*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 324*a*b*c*d^4*x^15
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a^
2*d^5*x^15*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*(-7*c + 6*d*x^3)*Hyperge
ometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 54*
c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2,
2}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(1820*c^6*x^13*(a + b*
x^3)^(2/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^{14} \left(d x^{3}+c \right)} \left(b x^{3}+a \right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{3}+a\right)^{\frac{4}{3}}}{\left(d x^{3}+c\right) x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

3.709 $\int \frac{x^6 (a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal. Leaf size=65

$$\frac{ax^7 \sqrt[3]{a+bx^3} F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (a*x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -4/3, 1, 10/3, -((b*x^3)/a), -((d*x^
3)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0559373, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{ax^7 \sqrt[3]{a+bx^3} F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -4/3, 1, 10/3, -((b*x^3)/a), -((d*x^
3)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6 \left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^7\sqrt[3]{a+bx^3}F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.603063, size = 343, normalized size = 5.28

$$\frac{x\left(-\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(8a^2bcd^2+a^3d^3-30ab^2c^2d+20b^3c^3\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2\left(a^2d^2-12abcd+10b^2c^2\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\ldots}\ldots\right)}{80bd^3\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(2*(a + b*x^3)*(2*a^2*d^2 + 3*a*b*d*(-8*c + 3*d*x^3) + b^2*(20*c^2 - 8*c
*d*x^3 + 5*d^2*x^6)) - ((20*b^3*c^3 - 30*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^3*
d^3)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)])/c + (16*a^2*c^2*(10*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*AppellF1[1/
3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])))))/(80*b*d^3*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{x^6}{dx^3+c}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

3.710 $\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=65

$$\frac{ax^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (a*x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -4/3, 1, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0560588, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{ax^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -4/3, 1, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3 \left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.445471, size = 280, normalized size = 4.31

$$\frac{x\left(\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(4a^2d^2-15abcd+10b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2(6ad-5bc)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};\right.\right.}\right.}{40d^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + 6*a*d + 2*b*d*x^3) + ((10*b^2*c^2 - 15*a*b*c*d
+ 4*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)
/a), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + 6*a*d)*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a)
, -((d*x^3)/c)])))))/(40*d^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x^3}{dx^3+c}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

3.711 $\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0273729, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.303343, size = 346, normalized size = 5.77

$$\frac{x\left(\frac{4\left(bx^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(2a^2d+abdx^3+b^2x^3\left(c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{bx^3\left(\frac{bx^3}{a}+\cdots\right.}{\cdots}\right.}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

3.712 $\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)} dx$

Optimal. Leaf size=65

$$-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -(a*(a + b*x^3)^(1/3)*AppellF1[-2/3, -4/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0570861, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*AppellF1[-2/3, -4/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^3\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^3\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.341863, size = 341, normalized size = 5.25

$$-\frac{bx^6\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-2bc)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4ac\left(x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{d}{}\right.\right.\right.}}{8c^2x^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -(b*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)] + (4*a*c*(-4*a*c*(a*c - 2*b*c*x^3 + 3*a*d*x^3 + b*
d*x^6)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a + b*
x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c +
 d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^
3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*App
ellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*c^2*x^2*(a + b*x
^3)^(2/3))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^3\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**3/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**3*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

3.713 $\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -(a*(a + b*x^3)^(1/3)*AppellF1[-5/3, -4/3, 1, -2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(5*c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0577736, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*AppellF1[-5/3, -4/3, 1, -2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(5*c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^6\left(c+dx^3\right)}\,dx=\frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^6\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$=-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.387243, size = 286, normalized size = 4.4

$$\frac{-\frac{16ax\left(10a^2d^2-15abcd+4b^2c^2\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}(5ad-6bc)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}}{40\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3))/(c^2*x^5) + (b*d*(-6*b*c
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c^3 - (16*a*(4*b^2*c^2 - 15*a*b*c*d + 10*a^2*d^2)*x*AppellF
1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^6\left(dx^3+c\right)}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^6\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**6/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**6*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

3.714 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=290

$$\frac{\left(a+bx^3\right)^{5/3}\left(3a^2d^2+2abcd+b^2c^2\right)}{5b^4d^3}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)\left(a^2d^2+b^2c^2\right)}{2b^4d^4}-\frac{\left(a+bx^3\right)^{8/3}(3ad+bc)}{8b^4d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{c^4}{6\cdots}$$

```
[Out] -((b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^4) + ((b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - ((b*c + 3*a*d)*
(a + b*x^3)^(8/3))/(8*b^4*d^2) + (a + b*x^3)^(11/3)/(11*b^4*d) - (c^4*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(6*d^(14/3)*(b*c - a*d)^
(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/
3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.316778, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{5/3}\left(3a^2d^2+2abcd+b^2c^2\right)}{5b^4d^3}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)\left(a^2d^2+b^2c^2\right)}{2b^4d^4}-\frac{\left(a+bx^3\right)^{8/3}(3ad+bc)}{8b^4d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{c^4}{6\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^4) + ((b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - ((b*c + 3*a*d)*
(a + b*x^3)^(8/3))/(8*b^4*d^2) + (a + b*x^3)^(11/3)/(11*b^4*d) - (c^4*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(6*d^(14/3)*(b*c - a*d)^
(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/
3)*(b*c - a*d)^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{(bc+ad)\left(-b^2c^2-a^2d^2\right)}{b^3d^4\sqrt[3]{a+bx}} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)(a+bx)^{2/3}}{b^3d^3} + \frac{(-bc-3ad)(a}{b^3d^2}\right.\right.$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\left(a\right.}{8b^4d^2}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\left(a\right.}{8b^4d^2}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\left(a\right.}{8b^4d^2}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\left(a\right.}{8b^4d^2}$$

Mathematica [C] time = 0.203575, size = 157, normalized size = 0.54

$$\frac{\left(a+bx^3\right)^{2/3}\left(\frac{9a^2bd^2\left(6dx^3-11c\right)-81a^3d^3-3ab^2d\left(44c^2-22cdx^3+15d^2x^6\right)+b^3\left(88c^2dx^3-220c^3-55cd^2x^6+40d^3x^9\right)}{b^4}+\frac{220c^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad}\right)}{440d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*((-81*a^3*d^3 + 9*a^2*b*d^2*(-11*c + 6*d*x^3) - 3*a*b^2*
d*(44*c^2 - 22*c*d*x^3 + 15*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*
d^2*x^6 + 40*d^3*x^9))/b^4 + (220*c^4*Hypergeometric2F1[2/3, 1, 5/3, (d*(a
+ b*x^3))/(-(b*c) + a*d)])/(b*c - a*d)))/(440*d^4)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^{14}}{d x^3+c} \frac{1}{\sqrt[3]{b x^3+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96175, size = 2228, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 + (
-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 4
40*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a
*d^3)^(1/3)) + 660*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt((-b*c*d^2 + a
*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(
2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a
*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)
/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c))
- 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d
^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*
```

```
b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a
^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7
), 1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 +
 (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 +
 a*d^3)^(1/3)) + 1320*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt(-(-b*c*d^2
 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*
c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(2
20*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d^5 - 8
1*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*b^3*c*
d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a^2*b^2
*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.26883, size = 613, normalized size = 2.11

$$-\frac{b^{48}c^{4}d^{7}\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{49}cd^{11}-ab^{48}d^{12}\right)}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c^{4}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^{6}-\sqrt{3}ad^{7}}+\frac{\left(-bcd^{2}+ad^{3}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^48*c^4*d^7*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^49*c*d^11 - a*b^48*d^12) - (-b*c*d^2 + a*d^3)^(2/3)*c
^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
```

```
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^6 - sqrt(3)*a*d^7) + 1/6*(-b*c*d^2 + a*d^3
)^(2/3)*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3
) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^6 - a*d^7) - 1/440*(220*(b*x^3 + a)^(2/3
)*b^43*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^42*c^2*d^8 + 220*(b*x^3 + a)^(2/3)*
a*b^42*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^41*c*d^9 - 176*(b*x^3 + a)^(5/3)*a*
b^41*c*d^9 + 220*(b*x^3 + a)^(2/3)*a^2*b^41*c*d^9 - 40*(b*x^3 + a)^(11/3)*b
^40*d^10 + 165*(b*x^3 + a)^(8/3)*a*b^40*d^10 - 264*(b*x^3 + a)^(5/3)*a^2*b^
40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^40*d^10)/(b^44*d^11)
```

3.715 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=244

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}-\frac{\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a}\right.}{2d^{11/3}\sqrt[3]{bc-ad}}$$

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - ((b*c + 2*a
*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (a + b*x^3)^(8/3)/(8*b^3*d) + (c^3*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d
)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.243404, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}-\frac{\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a}\right.}{2d^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```


```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - ((b*c + 2*a
*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (a + b*x^3)^(8/3)/(8*b^3*d) + (c^3*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d
)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)*(b*c - a*d)^(1/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3\sqrt[3]{a+bx}} + \frac{(-bc-2ad)(a+bx)^{2/3}}{b^2d^2} + \frac{(a+bx)^{5/3}}{b^2d} - \frac{c^3}{d^3\sqrt[3]{a+bx}(c+dx}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\,\mathrm{Subst}\left(\int \frac{}{\sqrt[3]{a}}\right.}{3}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} + \frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}}{\sqrt[3]{}}}{\sqrt{}}\right.}{\sqrt{3}d^{11/3}\sqrt[3]{bc}}$$

Mathematica [C] time = 0.0933834, size = 145, normalized size = 0.59

$$-\frac{\left(a+bx^3\right)^{2/3}\left(3a^2bd^2\left(c-2dx^3\right)+9a^3d^3+ab^2d\left(8c^2-2cdx^3+5d^2x^6\right)+20b^3c^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+b^3c\left(-20c^2+8c\right.\right.}{40b^3d^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(9*a^3*d^3 + 3*a^2*b*d^2*(c - 2*d*x^3) + b^3*c*(-20*c^2
 + 8*c*d*x^3 - 5*d^2*x^6) + a*b^2*d*(8*c^2 - 2*c*d*x^3 + 5*d^2*x^6) + 20*b^
3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]))/(40*
b^3*d^3*(b*c - a*d))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03154, size = 1904, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b*
c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*(b
*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(
1/3)) - 60*sqrt(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1
/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d
^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (
b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*
d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(20*b^3
*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*d^4 - a*b
^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*(b*x^3 + a
)^(2/3))/(b^4*c*d^5 - a*b^3*d^6), -1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^
3*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d +
 (b*c*d^2 - a*d^3)^(2/3)) - 40*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 +
 a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 120*sqrt(1/3)*(b^4*c^4*d - a*b^3*c
^3*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^
```

```
3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c
 - a*d))/d) - 3*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d
^5 + 5*(b^3*c*d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2
*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.25669, size = 501, normalized size = 2.05

$$\frac{b^{27}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{28}cd^8-ab^{27}d^9\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^5-\sqrt{3}ad^6}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^3)^(
2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/3)*b^2
3*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*b^22*c*
d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^7 + 20*(
b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

3.716 $\int \frac{x^8}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$

Optimal. Leaf size=203

$$-\frac{(a+bx^3)^{2/3}(ad+bc)}{2b^2d^2}+\frac{(a+bx^3)^{5/3}}{5b^2d}+\frac{c^2\log(c+dx^3)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}$$

```
[Out] -((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^2*d^2) + (a + b*x^3)^(5/3)/(5*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.21365, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 56, 617, 204, 31}

$$-\frac{(a+bx^3)^{2/3}(ad+bc)}{2b^2d^2}+\frac{(a+bx^3)^{5/3}}{5b^2d}+\frac{c^2\log(c+dx^3)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```


```
[Out] -((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^2*d^2) + (a + b*x^3)^(5/3)/(5*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(1/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{bd^2\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{bd} + \frac{c^2}{d^2\sqrt[3]{a+bx}(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2}\right.}{2d^3}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx\right.}}{2d^{8/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log}{}$$

Mathematica [C] time = 0.0608934, size = 103, normalized size = 0.51

$$\frac{\left(a+bx^3\right)^{2/3}\left(3a^2d^2 + 5b^2c^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right) + 2abd\left(c-dx^3\right) + b^2c\left(2dx^3-5c\right)\right)}{10b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(3*a^2*d^2 + 2*a*b*d*(c - d*x^3) + b^2*c*(-5*c + 2*d*x^3
) + 5*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)
]))/(10*b^2*d^2*(b*c - a*d))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^8}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.96938, size = 1716, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c
*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-
b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)
^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d^3)^
(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*
c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c
- a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(5
*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^
3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2
*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b
*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(b^3*c^3*d - a
*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
```

```
^(1/3)/(b*c - a*d))/d) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^
2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.22217, size = 423, normalized size = 2.08

$$-\frac{b^{12}c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{13}cd^5-ab^{12}d^6\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5}+\frac{\left(-bcd^2+ad\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^12*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^13*c*d^5 - a*b^12*d^6) - (-b*c*d^2 + a*d^3)^(2/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) + 1/6*(-b*c*d^2 + a*d^3)^
(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) - 1/10*(5*(b*x^3 + a)^(2/3)*b^9
*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^10
*d^5)
```

3.717 $\int \frac{x^5}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=168

$$-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}}+\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) - (c*Log[c +
 d*x^3])/(6*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.161212, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 80, 56, 617, 204, 31}

$$-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}}+\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) - (c*Log[c +
 d*x^3])/(6*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$
$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d}$$
$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{c\,\text{Subst}\Big(\int$$
$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{d^{5/3}\sqrt[3]{bc-a}}$$
$$= \frac{(a+bx^3)^{2/3}}{2bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] time = 0.0203736, size = 69, normalized size = 0.41

$$-\frac{(a+bx^3)^{2/3}\left(bc\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d(bx^3+a)}{ad-bc}\right) + ad - bc\right)}{2bd(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(-(b*c) + a*d + b*c*Hypergeometric2F1[2/3, 1, 5/3, (d*(
a + b*x^3))/(-(b*c) + a*d)]))/(2*b*d*(b*c - a*d))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^5}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70199, size = 1497, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a
*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a
*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*lo
g((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1
/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 -
a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*(b*x^3 +
 a)^(2/3))/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d +
(b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((b*c*d^
2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*
c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(b*c
*d^2 - a*d^3)*(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.23869, size = 347, normalized size = 2.07

$$\frac{\frac{2\,bcd\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^2-ad^3}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right.\right.}{bcd^3-ad^4}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*
d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3
*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1
/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^(2/3)/d)/b
```

3.718 $\int \frac{x^2}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal. Leaf size=145

$$\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(6*d^(2/3)*(b*c - a*d)^(
1/3)) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c
- a*d)^(1/3))
```

Rubi [A] time = 0.125213, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 56, 617, 204, 31}

$$\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(6*d^(2/3)*(b*c - a*d)^(
1/3)) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c
- a*d)^(1/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x,\right.}{2d^{2/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] time = 0.0147213, size = 50, normalized size = 0.34

$$\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) +
 a*d)])/(2*b*c - 2*a*d)
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{x^2}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.3586, size = 1357, normalized size = 9.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)
)*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2
/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c
*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^3)^(2
/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*
d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^
(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt(1/3)*(b
*c*d - a*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
 - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^
(1/3)))/(b*c*d^2 - a*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.27998, size = 305, normalized size = 2.1

$$-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right.\right.}{6\left(bcd^2-ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
- a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3)
+ 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-
(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3) - 1/3*(-(b
*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c - a*d)
```

3.719 $\int \frac{1}{x\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=244

$$-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{ac}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}\cdots}{\cdots}\right)}{\sqrt{3}\sqrt[3]{\cdots}}$$

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
c) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) - Log[x]/(2*a^(1/3)*c) - (d^(1/3)*L
og[c + d*x^3])/(6*c*(b*c - a*d)^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(
2*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*c*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.212111, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 86, 55, 617, 204, 31, 56}

$$-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{ac}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}\cdots}{\cdots}\right)}{\sqrt{3}\sqrt[3]{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
c) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) - Log[x]/(2*a^(1/3)*c) - (d^(1/3)*L
og[c + d*x^3])/(6*c*(b*c - a*d)^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(
2*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*c*(b*c - a*d)^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \cdots}\right.}{\cdots}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}} - \cdots$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a}\cdots\right.}{2\sqrt[3]{a}c}$$

Mathematica [C] time = 0.089987, size = 140, normalized size = 0.57

$$\frac{3\sqrt[3]{a}d\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)-(bc-ad)\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)}{6\sqrt[3]{a}c(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (3*a^(1/3)*d*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3
))/(-(b*c) + a*d)] - (b*c - a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/(6*a
^(1/3)*c*(-(b*c) + a*d))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(dx^{3}+c\right)}\frac{1}{\sqrt[3]{bx^{3}+a}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}}\left(dx^{3}+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)
```

Fricas [A] time = 1.48153, size = 1623, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 +
 a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*arct
an(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - a*(
d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/
3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(d/(b*c
 - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d)
 - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2
```

```
*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*a*(d/(b*c
 - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) -
 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*
(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(
1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (
b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A] time = 2.95327, size = 470, normalized size = 1.93

$$\frac{1}{6}\left(\frac{2\,d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^2c^2-abcd}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d-\sqrt{3}abcd^2}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^2 - a*b*c*d) + 6*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(
sqrt(3)*b^2*c^2*d - sqrt(3)*a*b*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^
```

```
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d - a*b*c*d^2) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a
)^(1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*b*c) - log((b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*b*c) + 2*log(abs((b*x^3 + a)^(1/3) -
 a^(1/3)))/(a^(1/3)*b*c))*b
```

3.720 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\left(c+dx^3\right)} \, dx$

Optimal. Leaf size=296

$$-\frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2}+\frac{\log(x)(3ad+bc)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\log\left(\right.}{}$$

```
[Out] -(a + b*x^3)^(2/3)/(3*a*c*x^3) - ((b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*
x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*c^2) - (d^(4/3)*ArcTan[(
1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^2
*(b*c - a*d)^(1/3)) + ((b*c + 3*a*d)*Log[x])/(6*a^(4/3)*c^2) + (d^(4/3)*Log
[c + d*x^3])/(6*c^2*(b*c - a*d)^(1/3)) - ((b*c + 3*a*d)*Log[a^(1/3) - (a +
b*x^3)^(1/3)])/(6*a^(4/3)*c^2) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.314165, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 103, 156, 55, 617, 204, 31, 56}

$$-\frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2}+\frac{\log(x)(3ad+bc)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(3*a*c*x^3) - ((b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*
x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*c^2) - (d^(4/3)*ArcTan[(
1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^2
*(b*c - a*d)^(1/3)) + ((b*c + 3*a*d)*Log[x])/(6*a^(4/3)*c^2) + (d^(4/3)*Log
[c + d*x^3])/(6*c^2*(b*c - a*d)^(1/3)) - ((b*c + 3*a*d)*Log[a^(1/3) - (a +
b*x^3)^(1/3)])/(6*a^(4/3)*c^2) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(1/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{3}(bc+3ad)+\frac{bdx}{3}}{x\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{d^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(bc+3ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{9ac^2}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} + \frac{d\operatorname{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx\right.}{2c^2}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} - \frac{(bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} - \frac{(bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{4/3}c^2} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}} + \frac{(bc+3ad)\log(}{6a^{4/3}c^2}$$

Mathematica [C] time = 0.365072, size = 156, normalized size = 0.53

$$-\frac{\frac{(3ad+bc)\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)}{a^{4/3}} - \frac{9d^2\left(a+bx^3\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad} + \frac{6c\left(a+bx^3\right)^{2/3}}{ax^3}}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((6*c*(a + b*x^3)^(2/3))/(a*x^3) - (9*d^2*(a + b*x^3)^(2/3)*Hypergeometric
2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(b*c - a*d) + ((b*c + 3*a
*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*Log[
x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/a^(4/3))/(18*c^2)
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(d x^{3}+c \right)} \frac{1}{\sqrt[3]{b x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{3}+a\right)^{\frac{1}{3}} \left(d x^{3}+c\right) x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)
```

Fricas [A] time = 2.02774, size = 2056, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3
 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c -
a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*
x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b
*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)
*d) + 3*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3
*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/
3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + (b*
c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1
/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) +
(-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a*c)/(a^2*c^2*x^3), 1/18*(6*sqrt(3)*a^2*d
*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c -
 a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log(-(b*x^
3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*
c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log((
b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*(a*b
*c + 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)
 - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x
^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d
)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*
a*c)/(a^2*c^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [A] time = 3.43343, size = 567, normalized size = 1.92

$$-\frac{1}{18}\left(\frac{6\,d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^3c^3-ab^2c^2d}+\frac{18\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c^3-\sqrt{3}ab^2c^2d}-\frac{3\left(-bcd^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/18*(6*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*
d)/d)^(1/3)))/(b^3*c^3 - a*b^2*c^2*d) + 18*(-b*c*d^2 + a*d^3)^(2/3)*arctan(
1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)
^(1/3))/(sqrt(3)*b^3*c^3 - sqrt(3)*a*b^2*c^2*d) - 3*(-b*c*d^2 + a*d^3)^(2/3
)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c
 - a*d)/d)^(2/3))/(b^3*c^3 - a*b^2*c^2*d) + 2*sqrt(3)*(a^(2/3)*b*c + 3*a^(5
/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*b^2
*c^2) + 2*(a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))
/(a^(5/3)*b^2*c^2) - (a^(2/3)*b*c + 3*a^(5/3)*d)*log((b*x^3 + a)^(2/3) + (b
*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^2*b^2*c^2) + 6*(b*x^3 + a)^(2/3)/(a*b
^2*c*x^3))*b^2
```

3.721 $\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=273

$$\frac{(ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}d^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^2}+\frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}}-\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2\sqrt[3]{bc-ad}}+\ldots$$

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d^2) + (c^(4/3)*ArcTan[(1 + (
2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d^2*
(b*c - a*d)^(1/3)) + (c^(4/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(1/3)) - (
c^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
 - a*d)^(1/3)) + ((3*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b
^(4/3)*d^2)
```

Rubi [A] time = 0.509068, antiderivative size = 394, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {494, 470, 522, 200, 31, 634, 617, 204, 628}

$$\frac{(ad+3bc)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2}-\frac{(ad+3bc)\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{4/3}d^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^2}-\frac{c^{4/3}\log\left(\sqrt[3]{c}-\ldots\right)}{3d^2\sqrt[3]{bc-ad}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d^2) + (c^(4/3)*ArcTan[(c^(1/
3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3
]*d^2*(b*c - a*d)^(1/3)) + ((3*b*c + a*d)*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(
1/3)])/(9*b^(4/3)*d^2) - ((3*b*c + a*d)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(4/3)*d^2) - (c^(4/3)*Log[c^(1
/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^(1/3)) +
 (c^(4/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3
)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = a^2\,\text{Subst}\left(\int \frac{x^6}{\left(1-bx^3\right)^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{c+(2bc+ad)x^3}{\left(1-bx^3\right)\left(c+(-bc+ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{1}{1-bx^3}\,dx, x, \ldots\right.}{3bd^2}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}\ldots}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-\ldots}\right.}{3d^2}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{c^{4/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + \frac{c^{5/3}\,\text{Subst}\left(\int \frac{\ldots}{c^{2/3}\ldots}\right.}{\ldots}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{(3bc+ad)\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{18b^{4/3}d^2} - \frac{c^{4}\ldots}{\ldots}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} - \frac{(3bc+ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^2} + \frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\sqrt[3]{bc-ad}} + \frac{(3bc+ad)\log\left(1\ldots\right.}{9b^{4/3}d^2\ldots}$$

Mathematica [C] time = 0.521259, size = 288, normalized size = 1.05

$$\frac{\frac{2\left(-a\sqrt[3]{c}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)+6x\left(a+bx^3\right)^{2/3}\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)-2\sqrt{3}a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}} - \frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}(ad+3bc)F_1\left(\frac{4}{3};\ldots\right.}{c\sqrt[3]{a+bx^3}\ldots}}{36bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*(3*b*c + a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)])/(c*(a + b*x^3)^(1/3)) + (2*(6*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x
)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*Log[c^(1/3) - ((b*c -
```

```
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)]))/(b*c - a*d)^(1/3))/(36*b*d)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{x^6}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [A] time = 2.02761, size = 2026, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*
```

```
d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)
/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))
^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 +
a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c + a*b*d
)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1
/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)
*sqrt(-1/b^(2/3)) + 2*a) - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3
)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d^2), -1/18*(6*sqrt(3)*b^2*c*(-
c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(
-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*c*(-c/(b*c - a
*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)
*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2) - 6*(b*x^
3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1
/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c + a*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.722 $\int \frac{x^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=233

$$-\frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}$$

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d) -
 (c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/
Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^(1/3)) - (c^(1/3)*Log[c + d*x^3])/(6*d*(b*
c - a*d)^(1/3)) + (c^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^
(1/3)])/(2*d*(b*c - a*d)^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*
b^(1/3)*d)
```

Rubi [A] time = 0.291295, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 481, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{b}d}-\frac{\sqrt[3]{c}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d) -
 (c^(1/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d*(b*c - a*d)^(1/3)) - Log[1 - (b^(1/3)*x)/(a + b*
x^3)^(1/3)]/(3*b^(1/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1
/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(1/3)*d) + (c^(1/3)*Log[c^(1/3) - ((b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d*(b*c - a*d)^(1/3)) - (c^(1/3)*Log[c^(2
/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3
)*x)/(a + b*x^3)^(1/3)])/(6*d*(b*c - a*d)^(1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = a\,\text{Subst}\left(\int \frac{x^3}{\left(1-bx^3\right)\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d} + \frac{\text{Subst}\left(\int \frac{2+\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d} - \frac{\sqrt[3]{c}\,\text{Subst}\left(\int \frac{}{\sqrt[3]{c}-}\right.}{}$$
$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2d} + \text{Sub}$$
$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}d} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c}\log\left(c^{2/3}\right.}{}$$
$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{bx}}{\sqrt[3]{a+b}}\right.}{6\sqrt[3]{b}d}$$

Mathematica [C] time = 0.0337986, size = 65, normalized size = 0.28

$$\frac{x^4\sqrt[3]{\frac{a+bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*((a + b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{x^3}{dx^3 + c} \frac{1}{\sqrt[3]{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3 + a\right)^{\frac{1}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [A] time = 1.4774, size = 1848, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-
b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*b*(c/
```

```
(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(
b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b
*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*log(((
b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(
b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((-b)^(1/3
)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^
(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(6*sqrt(1/3)*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3
)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*b*(c/(b*c -
 a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*
c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3)
+ (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)
*c)/x^2) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (-b)^(2
/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3
))/x^2))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.723 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=148

$$\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(
Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^
(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*
(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.127209, antiderivative size = 207, normalized size of antiderivative = 1.4, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/
3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)
*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c^(2/3) + ((b*c
- a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*
x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{\cdots}}{c^{2/3}+\sqrt[3]{\cdots}}\cdots\right)}{\cdots}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{bc\cdots}}{\sqrt[3]{c}\sqrt[3]{a\cdots}}\cdots\right)}{c^{2/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] time = 0.141825, size = 168, normalized size = 1.14

$$\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))
/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[
c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^
(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.724 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal. Leaf size=176

$$-\frac{d\log\left(c+dx^3\right)}{6c^{5/3}\sqrt[3]{bc-ad}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

```
[Out] -(a + b*x^3)^(2/3)/(2*a*c*x^2) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^
(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3)) - (
d*Log[c + d*x^3])/(6*c^(5/3)*(b*c - a*d)^(1/3)) + (d*Log[((b*c - a*d)^(1/3)
*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.226174, antiderivative size = 235, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(2*a*c*x^2) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*
x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3
)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(5/3)*
(b*c - a*d)^(1/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c -
a*d)^(1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^3\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}} - \frac{d\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad}\right.}{3c^{5/3}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx}}\right.}{2c^{4/3}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\,\text{Subst}}{}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}\right.}{6c^{5/3}\sqrt[3]{bc-a}}$$

Mathematica [C] time = 0.0572724, size = 124, normalized size = 0.7

$$\frac{-3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-4c\left(a+bx^3\right)\left(c+3dx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{8c^3x^2\left(a+bx^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-4*c*(a + b*x^3)*(c + 3*d*x^3)*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[4/
```

```
3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(8*c^3*x^2*(a + b*x^3)^(4/3)
)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(d x^{3}+c \right)} \frac{1}{\sqrt[3]{b x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{3}+a\right)^{\frac{1}{3}} \left(d x^{3}+c\right) x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

3.725 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx$

Optimal. Leaf size=214

$$\frac{(a+bx^3)^{2/3} (5ad+3bc)}{10a^2c^2x^2} + \frac{d^2 \log(c+dx^3)}{6c^{8/3} \sqrt[3]{bc-ad}} - \frac{d^2 \log\left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c} \sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3} c^{8/3} \sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{5acx^5}$$

```
[Out] -(a + b*x^3)^(2/3)/(5*a*c*x^5) + ((3*b*c + 5*a*d)*(a + b*x^3)^(2/3))/(10*a^
2*c^2*x^2) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^
(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) + (d^2*Log[c + d*x^3]
)/(6*c^(8/3)*(b*c - a*d)^(1/3)) - (d^2*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(2*c^(8/3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.286212, antiderivative size = 271, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$\frac{(a+bx^3)^{2/3} (ad+bc)}{2a^2c^2x^2} - \frac{(a+bx^3)^{5/3}}{5a^2cx^5} - \frac{d^2 \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \log\left(\frac{x^2 (bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{8/3} \sqrt[3]{bc-ad}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3} c^{8/3} \sqrt[3]{bc-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*a^2*c^2*x^2) - (a + b*x^3)^(5/3)/(5*a^2*
c*x^5) + (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/
(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) - (d^2*Log[c^(1/3)
- ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(8/3)*(b*c - a*d)^(1/3)) +
 (d^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(8/3)*(b*c - a*d)^(1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
```

```
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^2}{x^6\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^6}+\frac{-bc-ad}{c^2x^3}+\frac{a^2d^2}{c^2\left(c-(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c^2}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}} + \frac{d^2\,\text{Subst}}{}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}}\right.}{2c}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\right.}{6c^{8/3}\sqrt[3]{bc-ad}}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \cdots$$

Mathematica [C] time = 0.778289, size = 207, normalized size = 0.97

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\text{HypergeometricPFQ}\left(\left\{\frac{4}{3},2,2\right\},\left\{1,\frac{7}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)-3x^3\left(-c^2+8cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1}{40c^4x^5\left(a+bx^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(8*c*(a + b*x^3)*(c^2 - 3*c*d*x^3 - 9*d^2*x^6)*Hypergeometric2F1[1/3, 1, 4
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(-c^2 + 8*c*d*x^
3 + 9*d^2*x^6)*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 2}, {1,
7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*x^5*(a + b*x^3)^(4/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \left(d x^3+c\right)} \frac{1}{\sqrt[3]{b x^3+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^3+a\right)^{\frac{1}{3}} \left(d x^3+c\right) x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

3.726 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}(c+dx^3)} dx$

Optimal. Leaf size=262

$$-\frac{(a+bx^3)^{2/3}(20a^2d^2+12abcd+9b^2c^2)}{40a^3c^3x^2}+\frac{(a+bx^3)^{2/3}(4ad+3bc)}{20a^2c^2x^5}-\frac{d^3\log(c+dx^3)}{6c^{11/3}\sqrt[3]{bc-ad}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}}$$

```
[Out] -(a + b*x^3)^(2/3)/(8*a*c*x^8) + ((3*b*c + 4*a*d)*(a + b*x^3)^(2/3))/(20*a^
2*c^2*x^5) - ((9*b^2*c^2 + 12*a*b*c*d + 20*a^2*d^2)*(a + b*x^3)^(2/3))/(40*
a^3*c^3*x^2) - (d^3*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3
)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(11/3)*(b*c - a*d)^(1/3)) - (d^3*Log[c + d*x
^3])/(6*c^(11/3)*(b*c - a*d)^(1/3)) + (d^3*Log[((b*c - a*d)^(1/3)*x)/c^(1/3
) - (a + b*x^3)^(1/3)])/(2*c^(11/3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.324543, antiderivative size = 317, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{(a+bx^3)^{2/3}(a^2d^2+abcd+b^2c^2)}{2a^3c^3x^2}+\frac{(a+bx^3)^{5/3}(ad+2bc)}{5a^3c^2x^5}-\frac{(a+bx^3)^{8/3}}{8a^3cx^8}+\frac{d^3\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{11/3}\sqrt[3]{bc-ad}}-\frac{d^3\log\left(\frac{x^2(bc-ad)^2}{(a+bx^3)^{2}}\right.}{6c^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*a^3*c^3*x^2) + ((2*b*
c + a*d)*(a + b*x^3)^(5/3))/(5*a^3*c^2*x^5) - (a + b*x^3)^(8/3)/(8*a^3*c*x^
8) - (d^3*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c^(11/3)*(b*c - a*d)^(1/3)) + (d^3*Log[c^(1/3) - (
(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(11/3)*(b*c - a*d)^(1/3)) - (
d^3*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c
 - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(11/3)*(b*c - a*d)^(1/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
```

```
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^3}{x^9\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^9}+\frac{-2bc-ad}{c^2x^6}+\frac{b^2c^2+abcd+a^2d^2}{c^3x^3}+\frac{a^3d^3}{c^3\left(-c+(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^3}$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+\frac{d^3\,\text{Subst}}{}$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+\frac{d^3\,\text{Subst}}{}$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+\frac{d^3\log\left(\sqrt[3]{\right.}}{3c^{11/3}}$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+\frac{d^3\log\left(\sqrt[3]{\right.}}{3c^{11/3}}$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}-\frac{d^3\tan^{-1}\left(\right.}{\sqrt{3}c^{11/3}}$$

Mathematica [C] time = 2.29045, size = 821, normalized size = 3.13

$$-\frac{648bcd^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{12}-297ad^4\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{12}+297bcd^3\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{12}+648acd^3\,{}_2F}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(40*a*c^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 40*b*c^4*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] - 72*a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 72*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] + 216*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3,
((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 216*b*c^2*d^2*x^9*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*a*c*d^3*x^9*Hypergeomet
ric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*b*c*d^3*x^12*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*b*c^4*
x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*a
*c^3*d*x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] - 45*b*c^3*d*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + 45*a*c^2*d^2*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] + 243*b*c^2*d^2*x^9*Hypergeometric2F1[4/3, 2, 7/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 243*a*c*d^3*x^9*Hypergeometric2F1[4/3, 2,
7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F
1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeo
metric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c - a*d)*
x^3*(c - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 2}, {1, 7/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*Hyperge
ometricPFQ[{4/3, 2, 2, 2}, {1, 1, 7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])
/(320*c^5*x^8*(a + b*x^3)^(4/3))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{1}{x^{9}\left(dx^{3}+c\right)}\frac{1}{\sqrt[3]{bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}}\left(dx^{3}+c\right)x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

3.727 $\int \frac{x^7}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 1/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*c*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0526605, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 1/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*c*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.113332, size = 144, normalized size = 2.25

$$\frac{-2x^5\sqrt[3]{\frac{bx^3}{a}+1}(ad+2bc)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5acx^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(a+bx^3\right)}{20bcd\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*(2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(1
/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*b*c*d*(a +
b*x^3)^(1/3))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{x^7}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.728 $\int \frac{x^4}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0529023, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx=\frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$=\frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0306476, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt[3]{\frac{a+bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*((a + b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x^4}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.729 $\int \frac{x}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0395461, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0303273, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt[3]{\frac{a+bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*((a + b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.730 $\int \frac{1}{x^2 \sqrt[3]{a+bx^3}(c+dx^3)} dx$

Optimal. Leaf size=62

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 1/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*x*(a + b*x^3)^(1/3)))
```

Rubi [A] time = 0.0547463, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 1/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*x*(a + b*x^3)^(1/3)))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx=\frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.0965735, size = 141, normalized size = 2.27

$$\frac{2bdx^6\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5x^3\sqrt[3]{\frac{bx^3}{a}+1}(bc-ad)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-10c\left(a+bx^3\right)}{10ac^2x\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) + 5*(b*c - a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3,
1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/3)*
AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a*c^2*x*(a + b*
x^3)^(1/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(dx^3+c\right)}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

3.731 $\int \frac{1}{x^5 \sqrt[3]{a+bx^3}(c+dx^3)} dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 1/3, 1, -1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*x^4*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0544019, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 1/3, 1, -1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*x^4*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^5\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.171355, size = 183, normalized size = 2.86

$$\frac{5x^6\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2-2abcd-b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bdx^9\sqrt[3]{\frac{bx^3}{a}+1}(2ad+bc)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5c\left(\right.}{20a^2c^3x^4\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(a + b*x^3)*(-(a*c) + 2*b*c*x^3 + 4*a*d*x^3) + 5*(-(b^2*c^2) - 2*a*b*c
*d + 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x
^3)/a), -((d*x^3)/c)] - 2*b*d*(b*c + 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a^2*c^3*x^4*(a + b*x
^3)^(1/3))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(dx^3+c\right)}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

3.732 $\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3d^3}-\frac{\left(a+bx^3\right)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{\ldots}\right.}{2d^{10/3}(bc-ad)^{2/\ldots}}$$

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - ((b*c + 2*a*d
)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (a + b*x^3)^(7/3)/(7*b^3*d) + (c^3*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(6*d^(10/3)*(b*c - a*d)^
(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/
3)*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.263086, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 58, 617, 204, 31}

$$\frac{\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3d^3}-\frac{\left(a+bx^3\right)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{\ldots}\right.}{2d^{10/3}(bc-ad)^{2/\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```


```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - ((b*c + 2*a*d
)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (a + b*x^3)^(7/3)/(7*b^3*d) + (c^3*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(6*d^(10/3)*(b*c - a*d)^
(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/
3)*(b*c - a*d)^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3(a+bx)^{2/3}} + \frac{(-bc-2ad)\sqrt[3]{a+bx}}{b^2d^2} + \frac{(a+bx)^{4/3}}{b^2d} - \frac{c^3}{d^3(a+bx)^{2/3}(c\ldots}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} - \frac{c^3\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\log\left(c+d\ldots\right.}{6d^{10/3}(bc-a\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\log\left(c+d\ldots\right.}{6d^{10/3}(bc-a\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\tan^{-1}\left(\frac{1-\frac{2\ldots}{\ldots}}{\ldots}\right.}{\sqrt{3}d^{10/3}(bc-\ldots}$$

Mathematica [A] time = 0.49062, size = 251, normalized size = 1.04

$$\frac{\frac{84\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3} - \frac{21d\left(a+bx^3\right)^{4/3}(2ad+bc)}{b^3} + \frac{12d^2\left(a+bx^3\right)^{7/3}}{b^3} + \frac{14c^3\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\ldots\right.\right.}{\sqrt[3]{d}(bc-ad)^{2/3}}}{84d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((84*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/b^3 - (21*d*(b*c + 2*
a*d)*(a + b*x^3)^(4/3))/b^3 + (12*d^2*(a + b*x^3)^(7/3))/b^3 + (14*c^3*(2*S
qrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^
(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(
2/3)]))/(d^(1/3)*(b*c - a*d)^(2/3)))/(84*d^3)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{d x^{3}+c} \left(b x^{3}+a \right)^{-\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08065, size = 2795, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c
- a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28
*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*
(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 42*sqrt(1/3
)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/
3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*
sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2
- a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-
```

```
b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*
x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3
*b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 -
 (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*
x^3 + a)^(1/3))/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6), 1/84*(14*(-b^2
*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(2/3)*(b*c*d
 - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28*(-b^2*c^2*d +
2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
 - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 84*sqrt(1/3)*(b^4*c^4*d -
a*b^3*c^3*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*arctan(s
qrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(-b^2*c^2*d +
 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 3*(28*b
^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3*b*c*d^4 + 18*a^4*d
^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8
*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(1/3))/(b^
5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.24281, size = 502, normalized size = 2.08

$$\frac{b^{24}c^3d^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{25}cd^7-ab^{24}d^8\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5}-\frac{\left(-bcd^2+ad^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^24*c^3*d^4*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^25*c*d^7 - a*b^24*d^8) - (-b*c*d^2 + a*d^3)^(1/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(
1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 1/28*(28*(b*x^3 + a)^(1/3)*b^2
0*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^19*c*d^5 + 28*(b*x^3 + a)^(1/3)*a*b^19*c*
d^5 + 4*(b*x^3 + a)^(7/3)*b^18*d^6 - 14*(b*x^3 + a)^(4/3)*a*b^18*d^6 + 28*(
b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7)
```

3.733 $\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=201

$$-\frac{\sqrt[3]{a+bx^3}(ad+bc)}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}}$$

```
[Out] -(((b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2)) + (a + b*x^3)^(4/3)/(4*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(6*d^(7/3)*(b
*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(7/3)*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.211912, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 88, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}(ad+bc)}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(((b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2)) + (a + b*x^3)^(4/3)/(4*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(6*d^(7/3)*(b
*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(7/3)*(b*c - a*d)^(2/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)$$
$$=\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{bd^2(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{bd}+\frac{c^2}{d^2(a+bx)^{2/3}(c+dx)}\right)dx,x,x^3\right)$$
$$=-\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}+\frac{c^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3d^2}$$
$$=-\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx,x,\sqrt[3]{a}\right.}{2d^{7/3}(bc-ad)^{2/3}}$$
$$=-\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx\right.}}{2d^{7/3}(bc-ad)^{2/3}}$$
$$=-\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log}{}$$

Mathematica [A] time = 0.341205, size = 211, normalized size = 1.05

$$\frac{-\frac{12\sqrt[3]{a+bx^3}(ad+bc)}{b^2}+\frac{3d\left(a+bx^3\right)^{4/3}}{b^2}-\frac{2c^2\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right.\right.}{\sqrt[3]{d}(bc-ad)^{2/3}}}{12d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-12*(b*c + a*d)*(a + b*x^3)^(1/3))/b^2 + (3*d*(a + b*x^3)^(4/3))/b^2 - (2
*c^2*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*
c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)]))/(d^(1/3)*(b*c - a*d)^(2/3)))/(12*d^2)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^8}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.63623, size = 2496, normalized size = 12.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a
)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c -
a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*
d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*(b^3*
c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*l
og((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*sqrt(1/
3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(b^2*c^2*d
```

```
- 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c))
+ 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4 - (b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c^2*d^3 - 2*a*
b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log
(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3
+ a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*
a^3*d^4 - (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))
/(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.18087, size = 421, normalized size = 2.09

$$-\frac{b^{10}c^2d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{11}cd^4-ab^{10}d^5\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}+\frac{\left(-bcd^2+ad\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^10*c^2*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^11*c*d^4 - a*b^10*d^5) + (-b*c*d^2 + a*d^3)^(1/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) - 1/4*(4*(b*x^3 + a)^(1/3)*b^7*
c*d^2 - (b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4
)
```

3.734 $\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=165

$$\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}$$

```
[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[c + d
*x^3])/(6*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*d^(4/3)*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.155348, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 80, 58, 617, 204, 31}

$$\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```


```
[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[c + d
*x^3])/(6*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*d^(4/3)*(b*c - a*d)^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x]))] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{c\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subst}\left(\int \frac{}{\frac{(bc-ad)}{d^{2/3}}} \right.}{2}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, \right.}{d^{4/3}(bc-}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$

Mathematica [A] time = 0.103148, size = 202, normalized size = 1.22

$$\frac{bc\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}(bc-ad)^{2/3}-2bc\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\right.}{6bd^{4/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*b*c*ArcTan[(-1 +
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*b*c*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) - d^
(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*
b*d^(4/3)*(b*c - a*d)^(2/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^5}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.52908, size = 2276, normalized size = 13.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3)
*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) +
 (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^
2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^
2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^2*d -
 a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b^2*c^2
 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(2*(b*x^
3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(
b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*
sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*d + 2*a*b*
c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c)) + 6*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2 - 2*a*b^2*c
*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(
-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(
1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(
```

```
1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(
1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 6*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c -
a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt
(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.18109, size = 342, normalized size = 2.07

$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^2-ad^3}-\frac{2bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3\right.\right.\right.}{bcd-ad}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt
(3)*a*d^3) + (-b*c*d^2 + a*d^3)^(1/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3)
 - 2*b*c*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c*d - a*d^2) - 6*(b*x^3 + a)^(1/3)/d)/b
```

3.735 $\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=145

$$-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}$$

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(6*d^(1/3)*(b*c - a*d)^(
2/3)) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(1/3)*(b*c
- a*d)^(2/3))
```

Rubi [A] time = 0.123571, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 58, 617, 204, 31}

$$-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(6*d^(1/3)*(b*c - a*d)^(
2/3)) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx\right.}{2d^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right.}{\sqrt[3]{d}(bc-ad)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

Mathematica [A] time = 0.0460597, size = 164, normalized size = 1.13

$$\frac{-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+b}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right.}{6\sqrt[3]{d}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c -
a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(6*d^(1/3)*(b*c - a*d)^(2/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x^2}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49881, size = 2037, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3
)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3
 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(
d*x^3 + c)) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
 (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)
*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
```

```
+ (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) + 2*(b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.17035, size = 298, normalized size = 2.06

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
 a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) + 1
/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*
c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d - a*d^2) - 1/3*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c - a
*d)
```

3.736 $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=245

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}+\frac{d^{2/3}}{\cdots}$$

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*c)) + (d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(2/3)) - Log[x]/(2*a^(2/3)*c) + (d^(2/3
)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)
]/(2*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*c*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.210522, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 86, 57, 617, 204, 31, 58}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}+\frac{d^{2/3}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```


```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*c)) + (d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(2/3)) - Log[x]/(2*a^(2/3)*c) + (d^(2/3
)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)
]/(2*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*c*(b*c - a*d)^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\right.}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+b}\right.}{2c(bc-ad)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}c} + \frac{d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{\right.}}{}$$

Mathematica [A] time = 0.278001, size = 308, normalized size = 1.26

$$-\frac{-\frac{2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{a^{2/3}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{2/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}} - \frac{d^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{\right.}}{}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) +
 (2*Sqrt[3]*d^(2/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^
(1/3))/Sqrt[3]])/(b*c - a*d)^(2/3) - (2*Log[a^(1/3) - (a + b*x^3)^(1/3)])/a
^(2/3) + (2*d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(2/3) + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)]/a^(2/3) - (d^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/(6*c)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)
```

Fricas [B] time = 1.50184, size = 1115, normalized size = 4.55

$$-\frac{2\sqrt{3}a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/
3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(2/3) - sqrt(3)*d)/d) + a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
1/3)*log((b*x^3 + a)^(2/3)*d^2 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)*(-d^2/(b
```

```
^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-d^
2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 2*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*
d + a^2*d^2))^(1/3)*log((b*x^3 + a)^(1/3)*d - (b*c - a*d)*(-d^2/(b^2*c^2 -
2*a*b*c*d + a^2*d^2))^(1/3)) + 2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(a^2)^(1/
6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) +
 (a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a
^2)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/(a^2*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A] time = 2.61581, size = 462, normalized size = 1.89

$$\frac{1}{6}\left(\frac{2\,d\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^2c^2-abcd}-\frac{6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2-\sqrt{3}abcd}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*d*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^2 - a*b*c*d) - 6*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(
3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(
sqrt(3)*b^2*c^2 - sqrt(3)*a*b*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 +
```

```
a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/(b^2*c^2 - a*b*c*d) - 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3)
+ a^(1/3))/a^(1/3))/(a^(2/3)*b*c) - log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*a^(1/3) + a^(2/3))/(a^(2/3)*b*c) + 2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)
))/(a^(2/3)*b*c))*b
```

3.737 $\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=299

$$-\frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{(3ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2}+\frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}+\frac{d^{5/3}}{}$$

```
[Out] -(a + b*x^3)^(1/3)/(3*a*c*x^3) + ((2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*c^2) - (d^(5/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c
^2*(b*c - a*d)^(2/3)) + ((2*b*c + 3*a*d)*Log[x])/(6*a^(5/3)*c^2) - (d^(5/3)
*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(2/3)) - ((2*b*c + 3*a*d)*Log[a^(1/3) -
 (a + b*x^3)^(1/3)])/(6*a^(5/3)*c^2) + (d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(
1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.316702, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 103, 156, 57, 617, 204, 31, 58}

$$-\frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{(3ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2}+\frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}+\frac{d^{5/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```


```
[Out] -(a + b*x^3)^(1/3)/(3*a*c*x^3) + ((2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*c^2) - (d^(5/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c
^2*(b*c - a*d)^(2/3)) + ((2*b*c + 3*a*d)*Log[x])/(6*a^(5/3)*c^2) - (d^(5/3)
*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(2/3)) - ((2*b*c + 3*a*d)*Log[a^(1/3) -
 (a + b*x^3)^(1/3)])/(6*a^(5/3)*c^2) + (d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(
1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(2/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{3}(2bc+3ad)+\frac{2bdx}{3}}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{d^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(2bc+3ad)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx\right.}{9ac^2}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} + \frac{d^{5/3}\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x,\right.}{2c^2(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{6a^{5/3}c^2}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}c^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}} + \frac{(2bc+3ad}{6a^{5/3}c}$$

Mathematica [A] time = 0.416603, size = 303, normalized size = 1.01

$$\frac{\frac{(3ad+2bc)\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{a^{2/3}c} + \frac{3ad^{5/3}\left(-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}(a+bx\right.\right.}{c(}}{18ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-6*(a + b*x^3)^(1/3))/x^3 + ((2*b*c + 3*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a
 + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + L
og[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/(a^(2/3)*c) +
 (3*a*d^(5/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
 - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^
(2/3)*(a + b*x^3)^(2/3)]))/(c*(b*c - a*d)^(2/3)))/(18*a*c)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^{4} \left(d x^{3}+c \right)} \left(b x^{3}+a \right)^{-\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(b x^{3} + a\right)}^{\frac{2}{3}} {\left(d x^{3} + c\right)} x^{4}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

Fricas [B] time = 4.66328, size = 1296, normalized size = 4.33

$$-\frac{6\sqrt{3}a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+3\,a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\log\left(\left(bx^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*arct
an(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2))^(2/3) - sqrt(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(1/3)*x^3*log((b*x^3 + a)^(2/3)*d^2 - (b*x^3 + a)^(1/3)*(b*c*d - a*d
^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 6*a^3*d*(d^2/(b^2*c^2
 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(1/3)*d + (b*c - a*d)*(d
^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) - 2*sqrt(3)*(2*a*b*c + 3*a^2*d)*
x^3*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) - (-a^2)^(2/3)*(2*b*c
 + 3*a*d)*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(-a^2)^(2/3)) + 2*(-a^2)^(2/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a
- (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2*c)/(a^3*c^2*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [A] time = 2.36861, size = 567, normalized size = 1.9

$$-\frac{1}{18}\left(\frac{6\,d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^3c^3-ab^2c^2d}-\frac{18\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c^3-\sqrt{3}ab^2c^2d}-\frac{3\left(-bcd^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/18*(6*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*
d)/d)^(1/3)))/(b^3*c^3 - a*b^2*c^2*d) - 18*(-b*c*d^2 + a*d^3)^(1/3)*d*arcta
n(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/
d)^(1/3))/(sqrt(3)*b^3*c^3 - sqrt(3)*a*b^2*c^2*d) - 3*(-b*c*d^2 + a*d^3)^(1
/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-
(b*c - a*d)/d)^(2/3))/(b^3*c^3 - a*b^2*c^2*d) - 2*sqrt(3)*(2*a^(1/3)*b*c +
3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a
^2*b^2*c^2) + 2*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5
/3)*b^2*c^2) - (2*a^(1/3)*b*c + 3*a^(4/3)*d)*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^2*b^2*c^2) + 6*(b*x^3 + a)^(1/3)/(a*b^2*c
*x^3))*b^2
```

3.738 $\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=279

$$\frac{(2ad+3bc)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2}+\frac{(2ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d^2}+\frac{c^{5/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{2/3}}-\frac{c^{5/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{2/3}}$$

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d^2) - (c^(5/3)*ArcTan[(1
 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*
d^2*(b*c - a*d)^(2/3)) + (c^(5/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(2/3))
 + ((3*b*c + 2*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(5/3)*d^2) - (
c^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
 - a*d)^(2/3))
```

Rubi [A] time = 0.501508, antiderivative size = 400, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {494, 470, 584, 292, 31, 634, 617, 204, 628}

$$\frac{(2ad+3bc)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2}-\frac{(2ad+3bc)\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{5/3}d^2}+\frac{(2ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d^2}-\frac{c^{5/3}\log\left(\sqrt[3]{c}\right.}{3d^2(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d^2) - (c^(5/3)*ArcTan[(c
^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sq
rt[3]*d^2*(b*c - a*d)^(2/3)) + ((3*b*c + 2*a*d)*Log[1 - (b^(1/3)*x)/(a + b*
x^3)^(1/3)])/(9*b^(5/3)*d^2) - ((3*b*c + 2*a*d)*Log[1 + (b^(2/3)*x^2)/(a +
b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(5/3)*d^2) - (c^(5/3)*
Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^
(2/3)) + (c^(5/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) +
 (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(2/3)
```

```
)
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = a^2\,\text{Subst}\left(\int \frac{x^7}{\left(1-bx^3\right)^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{x\left(2c+(bc+2ad)x^3\right)}{\left(1-bx^3\right)\left(c+(-bc+ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \left(\frac{(3bc+2ad)x}{ad\left(1-bx^3\right)} + \frac{3bc^2x}{ad\left(-c+(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^2\,\text{Subst}\left(\int \frac{x}{-c+(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - \frac{(3bc+2ad)\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \cdots\right.}{3bd^2}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^{5/3}\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{c}+\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + \frac{c^{5/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{c}+\sqrt[3]{bc-ad}\cdots}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc\cdots}\right.}{3d^2\sqrt[3]{bc-a\cdots}}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{c^{5/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d^2(bc-ad)^{2/3}} + \frac{c^{5/3}\,\text{Subst}\left(\int \frac{\cdots}{c^{2/3}\cdots}\right.}{\cdots}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{(3bc+2ad)\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{18b^{5/3}d^2} - \cdots$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d^2} - \frac{c^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{2/3}} + \frac{(3bc+2ad)\log\left(\cdots\right.}{9b^{5/3}a\cdots}$$

Mathematica [C] time = 0.19103, size = 190, normalized size = 0.68

$$\frac{5cx^2\left(\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3} - a\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)\right) - x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(2ad+3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};\cdots\right.}{15bcd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-((3*b*c + 2*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1
[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]) + 5*c*x^2*((a + b*x^3)*(1 +
```

```
(d*x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(15*b*c*d*(a + b*x^3)^(2/3)*(1 +
(d*x^3)/c)^(2/3))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{x^7}{dx^3 + c} \left(bx^3 + a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3 + a\right)^{\frac{2}{3}} \left(dx^3 + c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [B] time = 4.63082, size = 1305, normalized size = 4.68

$$6\,\sqrt{3} b^3 c \left(-\frac{c^2}{b^2 c^2 - 2\,abcd + a^2 d^2}\right)^{\frac{1}{3}} \arctan\left(-\frac{2\,\sqrt{3} \left(bx^3 + a\right)^{\frac{1}{3}} (bc - ad) \left(-\frac{c^2}{b^2 c^2 - 2\,abcd + a^2 d^2}\right)^{\frac{2}{3}} + \sqrt{3} cx}{3\,cx}\right) + 6\left(bx^3 + a\right)^{\frac{1}{3}} b^2 dx^2 + 6\,b^3 c \left(-\frac{c^2}{b^2 c^2 - 2\,abcd - \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*b^
3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b*c - a*d)*(-c^2/(b^
2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x + (b*x^3 + a)^(1/3)*c)/x) - 3*b^3*c*(
-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*
c^2 - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(1/3)*x)/x^2) - 2*sqrt(3)*(3*b^2*c + 2*a*b*d)*(b^2)^(1/6)*arctan(1/3*(s
qrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/
6)/(b^2*x)) + 2*(b^2)^(2/3)*(3*b*c + 2*a*d)*log(-((b^2)^(2/3)*x - (b*x^3 +
a)^(1/3)*b)/x) - (b^2)^(2/3)*(3*b*c + 2*a*d)*log(((b^2)^(1/3)*b*x^2 + (b*x^
3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^3*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(a + bx^3\right)^{\frac{2}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3 + a\right)^{\frac{2}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.739 $\int \frac{x^4}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=234

$$-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}-\frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}}+\frac{c^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}+\frac{c^{2/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right.}{\sqrt{3}d(bc-ad)^{2/3}}$$

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d)
) + (c^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)
))/Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^(2/3)) - (c^(2/3)*Log[c + d*x^3])/(6*d*
(b*c - a*d)^(2/3)) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3)*d) + (c^
(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d*(b*c - a
*d)^(2/3))
```

Rubi [A] time = 0.308244, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 481, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}+\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}+\frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}}-\frac{c^{2/3}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\right.}{6d(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d)
) + (c^(2/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(
Sqrt[3]*c^(1/3))])/(Sqrt[3]*d*(b*c - a*d)^(2/3)) - Log[1 - (b^(1/3)*x)/(a +
 b*x^3)^(1/3)]/(3*b^(2/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b
^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(2/3)*d) + (c^(2/3)*Log[c^(1/3) - ((b*c -
 a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d*(b*c - a*d)^(2/3)) - (c^(2/3)*Log[c
^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(
1/3)*x)/(a + b*x^3)^(1/3)])/(6*d*(b*c - a*d)^(2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = a\,\text{Subst}\left(\int \frac{x^4}{\left(1-bx^3\right)\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} - \frac{\text{Subst}\left(\int \frac{1-\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} - \frac{c^{2/3}\,\text{Subst}\left(\int \ldots\right.}{3\ldots}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}d} - \ldots$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} - \frac{c^{2/3}\log\left(\ldots\right.}{\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{2/3}} - \frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\ldots\right.}{6b^{2/3}d}$$

Mathematica [C] time = 0.0345053, size = 65, normalized size = 0.28

$$\frac{x^5\left(\frac{a+bx^3}{a}\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*((a + b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^4}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [B] time = 1.52868, size = 1206, normalized size = 5.15

$$2\sqrt{3}b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}cx}{3\,cx}\right)+2\,b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)\left(\frac{c^2}{b^2c^2-2\,abcd\cdots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3*
(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 2*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2
))^(1/3)*log(-((b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x -
(b*x^3 + a)^(1/3)*c)/x) - b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*l
og(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
2/3)*x^2 + (b*x^3 + a)^(2/3)*c^2 + (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(c^2/(
b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x)/x^2) + 2*sqrt(3)*b*sqrt(-(-b^2)^(1
/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-
b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*log(-((-b^2)^(2/3
)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*
x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.740 $\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=149

$$\frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}$$

```
[Out] -(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]
/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*
d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/
3)*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.140244, antiderivative size = 208, normalized size of antiderivative = 1.4, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(
1/3))]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) - Log[c^(1/3) - ((b*c - a*d)^(1
/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(1/3)*(b*c - a*d)^(2/3)) + Log[c^(2/3) + ((b
*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a +
 b*x^3)^(1/3)]/(6*c^(1/3)*(b*c - a*d)^(2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
```

```
tQ[-1, p, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\sqrt[3]{bc-ad}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\text{Subst}\left(\int \frac{}{c^{2/3}+\cdots}\right.}{}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\cdots}{\sqrt[3]{\cdots}}\right.}{\sqrt[3]{c}(bc-ad)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}$$

Mathematica [C] time = 0.0251274, size = 83, normalized size = 0.56

$$\frac{x^2\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{2c\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)
*x^3)/(a*(c + d*x^3))])/(2*c*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{x}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.741 $\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=173

$$-\frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

```
[Out] -((a + b*x^3)^(1/3)/(a*c*x)) + (d*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3)) - (d*
Log[c + d*x^3])/(6*c^(4/3)*(b*c - a*d)^(2/3)) + (d*Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3)*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.22069, antiderivative size = 232, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*c*x)) + (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)
/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3))
 + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(4/3)*(b
*c - a*d)^(2/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/
3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(4/3)*(b*c - a*
d)^(2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\,\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}\sqrt[3]{bc-ad}} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-a}\right.}{3c^{4/3}\sqrt[3]{bc-a}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+b}}\right.}{6c^{4/3}(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{4/3}(bc-ad)^{2/3}} - \frac{d\,\text{Subst}}{}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}\right.}{6c^{4/3}(bc-ad}$$

Mathematica [C] time = 0.0560154, size = 128, normalized size = 0.74

$$-\frac{\frac{6x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{a+bx^3}+5c\left(2c+3dx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{10c^3x\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(5*c*(2*c + 3*d*x^3)*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))] + (6*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[5/3, 2, 8/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(a + b*x^3))/(10*c^3*x*(a + b*x^3)^(2
/3))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(d x^3 + c\right)} \left(b x^3 + a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^3 + a\right)^{\frac{2}{3}} \left(d x^3 + c\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + bx^3\right)^{\frac{2}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{2}{3}} \left(dx^3 + c\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

3.742 $\int \frac{1}{x^5 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{4a^2c^2x}+\frac{d^2\log\left(c+dx^3\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

```
[Out] -(a + b*x^3)^(1/3)/(4*a*c*x^4) + ((3*b*c + 4*a*d)*(a + b*x^3)^(1/3))/(4*a^2
*c^2*x) - (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/
3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)*(b*c - a*d)^(2/3)) + (d^2*Log[c + d*x^3])/(
6*c^(7/3)*(b*c - a*d)^(2/3)) - (d^2*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
+ b*x^3)^(1/3)])/(2*c^(7/3)*(b*c - a*d)^(2/3))
```

Rubi [A] time = 0.296604, antiderivative size = 269, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a+bx^3}(ad+bc)}{a^2c^2x}-\frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4}-\frac{d^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}}+\frac{d^2\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+}{\sqrt{3}\sqrt[3]{c}}\right.}{\sqrt{3}c^{7/3}(bc-ad)^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((b*c + a*d)*(a + b*x^3)^(1/3))/(a^2*c^2*x) - (a + b*x^3)^(4/3)/(4*a^2*c*x^
4) - (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d)^(2/3)) - (d^2*Log[c^(1/3) - ((
b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(7/3)*(b*c - a*d)^(2/3)) + (d^
2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]])/(6*c^(7/3)*(b*c - a*d)^(2/3))
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
```

```
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)} \, dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^2}{x^5\left(c-(bc-ad)x^3\right)} \, dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^5} + \frac{-bc-ad}{c^2x^2} + \frac{a^2d^2x}{c^2\left(c-(bc-ad)x^3\right)}\right) dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} + \frac{d^2\,\text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3} \, dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c^2}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} \, dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}\sqrt[3]{bc-ad}} - \frac{d^2\,\text{Subst}}{}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}}\right.}{6c^{7/3}(bc}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\right.}{6c^{7/3}(bc-ad)^{2}}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \cdots$$

Mathematica [C] time = 1.5319, size = 267, normalized size = 1.24

$$-\frac{216dx^6\left(c+dx^3\right)(ad-bc)\text{HypergeometricPFQ}\left(\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{8}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right) - 81x^3\left(c+dx^3\right)^2(bc-ad)\text{Hypergeome}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(-5*(2*c*(a + b*x^3)*(c^2 + 10*c*d*x^3 + 9*d^2*x^6) + (3*b*c*x^3*(-3*c^2 +
 2*c*d*x^3 + 9*d^2*x^6) + a*(-8*c^3 + 17*c^2*d*x^3 + 46*c*d^2*x^6 + 9*d^3*x
^9))*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + 2
16*d*(-(b*c) + a*d)*x^6*(c + d*x^3)*HypergeometricPFQ[{2/3, 2, 2}, {1, 8/3}
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*(b*c - a*d)*x^3*(c + d*x^3)^2*Hyp
ergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))])/(120*c^4*x^4*(a + b*x^3)^(5/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(d x^3 + c\right)} \left(b x^3 + a\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^3 + a\right)^{\frac{2}{3}} \left(d x^3 + c\right) x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

3.743 $\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x^7*(1 + (b*x^3)/a)^(2/3)*AppellF1[7/3, 2/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*c*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0575557, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```


```
[Out] (x^7*(1 + (b*x^3)/a)^(2/3)*AppellF1[7/3, 2/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*c*(a + b*x^3)^(2/3))
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.333766, size = 249, normalized size = 3.89

$$\frac{x\left(4\left(\frac{4a^2c^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{b\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{a}{b}+x^3\right)-\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+2bc)F_1\left(\frac{4}{3};\frac{2}{3},1;\cdots\right)}{bc}\cdots\right)}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-(((2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(b*c)) + 4*(a/b + x^3 + (4*a^2*c^2*AppellF1[1/3
, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(b*(c + d*x^3)*(-4*a*c*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2
/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{x^6}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.744 $\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0612628, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.034045, size = 65, normalized size = 1.02

$$\frac{x^4\left(\frac{a+bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*((a + b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x^3}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.745 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0310665, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.0346739, size = 161, normalized size = 2.73

$$-\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.746 $\int \frac{1}{x^3(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] -((1 + (b*x^3)/a)^(2/3)*AppellF1[-2/3, 2/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*c*x^2*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.062456, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(2/3)*AppellF1[-2/3, 2/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*c*x^2*(a + b*x^3)^(2/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.264103, size = 338, normalized size = 5.28

$$\frac{\frac{4c\left(x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(ac+3adx^3+2bcx^3+bdx^6\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)} - bdx^6\left(\frac{bx^3}{a}+1\right)^{2}}{8ac^2x^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (4*c*(-4*a*c*(a*c + 2*b*c*x^3 + 3*a*d*x^3 + b*d*x^6)*Appell
F1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a + b*x^3)*(c + d*x
^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*A
ppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*
c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*Appel
lF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*a*c^2*x^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

3.747 $\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=347

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}-\frac{a^4}{b^4\sqrt[3]{a+bx^3}(bc-ad)}+\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{a\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^4d^2}-\frac{\left(a+bx^3\right)^{5/3}(ad}{5b^4d^2}$$

```
[Out] -(a^4/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a^2*(a + b*x^3)^(2/3))/(2*b^4
*d) + (a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + ((b^2*c^2 + a*b*c*d +
 a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (2*a*(a + b*x^3)^(5/3))/(5*b^4*d
) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (a + b*x^3)^(8/3)/(8*b^4*
d) + (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(Sqrt[3]*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[c + d*x^3])/(6*d^(11/
3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.440589, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}-\frac{a^4}{b^4\sqrt[3]{a+bx^3}(bc-ad)}+\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{a\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^4d^2}-\frac{\left(a+bx^3\right)^{5/3}(ad}{5b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^4/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a^2*(a + b*x^3)^(2/3))/(2*b^4
*d) + (a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + ((b^2*c^2 + a*b*c*d +
 a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (2*a*(a + b*x^3)^(5/3))/(5*b^4*d
) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (a + b*x^3)^(8/3)/(8*b^4*
d) + (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(Sqrt[3]*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[c + d*x^3])/(6*d^(11/
3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
```

```
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a^4}{b^3(bc-ad)(a+bx)^{4/3}} + \frac{b^2c^2+abcd+a^2d^2}{b^3d^3\sqrt[3]{a+bx}} - \frac{(bc+ad)x}{b^2d^2\sqrt[3]{a+bx}} + \frac{x^2}{bd\sqrt[3]{a+bx}}\right.\right.$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} + \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3bd}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} - \frac{c^4\log\left(c+dx^3\right)}{6d^{11/3}(bc-ad)^{4/3}} + \frac{\text{Subst}}{}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{\left(b^2c^2+abcd+a^2d^2\right.}{2b^4d^3}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{\left(b^2c^2+abcd+a^2d^2\right.}{2b^4d^3}$$

Mathematica [C] time = 0.213262, size = 157, normalized size = 0.45

$$\frac{\frac{9a^2bd^2\left(8c+3dx^3\right)+81a^3d^3+3ab^2d\left(20c^2+8cdx^3-3d^2x^6\right)+b^3\left(20c^2dx^3+40c^3-8cd^2x^6+5d^3x^9\right)}{b^4} - \frac{40c^4\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad}}{40d^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((81*a^3*d^3 + 9*a^2*b*d^2*(8*c + 3*d*x^3) + 3*a*b^2*d*(20*c^2 + 8*c*d*x^3
- 3*d^2*x^6) + b^3*(40*c^3 + 20*c^2*d*x^3 - 8*c*d^2*x^6 + 5*d^3*x^9))/b^4 -
 (40*c^4*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(
b*c - a*d))/(40*d^4*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{x^{14}}{d x^3+c} \left(b x^3+a\right)^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24325, size = 2768, normalized size = 7.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c
^4*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 -
b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)
)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(
b*x^3 + a)^(1/3))/(d*x^3 + c)) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a
)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*
d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*
```

```
(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5
+ 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c
^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*
c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*
d^6)*x^3)*(b*x^3 + a)^(2/3))/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7
 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*(120*sqrt(1/3)*
(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2)*x^3)*sqrt(-(-b
*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d
+ (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d)
+ 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/
3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^
(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 +
 a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c
^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 -
 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a
^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^
2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(a
*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6
 + a^2*b^5*d^7)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.22618, size = 582, normalized size = 1.68

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^5-2\sqrt{3}abcd^6+\sqrt{3}a^2d^7}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-}{a}\right.\right.}{6\left(b^2c^2d^5-2\,abcd^6+a^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^5 - 2*sqrt(3
)*a*b*c*d^6 + sqrt(3)*a^2*d^7) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^4*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 1/3*c^4*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^3 - 2*a*
b*c*d^4 + a^2*d^5) - a^4/((b^5*c - a*b^4*d)*(b*x^3 + a)^(1/3)) + 1/40*(20*(
b*x^3 + a)^(2/3)*b^30*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^29*c*d^6 + 40*(b*x^3
+ a)^(2/3)*a*b^29*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^28*d^7 - 24*(b*x^3 + a)^(5/
3)*a*b^28*d^7 + 60*(b*x^3 + a)^(2/3)*a^2*b^28*d^7)/(b^32*d^8)
```

3.748 $\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=253

$$\frac{a^3}{b^3\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^3d^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{\left(a+bx^3\right)^{5/3}}{5b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\cdots\right.}{2d^{8/3}(bc-a\cdots}$$

```
[Out] a^3/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (a*(a + b*x^3)^(2/3))/(2*b^3*d) -
 ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (a + b*x^3)^(5/3)/(5*b^3*d)
- (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(4/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.340169, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{a^3}{b^3\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^3d^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{\left(a+bx^3\right)^{5/3}}{5b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\cdots\right.}{2d^{8/3}(bc-a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```


```
[Out] a^3/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (a*(a + b*x^3)^(2/3))/(2*b^3*d) -
 ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (a + b*x^3)^(5/3)/(5*b^3*d)
- (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(4/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(4/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^2(bc-ad)(a+bx)^{4/3}} + \frac{-bc-ad}{b^2d^2\sqrt[3]{a+bx}} + \frac{x}{bd\sqrt[3]{a+bx}} - \frac{c^3}{d^2(-bc+ad)\sqrt[3]{a+\ldots}}\right.\right.$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3bd} + \frac{c^3\,\text{Subst}\left(\ldots\right.}{3\ldots}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}} + \frac{\text{Subst}\left(\int\left(-\frac{a}{b\sqrt[3]{a+bx}}\ldots\right.\right.}{3b\ldots}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} + \frac{c^3\log\left(c\ldots\right.}{6d^{8/3}(bc\ldots}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} - \frac{c^3\tan^{-1}\ldots}{\sqrt{3}d^{8/3}\ldots}$$

Mathematica [C] time = 0.0878751, size = 147, normalized size = 0.58

$$\frac{3a^2bd^2\left(2dx^3-c\right)+18a^3d^3-ab^2d\left(5c^2+cdx^3+2d^2x^6\right)+10b^3c^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+b^3c\left(-10c^2-5cdx^3+2d^2x^6\right)}{10b^3d^3\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (18*a^3*d^3 + 3*a^2*b*d^2*(-c + 2*d*x^3) + b^3*c*(-10*c^2 - 5*c*d*x^3 + 2*d
^2*x^6) - a*b^2*d*(5*c^2 + c*d*x^3 + 2*d^2*x^6) + 10*b^3*c^3*Hypergeometric
2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(10*b^3*d^3*(b*c - a*d)*
(a + b*x^3)^(1/3))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{dx^3 + c} \left(bx^3 + a\right)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60631, size = 2417, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^
3*d^2)*x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b
*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*s
qrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^
3 + a)^(1/3))/(d*x^3 + c)) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (b*c*d^2 - a*d^3)^(2/3)) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d
^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 3*(5*a*b^3*c
^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 -
```

```
 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^
2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b
^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3),
1/30*(30*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^3*
d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d))/d) + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log
((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(5*a*b^3*c^3*d^2 - 2*a^
2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 - 2*a*b^3*c*d^
4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^2*b^2*c*d^4 +
 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b^4*c*d^5 + a^
3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.23657, size = 502, normalized size = 1.98

$$-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^4-2\sqrt{3}abcd^5+\sqrt{3}a^2d^6}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{b}{}\right.\right.}{6\left(b^2c^2d^4-2\,abcd^5+a^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^4 - 2*sqrt(
3)*a*b*c*d^5 + sqrt(3)*a^2*d^6) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)
^(2/3))/(b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6) - 1/3*c^3*(-(b*c - a*d)/d)^(2
/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4) + a^3/((b^4*c - a*b^3*d)*(b*x^3 + a)^(1/3)) - 1/10*(5*(
b*x^3 + a)^(2/3)*b^13*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^12*d^4 + 10*(b*x^3 + a)
^(2/3)*a*b^12*d^4)/(b^15*d^5)
```

3.749 $\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=203

$$-\frac{a^2}{b^2\sqrt[3]{a+bx^3}(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}}$$

```
[Out] -(a^2/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a + b*x^3)^(2/3)/(2*b^2*d) +
(c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(6*d^(5/3)*(b*c
 - a*d)^(4/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*d^(5/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.241595, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 87, 56, 617, 204, 31}

$$-\frac{a^2}{b^2\sqrt[3]{a+bx^3}(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```



```
[Out] -(a^2/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a + b*x^3)^(2/3)/(2*b^2*d) +
(c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(6*d^(5/3)*(b*c
 - a*d)^(4/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*d^(5/3)*(b*c - a*d)^(4/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^{4/3}} + \frac{1}{bd\sqrt[3]{a+bx}} + \frac{c^2}{d(-bc+ad)\sqrt[3]{a+bx}(c+dx)}\right) dx, x, \right.$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d(bc-ad)}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{\right.}}{2d^{5/3}(bc-ad)^{4/3}}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+\right.}}{2d^{5/3}(bc-ad)^{4/3}}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} + \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}} - \frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2}{}$$

Mathematica [C] time = 0.0599008, size = 101, normalized size = 0.5

$$\frac{-3a^2d^2 - 2b^2c^2\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right) + abd\left(c-dx^3\right) + b^2c\left(2c+dx^3\right)}{2b^2d^2\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (-3*a^2*d^2 + a*b*d*(c - d*x^3) + b^2*c*(2*c + d*x^3) - 2*b^2*c^2*Hypergeom
etric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(2*b^2*d^2*(b*c - a
*d)*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^8}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.54935, size = 2159, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*
d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c
*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*s
qrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x
^3 + a)^(1/3))/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*
c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*
d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5
 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*sqrt(1/3)*(a*b
^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*sqrt(-(-b*c*d
^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-
b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (b
```

```
^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
- 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3
)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*
d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*
b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4
+ a^2*b^3*d^5)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A] time = 1.19971, size = 439, normalized size = 2.16

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^3-2\sqrt{3}abcd^4+\sqrt{3}a^2d^5}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-}{}\right.\right.}{6\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^3 - 2*sqrt(3
)*a*b*c*d^4 + sqrt(3)*a^2*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 1/3*c^2*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) - a^2/((b^3*c - a*b^2*d)*(b*x^3 + a)^(1/3)) + 1/2*(b*x^3 +
```

```
a)^(2/3)/(b^2*d)
```

3.750 $\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=174

$$\frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)) +
(c*Log[c + d*x^3])/(6*d^(2/3)*(b*c - a*d)^(4/3)) - (c*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.171119, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 78, 56, 617, 204, 31}

$$\frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)) +
(c*Log[c + d*x^3])/(6*d^(2/3)*(b*c - a*d)^(4/3)) - (c*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(4/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} + \ldots$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} + \frac{c\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\ldots\right.}{2d^{2/3}(bc-ad)\ldots}$$

Mathematica [C] time = 0.0307372, size = 77, normalized size = 0.44

$$\frac{bc\left(a+bx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2a(bc-ad)}{2b\sqrt[3]{a+bx^3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (2*a*(b*c - a*d) + b*c*(a + b*x^3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b
*x^3))/(-(b*c) + a*d)])/(2*b*(b*c - a*d)^2*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^5}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.48355, size = 1890, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x
^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3
*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*
c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(
1/3))/(d*x^3 + c)) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3
 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 -
a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)
^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b
^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 +
 (b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c
*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)
^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2
/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*(a*b*c*d^2 - a^2
```

```
*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [B] time = 1.19788, size = 406, normalized size = 2.33

$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^2-2\sqrt{3}abcd^3+\sqrt{3}a^2d^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2abcd^3+a^2d^4}+\frac{2bc\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3\cdots\right.\right.\right.}{b^2c^2-2abcd\cdots}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2 -
2*sqrt(3)*a*b*c*d^3 + sqrt(3)*a^2*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 2*b*c*(-(b*c - a*d)/d)^(
2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2 - 2*a*b*
c*d + a^2*d^2) - 6*a/((b*x^3 + a)^(1/3)*(b*c - a*d)))/b
```

3.751 $\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=167

$$-\frac{1}{\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}}$$

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) - (
d^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) + (d^(1/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.165478, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {444, 51, 56, 617, 204, 31}

$$-\frac{1}{\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```


```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) - (
d^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) + (d^(1/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}} - \text{Subs}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}} - \sqrt[3]{d}\,\text{Subst}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\right.}{2(bc-ad}$$

Mathematica [C] time = 0.0145839, size = 50, normalized size = 0.3

$$-\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)]/((b*c - a
*d)*(a + b*x^3)^(1/3)))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{x^2}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33494, size = 599, normalized size = 3.59

$$-\frac{2\sqrt{3}\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\right.}{6\left(\left(b^2c-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b*x^3 + a)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^
3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - (b*x^3 + a)*(-d/(b*c -
 a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b
*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 2*(b*x^3 + a)*(-d
/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1
/3)*d) + 6*(b*x^3 + a)^(2/3))/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [B] time = 1.17892, size = 385, normalized size = 2.31

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d-2\sqrt{3}abcd^2+\sqrt{3}a^2d^3}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(b\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/
3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(
1/3))/(sqrt(3)*b^2*c^2*d - 2*sqrt(3)*a*b*c*d^2 + sqrt(3)*a^2*d^3) - 1/6*(-b
*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) -
 1/((b*x^3 + a)^(1/3)*(b*c - a*d))
```

3.752 $\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=271

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}-\frac{d^{4/3}}{}$$

```
[Out] b/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3)
)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(4/3
)) - Log[x]/(2*a^(4/3)*c) + (d^(4/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(4/3)
) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*
d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.316329, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 85, 156, 55, 617, 204, 31, 56}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}-\frac{d^{4/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```


```
[Out] b/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3)
)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(4/3
)) - Log[x]/(2*a^(4/3)*c) + (d^(4/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(4/3)
) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*
d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(4/3))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{-bc+ad-bdx}{x\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3ac} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} +$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} - \frac{d^{4/3}\log\left(\cdot\right.}{}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}c} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log}{6c(b}$$

Mathematica [C] time = 0.0296921, size = 86, normalized size = 0.32

$$\frac{ad\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+(bc-ad)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^3}{a}+1\right)}{ac\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*d*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + (b*c
 - a*d)*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^3)/a])/(a*c*(b*c - a*d)*(a
 + b*x^3)^(1/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)
```

Fricas [B] time = 1.66092, size = 2310, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c -
 a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(
```

```
2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 +
 a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a
*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3
*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(
2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a^3*d)
*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a^2*b*
d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^
3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*
c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3)
- 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a
^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*
d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a
^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(1/3)*d) + 6*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c - a^2
*b*d)*x^3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3
))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A] time = 2.10361, size = 552, normalized size = 2.04

$$-\frac{1}{6}\left(\frac{2\,d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^3c^3-2\,ab^2c^2d+a^2bcd^2}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c^3-2\,\sqrt{3}ab^2c^2d+\sqrt{3}a^2bcd^2}-\frac{\left(-bcd^2+ad^3\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(2*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d
)/d)^(1/3)))/(b^3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2) + 6*(-b*c*d^2 + a*d^3)
^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-
(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^3*c^3 - 2*sqrt(3)*a*b^2*c^2*d + sqrt(3)*a^
2*b*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(
1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^3*c^3 - 2*a*b^2*c^
2*d + a^2*b*c*d^2) - 6/((b*x^3 + a)^(1/3)*(a*b*c - a^2*d)) - 2*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*b*c) + log
((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*b*c) - 2
*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*b*c))*b
```

3.753 $\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=357

$$-\frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}-\frac{(3ad+4bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^2}+\frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2}-\frac{}{c^2\sqrt[3]{a+}}$$

```
[Out] -(d^2/(c^2*(b*c - a*d)*(a + b*x^3)^(1/3))) - (4*b*c + 3*a*d)/(3*a^2*c^2*(a
+ b*x^3)^(1/3)) - 1/(3*a*c*x^3*(a + b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan
[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*c^2
) + (d^(7/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(Sqrt[3]*c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[x])/(6*a^(7
/3)*c^2) - (d^(7/3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(4/3)) - ((4*b*c + 3
*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(7/3)*c^2) + (d^(7/3)*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.393891, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 51, 55, 617, 204, 31, 56}

$$-\frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}-\frac{(3ad+4bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^2}+\frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2}-\frac{}{c^2\sqrt[3]{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(d^2/(c^2*(b*c - a*d)*(a + b*x^3)^(1/3))) - (4*b*c + 3*a*d)/(3*a^2*c^2*(a
+ b*x^3)^(1/3)) - 1/(3*a*c*x^3*(a + b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan
[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*c^2
) + (d^(7/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(Sqrt[3]*c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[x])/(6*a^(7
/3)*c^2) - (d^(7/3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(4/3)) - ((4*b*c + 3
*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(7/3)*c^2) + (d^(7/3)*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(4/3))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}(c+dx)}\,dx,x,x^3\right)$$

$$=-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}-\frac{\text{Subst}\left(\int \frac{\frac{1}{3}(4bc+3ad)+\frac{4bdx}{3}}{x(a+bx)^{4/3}(c+dx)}\,dx,x,x^3\right)}{3ac}$$

$$=-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx,x,x^3\right)}{3c^2}-\frac{(4bc+3ad)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/}}\right.}{9ac^2}$$

$$=-\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}}-\frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}-\frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx\right.}{3c^2(bc-ad)}$$

$$=-\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}}-\frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}+\frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2}-\frac{d^{7/3}}{6c^2}$$

$$=-\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}}-\frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}+\frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2}-\frac{d^{7/3}}{6c^2}$$

$$=-\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}}-\frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}-\frac{(4bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+}}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{3\sqrt{3}a^{7/3}c^2}$$

Mathematica [C] time = 0.0475005, size = 117, normalized size = 0.33

$$\frac{3a^2d^2x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+(bc-ad)\left(x^3(3ad+4bc)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^3}{a}+1\right)+ac\right)}{3a^2c^2x^3\sqrt[3]{a+bx^3}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (3*a^2*d^2*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*
d)] + (b*c - a*d)*(a*c + (4*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/3, 1, 2/3
, 1 + (b*x^3)/a]))/(3*a^2*c^2*(-(b*c) + a*d)*x^3*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)
```

Fricas [B] time = 6.19937, size = 3062, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*
b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*s
qrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)
*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6*sqr
t(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3
)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c^2 - a
*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-
```

```
a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3))
 - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d
- 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(a^3*b
*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c
- a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c
- a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log
((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*(a^2*b*c^2 -
 a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3
- a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6*sqrt(1/3)*((4*a
*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2 - a^3*b*c*d - 3*
a^4*d^2)*x^3)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (
-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(
-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))
^(1/3) + 1/3*sqrt(3)) - ((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b
^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*
x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2
*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*
x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a
*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*(a^3*b*d^2*x^6 +
a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d) + 6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)
*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^
5*c^2*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [A] time = 2.89965, size = 699, normalized size = 1.96

$$\frac{1}{18}\left(\frac{6\,d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2}+\frac{18\left(-bcd^2+ad^3\right)^{\frac{2}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^4c^4-2\sqrt{3}ab^3c^3d+\sqrt{3}a^2b^2c^2d^2}-\frac{3\left(-bcd^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/18*(6*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d
)/d)^(1/3))))/(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2) + 18*(-b*c*d^2 + a
*d^3)^(2/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1
/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^4*c^4 - 2*sqrt(3)*a*b^3*c^3*d + sqr
t(3)*a^2*b^2*c^2*d^2) - 3*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^4*c
^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2) - 6*(4*(b*x^3 + a)*b*c - 3*a*b*c - (b
*x^3 + a)*a*d)/((a^2*b^2*c^2 - a^3*b*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^
(1/3)*a)) - 2*sqrt(3)*(4*a^(2/3)*b*c + 3*a^(5/3)*d)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^3*b^2*c^2) - 2*(4*a^(1/3)*b*c + 3*a
^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(8/3)*b^2*c^2) + (4*a^(2
/3)*b*c + 3*a^(5/3)*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) +
a^(2/3))/(a^3*b^2*c^2))*b^2
```

3.754 $\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=322

$$\frac{(4ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{7/3}d^2}-\frac{(4ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}d^2}+\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3b^2d(bc-ad)}+\frac{c^{7/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{4/3}}-\frac{c^{7/3}}{}$$

```
[Out] (a*x^4)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ((b*c - 4*a*d)*x*(a + b*x^3)^(2
/3))/(3*b^2*d*(b*c - a*d)) - ((3*b*c + 4*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)*d^2) + (c^(7/3)*ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d^2*(
b*c - a*d)^(4/3)) + (c^(7/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(4/3)) - (c
^(7/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
- a*d)^(4/3)) + ((3*b*c + 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*
b^(7/3)*d^2)
```

Rubi [C] time = 0.0603097, antiderivative size = 67, normalized size of antiderivative = 0.21, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^{10}\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^10*(1 + (b*x^3)/a)^(1/3)*AppellF1[10/3, 4/3, 1, 13/3, -((b*x^3)/a), -((d
*x^3)/c)])/(10*a*c*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^9}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^{10}\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac\sqrt[3]{a+bx^3}}$$

Mathematica [C] time = 0.828278, size = 504, normalized size = 1.57

$$-\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\sqrt[3]{bc-ad}\left(-4a^2d^2+abcd+3b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2c\left(-4a^2\sqrt[3]{c}d\sqrt[3]{a+bx^3}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(3*(b*c - a*d)^(1/3)*(3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*x^4*(1 + (b*x^3)/a)
^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-6*a*b
*c*(b*c - a*d)^(1/3)*x + 24*a^2*d*(b*c - a*d)^(1/3)*x - 6*b^2*c*(b*c - a*d)
^(1/3)*x^4 + 6*a*b*d*(b*c - a*d)^(1/3)*x^4 - 2*Sqrt[3]*a*c^(1/3)*(-(b*c) +
4*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*(-(b*c) + 4*a*d)*(a + b*x^3)^(1/3)*Lo
g[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + a*b*c^(4/3)*(a + b*x
^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3
)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - 4*a^2*c^(1/3)*d*(a + b*x^3)^(1/
```

```
3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c
- a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(36*b^2*c*d*(b*c - a*d)^(4/3)*(a + b*x
^3)^(1/3))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x^9}{dx^3+c} \left(bx^3+a\right)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [B] time = 6.05854, size = 2989, normalized size = 9.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 +
a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a
```

```
)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 -
2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4*c^2
*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c*d - 4
*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*
c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^
(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/
(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c -
 a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c -
 a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^2*d^2)*x^4 + (
a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*d^3 +
 (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^3 + a*b^3*c^2)*(
c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c
/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c
^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4
*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (
b*x^3 + a)^(2/3))/x^2) + 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*
log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - 3*(b^
4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c -
 a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*c)/x^2) - 6*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^
2 + (3*b^4*c^2 + a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*(b^(1/3)*
x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 6*((b^3*c*d - a*b^2*d^2)*x^
4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*
d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.755 $\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=260

$$-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{4/3}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d}-\frac{c^{4/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{4/3}}+\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}}-\frac{c^{4/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{4/3}}$$

```
[Out] (a*x)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(4/3)*d) - (c^(4/3)*ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^
(4/3)) - (c^(4/3)*Log[c + d*x^3])/(6*d*(b*c - a*d)^(4/3)) + (c^(4/3)*Log[((
b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d*(b*c - a*d)^(4/3)) -
 Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(4/3)*d)
```

Rubi [C] time = 0.0589555, antiderivative size = 67, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(1/3)*AppellF1[7/3, 4/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*a*c*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}$$

Mathematica [C] time = 0.322064, size = 309, normalized size = 1.19

$$\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}(bc-ad)^{4/3}F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2ac\left(\sqrt[3]{c}\sqrt[3]{a+bx^3}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)-2\sqrt[3]{c}\sqrt[3]{a+bx^3}\right.}{12bc\sqrt[3]{a+bx^3}(bc-ad)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (3*(b*c - a*d)^(4/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] - 2*a*c*(-6*(b*c - a*d)^(1/3)*x + 2*Sqrt[3]*c^(1
/3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)] + c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((
b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b
+ a*x^3)^(1/3)]))/(12*b*c*(b*c - a*d)^(4/3)*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{x^6}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [B] time = 1.99408, size = 2562, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*c
 - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^
(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*
x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3*c*x
^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*c
 - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c - a
*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/
3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(b*
c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1
/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*
x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d
))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d -
 a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1/3)*(a*b^2*c -
```

```
a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-
b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*sqrt(3)*(b^3*c
*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c -
 a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(
1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(
b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a
*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d
 - a*b^3*d^2)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.756 $\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=172

$$-\frac{x}{\sqrt[3]{a+bx^3}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}-\frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}}$$

```
[Out] -(x/((b*c - a*d)*(a + b*x^3)^(1/3))) + (c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3))
 + (c^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) - (c^(1/3)*Log[((b*c - a*
d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rubi [C] time = 0.0654884, antiderivative size = 92, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{4ac\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((c*((b*x^3)/a
 - (d*x^3)/c))/(c + d*x^3))])/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3
))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4ac\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}}$$

Mathematica [C] time = 0.034121, size = 86, normalized size = 0.5

$$\frac{x^4\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{4ac\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, ((-(b*c) + a*d)
*x^3)/(a*(c + d*x^3))])/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{x^3}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.757 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=179

$$-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
4/3)) - (d*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(4/3)) + (d*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.19578, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(4/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3
*c^(2/3)*(b*c - a*d)^(4/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a +
 b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3
)*(b*c - a*d)^(4/3))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-adx}}\right.}{3c^{2/3}(bc}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-adx}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-adx}+(bc-ad)^{2/3}x^2}\,dx, x\right.}{6c^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-adx}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-adx}}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d}{\ }$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-adx}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-adx}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad}{(a+b}\right.}{6c^{2/3}(bc}$$

Mathematica [C] time = 0.0827065, size = 256, normalized size = 1.43

$$-\frac{21c^2dx^3\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+28c^3\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-21c^2dx^3\left(a+bx^3\right)^2-28c^3\left(a+bx^3\right.}{7c^3x^2\left(a+bx^3\right)^{7/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^3)^2
*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*c^2
*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c -
```

```
 a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/
3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(7*c^3*(-(b*c) + a*d)*x^2*(a +
 b*x^3)^(7/3))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{4}{3}}\left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.758 $\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=229

$$-\frac{(a+bx^3)^{2/3}(3bc-ad)}{2a^2cx^2(bc-ad)}+\frac{d^2\log(c+dx^3)}{6c^{5/3}(bc-ad)^{4/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}}+\frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-}$$

```
[Out] b/(a*(b*c - a*d)*x^2*(a + b*x^3)^(1/3)) - ((3*b*c - a*d)*(a + b*x^3)^(2/3))
/(2*a^2*c*(b*c - a*d)*x^2) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) + (d^
2*Log[c + d*x^3])/(6*c^(5/3)*(b*c - a*d)^(4/3)) - (d^2*Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3)*(b*c - a*d)^(4/3))
```

Rubi [C] time = 1.28743, antiderivative size = 542, normalized size of antiderivative = 2.37, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-9c^2x^6(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-9d^2x^{12}(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-18cdx^9(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (28*c^4*(a + b*x^3)^2 + 168*c^3*d*x^3*(a + b*x^3)^2 + 126*c^2*d^2*x^6*(a +
b*x^3)^2 - 28*c^4*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 168*c^3*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 126*c^2*d^2*x^6*(a + b*x^3)^2
*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*c^2
*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 42*c*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] - 27*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1
[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^2*x^6
*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] - 18*c*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^2*x^12*HypergeometricPF
```

```
Q[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(14*c^4*(b*c
- a*d)*x^5*(a + b*x^3)^(7/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{28c^4\left(a+bx^3\right)^2+168c^3dx^3\left(a+bx^3\right)^2+126c^2d^2x^6\left(a+bx^3\right)^2-28c^4\left(a+bx^3\right)^2\,{}_2F_1\Big(}{}$$

Mathematica [C] time = 0.686156, size = 542, normalized size = 2.37

$$\frac{9c^2x^6(bc-ad)^2\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)+9d^2x^{12}(bc-ad)^2\text{HypergeometricPFQ}\Big(\Big\{2,2,\frac{7}{3}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (-28*c^4*(a + b*x^3)^2 - 168*c^3*d*x^3*(a + b*x^3)^2 - 126*c^2*d^2*x^6*(a +
 b*x^3)^2 + 28*c^4*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] + 168*c^3*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3
, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 126*c^2*d^2*x^6*(a + b*x^3)^
2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*c^
2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 42*c*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 27*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F
1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*c^2*(b*c - a*d)^2*x^
6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 18*c*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*d^2*(b*c - a*d)^2*x^12*HypergeometricP
FQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(14*c^4*(-(b
*c) + a*d)*x^5*(a + b*x^3)^(7/3))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{4}{3}} \left(dx^3 + c\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

3.759 $\int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=287

$$\frac{(a+bx^3)^{2/3}(-5a^2d^2-3abcd+18b^2c^2)}{10a^3c^2x^2(bc-ad)}-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5a^2cx^5(bc-ad)}-\frac{d^3\log(c+dx^3)}{6c^{8/3}(bc-ad)^{4/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}}-\ldots$$

```
[Out] b/(a*(b*c - a*d)*x^5*(a + b*x^3)^(1/3)) - ((6*b*c - a*d)*(a + b*x^3)^(2/3))
/(5*a^2*c*(b*c - a*d)*x^5) + ((18*b^2*c^2 - 3*a*b*c*d - 5*a^2*d^2)*(a + b*x
^3)^(2/3))/(10*a^3*c^2*(b*c - a*d)*x^2) - (d^3*ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)*(b*c - a*d)
^(4/3)) - (d^3*Log[c + d*x^3])/(6*c^(8/3)*(b*c - a*d)^(4/3)) + (d^3*Log[((b
*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(8/3)*(b*c - a*d)^(4/
3))
```

Rubi [C] time = 3.85749, antiderivative size = 950, normalized size of antiderivative = 3.31, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{297d^3(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{15}+162d^3(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{15}+27d^3(bc-ad)^2\,{}_4F_3\left(2,2,2,\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^6*(a +
 b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*x^3*(a
 + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeometric2
F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*d)^2*x
^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 171
*c^2*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 486*c*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 1
```

```
0/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*d^3*(b*c - a*d)^2*x^15*Hyperg
eometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c
- a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 216*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3},
 {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 351*c*d^2*(b*c - a*d)^2*x^
12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] + 162*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3},
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*Hypergeometr
icPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81
*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c*d^2*(b*c - a*d)^2*x^12*Hypergeometr
icPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27
*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))])/(70*c^5*(b*c - a*d)*x^8*(a + b*x^3)^(7/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^6\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{56c^5\left(a+bx^3\right)^2 - 252c^4dx^3\left(a+bx^3\right)^2 - 1512c^3d^2x^6\left(a+bx^3\right)^2 - 1134c^2d^3x^9\left(a+bx^3\right.}{}$$

Mathematica [C] time = 1.78511, size = 950, normalized size = 3.31

$$-\frac{297d^3(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{15}+162d^3(bc-ad)^2\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{15}+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^6*(a
+ b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hypergeo
metric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*x^3*(
a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*d)^2*
x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 17
1*c^2*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 486*c*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3,
10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*d^3*(b*c - a*d)^2*x^15*Hyper
geometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c
 - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 216*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}
, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 351*c*d^2*(b*c - a*d)^2*x
^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] + 162*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*Hypergeomet
ricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 8
1*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c*d^2*(b*c - a*d)^2*x^12*Hypergeomet
ricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2
7*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))])/(70*c^5*(-(b*c) + a*d)*x^8*(a + b*x^3)^(7
/3))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{x^6\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^6\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

3.760 $\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=351

$$-\frac{\left(a+bx^3\right)^{2/3}\left(-12a^2bcd^2-20a^3d^3-9ab^2c^2d+81b^3c^3\right)}{40a^4c^3x^2(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{20a^3c^2x^5(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8a^2cx^8(bc-ad)}$$

```
[Out] b/(a*(b*c - a*d)*x^8*(a + b*x^3)^(1/3)) - ((9*b*c - a*d)*(a + b*x^3)^(2/3))
/(8*a^2*c*(b*c - a*d)*x^8) + ((9*b*c - 4*a*d)*(3*b*c + a*d)*(a + b*x^3)^(2/
3))/(20*a^3*c^2*(b*c - a*d)*x^5) - ((81*b^3*c^3 - 9*a*b^2*c^2*d - 12*a^2*b*
c*d^2 - 20*a^3*d^3)*(a + b*x^3)^(2/3))/(40*a^4*c^3*(b*c - a*d)*x^2) + (d^4*
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/
(Sqrt[3]*c^(11/3)*(b*c - a*d)^(4/3)) + (d^4*Log[c + d*x^3])/(6*c^(11/3)*(b*
c - a*d)^(4/3)) - (d^4*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3
)])/(2*c^(11/3)*(b*c - a*d)^(4/3))
```

Rubi [C] time = 8.29549, antiderivative size = 1486, normalized size of antiderivative = 4.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (280*c^6*(a + b*x^3)^2 - 672*c^5*d*x^3*(a + b*x^3)^2 + 3024*c^4*d^2*x^6*(a
+ b*x^3)^2 + 18144*c^3*d^3*x^9*(a + b*x^3)^2 + 13608*c^2*d^4*x^12*(a + b*x^
3)^2 - 280*c^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] + 672*c^5*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3024*c^4*d^2*x^6*(a + b*x^3)^2*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18144*c^
3*d^3*x^9*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 13608*c^2*d^4*x^12*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 66*c^4*(b*c - a*d)^2*x^6*Hyperge
ometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 312*c^3*d*(b*
c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 2268*c^2*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 10/3, ((
```

```
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6696*c*d^3*(b*c - a*d)^2*x^15*Hypergeome
tric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4050*d^4*(b*c -
a*d)^2*x^18*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 189*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] - 108*c^3*d*(b*c - a*d)^2*x^9*Hypergeomet
ricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3618*c^
2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] - 6156*c*d^3*(b*c - a*d)^2*x^15*HypergeometricPF
Q[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2835*d^4*(b*
c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 54*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 7/
3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 648*c^3*d*(b*c - a*d
)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 2268*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2,
2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2376*c*d^3*(b*c
 - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 810*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2
, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*c^4*(b*c -
 a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 324*c^3*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{
2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 486
*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1,
10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*c*d^3*(b*c - a*d)^2*x^15*Hy
pergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] - 81*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3}
, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560*c^6*(b*c - a*d)
*x^11*(a + b*x^3)^(7/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^9 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^9\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)} dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{280c^6\left(a+bx^3\right)^2 - 672c^5dx^3\left(a+bx^3\right)^2 + 3024c^4d^2x^6\left(a+bx^3\right)^2 + 18144c^3d^3x^9\left(a+b\right.}{}$$

Mathematica [C] time = 3.74252, size = 1486, normalized size = 4.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (-280*c^6*(a + b*x^3)^2 + 672*c^5*d*x^3*(a + b*x^3)^2 - 3024*c^4*d^2*x^6*(a
 + b*x^3)^2 - 18144*c^3*d^3*x^9*(a + b*x^3)^2 - 13608*c^2*d^4*x^12*(a + b*x
^3)^2 + 280*c^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 672*c^5*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1
, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3024*c^4*d^2*x^6*(a + b*x^3)^2*
Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 18144*c
^3*d^3*x^9*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 13608*c^2*d^4*x^12*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1
, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 66*c^4*(b*c - a*d)^2*x^6*Hyperg
eometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 312*c^3*d*(b
*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] + 2268*c^2*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 10/3, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6696*c*d^3*(b*c - a*d)^2*x^15*Hypergeom
etric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 4050*d^4*(b*c -
 a*d)^2*x^18*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 189*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3},
((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 108*c^3*d*(b*c - a*d)^2*x^9*Hypergeome
tricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3618*c
^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] + 6156*c*d^3*(b*c - a*d)^2*x^15*HypergeometricP
FQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2835*d^4*(b
*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3
)/(c*(a + b*x^3))] - 54*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 7
```

```
/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*c^3*d*(b*c - a*
d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 2268*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2,
 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2376*c*d^3*(b*
c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] + 810*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2,
2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c^4*(b*c
- a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] + 324*c^3*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[
{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 48
6*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1,
 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 324*c*d^3*(b*c - a*d)^2*x^15*H
ypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))] + 81*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3
}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560*c^6*(-(b*c) +
a*d)*x^11*(a + b*x^3)^(7/3))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^9 \left(d x^3+c\right)} \left(b x^3+a\right)^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^3+a\right)^{\frac{4}{3}} \left(d x^3+c\right) x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

3.761 $\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=67

$$\frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

```
[Out] (x^11*(1 + (b*x^3)/a)^(1/3)*AppellF1[11/3, 4/3, 1, 14/3, -((b*x^3)/a), -((d
*x^3)/c)])/(11*a*c*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0577278, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^11*(1 + (b*x^3)/a)^(1/3)*AppellF1[11/3, 4/3, 1, 14/3, -((b*x^3)/a), -((d
*x^3)/c)])/(11*a*c*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^{10}}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.218091, size = 194, normalized size = 2.9

$$\frac{x^2\left(2x^3\sqrt[3]{\frac{bx^3}{a}+1}\left(5a^2d^2-abcd-2b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5c\left(-5a^2d+ab\left(c-dx^3\right)+b^2cx^3\right)+5ac\sqrt[3]{\frac{bx^3}{a}+1}(5}{20b^2cd\sqrt[3]{a+bx^3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(5*c*(-5*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)) + 5*a*c*(-(b*c) + 5*a*d)
*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*(-2*b^2*c^2 - a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(20*b^2*c*d*(b*c - a*d)*(
a + b*x^3)^(1/3))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{x^{10}}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**10/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{10}}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.762 $\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 4/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*a*c*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0567504, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```


```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 4/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*a*c*(a + b*x^3)^(1/3))
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.118502, size = 144, normalized size = 2.15

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}(bc-2ad)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5acx^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5acx^2}{5bc\sqrt[3]{a+bx^3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*a*c*x^2 - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((
b*x^3)/a), -((d*x^3)/c)] + (b*c - 2*a*d)*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1
[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(5*b*c*(b*c - a*d)*(a + b*x
^3)^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^7}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.763 $\int \frac{x^4}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=67

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 4/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*a*c*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0556653, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```



```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 4/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*a*c*(a + b*x^3)^(1/3))
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0844091, size = 129, normalized size = 1.93

$$\frac{x^2\left(dx^3\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5c\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5c\right)}{5c\sqrt[3]{a+bx^3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-5*c + 5*c*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)
/a), -((d*x^3)/c)] + d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3,
 -((b*x^3)/a), -((d*x^3)/c)]))/(5*c*(b*c - a*d)*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^4}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maximia [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.764 $\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 4/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*a*c*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0405861, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 4/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*a*c*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.107153, size = 141, normalized size = 2.1

$$\frac{x^2\left(2bdx^3\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5\sqrt[3]{\frac{bx^3}{a}+1}(ad+bc)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-10bc\right)}{10ac\sqrt[3]{a+bx^3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-10*b*c + 5*(b*c + a*d)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[
5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(10*a*c*(-(b*c) + a*d)*(a +
 b*x^3)^(1/3))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.765 $\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

```
[Out] -((((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c*x*(a + b*x^3)^(1/3)))
```

Rubi [A] time = 0.0594091, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -((((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c*x*(a + b*x^3)^(1/3)))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.196388, size = 193, normalized size = 2.97

$$\frac{-5x^3\sqrt[3]{\frac{bx^3}{a}+1}\left(a^2d^2-abcd+2b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+10c\left(-a^2d+ab\left(c-dx^3\right)+2b^2cx^3\right)+2bdx^6\sqrt[3]{\frac{bx^3}{a}+1}(a}{10a^2c^2x\sqrt[3]{a+bx^3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (10*c*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)) - 5*(2*b^2*c^2 - a*b*c*d +
 a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 2*b*d*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a^2*c^2*(-(b*c) + a*d)*x*
(a + b*x^3)^(1/3))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

3.766 $\int \frac{1}{x^5\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 4/3, 1, -1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*x^4*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0583748, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 4/3, 1, -1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*x^4*(a + b*x^3)^(1/3))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^5\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

Mathematica [B] time = 0.284689, size = 264, normalized size = 3.94

$$\frac{-2bdx^9\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2+abcd-5b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5x^6\sqrt[3]{\frac{bx^3}{a}+1}\left(-2a^2bcd^2+2a^3d^3-ab^2c^2d+5b^3c^3\right)}{20a^3c^3x^4\sqrt[3]{a+bx^3}(a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(-10*b^3*c^2*x^6 + a*b^2*c*x^3*(-5*c + 2*d*x^3) + a^3*d*(-c + 4*d*x^3)
 + a^2*b*(c^2 + c*d*x^3 + 4*d^2*x^6)) + 5*(5*b^3*c^3 - a*b^2*c^2*d - 2*a^2*
b*c*d^2 + 2*a^3*d^3)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -
((b*x^3)/a), -((d*x^3)/c)] - 2*b*d*(-5*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x^9*(
1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(20*a^3*c^3*(-(b*c) + a*d)*x^4*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(dx^3+c\right)}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

3.767 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=90

$$-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}-\frac{x^4(ad+bc)}{4b^2d^2}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

```
[Out] -((b*c + a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^3*
(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Rubi [A] time = 0.0955465, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}-\frac{x^4(ad+bc)}{4b^2d^2}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((b*c + a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^3*
(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)(c+dx)}\,dx,x,x^4\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d^2}+\frac{x}{bd}-\frac{a^3}{b^2(bc-ad)(a+bx)}-\frac{c^3}{d^2(-bc+ad)(c+dx)}\right)dx,x,x^4\right)$$
$$= -\frac{(bc+ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}$$

Mathematica [A] time = 0.0469224, size = 92, normalized size = 1.02

$$-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}+\frac{x^4(-ad-bc)}{4b^2d^2}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(b*c) - a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^
3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Maple [A] time = 0.007, size = 89, normalized size = 1.

$$\frac{x^8}{8\,bd}-\frac{x^4a}{4\,b^2d}-\frac{x^4c}{4\,bd^2}-\frac{c^3\ln\left(dx^4+c\right)}{4\,d^3\,(ad-bc)}+\frac{a^3\ln\left(bx^4+a\right)}{4\,b^3\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*x^8/b/d-1/4/b^2/d*x^4*a-1/4/b/d^2*x^4*c-1/4*c^3/d^3/(a*d-b*c)*ln(d*x^4+
c)+1/4*a^3/b^3/(a*d-b*c)*ln(b*x^4+a)
```

Maxima [A] time = 0.935028, size = 113, normalized size = 1.26

$$-\frac{a^3\log\left(bx^4+a\right)}{4\left(b^4c-ab^3d\right)}+\frac{c^3\log\left(dx^4+c\right)}{4\left(bcd^3-ad^4\right)}+\frac{bdx^8-2\,(bc+ad)x^4}{8\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^3
 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)
```

Fricas [A] time = 28.199, size = 196, normalized size = 2.18

$$\frac{\left(b^{3}cd^{2}-ab^{2}d^{3}\right)x^{8}-2\,a^{3}d^{3}\log\left(bx^{4}+a\right)+2\,b^{3}c^{3}\log\left(dx^{4}+c\right)-2\left(b^{3}c^{2}d-a^{2}bd^{3}\right)x^{4}}{8\left(b^{4}cd^{3}-ab^{3}d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*log
(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)
```

Sympy [B] time = 9.56232, size = 230, normalized size = 2.56

$$\frac{a^{3}\log\left(x^{4}+\frac{\frac{a^{5}d^{4}}{b(ad-bc)}-\frac{2a^{4}cd^{3}}{ad-bc}+\frac{a^{3}bc^{2}d^{2}}{ad-bc}+a^{3}cd^{2}+ab^{2}c^{3}}{a^{3}d^{3}+b^{3}c^{3}}\right)}{4b^{3}\left(ad-bc\right)}-\frac{c^{3}\log\left(x^{4}+\frac{a^{3}cd^{2}-\frac{a^{2}b^{2}c^{3}d}{ad-bc}+\frac{2ab^{3}c^{4}}{ad-bc}+ab^{2}c^{3}-\frac{b^{4}c^{5}}{d(ad-bc)}}{a^{3}d^{3}+b^{3}c^{3}}\right)}{4d^{3}\left(ad-bc\right)}+\frac{x^{8}}{8bd}-\frac{x^{4}\left(ad+bc\right)}{4b^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] a**3*log(x**4 + (a**5*d**4/(b*(a*d - b*c)) - 2*a**4*c*d**3/(a*d - b*c) + a*
*3*b*c**2*d**2/(a*d - b*c) + a**3*c*d**2 + a*b**2*c**3)/(a**3*d**3 + b**3*c
**3))/(4*b**3*(a*d - b*c)) - c**3*log(x**4 + (a**3*c*d**2 - a**2*b**2*c**3*
d/(a*d - b*c) + 2*a*b**3*c**4/(a*d - b*c) + a*b**2*c**3 - b**4*c**5/(d*(a*d
 - b*c)))/(a**3*d**3 + b**3*c**3))/(4*d**3*(a*d - b*c)) + x**8/(8*b*d) - x*
*4*(a*d + b*c)/(4*b**2*d**2)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.768 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)} dx$

Optimal. Leaf size=70

$$\frac{a^2 \log (a+bx^4)}{4b^2(bc-ad)}-\frac{c^2 \log (c+dx^4)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

```
[Out] x^4/(4*b*d) + (a^2*Log[a + b*x^4])/(4*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4
])/(4*d^2*(b*c - a*d))
```

Rubi [A] time = 0.0637166, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2 \log (a+bx^4)}{4b^2(bc-ad)}-\frac{c^2 \log (c+dx^4)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^4/(4*b*d) + (a^2*Log[a + b*x^4])/(4*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4
])/(4*d^2*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx, x, x^4\right)$$
$$= \frac{x^4}{4bd}+\frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)}-\frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}$$

Mathematica [A] time = 0.0331131, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log\left(a+bx^4\right)-b\left(dx^4(ad-bc)+bc^2\log\left(c+dx^4\right)\right)}{4b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))/
(4*b^2*d^2*(b*c - a*d))
```

Maple [A] time = 0.009, size = 65, normalized size = 0.9

$$\frac{x^4}{4\,bd}+\frac{c^2\ln\left(dx^4+c\right)}{4\,d^2\,(ad-bc)}-\frac{a^2\ln\left(bx^4+a\right)}{4\,b^2\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4*x^4/b/d+1/4*c^2/d^2/(a*d-b*c)*ln(d*x^4+c)-1/4*a^2/b^2/(a*d-b*c)*ln(b*x^
4+a)
```

Maxima [A] time = 0.928949, size = 92, normalized size = 1.31

$$\frac{x^4}{4\,bd}+\frac{a^2\log\left(bx^4+a\right)}{4\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(dx^4+c\right)}{4\left(bcd^2-ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x^
4 + c)/(b*c*d^2 - a*d^3)
```

Fricas [A] time = 7.86221, size = 142, normalized size = 2.03

$$\frac{\left(b^2cd-abd^2\right)x^4+a^2d^2\log\left(bx^4+a\right)-b^2c^2\log\left(dx^4+c\right)}{4\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4 +
 c))/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [B] time = 7.35492, size = 201, normalized size = 2.87

$$-\frac{a^2\log\left(x^4+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{4b^2\left(ad-bc\right)}+\frac{c^2\log\left(x^4+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{4d^2\left(ad-bc\right)}+\frac{x^4}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -a**2*log(x**4 + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a
**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(4
*b**2*(a*d - b*c)) + c**2*log(x**4 + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d
 + 2*a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*
d**2 + b**2*c**2))/(4*d**2*(a*d - b*c)) + x**4/(4*b*d)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.769 $\int \frac{x^7}{(a+bx^4)(c+dx^4)} \, dx$

Optimal. Leaf size=53

$$\frac{c \log \left(c+dx^4\right)}{4d(bc-ad)}-\frac{a \log \left(a+bx^4\right)}{4b(bc-ad)}$$

```
[Out] -(a*Log[a + b*x^4])/(4*b*(b*c - a*d)) + (c*Log[c + d*x^4])/(4*d*(b*c - a*d)
)
```

Rubi [A] time = 0.0499208, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c \log \left(c+dx^4\right)}{4d(bc-ad)}-\frac{a \log \left(a+bx^4\right)}{4b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(a*Log[a + b*x^4])/(4*b*(b*c - a*d)) + (c*Log[c + d*x^4])/(4*d*(b*c - a*d)
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\operatorname{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right)dx, x, x^4\right)$$
$$= -\frac{a\log(a+bx^4)}{4b(bc-ad)} + \frac{c\log(c+dx^4)}{4d(bc-ad)}$$

Mathematica [A] time = 0.0228855, size = 43, normalized size = 0.81

$$-\frac{ad\log(a+bx^4) - bc\log(c+dx^4)}{4b^2cd - 4abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))
```

Maple [A] time = 0.009, size = 50, normalized size = 0.9

$$-\frac{c\ln(dx^4+c)}{(4\,ad-4\,bc)\,d} + \frac{a\ln(bx^4+a)}{(4\,ad-4\,bc)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/4*c/(a*d-b*c)/d*ln(d*x^4+c)+1/4*a/(a*d-b*c)/b*ln(b*x^4+a)
```

Maxima [A] time = 0.932673, size = 66, normalized size = 1.25

$$-\frac{a\log(bx^4+a)}{4(b^2c-abd)} + \frac{c\log(dx^4+c)}{4(bcd-ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^2
)
```

Fricas [A] time = 2.84223, size = 92, normalized size = 1.74

$$-\frac{ad\log\left(bx^4+a\right)-bc\log\left(dx^4+c\right)}{4\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] time = 4.54636, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^4+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{4b\left(ad-bc\right)}-\frac{c\log\left(x^4+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{4d\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] a*log(x**4 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(4*b*(a*d - b*c)) - c*log(x**4 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(4*d*(a*d - b*c))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.770 $\int \frac{x^3}{(a+bx^4)(c+dx^4)} dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

```
[Out] Log[a + b*x^4]/(4*(b*c - a*d)) - Log[c + d*x^4]/(4*(b*c - a*d))
```

Rubi [A] time = 0.0314069, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[a + b*x^4]/(4*(b*c - a*d)) - Log[c + d*x^4]/(4*(b*c - a*d))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^4\right)$$
$$= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^4\right)}{4(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^4\right)}{4(bc-ad)}$$
$$= \frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

Mathematica [A] time = 0.0190943, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^4)-\log(c+dx^4)}{4bc-4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)
```

Maple [A] time = 0.007, size = 42, normalized size = 0.9

$$\frac{\ln(dx^4+c)}{4\,ad-4\,bc} - \frac{\ln(bx^4+a)}{4\,ad-4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4/(a*d-b*c)*ln(d*x^4+c)-1/4/(a*d-b*c)*ln(b*x^4+a)
```

Maxima [A] time = 0.927421, size = 55, normalized size = 1.22

$$\frac{\log(bx^4+a)}{4\,(bc-ad)} - \frac{\log(dx^4+c)}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)
```

Fricas [A] time = 1.29518, size = 69, normalized size = 1.53

$$\frac{\log\left(bx^4 + a\right) - \log\left(dx^4 + c\right)}{4\,(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)
```

Sympy [B] time = 1.71828, size = 138, normalized size = 3.07

$$\frac{\log\left(x^4 + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{4\,(ad - bc)} - \frac{\log\left(x^4 + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{4\,(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(4*(a*d - b*c))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.771 $\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=62

$$-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^4])/(4*a*(b*c - a*d)) + (d*Log[c + d*x^4])/(4
*c*(b*c - a*d))
```

Rubi [A] time = 0.0637304, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^4])/(4*a*(b*c - a*d)) + (d*Log[c + d*x^4])/(4
*c*(b*c - a*d))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^4\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^4\right)$$
$$= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}$$

Mathematica [A] time = 0.0295677, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^4\right)+ad\log\left(c+dx^4\right)-4ad\log(x)+4bc\log(x)}{4abc^2-4a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4*
a*b*c^2 - 4*a^2*c*d)
```

Maple [A] time = 0.007, size = 59, normalized size = 1.

$$-\frac{d\ln\left(dx^4+c\right)}{4\,c\,(ad-bc)}+\frac{b\ln\left(bx^4+a\right)}{4\,a\,(ad-bc)}+\frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/4*d/c/(a*d-b*c)*ln(d*x^4+c)+1/4*b/a/(a*d-b*c)*ln(b*x^4+a)+ln(x)/a/c
```

Maxima [A] time = 0.927327, size = 82, normalized size = 1.32

$$-\frac{b\log\left(bx^4+a\right)}{4\left(abc-a^2d\right)}+\frac{d\log\left(dx^4+c\right)}{4\left(bc^2-acd\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*d
) + 1/4*log(x^4)/(a*c)
```

Fricas [A] time = 9.1252, size = 123, normalized size = 1.98

$$-\frac{bc\log\left(bx^4+a\right)-ad\log\left(dx^4+c\right)-4\,(bc-ad)\log\left(x\right)}{4\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.772 $\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

```
[Out] -1/(4*a*c*x^4) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^4])/(4*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Rubi [A] time = 0.0899582, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```


```
[Out] -1/(4*a*c*x^4) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^4])/(4*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx, x, x^4\right)$$
$$= -\frac{1}{4acx^4}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}$$

Mathematica [A] time = 0.0413605, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^4\right)}{4a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(4*a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4])/(
4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Maple [A] time = 0.011, size = 87, normalized size = 1.

$$\frac{d^2\ln\left(dx^4+c\right)}{4\,c^2\,(ad-bc)}-\frac{b^2\ln\left(bx^4+a\right)}{4\,a^2\,(ad-bc)}-\frac{1}{4\,acx^4}-\frac{\ln(x)\,d}{c^2a}-\frac{\ln(x)\,b}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4*d^2/c^2/(a*d-b*c)*ln(d*x^4+c)-1/4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)-1/4/a/c
/x^4-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b
```

Maxima [A] time = 0.937868, size = 117, normalized size = 1.34

$$\frac{b^2\log\left(bx^4+a\right)}{4\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^4+c\right)}{4\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^4\right)}{4\,a^2c^2}-\frac{1}{4\,acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 -
a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)
```

Fricas [A] time = 38.2724, size = 200, normalized size = 2.3

$$\frac{b^2c^2x^4\log\left(bx^4+a\right)-a^2d^2x^4\log\left(dx^4+c\right)-4\left(b^2c^2-a^2d^2\right)x^4\log\left(x\right)-abc^2+a^2cd}{4\left(a^2bc^3-a^3c^2d\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2 -
 a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.773 $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)} \, dx$

Optimal. Leaf size=112

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}+\frac{x^6}{6bd}$$

```
[Out] -((b*c + a*d)*x^2)/(2*b^2*d^2) + x^6/(6*b*d) - (a^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*b^(5/2)*(b*c - a*d)) + (c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*d^(5/2)*(b*c - a*d))
```

Rubi [A] time = 0.273042, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 479, 582, 522, 205}

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}+\frac{x^6}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((b*c + a*d)*x^2)/(2*b^2*d^2) + x^6/(6*b*d) - (a^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*b^(5/2)*(b*c - a*d)) + (c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*d^(5/2)*(b*c - a*d))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^6}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)\\
&= \frac{x^6}{6bd} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3ac+3(bc+ad)x^2)}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6bd}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} + \frac{\operatorname{Subst}\left(\int \frac{3ac(bc+ad)+3(b^2c^2+ad(bc+ad))x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6b^2d^2}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b^2(bc-ad)} + \frac{c^3\operatorname{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d^2(bc-ad)}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} + \frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.158419, size = 104, normalized size = 0.93

$$\frac{1}{6}\left(\frac{3a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{5/2}(ad-bc)}+\frac{x^2\left(-3ad-3bc+bdx^4\right)}{b^2d^2}+\frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x^
2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqr
t[c]])/(d^(5/2)*(b*c - a*d)))/6
```

Maple [A] time = 0.011, size = 105, normalized size = 0.9

$$\frac{x^6}{6\,bd}-\frac{x^2a}{2\,b^2d}-\frac{x^2c}{2\,bd^2}-\frac{c^3}{2\,d^2\,(ad-bc)}\arctan\left(x^2d\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{a^3}{2\,b^2\,(ad-bc)}\arctan\left(bx^2\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/6*x^6/b/d-1/2/b^2/d*x^2*a-1/2/b/d^2*x^2*c-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/
2)*arctan(x^2*d/(c*d)^(1/2))+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2
/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 10.3222, size = 1125, normalized size = 10.04

$$\left[\frac{2\left(b^2cd-abd^2\right)x^6-3\,a^2d^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)-3\,b^2c^2\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)-6\left(b^2c^2-a^2d^2\right)x^2}{12\left(b^3cd^2-ab^2d^3\right)},\ \frac{2\left(b^2cd-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^
2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*
sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^
2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*x^2*
sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d
*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(b
^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*a
^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 6*(b^
2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b*d^2)*x^6
 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt(c/d)*arct
an(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24234, size = 722, normalized size = 6.45

$$-\frac{\left(\sqrt{ab}b^5c^2d^5x^4|b|+\sqrt{ab}ab^4cd^6x^4|b|+\sqrt{ab}a^2b^3d^7x^4|b|+\sqrt{ab}ab^4c^2d^5|b|+\sqrt{ab}a^2b^3cd^6|b|\right)\arctan\left(\frac{8\sqrt{\frac{1}{2}\cdots}}{\sqrt{\frac{16\,b^4cd^3+16\,ab^3d^4+\sqrt{-1024\,a\cdots}}{b^4a\cdots}}}\right.}{b^4cd^3\left|-b^4cd^3+ab^3d^4\right|+ab^3d^4\left|-b^4cd^3+ab^3d^4\right|+\left(b^4cd^3-ab^3d^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -(sqrt(a*b)*b^5*c^2*d^5*x^4*abs(b) + sqrt(a*b)*a*b^4*c*d^6*x^4*abs(b) + sqr
t(a*b)*a^2*b^3*d^7*x^4*abs(b) + sqrt(a*b)*a*b^4*c^2*d^5*abs(b) + sqrt(a*b)*
a^2*b^3*c*d^6*abs(b))*arctan(8*sqrt(1/2)*x^2/sqrt((16*b^4*c*d^3 + 16*a*b^3*
d^4 + sqrt(-1024*a*b^7*c*d^7 + 256*(b^4*c*d^3 + a*b^3*d^4)^2))/(b^4*d^4)))/
(b^4*c*d^3*abs(-b^4*c*d^3 + a*b^3*d^4) + a*b^3*d^4*abs(-b^4*c*d^3 + a*b^3*d
^4) + (b^4*c*d^3 - a*b^3*d^4)^2) + (sqrt(c*d)*b^7*c^2*d^3*x^4*abs(d) + sqrt
(c*d)*a*b^6*c*d^4*x^4*abs(d) + sqrt(c*d)*a^2*b^5*d^5*x^4*abs(d) + sqrt(c*d)
*a*b^6*c^2*d^3*abs(d) + sqrt(c*d)*a^2*b^5*c*d^4*abs(d))*arctan(8*sqrt(1/2)*
x^2/sqrt((16*b^4*c*d^3 + 16*a*b^3*d^4 - sqrt(-1024*a*b^7*c*d^7 + 256*(b^4*c
*d^3 + a*b^3*d^4)^2))/(b^4*d^4)))/(b^4*c*d^3*abs(-b^4*c*d^3 + a*b^3*d^4) +
a*b^3*d^4*abs(-b^4*c*d^3 + a*b^3*d^4) - (b^4*c*d^3 - a*b^3*d^4)^2) + 1/6*(b
^2*d^2*x^6 - 3*b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)
```

3.774 $\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=92

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

```
[Out] x^2/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d
)) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d))
```

Rubi [A] time = 0.118431, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 479, 522, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^2/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d
)) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)\\ &= \frac{x^2}{2bd}-\frac{\operatorname{Subst}\left(\int \frac{ac+(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{2bd}\\ &= \frac{x^2}{2bd}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b(bc-ad)}-\frac{c^2\operatorname{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d(bc-ad)}\\ &= \frac{x^2}{2bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.13363, size = 82, normalized size = 0.89

$$\frac{\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}}+x^2\left(\frac{c}{d}-\frac{a}{b}\right)-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{3/2}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)
```

Maple [A] time = 0.009, size = 81, normalized size = 0.9

$$\frac{x^2}{2\,bd}+\frac{c^2}{2\,d\,(ad-bc)}\arctan\left(x^2d\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{a^2}{2\,b\,(ad-bc)}\arctan\left(bx^2\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2*x^2/b/d+1/2*c^2/d/(a*d-b*c)/(c*d)^(1/2)*arctan(x^2*d/(c*d)^(1/2))-1/2*a
^2/b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54164, size = 848, normalized size = 9.22

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)-2\,(bc-ad)x^2}{4\left(b^2cd-abd^2\right)},\frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4}{\ldots}\right.}{4\left(b^2cd-abd^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + b
*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c -
a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/
a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) + 2*(
```

```
b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^2*sqr
t(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)
) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arctan(b*x^2
*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*d)*x^2)/
(b^2*c*d - a*b*d^2)]
```

Sympy [B] time = 10.8265, size = 932, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -sqrt(-a**3/b**3)*log(x**2 + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**
3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b*
*3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c
)**3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-
a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(4*(a*
d - b*c)) + sqrt(-a**3/b**3)*log(x**2 + (a**4*d**4*sqrt(-a**3/b**3)/(a*d -
b*c) + a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5
*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/
(a*d - b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**4*c
**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**
3))/(4*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x**2 + (-a**4*d**4*sqrt(-c**3/d*
*3)/(a*d - b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2*
b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d*
*3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)*
*3 - b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d +
 a*b**2*c**3))/(4*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x**2 + (a**4*d**4*sqr
t(-c**3/d**3)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)*
*3 - a**2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4
*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a
*d - b*c)**3 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*
b*c**2*d + a*b**2*c**3))/(4*(a*d - b*c)) + x**2/(2*b*d)
```

Giac [B] time = 1.19765, size = 482, normalized size = 5.24

$$\frac{\left(\sqrt{ab}bcd^2x^4|b| + \sqrt{ab}ad^3x^4|b| + \sqrt{ab}acd^2|b|\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x^2}{\sqrt{\frac{4b^2cd+4abd^2+\sqrt{-64ab^3cd^3+16\left(b^2cd+abd^2\right)^2}}{b^2d^2}}}\right)}{b^2cd\left|-b^2cd+abd^2\right| + abd^2\left|-b^2cd+abd^2\right| + \left(b^2cd-abd^2\right)^2} - \frac{\left(\sqrt{cd}b^3cx^4|d| + \sqrt{cd}ab^2dx^4|d|\right.}{b^2cd\left|-b^2cd+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] (sqrt(a*b)*b*c*d^2*x^4*abs(b) + sqrt(a*b)*a*d^3*x^4*abs(b) + sqrt(a*b)*a*c*
d^2*abs(b))*arctan(4*sqrt(1/2)*x^2/sqrt((4*b^2*c*d + 4*a*b*d^2 + sqrt(-64*a
*b^3*c*d^3 + 16*(b^2*c*d + a*b*d^2)^2))/(b^2*d^2)))/(b^2*c*d*abs(-b^2*c*d +
 a*b*d^2) + a*b*d^2*abs(-b^2*c*d + a*b*d^2) + (b^2*c*d - a*b*d^2)^2) - (sqr
t(c*d)*b^3*c*x^4*abs(d) + sqrt(c*d)*a*b^2*d*x^4*abs(d) + sqrt(c*d)*a*b^2*c*
abs(d))*arctan(4*sqrt(1/2)*x^2/sqrt((4*b^2*c*d + 4*a*b*d^2 - sqrt(-64*a*b^3
*c*d^3 + 16*(b^2*c*d + a*b*d^2)^2))/(b^2*d^2)))/(b^2*c*d*abs(-b^2*c*d + a*b
*d^2) + a*b*d^2*abs(-b^2*c*d + a*b*d^2) - (b^2*c*d - a*b*d^2)^2) + 1/2*x^2/
(b*d)
```

3.775 $\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}$$

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c - a*d)) + (Sqrt[c]
*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d))
```

Rubi [A] time = 0.0621678, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 481, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c - a*d)) + (Sqrt[c]
*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{a \operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2(bc-ad)} + \frac{c \operatorname{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2(bc-ad)}$$
$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)} + \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}$$

Mathematica [A] time = 0.038372, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt
[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)
```

Maple [A] time = 0.005, size = 60, normalized size = 0.8

$$-\frac{c}{2\,ad-2\,bc}\arctan\left(x^2d\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{a}{2\,ad-2\,bc}\arctan\left(bx^2\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/2*c/(a*d-b*c)/(c*d)^(1/2)*arctan(x^2*d/(c*d)^(1/2))+1/2*a/(a*d-b*c)/(a*b
)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34681, size = 657, normalized size = 8.32

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},-\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\frac{2\sqrt{\frac{c}{d}}\,\mathrm{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + sqrt(
-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), -1/4*
(2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sq
rt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(2*sqrt(c/d)*arctan(d*x^2*sqrt
(c/d)/c) - sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)))/(b
*c - a*d), -1/2*(sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x
^2*sqrt(c/d)/c))/(b*c - a*d)]
```

Sympy [B] time = 38.0516, size = 576, normalized size = 7.29

$$\frac{\sqrt{-\frac{a}{b}}\log\left(-\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x^2\right)}{4\,(ad-bc)}-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}\right.}{4\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] sqrt(-a/b)*log(-2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 + 4*a*b**2*c*d**
2*(-a/b)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-a/b)/(a*d - b*c) - 2*b**3*c**2*d
*(-a/b)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-a/b)/(a*d - b*c) + x**2)/(4*(a*d
- b*c)) - sqrt(-a/b)*log(2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 - 4*a*b
**2*c*d**2*(-a/b)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-a/b)/(a*d - b*c) + 2*b*
*3*c**2*d*(-a/b)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-a/b)/(a*d - b*c) + x**2)
/(4*(a*d - b*c)) + sqrt(-c/d)*log(-2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)*
*3 + 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-c/d)/(a*d - b
*c) - 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-c/d)/(a*d - b*
c) + x**2)/(4*(a*d - b*c)) - sqrt(-c/d)*log(2*a**2*b*d**3*(-c/d)**(3/2)/(a*
d - b*c)**3 - 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-c/d)
/(a*d - b*c) + 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-c/d)/
(a*d - b*c) + x**2)/(4*(a*d - b*c))
```

Giac [B] time = 1.14814, size = 271, normalized size = 3.43

$$-\frac{\sqrt{cd}bx^4|d|\arctan\left(\frac{2x^2}{\sqrt{\frac{2bc+2ad+\sqrt{-16abcd+4(bc+ad)^2}}{bd}}}\right)}{bcd|bc-ad|+ad^2|bc-ad|+(bc-ad)^2d}+\frac{\sqrt{ab}dx^4|b|\arctan\left(\frac{2x^2}{\sqrt{\frac{2bc+2ad-\sqrt{-16abcd+4(bc+ad)^2}}{bd}}}\right)}{b^2c|bc-ad|+abd|bc-ad|-(bc-ad)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -sqrt(c*d)*b*x^4*abs(d)*arctan(2*x^2/sqrt((2*b*c + 2*a*d + sqrt(-16*a*b*c*d
 + 4*(b*c + a*d)^2))/(b*d)))/(b*c*d*abs(b*c - a*d) + a*d^2*abs(b*c - a*d) +
 (b*c - a*d)^2*d) + sqrt(a*b)*d*x^4*abs(b)*arctan(2*x^2/sqrt((2*b*c + 2*a*d
 - sqrt(-16*a*b*c*d + 4*(b*c + a*d)^2))/(b*d)))/(b^2*c*abs(b*c - a*d) + a*b
*d*abs(b*c - a*d) - (b*c - a*d)^2*b)
```

3.776 $\int \frac{x}{(a+bx^4)(c+dx^4)} dx$

Optimal. Leaf size=79

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d))
```

Rubi [A] time = 0.0480111, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {465, 391, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)\\ &= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2(bc-ad)}-\frac{d\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2(bc-ad)}\\ &= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.0459221, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]
*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)
```

Maple [A] time = 0.008, size = 60, normalized size = 0.8

$$\frac{d}{2\,ad-2\,bc}\arctan\left(x^2d\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b}{2\,ad-2\,bc}\arctan\left(bx^2\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2*d/(a*d-b*c)/(c*d)^(1/2)*arctan(x^2*d/(c*d)^(1/2))-1/2*b/(a*d-b*c)/(a*b)
^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3518, size = 668, normalized size = 8.46

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, \frac{2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)}{4\,(bc-ad)}, -\frac{2\sqrt{\frac{b}{a}}\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + sqrt(
-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(
2*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - sqrt(-b/a)*log((b*x^4 - 2*a*x^2*s
qrt(-b/a) - a)/(b*x^4 + a)))/(b*c - a*d), -1/4*(2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*x^2)) + sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c))
)/(b*c - a*d), -1/2*(sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - sqrt(d/c)*arct
an(c*sqrt(d/c)/(d*x^2)))/(b*c - a*d)]
```

Sympy [B] time = 21.4931, size = 719, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] sqrt(-b/a)*log(x**2 + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c
**2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d
 - b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(
a*d - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(4*(a*d - b*c)) -
sqrt(-b/a)*log(x**2 + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c*
*2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d
- b*c)**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a
*d - b*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(4*(a*d - b*c)) + s
qrt(-d/c)*log(x**2 + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c*
*2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d
- b*c)**3 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a
*d - b*c)**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(4*(a*d - b*c)) - s
qrt(-d/c)*log(x**2 + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**
2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d -
 b*c)**3 + a**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*
d - b*c)**3 + b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(4*(a*d - b*c))
```

Giac [B] time = 1.17518, size = 261, normalized size = 3.3

$$-\frac{\sqrt{cd}b|d|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x^2}{\sqrt{\frac{bc+ad+\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{bcd|bc-ad|+ad^2|bc-ad|+(bc-ad)^2d}+\frac{\sqrt{ab}d|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x^2}{\sqrt{\frac{bc+ad-\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{b^2c|bc-ad|+abd|bc-ad|-(bc-ad)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -sqrt(c*d)*b*abs(d)*arctan(2*sqrt(1/2)*x^2/sqrt((b*c + a*d + sqrt(-4*a*b*c*
d + (b*c + a*d)^2))/(b*d)))/(b*c*d*abs(b*c - a*d) + a*d^2*abs(b*c - a*d) +
(b*c - a*d)^2*d) + sqrt(a*b)*d*abs(b)*arctan(2*sqrt(1/2)*x^2/sqrt((b*c + a*
d - sqrt(-4*a*b*c*d + (b*c + a*d)^2))/(b*d)))/(b^2*c*abs(b*c - a*d) + a*b*d
*abs(b*c - a*d) - (b*c - a*d)^2*b)
```

3.777 $\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=92

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}-\frac{1}{2acx^2}$$

```
[Out] -1/(2*a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(3/2)*(b*c -
a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d))
```

Rubi [A] time = 0.121941, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 480, 522, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(2*a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(3/2)*(b*c -
a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{2acx^2}+\frac{\mathrm{Subst}\left(\int \frac{-bc-ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)}{2ac}\\ &= -\frac{1}{2acx^2}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2a(bc-ad)}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2c(bc-ad)}\\ &= -\frac{1}{2acx^2}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.203156, size = 169, normalized size = 1.84

$$\frac{-\frac{b^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}}-\frac{b^{3/2}x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{3/2}}+\frac{b}{a}+\frac{d^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{3/2}}+\frac{d^{3/2}x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{c^{3/2}}-\frac{d}{c}}{2x^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2)
- (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2)*
```

```
x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)
```

Maple [A] time = 0.008, size = 81, normalized size = 0.9

$$-\frac{d^2}{2\,c\,(ad-bc)}\arctan\left(x^2d\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{1}{2\,acx^2}+\frac{b^2}{2\,a\,(ad-bc)}\arctan\left(bx^2\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/2*d^2/c/(a*d-b*c)/(c*d)^(1/2)*arctan(x^2*d/(c*d)^(1/2))-1/2/a/c/x^2+1/2*
b^2/a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.90047, size = 910, normalized size = 9.89

$$\left[-\frac{bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)+2\,bc-2\,ad}{4\left(abc^2-a^2cd\right)x^2},-\frac{2\,adx^2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)+bcx^2\sqrt{-\frac{b}{a}}\log}{4\left(abc^2-a^2c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a))
 + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) + 2
*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*a*d*x^2*sqrt(d/c)*arctan(c
*sqrt(d/c)/(d*x^2)) + b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) -
a)/(b*x^4 + a)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*b*c*x^2*
sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c
*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2
), 1/2*(b*c*x^2*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(d/c)*a
rctan(c*sqrt(d/c)/(d*x^2)) - b*c + a*d)/((a*b*c^2 - a^2*c*d)*x^2)]
```

Sympy [B] time = 11.9343, size = 1103, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -sqrt(-b**3/a**3)*log(x**2 + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*
c)**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2
*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**3)/
(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*
b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)/(
a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d - b*
c)) + sqrt(-b**3/a**3)*log(x**2 + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d
- b*c)**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*
b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3/a*
*3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a
**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a**
3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d
- b*c)) - sqrt(-d**3/c**3)*log(x**2 + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/
(a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2*
a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-d*
*3/c**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**
3 - a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d**
3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*
(a*d - b*c)) + sqrt(-d**3/c**3)*log(x**2 + (a**7*c**3*d**4*(-d**3/c**3)**(3
/2)/(a*d - b*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3
+ 2*a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt
(-d**3/c**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(
-d**3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))
```

```
/(4*(a*d - b*c)) - 1/(2*a*c*x**2)
```

Giac [B] time = 1.20184, size = 508, normalized size = 5.52

$$\frac{\left(\sqrt{cd}ab^2cdx^4|d| + \sqrt{cd}ab^2c^2|d| + \sqrt{cd}a^2bcd|d|\right)\arctan\left(\frac{2\,x^2}{\sqrt{\frac{2\,abc^2+2\,a^2cd+\sqrt{-16\,a^3bc^3d+4\left(abc^2+a^2cd\right)^2}}{abcd}}}\right)}{abc^2d\left|abc^2-a^2cd\right| + a^2cd^2\left|abc^2-a^2cd\right| + \left(abc^2-a^2cd\right)^2d} - \frac{\left(\sqrt{ab}abcd^2x^4|b| + \sqrt{ab}abc^2d|b\right.}{ab^2c^2\left|abc^2-a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] (sqrt(c*d)*a*b^2*c*d*x^4*abs(d) + sqrt(c*d)*a*b^2*c^2*abs(d) + sqrt(c*d)*a^
2*b*c*d*abs(d))*arctan(2*x^2/sqrt((2*a*b*c^2 + 2*a^2*c*d + sqrt(-16*a^3*b*c
^3*d + 4*(a*b*c^2 + a^2*c*d)^2))/(a*b*c*d)))/(a*b*c^2*d*abs(a*b*c^2 - a^2*c
*d) + a^2*c*d^2*abs(a*b*c^2 - a^2*c*d) + (a*b*c^2 - a^2*c*d)^2*d) - (sqrt(a
*b)*a*b*c*d^2*x^4*abs(b) + sqrt(a*b)*a*b*c^2*d*abs(b) + sqrt(a*b)*a^2*c*d^2
*abs(b))*arctan(2*x^2/sqrt((2*a*b*c^2 + 2*a^2*c*d - sqrt(-16*a^3*b*c^3*d +
4*(a*b*c^2 + a^2*c*d)^2))/(a*b*c*d)))/(a*b^2*c^2*abs(a*b*c^2 - a^2*c*d) + a
^2*b*c*d*abs(a*b*c^2 - a^2*c*d) - (a*b*c^2 - a^2*c*d)^2*b) - 1/2/(a*c*x^2)
```

3.778 $\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=112

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^2) + (b^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*c^(5/2)*(b*c - a*d))
```

Rubi [A] time = 0.218099, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 480, 583, 522, 205}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^2) + (b^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*c^(5/2)*(b*c - a*d))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
```

```
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3(bc+ad)-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\text{Subst}\left(\int \frac{-3\left(b^2c^2+abcd+a^2d^2\right)-3bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)}{6a^2c^2}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2a^2(bc-ad)}-\frac{d^3\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2c^2(bc-ad)}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}
\end{aligned}
$$

Mathematica [A] time = 0.213169, size = 193, normalized size = 1.72

$$\frac{-\frac{3b^2x^4}{a^2}+\frac{3b^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/2}}+\frac{3b^{5/2}x^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{5/2}}+\frac{b}{a}+\frac{3d^2x^4}{c^2}-\frac{3d^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/2}}-\frac{3d^{5/2}x^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{c^{5/2}}-\frac{d}{c}}{6x^6(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)
```

Maple [A] time = 0.012, size = 105, normalized size = 0.9

$$\frac{d^3}{2\,c^2\,(ad-bc)}\arctan\left(x^2d\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^3}{2\,a^2\,(ad-bc)}\arctan\left(bx^2\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{6\,acx^6}+\frac{d}{2\,c^2ax^2}+\frac{b}{2\,a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2*d^3/c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(x^2*d/(c*d)^(1/2))-1/2*b^3/a^2/(a*
d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/6/a/c/x^6+1/2/a/c^2/x^2*d+1/
2/a^2/c/x^2*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.4613, size = 1191, normalized size = 10.63

$$\left[-\frac{3\,b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+3\,a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-6\left(b^2c^2-a^2d^2\right)x^4+2\,abc^2-2\,a^2cd}{12\left(a^2bc^3-a^3c^2d\right)x^6},\ 6\,a^2d^2x^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^
4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*
x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3
- a^3*c^2*d)*x^6), 1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)
) - 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 +
a)) + 6*(b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3 - a^3*
c^2*d)*x^6), -1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) + 3
*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) -
 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d
)*x^6), -1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - 3*a^2*d
^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*(b^2*c^2 - a^2*d^2)*x^4 +
a*b*c^2 - a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.69473, size = 736, normalized size = 6.57

$$-\frac{\left(\sqrt{cd}a^2b^3c^3dx^4|d| + \sqrt{cd}a^3b^2c^2d^2x^4|d| + \sqrt{cd}a^2b^3c^4|d| + \sqrt{cd}a^3b^2c^3d|d| + \sqrt{cd}a^4bc^2d^2|d|\right)\arctan\left(\frac{2\,x^2}{\sqrt{\frac{2\,a^2bc^3+2\,a^3c^2d+\sqrt{-16\,a^5bc}}{a^2bc^2d}}}\right.}{a^2bc^3d\left|a^2bc^3 - a^3c^2d\right| + a^3c^2d^2\left|a^2bc^3 - a^3c^2d\right| + \left(a^2bc^3 - a^3c^2d\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -(sqrt(c*d)*a^2*b^3*c^3*d*x^4*abs(d) + sqrt(c*d)*a^3*b^2*c^2*d^2*x^4*abs(d)
 + sqrt(c*d)*a^2*b^3*c^4*abs(d) + sqrt(c*d)*a^3*b^2*c^3*d*abs(d) + sqrt(c*d
)*a^4*b*c^2*d^2*abs(d))*arctan(2*x^2/sqrt((2*a^2*b*c^3 + 2*a^3*c^2*d + sqrt
(-16*a^5*b*c^5*d + 4*(a^2*b*c^3 + a^3*c^2*d)^2))/(a^2*b*c^2*d)))/(a^2*b*c^3
*d*abs(a^2*b*c^3 - a^3*c^2*d) + a^3*c^2*d^2*abs(a^2*b*c^3 - a^3*c^2*d) + (a
^2*b*c^3 - a^3*c^2*d)^2*d) + (sqrt(a*b)*a^2*b^2*c^3*d^2*x^4*abs(b) + sqrt(a
*b)*a^3*b*c^2*d^3*x^4*abs(b) + sqrt(a*b)*a^2*b^2*c^4*d*abs(b) + sqrt(a*b)*a
^3*b*c^3*d^2*abs(b) + sqrt(a*b)*a^4*c^2*d^3*abs(b))*arctan(2*x^2/sqrt((2*a^
2*b*c^3 + 2*a^3*c^2*d - sqrt(-16*a^5*b*c^5*d + 4*(a^2*b*c^3 + a^3*c^2*d)^2)
)/(a^2*b*c^2*d)))/(a^2*b^2*c^3*abs(a^2*b*c^3 - a^3*c^2*d) + a^3*b*c^2*d*abs
(a^2*b*c^3 - a^3*c^2*d) - (a^2*b*c^3 - a^3*c^2*d)^2*b) + 1/6*(3*b*c*x^4 + 3
*a*d*x^4 - a*c)/(a^2*c^2*x^6)
```

3.779 $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=457

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{2\sqrt{2}b^{5/4}(bc-}$$

```
[Out] x/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rubi [A] time = 0.442611, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{2\sqrt{2}b^{5/4}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c
```

```
^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{bd}$$

$$= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^4}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^4}\,dx}{d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2b(bc-ad)} + \frac{a^{3/2}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2b(bc-ad)} - \frac{c^{3/2}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2d(bc-ad)} - \frac{c^{3/2}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)} - \frac{a^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{5/4}(bc-ad)} - \frac{a^{5/4}\int \frac{\sqrt{2}\ldots}{-\frac{\sqrt{a}}{\sqrt{b}}+\ldots}}{4\sqrt{2}b^{5/4}(\ldots}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\log\left(\sqrt{c}\ldots\right.}{4\sqrt{\ldots}}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\tan^{-1}\ldots}{2\sqrt{2}d^{5\ldots}}$$

Mathematica [A] time = 0.210264, size = 377, normalized size = 0.82

$$\frac{-\frac{\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{5/4}} + \frac{\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{5/4}} - \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{b^{5/4}} - \frac{8ax}{b} + \frac{\sqrt{2}c^{5/4}\,\mathrm{l}\ldots}{\ldots}}{8bc-8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/d^(5
/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)
```

Maple [A] time = 0.016, size = 328, normalized size = 0.7

$$\frac{x}{bd}+\frac{c\sqrt{2}}{8\,d\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{c\sqrt{2}}{4\,d\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] x/b/d+1/8/d*c/(a*d-b*c)*(1/d*c)^(1/4)*2^(1/2)*ln((x^2+(1/d*c)^(1/4)*x*2^(1/
2)+(1/d*c)^(1/2))/(x^2-(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))+1/4/d*c/(a*d
-b*c)*(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)+1/4/d*c/(a*d-
b*c)*(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)-1/8/b*a/(a*d-b
*c)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/
b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3188, size = 2708, normalized size = 5.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*
d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a
^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*x - (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5
*c*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 -
4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt((a^2*x^2 + (b^4*c^2 - 2*a*b^3*c*
d + a^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4
*a^3*b^6*c*d^3 + a^4*b^5*d^4)))/a^2))/a^4) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*b^3
*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*arctan((
(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4)*x
 - (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4
*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4
)*sqrt((c^2*x^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(-c^5/(b^4*c^4*
d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)))/c^2)
)/c^4) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d
^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*x + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a
^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (
-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b
^5*d^4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2
*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (-c^5/(b^4*
c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(
1/4)*b*d*log(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7
 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - (-c^5/(b^4*c^4*d^5 -
4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*l
og(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - 4*x)/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [C] time = 2.26359, size = 1670, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 -
4*a^3*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(b^13*c^8*d^5 - 8*a*b^12*c^7*d^6 + 28*
a^2*b^11*c^6*d^7 - 56*a^3*b^10*c^5*d^8 + 70*a^4*b^9*c^4*d^9 - 56*a^5*b^8*c^
3*d^10 + 28*a^6*b^7*c^2*d^11 - 8*a^7*b^6*c*d^12 + a^8*b^5*d^13))^(1/4)*log(
2*I*d*x + 2*(-c*d^3)^(1/4)) - 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^
8*c^8*d + 6*a^2*b^7*c^7*d^2 - 4*a^3*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(b^13*c^
8*d^5 - 8*a*b^12*c^7*d^6 + 28*a^2*b^11*c^6*d^7 - 56*a^3*b^10*c^5*d^8 + 70*a
^4*b^9*c^4*d^9 - 56*a^5*b^8*c^3*d^10 + 28*a^6*b^7*c^2*d^11 - 8*a^7*b^6*c*d^
12 + a^8*b^5*d^13))^(1/4)*log(-2*I*d*x + 2*(-c*d^3)^(1/4)) - 1/4*2^(1/4)*(1
/2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3*d^6 + 6*a^7*b^2*c^2*d^7 - 4*a^
8*b*c*d^8 + a^9*d^9)/(b^13*c^8*d^5 - 8*a*b^12*c^7*d^6 + 28*a^2*b^11*c^6*d^7
 - 56*a^3*b^10*c^5*d^8 + 70*a^4*b^9*c^4*d^9 - 56*a^5*b^8*c^3*d^10 + 28*a^6*
b^7*c^2*d^11 - 8*a^7*b^6*c*d^12 + a^8*b^5*d^13))^(1/4)*log(abs(2*b*x + 2*(-
a*b^3)^(1/4))) + 1/4*2^(1/4)*(1/2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3
*d^6 + 6*a^7*b^2*c^2*d^7 - 4*a^8*b*c*d^8 + a^9*d^9)/(b^13*c^8*d^5 - 8*a*b^1
2*c^7*d^6 + 28*a^2*b^11*c^6*d^7 - 56*a^3*b^10*c^5*d^8 + 70*a^4*b^9*c^4*d^9
- 56*a^5*b^8*c^3*d^10 + 28*a^6*b^7*c^2*d^11 - 8*a^7*b^6*c*d^12 + a^8*b^5*d^
13))^(1/4)*log(abs(-2*b*x + 2*(-a*b^3)^(1/4))) - 1/4*2^(1/4)*(1/2)^(1/4)*(-
(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 - 4*a^3*b^6*c^6*d^3 + a^4*b^5*
c^5*d^4)/(b^13*c^8*d^5 - 8*a*b^12*c^7*d^6 + 28*a^2*b^11*c^6*d^7 - 56*a^3*b^
10*c^5*d^8 + 70*a^4*b^9*c^4*d^9 - 56*a^5*b^8*c^3*d^10 + 28*a^6*b^7*c^2*d^11
 - 8*a^7*b^6*c*d^12 + a^8*b^5*d^13))^(1/4)*log(abs(2*d*x + 2*(-c*d^3)^(1/4)
)) + 1/4*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2
 - 4*a^3*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(b^13*c^8*d^5 - 8*a*b^12*c^7*d^6 +
```

```
28*a^2*b^11*c^6*d^7 - 56*a^3*b^10*c^5*d^8 + 70*a^4*b^9*c^4*d^9 - 56*a^5*b^8
*c^3*d^10 + 28*a^6*b^7*c^2*d^11 - 8*a^7*b^6*c*d^12 + a^8*b^5*d^13))^(1/4)*l
og(abs(-2*d*x + 2*(-c*d^3)^(1/4))) + x/(b*d)
```

3.780 $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{2}b^{3/4}(bc}$$

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/
4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rubi [A] time = 0.284764, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{2}b^{3/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/
4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
```

```
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{a\int \frac{x^2}{a+bx^4}\,dx}{bc-ad}+\frac{c\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{a\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{b}(bc-ad)}-\frac{a\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{b}(bc-ad)}-\frac{c\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{d}(bc-ad)}+\frac{c\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{d}(bc-ad)}$$

$$= -\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}}{4\sqrt{2}b^{3/4}(bc-ad}$$

$$= -\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{c^{3/4}\log\left(\sqrt{c}-\right.}{4\sqrt{2}d}$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan^{-1}\left(1\right.}{2\sqrt{2}d^{3/4}(bc}$$

Mathematica [A] time = 0.0919849, size = 340, normalized size = 0.76

$$\frac{-a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2a^{3/4}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(3
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
```

```
t[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

Maple [A] time = 0.006, size = 320, normalized size = 0.7

$$-\frac{c\sqrt{2}}{(8\,ad-8\,bc)\,d}\ln\left(\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{c\sqrt{2}}{(4\,ad-4\,bc)\,d}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8*c/(a*d-b*c)/d/(1/d*c)^(1/4)*2^(1/2)*ln((x^2-(1/d*c)^(1/4)*x*2^(1/2)+(1
/d*c)^(1/2))/(x^2+(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))-1/4*c/(a*d-b*c)/d
/(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)-1/4*c/(a*d-b*c)/d/
(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)+1/8*a/(a*d-b*c)/b/(
a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1
/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x+1)+1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71072, size = 2657, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4
*b^3*d^4))^(1/4)*arctan(((b^2*c - a*b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6
*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*x - (b^2*c - a*b*d
)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^
4*b^3*d^4))^(1/4)*sqrt((a*x^2 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-a
^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3
*d^4)))/a))/a) + (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 -
 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(((b*c*d - a*d^2)*(-c^3/(b^4*c^4*d^3
 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*x
- (b*c*d - a*d^2)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5
- 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*sqrt((c*x^2 - (b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a
^3*b*c*d^6 + a^4*d^7)))/c))/c) - 1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2
*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x + (b^5*c^3 -
 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^
3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*(-a^
3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*
d^4))^(1/4)*log(a^2*x - (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^
2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
 + a^4*b^3*d^4))^(3/4)) + 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*
b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x + (b^3*c^3*d^2 - 3*
a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d
^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/4*(-c^3/(b^4*
c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(
1/4)*log(c^2*x - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*
(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 +
a^4*d^7))^(3/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.781 $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\,\mathrm{lc}}{}$$

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/
4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rubi [A] time = 0.274268, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\,\mathrm{lc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)*(c + d*x^4)),x]
```


```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/
4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
```

```
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{a\int \frac{1}{a+bx^4}\,dx}{bc-ad}+\frac{c\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}-\frac{\sqrt{a}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{c}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}$$

$$= -\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}(bc-ad)}-\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}(bc-ad)}+\frac{\sqrt[4]{a}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}+\frac{\sqrt[4]{a}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2\ldots}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x\ldots}{\sqrt[4]{b}}}}{4\sqrt{2}\sqrt[4]{b}(bc-\ldots}$$

$$= \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\ldots\right.}{4\sqrt{2}\sqrt[4]{d}(\ldots}$$

$$= \frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\tan^{-1}\left(1+\ldots\right.}{2\sqrt{2}\sqrt[4]{d}(bc-\ldots}$$

Mathematica [A] time = 0.0933237, size = 340, normalized size = 0.76

$$\frac{\sqrt[4]{a}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\,\mathrm{t}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
```

```
d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

Maple [A] time = 0.006, size = 296, normalized size = 0.7

$$-\frac{\sqrt{2}}{8\,ad-8\,bc}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)-\frac{\sqrt{2}}{4\,ad-4\,bc}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{\sqrt{2}}{4\,ad-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8/(a*d-b*c)*(1/d*c)^(1/4)*2^(1/2)*ln((x^2+(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c
)^(1/2))/(x^2-(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))-1/4/(a*d-b*c)*(1/d*c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)-1/4/(a*d-b*c)*(1/d*c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)+1/8/(a*d-b*c)*(a/b)^(1/4)*2^(1/
2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2)))+1/4/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+
1)+1/4/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.3777, size = 2491, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4))^(1/4)*arctan(((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)
*x*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4
*b*d^4))^(3/4) - (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sq
rt(x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)))*(-a/(b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4))/a) - (-c/(
b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))
^(1/4)*arctan(((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(3/4) - (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(x^
2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)))*(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4))/c) - 1/4*(-a/(b
^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^
(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(b*c - a*d)*(
-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4))^(1/4) + x) + 1/4*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
- 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*
c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2
*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.782 $\int \frac{x^2}{(a+bx^4)(c+dx^4)} dx$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\,\mathrm{lo}}{}$$

```
[Out] -(b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c
- a*d)) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(1/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c
- a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rubi [A] time = 0.270049, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {482, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c
- a*d)) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(1/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c
- a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
```

```
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{b\int \frac{x^2}{a+bx^4}\,dx}{bc-ad}-\frac{d\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}\\
&= -\frac{\sqrt{b}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{b}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{d}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}-\frac{\sqrt{d}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}\\
&= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)}+\frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)}-\frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)}-\frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)}+\frac{\sqrt[4]{b}\int -\cdots}{4\sqrt{2}\cdots}\\
&= \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(\sqrt{c}-\sqrt{2}\cdots\right.}{4\sqrt{2}\sqrt[4]{c}(\cdots}\\
&= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\tan^{-1}\left(1+\cdots\right.}{2\sqrt{2}\sqrt[4]{c}(bc\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.117849, size = 340, normalized size = 0.76

$$
\frac{\sqrt[4]{b}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\,\mathrm{ta}\cdots}{\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*c^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*d^(1/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
```

```
rt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] + a^(1/4)*d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

Maple [A] time = 0.009, size = 296, normalized size = 0.7

$$\frac{\sqrt{2}}{8\,ad-8\,bc}\ln\left(\left(x^{2}-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^{2}+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}}{4\,ad-4\,bc}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{\sqrt{2}}{4\,ad-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8/(a*d-b*c)/(1/d*c)^(1/4)*2^(1/2)*ln((x^2-(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)
^(1/2))/(x^2+(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))+1/4/(a*d-b*c)/(1/d*c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)+1/4/(a*d-b*c)/(1/d*c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)-1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2
)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2)))-1/4/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)-1/4/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.46707, size = 2515, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*
d^4))^(1/4)*arctan((b*c - a*d)*x*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b
^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4) - (b*c - a*d)*sqrt((b*x^2 - (a
*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)))/b)*(-b/(a*b^4*c^4 - 4*a^2*b^3*
c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)) - (-d/(b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*
arctan((b*c - a*d)*x*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a
^3*b*c^2*d^3 + a^4*c*d^4))^(1/4) - (b*c - a*d)*sqrt((d*x^2 - (b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2)*sqrt(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2
- 4*a^3*b*c^2*d^3 + a^4*c*d^4)))/d)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^
2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)) + 1/4*(-b/(a*b^4*c^4 - 4*a
^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x
+ (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 -
4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/
4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^
5*d^4))^(1/4)*log(b*x - (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*
d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4))^(3/4)) - 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x + (b^3*c^4 - 3*a*b^2*c^3*d + 3*
a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2
 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d
+ 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x - (b^3*c^
4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4
*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.783 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}}{}\right.}{2\sqrt{2}a^{3/4}(bc-}$$

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rubi [A] time = 0.259183, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}}{}\right.}{2\sqrt{2}a^{3/4}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```


```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
```

```
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\ldots}}{4\sqrt{2}a^{3/4}(bc-\ldots}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\log\left(\sqrt{c}-\ldots\right.}{4\sqrt{2}c\ldots}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\tan^{-1}\left(\ldots\right.}{2\sqrt{2}c^{3/4}(\ldots}$$

Mathematica [A] time = 0.113015, size = 340, normalized size = 0.76

$$\frac{a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a^{3/4}d\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
```

```
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Maple [A] time = 0.007, size = 320, normalized size = 0.7

$$\frac{d\sqrt{2}}{(8\,ad-8\,bc)\,c}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{d\sqrt{2}}{(4\,ad-4\,bc)\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{}{(4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d/(a*d-b*c)*(1/d*c)^(1/4)/c*2^(1/2)*ln((x^2+(1/d*c)^(1/4)*x*2^(1/2)+(1/
d*c)^(1/2))/(x^2-(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))+1/4*d/(a*d-b*c)*(1
/d*c)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)+1/4*d/(a*d-b*c)*(1/
d*c)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)-1/8*b/(a*d-b*c)*(a/b
)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/
4)*x*2^(1/2)+(a/b)^(1/2)))-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03573, size = 2668, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4))^(3/4)*sqrt((b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4)))/b^2))/b^2) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5 - 3*a*b^
2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3*a
*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt((d^2*x^2 +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))/d^2))/d^2) + 1/4*(-b^
3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*
d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d
+ 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^
2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3
*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
+ (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^
3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 -
 a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^
3 + a^4*c^3*d^4))^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.784 $\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=460

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{2\sqrt{2}a^{5/4}(bc-\ldots}$$

```
[Out] -(1/(a*c*x)) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)) +
(b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rubi [A] time = 0.445054, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{2\sqrt{2}a^{5/4}(bc-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)) +
(b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt
```

```
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{1}{acx}+\frac{\int \frac{x^2\left(-bc-ad-bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{ac}$$

$$= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx^2}{(bc-ad)\left(a+bx^4\right)}-\frac{ad^2x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx}{ac}$$

$$= -\frac{1}{acx}-\frac{b^2\int \frac{x^2}{a+bx^4}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x^2}{c+dx^4}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{acx}+\frac{b^{3/2}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a(bc-ad)}-\frac{b^{3/2}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a(bc-ad)}-\frac{d^{3/2}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2c(bc-ad)}+\frac{d^{3/2}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2c(bc-ad)}$$

$$= -\frac{1}{acx}-\frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)}-\frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)}-\frac{b^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{5/4}(bc-ad)}-\cdots$$

$$= -\frac{1}{acx}-\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\cdots$$

$$= -\frac{1}{acx}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)}+\cdots$$

Mathematica [A] time = 0.209379, size = 385, normalized size = 0.84

$$\frac{\frac{\sqrt{2}b^{5/4}x\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{5/4}}-\frac{\sqrt{2}b^{5/4}x\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{5/4}}-\frac{2\sqrt{2}b^{5/4}x\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{2\sqrt{2}b^{5/4}x\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{5/4}}+\frac{8b}{a}-\frac{\sqrt{2}d^{5/4}}{}}{8adx-8bcx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c
^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(-8*b*c*x + 8*a*d*x)
```

Maple [A] time = 0.012, size = 331, normalized size = 0.7

$$-\frac{d\sqrt{2}}{8c(ad-bc)}\ln\left(\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{d\sqrt{2}}{4c(ad-bc)}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}-\frac{}{4c(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8*d/c/(a*d-b*c)/(1/d*c)^(1/4)*2^(1/2)*ln((x^2-(1/d*c)^(1/4)*x*2^(1/2)+(1
/d*c)^(1/2))/(x^2+(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))-1/4*d/c/(a*d-b*c)
/(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)-1/4*d/c/(a*d-b*c)/
(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)-1/a/c/x+1/8*b/a/(a*
d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+
(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62832, size = 2730, normalized size = 5.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(1/4)*a*c*x*arctan(((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d
+ 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*(a*b*c - a^2*d)*x - (
-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a
^9*d^4))^(1/4)*(a*b*c - a^2*d)*sqrt((b^3*x^2 - (a^3*b^2*c^2 - 2*a^4*b*c*d +
 a^5*d^2)*sqrt(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*
a^8*b*c*d^3 + a^9*d^4)))/b^3))/b) - 4*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^
2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*arctan(((-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
))^(1/4)*(b*c^2 - a*c*d)*x - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7
*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*(b*c^2 - a*c*d)*sqrt((d^3*x^2
- (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d
+ 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4)))/d^3))/d) + (-b^5/(a^
5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))
^(1/4)*a*c*x*log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a
^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2
*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x - (a^4*b^3*c^3 -
 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*
c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c
^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1
/4)*a*c*x*log(d^4*x + (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*
d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4))^(3/4)) + (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2
```

```
- 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x - (b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6
*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + 4)/(a*c*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [C] time = 4.77351, size = 2279, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*I*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 -
 4*a^3*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28
*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*
c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log
(8^(3/4)*a + 4*I*2^(1/4)*(-a^3*b)^(1/4)*x) + 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(b
^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 - 4*a^3*b^6*c^6*d^3 + a^4*b^5*c^
5*d^4)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*b^5*
c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d^6 -
 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log(8^(3/4)*a - 4*I*2^(1/4)*(-a^3*
b)^(1/4)*x) - 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3*
d^6 + 6*a^7*b^2*c^2*d^7 - 4*a^8*b*c*d^8 + a^9*d^9)/(a^5*b^8*c^13 - 8*a^6*b^
7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 -
 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^
8))^(1/4)*log(8^(3/4)*c + 4*I*2^(1/4)*(-c^3*d)^(1/4)*x) + 1/4*I*2^(1/4)*(1/
2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3*d^6 + 6*a^7*b^2*c^2*d^7 - 4*a^8
*b*c*d^8 + a^9*d^9)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2
- 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*
```

```
b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log(8^(3/4)*c - 4*I*2
^(1/4)*(-c^3*d)^(1/4)*x) - 1/4*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^8*c^8
*d + 6*a^2*b^7*c^7*d^2 - 4*a^3*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(a^5*b^8*c^13
 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^
4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 +
a^13*c^5*d^8))^(1/4)*log(abs(8^(3/4)*a + 4*2^(1/4)*(-a^3*b)^(1/4)*x)) + 1/4
*2^(1/4)*(1/2)^(1/4)*(-(b^9*c^9 - 4*a*b^8*c^8*d + 6*a^2*b^7*c^7*d^2 - 4*a^3
*b^6*c^6*d^3 + a^4*b^5*c^5*d^4)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28*a^7*b
^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*c^8*d^
5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log(abs(8
^(3/4)*a - 4*2^(1/4)*(-a^3*b)^(1/4)*x)) - 1/4*2^(1/4)*(1/2)^(1/4)*(-(a^5*b^
4*c^4*d^5 - 4*a^6*b^3*c^3*d^6 + 6*a^7*b^2*c^2*d^7 - 4*a^8*b*c*d^8 + a^9*d^9
)/(a^5*b^8*c^13 - 8*a^6*b^7*c^12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*
d^3 + 70*a^9*b^4*c^9*d^4 - 56*a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^
12*b*c^6*d^7 + a^13*c^5*d^8))^(1/4)*log(abs(8^(3/4)*c + 4*2^(1/4)*(-c^3*d)^
(1/4)*x)) + 1/4*2^(1/4)*(1/2)^(1/4)*(-(a^5*b^4*c^4*d^5 - 4*a^6*b^3*c^3*d^6
+ 6*a^7*b^2*c^2*d^7 - 4*a^8*b*c*d^8 + a^9*d^9)/(a^5*b^8*c^13 - 8*a^6*b^7*c^
12*d + 28*a^7*b^6*c^11*d^2 - 56*a^8*b^5*c^10*d^3 + 70*a^9*b^4*c^9*d^4 - 56*
a^10*b^3*c^8*d^5 + 28*a^11*b^2*c^7*d^6 - 8*a^12*b*c^6*d^7 + a^13*c^5*d^8))^
(1/4)*log(abs(8^(3/4)*c - 4*2^(1/4)*(-c^3*d)^(1/4)*x)) - 1/(a*c*x)
```

3.785 $\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=462

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{7/4}(bc-ac}$$

```
[Out] -1/(3*a*c*x^3) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/
c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d))
- (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rubi [A] time = 0.432014, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{7/4}(bc-ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/
c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d))
- (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] + Sq
```

```
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{1}{3acx^3}+\frac{\int \frac{-3(bc+ad)-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{3ac}$$

$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{1}{a+bx^4}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^4}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}-\frac{b^2\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}+\frac{d^2\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2c^{3/2}(bc-ad)}+\frac{d^2\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2c^{3/2}(bc-ad)}$$

$$= -\frac{1}{3acx^3}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}+\frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\ \cdots}{4\cdots}$$

$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}-\frac{d^{7/4}\ \cdots}{\cdots}$$

$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\ \cdots}{\cdots}$$

Mathematica [A] time = 0.225688, size = 406, normalized size = 0.88

$$\frac{-\frac{3\sqrt{2}b^{7/4}x^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}b^{7/4}x^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{6\sqrt{2}b^{7/4}x^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}b^{7/4}x^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8b}{a}\cdots}{24x^3(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[2]*d^(7/
4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(24
*(-(b*c) + a*d)*x^3)
```

Maple [A] time = 0.01, size = 343, normalized size = 0.7

$$-\frac{d^2\sqrt{2}}{8\,c^2\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)-\frac{d^2\sqrt{2}}{4\,c^2\,(ad-bc)}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8/c^2*d^2/(a*d-b*c)*(1/d*c)^(1/4)*2^(1/2)*ln((x^2+(1/d*c)^(1/4)*x*2^(1/2
)+(1/d*c)^(1/2))/(x^2-(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))-1/4/c^2*d^2/(
a*d-b*c)*(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)-1/4/c^2*d^
2/(a*d-b*c)*(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)+1/8/a^2
*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/3/a/c/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.4758, size = 2822, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/12*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*
b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*arctan(((a^5*b^3*c^3 - 3*a^6*b^2*c^2*d +
 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*x - (a^5*b^3*c^3 - 3*a^6*b^2*c^
2*d + 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9
*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt((b^4*x^2 + (a^4*b^2*c
^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^
9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4)))/b^4))/b^5) - 12*(-d^7/(b^4*c^1
1 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1
/4)*a*c*x^3*arctan(((b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^
3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 +
 a^4*c^7*d^4))^(3/4)*x - (b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c
^5*d^3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(3/4)*sqrt((d^4*x^2 + (b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*
d^2)*sqrt(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8
*d^3 + a^4*c^7*d^4)))/d^4))/d^5) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d +
 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x +
(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 +
 a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3
*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*
x - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d
^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*log(
d^2*x + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^3*
c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*
log(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*
c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 4)/(a*c*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [C] time = 5.52145, size = 2225, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*I*2^(1/4)*(1/2)^(1/4)*(-(b^11*c^11 - 4*a*b^10*c^10*d + 6*a^2*b^9*c^9*d
^2 - 4*a^3*b^8*c^8*d^3 + a^4*b^7*c^7*d^4)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d
+ 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^
12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(
1/4)*log(2*I*b*x + 2*(-a*b^3)^(1/4)) + 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(b^11*c^
11 - 4*a*b^10*c^10*d + 6*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^8*d^3 + a^4*b^7*c^7*
d^4)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c
^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6
 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(-2*I*b*x + 2*(-a*b^3)^(1/4))
 - 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(a^7*b^4*c^4*d^7 - 4*a^8*b^3*c^3*d^8 + 6*a^9
*b^2*c^2*d^9 - 4*a^10*b*c*d^10 + a^11*d^11)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*
d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*
a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))
^(1/4)*log(2*I*d*x + 2*(-c*d^3)^(1/4)) + 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(a^7*b
^4*c^4*d^7 - 4*a^8*b^3*c^3*d^8 + 6*a^9*b^2*c^2*d^9 - 4*a^10*b*c*d^10 + a^11
*d^11)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5
*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d
^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(-2*I*d*x + 2*(-c*d^3)^(1/4
)) + 1/4*2^(1/4)*(1/2)^(1/4)*(-(b^11*c^11 - 4*a*b^10*c^10*d + 6*a^2*b^9*c^9
*d^2 - 4*a^3*b^8*c^8*d^3 + a^4*b^7*c^7*d^4)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*
d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*
a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))
^(1/4)*log(abs(2*b*x + 2*(-a*b^3)^(1/4))) - 1/4*2^(1/4)*(1/2)^(1/4)*(-(b^11
*c^11 - 4*a*b^10*c^10*d + 6*a^2*b^9*c^9*d^2 - 4*a^3*b^8*c^8*d^3 + a^4*b^7*c
```

```
^7*d^4)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^
5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*
d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(abs(-2*b*x + 2*(-a*b^3)^(
1/4))) + 1/4*2^(1/4)*(1/2)^(1/4)*(-(a^7*b^4*c^4*d^7 - 4*a^8*b^3*c^3*d^8 + 6
*a^9*b^2*c^2*d^9 - 4*a^10*b*c*d^10 + a^11*d^11)/(a^7*b^8*c^15 - 8*a^8*b^7*c
^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^10*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 -
 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d
^8))^(1/4)*log(abs(2*d*x + 2*(-c*d^3)^(1/4))) - 1/4*2^(1/4)*(1/2)^(1/4)*(-(
a^7*b^4*c^4*d^7 - 4*a^8*b^3*c^3*d^8 + 6*a^9*b^2*c^2*d^9 - 4*a^10*b*c*d^10 +
 a^11*d^11)/(a^7*b^8*c^15 - 8*a^8*b^7*c^14*d + 28*a^9*b^6*c^13*d^2 - 56*a^1
0*b^5*c^12*d^3 + 70*a^11*b^4*c^11*d^4 - 56*a^12*b^3*c^10*d^5 + 28*a^13*b^2*
c^9*d^6 - 8*a^14*b*c^8*d^7 + a^15*c^7*d^8))^(1/4)*log(abs(-2*d*x + 2*(-c*d^
3)^(1/4))) - 1/3/(a*c*x^3)
```

3.786 $\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=479

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{9/4}(bc-}$$

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(a^2*c^2*x) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^(9
/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)
) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)) +
 (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(9/4)*(b*c - a*d))
```

Rubi [A] time = 0.596542, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{9/4}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(a^2*c^2*x) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^(9
/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)
) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)) +
```

```
(d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(9/4)*(b*c - a*d))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{1}{5acx^5} + \frac{\int \frac{-5(bc+ad)-5bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{5ac}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{\int \frac{x^2\left(-5\left(b^2c^2+abcd+a^2d^2\right)-5bd(bc+ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{5a^2c^2}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{\int \left(-\frac{5b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{5a^2d^3x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx}{5a^2c^2}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} + \frac{b^3\int \frac{x^2}{a+bx^4}\,dx}{a^2(bc-ad)} - \frac{d^3\int \frac{x^2}{c+dx^4}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{b^{5/2}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a^2(bc-ad)} + \frac{b^{5/2}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a^2(bc-ad)} + \frac{d^{5/2}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2c^2(bc-ad)} - \frac{d^{5/2}\int \frac{\sqrt{}}{}}{2c^2(b}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} + \frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^2(bc-ad)} + \frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^2(bc-ad)} + \frac{b^{9/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{9/4}(bc-ad)}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} + \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} - \frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}\right.}{4\sqrt{2}a^{9/4}(bc-ad)}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} + \frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} + \frac{d^{9/4}\tan^{-1}\left(1-\right.}{2\sqrt{2}c^{9/4}(bc-}$$

Mathematica [A] time = 0.253502, size = 428, normalized size = 0.89

$$\frac{-\frac{40b^2x^4}{a^2} - \frac{5\sqrt{2}b^{9/4}x^5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{9/4}} + \frac{5\sqrt{2}b^{9/4}x^5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{9/4}} + \frac{10\sqrt{2}b^{9/4}x^5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}} - \frac{10\sqrt{2}b^{9/4}x^5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{a^{9/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*d^(9
```

```
/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*b^(9/
4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4) + (5
*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)
```

Maple [A] time = 0.013, size = 365, normalized size = 0.8

$$\frac{d^2\sqrt{2}}{8\,c^2\,(ad-bc)}\ln\left(\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{d^2\sqrt{2}}{4\,c^2\,(ad-bc)}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)\frac{1}{\sqrt[4]{\frac{c}{d}}}+\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d^2/c^2/(a*d-b*c)/(1/d*c)^(1/4)*2^(1/2)*ln((x^2-(1/d*c)^(1/4)*x*2^(1/2)
+(1/d*c)^(1/2))/(x^2+(1/d*c)^(1/4)*x*2^(1/2)+(1/d*c)^(1/2)))+1/4*d^2/c^2/(a
*d-b*c)/(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x+1)+1/4*d^2/c^2
/(a*d-b*c)/(1/d*c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/d*c)^(1/4)*x-1)-1/8*b^2/
a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
)/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/5/a/c/x^5+1/a/c^2/x*d+1/a^2/c/x*
b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 36.764, size = 2942, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*arctan(((-b^9/(a^9*b^4*c^4 - 4*a^1
0*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b
*c - a^3*d)*x - (-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2
- 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b*c - a^3*d)*sqrt((b^5*x^2 - (a^5*
b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d
+ 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)))/b^5))/b^2) - 20*(-d^9/(
b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9
*d^4))^(1/4)*a^2*c^2*x^5*arctan(((-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b
^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(b*c^3 - a*c^2*d)*x -
(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 +
a^4*c^9*d^4))^(1/4)*(b*c^3 - a*c^2*d)*sqrt((d^5*x^2 - (b^2*c^7 - 2*a*b*c^6*
d + a^2*c^5*d^2)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2
- 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)))/d^5))/d^2) + 5*(-b^9/(a^9*b^4*c^4 - 4*a
^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*
c^2*x^5*log(b^7*x + (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d
^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c
*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11
*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x - (a
^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4
 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4
)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10
*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x + (b^3*c^10 - 3*a*b^2*c^9*
d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2
*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/(b^4*c^13
 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(
1/4)*a^2*c^2*x^5*log(d^7*x - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 -
a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*
b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 20*(b*c + a*d)*x^4 - 4*a*c)/(a^2*c^2*x^
5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [C] time = 3.52955, size = 1737, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*I*2^(1/4)*(1/2)^(1/4)*(-(b^13*c^13 - 4*a*b^12*c^12*d + 6*a^2*b^11*c^11
*d^2 - 4*a^3*b^10*c^10*d^3 + a^4*b^9*c^9*d^4)/(a^9*b^8*c^17 - 8*a^10*b^7*c^
16*d + 28*a^11*b^6*c^15*d^2 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 -
 56*a^14*b^3*c^12*d^5 + 28*a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9
*d^8))^(1/4)*log(8^(3/4)*a + 4*I*2^(1/4)*(-a^3*b)^(1/4)*x) + 1/4*I*2^(1/4)*
(1/2)^(1/4)*(-(b^13*c^13 - 4*a*b^12*c^12*d + 6*a^2*b^11*c^11*d^2 - 4*a^3*b^
10*c^10*d^3 + a^4*b^9*c^9*d^4)/(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28*a^11*
b^6*c^15*d^2 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*b^3*c^
12*d^5 + 28*a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1/4)*lo
g(8^(3/4)*a - 4*I*2^(1/4)*(-a^3*b)^(1/4)*x) - 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(
a^9*b^4*c^4*d^9 - 4*a^10*b^3*c^3*d^10 + 6*a^11*b^2*c^2*d^11 - 4*a^12*b*c*d^
12 + a^13*d^13)/(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28*a^11*b^6*c^15*d^2 -
56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*b^3*c^12*d^5 + 28*a^1
5*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1/4)*log(8^(3/4)*c + 4
*I*2^(1/4)*(-c^3*d)^(1/4)*x) + 1/4*I*2^(1/4)*(1/2)^(1/4)*(-(a^9*b^4*c^4*d^9
 - 4*a^10*b^3*c^3*d^10 + 6*a^11*b^2*c^2*d^11 - 4*a^12*b*c*d^12 + a^13*d^13)
/(a^9*b^8*c^17 - 8*a^10*b^7*c^16*d + 28*a^11*b^6*c^15*d^2 - 56*a^12*b^5*c^1
4*d^3 + 70*a^13*b^4*c^13*d^4 - 56*a^14*b^3*c^12*d^5 + 28*a^15*b^2*c^11*d^6
- 8*a^16*b*c^10*d^7 + a^17*c^9*d^8))^(1/4)*log(8^(3/4)*c - 4*I*2^(1/4)*(-c^
3*d)^(1/4)*x) - 1/4*2^(1/4)*(1/2)^(1/4)*(-(b^13*c^13 - 4*a*b^12*c^12*d + 6*
a^2*b^11*c^11*d^2 - 4*a^3*b^10*c^10*d^3 + a^4*b^9*c^9*d^4)/(a^9*b^8*c^17 -
8*a^10*b^7*c^16*d + 28*a^11*b^6*c^15*d^2 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b
^4*c^13*d^4 - 56*a^14*b^3*c^12*d^5 + 28*a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d
^7 + a^17*c^9*d^8))^(1/4)*log(abs(8^(3/4)*a + 4*2^(1/4)*(-a^3*b)^(1/4)*x))
+ 1/4*2^(1/4)*(1/2)^(1/4)*(-(b^13*c^13 - 4*a*b^12*c^12*d + 6*a^2*b^11*c^11*
d^2 - 4*a^3*b^10*c^10*d^3 + a^4*b^9*c^9*d^4)/(a^9*b^8*c^17 - 8*a^10*b^7*c^1
6*d + 28*a^11*b^6*c^15*d^2 - 56*a^12*b^5*c^14*d^3 + 70*a^13*b^4*c^13*d^4 -
56*a^14*b^3*c^12*d^5 + 28*a^15*b^2*c^11*d^6 - 8*a^16*b*c^10*d^7 + a^17*c^9*
```

```
d^8))^(1/4)*log(abs(8^(3/4)*a - 4*2^(1/4)*(-a^3*b)^(1/4)*x)) + 1/5*(5*b*c*x
^4 + 5*a*d*x^4 - a*c)/(a^2*c^2*x^5)
```

3.787 $\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=93

$$-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}$$

```
[Out] -(a*Sqrt[c + d*x^4])/(2*b^2) + (c + d*x^4)^(3/2)/(6*b*d) + (a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

Rubi [A] time = 0.0824442, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```



```
[Out] -(a*Sqrt[c + d*x^4])/(2*b^2) + (c + d*x^4)^(3/2)/(6*b*d) + (a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)\\
&= \frac{\left(c+dx^4\right)^{3/2}}{6bd} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)}{4b}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{\left(c+dx^4\right)^{3/2}}{6bd} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{\left(c+dx^4\right)^{3/2}}{6bd} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2b^2d}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{\left(c+dx^4\right)^{3/2}}{6bd} + \frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0650025, size = 88, normalized size = 0.95

$$\frac{\sqrt{c+dx^4}\left(b\left(c+dx^4\right)-3ad\right)}{6b^2d}+\frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-3*a*d + b*(c + d*x^4)))/(6*b^2*d) + (a*Sqrt[b*c - a*d]*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

Maple [B] time = 0.054, size = 1015, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d-1/4*a/b^2*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2
)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/4*a/b^3*d^(1/2)*(-a*b)^(1/2)*
ln((d*(-a*b)^(1/2)/b+(x^2-(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^
2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^3
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/
2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(
x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4*a/b^2/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)
/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^
2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c-1/4*a/b^2*((x
^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)
^(1/2)+1/4*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x^2+(-a*b)^(1/
2)/b)*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(
1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(
-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/
2))/(x^2+(-a*b)^(1/2)/b))*d+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a
*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)
)/(x^2+(-a*b)^(1/2)/b))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47033, size = 428, normalized size = 4.6

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\left(bdx^4+bc-3\,ad\right)\sqrt{dx^4+c}}{12\,b^2d}, \frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(\ldots}{6\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
+ c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(b*d*x^4 + b*c - 3*a*d)*sqrt(d
*x^4 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^4 + b*c - 3*a*d)*sqrt(d*x^4 +
 c))/(b^2*d)]
```

Sympy [A] time = 20.6882, size = 90, normalized size = 0.97

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^4\right)^{\frac{3}{2}}}{12b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**4)/(4*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**4)/sqrt((a*d - b*c)/b))/(4*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**4)**(3
/2)/(12*b))/d**2
```

Giac [A] time = 1.10728, size = 130, normalized size = 1.4

$$-\frac{\frac{3\left(abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2}-\frac{\left(dx^4+c\right)^{\frac{3}{2}}b^2-3\sqrt{dx^4+c}abd}{b^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/6*(3*(a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/
(sqrt(-b^2*c + a*b*d)*b^2) - ((d*x^4 + c)^(3/2)*b^2 - 3*sqrt(d*x^4 + c)*a*b
*d)/b^3)/d
```

3.788 $\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=120

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}+\frac{x^2\sqrt{c+dx^4}}{4b}$$

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a
*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2) + ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Rubi [A] time = 0.154238, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 478, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}+\frac{x^2\sqrt{c+dx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a
*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2) + ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\text{Subst}\left(\int \frac{ac+(-bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^2} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^2} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

Mathematica [A] time = 0.149976, size = 114, normalized size = 0.95

$$\frac{\frac{(bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}} - 2\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right) + bx^2\sqrt{c+dx^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*
x^2)/(Sqrt[a]*Sqrt[c + d*x^4])] + ((b*c - 2*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c
 + d*x^4]])/Sqrt[d])/(4*b^2)
```

Maple [B] time = 0.025, size = 1066, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/4*x^2*(d*x^4+c)^(1/2)/b+1/4/b*c/d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))-1
/4*a/b/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b
)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x^2-(
-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2
-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/
2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4*a/b/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)
+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(
-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c+1/4*a/b/(-a*b)^(
1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d
-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x^2+(-a*b)^(1/2)/b)
*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/
b)-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)
*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c
)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4*a/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)
/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)
-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35321, size = 1577, normalized size = 13.14

$$\left[\frac{2\,\sqrt{dx^4+c}bdx^2-(bc-2\,ad)\sqrt{d}\log\left(-2\,dx^4+2\,\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-abc+a^2d}d\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,\cdots}{\cdots}\right.}{8\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sq
rt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c
 - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2
*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - 2*(b*c - 2*a*d)
*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + sqrt(-a*b*c + a^2*d)*d*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a
^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*
d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 -
 2*sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))
 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))
/(b^2*d), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt
(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)
*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 +
(a*b*c^2 - a^2*c*d)*x^2)))/(b^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \sqrt{c + dx^4}}{a + bx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [A] time = 1.58648, size = 136, normalized size = 1.13

$$\frac{\sqrt{dx^4 + c}b^2x^2}{384\, d^3} + \frac{\sqrt{abc - a^2d}\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{2\, b^2} - \frac{\left(b^2c - 2\, abd\right)\arctan\left(\frac{\sqrt{d+\frac{c}{x^4}}}{\sqrt{-d}}\right)}{384\, \sqrt{-d}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(d*x^4 + c)*b^2*x^2/d^3 + 1/2*sqrt(a*b*c - a^2*d)*arctan(a*sqrt(d
 + c/x^4)/sqrt(a*b*c - a^2*d))/b^2 - 1/384*(b^2*c - 2*a*b*d)*arctan(sqrt(d
+ c/x^4)/sqrt(-d))/(sqrt(-d)*d^3)
```

3.789 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

```
[Out] Sqrt[c + d*x^4]/(2*b) - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[b*c - a*d]])/(2*b^(3/2))
```

Rubi [A] time = 0.0610729, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b) - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[b*c - a*d]])/(2*b^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^4\right)\\ &= \frac{\sqrt{c+dx^4}}{2b}+\frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b}\\ &= \frac{\sqrt{c+dx^4}}{2b}+\frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2bd}\\ &= \frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0315341, size = 69, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqr
t[b*c - a*d]])/b^(3/2))/2
```

Maple [B] time = 0.006, size = 988, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/4/b*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*
d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x^2-(-a*
b)^(1/2)/b)*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-
a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2
-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(
1/2))/(x^2-(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a
*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)
)/(x^2-(-a*b)^(1/2)/b))*c+1/4/b*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/
b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln((
-d*(-a*b)^(1/2)/b+(x^2+(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d
-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2
*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a
*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(
a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^
(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55901, size = 331, normalized size = 4.73

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}}{4\,b},-\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^4+c}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^4 + c))/b]
```

Sympy [A] time = 7.51617, size = 65, normalized size = 0.93

$$\frac{2\left(\frac{d\sqrt{c+dx^4}}{4b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d
 - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A] time = 1.1754, size = 89, normalized size = 1.27

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+cb}}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abdb}}+\frac{\sqrt{dx^4+c}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b
```

3.790 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(
2*Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(2*b)
```

Rubi [A] time = 0.0763402, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {465, 402, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```



```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(
2*Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(2*b)
```


Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx, x, x^2\right)\\
&= \frac{d\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b}\\
&= \frac{d\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\
&= \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.0423023, size = 89, normalized size = 0.98

$$\frac{\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{\sqrt{a}} + \sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] ((Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/
Sqrt[a] + Sqrt[d]*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/(2*b)
```

Maple [B] time = 0.006, size = 1000, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/4/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(
1/2)/b)-(a*d-b*c)/b)^(1/2)+1/4*d^(1/2)/b*ln((d*(-a*b)^(1/2)/b+(x^2-(-a*b)^(
1/2)/b)*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)
^(1/2)/b)-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2
)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*
c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*a*d-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/
b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-
(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c-1/4/(-a*b)^(1/2)*((x^2+(-a*b)^(
1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/4*
d^(1/2)/b*ln((-d*(-a*b)^(1/2)/b+(x^2+(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2+(-a*b
)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-
1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)
/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*
d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)
/b))*a*d+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b
)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)
^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b
)^(1/2)/b))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80825, size = 1310, normalized size = 14.4

$$\left[\frac{2\sqrt{d}\log\left(-2\,dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+\cdots}}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-(
b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, -1/8*(4*sqrt(-d
)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4
*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d
)*x^2 - c))/b, -1/4*(2*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt
((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((
b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [A] time = 1.19227, size = 157, normalized size = 1.73

$$-\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}b}-\frac{\sqrt{d}\log\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b) - 1
/4*sqrt(d)*log((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2)/b
```

3.791 $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

```
[Out] -(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a) + (Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b])
```

Rubi [A] time = 0.0746659, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a) + (Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx,x,x^4\right)\\ &= \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^4\right)}{4a} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}\\ &= \frac{c\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad}\\ &= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a} + \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0350757, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] (-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/Sqrt[b])/(2*a)
```

Maple [B] time = 0.016, size = 1037, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x/(b*x^4+a),x)
```

```
[Out] -1/4/a*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a
*d-b*c)/b)^(1/2)-1/4/b/a*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x^2-(-a
*b)^(1/2)/b)*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(
-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b
*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-
(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1
/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+
2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)
^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(
x^2-(-a*b)^(1/2)/b))*c-1/4/a*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(
x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/4/b/a*d^(1/2)*(-a*b)^(1/2)*ln((-d*
(-a*b)^(1/2)/b+(x^2+(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*
d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4/b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a
*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(
1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d+1/4/a/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c
)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b
)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c+1/2/a*(d*x^4+c)^(1/2)-1/2/a*c
^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}}{(bx^4 + a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)
```

Fricas [A] time = 1.65702, size = 851, normalized size = 10.01

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,a},\frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{c}\log\left(\right.}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*
sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 +
c)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^4 + a)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
4 + c)*sqrt(-c)/c))/a]
```

Sympy [A] time = 11.2061, size = 82, normalized size = 0.96

$$\frac{2\left(\frac{cd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}+\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x/(b*x**4+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/b)))/d
```

Giac [A] time = 1.14372, size = 117, normalized size = 1.38

$$-\frac{1}{2}d\left(\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{c\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d*((b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a*d) - c*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.792 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

```
[Out] -Sqrt[c + d*x^4]/(2*a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/
(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2))
```

Rubi [A] time = 0.0862588, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 475, 12, 377, 205}

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(2*a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/
(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-bc+ad}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0177218, size = 53, normalized size = 0.7

$$-\frac{\sqrt{c+dx^4}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{(ad-bc)x^4}{a\left(dx^4+c\right)}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c + d*x^4]*Hypergeometric2F1[-1/2, 1, 1/2, ((-(b*c) + a*d)*x^4)/(a*(
c + d*x^4))])/(2*a*x^2)
```

Maple [B] time = 0.016, size = 1075, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x)
```

```
[Out] -1/2/a/c/x^2*(d*x^4+c)^(3/2)+1/2/a*d/c*x^2*(d*x^4+c)^(1/2)+1/2/a*d^(1/2)*ln
(x^2*d^(1/2)+(d*x^4+c)^(1/2))-1/4*b/a/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*
d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/4/a*d^(1/2)*
ln((d*(-a*b)^(1/2)/b+(x^2-(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^
2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^
(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b
)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2
)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4*b/
a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(
x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-
a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
*c+1/4*b/a/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(
-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/4/a*d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x^2+
(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^
2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b
)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(
a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4*b/a/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a
*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(
1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}}{\left(bx^4 + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^3), x)
```

Fricas [A] time = 1.65851, size = 586, normalized size = 7.71

$$\left[\frac{x^2\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)-4\sqrt{dx^4+c}}{8\,ax^2}, -\frac{x^2\sqrt{\frac{bc-ad}{a}}\operatorname{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(x^2*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c
*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)) -
4*sqrt(d*x^4 + c))/(a*x^2), -1/4*(x^2*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + 2*sqrt(d*x^4 + c))/(a*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)
```

Giac [A] time = 1.13476, size = 89, normalized size = 1.17

$$\frac{(bc-ad)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{2\sqrt{abc-a^2d}a}-\frac{\sqrt{d+\frac{c}{x^4}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c - a*d)*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c -
 a^2*d)*a) - 1/2*sqrt(d + c/x^4)/a
```

3.793 $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$

Optimal. Leaf size=115

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

```
[Out] -Sqrt[c + d*x^4]/(4*a*x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]
])/(4*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^4])/Sqrt[b*c - a*d]])/(2*a^2)
```

Rubi [A] time = 0.123727, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*a*x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]
])/(4*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^4])/Sqrt[b*c - a*d]])/(2*a^2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
```

```
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{8a^2}\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x,\right.}{4a^2d}\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.120966, size = 107, normalized size = 0.93

$$\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}-2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)-\frac{a\sqrt{c+dx^4}}{x^4}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^4])/x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/S
qrt[b*c - a*d]])/(4*a^2)
```

Maple [B] time = 0.016, size = 1107, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x)
```

```
[Out] 1/4*b/a^2*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)
-(a*d-b*c)/b)^(1/2)+1/4/a^2*d^(1/2)*(-a*b)^(1/2)*ln((d*(-a*b)^(1/2)/b+(x^2-
(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^
2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*
d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x
^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)
^(1/2))/(x^2-(-a*b)^(1/2)/b))*d-1/4*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2
-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(
1/2))/(x^2-(-a*b)^(1/2)/b))*c+1/4*b/a^2*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b
)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/4/a^2*d^(1/2)*(-a*b)^(1
/2)*ln((-d*(-a*b)^(1/2)/b+(x^2+(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2+(-a*b)^(1/2
)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))+1/4/a/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2
)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x
^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4*b/a^2/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/
b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2
+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c-1/4/a/c/x^4*(d
```

```
*x^4+c)^(3/2)-1/4/a*d/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)+1/4/a
*d/c*(d*x^4+c)^(1/2)-1/2/a^2*b*(d*x^4+c)^(1/2)+1/2/a^2*b*c^(1/2)*ln((2*c+2*
c^(1/2)*(d*x^4+c)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)
```

Fricas [A] time = 1.68249, size = 1160, normalized size = 10.09

$$\left[\frac{2\sqrt{b^2c-abd}cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-(2\,bc-ad)\sqrt{c}x^4\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{dx^4+c}ac}{8\,a^2cx^4},\ \frac{4\,\sqrt{-b^2c+\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4
 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*
x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c
*x^4), 1/8*(4*sqrt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*x^4 - 2*sqrt(
d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), -1/4*(
(2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c -
a*b*d))/(b*x^4 + a)) + sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), 1/4*(2*sqrt(-b^2*c
 + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(d*
x^4 + c)*a*c)/(a^2*c*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^4}}{x^5 \left(a + bx^4\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)
```

Giac [A] time = 1.13654, size = 163, normalized size = 1.42

$$\frac{1}{4} d^2 \left(\frac{2 \left(b^2 c - abd\right) \arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c + abd}a^2 d^2} - \frac{(2\,bc - ad) \arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{a^2 \sqrt{-c} d^2} - \frac{\sqrt{dx^4 + c}}{ad^2 x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*d^2*(2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
sqrt(-b^2*c + a*b*d)*a^2*d^2) - (2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-
c))/(a^2*sqrt(-c)*d^2) - sqrt(d*x^4 + c)/(a*d^2*x^4))
```

3.794 $\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{c+dx^4}(3bc-ad)}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^4}}{6ax^6}$$

```
[Out] -Sqrt[c + d*x^4]/(6*a*x^6) + ((3*b*c - a*d)*Sqrt[c + d*x^4])/(6*a^2*c*x^2)
+ (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])
])/(2*a^(5/2))
```

Rubi [A] time = 0.15976, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 475, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}(3bc-ad)}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(6*a*x^6) + ((3*b*c - a*d)*Sqrt[c + d*x^4])/(6*a^2*c*x^2)
+ (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])
])/(2*a^(5/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
```

```
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{\text{Subst}\left(\int \frac{-3bc+ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} - \frac{\text{Subst}\left(\int -\frac{3bc(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a^2c}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}$$

Mathematica [C] time = 1.06978, size = 221, normalized size = 2.01

$$-\frac{\sqrt{c+dx^4}\left(\frac{dx^4}{c}+1\right)\left(\frac{4x^4\left(c+dx^4\right)(ad-bc)\,{}_2F_1\left(2,2;\frac{3}{2};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)}{c^2\left(a+bx^4\right)}+\frac{\left(c-2dx^4\right)\left(\sqrt{\frac{a\left(c+dx^4\right)}{c\left(a+bx^4\right)}}+\sqrt{\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}}\sin^{-1}\left(\sqrt{\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}}\right)\right)}{c\left(\frac{a\left(c+dx^4\right)}{c\left(a+bx^4\right)}\right)^{3/2}}\right)}{6x^6\left(a+bx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c + d*x^4]*(1 + (d*x^4)/c)*(((c - 2*d*x^4)*(Sqrt[(a*(c + d*x^4))/(c*
(a + b*x^4))] + Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]*ArcSin[Sqrt[((b*c -
 a*d)*x^4)/(c*(a + b*x^4))]]))/(c*((a*(c + d*x^4))/(c*(a + b*x^4)))^(3/2))
+ (4*(-(b*c) + a*d)*x^4*(c + d*x^4)*Hypergeometric2F1[2, 2, 3/2, ((b*c - a*
d)*x^4)/(c*(a + b*x^4))])/(c^2*(a + b*x^4))))/(6*x^6*(a + b*x^4))
```

Maple [B] time = 0.014, size = 1116, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x)
```

```
[Out] 1/2/a^2*b/c/x^2*(d*x^4+c)^(3/2)-1/2/a^2*b*d/c*x^2*(d*x^4+c)^(1/2)-1/2/a^2*b
*d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))+1/4*b^2/a^2/(-a*b)^(1/2)*((x^2-(-a
*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)
+1/4*b/a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+(x^2-(-a*b)^(1/2)/b)*d)/d^(1/2)+((x
^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)
^(1/2))+1/4*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-
a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)
/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-
a*b)^(1/2)/b))*d-1/4*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-
b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2
-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(
1/2))/(x^2-(-a*b)^(1/2)/b))*c-1/4*b^2/a^2/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b
)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/4*b/a^2*
d^(1/2)*ln((-d*(-a*b)^(1/2)/b+(x^2+(-a*b)^(1/2)/b)*d)/d^(1/2)+((x^2+(-a*b)^
(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/
4*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)
/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*
d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)
/b))*d+1/4*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d
*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1
/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2
+(-a*b)^(1/2)/b))*c-1/6/a*(d*x^4+c)^(3/2)/x^6/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}}{\left(bx^4 + a\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)
```

Fricas [A] time = 2.00939, size = 684, normalized size = 6.22

$$\left[\frac{3\,bcx^6\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)+4\left((3\,bc-ad)x^4-ac\right)\sqrt{}}{24\,a^2cx^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6
- a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a
^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(3*b*
c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
 c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*((
3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)
```

Giac [A] time = 1.21403, size = 131, normalized size = 1.19

$$-\frac{\frac{3\left(b^2c^2-abcd\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}\,a^2}-\frac{3\,abc\sqrt{d+\frac{c}{x^4}}-a^2\left(d+\frac{c}{x^4}\right)^{\frac{3}{2}}}{a^3}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/6*(3*(b^2*c^2 - a*b*c*d)*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/(
sqrt(a*b*c - a^2*d)*a^2) - (3*a*b*c*sqrt(d + c/x^4) - a^2*(d + c/x^4)^(3/2)
)/a^3)/c
```

$\mathbf{3.795} \qquad \int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=857

$$\frac{\sqrt{dx^4+c}\,x^3}{5b}+\frac{(2bc-5ad)\sqrt{dx^4+c}\,x}{5b^2\sqrt{d}\left(\sqrt{d}x^2+\sqrt{c}\right)}-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4b^2}-\frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4b^2}-\frac{\sqrt[4]{c}(2bc-5ad)\left(\cdots\right.}{}$$

```
[Out] (x^3*Sqrt[c + d*x^4])/(5*b) + ((2*b*c - 5*a*d)*x*Sqrt[c + d*x^4])/(5*b^2*Sq
rt[d]*(Sqrt[c] + Sqrt[d]*x^2)) - (a*Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]
*ArcTan[(Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]*x)/Sqrt[c + d*x^4]])/(4*b^
2) - (a*Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt
[-a]*Sqrt[b])]*x)/Sqrt[c + d*x^4]])/(4*b^2) - (c^(1/4)*(2*b*c - 5*a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(5*b^2*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1
/4)*(b^2*c^2 + a*b*c*d - 5*a^2*d^2)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(5*b^2*d^(3/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] - Sqrt[-
a]*Sqrt[d])*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[
-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/
2)*c^(1/4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] - a*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4])
- (a*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^
2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[c]*(Sqrt[b
] - (Sqrt[-a]*Sqrt[d])/Sqrt[c])^2)/(4*Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a
*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.85504, antiderivative size = 1067, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {478, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{\sqrt{dx^4+c}\,x^3}{5b}+\frac{(2bc-5ad)\sqrt{dx^4+c}\,x}{5b^2\sqrt{d}\left(\sqrt{d}x^2+\sqrt{c}\right)}+\frac{(-a)^{3/4}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{9/4}}+\frac{(-a)^{3/4}\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{9/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^3*Sqrt[c + d*x^4])/(5*b) + ((2*b*c - 5*a*d)*x*Sqrt[c + d*x^4])/(5*b^2*Sq
rt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b
*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(9/4)) + ((-a)^(3/
4)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^4])])/(4*b^(9/4)) - (c^(1/4)*(2*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(5*b^2*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(2*b*c - 5*a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(10*b^2*d^(3/4)*Sqrt[c + d*x^4]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
+ (a*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) +
(Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*
Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c
 + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
```

```
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \frac{x^2(3ac+(-2bc+5ad)x^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{5b}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{c+dx^4}} - \frac{5(-abc+a^2d)x^2}{b(a+bx^4)\sqrt{c+dx^4}}\right)dx}{5b}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{5b^2} - \frac{(a(bc-ad))\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{\left(\sqrt{c}(2bc-5ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{5b^2\sqrt{d}} - \frac{\left(\sqrt{c}(2bc-5ad)\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{5b^2\sqrt{d}} + \frac{(a(bc-ad))\int \frac{}{(\sqrt{-a}-\sqrt{b}}}{2b^{5/2}}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right.}{5b^2d^{3/4}\sqrt{c+dx^4}}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{(-a)^{3/4}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{9/4}} + \frac{(-a)^{3/4}\sqrt{-bc+ad}\,\mathrm{t}}{4b}$$

Mathematica [C] time = 0.103769, size = 141, normalized size = 0.16

$$\frac{x^7\sqrt{\frac{dx^4}{c}+1}(2bc-5ad)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) - 7acx^3\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + 7ax^3\left(c+dx^4\right)}{35ab\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (7*a*x^3*(c + d*x^4) - 7*a*c*x^3*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4, 1/2, 1,
7/4, -((d*x^4)/c), -((b*x^4)/a)] + (2*b*c - 5*a*d)*x^7*Sqrt[1 + (d*x^4)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(35*a*b*Sqrt[c + d
*x^4])
```

Maple [C] time = 0.061, size = 421, normalized size = 0.5

$$\frac{1}{b}\left(\frac{x^3}{5}\sqrt{dx^4+c}+\frac{2i}{5}c^{\frac{3}{2}}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/b*(1/5*x^3*(d*x^4+c)^(1/2)+2/5*I*c^(3/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c
^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d
^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(
1/2))^(1/2),I)))-a/b*(I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^
(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(E
llipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/
2),I))-1/8/b^2*sum((a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2
*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/
2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1
/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4+c}\,x^6}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6\sqrt{c + dx^4}}{a + bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**6*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}x^6}{bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

3.796 $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=700

$$\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(bc-2ad)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3b\sqrt[4]{d}\sqrt{c+dx^4}(ad+bc)}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{(bc-ad)\tan^{-1}\cdots}{4b^2\sqrt{\cdots}}$$

```
[Out] (x*Sqrt[c + d*x^4])/(3*b) - ((b*c - a*d)*ArcTan[(Sqrt[(Sqrt[-a]*((b*c)/a -
d))/Sqrt[b]]*x)/Sqrt[c + d*x^4]])/(4*b^2*Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[
b]))]) - ((b*c - a*d)*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt[
c + d*x^4]])/(4*b^2*Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]) + (c^(3/4)*(b*c -
 2*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(3*b*d^(1/4)*(b*c + a*d)*Sq
rt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(b*c - a*d)*(Sqrt[c]
 + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*(Sqrt[b]*Sqrt[c] - Sqrt[
-a]*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d
])*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*
x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.2479, antiderivative size = 904, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {478, 523, 220, 409, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt{-a}\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b) - ((-a)^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)) + ((-a)^(1/4)*Sq
```

```
rt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])]])/(4*b^(7/4)) + ((2*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(6*b^2*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*S
qrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b
^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[
-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sq
rt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^
2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
```

```
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x\sqrt{c+dx^4}}{3b} - \frac{\int \frac{ac+(-2bc+3ad)x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{3b}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2} - \frac{(a(bc-ad))\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{b^2}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{(bc-ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+\ldots}}}{2b^2}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\ldots\right.}{2b^{3/\ldots}}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} - \frac{\sqrt[4]{-a}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}} + \frac{\sqrt[4]{-a}\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}} + \frac{(2bc-\ldots}{\ldots}$$

Mathematica [C] time = 0.410914, size = 241, normalized size = 0.34

$$\frac{x\left(5\left(\frac{5a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}+c+dx^4\right)+\frac{x^4\sqrt{\frac{dx^4}{c}+1}(2bc-3ad)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4}\right.}{a}}{15b\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*(((2*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/a + 5*(c + d*x^4 + (5*a^2*c^2*AppellF1[1/4, 1/2,
1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2
, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2,
9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)
/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Maple [C] time = 0.019, size = 368, normalized size = 0.5

$$\frac{1}{b}\left(\frac{x}{3}\sqrt{dx^4+c}+\frac{2c}{3}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{1}{\sqrt{dx^4+c}}\right)-\frac{a}{b}\left(\frac{d}{b}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/b*(1/3*x*(d*x^4+c)^(1/2)+2/3*c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(
1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x
*(I/c^(1/2)*d^(1/2))^(1/2),I))-a/b*(d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1
/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Elli
pticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-
a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(
d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/
2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*
(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2
))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}x^4}{bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{c + dx^4}}{a + bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.797 $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=786

$$\frac{a\sqrt[4]{c}d^{5/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{b\sqrt{c+dx^4}(ad+bc)}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-a}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}\left(\sqrt{-a}\right.}$$

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(b*(Sqrt[c] + Sqrt[d]*x^2)) + (Sqrt[-((b*c - a*
d)/(Sqrt[-a]*Sqrt[b]))]*ArcTan[(Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]*x)/
Sqrt[c + d*x^4]])/(4*b) + (Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*ArcTan[(Sqr
t[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt[c + d*x^4]])/(4*b) - (c^(1/4)*d^(
1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^4]) + (a*c^(1/
4)*d^(5/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*(b*c + a*d)*Sqrt[c +
d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(b*c - a*d)*(Sqrt[c] + Sqrt
[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sq
rt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[
(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] -
a*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])
*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^
2)^2]*EllipticPi[-(Sqrt[c]*(Sqrt[b] - (Sqrt[-a]*Sqrt[d])/Sqrt[c])^2)/(4*Sqr
t[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(
1/4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.03788, antiderivative size = 1012, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {491, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc}}{\sqrt[4]{-a}\sqrt[4]{b}}\right.}{4\sqrt[4]{-a}b^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(b*(Sqrt[c] + Sqrt[d]*x^2)) + (Sqrt[b*c - a*d]*
ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(
1/4)*b^(5/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)) - (c^(1/4)*d^(1/4)*(Sq
rt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)
*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*Sqrt[c + d*x^4]) - ((Sqrt[c]
- (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*S
qrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c
^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqr
t[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(
4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]
*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
+ d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/
4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 491

```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Dist[d/b, Int[x^2/Sqrt[c + d*x^4], x], x] + Dist[(b*c - a*d)/b, Int[x^2/((a
 + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a
*d, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
```

```
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} + \frac{(bc-ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{b} - \frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} + \frac{(bc-ad)\int \frac{}{\left(\sqrt{-a}+\sqrt{b}\right.}}{2b^{3/2}}$$

$$= \frac{\sqrt{d}x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{c+dx^4}} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{}{(\sqrt{c}}}}{2b\sqrt{}}$$

$$= \frac{\sqrt{d}x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}b^{5/4}} + \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}b^{5/4}} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\right.}{}$$

Mathematica [C] time = 0.0307859, size = 65, normalized size = 0.08

$$\frac{x^3\sqrt{c+dx^4}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^3*Sqrt[c + d*x^4]*AppellF1[3/4, -1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)
])/(3*a*Sqrt[(c + d*x^4)/c])
```

Maple [C] time = 0.006, size = 299, normalized size = 0.4

$$\frac{i}{b}\sqrt{d}\sqrt{c}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{1}{\sqrt{dx^4+c}}-\frac{1}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/
2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/b^2*sum((a*
d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/(
(-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b
/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c
)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a
*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^
4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}x^2}{bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{c + dx^4}}{a + bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}x^2}{bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

3.798 $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=679

$$\frac{c^{3/4}d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{\sqrt{c+dx^4}(ad+bc)}+\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\cdots}}{\sqrt{c-\cdots}}\right)}{4ab\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$

```
[Out] ((b*c - a*d)*ArcTan[(Sqrt[(Sqrt[-a]*((b*c)/a - d))/Sqrt[b]]*x)/Sqrt[c + d*x
^4]])/(4*a*b*Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]) + ((b*c - a*d)*ArcTan
[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt[c + d*x^4]])/(4*a*b*Sqrt[(b*
c - a*d)/(Sqrt[-a]*Sqrt[b])]) + (c^(3/4)*d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sq
rt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^
(1/4)], 1/2])/((b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*
Sqrt[d])*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a
]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*b*c^(
1/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4]) + ((Sqrt
[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1
/4)], 1/2])/(8*a*b*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*Sqr
t[c + d*x^4])
```

Rubi [A] time = 0.9782, antiderivative size = 881, normalized size of antiderivative = 1.3, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {406, 220, 409, 1217, 1707}

$$\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8ab\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] -(Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d
*x^4])])/(4*(-a)^(3/4)*b^(3/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) +
```

```
 a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*b^(3/4)) + (d
^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*Sqrt[c + d*x^4]
) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4
)*x)/c^(1/4)], 1/2])/(4*Sqrt[-a]*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] -
Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + S
qrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{(-bc+ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}} - \frac{(-bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2ab} - \frac{(-bc+ad)\int}{}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+}}}{2a\sqrt{b}(bc+ad)}$$

$$= -\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} + \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}}{2b\sqrt[4]{c}\sqrt{c+}}$$

Mathematica [C] time = 0.148833, size = 161, normalized size = 0.24

$$\frac{5acx\sqrt{c+dx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{\left(a+bx^4\right)\left(2x^4\left(adF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-2bcF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[c + d*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4
)/a)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(-2*b*c*AppellF1[5/4, -1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
```

```
/a)] + a*d*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [C] time = 0.005, size = 273, normalized size = 0.4

$$\frac{d}{b}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{1}{\sqrt{dx^4+c}}-\frac{1}{8\,b^2}\sum_{\_alpha=RootOf(\_Z^4b+a)}\frac{ad-bc}{\_alpha^3}\left(-A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_al
pha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2)
)^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x
^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/
d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)))
,_alpha=RootOf(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.799 $\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=809

$$\frac{b\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)c^{5/4}}{a(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\sqrt[4]{c}}{a\sqrt{dx^4+c}}$$

```
[Out] -(Sqrt[c + d*x^4]/(a*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*(Sqrt[c] + Sqrt[d
]*x^2)) - (Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]*ArcTan[(Sqrt[-((b*c - a*
d)/(Sqrt[-a]*Sqrt[b]))]*x)/Sqrt[c + d*x^4]])/(4*a) - (Sqrt[(b*c - a*d)/(Sqr
t[-a]*Sqrt[b])]*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt[c + d*
x^4]])/(4*a) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*
Sqrt[c + d*x^4]) + (b*c^(5/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2
])/(a*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*
(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2
)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*
Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[b]*c^(1/4)*(
(-a)^(3/2)*Sqrt[b]*Sqrt[c] + a^2*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4]) - ((Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[c]*(Sqrt[b] - (Sqrt[
-a]*Sqrt[d])/Sqrt[c])^2)/(4*Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)
/c^(1/4)], 1/2])/(8*a*Sqrt[b]*c^(1/4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d]
)*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.50647, antiderivative size = 1031, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {475, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-a}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{}}\right)}{4(-a)^{5/4}\sqrt[4]{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*(Sqrt[c] + Sqrt[d
]*x^2)) + (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*S
qrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt
[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1
/4)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*Sqrt[c +
 d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*Sq
rt[c + d*x^4]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c +
 d*x^4]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sq
rt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/
4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = -\frac{\sqrt{c+dx^4}}{ax} + \frac{\int \frac{x^2\left(-bc+2ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+ad)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{a} + \frac{(-bc+ad)\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{a} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{a} + \frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a\sqrt{b}} - \frac{(bc}{}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{c+dx^4}} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{}\right.}{}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt[4]{b}} + \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt[4]{b}}$$

Mathematica [C] time = 0.0920126, size = 138, normalized size = 0.17

$$\frac{3bdx^8\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) - 7x^4\sqrt{\frac{dx^4}{c}+1}(bc-2ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) - 21a\left(c+dx^4\right)}{21a^2x\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

```
[Out] (-21*a*(c + d*x^4) - 7*(b*c - 2*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*Ap
pellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*x*Sqrt[c + d
*x^4])
```

Maple [C] time = 0.014, size = 421, normalized size = 0.5

$$\frac{1}{a}\left(-\frac{1}{x}\sqrt{dx^4+c}+2i\sqrt{c}\sqrt{d}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{d}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x)
```

```
[Out] 1/a*(-1/x*(d*x^4+c)^(1/2)+2*I*d^(1/2)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))
^(1/2),I)))-b/a*(I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(Ellipt
icF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
)-1/8/b^2*sum((a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alp
ha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))
^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^
2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d
^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),
_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**2/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**2*(a + b*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.800 $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$

Optimal. Leaf size=703

$$-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(2bc-ad)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3a\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{(bc-ad)\tan^{-1}}{4a^2\sqrt{\cdots}}$$

```
[Out] -Sqrt[c + d*x^4]/(3*a*x^3) - ((b*c - a*d)*ArcTan[(Sqrt[(Sqrt[-a]*((b*c)/a -
 d))/Sqrt[b]]*x)/Sqrt[c + d*x^4]])/(4*a^2*Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt
[b]))]) - ((b*c - a*d)*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt
[c + d*x^4]])/(4*a^2*Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]) - (d^(3/4)*(2*b*
c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(3*a*c^(1/4)*(b*c + a*d)*S
qrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(b*c - a*d)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(S
qrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(Sqrt[b]*Sqrt[c] - Sqrt
[-a]*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt
[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(
Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.19046, antiderivative size = 893, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {475, 523, 220, 409, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(3*a*x^3) - (b^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*
d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)) + (b^(1/4)*Sqrt
```

```
[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d
*x^4])]])/(4*(-a)^(7/4)) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
(6*a*c^(1/4)*Sqrt[c + d*x^4]) - (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d
]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c
+ a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*
c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(-a)^(3/2)*c^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]
*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*
Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c
+ a*d)*Sqrt[c + d*x^4])
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
```

```
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx = -\frac{\sqrt{c+dx^4}}{3ax^3}+\frac{\int \frac{-3bc+2ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3a}-\frac{(bc-ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{c+dx^4}}-\frac{(bc-ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2}-$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{c+dx^4}}-\frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc\right.}{2a^2(bc+}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{\sqrt[4]{b}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}+\frac{\sqrt[4]{b}\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}-\frac{d^{3/4}\left(\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.266061, size = 333, normalized size = 0.47

$$\frac{\frac{a\left(25ac\left(ac-adx^4+4bcx^4+bdx^8\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-10x^4\left(a+bx^4\right)\left(c+dx^4\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}{\left(a+bx^4\right)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}-bdx^8\sqrt{\frac{dx^4}{c}+1}}{15a^2x^3\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (a*(25*a*c*(a*c + 4*b*c*x^4 - a*d*x^4 + b*d*x^8)*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 10*x^4*(a + b*x^4)*(c + d*x^4)
*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*Appell
F1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(-5*a*c*Ap
pellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF
1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1,
 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*x^3*Sqrt[c + d*x^4])
```

Maple [C] time = 0.016, size = 370, normalized size = 0.5

$$-\frac{b}{a}\left(\frac{d}{b}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\text{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{1}{\sqrt{dx^4+c}}-\frac{1}{8\,b^2}\sum_{\_alpha=\text{RootOf}\left(\_Z^4b+a\right)}\frac{ad-bc}{\_alpha^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x)
```

```
[Out] -b/a*(d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^
(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1
/2),I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*
(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d
^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(
1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^
(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(
1/2))),_alpha=RootOf(_Z^4*b+a)))+1/a*(-1/3/x^3*(d*x^4+c)^(1/2)+2/3*d/(I/c^(
1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^
2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**4/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**4*(a + b*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.801 $\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=71

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] (2*(e*x)^(5/2)*Sqrt[c + d*x^4]*AppellF1[5/8, 1, -1/2, 13/8, -((b*x^4)/a), -
((d*x^4)/c)])/(5*a*e*Sqrt[1 + (d*x^4)/c])
```

Rubi [A] time = 0.11706, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 511, 510}

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(5/2)*Sqrt[c + d*x^4]*AppellF1[5/8, 1, -1/2, 13/8, -((b*x^4)/a), -
((d*x^4)/c)])/(5*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{x^4\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}\\ &= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}\\ &= \frac{2(ex)^{5/2}\sqrt{c+dx^4}F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}\end{aligned}$$

Mathematica [A] time = 0.0354186, size = 70, normalized size = 0.99

$$\frac{2x(ex)^{3/2}\sqrt{c+dx^4}F_1\left(\frac{5}{8};-\frac{1}{2},1;\frac{13}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*x*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[5/8, -1/2, 1, 13/8, -((d*x^4)/c),
 -((b*x^4)/a)])/(5*a*Sqrt[(c + d*x^4)/c])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{b x^{4}+a} (e x)^{\frac{3}{2}} \sqrt{d x^{4}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d x^{4}+c}\, (e x)^{\frac{3}{2}}}{b x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d x^{4}+c}\sqrt{e x} e x}{b x^{4}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*sqrt(e*x)*e*x/(b*x^4 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e x)^{\frac{3}{2}} \sqrt{c + d x^{4}}}{a + b x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral((e*x)**(3/2)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}\,(ex)^{\frac{3}{2}}}{bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

3.802 $\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=71

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] (2*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[3/8, 1, -1/2, 11/8, -((b*x^4)/a), -
((d*x^4)/c)])/(3*a*e*Sqrt[1 + (d*x^4)/c])
```

Rubi [A] time = 0.114422, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 511, 510}

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[3/8, 1, -1/2, 11/8, -((b*x^4)/a), -
((d*x^4)/c)])/(3*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{2\,\mathrm{Subst}\left(\int \frac{x^2\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}\\ &= \frac{\left(2\sqrt{c+dx^4}\right)\mathrm{Subst}\left(\int \frac{x^2\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}\\ &= \frac{2(ex)^{3/2}\sqrt{c+dx^4}F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}\end{aligned}$$

Mathematica [A] time = 0.0333433, size = 70, normalized size = 0.99

$$\frac{2x\sqrt{ex}\sqrt{c+dx^4}F_1\left(\frac{3}{8};-\frac{1}{2},1;\frac{11}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*x*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[3/8, -1/2, 1, 11/8, -((d*x^4)/c), -
((b*x^4)/a)])/(3*a*Sqrt[(c + d*x^4)/c])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{b x^4 + a}\sqrt{e x}\,\sqrt{d x^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d x^4 + c}\,\sqrt{e x}}{b x^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e x}\,\sqrt{c + d x^4}}{a + b x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(e*x)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

3.803 $\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{ex}\sqrt{c+dx^4}F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] (2*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[1/8, 1, -1/2, 9/8, -((b*x^4)/a), -((d
*x^4)/c)])/(a*e*Sqrt[1 + (d*x^4)/c])
```

Rubi [A] time = 0.0729746, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 430, 429}

$$\frac{2\sqrt{ex}\sqrt{c+dx^4}F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

```
[Out] (2*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[1/8, 1, -1/2, 9/8, -((b*x^4)/a), -((d
*x^4)/c)])/(a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\left(a+bx^4\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$

$$= \frac{2\sqrt{ex}\sqrt{c+dx^4}F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [A] time = 0.0311815, size = 68, normalized size = 0.99

$$\frac{2x\sqrt{c+dx^4}F_1\left(\frac{1}{8};-\frac{1}{2},1;\frac{9}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a\sqrt{ex}\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

```
[Out] (2*x*Sqrt[c + d*x^4]*AppellF1[1/8, -1/2, 1, 9/8, -((d*x^4)/c), -((b*x^4)/a)
])/(a*Sqrt[e*x]*Sqrt[(c + d*x^4)/c])
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{b x^4 + a}\sqrt{d x^4 + c}\,\frac{1}{\sqrt{e x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d x^4 + c}}{\left(b x^4 + a\right)\sqrt{e x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d x^4}}{\sqrt{e x}\left(a + b x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(sqrt(e*x)*(a + b*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}}{\left(bx^4 + a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

3.804 $\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=69

$$-\frac{2\sqrt{c+dx^4}F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] (-2*Sqrt[c + d*x^4]*AppellF1[-1/8, 1, -1/2, 7/8, -((b*x^4)/a), -((d*x^4)/c)
])/(a*e*Sqrt[e*x]*Sqrt[1 + (d*x^4)/c])
```

Rubi [A] time = 0.118679, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 511, 510}

$$-\frac{2\sqrt{c+dx^4}F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

```
[Out] (-2*Sqrt[c + d*x^4]*AppellF1[-1/8, 1, -1/2, 7/8, -((b*x^4)/a), -((d*x^4)/c)
])/(a*e*Sqrt[e*x]*Sqrt[1 + (d*x^4)/c])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\left(a+bx^4\right)}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e} \\
&= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}} \\
&= -\frac{2\sqrt{c+dx^4}F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}
\end{aligned}$$

Mathematica [B] time = 0.113718, size = 143, normalized size = 2.07

$$\frac{x\left(14bdx^8\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{15}{8};\frac{1}{2},1;\frac{23}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-10x^4\sqrt{\frac{dx^4}{c}+1}(bc-4ad)F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-70a\left(c+dx^4\right)\right)}{35a^2(ex)^{3/2}\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

```
[Out] (x*(-70*a*(c + d*x^4) - 10*(b*c - 4*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^4)/c), -((b*x^4)/a)] + 14*b*d*x^8*Sqrt[1 + (d*x^4)
/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^4)/c), -((b*x^4)/a)]))/(35*a^2*(e*x
)^(3/2)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{b x^4 + a}\sqrt{d x^4 + c}\,(e x)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d x^4 + c}}{\left(b x^4 + a\right)(e x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d x^4 + c}\sqrt{e x}}{b e^2 x^6 + a e^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*sqrt(e*x)/(b*e^2*x^6 + a*e^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^4}}{(ex)^{\frac{3}{2}} (a + bx^4)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(3/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/((e*x)**(3/2)*(a + b*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^4 + c}}{(bx^4 + a)(ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

3.805 $\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}(ad+bc)}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}$$

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^4])/(2*b^2*d^2) + (c + d*x^4)^(3/2)/(6*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*
c - a*d])
```

Rubi [A] time = 0.111793, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}(ad+bc)}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^4])/(2*b^2*d^2) + (c + d*x^4)^(3/2)/(6*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*
c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^4\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.181091, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^4}\left(-3ad-2bc+bdx^4\right)}{6b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

Maple [B] time = 0.033, size = 378, normalized size = 3.6

$$\frac{x^4}{6\,bd}\sqrt{dx^4+c}-\frac{c}{3\,bd^2}\sqrt{dx^4+c}-\frac{a}{2\,b^2d}\sqrt{dx^4+c}-\frac{a^2}{4\,b^3}\ln\left(\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/6/b*(d*x^4+c)^(1/2)/d*x^4-1/3/b*(d*x^4+c)^(1/2)/d^2*c-1/2/b^2*a/d*(d*x^4+
c)^(1/2)-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/
2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+
2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/
2)/b))-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)
/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*
d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)
/b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72938, size = 594, normalized size = 5.71

$$\left[\frac{3\,\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^4\right)\sqrt{dx^4+c}}{12\left(b^4cd^2-ab^3d^3\right)},\ \frac{3\,\sqrt{-b^2c}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*
x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a
^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*
d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.09915, size = 143, normalized size = 1.38

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^2}+\frac{\left(dx^4+c\right)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^4+c}b^2cd^4-3\sqrt{dx^4+c}abd^5}{6b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^4 + c)*b^2*c*d^4 - 3*s
qrt(d*x^4 + c)*a*b*d^5)/(b^3*d^6)
```

3.806 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{2bd}$$

```
[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0648049, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}\\
&= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}\\
&= \frac{\sqrt{c+dx^4}}{2bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.0630285, size = 72, normalized size = 0.97

$$\frac{1}{2}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/2
```

Maple [B] time = 0.009, size = 335, normalized size = 4.5

$$\frac{1}{2bd}\sqrt{dx^4+c}+\frac{a}{4b^2}\ln\left(\left(\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+2\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b/d+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2
*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^
(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x
^2-(-a*b)^(1/2)/b))+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(
-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2
)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(
-a*b)^(1/2)/b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57063, size = 437, normalized size = 5.91

$$\left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\sqrt{dx^4+c}\left(b^2c-abd\right)}{4\left(b^3cd-ab^2d^2\right)},-\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-\sqrt{}}{2\left(b^3cd-ab^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c
)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.09834, size = 86, normalized size = 1.16

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^4+c}}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^4 + c)/b)/d
```

3.807 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(2*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rubi [A] time = 0.0485695, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(2*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0157814, size = 51, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(2*Sqrt[b]*Sqrt[b*c - a
*d])
```

Maple [B] time = 0.006, size = 316, normalized size = 6.2

$$-\frac{1}{4b}\ln\left(\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*
b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/
2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/4/b/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)
/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^
2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50313, size = 288, normalized size = 5.65

$$\left[\frac{\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)}{4\,\sqrt{b^2c-abd}},\ \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)}{2\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b
*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] time = 11.8233, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.11472, size = 54, normalized size = 1.06

$$\frac{\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.808 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

```
[Out] -ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0731661, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}-\frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.0769461, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^4])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(2*a)
```

Maple [B] time = 0.013, size = 347, normalized size = 4.1

$$\frac{1}{4a}\ln\left(\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+2\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b
)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2
)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/4/a/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/
b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2
+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/2/a/c^(1/2)*ln
((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)
```

Fricas [A] time = 1.65892, size = 948, normalized size = 11.15

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,ac},\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+\right.}{4\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt
(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt
(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-
c)/c))/(a*c), 1/2*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^4 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A] time = 16.3892, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**4)/sqrt(-c))/(2*a*sqrt(-c))
```

Giac [A] time = 1.06789, size = 107, normalized size = 1.26

$$-\frac{1}{2}d\left(\frac{b\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d*(b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*a*d) - arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.809 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

```
[Out] -Sqrt[c + d*x^4]/(4*a*c*x^4) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[
c]])/(4*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c
- a*d]])/(2*a^2*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.119991, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*a*c*x^4) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[
c]])/(4*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c
- a*d]])/(2*a^2*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{8a^2c}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{\right.}{4a^2cd}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.115368, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2(ad-bc)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4ac^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*a*c*x^4) + (b*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a^2*
Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a*c^(3/2)) + (b^(3/2)*Sq
rt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a^2*(-
(b*c) + a*d))
```

Maple [B] time = 0.016, size = 402, normalized size = 3.4

$$-\frac{b}{4\,a^2}\ln\left(\left(\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-
(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)
^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/4
*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*
b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/
2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4*(d*
x^4+c)^(1/2)/a/c/x^4+1/4/a*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2
)+1/2/a^2*b/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+cx^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)
```

Fricas [A] time = 1.70135, size = 1261, normalized size = 10.78

$$\left[\frac{2\,bc^2x^4\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+(2\,bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{dx^4+c}ac}{8\,a^2c^2x^4}, -\frac{4\,bc^2x^4 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d
*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + (2*b*c + a*d)*sqr
t(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4
+ c)*a*c)/(a^2*c^2*x^4), -1/8*(4*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*sqrt
(d*x^4 + c)*a*c)/(a^2*c^2*x^4), 1/4*(b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d
*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^4 + a)) - (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) -
 sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4), -1/4*(2*b*c^2*x^4*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + sqrt(d*x^
4 + c)*a*c)/(a^2*c^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.09855, size = 159, normalized size = 1.36

$$\frac{1}{4}\,d^2\left(\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}cd^2}-\frac{\sqrt{dx^4+c}}{acd^2x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^2*(2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*a^2*d^2) - (2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqr
t(-c)*c*d^2) - sqrt(d*x^4 + c)/(a*c*d^2*x^4))
```

3.810 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt
[a]*Sqrt[c + d*x^4])])/(2*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))
```

Rubi [A] time = 0.149109, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt
[a]*Sqrt[c + d*x^4])])/(2*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
```

```
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} - \frac{\mathrm{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4bd}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2} - \frac{(bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^2d}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2} - \frac{(bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+d}}\right.}{4b^2d}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}$$

Mathematica [A] time = 0.129691, size = 118, normalized size = 0.96

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{d^{3/2}} + \frac{bx^2\sqrt{c+dx^4}}{d}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*x^2*Sqrt[c + d*x^4])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[
a]*Sqrt[c + d*x^4])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^2 + Sqrt[d]*
Sqrt[c + d*x^4]])/d^(3/2))/(4*b^2)
```

Maple [B] time = 0.027, size = 408, normalized size = 3.3

$$\frac{x^2}{4\,bd}\sqrt{dx^4+c} - \frac{c}{4\,b}\ln\left(x^2\sqrt{d}+\sqrt{dx^4+c}\right)d^{-\frac{3}{2}} - \frac{a}{2\,b^2}\ln\left(x^2\sqrt{d}+\sqrt{dx^4+c}\right)\frac{1}{\sqrt{d}} - \frac{a^2}{4\,b^2}\ln\left(\left(\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4*x^2*(d*x^4+c)^(1/2)/b/d-1/4/b*c/d^(3/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))
-1/2/b^2*a*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)-1/4*a^2/b^2/(-a*b)^(1/2)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/
2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(
x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/4*a^2/b^2/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2
+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b
)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [A] time = 2.4851, size = 1623, normalized size = 13.2

$$\left[\frac{2\sqrt{dx^4+c}bdx^2 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^6-\left(abc^2-a^2cd\right)x^2\right)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc}}}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^
4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/8*(2
*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*
```

```
sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-
d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2), 1/8*(2*sqrt(d*x^4 + c)*
b*d*x^2 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/4*(s
qrt(d*x^4 + c)*b*d*x^2 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*
d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b
*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.33182, size = 140, normalized size = 1.14

$$\frac{\sqrt{dx^4+cx^2}}{4\,bd}-\frac{a^2\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{2\sqrt{abc-a^2d}b^2}+\frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{d+\frac{c}{x^4}}}{\sqrt{-d}}\right)}{4\,b^2\sqrt{-d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*x^2/(b*d) - 1/2*a^2*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c
 - a^2*d))/(sqrt(a*b*c - a^2*d)*b^2) + 1/4*(b*c + 2*a*d)*arctan(sqrt(d + c/
x^4)/sqrt(-d))/(b^2*sqrt(-d)*d)
```

3.811 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b*Sqrt[d])
```

Rubi [A] time = 0.0874657, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b*Sqrt[d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}-\frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0571033, size = 90, normalized size = 0.99

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{\sqrt{bc-ad}}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/Sqrt[b
*c - a*d]) + Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]]/Sqrt[d])/(2*b)
```

Maple [B] time = 0.01, size = 356, normalized size = 3.9

$$\frac{1}{2b}\ln\left(x^2\sqrt{d}+\sqrt{dx^4+c}\right)\frac{1}{\sqrt{d}}+\frac{a}{4b}\ln\left(\left(-2\frac{ad-bc}{b}+2\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+2\frac{d\sqrt{}}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/b*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)+1/4*a/b/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(
-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b
)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/4*a/b/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)
/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^
2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [A] time = 2.22666, size = 1358, normalized size = 14.92

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^6-\left(abc^2-a^2cd\right)x^2\right)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2\,abx^4+a^2}\right)+2\,\sqrt{d}\log\left(-2\,dx\right.}{8\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^8 + 2*a*b*x^4 + a^2)) + 2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt
(d)*x^2 - c))/(b*d), 1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 -
3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(
-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*
x^2/sqrt(d*x^4 + c)))/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c -
 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2))
 + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b*d), 1/4*(d
*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - 2*sqrt(-d)*arctan(sqrt(-d)*x^2/s
qrt(d*x^4 + c)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.13233, size = 107, normalized size = 1.18

$$\frac{1}{2}c\left(\frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}\,bc}-\frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^4}}}{\sqrt{-d}}\right)}{bc\sqrt{-d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*(a*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*b*c) - arctan(sqrt(d + c/x^4)/sqrt(-d))/(b*c*sqrt(-d)))
```

3.812 $\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rubi [A] time = 0.0436755, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```


```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*c
 - a*d])
```


Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.0562158, size = 95, normalized size = 1.76

$$\frac{x^2\sqrt{\frac{dx^4}{c}+1}\tanh^{-1}\left(\frac{\sqrt{\frac{dx^4}{c}-\frac{bx^4}{a}}}{\sqrt{\frac{dx^4}{c}+1}}\right)}{2a\sqrt{c+dx^4}\sqrt{\frac{dx^4}{c}-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^4)/c]*ArcTanh[Sqrt[-((b*x^4)/a) + (d*x^4)/c]/Sqrt[1 + (d
*x^4)/c]])/(2*a*Sqrt[c + d*x^4]*Sqrt[-((b*x^4)/a) + (d*x^4)/c])
```

Maple [B] time = 0.006, size = 322, normalized size = 6.

$$-\frac{1}{4}\ln\left(\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/
b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d
*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/
b))+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-
2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/
2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^4 + a\right)\sqrt{dx^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [B] time = 2.05415, size = 518, normalized size = 9.59

$$\left[-\frac{\sqrt{-abc + a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{8\left(abc - a^2d\right)}, \frac{\arctan\left(\frac{\left((bc-2\,ad)x^4-ac\right)\sqrt{dx^4+ \cdots}}{2\left(\left(abcd-a^2d^2\right)x^6+\left(abc \cdots\right.\right.}\right.}{4\sqrt{abc - a^2d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt
(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))/(a*b*c - a^2
*d), 1/4*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c -
a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/sqrt(a*b*c - a^
2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + bx^4\right)\sqrt{c + dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.14069, size = 97, normalized size = 1.8

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd - a^2d^2}}\right)}{2\sqrt{abcd - a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.813 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

```
[Out] -Sqrt[c + d*x^4]/(2*a*c*x^2) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqr
t[c + d*x^4])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0880034, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(2*a*c*x^2) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqr
t[c + d*x^4])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2ac}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 0.685292, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^4}{c}+1\right)\left(\frac{4x^4\left(c+dx^4\right)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)}{3c^2\left(a+bx^4\right)}+\frac{\left(c+2dx^4\right)\sin^{-1}\left(\sqrt{\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}}\right)}{c\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}}}\right)}{2x^2\left(a+bx^4\right)\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((1 + (d*x^4)/c)*(((c + 2*d*x^4)*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x
^4))]])/(c*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)^2)]) + (4*
(b*c - a*d)*x^4*(c + d*x^4)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x^4)/
(c*(a + b*x^4))])/(3*c^2*(a + b*x^4))))/(2*x^2*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [B] time = 0.015, size = 350, normalized size = 4.4

$$-\frac{1}{2\,acx^2}\sqrt{dx^4+c}+\frac{b}{4\,a}\ln\left(\left(\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+2\,\frac{d\sqrt{-ab}}{b}\left(x^2-\frac{\sqrt{-ab}}{b}\right)\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/2*(d*x^4+c)^(1/2)/a/c/x^2+1/4*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/
2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b
*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/4*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)
/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)
-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)
```

Fricas [B] time = 2.25124, size = 698, normalized size = 8.72

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^2\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\,\sqrt{dx^4+c}\left(abc-a^2\right.}{8\,(a^2bc^2-a^3cd)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*b*c + a^2*d)*b*c*x^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*
x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4
*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -1/4*(sqrt(a
*b*c - a^2*d)*b*c*x^2*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) +
2*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.09911, size = 86, normalized size = 1.08

$$\frac{\frac{bc\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}a}-\frac{\sqrt{d+\frac{c}{x^4}}}{a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*c*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*a) - sqrt(d + c/x^4)/a)/c
```

3.814 $\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

```
[Out] -Sqrt[c + d*x^4]/(6*a*c*x^6) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(6*a^2*c^2
*x^2) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^
(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.163556, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(6*a*c*x^6) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(6*a^2*c^2
*x^2) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^
(5/2)*Sqrt[b*c - a*d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^7 \left(a+bx^4\right) \sqrt{c+dx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4 \left(a+bx^2\right) \sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a^2c^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

Mathematica [C] time = 1.58436, size = 253, normalized size = 2.2

$$-\frac{\left(\frac{dx^4}{c}+1\right)\left(-\frac{8x^4\left(c+dx^4\right)^2(bc-ad)\,\text{HypergeometricPFQ}\left(\{2,2,2\},\left\{1,\frac{5}{2}\right\},\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)}{a+bx^4} + \frac{3c\left(c^2-4cdx^4-8d^2x^8\right)\sin^{-1}\left(\sqrt{\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}}\right)}{\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}}} + \frac{24dx^8\left(c+dx^4\right)(ad-bc)\,{}_2\ldots}{a+bx\ldots}\right.}{18c^3x^6\left(a+bx^4\right)\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((1 + (d*x^4)/c)*((3*c*(c^2 - 4*c*d*x^4 - 8*d^2*x^8)*ArcSin[Sqrt[((b*c - a
*d)*x^4)/(c*(a + b*x^4))]])/Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a +
b*x^4)^2)] + (24*d*(-(b*c) + a*d)*x^8*(c + d*x^4)*Hypergeometric2F1[2, 2, 5
/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(a + b*x^4) - (8*(b*c - a*d)*x^4*(c
 + d*x^4)^2*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((b*c - a*d)*x^4)/(c*(a
+ b*x^4))])/(a + b*x^4)))/(18*c^3*x^6*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [B] time = 0.015, size = 383, normalized size = 3.3

$$\frac{b}{2\,a^{2}x^{2}c}\sqrt{dx^{4}+c}-\frac{b^{2}}{4\,a^{2}}\ln\left(\left(\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x^{2}-\sqrt{\right.}\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/a^2*b*(d*x^4+c)^(1/2)/x^2/c-1/4*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)
/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)
-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/4*b^2/a^2/(-a*b)^(1/2)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*
(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*
b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/6/a*(d*x^4+c)^(1/2)
*(-2*d*x^4+c)/x^6/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{4}+a\right)\sqrt{dx^{4}+c}x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)
```

Fricas [A] time = 2.3072, size = 856, normalized size = 7.44

$$\left[-\frac{3\,\sqrt{-abc+a^{2}d}b^{2}c^{2}x^{6}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{8}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{4}+a^{2}c^{2}-4\left((bc-2\,ad)x^{6}-acx^{2}\right)\sqrt{dx^{4}+c}\sqrt{-abc+a^{2}d}}{b^{2}x^{8}+2\,abx^{4}+a^{2}}\right)+4\left(a^{2}bc^{2}-a^{3}cd-\right.}{24\left(a^{3}bc^{3}-a^{4}c^{2}d\right)x^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6
 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^
2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*s
qrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^2*d)*
b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(a^2*b*c
^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^4 + c))/
((a^3*b*c^3 - a^4*c^2*d)*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7 \left(a + bx^4\right) \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [A] time = 1.10981, size = 144, normalized size = 1.25

$$-\frac{b^2 \arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{2\sqrt{abc-a^2d}a^2} + \frac{3\,abc^5\sqrt{d+\frac{c}{x^4}} - a^2c^4\left(d+\frac{c}{x^4}\right)^{\frac{3}{2}} + 3\,a^2c^4\sqrt{d+\frac{c}{x^4}}d}{6\,a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*a^2) + 1/6*(3*a*b*c^5*sqrt(d + c/x^4) - a^2*c^4*(d + c/x^4)^(3/2) + 3*a^2*
c^4*sqrt(d + c/x^4)*d)/(a^3*c^6)
```

3.815 $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=872

$$\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)a^2}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{}{(\sqrt{d}}}}{4b^2\sqrt[4]{c}(bc+ad}$$

```
[Out] (x*Sqrt[c + d*x^4])/(3*b*d) - ((-a)^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)*Sqrt[b*c - a*d]) - ((-a)^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(
7/4)*Sqrt[-(b*c) + a*d]) + (a^2*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (a*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
 ((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(6*b^2*c^(1/4)*d^(5
/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt
[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c
 + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.01903, antiderivative size = 872, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {479, 523, 220, 409, 1217, 1707}

$$\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)a^2}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}F}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b*d) - ((-a)^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)*Sqrt[b*c - a*d]) - ((-a)^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(
7/4)*Sqrt[-(b*c) + a*d]) + (a^2*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (a*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
 ((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(6*b^2*c^(1/4)*d^(5
/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt
[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c
 + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x\sqrt{c+dx^4}}{3bd} - \frac{\int \frac{ac+(bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3bd}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} + \frac{a^2\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2} - \frac{(bc+3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2d}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^4}} + \frac{a\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx}}}{2b^2}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^4}} + \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}-\right.\right.}{2}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{bc-ad}} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}} + \frac{a^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right.}{}$$

Mathematica [C] time = 0.330292, size = 249, normalized size = 0.29

$$\frac{x\left(5\left(\frac{5a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{d(a+bx^4)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}+\frac{c}{d}+x^4\right)-\frac{x^4\sqrt{\frac{dx^4}{c}+1}(3ad+bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\right.}{ad}\right.}{15b\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(-(((b*c + 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*d)) + 5*(c/d + x^4 + (5*a^2*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(d*(a + b*x^4)*(-5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1
/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Maple [C] time = 0.023, size = 363, normalized size = 0.4

$$\frac{1}{b^2}\left(b\left(\frac{x}{3d}\sqrt{dx^4+c}-\frac{c}{3d}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\text{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{1}{\sqrt{dx^4+c}}\right)-a\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/b^2*(b*(1/3/d*x*(d*x^4+c)^(1/2)-1/3/d*c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^
(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*El
lipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-a/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^
(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*El
lipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))+1/8*a^2/b^3*sum(1/_alpha^3*(-1/((-a
*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d
*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2
)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(
I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2)
)^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.816 $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=638

$$\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{d}\sqrt{c+dx^4}(ad+bc)}-\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\cdots}}{\cdots}$$

```
[Out] -ArcTan[(Sqrt[(Sqrt[-a]*((b*c)/a - d))/Sqrt[b]]*x)/Sqrt[c + d*x^4]]/(4*b*Sq
rt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]]) - ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]
*Sqrt[b])]*x)/Sqrt[c + d*x^4]]/(4*b*Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]) +
 (c^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*d^(1/4)*(b*c + a*d)*Sq
rt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c
] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d
^(1/4)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b
]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Arc
Tan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sq
rt[d])*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.681599, antiderivative size = 837, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {483, 220, 409, 1217, 1707}

$$-\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{-a}\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4]
)])/(4*b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)
```

```
/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(3/4)*Sqrt[-(b*c) + a*d]) + ((
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]
) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[-a]*S
qrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*S
qrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d])
, 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqr
t[c + d*x^4])
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
```

```
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b} - \frac{\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b} \cdots$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)} - \cdots$$

$$= -\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{-bc+ad}} + \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan \cdots\right.}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4} \cdots}$$

Mathematica [C] time = 0.0325162, size = 65, normalized size = 0.1

$$\frac{x^5\sqrt{\frac{c+dx^4}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)
/a)])/(5*a*Sqrt[c + d*x^4])
```

Maple [C] time = 0.007, size = 265, normalized size = 0.4

$$\frac{1}{b}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{1}{\sqrt{dx^4+c}}-\frac{a}{8\,b^2}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{1}{\_alpha^3}\Bigg(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
-1/8*a/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*
d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2
)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1
/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2
)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.817 $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=638

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}+\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)}{}$$

```
[Out] ArcTan[(Sqrt[(Sqrt[-a]*((b*c)/a - d))/Sqrt[b]]*x)/Sqrt[c + d*x^4]]/(4*a*Sqr
t[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]]) + ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*
Sqrt[b])]*x)/Sqrt[c + d*x^4]]/(4*a*Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]]) +
(d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqr
t[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(Sqrt[c] + Sqrt[d]*x^
2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c]
 - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^
(1/4)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(Sqrt[c] + S
qrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.601565, antiderivative size = 742, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {409, 1217, 220, 1707}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\right.}{4\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])
/(4*(-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((
```

```
-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*Sqrt[-(b*c) + a*d]) + ((
(Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/
4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*Sqrt[b]*Sqr
t[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1
/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*
(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c + a*d
)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
4])
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

```
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a}$$

$$= \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a(bc+ad)} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+}}}{2a(bc+ad)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{-bc+ad}} + \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{}}{4\sqrt[4]{c}(bc+ad)}$$

Mathematica [C] time = 0.0448179, size = 161, normalized size = 0.25

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\sqrt{c+dx^4}\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x
^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a
)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [C] time = 0.004, size = 191, normalized size = 0.3

$$\frac{1}{8b}\sum_{\_alpha=RootOf(\_Z^4b+a)}\frac{1}{\_alpha^3}\left(-\mathrm{Artanh}\left(\frac{2\_alpha^2dx^2+2c}{2}\frac{1}{\sqrt{\frac{-ad+bc}{b}}}\frac{1}{\sqrt{dx^4+c}}\right)\frac{1}{\sqrt{\frac{-ad+bc}{b}}}+2\frac{\_alpha^3b}{a\sqrt{dx^4+c}}\sqrt{1-\frac{i\sqrt{}}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.818 $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=677

$$-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(ad+4bc)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}(ad+bc)}-\frac{b\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{b\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\ldots$$

```
[Out] -Sqrt[c + d*x^4]/(3*a*c*x^3) - (b*ArcTan[(Sqrt[(Sqrt[-a]*((b*c)/a - d))/Sqr
t[b]]*x)/Sqrt[c + d*x^4]])/(4*a^2*Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))])
- (b*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt[c + d*x^4]])/(4*a
^2*Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]) - (d^(3/4)*(4*b*c + a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(6*a*c^(5/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
(b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sq
rt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(8*a^2*c^(1/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*Sqrt[c
 + d*x^4]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] +
 Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4
)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^
(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.991279, antiderivative size = 864, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {480, 523, 220, 409, 1217, 1707}

$$-\frac{b\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{7/4}\sqrt{bc-ad}}-\ldots$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(3*a*c*x^3) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1
/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4)*Ar
```

```
cTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^
(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(6*a*c^(5/4)*Sqrt[c + d*x^4]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])
*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*S
qrt[c + d*x^4]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[
-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c
^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d]
)^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
```

```
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{3acx^3}+\frac{\int \frac{-3bc-ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{3ac}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}-\frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3ac}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}}-\frac{b\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2}-$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}}-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.\right.}{2a^2(b}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{bc-ad}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{-bc+ad}}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)}{6}$$

Mathematica [C] time = 0.256109, size = 337, normalized size = 0.5

$$\frac{\frac{5a\left(2x^4(a+bx^4)(c+dx^4)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5ac(ac+2adx^4+4bcx^4+bdx^8)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}{(a+bx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}-bdx^8\sqrt{\frac{dx^4}{c}+1}\ldots}{15a^2cx^3\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (5*a*(-5*a*c*(a*c + 4*b*c*x^4 + 2*a*d*x^4 + b*d*x^8)*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(a + b*x^4)*(c + d*x
^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*App
ellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(5*a*c*
AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 2*x^4*(2*b*c*Appel
lF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2,
1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*c*x^3*Sqrt[c + d*x^4])
```

Maple [C] time = 0.015, size = 288, normalized size = 0.4

$$-\frac{1}{8a}\sum_{\_alpha=RootOf(\_Z^4b+a)}\frac{1}{\_alpha^3}\left(-\mathrm{Artanh}\left(\frac{2\_alpha^2dx^2+2c}{2}\frac{1}{\sqrt{\frac{-ad+bc}{b}}}\frac{1}{\sqrt{dx^4+c}}\right)\frac{1}{\sqrt{\frac{-ad+bc}{b}}}+2\frac{\_alpha^3b}{a\sqrt{dx^4+c}}\sqrt{1-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/a*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^
2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_a
lpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/
(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_a
lpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*b+a))+1/a*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3*d/c/(I/c^(1/2)*d^(1/2))
^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x
^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

3.819 $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=804

$$\frac{\sqrt{dx^4+c}x}{b\sqrt{d}\left(\sqrt{d}x^2+\sqrt{c}\right)}-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{dx^4+c}}\right)}{4b(bc-ad)}-\frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{dx^4+c}}\right)}{4b(bc-ad)}-\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\tan\right.}{bd^{3/4}\sqrt{dx^4+c}}$$

[Out] (x*Sqrt[c + d*x^4])/(b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^2)) - (a*Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]*ArcTan[(Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]*x)/Sqrt[c + d*x^4]])/(4*b*(b*c - a*d)) - (a*Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt[c + d*x^4]])/(4*b *(b*c - a*d)) - (c^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*d^(3/4) *Sqrt[c + d*x^4]) + (c^(1/4)*(b*c + 2*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)] , 1/2])/(2*b*d^(3/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] - S qrt[-a]*Sqrt[d])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqr t[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1 /4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] - a*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4]) - (a*(S qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4) /(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[c]*(Sqrt[b] - (Sqrt[-a]*Sqrt[d])/Sqrt[c])^2)/(4*Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)] , 1/2])/(8*b^(3/2)*c^(1/4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*S qrt[c + d*x^4])

Rubi [A] time = 0.962957, antiderivative size = 982, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {483, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{-a}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{5/4}\sqrt{bc-ad}}$$

Warning: Unable to verify antiderivative.

[In] Int[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]

```
[Out] (x*Sqrt[c + d*x^4])/(b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*ArcTa
n[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqr
t[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[c
] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (a*(Sqrt[c] + (Sqrt[-a]*Sq
rt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4
)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(1/
4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(1/
4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{a\int \frac{1}{(\sqrt{-a}-\sqrt{b}x^2)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} - \frac{a\int \frac{1}{(\sqrt{-a}+\sqrt{b}x^2)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} + \frac{\sqrt{c}\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b\sqrt{d}} - \frac{\sqrt{c}\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{b\sqrt{d}}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{c+dx^4}} + \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)}{}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{5/4}\sqrt{bc-ad}} - \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{5/4}\sqrt{-bc+ad}} - \frac{\sqrt[4]{c}\left(\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.0348157, size = 65, normalized size = 0.08

$$\frac{x^7\sqrt{\frac{c+dx^4}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{7a\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^7*Sqrt[(c + d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4
)/a)])/(7*a*Sqrt[c + d*x^4])
```

Maple [C] time = 0.018, size = 292, normalized size = 0.4

$$\frac{i}{b}\sqrt{c}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{1}{\sqrt{dx^4+c}}\frac{1}{\sqrt{d}} - \frac{}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] I/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/
c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8*a/b^2*sum(1
/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+
b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I
/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,x^6}{bdx^8+(bc+ad)x^4+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*x^6/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**6/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^4 + a)\sqrt{dx^4 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.820 $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=656

$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt{c+dx^4}(ad+bc)}+\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{x\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}+\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{x\sqrt{\frac{bc-}{\sqrt{-}}}}{\sqrt{c+d}}\right)}{4(bc-ad)}$$

```
[Out] (Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]*ArcTan[(Sqrt[-((b*c - a*d)/(Sqrt[-
a]*Sqrt[b]))]*x)/Sqrt[c + d*x^4]])/(4*(b*c - a*d)) + (Sqrt[(b*c - a*d)/(Sqr
t[-a]*Sqrt[b])]*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*x)/Sqrt[c + d*
x^4]])/(4*(b*c - a*d)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(2*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*
Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[b]*c^(1/4)*(Sqrt[-a]
*Sqrt[b]*Sqrt[c] - a*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c]
+ Sqrt[-a]*Sqrt[d])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticPi[-(Sqrt[c]*(Sqrt[b] - (Sqrt[-a]*Sqrt[d])/Sqrt[c])^2)
/(4*Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt
[b]*c^(1/4)*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.709563, antiderivative size = 756, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {490, 1217, 220, 1707}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}-\frac{\sqrt[4]{d}(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1
/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^
(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((Sqrt[
```

```
c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(S
qrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^4])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

```
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}} + \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx}}}{2(bc+ad)}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}} - \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}}{4\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] time = 0.032795, size = 65, normalized size = 0.1

$$\frac{x^3\sqrt{\frac{c+dx^4}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^3*Sqrt[(c + d*x^4)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)
/a)])/(3*a*Sqrt[c + d*x^4])
```

Maple [C] time = 0.007, size = 191, normalized size = 0.3

$$\frac{1}{8b}\sum_{\_alpha=RootOf\left(\_Z^4b+a\right)}\frac{1}{\_alpha}\left(-\mathrm{Artanh}\left(\frac{2\_alpha^2dx^2+2c}{2}\frac{1}{\sqrt{\frac{-ad+bc}{b}}}\frac{1}{\sqrt{dx^4+c}}\right)\frac{1}{\sqrt{\frac{-ad+bc}{b}}}+2\frac{\_alpha^3b}{a\sqrt{dx^4+c}}\sqrt{1-\frac{i\sqrt{d}x}{\sqrt{c}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2
*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alph
a^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*
x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alph
a^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootO
f(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.821 $\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=833

$$\frac{\sqrt{d}\sqrt{dx^4+cx}}{ac\left(\sqrt{d}x^2+\sqrt{c}\right)} - \frac{b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{dx^4+c}}\right)}{4a(bc-ad)} - \frac{b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{dx^4+c}}\right)}{4a(bc-ad)} - \frac{\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\tan\right.}{ac^{3/4}\sqrt{dx^4+c}}$$

```
[Out] -(Sqrt[c + d*x^4]/(a*c*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*c*(Sqrt[c] + Sq
rt[d]*x^2)) - (b*Sqrt[-((b*c - a*d)/(Sqrt[-a]*Sqrt[b]))]*ArcTan[(Sqrt[-((b*
c - a*d)/(Sqrt[-a]*Sqrt[b]))]*x)/Sqrt[c + d*x^4]])/(4*a*(b*c - a*d)) - (b*S
qrt[(b*c - a*d)/(Sqrt[-a]*Sqrt[b])]*ArcTan[(Sqrt[(b*c - a*d)/(Sqrt[-a]*Sqrt
[b])]*x)/Sqrt[c + d*x^4]])/(4*a*(b*c - a*d)) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4
)*x)/c^(1/4)], 1/2])/(a*c^(3/4)*Sqrt[c + d*x^4]) + (d^(1/4)*(2*b*c + a*d)*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*c^(3/4)*(b*c + a*d)*Sqrt[c + d*
x^4]) + (Sqrt[b]*((Sqrt[b]*c^(1/4))/d^(1/4) - (Sqrt[-a]*d^(1/4))/c^(1/4))*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*(Sqrt[-a]*Sqrt[b]*Sqrt[c] -
a*Sqrt[d])*Sqrt[c + d*x^4]) - (Sqrt[b]*((Sqrt[b]*c^(1/4))/d^(1/4) + (Sqrt[-
a]*d^(1/4))/c^(1/4))*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticPi[-(Sqrt[c]*(Sqrt[b] - (Sqrt[-a]*Sqrt[d])/Sqrt[c])^2
)/(4*Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*(
Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.30853, antiderivative size = 1007, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {480, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{b}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}} + \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{5/4}\sqrt{bc-ad}} - \ldots$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*c*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*c*(Sqrt[c] + Sq
rt[d]*x^2)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[
c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[-(b*c)
 + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*c^(3/4)*Sqrt
[c + d*x^4]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*c^(3/4
)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]
*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{acx}+\frac{\int \frac{x^2\left(-bc+ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}}-\frac{bcx^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}-\frac{b\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}+\frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a}-\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a}+\frac{\sqrt{d}\int \frac{1}{\sqrt{c+dx^4}}\,dx}{a\sqrt{c}}-$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}x^2\right)}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{ac^{3/4}\sqrt{c+dx^4}}+$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}x^2\right)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt{-bc+ad}}-$$

Mathematica [C] time = 0.100521, size = 141, normalized size = 0.17

$$\frac{3bdx^8\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+7x^4\sqrt{\frac{dx^4}{c}+1}(ad-bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-21a\left(c+dx^4\right)}{21a^2cx\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-21*a*(c + d*x^4) + 7*(-(b*c) + a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*A
ppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*c*x*Sqrt[c
+ d*x^4])
```

Maple [C] time = 0.015, size = 310, normalized size = 0.4

$$\frac{1}{a}\left(-\frac{1}{cx}\sqrt{dx^4+c}+i\sqrt{d}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\right)\frac{1}{\sqrt{c}}\frac{1}{\sqrt{i\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/a*(-1/c*(d*x^4+c)^(1/2)/x+I*d^(1/2)/c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))
^(1/2),I)))-1/8/a*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alp
ha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))
^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^
2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d
^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),
_alpha=RootOf(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + bx^4\right) \sqrt{c + dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.822 $\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=175

$$-\frac{\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{12b^3d^2(bc-ad)}-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}+\frac{ax^8\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}$$

```
[Out] (a*x^8*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[c + d*x^4]*(4
*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(12*b^3*d^2*(
b*c - a*d)) - (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.173387, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 147, 63, 208}

$$-\frac{\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{12b^3d^2(bc-ad)}-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}+\frac{ax^8\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^8*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[c + d*x^4]*(4
*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(12*b^3*d^2*(
b*c - a*d)) - (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{x^3}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{x\left(2ac+\frac{1}{2}(-2bc+5ad)x\right)}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)} + \frac{\left(a^2(6bc\right.}{}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)} + \frac{\left(a^2(6bc\right.}{}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)} - \frac{a^2(6bc-}{}$$

Mathematica [A] time = 0.190138, size = 175, normalized size = 1.

$$\frac{a^2(5ad-6bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}\left(2a^2bd\left(4c-5dx^4\right)-15a^3d^2+2ab^2\left(2c^2+3cdx^4+d^2x^8\right)+2b^3cx^4\left(2c-\right.\right.}{12b^3d^2\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*x^4*(2
*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(12*b^3*d^2*(b*c - a*
d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/S
qrt[b*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.043, size = 923, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/6/b^2*(d*x^4+c)^(1/2)/d*x^4-1/3/b^2*(d*x^4+c)^(1/2)/d^2*c-1/b^3*a/d*(d*x^
4+c)^(1/2)-3/4*a^2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(
1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*
d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(
1/2)/b))-3/4*a^2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/
2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-
2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/
2)/b))-1/8*a^2/b^4*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)
^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/
8*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1
/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d
-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1
/2)/b))+1/8*a^2/b^4*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b
)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1
/8*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(
1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*
d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(
1/2)/b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63743, size = 1270, normalized size = 7.26

$$\left[\frac{3\,(6\,a^3bcd^2-5\,a^4d^3+(6\,a^2b^2cd^2-5\,a^3bd^3)x^4)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,(2\,(b^5c^2d-2\,ab^4cd^2+a}{24\,(ab^6c^2d^2-2\,a^2b^5cd^3+a^3b^4d^4+(b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)*
sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2
*c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*
x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 - 2*(
2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqrt(d*x^4
+ c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b
^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b
^2*c*d^2 - 5*a^3*b*d^3)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b
^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d
^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqr
t(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2
 - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.02784, size = 243, normalized size = 1.39

$$\frac{\sqrt{dx^4+c}\,a^3d}{4\left(b^4c-ab^3d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}+\frac{\left(6\,a^2bc-5\,a^3d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^4c-ab^3d\right)\sqrt{-b^2c+abd}}+\frac{\left(dx^4+c\right)^{\frac{3}{2}}b^4d^4-3\sqrt{dx^4+c}\,b^4cd^4-6\cdots}{6\,b^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d)) +
 1/4*(6*a^2*b*c - 5*a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
(b^4*c - a*b^3*d)*sqrt(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^(3/2)*b^4*d^4 -
```

```
3*sqrt(d*x^4 + c)*b^4*c*d^4 - 6*sqrt(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6)
```

3.823 $\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^4}}{4b^2\left(a+bx^4\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}}{2b^2d}$$

```
[Out] Sqrt[c + d*x^4]/(2*b^2*d) - (a^2*Sqrt[c + d*x^4])/(4*b^2*(b*c - a*d)*(a + b
*x^4)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(4*b^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.146328, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^4}}{4b^2\left(a+bx^4\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b^2*d) - (a^2*Sqrt[c + d*x^4])/(4*b^2*(b*c - a*d)*(a + b
*x^4)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(4*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)$$

$$=-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}+\frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^4}}{2b^2d}-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}-\frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{8b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^4}}{2b^2d}-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}-\frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{4b^2d(bc-ad)}$$

$$=\frac{\sqrt{c+dx^4}}{2b^2d}-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.228681, size = 107, normalized size = 0.87

$$\frac{1}{4}\left(\frac{\sqrt{c+dx^4}\left(\frac{a^2}{\left(a+bx^4\right)(ad-bc)}+\frac{2}{d}\right)}{b^2}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((Sqrt[c + d*x^4]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^4))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/4
```

Maple [B] time = 0.014, size = 876, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b^2/d+1/2*a/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b
)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/
(x^2-(-a*b)^(1/2)/b))+1/2*a/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d
*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1
/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2
+(-a*b)^(1/2)/b))+1/8*a/b^3*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x
^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)
^(1/2)-1/8*a^2/b^3*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*
(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/
2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+
(-a*b)^(1/2)/b))-1/8*a/b^3*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^
2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^
(1/2)-1/8*a^2/b^3*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(
-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2
)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(
-a*b)^(1/2)/b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.58557, size = 983, normalized size = 7.99

$$\left[\frac{\left(4\,a^{2}bcd - 3\,a^{3}d^{2} + \left(4\,ab^{2}cd - 3\,a^{2}bd^{2}\right)x^{4}\right)\sqrt{b^{2}c - abd}\log\left(\frac{bdx^{4} + 2\,bc - ad + 2\,\sqrt{dx^{4} + c}\sqrt{b^{2}c - abd}}{bx^{4} + a}\right) + 2\left(2\,ab^{3}c^{2} - 5\,a^{2}b^{2}cd + 3\,a^{3}bd^{2}\right.}{8\left(ab^{5}c^{2}d - 2\,a^{2}b^{4}cd^{2} + a^{3}b^{3}d^{3} + \left(b^{6}c^{2}d - 2\,ab^{5}cd^{2} + a^{2}b^{4}d^{3}\right)x^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2*
c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*
d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4), -1
/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.55646, size = 181, normalized size = 1.47

$$-\frac{\sqrt{dx^4+c}a^2d}{4\left(b^3c-ab^2d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^4+c}}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d*x^4 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^4 + c)*b - b*c + a*d))
- 1/4*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/2*sqrt(d*x^4 + c)/(b^2*d)
```

3.824 $\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0867293, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0891622, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*b^(
3/2))
```

Maple [B] time = 0.012, size = 851, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-
a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(
1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/4/b
^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(
1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b
*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8/b^2*(-a
*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*
b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8*a/b^2*d/(a*d-b*c)/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/
b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2
+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8/b^2*(-a*b)^(
1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1
/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8*a/b^2*d/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*
(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*
b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56791, size = 722, normalized size = 7.29

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^4+2\,abc-a^2d\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)}{8\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^4\right)},\frac{\left(\left(2\,b^2c-abd\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) +
 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*b*d)
*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^4)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.32936, size = 157, normalized size = 1.59

$$\frac{\frac{\sqrt{dx^4+c}ad^2}{\left(b^2c-abd\right)\left(\left(dx^4+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.825 $\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^4]/(4*(b*c - a*d)*(a + b*x^4)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0735342, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*(b*c - a*d)*(a + b*x^4)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)\\ &= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{8(bc-ad)}\\ &= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{4(bc-ad)}\\ &= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0865039, size = 85, normalized size = 0.98

$$\frac{1}{4}\left(\frac{\sqrt{c+dx^4}}{\left(a+bx^4\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/((-(b*c) + a*d)*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```

Maple [B] time = 0.006, size = 541, normalized size = 6.2

$$\frac{1}{8\,ab\,(ad-bc)}\sqrt{-ab}\sqrt{\left(x^2+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x^2+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\left(x^2+\frac{1}{b}\sqrt{-ab}\right)^{-1}-\frac{d}{8\,b\,(ad-bc)}\ln\left(\left(-2\,\frac{ad}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2
*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)
^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)
/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*(-a*b
)^(1/2)/a/b/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-
a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)
+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(
-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57579, size = 639, normalized size = 7.34

$$\left[-\frac{\left(bdx^4+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(b^2c-abd\right)}{8\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4\right)},-\frac{\left(bdx^4+ad\right)\sqrt{-b^2c+abd}\arctan\left(\right.}{4\left(ab^3c^2-2\,a^2b^2cd+a^3bd\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*s
qrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^4)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.51874, size = 124, normalized size = 1.43

$$-\frac{1}{4}\,d\left(\frac{\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{\sqrt{dx^4+c}}{\left(\left(dx^4+c\right)b-bc+ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*(arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) + sqrt(d*x^4 + c)/(((d*x^4 + c)*b - b*c + a*d)*(b*c - a*d)))
```

3.826 $\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

```
[Out] (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]]/(2*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^2*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.141509, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```


```
[Out] (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]]/(2*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^2*(b*c - a*d)^(3/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\mathrm{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(b(2bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{8a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(b(2bc-3ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}}\right.}{4a^2d(bc-a}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.237257, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - (2*ArcTanh[Sqrt[c + d*x^
4]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*a^2)
```

Maple [B] time = 0.013, size = 880, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a
*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1
/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/4/a^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1
/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*
(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8/a^2*(-a*
b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b
)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8/a*d/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*
(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*
b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8/a^2*(-a*b)^(1/2)/
(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b
*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*
c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/
b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/2/a^2/c^(1/2)*ln((2*c+2*c^(1
/2)*(d*x^4+c)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x), x)
```

Fricas [A] time = 1.70359, size = 1831, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 -
3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
```

```
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^4 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4
 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqr
t(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^4 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a*b
*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c
 - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*
a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 +
 b*c)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.37324, size = 207, normalized size = 1.57

$$-\frac{1}{4}\,d^2\left(\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx^4+c}b}{\left(abcd-a^2d^2\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{2\,\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d^2*((2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d)
)/((a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x^4 + c)*b/((a*b*
```

```
c*d - a^2*d^2)*((d*x^4 + c)*b - b*c + a*d)) - 2*arctan(sqrt(d*x^4 + c)/sqrt
(-c))/(a^2*sqrt(-c)*d^2))
```

3.827 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b\sqrt{c+dx^4}(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(a + b*x^4)) - Sqrt
[c + d*x^4]/(4*a*c*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4
]/Sqrt[c]])/(4*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^3*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.250843, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b\sqrt{c+dx^4}(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(a + b*x^4)) - Sqrt
[c + d*x^4]/(4*a*c*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4
]/Sqrt[c]])/(4*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^3*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, \right.}{4a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x\right.}{8a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \right.}{4a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b^{3/2}(4bc-5}{4a}$$

Mathematica [A] time = 0.547311, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^4}\left(a^2d+ab\left(dx^4-c\right)-2b^2cx^4\right)}{x^4\left(a+bx^4\right)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[c + d*x^4]*(a^2*d - 2*b^2*c*x^4 + a*b*(-c + d*x^4)))/((b*c - a*d)*
x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(4*a^3*c)
```

Maple [B] time = 0.012, size = 938, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/2/a^3*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-
(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)
^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/2
/a^3*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*
b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/
2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8*b/a
^3*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*
d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8*b/a^2*d/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^
(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/
b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*b/a^3*
(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(
-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8*b/a^2*d/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/
2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(
x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/4/a^2/c/x^4
*(d*x^4+c)^(1/2)+1/4/a^2*d/c^(3/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)+
1/a^3*b/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

Fricas [A] time = 1.59956, size = 2504, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)
*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 +
2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a
^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*l
og((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arcta
n(sqrt(d*x^4 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d -
 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/4*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.66869, size = 362, normalized size = 1.96

$$\frac{1}{4}d^3\left(\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^4+c\right)^{\frac{3}{2}}b^2c-2\sqrt{dx^4+c}b^2c^2-\left(dx^4+c\right)^{\frac{3}{2}}abd+2\sqrt{dx^4+c}abcd-\sqrt{dx}}{\left(a^2bc^2d^2-a^3cd^3\right)\left(\left(dx^4+c\right)^2b-2\left(dx^4+c\right)bc+bc^2+\left(dx^4+c\right)ad-a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^3*((4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d
))/((a^3*b*c*d^3 - a^4*d^4)*sqrt(-b^2*c + a*b*d)) - (2*(d*x^4 + c)^(3/2)*b^
2*c - 2*sqrt(d*x^4 + c)*b^2*c^2 - (d*x^4 + c)^(3/2)*a*b*d + 2*sqrt(d*x^4 +
c)*a*b*c*d - sqrt(d*x^4 + c)*a^2*d^2)/((a^2*b*c^2*d^2 - a^3*c*d^3)*((d*x^4
+ c)^2*b - 2*(d*x^4 + c)*b*c + b*c^2 + (d*x^4 + c)*a*d - a*c*d)) - (4*b*c +
 a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^3*sqrt(-c)*c*d^3))
```

3.828 $\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=191

$$\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}+\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}$$

```
[Out] ((b*c - 2*a*d)*x^2*Sqrt[c + d*x^4])/(4*b^2*d*(b*c - a*d)) + (a*x^6*Sqrt[c +
 d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^3*(b*c - a*d)^(3/2)) -
 ((b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^3*d^(3/2))
```

Rubi [A] time = 0.365462, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {465, 470, 582, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}+\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*c - 2*a*d)*x^2*Sqrt[c + d*x^4])/(4*b^2*d*(b*c - a*d)) + (a*x^6*Sqrt[c +
 d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^3*(b*c - a*d)^(3/2)) -
 ((b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^3*d^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3ac-2(bc-2ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{-2ac(bc-2ad)-2(bc-ad)(bc+4ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x\right.}{8b^2d(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,\right.}{4b^3(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x\right.}{4b^3(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}} - \frac{(bc+}{}$$

Mathematica [A] time = 0.365008, size = 150, normalized size = 0.79

$$\frac{bx^2\sqrt{c+dx^4}\left(\frac{a^2}{\left(a+bx^4\right)(ad-bc)}+\frac{1}{d}\right)+\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{(bc-ad)^{3/2}}-\frac{(4ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{d^{3/2}}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4]*(d^(-1) + a^2/((-(b*c) + a*d)*(a + b*x^4))) + (a^(3/
2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])
/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/d
^(3/2))/(4*b^3)
```

Maple [B] time = 0.027, size = 953, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/b^2*x^2/d*(d*x^4+c)^(1/2)-1/4/b^2*c/d^(3/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1
/2))-1/b^3*a*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)-5/8*a^2/b^3/(-a*b)^(1/
2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(
1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b
*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+5/8*a^2/b^3
/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x
^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a
*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+
1/8*a^2/b^3/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-
a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8*a^2/b^4*d*(-a*b)^(
1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-
a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
+1/8*a^2/b^3/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(
-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8*a^2/b^4*d*(-a*b)^
(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*
(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(
-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{13}}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [A] time = 4.94912, size = 2853, normalized size = 14.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*
a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) +
 (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2
- a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqr
t(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), 1/
16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2
*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^
2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)
*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*((b
^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqrt(d*x^4 + c))/(
a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), -1/8*((5*a^2*b*c*
d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*ar
ctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*
d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b
^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)
*x^2 - c) - 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*s
qrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4),
1/8*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^
2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - (5*a^2*b*c*d^
2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*
x^6 + a*c*x^2)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*
x^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.08435, size = 238, normalized size = 1.25

$$-\frac{a^2c\sqrt{d+\frac{c}{x^4}}}{4\left(b^3c-ab^2d\right)\left(bc+a\left(d+\frac{c}{x^4}\right)-ad\right)}-\frac{\left(5\,a^2bc-4\,a^3d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{4\left(b^4c-ab^3d\right)\sqrt{abc-a^2d}}+\frac{\sqrt{dx^4+c}x^2}{4\,b^2d}+\frac{(bc+4\,ad)\arctan\left(\frac{\sqrt{d+\frac{c}{x^4}}}{\sqrt{-d}}\right)}{4\,b^3\sqrt{-d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*a^2*c*sqrt(d + c/x^4)/((b^3*c - a*b^2*d)*(b*c + a*(d + c/x^4) - a*d))
- 1/4*(5*a^2*b*c - 4*a^3*d)*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/(
(b^4*c - a*b^3*d)*sqrt(a*b*c - a^2*d)) + 1/4*sqrt(d*x^4 + c)*x^2/(b^2*d) +
1/4*(b*c + 4*a*d)*arctan(sqrt(d + c/x^4)/sqrt(-d))/(b^3*sqrt(-d)*d)
```

3.829 $\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{ax^2\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b^2*Sqrt[d])
```

Rubi [A] time = 0.162205, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{ax^2\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b^2*Sqrt[d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\right.}{4b^2(bc-ad)}$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x}\right.}{4b^2(bc-ad)}$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Mathematica [A] time = 0.197653, size = 135, normalized size = 0.96

$$\frac{\frac{abx^2\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)} + \frac{\sqrt{a}(2ad-3bc)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/Sqrt[d])/(4*b^2)
```

Maple [B] time = 0.011, size = 893, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/b^2*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))/d^(1/2)+3/8*a/b^2/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)
+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(
-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-3/8*a/b^2/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)
^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)
/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*a/b^2
/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/
b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1
/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*
(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*a/b^2/(a
*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(
x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8*a/b^3*d*(-a*b)^(1/2)/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)
/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^
2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [A] time = 2.74274, size = 2271, normalized size = 16.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d -
```

```
 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4
*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x
^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^
2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 -
 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(
d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a
/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c
^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x
^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*s
qrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2
*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
+ c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2*c - a*b*d)*x^4 + a*
b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(a*
b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*
a*b*d*x^2 - 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d
)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^
4*c*d - a*b^3*d^2)*x^4)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.16553, size = 205, normalized size = 1.45

$$\frac{1}{4}\,c^{2}\left(\frac{\left(3\,abc-2\,a^{2}d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^{4}}}}{\sqrt{abc-a^{2}d}}\right)}{\left(b^{3}c^{3}-ab^{2}c^{2}d\right)\sqrt{abc-a^{2}d}}+\frac{a\sqrt{d+\frac{c}{x^{4}}}}{\left(b^{2}c^{2}-abcd\right)\left(bc+a\left(d+\frac{c}{x^{4}}\right)-ad\right)}-\frac{2\arctan\left(\frac{\sqrt{d+\frac{c}{x^{4}}}}{\sqrt{-d}}\right)}{b^{2}c^{2}\sqrt{-d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*c^2*((3*a*b*c - 2*a^2*d)*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/
((b^3*c^3 - a*b^2*c^2*d)*sqrt(a*b*c - a^2*d)) + a*sqrt(d + c/x^4)/((b^2*c^2
 - a*b*c*d)*(b*c + a*(d + c/x^4) - a*d)) - 2*arctan(sqrt(d + c/x^4)/sqrt(-d
))/(b^2*c^2*sqrt(-d)))
```

3.830 $\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

```
[Out] -(x^2*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0915647, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(x^2*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\operatorname{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.445674, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^4}\left(-\frac{x^4(bc-ad)}{a+bx^4}-\frac{c\sqrt{x^4\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^4\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^4}{c}+1}}\right)}{\sqrt{\frac{dx^4}{c}+1}}\right)}{4x^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-(((b*c - a*d)*x^4)/(a + b*x^4)) - (c*Sqrt[(-(b/a) + d/c)
*x^4]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^4]/Sqrt[1 + (d*x^4)/c]])/Sqrt[1 + (d*x^
4)/c]))/(4*(b*c - a*d)^2*x^2)
```

Maple [B] time = 0.01, size = 861, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2
)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2
*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2
)/b))+1/8/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)
^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^
2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)
^(1/2)/b))+1/8/b/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2
*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8/b^2*d*(-a*b)^
(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*
(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(
-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)
)+1/8/b/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)
^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8/b^2*d*(-a*b)^(1/2)/(a*
d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*
b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/
2)/b*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [B] time = 1.58216, size = 883, normalized size = 9.49

$$\left[-\frac{4\sqrt{dx^4+c}\left(abc-a^2d\right)x^2-\left(bcx^4+ac\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4\cdots}}{b^2x^8+2\,abx^4+a^2}\cdots\right.}{16\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^4\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(
-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d +
 a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(d*x^4
 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*
((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a
^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d
^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.2533, size = 124, normalized size = 1.33

$$-\frac{1}{4}c\left(\frac{\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}(bc-ad)}+\frac{\sqrt{d+\frac{c}{x^4}}}{\left(bc+a\left(d+\frac{c}{x^4}\right)-ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*c*(arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*
(b*c - a*d)) + sqrt(d + c/x^4)/((b*c + a*(d + c/x^4) - a*d)*(b*c - a*d)))
```

3.831 $\int \frac{x}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

```
[Out] (b*x^2*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rubi [A] time = 0.0941179, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.772124, size = 407, normalized size = 3.91

$$\frac{x^2\sqrt{c+dx^4}\left(-30dx^4\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}}-45c\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}}+16dx^4\left(\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^4\right)}{c\left(a+bx^4\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+16c\cdots}{60c^2\left(a+bx^4\right)^2\left(\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)^{3/2}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4]*(-45*c*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a +
b*x^4)^2)] - 30*d*x^4*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 30*d*x^4*ArcS
in[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 16*c*(((b*c - a*d)*x^4)/(c*(a
 + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 16*d*x^4*(((b*c - a*d)*x^4)/
(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]))/(60*c^2*(((b*c - a*d)*x
^4)/(c*(a + b*x^4)))^(3/2)*(a + b*x^4)^2*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4
))])
```

Maple [B] time = 0.006, size = 867, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/a/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^
(1/2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8/b/a*d*(-a*b)^(1/2)/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b
)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2
)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8/a/(a
*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(
x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8/b/a*d*(-a*b)^(1/2)/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b
)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-
(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8/a/(-a*b)^(1/2
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1
/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*
(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*
b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/
2)/b*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [B] time = 1.74093, size = 967, normalized size = 9.3

$$\left[\frac{4\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)x^2-\left(\left(b^2c-2\,abd\right)x^4+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\,(}{b^2x^8+2\,abx^4+a^2}\right.}{16\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a
*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))
/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c - 2*a
*b*d)*x^4 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*
x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (
a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.56314, size = 320, normalized size = 3.08

$$-\frac{1}{4}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc-2\left(\sqrt{d}x^2-\sqrt{dx^4+\right.\right.}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*
d - b*c^2)/((((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

3.832 $\int \frac{1}{x^3(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{c+dx^4}(3bc-2ad)}{4a^2cx^2(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^4}}{4ax^2\left(a+bx^4\right)(bc-ad)}$$

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x^2) + (b*Sqrt[c +
d*x^4])/(4*a*(b*c - a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.198745, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^4}(3bc-2ad)}{4a^2cx^2(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^4}}{4ax^2\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x^2) + (b*Sqrt[c +
d*x^4])/(4*a*(b*c - a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2 \left(a+bx^2\right)^2 \sqrt{c+dx^2}}\,dx,x,x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\right.}{4a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{4a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

<u>Mathematica</u> [C] time = 1.56581, size = 869, normalized size = 5.83

$$-\frac{\sqrt{dx^4+c}\left(120d^2\sin^{-1}\left(\sqrt{\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}}\right)x^8 + 96d^2\left(\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)^{5/2}\sqrt{\frac{a\left(dx^4+c\right)}{c\left(bx^4+a\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)x^8 + 32d^2\left(\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)^{5/2}\sqrt{\frac{a(d}{c(b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[c + d*x^4]*(-45*c^2*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b
*x^4)^2)] - 180*c*d*x^4*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^
4)^2)] - 120*d^2*x^8*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)^
2)] + 45*c^2*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 180*c*d*x^4*
ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 120*d^2*x^8*ArcSin[Sqrt[(
(b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 64*c^2*(((b*c - a*d)*x^4)/(c*(a + b*x^
4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric2F1[2, 3, 7/
```

```
2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 160*c*d*x^4*(((b*c - a*d)*x^4)/(c*(
a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric2F1[
2, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 96*d^2*x^8*(((b*c - a*d)*x^
4)/(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeomet
ric2F1[2, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 32*c^2*(((b*c - a*d)
*x^4)/(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeo
metricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 64*c*d*
x^4*(((b*c - a*d)*x^4)/(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a +
b*x^4))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^4)/(c*(a + b
*x^4))] + 32*d^2*x^8*(((b*c - a*d)*x^4)/(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c +
 d*x^4))/(c*(a + b*x^4))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*
d)*x^4)/(c*(a + b*x^4))]))/(60*c^3*x^2*(((b*c - a*d)*x^4)/(c*(a + b*x^4)))^
(3/2)*(a + b*x^4)^2*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))])
```

Maple [B] time = 0.015, size = 885, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/2/a^2*(d*x^4+c)^(1/2)/x^2/c+3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)
^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a
*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*
d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1
/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8*b/a^2/(a*d-b*c)/(x^2+(
-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/
2)/b)-(a*d-b*c)/b)^(1/2)+1/8/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(a*d-b*c
)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b
)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8*b/a^2/(a*d-b*c)/(x^2-(-a*b)
^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)
-(a*d-b*c)/b)^(1/2)-1/8/a^2*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^
(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(a*
d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4 + a\right)^2 \sqrt{dx^4 + cx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

Fricas [B] time = 1.84819, size = 1256, normalized size = 8.43

$$\left[-\frac{\left(\left(3\,b^3c^2 - 4\,ab^2cd\right)x^6 + \left(3\,ab^2c^2 - 4\,a^2bcd\right)x^2\right)\sqrt{-abc + a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left((bc-2\,ad)x^6-acx^2\right.}{b^2x^8+2\,abx^4+a^2}\right.}{16\left(\left(a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bcd^2\right)x^6 + \left(a^4b^2c^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*c^2
- 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4
)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^
4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a*b^2*c*
d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*((
b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2
*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*
a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(d*x^4 + c))
/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*c^3 - 2*a^5*
b*c^2*d + a^6*c*d^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.60352, size = 181, normalized size = 1.21

$$-\frac{b^2c\sqrt{d+\frac{c}{x^4}}}{4\left(a^2bc-a^3d\right)\left(bc+a\left(d+\frac{c}{x^4}\right)-ad\right)}+\frac{\left(3\,b^2c-4\,abd\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{4\left(a^2bc-a^3d\right)\sqrt{abc-a^2d}}-\frac{\sqrt{d+\frac{c}{x^4}}}{2\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*c*sqrt(d + c/x^4)/((a^2*b*c - a^3*d)*(b*c + a*(d + c/x^4) - a*d))
+ 1/4*(3*b^2*c - 4*a*b*d)*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/((a
^2*b*c - a^3*d)*sqrt(a*b*c - a^2*d)) - 1/2*sqrt(d + c/x^4)/(a^2*c)
```

3.833 $\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=208

$$\frac{\sqrt{c+dx^4}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{12a^3c^2x^2(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6\left(a+bx^4\right)(bc-ad)}$$

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^6) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^4])/(12*a^3*c^2*(b*c - a*d)*x^2) +
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rubi [A] time = 0.328009, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{12a^3c^2x^2(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^6) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^4])/(12*a^3*c^2*(b*c - a*d)*x^2) +
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc-2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\right.}{12a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} -$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} -$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} -$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} -$$

Mathematica [A] time = 5.63878, size = 175, normalized size = 0.84

$$\frac{a^2\left(c+dx^4\right)\left(\frac{3b^3x^8}{\left(a+bx^4\right)(bc-ad)}+\frac{4x^4(ad+3bc)}{c^2}-\frac{2a}{c}\right)+\frac{3b^2x^{12}\sqrt{\frac{dx^4}{c}+1}(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{c\left(\frac{x^4(bc-ad)}{ac}\right)^{3/2}}}{12a^5x^6\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a^2*(c + d*x^4)*((-2*a)/c + (4*(3*b*c + a*d)*x^4)/c^2 + (3*b^3*x^8)/((b*c
- a*d)*(a + b*x^4))) + (3*b^2*(5*b*c - 6*a*d)*x^12*Sqrt[1 + (d*x^4)/c]*ArcS
in[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(c*(((b*c - a*d)*x^4)/(a*c))
```

```
^(3/2)))/(12*a^5*x^6*Sqrt[c + d*x^4])
```

Maple [B] time = 0.014, size = 923, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/a^3*b*(d*x^4+c)^(1/2)/x^2/c-5/8/a^3*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b
)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)-(
a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+5/8/a^3*b^2/(-a*b)^(1/2)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-
(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)
^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*b^2/a^3/(a*d-b*c)/(
x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b
)^(1/2)/b)-(a*d-b*c)/b)^(1/2)-1/8*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^(1/2)/b)+2*(-(
a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)^(1/2)/b*(x^2+(-a*b)^
(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*b^2/a^3/(a*d-b*c)/(x
^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)
^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/8*b/a^3*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(1/2)/b)+2*(-(a
*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x^2-(-a*b)^(
1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/6/a^2*(d*x^4+c)^(1/2)*(
-2*d*x^4+c)/x^6/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)
```

Fricas [A] time = 2.65047, size = 1547, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sq
rt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((15*a
*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a^3*b^
2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d +
a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^
3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^6
), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^
4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x
^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)
*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a
^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4
 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a
^7*c^2*d^2)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.57454, size = 244, normalized size = 1.17

$$\frac{b^3c\sqrt{d+\frac{c}{x^4}}}{4\left(a^3bc-a^4d\right)\left(bc+a\left(d+\frac{c}{x^4}\right)-ad\right)}-\frac{\left(5\,b^3c-6\,ab^2d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^4}}}{\sqrt{abc-a^2d}}\right)}{4\left(a^3bc-a^4d\right)\sqrt{abc-a^2d}}+\frac{6\,a^3bc^5\sqrt{d+\frac{c}{x^4}}-a^4c^4\left(d+\frac{c}{x^4}\right)^{\frac{3}{2}}+3\,a^4c^4\sqrt{\cdots}}{6\,a^6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^3*c*sqrt(d + c/x^4)/((a^3*b*c - a^4*d)*(b*c + a*(d + c/x^4) - a*d)) -
 1/4*(5*b^3*c - 6*a*b^2*d)*arctan(a*sqrt(d + c/x^4)/sqrt(a*b*c - a^2*d))/((
a^3*b*c - a^4*d)*sqrt(a*b*c - a^2*d)) + 1/6*(6*a^3*b*c^5*sqrt(d + c/x^4) -
a^4*c^4*(d + c/x^4)^(3/2) + 3*a^4*c^4*sqrt(d + c/x^4)*d)/(a^6*c^6)
```

3.834 $\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=996

result too large to display

```
[Out] (a*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((-a)^(1/4)*(5*b*c -
3*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(1
6*b^(7/4)*(b*c - a*d)^(3/2)) + ((-a)^(1/4)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*b^(7/4)*(-(b*c) +
a*d)^(3/2)) + ((4*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b
^2*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqr
t[-a] + Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c +
d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 3*a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqr
t[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c
 + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c -
 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.30812, antiderivative size = 996, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {470, 523, 220, 409, 1217, 1707}

$$-\frac{(5bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt{-a}\sqrt[4]{d}(5bc-3ad)\left(\sqrt{\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((-a)^(1/4)*(5*b*c -
3*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(1
6*b^(7/4)*(b*c - a*d)^(3/2)) + ((-a)^(1/4)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*b^(7/4)*(-(b*c) +
a*d)^(3/2)) + ((4*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b
^2*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqr
t[-a] + Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c +
d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 3*a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqr
t[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c
 + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c -
 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{ac+(-4bc+3ad)x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b^2(bc-ad)} - \frac{(a(5bc-3ad))\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b^2(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} - \frac{(5bc}{}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} - \frac{(\sqrt{c}}{}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt[4]{-a}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(bc-ad)^{3/2}} + \frac{\sqrt[4]{-a}(5bc-3ad)\tan^{-1}\left(\right.}{16b^{7/4}(-bc+ad}$$

Mathematica [C] time = 0.279361, size = 253, normalized size = 0.25

$$\frac{x\left(\frac{5a\left(\frac{5ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}+c+dx^4\right)}{b\left(a+bx^4\right)}+\frac{x^4\sqrt{\frac{dx^4}{c}+1}(4bc-3ad)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ab}\right)}{20\sqrt{c+dx^4}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(((4*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*b) + (5*a*(c + d*x^4 + (5*a*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]))))/(b*(a + b*x^4))))/(20*(b*c - a*d)*Sqrt[c + d*x^4])
```

Maple [C] time = 0.03, size = 604, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/b^2/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2
)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),
I)-1/4*a/b^3*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^
2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1
/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^
(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1
/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_al
pha=RootOf(_Z^4*b+a))+a^2/b^2*(-1/4*b/(a*d-b*c)/a*x*(d*x^4+c)^(1/2)/(b*x^4+
a)-1/4*d/(a*d-b*c)/a/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1
/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d
+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x
^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*
x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/
c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^
(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.835 $\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=908

result too large to display

```
[Out] -(x*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) - ((b*c + a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/
4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1
/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^(3/2))
 + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/
c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*Sqr
t[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/
(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
16*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4
*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(3
2*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sq
rt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[
d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sq
rt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a*b*c^(1/4
)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.906366, antiderivative size = 908, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {471, 523, 220, 409, 1217, 1707}

$$\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^4+c}}-\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+ -$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(x*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) - ((b*c + a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/
4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1
/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^(3/2))
 + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/
c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*Sqr
t[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/
(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
16*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4
*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(3
2*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sq
rt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[
d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sq
rt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a*b*c^(1/4
)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4])
```

<u>Rule 471</u>

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

<u>Rule 523</u>

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

<u>Rule 220</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} + \frac{\int \frac{c-dx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)} + \frac{(bc+ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \frac{(bc+ad)\int}{8ab}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \frac{\left(\sqrt{c}\left(\sqrt{c}-\right.\right.}{}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$

Mathematica [C] time = 0.195443, size = 238, normalized size = 0.26

$$\frac{x\left(\frac{5\left(\frac{5ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}+c+dx^4\right)}{a+bx^4}+\frac{dx^4\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}\right)}{20\sqrt{c+dx^4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*((d*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)])/a + (5*(c + d*x^4 + (5*a*c^2*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^
4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b
*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(a +
b*x^4)))/(20*(-(b*c) + a*d)*Sqrt[c + d*x^4])
```

Maple [C] time = 0.011, size = 530, normalized size = 0.6

$$\frac{1}{8\,b^2} \sum_{\_alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} \frac{1}{\_alpha^3}\left(-\mathrm{Artanh}\left(\frac{2\_alpha^2 d x^2+2\,c}{2}\frac{1}{\sqrt{\frac{-ad+bc}{b}}}\frac{1}{\sqrt{dx^4+c}}\right)\frac{1}{\sqrt{\frac{-ad+bc}{b}}}+2\,\frac{\_alpha^3 b}{a\sqrt{dx^4+c}}\sqrt{1-\frac{i\sqrt{}}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x
^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_
alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)
/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_
alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=R
ootOf(_Z^4*b+a))-a/b*(-1/4*b/(a*d-b*c)/a*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/
(a*d-b*c)/a/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/
c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^
(1/2),I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^
(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/
2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2
)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d
^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/
c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.836 $\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=983

result too large to display

```
[Out] (b*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + (b^(1/4)*(3*b*c - 5*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(
-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(7/4)*(-(b*c) +
a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)
*(b*c - a*d)*Sqrt[c + d*x^4]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a*c^(1/4)*
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*(
-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/
4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
 + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[
d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqr
t[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^4])
```

Rubi [A] time = 1.03265, antiderivative size = 983, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {414, 523, 220, 409, 1217, 1707}

$$\frac{(3bc-5ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt[4]{d}(3bc-5ad)\left(\sqrt{d}x^2+\right.}{16(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + (b^(1/4)*(3*b*c - 5*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(
-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(7/4)*(-(b*c) +
a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)
*(b*c - a*d)*Sqrt[c + d*x^4]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a*c^(1/4)*
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*(
-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/
4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
 + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[
d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqr
t[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^4])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{-3bc+4ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(3bc-5ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \frac{(3bc-5ad)\cdots}{8a\cdots}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\cdots\right.\right.}{\cdots}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt[4]{-a}\cdots}\right)}{16(-a)^{7/4}(-bc+ad\cdots}$$

Mathematica [C] time = 0.240779, size = 392, normalized size = 0.4

$$\frac{2bx^5\left(dx^4\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+5a\left(c+dx^4\right)\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\cdots\right.\right.}{20a^2\left(a+bx^4\right)\sqrt{c+dx^4}(bc-ad)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\cdots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]*(5*a*(4*b*
c - 4*a*d + b*d*x^4) + b*d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4
, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]) + 2*b*x^5*(5*a*(c + d*x^4) + d*
x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)])*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/
a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))/(20*a^2*
(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4,
-((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*
x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*
```

```
x^4)/a)])))
```

Maple [C] time = 0.007, size = 333, normalized size = 0.3

$$-\frac{bx}{(4\,ad-4\,bc)\,a\,(bx^4+a)}\sqrt{dx^4+c}-\frac{d}{(4\,ad-4\,bc)\,a}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)\frac{1}{\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}}}\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4*b/(a*d-b*c)/a*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/(a*d-b*c)/a/(I/c^(1/2)
*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(
1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b/a*sum(
(-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_
alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/
2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)
*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2
)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)
)),_alpha=RootOf(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.837 $\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1046

result too large to display

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c +
 d*x^4])/(4*a*(b*c - a*d)*x^3*(a + b*x^4)) + (b^(5/4)*(7*b*c - 9*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4
)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4)*(-(b*c) + a*d)^(3
/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(24
*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(1
6*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(16*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d
*x^4]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d)*(Sqrt[c]
 + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*
c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(7*b*
c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.55713, antiderivative size = 1046, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {472, 583, 523, 220, 409, 1217, 1707}

$$-\frac{b(7bc-9ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{b\sqrt[4]{d}(7bc-9ad)\left(\sqrt{dx\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c +
 d*x^4])/(4*a*(b*c - a*d)*x^3*(a + b*x^4)) + (b^(5/4)*(7*b*c - 9*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4
)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4)*(-(b*c) + a*d)^(3
/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(24
*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(1
6*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(16*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d
*x^4]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d)*(Sqrt[c]
 + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*
c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(7*b*
c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{\int \frac{-7bc+4ad-5bdx^4}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} + \frac{\int \frac{-21b^2c^2+20abcd+4a^2d^2-bd(7bc-4ad)x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{12a^2c(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{(b(7bc-9ad))\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a^2(bc-ad)} -$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c}{\left(\sqrt{c}+\right.}}}{24a^2c^{5/4}(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c}{\left(\sqrt{c}+\right.}}}{24a^2c^{5/4}(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} + \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right.}{16(-a)^{11/4}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.562522, size = 408, normalized size = 0.39

$$\frac{\frac{a\left(25ac\left(4a^2d\left(c+2dx^4\right)+4ab\left(-c^2+5cdx^4+d^2x^8\right)-7b^2cx^4\left(4c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+10x^4\left(c+dx^4\right)\left(-4a^2d+4ab\left(c-dx^4\right)+7b^2cx^4\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{b}{}\right.\right.\right.}{\left(a+bx^4\right)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}}{60a^3cx^3\sqrt{c+dx^4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*d*(7*b*c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + (a*(25*a*c*(-7*b^2*c*x^4*(4*c + d*x^4) + 4*a^2*d
*(c + 2*d*x^4) + 4*a*b*(-c^2 + 5*c*d*x^4 + d^2*x^8))*AppellF1[1/4, 1/2, 1,
5/4, -((d*x^4)/c), -((b*x^4)/a)] + 10*x^4*(c + d*x^4)*(-4*a^2*d + 7*b^2*c*x
^4 + 4*a*b*(c - d*x^4))*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((
b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/
```

```
((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]
 + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*
d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(60*a^3*c*(-(b
*c) + a*d)*x^3*Sqrt[c + d*x^4])
```

Maple [C] time = 0.013, size = 626, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/a^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*
x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*
_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2
)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*
_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*b+a))-b/a*(-1/4*b/(a*d-b*c)/a*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d
/(a*d-b*c)/a/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I
/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))
^(1/2),I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)
^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1
/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/
2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*
d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I
/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))+1/a^2*(-1/3/c*(d*x^4+c)
^(1/2)/x^3-1/3*d/c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2
)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^
(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

3.838 $\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1146

result too large to display

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) - (x^
3*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + ((3*b*c - a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/
4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d)^(3/2
)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*(b*c - a
*d)*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(8*b*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqr
t[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*b*c
^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(16*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c
] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c
+ d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c
 - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.56284, antiderivative size = 1146, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {471, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{dx^4+c}\,x^3}{4(bc-ad)\left(bx^4+a\right)}+\frac{\sqrt{d}\sqrt{dx^4+c}\,x}{4b(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}b^{5/4}(ad-bc)^{3/2}}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) - (x^
3*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + ((3*b*c - a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/
4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d)^(3/2
)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*(b*c - a
*d)*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(8*b*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqr
t[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*b*c
^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(16*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c
] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c
+ d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c
 - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
```

```
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\int \frac{x^2\left(3c+dx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\int\left(\frac{dx^2}{b\sqrt{c+dx^4}}+\frac{(3bc-ad)x^2}{b\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{4(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}+\frac{(3bc-ad)\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}-\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}-\frac{(3bc-ad)\int \frac{}{\left(\sqrt{-a}-\right.}}{8b^{3/2}(bc-}$$

$$= \frac{\sqrt{d}x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)}-\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\,\mathrm{t}\right.}{4b(bc-ad)\sqrt{c+dx^4}}$$

$$= \frac{\sqrt{d}x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)}-\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}}+\frac{(3b}{}$$

Mathematica [C] time = 0.122553, size = 162, normalized size = 0.14

$$\frac{dx^7\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+7cx^3\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-7ax^3\left(c+dx^4\right)}{28a\left(a+bx^4\right)\sqrt{c+dx^4}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-7*a*x^3*(c + d*x^4) + 7*c*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + d*x^7*(a + b*x^4)*Sqrt[1 + (d
```

```
*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(28*a*(b*
c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [C] time = 0.026, size = 556, normalized size = 0.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b^2*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*b+a))-a/b*(-1/4*b/(a*d-b*c)/a*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I*
d^(1/2)/(a*d-b*c)/a*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*
x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/
c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b/
a*sum((-3*a*d+b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2
*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/
2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1
/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.839 $\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1144

result too large to display

```
[Out] -(Sqrt[d]*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) + (b
*x^3*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ((b*c - 3*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*
b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(
(-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*b^(1/4)*(-(b*c) + a*d)
^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*(b*
c - a*d)*Sqrt[c + d*x^4]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(
c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4
)], 1/2])/(8*a*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d]
)/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(1
6*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a
]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(16*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
+ d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]
*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1
/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticP
i[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(
1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.4407, antiderivative size = 1144, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {472, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{b\sqrt{dx^4+c}\,x^3}{4a(bc-ad)\left(bx^4+a\right)}-\frac{\sqrt{d}\sqrt{dx^4+c}\,x}{4a(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(ad-bc)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[d]*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) + (b
*x^3*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ((b*c - 3*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*
b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(
(-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*b^(1/4)*(-(b*c) + a*d)
^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*(b*
c - a*d)*Sqrt[c + d*x^4]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(
c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4
)], 1/2])/(8*a*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d]
)/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(1
6*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a
]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(16*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
+ d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]
*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1
/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticP
i[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(
1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
```

```
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{x^2\left(-bc+4ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(bc-3ad)\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} - \frac{(bc-3ad)\int \frac{}{\left(\sqrt{-a}-\right.}}{8a\sqrt{b}(bc-}$$

$$= -\frac{\sqrt{d}x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(\right.}{4a(bc-ad)\sqrt{c+dx}}$$

$$= -\frac{\sqrt{d}x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}} - \ldots$$

Mathematica [C] time = 0.146309, size = 172, normalized size = 0.15

$$\frac{-3bdx^7\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + 7x^3\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}(bc-4ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + 21a}{84a^2\left(a+bx^4\right)\sqrt{c+dx^4}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (21*a*b*x^3*(c + d*x^4) + 7*(b*c - 4*a*d)*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/
c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] - 3*b*d*x^7*(a +
```

```
b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x
^4)/a)])/(84*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [C] time = 0.007, size = 359, normalized size = 0.3

$$-\frac{bx^3}{(4\,ad-4\,bc)\,a\left(bx^4+a\right)}\sqrt{dx^4+c}+\frac{\frac{i}{4}}{(ad-bc)\,a}\sqrt{d}\sqrt{c}\sqrt{1-ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\sqrt{1+ix^2\sqrt{d}\frac{1}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{i\sqrt{d}\frac{1}{\sqrt{c}}},i\right)-\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4*b/(a*d-b*c)/a*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I*d^(1/2)/(a*d-b*c)/a*
c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1
/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/
2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b/a*sum((-3*a*d+b*c)/(
a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)
/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3
*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4
+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2
/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.840 $\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1225

result too large to display

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x) + (Sqrt[d]*(5*b*
c - 4*a*d)*x*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2))
 + (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x*(a + b*x^4)) - (b^(3/4)*(5*b*c -
7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]])/(1
6*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]])/(16*(-a)^(9/4)*(-(b*c)
 + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(4*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (d^(1/4)*(5*b*c - 4*a*
d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[
c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*
a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)
*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqr
t[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^
2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) +
 (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)
*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rubi [A] time = 1.89823, antiderivative size = 1225, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {472, 583, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{\sqrt{b}(5bc-7ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}} - \frac{b^{3/4}(5bc-7ad)\tan^{-1}}{16(-a)^{9/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x) + (Sqrt[d]*(5*b*
c - 4*a*d)*x*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2))
 + (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x*(a + b*x^4)) - (b^(3/4)*(5*b*c -
7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(1
6*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(9/4)*(-(b*c)
 + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(4*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (d^(1/4)*(5*b*c - 4*a*
d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[
c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*
a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)
*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqr
t[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^
2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) +
 (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)
*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
```

```
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{\int \frac{-5bc+4ad-3bdx^4}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\int \frac{x^2\left(-(bc-2ad)(5bc-2ad)+bd(5bc-4ad)x^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\int \left(\frac{d(5bc-4ad)x^2}{\sqrt{c+dx^4}} + \frac{\left(-5b^2c^2+7abcd\right)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{4a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{(b(5bc-7ad))\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a^2(bc-ad)} + \frac{(d}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\left(\sqrt{b}(5bc-7ad)\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{\sqrt[4]{d}(5bc}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{b^{3/4}(5bc}{}$$

Mathematica [C] time = 0.25443, size = 226, normalized size = 0.18

$$\frac{-7x^4\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}\left(4a^2d^2-12abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+21a\left(c+dx^4\right)\left(4a^2d-4ab\left(c-dx^4\right)-5b^2c\right.}{84a^3cx\left(a+bx^4\right)\sqrt{c+dx^4}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (21*a*(c + d*x^4)*(4*a^2*d - 5*b^2*c*x^4 - 4*a*b*(c - d*x^4)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^8*(a
+ b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b
```

```
*x^4)/a)])/(84*a^3*c*(b*c - a*d)*x*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [C] time = 0.013, size = 674, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/a^2*(-1/c*(d*x^4+c)^(1/2)/x+I*d^(1/2)/c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(
1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1
/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2
))^(1/2),I)))-1/8/a^2*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*
_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1
/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2
)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/
2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2
))),_alpha=RootOf(_Z^4*b+a))-b/a*(-1/4*b/(a*d-b*c)/a*x^3*(d*x^4+c)^(1/2)/(b
*x^4+a)+1/4*I*d^(1/2)/(a*d-b*c)/a*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^
(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(E
llipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/
2),I))-1/32/b/a*sum((-3*a*d+b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*
arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/
(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I
/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2)
)^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2
)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

3.841 $\int \frac{(ex)^m\left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=200

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(a^2d^2(m+3)(m+7)+bc(m+1)(bc(m+5)-2ad(m+7))\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{d^2e(m+1)(m+3)(m+7)\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}(ex)^{m}\cdots}{d^2\cdots}$$

```
[Out] -((b*(b*c*(5 + m) - 2*a*d*(7 + m))*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d^2*e*(3
 + m)*(7 + m))) + (b^2*(e*x)^(5 + m)*Sqrt[c + d*x^4])/(d*e^5*(7 + m)) + ((a
^2*d^2*(3 + m)*(7 + m) + b*c*(1 + m)*(b*c*(5 + m) - 2*a*d*(7 + m)))*(e*x)^(
1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4, -((
d*x^4)/c)])/(d^2*e*(1 + m)*(3 + m)*(7 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.212091, antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {464, 459, 365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(\frac{a^2d^2(m+7)}{m+1}+\frac{bc(bc(m+5)-2ad(m+7))}{m+3}\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{d^2e(m+7)\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(m+7))\cdots}{d^2e(m+3)(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] -((b*(b*c*(5 + m) - 2*a*d*(7 + m))*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d^2*e*(3
 + m)*(7 + m))) + (b^2*(e*x)^(5 + m)*Sqrt[c + d*x^4])/(d*e^5*(7 + m)) + (((
a^2*d^2*(7 + m))/(1 + m) + (b*c*(b*c*(5 + m) - 2*a*d*(7 + m)))/(3 + m))*(e*
x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)])/(d^2*e*(7 + m)*Sqrt[c + d*x^4])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx = \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\int \frac{(ex)^m\left(a^2d(7+m)-b(bc(5+m)-2ad(7+m))x^4\right)}{\sqrt{c+dx^4}}\,dx}{d(7+m)}$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} - \left(-a^2 - \frac{bc(1+m)(bc(5+ \ldots}{d^2(3+m \ldots}\right.$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} - \frac{\left(\left(-a^2 - \frac{bc(1+m)(bc(5+m)-2a \ldots}{d^2(3+m)(7+m \ldots}\right.\right.}{\sqrt{\ldots}}$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\left(a^2 + \frac{bc(1+m)(bc(5+m)-2ad(7 \ldots}{d^2(3+m)(7+m)}\right.}{\ldots}$$

Mathematica [A] time = 0.137103, size = 164, normalized size = 0.82

$$\frac{x\sqrt{\frac{dx^4}{c}+1}(ex)^m\left(a^2\left(m^2+14m+45\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)+b(m+1)x^4\left(2a(m+9)\,{}_2F_1\left(\frac{1}{2},\frac{m+5}{4};\frac{m+9}{4};-\frac{dx^4}{c}\right)+b(m+5 \ldots\right.\right.}{(m+1)(m+5)(m+9)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[1/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
metric2F1[1/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[1/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/((1 + m)*(5 + m)*(9
+ m)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (ex)^m \left(bx^4+a\right)^2 \frac{1}{\sqrt{dx^4+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^2 (ex)^m}{\sqrt{dx^4+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^8+2\,abx^4+a^2\right)(ex)^m}{\sqrt{dx^4+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] time = 55.6202, size = 185, normalized size = 0.92

$$\frac{a^2e^m x x^m\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}+\frac{abe^m x^5x^m\Gamma\left(\frac{m}{4}+\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}+\frac{b^2e^m x^9x^m\Gamma\left(\frac{m}{4}+\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{9}{4}\\ \frac{m}{4}+\frac{13}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] a**2*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x
**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + a*b*e**m*x**5*x**m*ga
mma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)
/c)/(2*sqrt(c)*gamma(m/4 + 9/4)) + b**2*e**m*x**9*x**m*gamma(m/4 + 9/4)*hyp
er((1/2, m/4 + 9/4), (m/4 + 13/4,), d*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*ga
mma(m/4 + 13/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.842 $\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}-\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}(bc(m+1)-ad(m+3))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{de(m+1)(m+3)\sqrt{c+dx^4}}$$

```
[Out] (b*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m)) - ((b*c*(1 + m) - a*d*(3 +
m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5
+ m)/4, -((d*x^4)/c)])/(d*e*(1 + m)*(3 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0590184, antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(\frac{a}{m+1}-\frac{bc}{d(m+3)}\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e\sqrt{c+dx^4}}+\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

```
[Out] (b*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m)) + ((a/(1 + m) - (b*c)/(d*(3
 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)])/(e*Sqrt[c + d*x^4])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m\left(a+bx^4\right)}{\sqrt{c+dx^4}}\,dx &= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)}-\left(-a+\frac{bc(1+m)}{d(3+m)}\right)\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx\\ &= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)}-\frac{\left(\left(-a+\frac{bc(1+m)}{d(3+m)}\right)\sqrt{1+\frac{dx^4}{c}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}\\ &= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)}+\frac{\left(a-\frac{bc(1+m)}{d(3+m)}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [A] time = 0.0849124, size = 110, normalized size = 0.89

$$\frac{x\sqrt{\frac{dx^4}{c}+1}(ex)^m\left(a(m+5)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)+b(m+1)x^4\,{}_2F_1\left(\frac{1}{2},\frac{m+5}{4};\frac{m+9}{4};-\frac{dx^4}{c}\right)\right)}{(m+1)(m+5)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[1/2, (5 + m)/4,
 (9 + m)/4, -((d*x^4)/c)]))/((1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (ex)^m\left(bx^4+a\right)\frac{1}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)(ex)^m}{\sqrt{dx^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 + a)(ex)^m}{\sqrt{dx^4 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] time = 6.7225, size = 119, normalized size = 0.97

$$\frac{ae^m x x^m \Gamma\left(\frac{m}{4} + \frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4} + \frac{1}{4} \\ \frac{m}{4} + \frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4} + \frac{5}{4}\right)} + \frac{be^m x^5 x^m \Gamma\left(\frac{m}{4} + \frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4} + \frac{5}{4} \\ \frac{m}{4} + \frac{9}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4} + \frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] a*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4
*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + b*e**m*x**5*x**m*gamma(m
/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)/c)/(
4*sqrt(c)*gamma(m/4 + 9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)(ex)^m}{\sqrt{dx^4 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

3.843 $\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0194269, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/Sqrt[c + d*x^4],x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.0155782, size = 66, normalized size = 0.97

$$\frac{x\sqrt{\frac{dx^4}{c}+1}(ex)^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+1}{4}+1;-\frac{dx^4}{c}\right)}{(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/Sqrt[c + d*x^4],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/((1 + m)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int (ex)^m \frac{1}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(d*x^4+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{\sqrt{dx^4 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] time = 0.95311, size = 56, normalized size = 0.82

$$\frac{e^m x x^m \Gamma\left(\frac{m}{4} + \frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4} + \frac{1}{4} \\ \frac{m}{4} + \frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4} + \frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(d*x**4+c)**(1/2),x)
```

```
[Out] e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4*e
xp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sqrt{dx^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

3.844 $\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0552466, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};1,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.0950747, size = 125, normalized size = 1.54

$$\frac{x\sqrt{c+dx^4}(ex)^m\left(bcF_1\left(\frac{m+1}{4};-\frac{1}{2},1;\frac{m+5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-ad\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)\right)}{ac(m+1)\sqrt{\frac{dx^4}{c}+1}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b*c*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4, -((
d*x^4)/c), -((b*x^4)/a)] - a*d*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)]))/(a*c*(b*c - a*d)*(1 + m)*Sqrt[1 + (d*x^4)/c])
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{bx^4+a}\frac{1}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{bdx^8+(bc+ad)x^4+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

3.845 $\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0563393, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};2,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}$$

Mathematica [B] time = 0.138022, size = 179, normalized size = 2.21

$$\frac{x\sqrt{c+dx^4}(ex)^m\left(a^2d^2\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)-abcdF_1\left(\frac{m+1}{4};-\frac{1}{2},1;\frac{m+5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+bc(bc-ad)F_1\left(\frac{m+1}{4};2,-\frac{1}{2};\frac{m+5}{4};-\right.}{a^2c(m+1)\sqrt{\frac{dx^4}{c}+1}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(-(a*b*c*d*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/
4, -((d*x^4)/c), -((b*x^4)/a)]) + b*c*(b*c - a*d)*AppellF1[(1 + m)/4, 2, -1
/2, (5 + m)/4, -((b*x^4)/a), -((d*x^4)/c)] + a^2*d^2*Hypergeometric2F1[1/2,
 (1 + m)/4, (5 + m)/4, -((d*x^4)/c)]))/(a^2*c*(b*c - a*d)^2*(1 + m)*Sqrt[1
+ (d*x^4)/c])
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^2}\frac{1}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^2dx^{12}+(b^2c+2\,abd)x^8+(2\,abc+a^2d)x^4+a^2c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d*x^12 + (b^2*c + 2*a*b*d)*x^8 + (2*a
*b*c + a^2*d)*x^4 + a^2*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

3.846 $\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0564448, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};3,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.0601313, size = 77, normalized size = 0.95

$$\frac{x\sqrt{\frac{dx^4}{c}+1}(ex)^m F_1\left(\frac{m+1}{4};3,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -((b*
x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)^3}\frac{1}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^3dx^{16}+\left(b^3c+3\,ab^2d\right)x^{12}+3\left(ab^2c+a^2bd\right)x^8+\left(3\,a^2bc+a^3d\right)x^4+a^3c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d*x^16 + (b^3*c + 3*a*b^2*d)*x^12 + 3
*(a*b^2*c + a^2*b*d)*x^8 + (3*a^2*b*c + a^3*d)*x^4 + a^3*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

3.847 $\int \frac{(ex)^m\left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cd^2e(m+1)(m+3)\sqrt{c+dx^4}}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}+$$

```
[Out] ((b*c - a*d)^2*(e*x)^(1 + m))/(2*c*d^2*e*Sqrt[c + d*x^4]) + (b^2*(e*x)^(1 +
 m)*Sqrt[c + d*x^4])/(d^2*e*(3 + m)) - ((2*b^2*c^2*(1 + m) - (3 + m)*(2*a^2
*d^2 - (b*c - a*d)^2*(1 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeome
tric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d^2*e*(1 + m)*(3 + m
)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.17849, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {463, 459, 365, 364}

$$-\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cd^2e(m+1)(m+3)\sqrt{c+dx^4}}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```


```
[Out] ((b*c - a*d)^2*(e*x)^(1 + m))/(2*c*d^2*e*Sqrt[c + d*x^4]) + (b^2*(e*x)^(1 +
 m)*Sqrt[c + d*x^4])/(d^2*e*(3 + m)) - ((2*b^2*c^2*(1 + m) - (3 + m)*(2*a^2
*d^2 - (b*c - a*d)^2*(1 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeome
tric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d^2*e*(1 + m)*(3 + m
)*Sqrt[c + d*x^4])
```


Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
```

```
IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} - \frac{\int \frac{(ex)^m\left(-2a^2d^2+(bc-ad)^2(1+m)-2b^2cdx^4\right)}{\sqrt{c+dx^4}}\,dx}{2cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} - \frac{\left(-a^2d^2(1-m)-2abcd(1+m)+\frac{b^2c^2(1+m)(5+m)}{3+m}\right)\int \frac{(e}{\sqrt{c}}}{2cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} - \frac{\left(\left(-a^2d^2(1-m)-2abcd(1+m)+\frac{b^2c^2(1+m)(5+m)}{3+m}\right)\sqrt{1}\right.}{2cd^2\sqrt{c+dx^4}}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} + \frac{\left(a^2d^2(1-m)+2abcd(1+m)-\frac{b^2c^2(1+m)(5+m)}{3+m}\right)(ex)^{1+}}{2cd^2e(1+m)\sqrt{c+d}}$$

Mathematica [A] time = 0.14129, size = 167, normalized size = 0.84

$$\frac{x\sqrt{\frac{dx^4}{c}+1}(ex)^m\left(a^2\left(m^2+14m+45\right){}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)+b(m+1)x^4\left(2a(m+9){}_2F_1\left(\frac{3}{2},\frac{m+5}{4};\frac{m+9}{4};-\frac{dx^4}{c}\right)+b(m+\right.\right.}{c(m+1)(m+5)(m+9)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[3/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
metric2F1[3/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[3/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/(c*(1 + m)*(5 + m)*(
9 + m)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (ex)^m \left(bx^4+a\right)^2 \left(dx^4+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^2 (ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^8 + 2\,abx^4 + a^2\right)\sqrt{dx^4 + c}\,(ex)^m}{d^2x^8 + 2\,cdx^4 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c
*d*x^4 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^2 (ex)^m}{\left(dx^4 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.848 $\int \frac{(ex)^m(a+bx^4)}{(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}(ad(1-m)+bc(m+1))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

```
[Out] -((b*c - a*d)*(e*x)^(1 + m))/(2*c*d*e*Sqrt[c + d*x^4]) + ((a*d*(1 - m) + b*
c*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)
/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0599819, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}(ad(1-m)+bc(m+1))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```


```
[Out] -((b*c - a*d)*(e*x)^(1 + m))/(2*c*d*e*Sqrt[c + d*x^4]) + ((a*d*(1 - m) + b*
c*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)
/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d*e*(1 + m)*Sqrt[c + d*x^4])
```


Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ex)^m\left(a+bx^4\right)}{\left(c+dx^4\right)^{3/2}}\,dx &= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{(-ad(-1+m)+bc(1+m))\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx}{2cd}\\
&= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{\left((-ad(-1+m)+bc(1+m))\sqrt{1+\frac{dx^4}{c}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{2cd\sqrt{c+dx^4}}\\
&= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{(ad(1-m)+bc(1+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{2cde(1+m)\sqrt{c+dx^4}}
\end{aligned}
$$

Mathematica [A] time = 0.0914583, size = 113, normalized size = 0.86

$$
\frac{x\sqrt{\frac{dx^4}{c}+1}(ex)^m\left(a(m+5)\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)+b(m+1)x^4\,{}_2F_1\left(\frac{3}{2},\frac{m+5}{4};\frac{m+9}{4};-\frac{dx^4}{c}\right)\right)}{c(m+1)(m+5)\sqrt{c+dx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[3/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[3/2, (5 + m)/4,
 (9 + m)/4, -((d*x^4)/c)]))/(c*(1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (ex)^m \left(bx^4+a\right) \left(dx^4+c\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^4+a\right)\sqrt{dx^4+c}\,(ex)^m}{d^2x^8+2\,cdx^4+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x
)
```

Sympy [C] time = 163.386, size = 119, normalized size = 0.9

$$\frac{ae^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix} \middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^5 x^m \Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{5}{4} \\ \frac{m}{4}+\frac{9}{4} \end{matrix} \middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

```
[Out] a*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4
*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4)) + b*e**m*x**5*x**m*gamma(
m/4 + 5/4)*hyper((3/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)/c)/
(4*c**(3/2)*gamma(m/4 + 9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.849 $\int \frac{(ex)^m}{\left(c+dx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0231556, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/(c + d*x^4)^(3/2),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$

$$=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.0175721, size = 69, normalized size = 0.97

$$\frac{x\sqrt{\frac{dx^4}{c}+1}(ex)^m\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+1}{4}+1;-\frac{dx^4}{c}\right)}{c(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/(c*(1 + m)*Sqrt[c + d*x^4])
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int (ex)^m\left(dx^4+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(d*x^4+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{d^2x^8+2\,cdx^4+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [C] time = 1.6566, size = 56, normalized size = 0.79

$$\frac{e^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(d*x**4+c)**(3/2),x)
```

```
[Out] e**m*x*x**m*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4*e
xp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

3.850 $\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0601804, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\left(a+bx^4\right)\left(c+dx^4\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};1,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}$$

Mathematica [B] time = 0.136401, size = 169, normalized size = 2.01

$$\frac{x\sqrt{c+dx^4}(ex)^m\left(b^2c^2F_1\left(\frac{m+1}{4};-\frac{1}{2},1;\frac{m+5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\left((ad-bc)\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)-bc\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right.\right.\right.}{ac^2(m+1)\sqrt{\frac{dx^4}{c}+1}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b^2*c^2*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4,
 -((d*x^4)/c), -((b*x^4)/a)] + a*d*(-(b*c*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)]) + (-(b*c) + a*d)*Hypergeometric2F1[3/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)])))/(a*c^2*(b*c - a*d)^2*(1 + m)*Sqrt[1 + (d*x^4
)/c])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{bx^4+a}\left(dx^4+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{bd^2x^{12}+(2\,bcd+ad^2)x^8+(bc^2+2\,acd)x^4+ac^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d^2*x^12 + (2*b*c*d + a*d^2)*x^8 + (b*c
^2 + 2*a*c*d)*x^4 + a*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

3.851 $\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0603985, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^2\left(c+dx^4\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)^2\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};2,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.0688856, size = 77, normalized size = 0.92

$$\frac{x\left(\frac{dx^4}{c}+1\right)^{3/2}(ex)^mF_1\left(\frac{m+1}{4};2,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2(m+1)\left(c+dx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + m)*(c + d*x^4)^(3/2))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^2}\left(dx^4+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^2d^2x^{16}+2\left(b^2cd+abd^2\right)x^{12}+\left(b^2c^2+4\,abcd+a^2d^2\right)x^8+2\left(abc^2+a^2cd\right)x^4+a^2c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d^2*x^16 + 2*(b^2*c*d + a*b*d^2)*x^12
 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 2*(a*b*c^2 + a^2*c*d)*x^4 + a^2*c^
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^2\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

3.852 $\int \frac{(ex)^m}{\left(a+bx^4\right)^3\left(c+dx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] time = 0.0618125, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{\left(a+bx^4\right)^3\left(c+dx^4\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)^3\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};3,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.078791, size = 77, normalized size = 0.92

$$\frac{x\left(\frac{dx^4}{c}+1\right)^{3/2}(ex)^m F_1\left(\frac{m+1}{4};3,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(m+1)\left(c+dx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*(c + d*x^4)^(3/2))
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^3}\left(dx^4+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^3\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^3d^2x^{20}+\left(2\,b^3cd+3\,ab^2d^2\right)x^{16}+\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^{12}+\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^8+a^3c^2+\left(3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d^2*x^20 + (2*b^3*c*d + 3*a*b^2*d^2)*
x^16 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^12 + (3*a*b^2*c^2 + 6*a^2*b*
c*d + a^3*d^2)*x^8 + a^3*c^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^3\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

3.853 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}(ad+bc)}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}$$

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^6])/(3*b^2*d^2) + (c + d*x^6)^(3/2)/(9*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*
c - a*d])
```

Rubi [A] time = 0.107156, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}(ad+bc)}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^6])/(3*b^2*d^2) + (c + d*x^6)^(3/2)/(9*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*
c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{17}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx, x, x^6\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.177068, size = 91, normalized size = 0.88

$$
\frac{\sqrt{c+dx^6}\left(-3ad-2bc+bdx^6\right)}{9b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x^{17}}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.0947, size = 594, normalized size = 5.71

$$\left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\left(\left(b^3cd-ab^2d^2\right)x^6-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{dx^6+c}}{18\left(b^4cd^2-ab^3d^3\right)}, \frac{3\sqrt{-b^2\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*
x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 -
```

```
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 - a*b^2
*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{17}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**17/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A] time = 1.14837, size = 143, normalized size = 1.38

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^2}+\frac{\left(dx^6+c\right)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^6+c}\,b^2cd^4-3\sqrt{dx^6+c}\,abd^5}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*a^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/9*((d*x^6 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^6 + c)*b^2*c*d^4 - 3*s
qrt(d*x^6 + c)*a*b*d^5)/(b^3*d^6)
```

3.854 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{3bd}$$

```
[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a
*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0632217, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a
*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)\\ &= \frac{\sqrt{c+dx^6}}{3bd}-\frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6b}\\ &= \frac{\sqrt{c+dx^6}}{3bd}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3bd}\\ &= \frac{\sqrt{c+dx^6}}{3bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0628313, size = 72, normalized size = 0.97

$$\frac{1}{3}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{b x^{6}+a}\frac{1}{\sqrt{d x^{6}+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.05067, size = 437, normalized size = 5.91

$$\left[\frac{\sqrt{b^{2}c-abd}ad\log\left(\frac{bdx^{6}+2\,bc-ad+2\,\sqrt{dx^{6}+c}\sqrt{b^{2}c-abd}}{bx^{6}+a}\right)+2\,\sqrt{dx^{6}+c}\left(b^{2}c-abd\right)}{6\left(b^{3}cd-ab^{2}d^{2}\right)},-\frac{\sqrt{-b^{2}c+abd}ad\arctan\left(\frac{\sqrt{dx^{6}+c}\sqrt{-b^{2}c+abd}}{bdx^{6}+bc}\right)-}{3\left(b^{3}cd-ab^{2}d^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c
)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A] time = 1.12361, size = 86, normalized size = 1.16

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^6+c}}{b}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^6 + c)/b)/d
```

3.855 $\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(3*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rubi [A] time = 0.0459977, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```


```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(3*Sqrt[b]*Sqrt[b*c - a
*d])
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0154694, size = 51, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(3*Sqrt[b]*Sqrt[b*c - a
*d])
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{x^5}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.0487, size = 288, normalized size = 5.65

$$\left[\frac{\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)}{6\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)}{3\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b
*x^6 + a))/sqrt(b^2*c - a*b*d), 1/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] time = 18.6846, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.16337, size = 54, normalized size = 1.06

$$\frac{\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.856 $\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

```
[Out] -ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0741793, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```



```
[Out] -ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{6a}-\frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3ad}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3ad}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0759765, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^6])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(3*a)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(bx^6+a\right)}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)
```

Fricas [A] time = 1.104, size = 948, normalized size = 11.15

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+\sqrt{c}\log\left(\frac{dx^6-2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)}{6\,ac},\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)}{6\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sqrt
(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(2*c*sqrt(-b/(b*c - a*d))*arcta
```

```
n(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt
(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-
c)/c))/(a*c), 1/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^6 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A] time = 24.8035, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**6)/sqrt(-c))/(3*a*sqrt(-c))
```

Giac [A] time = 1.09068, size = 107, normalized size = 1.26

$$-\frac{1}{3}d\left(\frac{b\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d*(b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*a*d) - arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.857 $\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

```
[Out] -Sqrt[c + d*x^6]/(6*a*c*x^6) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[
c]])/(6*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c
- a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.118752, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```


```
[Out] -Sqrt[c + d*x^6]/(6*a*c*x^6) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[
c]])/(6*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c
- a*d]])/(3*a^2*Sqrt[b*c - a*d])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6}-\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6ac}\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6a^2}-\frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^6\right)}{12a^2c}\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3a^2d}-\frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6a^2cd}\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.117471, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2(ad-bc)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6ac^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(6*a*c*x^6) + (b*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(3*a^2*
Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(6*a*c^(3/2)) + (b^(3/2)*Sq
rt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a^2*(-
(b*c) + a*d))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^7\left(bx^6+a\right)}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)
```

Fricas [A] time = 1.33045, size = 1264, normalized size = 10.8

$$\left[\frac{2\,bc^2x^6\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+(2\,bc+ad)\sqrt{c}x^6\log\left(\frac{dx^6+2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)-2\,\sqrt{dx^6+c}ac}{12\,a^2c^2x^6},-\frac{4\,bc^2x^6\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(
d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*sq
rt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*x^6
 + c)*a*c)/(a^2*c^2*x^6), -1/12*(4*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*sq
rt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), 1/6*(b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^6 + a)) - (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c)
 - sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/6*(2*b*c^2*x^6*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + sqrt(d*
x^6 + c)*a*c)/(a^2*c^2*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A] time = 1.15329, size = 159, normalized size = 1.36

$$\frac{1}{6}d^2\left(\frac{2b^2\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{(2bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}cd^2}-\frac{\sqrt{dx^6+c}}{acd^2x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*d^2*(2*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*a^2*d^2) - (2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqr
t(-c)*c*d^2) - sqrt(d*x^6 + c)/(a*c*d^2*x^6))
```

3.858 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}+\frac{x^3\sqrt{c+dx^6}}{6bd}$$

```
[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt
[a]*Sqrt[c + d*x^6])])/(3*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^3)/Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))
```

Rubi [A] time = 0.153334, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}+\frac{x^3\sqrt{c+dx^6}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt
[a]*Sqrt[c + d*x^6])])/(3*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^3)/Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
```

```
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} - \frac{\mathrm{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6bd}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2} - \frac{(bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6b^2d}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2} - \frac{(bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+d}}\right.}{6b^2d}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}$$

Mathematica [A] time = 0.135248, size = 118, normalized size = 0.96

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^6}+dx^3\right)}{d^{3/2}} + \frac{bx^3\sqrt{c+dx^6}}{d}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ((b*x^3*Sqrt[c + d*x^6])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[
a]*Sqrt[c + d*x^6])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^3 + Sqrt[d]*
Sqrt[c + d*x^6]])/d^(3/2))/(6*b^2)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{x^{14}}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [A] time = 1.78512, size = 1632, normalized size = 13.27

$$\left[\frac{2\sqrt{dx^6+c}bdx^3 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{b}}}{b^2x^{12}+2\,abx^6+a^2}\right.}{12\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2*a*d
)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/1
2*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6
+ c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c + 2*a*d)*
sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2*sqrt(d*x^
6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6
 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + (b*c + 2
*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2),
 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c
 - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3
```

```
)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.859 $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}} dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

Rubi [A] time = 0.0873107, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^3\right)}{3b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{3b}\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{3b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{3b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}
\end{aligned}
$$

Mathematica [A] time = 0.0582949, size = 90, normalized size = 0.99

$$
\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^6}+dx^3\right)}{\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{\sqrt{bc-ad}}}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/Sqrt[b
*c - a*d]) + Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]]/Sqrt[d])/(3*b)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x^8}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [A] time = 1.4812, size = 1366, normalized size = 15.01

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2\,abx^6+a^2}\right)+2\sqrt{d}\log\left(-2\right.}{12\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*s
qrt(d)*x^3 - c))/(b*d), 1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*sqrt(-d)*arctan(sqr
t(-d)*x^3/sqrt(d*x^6 + c)))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*(
(b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c
*x^3)) + sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b*d),
1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) - 2*sqrt(-d)*arctan(sqrt(-d)
*x^3/sqrt(d*x^6 + c)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [B] time = 1.20023, size = 223, normalized size = 2.45

$$\frac{1}{3}c\left(\frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}\,bc\,\mathrm{sgn}(x)}-\frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{bc\sqrt{-d}\,\mathrm{sgn}(x)}\right)-\frac{\left(a\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{3\sqrt{abc-a^2d}\,b\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*c*(a*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*b*c*sgn(x)) - arctan(sqrt(d + c/x^6)/sqrt(-d))/(b*c*sqrt(-d)*sgn(x))) - 1/
3*(a*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*d)*a
rctan(sqrt(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b*sqrt(-d))
```

3.860 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}} dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rubi [A] time = 0.0487174, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.0617062, size = 95, normalized size = 1.76

$$
\frac{x^3\sqrt{\frac{dx^6}{c}+1}\,\tanh^{-1}\left(\frac{\sqrt{\frac{dx^6}{c}-\frac{bx^6}{a}}}{\sqrt{\frac{dx^6}{c}+1}}\right)}{3a\sqrt{c+dx^6}\sqrt{\frac{dx^6}{c}-\frac{bx^6}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^6)/c]*ArcTanh[Sqrt[-((b*x^6)/a) + (d*x^6)/c]/Sqrt[1 + (d
*x^6)/c]])/(3*a*Sqrt[c + d*x^6]*Sqrt[-((b*x^6)/a) + (d*x^6)/c])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$
\int \frac{x^2}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [B] time = 1.37804, size = 522, normalized size = 9.67

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left(\left(bc-2\,ad\right)x^9-acx^3\right)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{12\left(abc-a^2d\right)},\ \frac{\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^6-ac\right)\sqrt{dx^6\cdots}}{2\left(\left(abcd-a^2d^2\right)x^9+\left(abc\cdots\right.\right.}\right.}{6\sqrt{abc-a^2d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sq
rt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))/(a*b*c -
a^2*d), 1/6*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/sqrt(a*b*c -
 a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [A] time = 1.16987, size = 97, normalized size = 1.8

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{3\,\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.861 $\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

```
[Out] -Sqrt[c + d*x^6]/(3*a*c*x^3) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqr
t[c + d*x^6])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0887366, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(3*a*c*x^3) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqr
t[c + d*x^6])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3ac}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3a}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3a}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 0.693257, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^6}{c}+1\right)\left(\frac{4x^6(c+dx^6)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^6}{c(bx^6+a)}\right)}{3c^2(a+bx^6)}+\frac{(c+2dx^6)\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c(a+bx^6)}}\right)}{c\sqrt{\frac{ax^6(c+dx^6)(bc-ad)}{c^2(a+bx^6)^2}}}\right)}{3x^3\left(a+bx^6\right)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((1 + (d*x^6)/c)*(((c + 2*d*x^6)*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x
^6))]])/(c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2)]) + (4*
(b*c - a*d)*x^6*(c + d*x^6)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x^6)/
(c*(a + b*x^6))])/(3*c^2*(a + b*x^6))))/(3*x^3*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(bx^6+a\right)}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)
```

Fricas [B] time = 1.41206, size = 702, normalized size = 8.78

$$\left[-\frac{\sqrt{-abc+a^2d}bcx^3\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left((bc-2\,ad)x^9-acx^3\right)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+4\sqrt{dx^6+c}\left(abc-a\right.}{12\left(a^2bc^2-a^3cd\right)x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(sqrt(-a*b*c + a^2*d)*b*c*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*
c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))
+ 4*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*(sqr
t(a*b*c - a^2*d)*b*c*x^3*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))
 + 2*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [B] time = 1.22178, size = 185, normalized size = 2.31

$$\frac{\frac{bc\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}a\mathrm{sgn}(x)}-\frac{\sqrt{d+\frac{c}{x^6}}}{a\mathrm{sgn}(x)}}{3\,c}-\frac{\left(bc\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\sqrt{d}\right)\mathrm{sgn}\left(x\right)}{3\sqrt{abc-a^2d}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*a*sgn(x)) - sqrt(d + c/x^6)/(a*sgn(x)))/c - 1/3*(b*c*arctan(a*sqrt(d)/sqrt
(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)
*a*c)
```

3.862 $\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}} dx$

Optimal. Leaf size=115

$$\frac{b^2 \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}(2ad+3bc)}{9a^2c^2x^3} - \frac{\sqrt{c+dx^6}}{9acx^9}$$

```
[Out] -Sqrt[c + d*x^6]/(9*a*c*x^9) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(9*a^2*c^2
*x^3) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*a^
(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.162659, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}(2ad+3bc)}{9a^2c^2x^3} - \frac{\sqrt{c+dx^6}}{9acx^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(9*a*c*x^9) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(9*a^2*c^2
*x^3) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*a^
(5/2)*Sqrt[b*c - a*d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{\operatorname{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{9ac}$$
$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} - \frac{\operatorname{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{9a^2c^2}$$
$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3a^2}$$
$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3a^2}$$
$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

Mathematica [C] time = 1.73022, size = 253, normalized size = 2.2

$$-\frac{\left(\frac{dx^6}{c}+1\right)\left(-\frac{8x^6\left(c+dx^6\right)^2(bc-ad)\text{HypergeometricPFQ}\left(\{2,2,2\},\left\{1,\frac{5}{2}\right\},\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}\right)}{a+bx^6}+\frac{3c\left(c^2-4cdx^6-8d^2x^{12}\right)\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}}\right)}{\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}}+\frac{24dx^{12}\left(c+dx^6\right)(ad-bc)\ldots}{a+b\ldots}\right)}{27c^3x^9\left(a+bx^6\right)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((1 + (d*x^6)/c)*((3*c*(c^2 - 4*c*d*x^6 - 8*d^2*x^12)*ArcSin[Sqrt[((b*c -
a*d)*x^6)/(c*(a + b*x^6))]])/Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a +
 b*x^6)^2)] + (24*d*(-(b*c) + a*d)*x^12*(c + d*x^6)*Hypergeometric2F1[2, 2,
 5/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))])/(a + b*x^6) - (8*(b*c - a*d)*x^6*
(c + d*x^6)^2*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((b*c - a*d)*x^6)/(c*(
a + b*x^6))])/(a + b*x^6)))/(27*c^3*x^9*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10} \left(b x^{6}+a\right)} \frac{1}{\sqrt{d x^{6}+c}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{6}+a\right) \sqrt{d x^{6}+c} x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)
```

Fricas [A] time = 1.53969, size = 859, normalized size = 7.47

$$\left[-\frac{3 \sqrt{-abc + a^2 d} b^2 c^2 x^9 \log\left(\frac{\left(b^2 c^2 - 8\,abcd + 8\,a^2 d^2\right) x^{12} - 2\left(3\,abc^2 - 4\,a^2 cd\right) x^6 + a^2 c^2 - 4\left((bc - 2\,ad) x^9 - acx^3\right)\sqrt{dx^6 + c}\sqrt{-abc + a^2 d}}{b^2 x^{12} + 2\,abx^6 + a^2}\right) - 4\left(\left(3\,ab^2 c^2 - a^2 bc\right.\right.}{36\left(a^3 bc^3 - a^4 c^2 d\right) x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^
9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 +
a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)
*sqrt(d*x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c - a^2*d
```

```
)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b
*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*
b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^6 + c)
)/((a^3*b*c^3 - a^4*c^2*d)*x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [B] time = 1.32403, size = 277, normalized size = 2.41

$$-\frac{b^2\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}\,a^2\operatorname{sgn}(x)}+\frac{\left(3\,b^2c^2\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-3\sqrt{abc-a^2d}\,bc\sqrt{d}-2\sqrt{abc-a^2d}\,ad^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{9\sqrt{abc-a^2d}\,a^2c^2}+\frac{3\,abc^5\sqrt{d+\frac{c}{x^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*b^2*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*a^2*sgn(x)) + 1/9*(3*b^2*c^2*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 3*sqr
t(a*b*c - a^2*d)*b*c*sqrt(d) - 2*sqrt(a*b*c - a^2*d)*a*d^(3/2))*sgn(x)/(sqr
t(a*b*c - a^2*d)*a^2*c^2) + 1/9*(3*a*b*c^5*sqrt(d + c/x^6) - a^2*c^4*(d + c
/x^6)^(3/2) + 3*a^2*c^4*sqrt(d + c/x^6)*d)/(a^3*c^6*sgn(x))
```

3.863 $\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 1, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0517917, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 1, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a*Sqrt[c + d*x^6])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{\left(a+bx^6\right)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

Mathematica [A] time = 0.0397645, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{c+dx^6}{c}}F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{5a\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^6)/c]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6
)/a)])/(5*a*Sqrt[c + d*x^6])
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x^4}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.864 $\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0714221, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

Mathematica [A] time = 0.0362421, size = 65, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^6}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{4a\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)
/a)])/(4*a*Sqrt[c + d*x^6])
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{x^3}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^6 + a\right)\sqrt{dx^6 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.865 $\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0544657, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 430, 429}

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

Mathematica [A] time = 0.0339866, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{c+dx^6}{c}}F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{2a\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^6)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)
/a)])/(2*a*Sqrt[c + d*x^6])
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.866 $\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 1, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0248053, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 1, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a*Sqrt[c + d*x^6])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.165643, size = 161, normalized size = 2.73

$$-\frac{7acxF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{(a+bx^6)\sqrt{c+dx^6}\left(3x^6\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)-7acF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-7*a*c*x*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)])/((a + b*x
^6)*Sqrt[c + d*x^6]*(-7*a*c*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x
^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/
a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{bx^6+a}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

3.867 $\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 1, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a*x*Sqrt[c + d*x^6]))
```

Rubi [A] time = 0.0503759, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 1, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a*x*Sqrt[c + d*x^6]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.113662, size = 141, normalized size = 2.27

$$\frac{10bdx^{12}\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-11x^6\sqrt{\frac{dx^6}{c}+1}(bc-2ad)F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-55a\left(c+dx^6\right)}{55a^2cx\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-55*a*(c + d*x^6) - 11*(b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6,
 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*x^12*Sqrt[1 + (d*x^6)/c
]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*x^6)/a)])/(55*a^2*c*x*Sqr
t[c + d*x^6])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(bx^6+a\right)}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^6+c}}{bdx^{14}+(bc+ad)x^8+acx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^6 + c)/(b*d*x^14 + (b*c + a*d)*x^8 + a*c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

3.868 $\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^6)/a), -((d*x^6)/c
)])/(2*a*x^2*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0868468, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^6)/a), -((d*x^6)/c
)])/(2*a*x^2*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$
$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.109684, size = 141, normalized size = 2.2

$$\frac{2bdx^{12}\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+5x^6\sqrt{\frac{dx^6}{c}+1}(ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-20a\left(c+dx^6\right)}{40a^2cx^2\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-20*a*(c + d*x^6) + 5*(-2*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*x^12*Sqrt[1 + (d*x^6)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*x^6)/a)])/(40*a^2*c*x^2*Sqrt[
c + d*x^6])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(bx^6+a\right)}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

3.869 $\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^6)/a), -((d*x^6)/c
)])/(4*a*x^4*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0838677, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^6)/a), -((d*x^6)/c
)])/(4*a*x^4*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.115749, size = 141, normalized size = 2.2

$$\frac{-bdx^{12}\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-4x^6\sqrt{\frac{dx^6}{c}+1}(ad+4bc)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-8a\left(c+dx^6\right)}{32a^2cx^4\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-8*a*(c + d*x^6) - 4*(4*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] - b*d*x^12*Sqrt[1 + (d*x^6)/c]*Appe
llF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6)/a)])/(32*a^2*c*x^4*Sqrt[c +
d*x^6])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(bx^6+a\right)}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 (a + bx^6) \sqrt{c + dx^6}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

3.870 $\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^6}}{6b^2\left(a+bx^6\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}}{3b^2d}$$

```
[Out] Sqrt[c + d*x^6]/(3*b^2*d) - (a^2*Sqrt[c + d*x^6])/(6*b^2*(b*c - a*d)*(a + b
*x^6)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(6*b^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.139582, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^6}}{6b^2\left(a+bx^6\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b^2*d) - (a^2*Sqrt[c + d*x^6])/(6*b^2*(b*c - a*d)*(a + b
*x^6)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(6*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{12b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.236802, size = 107, normalized size = 0.87

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^6}\left(\frac{a^2}{\left(a+bx^6\right)(ad-bc)}+\frac{2}{d}\right)}{b^2} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((Sqrt[c + d*x^6]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^6))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/6
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{x^{17}}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15194, size = 984, normalized size = 8.

$$\left[\frac{\left(\left(4\,ab^{2}cd-3\,a^{2}bd^{2}\right)x^{6}+4\,a^{2}bcd-3\,a^{3}d^{2}\right)\sqrt{b^{2}c-abd}\log\left(\frac{bdx^{6}+2\,bc-ad+2\,\sqrt{dx^{6}+c}\sqrt{b^{2}c-abd}}{bx^{6}+a}\right)+2\left(2\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)\right.}{12\left(ab^{5}c^{2}d-2\,a^{2}b^{4}cd^{2}+a^{3}b^{3}d^{3}+\left(b^{6}c^{2}d-2\,ab^{5}cd^{2}+a^{2}b^{4}d^{3}\right)x^{6}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b
*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6), -
1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14323, size = 181, normalized size = 1.47

$$-\frac{\sqrt{dx^6+c}a^2d}{6\left(b^3c-ab^2d\right)\left(\left(dx^6+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^6+c}}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(d*x^6 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*d))
- 1/6*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/3*sqrt(d*x^6 + c)/(b^2*d)
```

3.871 $\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0803535, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)\\ &= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12b(bc-ad)}\\ &= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6bd(bc-ad)}\\ &= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0878818, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)}+\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{6b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*b^(
3/2))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(bx^{6}+a\right)^{2}} \frac{1}{\sqrt{dx^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88782, size = 724, normalized size = 7.31

$$\left[\frac{\left(\left(2\,b^{2}c-abd\right)x^{6}+2\,abc-a^{2}d\right)\sqrt{b^{2}c-abd}\log\left(\frac{bdx^{6}+2\,bc-ad-2\,\sqrt{dx^{6}+c}\sqrt{b^{2}c-abd}}{bx^{6}+a}\right)+2\,\sqrt{dx^{6}+c}\left(ab^{2}c-a^{2}bd\right)}{12\left(ab^{4}c^{2}-2\,a^{2}b^{3}cd+a^{3}b^{2}d^{2}+\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)x^{6}\right)}, \frac{\left(\left(2\,b^{2}c-abd\right)x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a))
+ 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b
^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a*b*d
)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11683, size = 157, normalized size = 1.59

$$\frac{\frac{\sqrt{dx^6+c}ad^2}{\left(b^2c-abd\right)\left(\left(dx^6+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.872 $\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^6]/(6*(b*c - a*d)*(a + b*x^6)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[b*c - a*d]])/(6*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0685971, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(6*(b*c - a*d)*(a + b*x^6)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[b*c - a*d]])/(6*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}-\frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12(bc-ad)}\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}-\frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6(bc-ad)}\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.088276, size = 85, normalized size = 0.98

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^6}}{\left(a+bx^6\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/((-(b*c) + a*d)*(a + b*x^6)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.53353, size = 640, normalized size = 7.36

$$\left[-\frac{\left(bdx^6+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\left(b^2c-abd\right)}{12\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)},-\frac{\left(bdx^6+ad\right)\sqrt{-b^2c+abd}\arctan\left(\right.}{6\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2*
sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*
```

```
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12672, size = 124, normalized size = 1.43

$$-\frac{1}{6}\,d\left(\frac{\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{\sqrt{dx^6+c}}{\left(\left(dx^6+c\right)b-bc+ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d*(arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) + sqrt(d*x^6 + c)/(((d*x^6 + c)*b - b*c + a*d)*(b*c - a*d)))
```

3.873 $\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

```
[Out] (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) - ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^2*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.127238, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) - ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^2*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{12a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3a^2d} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}}\right.}{6a^2d(bc-a}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.233277, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - (2*ArcTanh[Sqrt[c + d*x^
6]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*a^2)
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x), x)
```

Fricas [A] time = 1.9033, size = 1833, normalized size = 13.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x^6
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
6 + b*c)) + ((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sq
rt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c
^2 - a^4*c*d), 1/12*(2*sqrt(d*x^6 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^6 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b
*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6
```

```
 + b*c)) + 2*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^6 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14145, size = 207, normalized size = 1.57

$$-\frac{1}{6}\,d^2\left(\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx^6+c}b}{\left(abcd-a^2d^2\right)\left(\left(dx^6+c\right)b-bc+ad\right)}-\frac{2\,\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^2*((2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d)
)/((a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x^6 + c)*b/((a*b*
c*d - a^2*d^2)*((d*x^6 + c)*b - b*c + a*d)) - 2*arctan(sqrt(d*x^6 + c)/sqrt
(-c))/(a^2*sqrt(-c)*d^2))
```

3.874 $\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}(2bc-ad)}{6a^2c\left(a+bx^6\right)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}$$

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*(a + b*x^6)) - Sqrt
[c + d*x^6]/(6*a*c*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6
]/Sqrt[c]])/(6*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^3*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.227679, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}(2bc-ad)}{6a^2c\left(a+bx^6\right)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```


```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*(a + b*x^6)) - Sqrt
[c + d*x^6]/(6*a*c*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6
]/Sqrt[c]])/(6*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^3*(b*c - a*d)^(3/2))
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)}{6ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x\right.}{6a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,\right.}{12a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{6a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} - \frac{b^{3/2}(4bc-}{}$$

Mathematica [A] time = 0.537189, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^6}\left(a^2d+ab\left(dx^6-c\right)-2b^2cx^6\right)}{x^6\left(a+bx^6\right)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[c + d*x^6]*(a^2*d - 2*b^2*c*x^6 + a*b*(-c + d*x^6)))/((b*c - a*d)*
x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(6*a^3*c)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{x^7 \left(bx^6+a\right)^2} \frac{1}{\sqrt{dx^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2 \sqrt{dx^6+c}\, x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)
```

Fricas [A] time = 2.36195, size = 2525, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6
 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12
+ (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
```

```
6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sq
rt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt
(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*
b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/6*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^6 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19547, size = 362, normalized size = 1.96

$$\frac{1}{6}\,d^3\left(\frac{\left(4\,b^3c - 5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3bcd^3 - a^4d^4\right)\sqrt{-b^2c + abd}} - \frac{2\left(dx^6 + c\right)^{\frac{3}{2}}b^2c - 2\,\sqrt{dx^6 + c}b^2c^2 - \left(dx^6 + c\right)^{\frac{3}{2}}abd + 2\,\sqrt{dx^6 + c}abcd - \sqrt{dx}}{\left(a^2bc^2d^2 - a^3cd^3\right)\left(\left(dx^6 + c\right)^2b - 2\left(dx^6 + c\right)bc + bc^2 + \left(dx^6 + c\right)ad - a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*d^3*((4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d
))/((a^3*b*c*d^3 - a^4*d^4)*sqrt(-b^2*c + a*b*d)) - (2*(d*x^6 + c)^(3/2)*b^
2*c - 2*sqrt(d*x^6 + c)*b^2*c^2 - (d*x^6 + c)^(3/2)*a*b*d + 2*sqrt(d*x^6 +
c)*a*b*c*d - sqrt(d*x^6 + c)*a^2*d^2)/((a^2*b*c^2*d^2 - a^3*c*d^3)*((d*x^6
+ c)^2*b - 2*(d*x^6 + c)*b*c + b*c^2 + (d*x^6 + c)*a*d - a*c*d)) - (4*b*c +
 a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^3*sqrt(-c)*c*d^3))
```

3.875 $\int \frac{x^{14}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{ax^3\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b^2*Sqrt[d])
```

Rubi [A] time = 0.157584, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{ax^3\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b^2*Sqrt[d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)$$

$$=\frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}-\frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6b(bc-ad)}$$

$$=\frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^3\right)}{3b^2}-\frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx}}\right.}{6b^2(bc-ad)}$$

$$=\frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}+\frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2}-\frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad}\right.}{6b^2(bc-ad)}$$

$$=\frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Mathematica [A] time = 0.210232, size = 135, normalized size = 0.96

$$\frac{\frac{abx^3\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)}+\frac{\sqrt{a}(2ad-3bc)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{(bc-ad)^{3/2}}+\frac{2\log\left(\sqrt{d}\sqrt{c+dx^6}+dx^3\right)}{\sqrt{d}}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]])/Sqrt[d])/(6*b^2)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{14}}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [A] time = 4.16804, size = 2279, normalized size = 16.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a
*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3
 - 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqr
t(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(
-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*
b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a
*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12
*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d -
2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + 2*((b^2*c - a*b*d)*x^6
```

```
 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c)
)/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12*(2*sqrt(d*x^6
 + c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(s
qrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2
*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*
x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)))/((b^4*c*d - a*b^3*d^2)*x
^6 + a*b^3*c*d - a^2*b^2*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24882, size = 482, normalized size = 3.42

$$\frac{1}{6}c^2\left(\frac{\left(3abc-2a^2d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{\left(b^3c^3\mathrm{sgn}\left(x\right)-ab^2c^2d\mathrm{sgn}\left(x\right)\right)\sqrt{abc-a^2d}}+\frac{a\sqrt{d+\frac{c}{x^6}}}{\left(b^2c^2\mathrm{sgn}\left(x\right)-abcd\mathrm{sgn}\left(x\right)\right)\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)}-\frac{2\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{b^2c^2\sqrt{-d}\mathrm{sgn}\left(x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*c^2*((3*a*b*c - 2*a^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/
((b^3*c^3*sgn(x) - a*b^2*c^2*d*sgn(x))*sqrt(a*b*c - a^2*d)) + a*sqrt(d + c/
x^6)/((b^2*c^2*sgn(x) - a*b*c*d*sgn(x))*(b*c + a*(d + c/x^6) - a*d)) - 2*ar
ctan(sqrt(d + c/x^6)/sqrt(-d))/(b^2*c^2*sqrt(-d)*sgn(x))) - 1/6*(3*a*b*c*sq
rt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(-d)*d*arctan(a*sq
rt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*arctan(sqrt(d)/sqrt(
-d)) + 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)/sqrt(-d)) + sqrt(a*b*c - a^
2*d)*a*sqrt(-d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^3*c*sqrt(-d) - sqrt(
a*b*c - a^2*d)*a*b^2*sqrt(-d)*d)
```

3.876 $\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

```
[Out] -(x^3*Sqrt[c + d*x^6])/(6*(b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.09026, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(x^3*Sqrt[c + d*x^6])/(6*(b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{\operatorname{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.443684, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^6}\left(-\frac{x^6(bc-ad)}{a+bx^6}-\frac{c\sqrt{x^6\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^6\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^6}{c}+1}}\right)}{\sqrt{\frac{dx^6}{c}+1}}\right)}{6x^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-(((b*c - a*d)*x^6)/(a + b*x^6)) - (c*Sqrt[(-(b/a) + d/c)
*x^6]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^6]/Sqrt[1 + (d*x^6)/c]])/Sqrt[1 + (d*x^
6)/c]))/(6*(b*c - a*d)^2*x^3)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^8}{(bx^6+a)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

[Out] integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

Fricas [B] time = 2.44086, size = 887, normalized size = 9.54

$$\left[-\frac{4\sqrt{dx^6+c}\left(abc-a^2d\right)x^3-\left(bcx^6+ac\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left(\left(bc-2\,ad\right)x^9-acx^3\right)\sqrt{dx^6}}{b^2x^{12}+2\,abx^6+a^2}\right)}{24\left(\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Timed out

Giac [B] time = 1.27726, size = 248, normalized size = 2.67

$$-\frac{1}{6}\,c\left(\frac{\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}\left(bc\,\mathrm{sgn}\left(x\right)-ad\,\mathrm{sgn}\left(x\right)\right)}+\frac{\sqrt{d+\frac{c}{x^6}}}{\left(bc\,\mathrm{sgn}\left(x\right)-ad\,\mathrm{sgn}\left(x\right)\right)\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)}\right)+\frac{\left(bc\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)+\sqrt{}\right.}{6\left(\sqrt{abc-a^2d}\,db^2c-\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*c*(arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*
(b*c*sgn(x) - a*d*sgn(x))) + sqrt(d + c/x^6)/((b*c*sgn(x) - a*d*sgn(x))*(b*
c + a*(d + c/x^6) - a*d))) + 1/6*(b*c*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d))
 + sqrt(a*b*c - a^2*d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^2*c - sqrt(a*
b*c - a^2*d)*a*b*d)
```

3.877 $\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

```
[Out] (b*x^3*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rubi [A] time = 0.0981687, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*x^3*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)\\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6a(bc-ad)}\\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{6a(bc-ad)}\\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.771697, size = 407, normalized size = 3.91

$$\frac{x^3\sqrt{c+dx^6}\left(-30dx^6\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}-45c\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}+16dx^6\left(\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^6\right)}{c\left(a+bx^6\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)+16c\cdots\right.}{90c^2\left(a+bx^6\right)^2\left(\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}\right)^{3/2}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6]*(-45*c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a +
b*x^6)^2)] - 30*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 30*d*x^6*ArcS
in[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 16*c*(((b*c - a*d)*x^6)/(c*(a
 + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 16*d*x^6*(((b*c - a*d)*x^6)/
(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))]))/(90*c^2*(((b*c - a*d)*x
^6)/(c*(a + b*x^6)))^(3/2)*(a + b*x^6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6
))])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [B] time = 2.77535, size = 971, normalized size = 9.34

$$\left[\frac{4\,\sqrt{dx^6+c}\left(ab^2c-a^2bd\right)x^3-\left(\left(b^2c-2\,abd\right)x^6+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4}{b^2x^{12}+2\,abx^6+a}\right.}{24\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 -
a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)
))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^
4*b*d^2)*x^6), 1/12*(2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^2*c -
2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*
d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9
+ (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.183, size = 320, normalized size = 3.08

$$-\frac{1}{6}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2bc-2\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right.\right.}{\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^4b-2\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2bc+4\left(\sqrt{d}x^3-\sqrt{dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqrt(d*
x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

3.878 $\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{c+dx^6}(3bc-2ad)}{6a^2cx^3(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^6}}{6ax^3\left(a+bx^6\right)(bc-ad)}$$

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c +
d*x^6])/(6*a*(b*c - a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.193936, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^6}(3bc-2ad)}{6a^2cx^3(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^6}}{6ax^3\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```


```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c +
d*x^6])/(6*a*(b*c - a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(5/2)*(b*c - a*d)^(3/2))
```


Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2 \left(a+bx^2\right)^2 \sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\right.}{6a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{6a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 1.59069, size = 869, normalized size = 5.83

$$-\frac{\sqrt{dx^6+c}\left(120d^2\sin^{-1}\left(\sqrt{\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}}\right)x^{12} + 96d^2\left(\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)^{5/2}\sqrt{\frac{a\left(dx^6+c\right)}{c\left(bx^6+a\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)x^{12} + 32d^2\left(\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)^{5/2}\sqrt{\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[c + d*x^6]*(-45*c^2*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b
*x^6)^2)] - 180*c*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^
6)^2)] - 120*d^2*x^12*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)
^2)] + 45*c^2*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 180*c*d*x^6
*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 120*d^2*x^12*ArcSin[Sqrt
[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 64*c^2*(((b*c - a*d)*x^6)/(c*(a + b*
x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2, 3,
```

```
7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 160*c*d*x^6*(((b*c - a*d)*x^6)/(c
*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F
1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 96*d^2*x^12*(((b*c - a*d)
*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeo
metric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 32*c^2*(((b*c - a
*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hyper
geometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 64*c
*d*x^6*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a
 + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a
+ b*x^6))] + 32*d^2*x^12*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*
(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c
- a*d)*x^6)/(c*(a + b*x^6))]))/(90*c^3*x^3*(((b*c - a*d)*x^6)/(c*(a + b*x^6
)))^(3/2)*(a + b*x^6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))])
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \left(bx^6+a\right)^2} \frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2 \sqrt{dx^6+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)
```

Fricas [B] time = 2.86221, size = 1260, normalized size = 8.46

$$\left[-\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^9+\left(3\,ab^2c^2-4\,a^2bcd\right)x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left((bc-2\,ad)x^9-acx\right.}{b^2x^{12}+2\,abx^6+a^2}\right.}{24\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)x^9+\left(a^4b^2c^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6
 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3*c^2
 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d
^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (
a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 - 4*a*b^2
*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d -
a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2*c*d
+ 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^6 +
c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b^2*c^3 - 2*a
^5*b*c^2*d + a^6*c*d^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38362, size = 386, normalized size = 2.59

$$-\frac{b^2c\sqrt{d+\frac{c}{x^6}}}{6\left(a^2bc\operatorname{sgn}(x)-a^3d\operatorname{sgn}(x)\right)\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)}-\frac{\left(3\,b^2c^2\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-4\,abcd\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-3\sqrt{abc-a^2\right.}{6\left(\sqrt{abc-a^2d}a^2bc^2-\sqrt{abc-a^2d}a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*b^2*c*sqrt(d + c/x^6)/((a^2*b*c*sgn(x) - a^3*d*sgn(x))*(b*c + a*(d + c
/x^6) - a*d)) - 1/6*(3*b^2*c^2*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 4*a*
b*c*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 3*sqrt(a*b*c - a^2*d)*b*c*sqr
t(d) + 2*sqrt(a*b*c - a^2*d)*a*d^(3/2))*sgn(x)/(sqrt(a*b*c - a^2*d)*a^2*b*c
^2 - sqrt(a*b*c - a^2*d)*a^3*c*d) + 1/6*(3*b^2*c - 4*a*b*d)*arctan(a*sqrt(d
 + c/x^6)/sqrt(a*b*c - a^2*d))/((a^2*b*c*sgn(x) - a^3*d*sgn(x))*sqrt(a*b*c
- a^2*d)) - 1/3*sqrt(d + c/x^6)/(a^2*c*sgn(x))
```

3.879 $\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=208

$$\frac{\sqrt{c+dx^6}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{18a^3c^2x^3(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{18a^2cx^9(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^9\left(a+bx^6\right)(bc-ad)}$$

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^6])/(18*a^2*c*(b*c - a*d)*x^9) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^6])/(18*a^3*c^2*(b*c - a*d)*x^3) +
(b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rubi [A] time = 0.308015, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^6}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{18a^3c^2x^3(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{18a^2cx^9(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^9\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^6])/(18*a^2*c*(b*c - a*d)*x^9) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^6])/(18*a^3*c^2*(b*c - a*d)*x^3) +
(b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} - \frac{\operatorname{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} + \frac{\operatorname{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc-}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\right.}{18a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)}$$

Mathematica [C] time = 2.73896, size = 1535, normalized size = 7.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(45*c^3*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x
^6)^2)] - 270*c^2*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^
6)^2)] - 1080*c*d^2*x^12*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x
^6)^2)] - 720*d^3*x^18*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6
)^2)] - 45*c^3*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 270*c^2*d*
x^6*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 1080*c*d^2*x^12*ArcSi
n[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 720*d^3*x^18*ArcSin[Sqrt[((b*c
 - a*d)*x^6)/(c*(a + b*x^6))]] - 16*c^3*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))
```

```
^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2, 3, 7/2, (
(b*c - a*d)*x^6)/(c*(a + b*x^6))] + 528*c^2*d*x^6*(((b*c - a*d)*x^6)/(c*(a
+ b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2,
 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 1248*c*d^2*x^12*(((b*c - a*d)
*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeo
metric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 704*d^3*x^18*(((b
*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]
*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 96*c^3*(
((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6
))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6)
)] + 576*c^2*d*x^6*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d
*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)
*x^6)/(c*(a + b*x^6))] + 864*c*d^2*x^12*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))
^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {
1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 384*d^3*x^18*(((b*c - a*d)*x^
6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeomet
ricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 64*c^3*(((
b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))
]*HypergeometricPFQ[{2, 2, 2, 3}, {1, 1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*
x^6))] + 192*c^2*d*x^6*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c
 + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 2, 3}, {1, 1, 7/2}, ((
b*c - a*d)*x^6)/(c*(a + b*x^6))] + 192*c*d^2*x^12*(((b*c - a*d)*x^6)/(c*(a
+ b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2
, 2, 2, 3}, {1, 1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 64*d^3*x^18*(
((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6
))]*HypergeometricPFQ[{2, 2, 2, 3}, {1, 1, 7/2}, ((b*c - a*d)*x^6)/(c*(a +
b*x^6))]))/(270*c^4*x^9*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(3/2)*(a + b*x^
6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))])
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{x^{10}\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+cx^{10}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)
```

Fricas [A] time = 3.53685, size = 1553, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^9)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*s
qrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*((15
*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^12 - 2*a^3
*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d
 + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3*c^4 - 2*a^5*b^2
*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*
x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*
c^2*d)*x^9)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d
)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d
^3)*x^12 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 -
 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3
*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d
 + a^7*c^2*d^2)*x^9)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43885, size = 494, normalized size = 2.38

$$\frac{b^3c\sqrt{d+\frac{c}{x^6}}}{6\left(a^3bc\operatorname{sgn}(x)-a^4d\operatorname{sgn}(x)\right)\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)}+\frac{\left(15\,b^3c^3\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-18\,ab^2c^2d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-15\sqrt{ab\cdots}\right.}{18\left(\sqrt{abc-a^2d}a^3bc^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b^3*c*sqrt(d + c/x^6)/((a^3*b*c*sgn(x) - a^4*d*sgn(x))*(b*c + a*(d + c/
x^6) - a*d)) + 1/18*(15*b^3*c^3*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 18*
a*b^2*c^2*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 15*sqrt(a*b*c - a^2*d)*
b^2*c^2*sqrt(d) + 8*sqrt(a*b*c - a^2*d)*a*b*c*d^(3/2) + 4*sqrt(a*b*c - a^2*
d)*a^2*d^(5/2))*sgn(x)/(sqrt(a*b*c - a^2*d)*a^3*b*c^3 - sqrt(a*b*c - a^2*d)
*a^4*c^2*d) - 1/6*(5*b^3*c - 6*a*b^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c
 - a^2*d))/((a^3*b*c*sgn(x) - a^4*d*sgn(x))*sqrt(a*b*c - a^2*d)) + 1/9*(6*a
^3*b*c^5*sqrt(d + c/x^6) - a^4*c^4*(d + c/x^6)^(3/2) + 3*a^4*c^4*sqrt(d + c
/x^6)*d)/(a^6*c^6*sgn(x))
```

3.880 $\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 2, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a^2*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0512849, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 2, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a^2*Sqrt[c + d*x^6])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.171658, size = 169, normalized size = 2.64

$$\frac{x^5\left(-10bdx^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+11\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}(bc-6ad)F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right.}{330a^2\left(a+bx^6\right)\sqrt{c+dx^6}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*(55*a*b*(c + d*x^6) + 11*(b*c - 6*a*d)*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]
*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] - 10*b*d*x^6*(a +
b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*
x^6)/a)]))/(330*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^6+c}\,x^4}{b^2dx^{18}+(b^2c+2\,abd)x^{12}+(2\,abc+a^2d)x^6+a^2c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^6 + c)*x^4/(b^2*d*x^18 + (b^2*c + 2*a*b*d)*x^12 + (2*a*b*
c + a^2*d)*x^6 + a^2*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.881 $\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a^2*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0730317, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a^2*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.16944, size = 168, normalized size = 2.62

$$-\frac{x^4\left(bdx^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-5\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}(bc-3ad)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-10a\right.}{60a^2\left(a+bx^6\right)\sqrt{c+dx^6}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(x^4*(-10*a*b*(c + d*x^6) - 5*(b*c - 3*a*d)*(a + b*x^6)*Sqrt[1 + (d*x^6)/c
]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^6*(a + b*x
^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*x^6)/
a)]))/(60*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.882 $\int \frac{x}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a^2*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0540988, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 430, 429}

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a^2*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx,x,x^2\right)}{2\sqrt{c+dx^6}}\\ &= \frac{x^2\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [B] time = 0.154948, size = 172, normalized size = 2.69

$$\frac{bdx^8\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+8x^2\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}(2bc-3ad)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+8abx^2}{48a^2\left(a+bx^6\right)\sqrt{c+dx^6}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (8*a*b*x^2*(c + d*x^6) + 8*(2*b*c - 3*a*d)*x^2*(a + b*x^6)*Sqrt[1 + (d*x^6)
/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^8*(a + b
*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6
)/a)])/(48*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.883 $\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 2, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a^2*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.02523, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 2, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a^2*Sqrt[c + d*x^6])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{\left(a+bx^6\right)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.28211, size = 329, normalized size = 5.58

$$\frac{x\left(-2bdx^6\sqrt{\frac{dx^6}{c}+1}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-\frac{7a\left(3bx^6\left(c+dx^6\right)\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)+7ac\left(6ad-b\left(6c+d\right.\right.\right.}{\left(a+bx^6\right)\left(3x^6\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)-7acF_1\left(\frac{1}{6}\right.\right.}\right.}{42a^2\sqrt{c+dx^6}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*(-2*b*d*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[7/6, 1/2, 1, 13/6, -((d*x^6)/c)
, -((b*x^6)/a)] - (7*a*(7*a*c*(6*a*d - b*(6*c + d*x^6))*AppellF1[1/6, 1/2,
1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*b*x^6*(c + d*x^6)*(2*b*c*AppellF1[7
/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)])))/((a + b*x^6)*(-7*a*c*AppellF1[1/6, 1/2,
 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6
)/c), -((b*x^6)/a)])))))/(42*a^2*(-(b*c) + a*d)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

3.884 $\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 2, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a^2*x*Sqrt[c + d*x^6]))
```

Rubi [A] time = 0.0524712, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 2, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a^2*x*Sqrt[c + d*x^6]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= -\frac{\sqrt{1+\frac{dx^6}{c}}F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.261364, size = 226, normalized size = 3.65

$$\frac{-11x^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\left(12a^2d^2-24abcd+7b^2c^2\right)F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+55a\left(c+dx^6\right)\left(6a^2d-6ab\left(c-dx^6\right)-\right.}{330a^3cx\left(a+bx^6\right)\sqrt{c+dx^6}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (55*a*(c + d*x^6)*(6*a^2*d - 7*b^2*c*x^6 - 6*a*b*(c - d*x^6)) - 11*(7*b^2*c
^2 - 24*a*b*c*d + 12*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[
5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*(7*b*c - 6*a*d)*x^1
2*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c)
, -((b*x^6)/a)])/(330*a^3*c*(b*c - a*d)*x*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(bx^6+a\right)^2}\frac{1}{\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^6+c}}{b^2dx^{20}+\left(b^2c+2\,abd\right)x^{14}+\left(2\,abc+a^2d\right)x^8+a^2cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^6 + c)/(b^2*d*x^20 + (b^2*c + 2*a*b*d)*x^14 + (2*a*b*c +
a^2*d)*x^8 + a^2*c*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

3.885 $\int \frac{1}{x^3(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^6)/a), -((d*x^6)/c
)])/(2*a^2*x^2*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.0875388, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^6)/a), -((d*x^6)/c
)])/(2*a^2*x^2*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx,x,x^2\right)\\&=\frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx,x,x^2\right)}{2\sqrt{c+dx^6}}\\&=-\frac{\sqrt{1+\frac{dx^6}{c}}F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [B] time = 0.24826, size = 226, normalized size = 3.53

$$\frac{-5x^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\left(3a^2d^2-15abcd+8b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+20a\left(c+dx^6\right)\left(3a^2d-3ab\left(c-dx^6\right)-4b^2\right.}{120a^3cx^2\left(a+bx^6\right)\sqrt{c+dx^6}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (20*a*(c + d*x^6)*(3*a^2*d - 4*b^2*c*x^6 - 3*a*b*(c - d*x^6)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b
*x^6)/a)])/(120*a^3*c*(b*c - a*d)*x^2*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(b x^{6}+a \right)^{2}} \frac{1}{\sqrt{d x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{6}+a\right)^{2}\sqrt{dx^{6}+c}x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

3.886 $\int \frac{1}{x^5(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^6)/a), -((d*x^6)/c
)])/(4*a^2*x^4*Sqrt[c + d*x^6])
```

Rubi [A] time = 0.082107, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^6)/a), -((d*x^6)/c
)])/(4*a^2*x^4*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx,x,x^2\right)}{2\sqrt{c+dx^6}}$$
$$=-\frac{\sqrt{1+\frac{dx^6}{c}}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.259602, size = 225, normalized size = 3.52

$$\frac{4x^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\left(3a^2d^2+21abcd-20b^2c^2\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+8a\left(c+dx^6\right)\left(3a^2d-3ab\left(c-dx^6\right)-5b^2c\right.}{96a^3cx^4\left(a+bx^6\right)\sqrt{c+dx^6}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (8*a*(c + d*x^6)*(3*a^2*d - 5*b^2*c*x^6 - 3*a*b*(c - d*x^6)) + 4*(-20*b^2*c
^2 + 21*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[1
/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*(-5*b*c + 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b
*x^6)/a)])/(96*a^3*c*(b*c - a*d)*x^4*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{x^{5} \left(b x^{6}+a\right)^{2}} \frac{1}{\sqrt{d x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{6}+a\right)^{2} \sqrt{d x^{6}+c} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

3.887 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^8])/(4*b^2*d^2) + (c + d*x^8)^(3/2)/(12*b*d^2) -
 (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b
*c - a*d])
```

Rubi [A] time = 0.0985776, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^8])/(4*b^2*d^2) + (c + d*x^8)^(3/2)/(12*b*d^2) -
 (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b
*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{23}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^8\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{\left(c+dx^8\right)^{3/2}}{12bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{\left(c+dx^8\right)^{3/2}}{12bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{4b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{\left(c+dx^8\right)^{3/2}}{12bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.17129, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^8}\left(-3ad-2bc+bdx^8\right)}{12b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) - (a^2*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b*c - a*d])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x^{23}}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36578, size = 595, normalized size = 5.72

$$\left[\frac{3\,\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\left(\left(b^3cd-ab^2d^2\right)x^8-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{dx^8+c}}{24\left(b^4cd^2-ab^3d^3\right)}, \frac{3\,\sqrt{-b^2c}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*
x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8 -
```

```
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*
c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 - a*b^
2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17604, size = 143, normalized size = 1.38

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}b^2}+\frac{\left(dx^8+c\right)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^8+c}b^2cd^4-3\sqrt{dx^8+c}abd^5}{12\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/12*((d*x^8 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^8 + c)*b^2*c*d^4 - 3*
sqrt(d*x^8 + c)*a*b*d^5)/(b^3*d^6)
```

3.888 $\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}}{4bd}$$

```
[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a
*d]])/(4*b^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0610699, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a
*d]])/(4*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\ &= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b}\\ &= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4bd}\\ &= \frac{\sqrt{c+dx^8}}{4bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0629486, size = 72, normalized size = 0.97

$$\frac{1}{4}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^8}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/4
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^{15}}{b x^8+a}\frac{1}{\sqrt{d x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30984, size = 437, normalized size = 5.91

$$\left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\left(b^2c-abd\right)}{8\left(b^3cd-ab^2d^2\right)},-\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-\sqrt{}}{4\left(b^3cd-ab^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c
)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A] time = 1.12297, size = 86, normalized size = 1.16

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^8+c}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^8 + c)/b)/d
```

3.889 $\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(4*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rubi [A] time = 0.0468158, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(4*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.0147228, size = 51, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(4*Sqrt[b]*Sqrt[b*c - a
*d])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{x^7}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30495, size = 288, normalized size = 5.65

$$\left[\frac{\log\left(\frac{bdx^{8}+2\,bc-ad-2\,\sqrt{dx^{8}+c}\sqrt{b^{2}c-abd}}{bx^{8}+a}\right)}{8\,\sqrt{b^{2}c-abd}}, \frac{\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{dx^{8}+c}\sqrt{-b^{2}c+abd}}{bdx^{8}+bc}\right)}{4\left(b^{2}c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b
*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] time = 37.5565, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^{8}}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c)/b))
```

Giac [A] time = 1.11318, size = 54, normalized size = 1.06

$$\frac{\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.890 $\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

```
[Out] -ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0792753, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a*Sqrt[b*c - a*d])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a} - \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.0772861, size = 81, normalized size = 0.95

$$
\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^8])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(4*a)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(bx^{8}+a\right)}\frac{1}{\sqrt{dx^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{8}+a\right)\sqrt{dx^{8}+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)
```

Fricas [A] time = 1.3874, size = 948, normalized size = 11.15

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{8}+2\,bc-ad+2\,\sqrt{dx^{8}+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^{8}+a}\right)+\sqrt{c}\log\left(\frac{dx^{8}-2\,\sqrt{dx^{8}+c}\sqrt{c}+2\,c}{x^{8}}\right)}{8\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^{8}+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^{8}+bc}\right)+\right.}{8\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sqrt
(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(2*c*sqrt(-b/(b*c - a*d))*arcta
```

```
n(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt
(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-
c)/c))/(a*c), 1/4*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^8 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A] time = 44.69, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**8)/sqrt(-c))/(4*a*sqrt(-c))
```

Giac [A] time = 1.14335, size = 107, normalized size = 1.26

$$-\frac{1}{4}d\left(\frac{b\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ad}-\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*(b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*a*d) - arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c)*d))
```

3.891 $\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

```
[Out] -Sqrt[c + d*x^8]/(8*a*c*x^8) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[
c]])/(8*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c
- a*d]])/(4*a^2*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.112717, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```


```
[Out] -Sqrt[c + d*x^8]/(8*a*c*x^8) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[
c]])/(8*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c
- a*d]])/(4*a^2*Sqrt[b*c - a*d])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}-\frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8ac}\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a^2}-\frac{(2bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{16a^2c}\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4a^2d}-\frac{(2bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x,\right.}{8a^2cd}\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.116601, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2(ad-bc)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8ac^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(8*a*c*x^8) + (b*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(4*a^2*
Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a*c^(3/2)) + (b^(3/2)*Sq
rt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a^2*(-
(b*c) + a*d))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^9\left(bx^8+a\right)}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)
```

Fricas [A] time = 1.44823, size = 1264, normalized size = 10.8

$$\left[\frac{2\,bc^2x^8\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+(2\,bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)-2\,\sqrt{dx^8+c}ac}{16\,a^2c^2x^8},-\frac{4\,bc^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(
d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + (2*b*c + a*d)*sq
rt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*x^8
 + c)*a*c)/(a^2*c^2*x^8), -1/16*(4*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*sq
rt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), 1/8*(b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^8 + a)) - (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c)
 - sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), -1/8*(2*b*c^2*x^8*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + sqrt(d*
x^8 + c)*a*c)/(a^2*c^2*x^8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16155, size = 159, normalized size = 1.36

$$\frac{1}{8}\,d^2\left(\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2d^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}cd^2}-\frac{\sqrt{dx^8+c}}{acd^2x^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^2*(2*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*a^2*d^2) - (2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqr
t(-c)*c*d^2) - sqrt(d*x^8 + c)/(a*c*d^2*x^8))
```

3.892 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}+\frac{x^4\sqrt{c+dx^8}}{8bd}$$

```
[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt
[a]*Sqrt[c + d*x^8])])/(4*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^4)/Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))
```

Rubi [A] time = 0.130621, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}+\frac{x^4\sqrt{c+dx^8}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt
[a]*Sqrt[c + d*x^8])])/(4*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^4)/Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
```

```
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{19}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} - \frac{\operatorname{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8bd}\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8b^2d}\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c-}}\right.}{8b^2d}\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.131722, size = 118, normalized size = 0.96

$$
\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^8}+dx^4\right)}{d^{3/2}} + \frac{bx^4\sqrt{c+dx^8}}{d}}{8b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ((b*x^4*Sqrt[c + d*x^8])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[
a]*Sqrt[c + d*x^8])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^4 + Sqrt[d]*
Sqrt[c + d*x^8]])/d^(3/2))/(8*b^2)
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$
\int \frac{x^{19}}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{19}}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [A] time = 1.75201, size = 1638, normalized size = 13.32

$$\left[\frac{2\sqrt{dx^8+c}bdx^4 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^{12}-\left(abc^2-a^2cd\right)x^4\right)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc}}}{b^2x^{16}+2\,abx^8+a^2}\right.}{16\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c + 2*a*
d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^
8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*c + 2*a*d
)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x
^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + (b*c
+ 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^
2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((
b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c
```

```
*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*
d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39152, size = 140, normalized size = 1.14

$$\frac{\sqrt{dx^8+c}x^4}{8\,bd}-\frac{a^2\arctan\left(\frac{a\sqrt{d+\frac{c}{x^8}}}{\sqrt{abc-a^2d}}\right)}{4\,\sqrt{abc-a^2d}b^2}+\frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{d+\frac{c}{x^8}}}{\sqrt{-d}}\right)}{8\,b^2\sqrt{-d}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^8 + c)*x^4/(b*d) - 1/4*a^2*arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c
 - a^2*d))/(sqrt(a*b*c - a^2*d)*b^2) + 1/8*(b*c + 2*a*d)*arctan(sqrt(d + c/
x^8)/sqrt(-d))/(b^2*sqrt(-d)*d)
```

3.893 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b*Sqrt[d])
```

Rubi [A] time = 0.110115, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b*Sqrt[d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0562687, size = 90, normalized size = 0.99

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^8}+dx^4\right)}{\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{\sqrt{bc-ad}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/Sqrt[b
*c - a*d]) + Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]]/Sqrt[d])/(4*b)
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [A] time = 1.43658, size = 1372, normalized size = 15.08

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^{12}-\left(abc^2-a^2cd\right)x^4\right)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2\,abx^8+a^2}\right)+2\sqrt{d}\log\left(-2\,a\right.}{16\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*
sqrt(d)*x^4 - c))/(b*d), 1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c
)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*sqrt(-d)*arctan(s
qrt(-d)*x^4/sqrt(d*x^8 + c)))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2
*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 +
a*c*x^4)) + sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b*d
), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*
x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) - 2*sqrt(-d)*arctan(sqrt
(-d)*x^4/sqrt(d*x^8 + c)))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{(a + bx^8)\sqrt{c + dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A] time = 1.24553, size = 107, normalized size = 1.18

$$\frac{1}{4}\,c\left(\frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^8}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}bc} - \frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^8}}}{\sqrt{-d}}\right)}{bc\sqrt{-d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*c*(a*arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*b*c) - arctan(sqrt(d + c/x^8)/sqrt(-d))/(b*c*sqrt(-d)))
```

3.894 $\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rubi [A] time = 0.0546701, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.0660526, size = 95, normalized size = 1.76

$$\frac{x^4\sqrt{\frac{dx^8}{c}+1}\tanh^{-1}\left(\frac{\sqrt{\frac{dx^8}{c}-\frac{bx^8}{a}}}{\sqrt{\frac{dx^8}{c}+1}}\right)}{4a\sqrt{c+dx^8}\sqrt{\frac{dx^8}{c}-\frac{bx^8}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^8)/c]*ArcTanh[Sqrt[-((b*x^8)/a) + (d*x^8)/c]/Sqrt[1 + (d
*x^8)/c]])/(4*a*Sqrt[c + d*x^8]*Sqrt[-((b*x^8)/a) + (d*x^8)/c])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^3}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(bx^8 + a)\sqrt{dx^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [B] time = 1.28894, size = 525, normalized size = 9.72

$$\left[-\frac{\sqrt{-abc + a^2d}\log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^{16} - 2\left(3\,abc^2 - 4\,a^2cd\right)x^8 + a^2c^2 - 4\left((bc - 2\,ad)x^{12} - acx^4\right)\sqrt{dx^8 + c}\sqrt{-abc + a^2d}}{b^2x^{16} + 2\,abx^8 + a^2}\right)}{16\left(abc - a^2d\right)}, \frac{\arctan\left(\frac{\left((bc - 2\,ad)x^8 - ac\right)\sqrt{a}}{2\left(\left(abcd - a^2d^2\right)x^{12} +\right.}\right.}{8\sqrt{abc - a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*s
qrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*c -
 a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*
c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(a*b*c
 - a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + bx^8)\sqrt{c + dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A] time = 1.20488, size = 97, normalized size = 1.8

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{4\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.895 $\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

```
[Out] -Sqrt[c + d*x^8]/(4*a*c*x^4) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqr
t[c + d*x^8])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0873839, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(4*a*c*x^4) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqr
t[c + d*x^8])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4ac}\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4a}\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4a}\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 0.699402, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^8}{c}+1\right)\left(\frac{4x^8(c+dx^8)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^8}{c(bx^8+a)}\right)}{3c^2(a+bx^8)}+\frac{(c+2dx^8)\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c(a+bx^8)}}\right)}{c\sqrt{\frac{ax^8(c+dx^8)(bc-ad)}{c^2(a+bx^8)^2}}}\right)}{4x^4\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((1 + (d*x^8)/c)*(((c + 2*d*x^8)*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x
^8))]])/(c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)^2)]) + (4*
(b*c - a*d)*x^8*(c + d*x^8)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x^8)/
(c*(a + b*x^8))])/(3*c^2*(a + b*x^8))))/(4*x^4*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(bx^8+a\right)}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)
```

Fricas [B] time = 1.42864, size = 705, normalized size = 8.81

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^4\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4\left(\left(bc-2\,ad\right)x^{12}-acx^4\right)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)+4\sqrt{dx^8+c}\left(abc-a^2\right.}{16\left(a^2bc^2-a^3cd\right)x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a*b*c + a^2*d)*b*c*x^4*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a
*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))
 + 4*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8*(sq
rt(a*b*c - a^2*d)*b*c*x^4*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 +
 c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4
)) + 2*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [A] time = 1.21025, size = 86, normalized size = 1.08

$$\frac{\frac{bc\arctan\left(\frac{a\sqrt{d+\frac{c}{x^8}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}\,a}-\frac{\sqrt{d+\frac{c}{x^8}}}{a}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*c*arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*a) - sqrt(d + c/x^8)/a)/c
```

3.896 $\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}(2ad+3bc)}{12a^2c^2x^4}-\frac{\sqrt{c+dx^8}}{12acx^{12}}$$

```
[Out] -Sqrt[c + d*x^8]/(12*a*c*x^12) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(12*a^2*
c^2*x^4) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4
*a^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.16105, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}(2ad+3bc)}{12a^2c^2x^4}-\frac{\sqrt{c+dx^8}}{12acx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```


```
[Out] -Sqrt[c + d*x^8]/(12*a*c*x^12) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(12*a^2*
c^2*x^4) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4
*a^(5/2)*Sqrt[b*c - a*d])
```


Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{12ac}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{12a^2c^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4a^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4a^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

Mathematica [C] time = 1.75096, size = 253, normalized size = 2.2

$$-\frac{\left(\frac{dx^8}{c}+1\right)\left(-\frac{8x^8\left(c+dx^8\right)^2(bc-ad)\text{HypergeometricPFQ}\left(\{2,2,2\},\left\{1,\frac{5}{2}\right\},\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}\right)}{a+bx^8} + \frac{3c\left(c^2-4cdx^8-8d^2x^{16}\right)\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}}\right)}{\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}} + \frac{24dx^{16}\left(c+dx^8\right)(ad-bc)\,{}_2\ldots}{a+bx^8}\ldots\right)}{36c^3x^{12}\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((1 + (d*x^8)/c)*((3*c*(c^2 - 4*c*d*x^8 - 8*d^2*x^16)*ArcSin[Sqrt[((b*c -
a*d)*x^8)/(c*(a + b*x^8))]])/Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a +
 b*x^8)^2)] + (24*d*(-(b*c) + a*d)*x^16*(c + d*x^8)*Hypergeometric2F1[2, 2,
 5/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))])/(a + b*x^8) - (8*(b*c - a*d)*x^8*
(c + d*x^8)^2*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((b*c - a*d)*x^8)/(c*(
a + b*x^8))])/(a + b*x^8)))/(36*c^3*x^12*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13} \left(b x^{8}+a\right)} \frac{1}{\sqrt{d x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{8}+a\right) \sqrt{d x^{8}+c} x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^13), x)
```

Fricas [A] time = 1.54158, size = 867, normalized size = 7.54

$$\left[-\frac{3 \sqrt{-abc+a^{2}d} b^{2} c^{2} x^{12} \log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{16}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{8}+a^{2}c^{2}-4\left(\left(bc-2\,ad\right)x^{12}-acx^{4}\right)\sqrt{dx^{8}+c}\sqrt{-abc+a^{2}d}}{b^{2}x^{16}+2\,abx^{8}+a^{2}}\right)-4\left(\left(3\,ab^{2}c^{2}-\right.\right.}{48\left(a^{3}bc^{3}-a^{4}c^{2}d\right)x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x
^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8
+ a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*
d)*sqrt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c - a^
```

```
2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt
(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*(
(3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^8
 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1727, size = 144, normalized size = 1.25

$$-\frac{b^2 \arctan\left(\frac{a\sqrt{d+\frac{c}{x^8}}}{\sqrt{abc-a^2d}}\right)}{4\sqrt{abc-a^2d}a^2}+\frac{3\,abc^5\sqrt{d+\frac{c}{x^8}}-a^2c^4\left(d+\frac{c}{x^8}\right)^{\frac{3}{2}}+3\,a^2c^4\sqrt{d+\frac{c}{x^8}}d}{12\,a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)
*a^2) + 1/12*(3*a*b*c^5*sqrt(d + c/x^8) - a^2*c^4*(d + c/x^8)^(3/2) + 3*a^2
*c^4*sqrt(d + c/x^8)*d)/(a^3*c^6)
```

3.897 $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=851

$$-\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}-\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8b^{3/4}\sqrt{bc-ad}}-$$

```
[Out] -((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
8])])/(8*b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*
x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(3/4)*Sqrt[-(b*c) + a*d])
+ ((Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^8]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt
[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^
(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sq
rt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] +
Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]
*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(
1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sq
rt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2
]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*d^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.10131, antiderivative size = 851, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 483, 220, 409, 1217, 1707}

$$-\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}-\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8b^{3/4}\sqrt{bc-ad}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
8])])/(8*b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*
x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(3/4)*Sqrt[-(b*c) + a*d])
+ ((Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^8]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt
[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^
(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sq
rt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] +
Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]
*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(
1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sq
rt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2
]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*d^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
```

```
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} - \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b} - \ldots$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} - \frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+\ldots}}\ldots\right.}{4(bc+ad)}$$

$$= -\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}} + \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\ldots\right.}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+d\ldots}}$$

Mathematica [C] time = 0.0399451, size = 65, normalized size = 0.08

$$\frac{x^{10}\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{10a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^10*Sqrt[(c + d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8
)/a)])/(10*a*Sqrt[c + d*x^8])
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{x^9}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**9/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.898 $\int \frac{x}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=754

$$\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+\right.}{8a\sqrt[4]{c}\sqrt{c+dx^8}(}$$

```
[Out] -(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])
])/(8*(-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2
)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(3/4)*Sqrt[-(b*c) + a*d])
+ (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[-a]*Sqrt[
b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(S
qrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.765545, antiderivative size = 754, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 409, 1217, 220, 1707}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right.}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])
])/(8*(-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2
```

```
)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(3/4)*Sqrt[-(b*c) + a*d])
+ (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[-a]*Sqrt[
b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(S
qrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a}$$

$$= \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a(bc+ad)} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\text{S}\cdots}{4a(\cdots}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}} + \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\cdots}}{8\sqrt[4]{c}(bc+ad)\cdots}$$

Mathematica [C] time = 0.0344861, size = 65, normalized size = 0.09

$$\frac{x^2\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{2a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^8)/c]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)
/a)])/(2*a*Sqrt[c + d*x^8])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.899 $\int \frac{1}{x^7\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=878

$$-\frac{b\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}}-\ b^{5}$$

```
[Out] -Sqrt[c + d*x^8]/(6*a*c*x^6) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(
-a)^(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(12*a*c^(5/4)*Sqrt[c + d*x^8]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + S
qrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]
*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*
x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x
^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.22159, antiderivative size = 878, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 480, 523, 220, 409, 1217, 1707}

$$-\frac{b\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}}-\ b^{5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(6*a*c*x^6) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(
-a)^(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(12*a*c^(5/4)*Sqrt[c + d*x^8]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + S
qrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]
*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*
x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x
^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^7 \left(a+bx^8\right) \sqrt{c+dx^8}} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^4 \left(a+bx^4\right) \sqrt{c+dx^4}} \, dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} + \frac{\text{Subst}\left(\int \frac{-3bc-ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}} \, dx, x, x^2\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}} \, dx, x, x^2\right)}{2a} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}} \, dx, x, x^2\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{\cdots}}\cdots\right)}{4a^2}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}} - \frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-\cdots}\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\cdots}{12\cdots}$$

Mathematica [C] time = 0.113962, size = 141, normalized size = 0.16

$$\frac{-bdx^{16}\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5x^8\sqrt{\frac{dx^8}{c}+1}(ad+3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5a\left(c+dx^8\right)}{30a^2cx^6\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-5*a*(c + d*x^8) - 5*(3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4, 1
/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] - b*d*x^16*Sqrt[1 + (d*x^8)/c]*Appe
llF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)])/(30*a^2*c*x^6*Sqrt[c +
d*x^8])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^{7} \left(b x^{8}+a \right)} \frac{1}{\sqrt{d x^{8}+c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{8}+a\right) \sqrt{d x^{8}+c} x^{7}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{7} \left(a + b x^{8}\right) \sqrt{c + d x^{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

3.900 $\int \frac{x^{13}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1005

result too large to display

```
[Out] (x^2*Sqrt[c + d*x^8])/(2*b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^4)) + ((-a)^(3/4)*A
rcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/
4)*Sqrt[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^8]) + (c^(
1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*d^(3/4)*Sqrt[c + d*x^8]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (a*(Sqrt[c
] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c]
+ Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqr
t[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[-a]*(Sqrt[b]*S
qrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*
Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
16*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.36333, antiderivative size = 1005, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {465, 483, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{-a}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}+\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+}\cdots}\right.}{8b^{5/4}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^8])/(2*b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^4)) + ((-a)^(3/4)*A
rcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/
4)*Sqrt[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^8]) + (c^(
1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*d^(3/4)*Sqrt[c + d*x^8]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (a*(Sqrt[c
] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c]
+ Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqr
t[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[-a]*(Sqrt[b]*S
qrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*
Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
16*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b}$$

$$= \frac{a\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-a}-\sqrt{b}x^2)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b^{3/2}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(\sqrt{-a}+\sqrt{b}x^2)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b^{3/2}} + \frac{\sqrt{c}\operatorname{Subst}}{}$$

$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^8}} + \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)}{}$$

$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{bc-ad}} - \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{-bc+ad}} - \frac{\sqrt[4]{c}\left(\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.0428873, size = 65, normalized size = 0.06

$$\frac{x^{14}\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{14a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^14*Sqrt[(c + d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^
8)/a)])/(14*a*Sqrt[c + d*x^8])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^{13}}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{13}}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{13}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**13/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{13}}{(bx^8 + a)\sqrt{dx^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

3.901 $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=768

$$-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}+\right.}{8\sqrt[4]{c}\sqrt{c+dx^8}(}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^
(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4
)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1
/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] + (Sqrt[-a]
*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c
] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*c^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sq
rt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1/4)
*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d
])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]
*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1
/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.935099, antiderivative size = 768, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 490, 1217, 220, 1707}

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^
(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4
```

```
)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1
/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] + (Sqrt[-a]
*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c
] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*c^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sq
rt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1/4)
*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d
])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]
*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1
/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4\sqrt{b}}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\text{S}\cdots}{4\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}} - \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\cdots\right.}}\cdots}{8\sqrt[4]{c}(bc+ad)\sqrt{c+d\cdots}}$$

Mathematica [C] time = 0.0365615, size = 65, normalized size = 0.08

$$\frac{x^6\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{6a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*Sqrt[(c + d*x^8)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)
/a)])/(6*a*Sqrt[c + d*x^8])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x^5}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.902 $\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1032

result too large to display

```
[Out] -Sqrt[c + d*x^8]/(2*a*c*x^2) + (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(2*a*c*(Sqrt[c
] + Sqrt[d]*x^4)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(Sqr
t[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*S
qrt[-(b*c) + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*
a*c^(3/4)*Sqrt[c + d*x^8]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x
^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1
/2])/(4*a*c^(3/4)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[
b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4
)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a
*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(S
qrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF
[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d
*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^8]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + S
qrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.61168, antiderivative size = 1032, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {465, 480, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{b}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(2*a*c*x^2) + (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(2*a*c*(Sqrt[c
] + Sqrt[d]*x^4)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(Sqr
t[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*S
qrt[-(b*c) + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*
a*c^(3/4)*Sqrt[c + d*x^8]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x
^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1
/2])/(4*a*c^(3/4)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[
b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4
)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a
*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(S
qrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF
[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d
*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^8]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + S
qrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
```

```
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\text{Subst}\left(\int \frac{x^2\left(-bc+ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2ac}\\
&= -\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}}-\frac{bcx^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx,x,x^2\right)}{2ac}\\
&= -\frac{\sqrt{c+dx^8}}{2acx^2}-\frac{b\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2a}+\frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2ac}\\
&= -\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a}-\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,d\right.}{4a}\\
&= -\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}x^4\right)}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{c+dx^8}}+\\
&= -\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}x^4\right)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{-bc+ad}}\end{aligned}$$

Mathematica [C] time = 0.105386, size = 141, normalized size = 0.14

$$\frac{3bdx^{16}\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7x^8\sqrt{\frac{dx^8}{c}+1}(ad-bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-21a\left(c+dx^8\right)}{42a^2cx^2\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-21*a*(c + d*x^8) + 7*(-(b*c) + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)])/(42*a^2*c*x^2*Sqrt
[c + d*x^8])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(b x^{8}+a\right)}\frac{1}{\sqrt{d x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{8}+a\right)\sqrt{d x^{8}+c}\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + bx^8) \sqrt{c + dx^8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.903 $\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 1, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.051597, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```


```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 1, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a*Sqrt[c + d*x^8])
```


Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

Mathematica [A] time = 0.0378513, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{5a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^8)/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8
)/a)])/(5*a*Sqrt[c + d*x^8])
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \frac{x^4}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.904 $\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 1, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.053032, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 1, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a*Sqrt[c + d*x^8])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x^3\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

Mathematica [A] time = 0.037005, size = 65, normalized size = 1.02

$$\frac{x^3\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{3}{8};\frac{1}{2},1;\frac{11}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{3a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[(c + d*x^8)/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8
)/a)])/(3*a*Sqrt[c + d*x^8])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^2}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^8 + a)\sqrt{dx^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + bx^8)\sqrt{c + dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.905 $\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 1, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.0272912, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 1, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a*Sqrt[c + d*x^8])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.16582, size = 161, normalized size = 2.73

$$-\frac{9acxF_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{(a+bx^8)\sqrt{c+dx^8}\left(4x^8\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)-9acF_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-9*a*c*x*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)])/((a + b*x
^8)*Sqrt[c + d*x^8]*(-9*a*c*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x
^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/
a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{bx^8+a}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + bx^8\right)\sqrt{c + dx^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.906 $\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 1, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a*x*Sqrt[c + d*x^8]))
```

Rubi [A] time = 0.0586223, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 1, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a*x*Sqrt[c + d*x^8]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$=-\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.117021, size = 141, normalized size = 2.27

$$\frac{7bdx^{16}\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{15}{8};\frac{1}{2},1;\frac{23}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5x^8\sqrt{\frac{dx^8}{c}+1}(bc-3ad)F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-35a\left(c+dx^8\right)}{35a^2cx\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-35*a*(c + d*x^8) - 5*(b*c - 3*a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[7/8,
1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c), -((b*x^8)/a)])/(35*a^2*c*x*Sqrt[
c + d*x^8])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(bx^8+a\right)}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^8+c}}{bdx^{18}+(bc+ad)x^{10}+acx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^8 + c)/(b*d*x^18 + (b*c + a*d)*x^10 + a*c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.907 $\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

```
[Out] -(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 1, 1/2, 5/8, -((b*x^8)/a), -((d*x^8)/c
)])/(3*a*x^3*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.05565, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 1, 1/2, 5/8, -((b*x^8)/a), -((d*x^8)/c
)])/(3*a*x^3*Sqrt[c + d*x^8])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$=-\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.113915, size = 141, normalized size = 2.2

$$\frac{5bdx^{16}\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{13}{8};\frac{1}{2},1;\frac{21}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+13x^8\sqrt{\frac{dx^8}{c}+1}(ad-3bc)F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-65a\left(c+dx^8\right)}{195a^2cx^3\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-65*a*(c + d*x^8) + 13*(-3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8
, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b*x^8)/a)])/(195*a^2*c*x^3*
Sqrt[c + d*x^8])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(bx^8+a\right)}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^8+a)\sqrt{dx^8+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.908 $\int \frac{x^{23}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^8}}{8b^2\left(a+bx^8\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

```
[Out] Sqrt[c + d*x^8]/(4*b^2*d) - (a^2*Sqrt[c + d*x^8])/(8*b^2*(b*c - a*d)*(a + b
*x^8)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.142471, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^8}}{8b^2\left(a+bx^8\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b^2*d) - (a^2*Sqrt[c + d*x^8])/(8*b^2*(b*c - a*d)*(a + b
*x^8)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{23}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{8}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)$$

$$=-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}+\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}-\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{16b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}-\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{8b^2d(bc-ad)}$$

$$=\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.236838, size = 107, normalized size = 0.87

$$\frac{1}{8}\left(\frac{\sqrt{c+dx^8}\left(\frac{a^2}{\left(a+bx^8\right)(ad-bc)}+\frac{2}{d}\right)}{b^2}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((Sqrt[c + d*x^8]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^8))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/8
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x^{23}}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70156, size = 984, normalized size = 8.

$$\left[\frac{\left(\left(4\,ab^2cd-3\,a^2bd^2\right)x^8+4\,a^2bcd-3\,a^3d^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\left(2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x\right.}{16\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)x^8\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b
*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8), -
1/8*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47548, size = 181, normalized size = 1.47

$$-\frac{\sqrt{dx^8+c}a^2d}{8\left(b^3c-ab^2d\right)\left(\left(dx^8+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^8+c}}{4\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(d*x^8 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^8 + c)*b - b*c + a*d))
- 1/8*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/4*sqrt(d*x^8 + c)/(b^2*d)
```

3.909 $\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0829682, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)}+\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{16b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)}+\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{8bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0878775, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)}+\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*b^(
3/2))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^{15}}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64851, size = 724, normalized size = 7.31

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^8+2\,abc-a^2d\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\left(ab^2c-a^2bd\right)}{16\left(\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^8+ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)},\frac{\left(\left(2\,b^2c-abd\right)x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a))
+ 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a*b*d
)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4907, size = 157, normalized size = 1.59

$$\frac{\frac{\sqrt{dx^8+c}ad^2}{\left(b^2c-abd\right)\left(\left(dx^8+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.910 $\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

```
[Out] -Sqrt[c + d*x^8]/(8*(b*c - a*d)*(a + b*x^8)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0733321, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(8*(b*c - a*d)*(a + b*x^8)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)\\ &= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}-\frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{16(bc-ad)}\\ &= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}-\frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{8(bc-ad)}\\ &= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0878624, size = 85, normalized size = 0.98

$$\frac{1}{8}\left(\frac{\sqrt{c+dx^8}}{\left(a+bx^8\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/((-(b*c) + a*d)*(a + b*x^8)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62791, size = 640, normalized size = 7.36

$$\left[-\frac{\left(bdx^8+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\left(b^2c-abd\right)}{16\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)},-\frac{\left(bdx^8+ad\right)\sqrt{-b^2c+abd}\arctan\left(\right.}{8\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2*
sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*
```

```
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46773, size = 124, normalized size = 1.43

$$-\frac{1}{8}\,d\left(\frac{\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{\sqrt{dx^8+c}}{\left(\left(dx^8+c\right)b-bc+ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d*(arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) + sqrt(d*x^8 + c)/(((d*x^8 + c)*b - b*c + a*d)*(b*c - a*d)))
```

3.911 $\int \frac{1}{x(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

```
[Out] (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]]/(4*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^2*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.137263, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]]/(4*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^2*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a^2} - \frac{(b(2bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{16a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4a^2d} - \frac{(b(2bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}}\right.}{8a^2d(bc-a}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.241851, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - (2*ArcTanh[Sqrt[c + d*x^
8]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*a^2)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x), x)
```

Fricas [A] time = 1.65469, size = 1833, normalized size = 13.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x^8
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
8 + b*c)) + ((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sq
rt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c
^2 - a^4*c*d), 1/16*(2*sqrt(d*x^8 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^8 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b
*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8
```

```
 + b*c)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^8 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61945, size = 207, normalized size = 1.57

$$-\frac{1}{8}\,d^2\left(\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx^8+c}b}{\left(abcd-a^2d^2\right)\left(\left(dx^8+c\right)b-bc+ad\right)}-\frac{2\,\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d^2*((2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d)
)/((a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x^8 + c)*b/((a*b*
c*d - a^2*d^2)*((d*x^8 + c)*b - b*c + a*d)) - 2*arctan(sqrt(d*x^8 + c)/sqrt
(-c))/(a^2*sqrt(-c)*d^2))
```

3.912 $\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b\sqrt{c+dx^8}(2bc-ad)}{8a^2c\left(a+bx^8\right)(bc-ad)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}$$

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(a + b*x^8)) - Sqrt
[c + d*x^8]/(8*a*c*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8
]/Sqrt[c]])/(8*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^3*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.229378, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b\sqrt{c+dx^8}(2bc-ad)}{8a^2c\left(a+bx^8\right)(bc-ad)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(a + b*x^8)) - Sqrt
[c + d*x^8]/(8*a*c*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8
]/Sqrt[c]])/(8*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^3*(b*c - a*d)^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)}{8ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} - \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x\right.}{8a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} + \frac{\left(b^2(4bc-5ad)\right)\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,\right.}{16a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} + \frac{\left(b^2(4bc-5ad)\right)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{8a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}} - \frac{b^{3/2}(4bc-}{8}$$

Mathematica [A] time = 0.569221, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^8}\left(a^2d+ab\left(dx^8-c\right)-2b^2cx^8\right)}{x^8\left(a+bx^8\right)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[c + d*x^8]*(a^2*d - 2*b^2*c*x^8 + a*b*(-c + d*x^8)))/((b*c - a*d)*
x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(8*a^3*c)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{x^9 \left(b x^8+a\right)^2} \frac{1}{\sqrt{d x^8+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^8+a\right)^2 \sqrt{d x^8+c} x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^9), x)
```

Fricas [A] time = 1.89445, size = 2525, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8
 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16
+ (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
```

```
8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sq
rt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt
(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*
b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/8*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^8 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41247, size = 362, normalized size = 1.96

$$\frac{1}{8}\,d^3\left(\frac{\left(4\,b^3c - 5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\left(a^3bcd^3 - a^4d^4\right)\sqrt{-b^2c+abd}} - \frac{2\left(dx^8+c\right)^{\frac{3}{2}}b^2c - 2\sqrt{dx^8+c}b^2c^2 - \left(dx^8+c\right)^{\frac{3}{2}}abd + 2\sqrt{dx^8+c}abcd - \sqrt{dx}}{\left(a^2bc^2d^2 - a^3cd^3\right)\left(\left(dx^8+c\right)^2b - 2\left(dx^8+c\right)bc + bc^2 + \left(dx^8+c\right)ad - a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^3*((4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d
))/((a^3*b*c*d^3 - a^4*d^4)*sqrt(-b^2*c + a*b*d)) - (2*(d*x^8 + c)^(3/2)*b^
2*c - 2*sqrt(d*x^8 + c)*b^2*c^2 - (d*x^8 + c)^(3/2)*a*b*d + 2*sqrt(d*x^8 +
c)*a*b*c*d - sqrt(d*x^8 + c)*a^2*d^2)/((a^2*b*c^2*d^2 - a^3*c*d^3)*((d*x^8
+ c)^2*b - 2*(d*x^8 + c)*b*c + b*c^2 + (d*x^8 + c)*a*d - a*c*d)) - (4*b*c +
 a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^3*sqrt(-c)*c*d^3))
```

3.913 $\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])
```

Rubi [A] time = 0.159961, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```


```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])
```


Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8b(bc-ad)}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx}}\right.}{8b^2(bc-ad)}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad}\right.}{8b^2(bc-ad)}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Mathematica [A] time = 0.210382, size = 135, normalized size = 0.96

$$\frac{\frac{abx^4\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)} + \frac{\sqrt{a}(2ad-3bc)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\sqrt{c+dx^8}+dx^4\right)}{\sqrt{d}}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]])/Sqrt[d])/(8*b^2)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^{19}}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{19}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^19/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [A] time = 3.75799, size = 2284, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d -
a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^
4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sq
rt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt
(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (
a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2
*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + 2*((b^2*c - a*b*d)*
```

```
x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 -
 c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*a*b*d*x^4 - 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)))/((b^4*c*d - a*b^3*d^
2)*x^8 + a*b^3*c*d - a^2*b^2*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74024, size = 205, normalized size = 1.45

$$\frac{1}{8}c^2\left(\frac{(3\,abc-2\,a^2d)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^8}}}{\sqrt{abc-a^2d}}\right)}{(b^3c^3-ab^2c^2d)\sqrt{abc-a^2d}}+\frac{a\sqrt{d+\frac{c}{x^8}}}{(b^2c^2-abcd)\left(bc+a\left(d+\frac{c}{x^8}\right)-ad\right)}-\frac{2\arctan\left(\frac{\sqrt{d+\frac{c}{x^8}}}{\sqrt{-d}}\right)}{b^2c^2\sqrt{-d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*c^2*((3*a*b*c - 2*a^2*d)*arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c - a^2*d))/
((b^3*c^3 - a*b^2*c^2*d)*sqrt(a*b*c - a^2*d)) + a*sqrt(d + c/x^8)/((b^2*c^2
 - a*b*c*d)*(b*c + a*(d + c/x^8) - a*d)) - 2*arctan(sqrt(d + c/x^8)/sqrt(-d
))/(b^2*c^2*sqrt(-d)))
```

3.914 $\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

```
[Out] -(x^4*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.0856842, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(x^4*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{\operatorname{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.472277, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^8}\left(-\frac{x^8(bc-ad)}{a+bx^8}-\frac{c\sqrt{x^8\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^8\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^8}{c}+1}}\right)}{\sqrt{\frac{dx^8}{c}+1}}\right)}{8x^4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-(((b*c - a*d)*x^8)/(a + b*x^8)) - (c*Sqrt[(-(b/a) + d/c)
*x^8]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^8]/Sqrt[1 + (d*x^8)/c]])/Sqrt[1 + (d*x^
8)/c]))/(8*(b*c - a*d)^2*x^4)
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [B] time = 2.0048, size = 890, normalized size = 9.57

$$\left[-\frac{4\sqrt{dx^8+c}\left(abc-a^2d\right)x^4-\left(bcx^8+ac\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4\left(\left(bc-2\,ad\right)x^{12}-acx^4\right)\sqrt{dx}}{b^2x^{16}+2\,abx^8+a^2}\right.}{32\left(\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqr
t(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c
*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*sqrt(d
*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*arctan(
1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
 - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32557, size = 124, normalized size = 1.33

$$-\frac{1}{8}\,c\left(\frac{\arctan\left(\frac{a\sqrt{d+\frac{c}{x^8}}}{\sqrt{abc-a^2d}}\right)}{\sqrt{abc-a^2d}(bc-ad)}+\frac{\sqrt{d+\frac{c}{x^8}}}{\left(bc+a\left(d+\frac{c}{x^8}\right)-ad\right)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*c*(arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*
(b*c - a*d)) + sqrt(d + c/x^8)/((b*c + a*(d + c/x^8) - a*d)*(b*c - a*d)))
```

3.915 $\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

```
[Out] (b*x^4*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rubi [A] time = 0.090251, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^4*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)\\ &= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}\\ &= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8a(bc-ad)}\\ &= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.778745, size = 407, normalized size = 3.91

$$\frac{x^4\sqrt{c+dx^8}\left(-30dx^8\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}-45c\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}+16dx^8\left(\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^8\right)}{c\left(a+bx^8\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)+16c\right.}{120c^2\left(a+bx^8\right)^2\left(\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8]*(-45*c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a +
b*x^8)^2)] - 30*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 30*d*x^8*ArcS
in[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 16*c*(((b*c - a*d)*x^8)/(c*(a
 + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 16*d*x^8*(((b*c - a*d)*x^8)/
(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))]))/(120*c^2*(((b*c - a*d)*
x^8)/(c*(a + b*x^8)))^(3/2)*(a + b*x^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^
8))])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [B] time = 2.26653, size = 973, normalized size = 9.36

$$\left[\frac{4\,\sqrt{dx^8+c}\left(ab^2c-a^2bd\right)x^4-\left(\left(b^2c-2\,abd\right)x^8+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4}{b^2x^{16}+2\,abx^8+}\right.}{32\left(\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^8+a^3b^2c^2-2\,a^4bcd+a^5d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 -
 a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2
)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*
c*d + a^5*d^2), 1/16*(2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b^2*c -
 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a
*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^1
2 + (a*b*c^2 - a^2*c*d)*x^4)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x
^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21005, size = 320, normalized size = 3.08

$$-\frac{1}{8}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right.\right.}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}x^4-\sqrt{dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*
x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

3.916 $\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{c+dx^8}(3bc-2ad)}{8a^2cx^4(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^8}}{8ax^4\left(a+bx^8\right)(bc-ad)}$$

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^4) + (b*Sqrt[c +
d*x^8])/(8*a*(b*c - a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.184842, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^8}(3bc-2ad)}{8a^2cx^4(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^8}}{8ax^4\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^4) + (b*Sqrt[c +
d*x^8])/(8*a*(b*c - a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)$$

$$=\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a(bc-ad)}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a^2c(bc-ad)}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\right.}{8a^2(bc-ad)}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{8a^2(bc-ad)}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 1.57073, size = 869, normalized size = 5.83

$$-\frac{\sqrt{dx^8+c}\left(120d^2\sin^{-1}\left(\sqrt{\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}}\right)x^{16}+96d^2\left(\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)^{5/2}\sqrt{\frac{a\left(dx^8+c\right)}{c\left(bx^8+a\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)x^{16}+32d^2\left(\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)^{5/2}\sqrt{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[c + d*x^8]*(-45*c^2*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b
*x^8)^2)] - 180*c*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^
8)^2)] - 120*d^2*x^16*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)
^2)] + 45*c^2*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 180*c*d*x^8
*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 120*d^2*x^16*ArcSin[Sqrt
[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 64*c^2*(((b*c - a*d)*x^8)/(c*(a + b*
x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2, 3,
```

```
7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 160*c*d*x^8*(((b*c - a*d)*x^8)/(c
*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F
1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 96*d^2*x^16*(((b*c - a*d)
*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeo
metric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 32*c^2*(((b*c - a
*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hyper
geometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 64*c
*d*x^8*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a
 + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a
+ b*x^8))] + 32*d^2*x^16*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*
(c + d*x^8))/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c
- a*d)*x^8)/(c*(a + b*x^8))]))/(120*c^3*x^4*(((b*c - a*d)*x^8)/(c*(a + b*x^
8)))^(3/2)*(a + b*x^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{x^{5} \left(b x^{8}+a \right)^{2}} \frac{1}{\sqrt{d x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(b x^{8} + a\right)}^{2} \sqrt{d x^{8} + c} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)
```

Fricas [B] time = 2.49406, size = 1268, normalized size = 8.51

$$\left[-\frac{\left(\left(3\,b^{3}c^{2}-4\,ab^{2}cd\right)x^{12}+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd\right)x^{4}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{16}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{8}+a^{2}c^{2}+4\left((bc-2\,ad)x^{12}-\right.}{b^{2}x^{16}+2\,abx^{8}+a^{2}}\right.}{32\left(\left(a^{3}b^{3}c^{3}-2\,a^{4}b^{2}c^{2}d+a^{5}bcd^{2}\right)x^{12}+\left(a^{4}b^{2}c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x
^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b^3*c
^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4
*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12
+ (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2 - 4*a*
b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d)*arctan
(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*
d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2
*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x
^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 + (a^4*b^2*c^3
 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21622, size = 181, normalized size = 1.21

$$-\frac{b^{2}c\sqrt{d+\frac{c}{x^{8}}}}{8\left(a^{2}bc-a^{3}d\right)\left(bc+a\left(d+\frac{c}{x^{8}}\right)-ad\right)}+\frac{\left(3\,b^{2}c-4\,abd\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^{8}}}}{\sqrt{abc-a^{2}d}}\right)}{8\left(a^{2}bc-a^{3}d\right)\sqrt{abc-a^{2}d}}-\frac{\sqrt{d+\frac{c}{x^{8}}}}{4\,a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*b^2*c*sqrt(d + c/x^8)/((a^2*b*c - a^3*d)*(b*c + a*(d + c/x^8) - a*d))
+ 1/8*(3*b^2*c - 4*a*b*d)*arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c - a^2*d))/((a
^2*b*c - a^3*d)*sqrt(a*b*c - a^2*d)) - 1/4*sqrt(d + c/x^8)/(a^2*c)
```

3.917 $\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=208

$$\frac{\sqrt{c+dx^8}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{24a^3c^2x^4(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{24a^2cx^{12}(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^{12}\left(a+bx^8\right)(bc-ad)}$$

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^12) + ((15*b^2*c
^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^8])/(24*a^3*c^2*(b*c - a*d)*x^4) +
 (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rubi [A] time = 0.297487, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^8}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{24a^3c^2x^4(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{24a^2cx^{12}(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^{12}\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^12) + ((15*b^2*c
^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^8])/(24*a^3*c^2*(b*c - a*d)*x^4) +
 (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} - \frac{\operatorname{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc\cdots}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\cdots\right.}{24a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)}$$

Mathematica [C] time = 2.75445, size = 1535, normalized size = 7.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(45*c^3*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x
^8)^2)] - 270*c^2*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^
8)^2)] - 1080*c*d^2*x^16*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x
^8)^2)] - 720*d^3*x^24*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8
)^2)] - 45*c^3*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 270*c^2*d*
x^8*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 1080*c*d^2*x^16*ArcSi
n[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 720*d^3*x^24*ArcSin[Sqrt[((b*c
 - a*d)*x^8)/(c*(a + b*x^8))]] - 16*c^3*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))
```

```
^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2, 3, 7/2, (
(b*c - a*d)*x^8)/(c*(a + b*x^8))] + 528*c^2*d*x^8*(((b*c - a*d)*x^8)/(c*(a
+ b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2,
 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 1248*c*d^2*x^16*(((b*c - a*d)
*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeo
metric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 704*d^3*x^24*(((b
*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]
*Hypergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 96*c^3*(
((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8
))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8)
)] + 576*c^2*d*x^8*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d
*x^8))/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)
*x^8)/(c*(a + b*x^8))] + 864*c*d^2*x^16*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))
^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {
1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 384*d^3*x^24*(((b*c - a*d)*x^
8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeomet
ricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 64*c^3*(((
b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))
]*HypergeometricPFQ[{2, 2, 2, 3}, {1, 1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*
x^8))] + 192*c^2*d*x^8*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c
 + d*x^8))/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 2, 3}, {1, 1, 7/2}, ((
b*c - a*d)*x^8)/(c*(a + b*x^8))] + 192*c*d^2*x^16*(((b*c - a*d)*x^8)/(c*(a
+ b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*HypergeometricPFQ[{2
, 2, 2, 3}, {1, 1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 64*d^3*x^24*(
((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8
))]*HypergeometricPFQ[{2, 2, 2, 3}, {1, 1, 7/2}, ((b*c - a*d)*x^8)/(c*(a +
b*x^8))]))/(360*c^4*x^12*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(3/2)*(a + b*x
^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^{13} \left(b x^8 + a\right)^2} \frac{1}{\sqrt{d x^8 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)
```

Fricas [A] time = 3.53749, size = 1561, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^12)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)
*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*((
15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^16 + 2*(
5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*
c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b
^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2
)*x^12), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b
^2*c^2*d)*x^12)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sq
rt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^
2*c*d)*x^4)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^16 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^
3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^
4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*
c^3*d + a^7*c^2*d^2)*x^12)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18832, size = 244, normalized size = 1.17

$$\frac{b^3c\sqrt{d+\frac{c}{x^8}}}{8\left(a^3bc-a^4d\right)\left(bc+a\left(d+\frac{c}{x^8}\right)-ad\right)}-\frac{\left(5\,b^3c-6\,ab^2d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^8}}}{\sqrt{abc-a^2d}}\right)}{8\left(a^3bc-a^4d\right)\sqrt{abc-a^2d}}+\frac{6\,a^3bc^5\sqrt{d+\frac{c}{x^8}}-a^4c^4\left(d+\frac{c}{x^8}\right)^{\frac{3}{2}}+3\,a^4c^4\sqrt{\ldots}}{12\,a^6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b^3*c*sqrt(d + c/x^8)/((a^3*b*c - a^4*d)*(b*c + a*(d + c/x^8) - a*d)) -
 1/8*(5*b^3*c - 6*a*b^2*d)*arctan(a*sqrt(d + c/x^8)/sqrt(a*b*c - a^2*d))/((
a^3*b*c - a^4*d)*sqrt(a*b*c - a^2*d)) + 1/12*(6*a^3*b*c^5*sqrt(d + c/x^8) -
 a^4*c^4*(d + c/x^8)^(3/2) + 3*a^4*c^4*sqrt(d + c/x^8)*d)/(a^6*c^6)
```

3.918 $\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=924

result too large to display

```
[Out] -(x^2*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) - ((b*c + a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b
^(3/4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((
-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^
(3/2)) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sqrt
[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
 d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)
], 1/2])/(32*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) - (d^(3/4)*(Sqrt[c] +
 Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
+ ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
 d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sqrt[
b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(
Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2
])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.2877, antiderivative size = 924, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 471, 523, 220, 409, 1217, 1707}

$$\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^8+c}}-\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(x^2*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) - ((b*c + a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b
^(3/4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((
-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^
(3/2)) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sqrt
[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
 d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)
], 1/2])/(32*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) - (d^(3/4)*(Sqrt[c] +
 Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
+ ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
 d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sqrt[
b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(
Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2
])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{c-dx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8b(bc-ad)} + \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,d\right.}{8b(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} + \frac{(bc+ad)\,\text{S}}{}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} + \frac{\left(\sqrt{c}\left(\sqrt{c}-\right.\right.}{}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$

Mathematica [C] time = 0.139311, size = 159, normalized size = 0.17

$$-\frac{x^2\left(dx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5c\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5a\left(c+dx^8\right)\right)}{40a\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(x^2*(5*a*(c + d*x^8) - 5*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 + (d*x^
8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)]))/(40*a*(b*c -
 a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.919 $\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=999

result too large to display

```
[Out] (b*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + (b^(1/4)*(3*b*c - 5
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(
32*(-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(
b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(7/4)*(-(b
*c) + a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
a*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32
*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - S
qrt[-a]*Sqrt[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(32*(-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c
] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*S
qrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(
Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.40638, antiderivative size = 999, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {465, 414, 523, 220, 409, 1217, 1707}

$$\frac{(3bc-5ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt[4]{d}(3bc-5ad)\left(\sqrt{d}x^4+\right.}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + (b^(1/4)*(3*b*c - 5
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(
32*(-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(
b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(7/4)*(-(b
*c) + a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
a*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32
*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - S
qrt[-a]*Sqrt[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(32*(-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c
] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*S
qrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(
Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)} + \frac{(3bc-5ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right.}{8a(bc-ad)}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} + \frac{(3bc-5ad}{}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{\right.\right.}{}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-a}}\right.}{32(-a)^{7/4}(-bc+ad}$$

Mathematica [C] time = 0.157496, size = 169, normalized size = 0.17

$$\frac{x^2\left(bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}(3bc-4ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5ab\right.}{40a^2\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*(5*a*b*(c + d*x^8) + 5*(3*b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]
*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*x^8*(a + b*x^
8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a
)]))/(40*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{x}{\left(b x^{8}+a\right)^{2}} \frac{1}{\sqrt{d x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(b x^{8}+a\right)^{2} \sqrt{d x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.920 $\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1060

result too large to display

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^6) + (b*Sqrt[c +
 d*x^8])/(8*a*(b*c - a*d)*x^6*(a + b*x^8)) + (b^(5/4)*(7*b*c - 9*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11
/4)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d
]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11/4)*(-(b*c) + a*d
)^(3/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8
)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2
])/(48*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellip
ticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*S
qrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3*c^(1/4)*d^(1/4)*(b*
c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] +
Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-
a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3
*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.98252, antiderivative size = 1060, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {465, 472, 583, 523, 220, 409, 1217, 1707}

$$-\frac{b(7bc-9ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}+\frac{b\sqrt[4]{d}(7bc-9ad)\left(\sqrt{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^6) + (b*Sqrt[c +
 d*x^8])/(8*a*(b*c - a*d)*x^6*(a + b*x^8)) + (b^(5/4)*(7*b*c - 9*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11
/4)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d
]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11/4)*(-(b*c) + a*d
)^(3/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8
)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2
])/(48*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellip
ticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*S
qrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3*c^(1/4)*d^(1/4)*(b*
c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] +
Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-
a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3
*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

```
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-5bdx^4}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{-21b^2c^2+20abcd+4a^2d^2-bd(7bc-}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right.}{24a^2c(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{(b(7bc-9ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+a}}\right.}{8a^2(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c}{\left(\sqrt{c}+\right.}}}{48a^2c^{5/4}(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c}{\left(\sqrt{c}+\right.}}}{48a^2c^{5/4}(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} + \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right.}{32(-a)^{11/4}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.272972, size = 225, normalized size = 0.21

$$\frac{5x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(4a^2d^2+20abcd-21b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5a\left(c+dx^8\right)\left(4a^2d-4ab\left(c-dx^8\right)-7b^2c\right.}{120a^3cx^6\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (5*a*(c + d*x^8)*(4*a^2*d - 7*b^2*c*x^8 - 4*a*b*(c - d*x^8)) + 5*(-21*b^2*c
^2 + 20*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*(-7*b*c + 4*a*d)*x^16*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b
*x^8)/a)])/(120*a^3*c*(b*c - a*d)*x^6*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^7 \left(bx^8 + a\right)^2} \frac{1}{\sqrt{dx^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + c}\, x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

3.921 $\int \frac{x^{13}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1164

result too large to display

```
[Out] (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) - (
x^6*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + ((3*b*c - a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b
^(5/4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d
)^(3/2)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*
(b*c - a*d)*Sqrt[c + d*x^8]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqr
t[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c
^(1/4)], 1/2])/(16*b*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[-a]*S
qrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] +
(Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c]
- Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c
 - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c
^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.93553, antiderivative size = 1164, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {465, 471, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{dx^8+c}\,x^6}{8(bc-ad)\left(bx^8+a\right)}+\frac{\sqrt{d}\sqrt{dx^8+c}\,x^2}{8b(bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}b^{5/4}(ad-bc)^{3/2}}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) - (
x^6*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + ((3*b*c - a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b
^(5/4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d
)^(3/2)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*
(b*c - a*d)*Sqrt[c + d*x^8]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqr
t[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c
^(1/4)], 1/2])/(16*b*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[-a]*S
qrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] +
(Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c]
- Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c
 - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c
^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3c+dx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \left(\frac{dx^2}{b\sqrt{c+dx^4}} + \frac{(3bc-ad)x^2}{b\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx,x,x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8b(bc-ad)} + \frac{(3bc-ad)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right.}{8b(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8b(bc-ad)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx\right.}{8b(bc-ad)}$$

$$= \frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} - \frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\,\text{t}\right.}{8b(bc-ad)\sqrt{c+dx^8}}$$

$$= \frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} - \frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}} + \frac{(3b}{}$$

Mathematica [C] time = 0.133636, size = 159, normalized size = 0.14

$$\frac{x^6\left(dx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7c\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-7a\left(c+dx^8\right)\right)}{56a\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(-7*a*(c + d*x^8) + 7*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 + (d*x^
8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)]))/(56*a*(b*c
- a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{x^{13}}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{13}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{13}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.922 $\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1162

result too large to display

```
[Out] -(Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) +
(b*x^6*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ((b*c - 3*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5
/4)*b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*
x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5/4)*b^(1/4)*(-(b*c)
+ a*d)^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(
Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/
(8*a*(b*c - a*d)*Sqrt[c + d*x^8]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
^2)/c^(1/4)], 1/2])/(16*a*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[
-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[
c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x^2)/c^(1/4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]
) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d
]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2
*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*
Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 1.90973, antiderivative size = 1162, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {465, 472, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{b\sqrt{dx^8+cx^6}}{8a(bc-ad)\left(bx^8+a\right)}-\frac{\sqrt{d}\sqrt{dx^8+cx^2}}{8a(bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(ad-bc)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) +
(b*x^6*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ((b*c - 3*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5
/4)*b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*
x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5/4)*b^(1/4)*(-(b*c)
+ a*d)^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(
Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/
(8*a*(b*c - a*d)*Sqrt[c + d*x^8]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
^2)/c^(1/4)], 1/2])/(16*a*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[
-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[
c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x^2)/c^(1/4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]
) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d
]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2
*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*
Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
```

```
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+4ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx,x,x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,d\right.}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx\right.}{8a(bc-ad)}$$

$$= -\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(\right.}{8a(bc-ad)\sqrt{c+dx}}$$

$$= -\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}} - \cdots$$

Mathematica [C] time = 0.161663, size = 169, normalized size = 0.15

$$\frac{x^6\left(-3bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}(bc-4ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+21a\right.}{168a^2\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(21*a*b*(c + d*x^8) + 7*(b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*
AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] - 3*b*d*x^8*(a + b*x
^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)
/a)]))/(168*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.923 $\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1243

result too large to display

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^2) + (Sqrt[d]*(5*
b*c - 4*a*d)*x^2*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x
^4)) + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^2*(a + b*x^8)) - (b^(3/4)*(5*
b*c - 7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
8])])/(32*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(
Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(9/
4)*(-(b*c) + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^
2)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (d^(1/4)*(
5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(3/4)*(b
*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32*a^2*c^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^
8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/
2])/(32*a^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[b]*(Sq
rt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4
)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*
d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*
c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)
^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1
/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] time = 2.38384, antiderivative size = 1243, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {465, 472, 583, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{\sqrt{b}(5bc-7ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}-\frac{b^{3/4}(5bc-7ad)\tan^{-}\cdots}{32(-a)^{9/4}(bc\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^2) + (Sqrt[d]*(5*
b*c - 4*a*d)*x^2*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x
^4)) + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^2*(a + b*x^8)) - (b^(3/4)*(5*
b*c - 7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
8])])/(32*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(
Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(9/
4)*(-(b*c) + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^
2)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (d^(1/4)*(
5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(3/4)*(b
*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32*a^2*c^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^
8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/
2])/(32*a^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[b]*(Sq
rt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4
)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*
d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*
c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)
^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1
/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
```

```
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-3bdx^4}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-(bc-2ad)(5bc-2ad)+bd(5bc-4a\right.}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right.}{8a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \left(\frac{d(5bc-4ad)x^2}{\sqrt{c+dx^4}} + \frac{\left(-5b^2c^2+7abcd\right.}{\left(a+bx^4\right)\sqrt{c+d}}\right.\right.}{8a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{(b(5bc-7ad))\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right.}{8a^2(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\left(\sqrt{b}(5bc-7ad)\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right.}\right.}{16a^2(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{\sqrt{d}(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{\sqrt[4]{d}(5b}{}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{\sqrt{d}(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{b^{3/4}(5b}{}$$

Mathematica [C] time = 0.263444, size = 226, normalized size = 0.18

$$\frac{-7x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(4a^2d^2-12abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+21a\left(c+dx^8\right)\left(4a^2d-4ab\left(c-dx^8\right)-5b^2c\right.}{168a^3cx^2\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (21*a*(c + d*x^8)*(4*a^2*d - 5*b^2*c*x^8 - 4*a*b*(c - d*x^8)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^16*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((
b*x^8)/a)])/(168*a^3*c*(b*c - a*d)*x^2*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(b x^{8}+a \right)^{2}} \frac{1}{\sqrt{d x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(b x^{8} + a\right)}^{2} \sqrt{d x^{8} + c} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

3.924 $\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 2, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a^2*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.05191, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 2, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a^2*Sqrt[c + d*x^8])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.177796, size = 170, normalized size = 2.66

$$\frac{x^5\left(-5bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{13}{8};\frac{1}{2},1;\frac{21}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+13\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}(3bc-8ad)F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+\right.}{520a^2\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*(65*a*b*(c + d*x^8) + 13*(3*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] - 5*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(520*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.925 $\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 2, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a^2*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.0530837, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 2, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a^2*Sqrt[c + d*x^8])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x^3\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.17256, size = 170, normalized size = 2.66

$$\frac{x^3\left(3bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{11}{8};\frac{1}{2},1;\frac{19}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+11\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}(5bc-8ad)F_1\left(\frac{3}{8};\frac{1}{2},1;\frac{11}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3\right.}{264a^2\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*(33*a*b*(c + d*x^8) + 11*(5*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[11/8, 1/2, 1, 19/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(264*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.926 $\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 2, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a^2*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.0270407, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```



```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 2, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a^2*Sqrt[c + d*x^8])
```


Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= \frac{x\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.301031, size = 328, normalized size = 5.56

$$-\frac{x\left(bdx^8\sqrt{\frac{dx^8}{c}+1}F_1\left(\frac{9}{8};\frac{1}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+\frac{3a\left(4bx^8\left(c+dx^8\right)\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)+9ac\left(8ad-b\left(8c+dx^8\right)\right)}{\left(a+bx^8\right)\left(4x^8\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)-9acF_1\left(\frac{1}{8};\frac{1}{2},1;\right.}\right.}{24a^2\sqrt{c+dx^8}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(x*(b*d*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[9/8, 1/2, 1, 17/8, -((d*x^8)/c),
-((b*x^8)/a)] + (3*a*(9*a*c*(8*a*d - b*(8*c + d*x^8))*AppellF1[1/8, 1/2, 1,
 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*b*x^8*(c + d*x^8)*(2*b*c*AppellF1[9/8
, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/
8, -((d*x^8)/c), -((b*x^8)/a)])))/((a + b*x^8)*(-9*a*c*AppellF1[1/8, 1/2, 1
, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2, 17/
8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*x^8)/
c), -((b*x^8)/a)])))))/(24*a^2*(-(b*c) + a*d)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

3.927 $\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 2, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a^2*x*Sqrt[c + d*x^8]))
```

Rubi [A] time = 0.0544516, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 2, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a^2*x*Sqrt[c + d*x^8]))
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= -\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.263178, size = 226, normalized size = 3.65

$$\frac{-5x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(24a^2d^2-40abcd+9b^2c^2\right)F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+35a\left(c+dx^8\right)\left(8a^2d-8ab\left(c-dx^8\right)-9b\right.}{280a^3cx\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (35*a*(c + d*x^8)*(8*a^2*d - 9*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 5*(9*b^2*c^
2 - 40*a*b*c*d + 24*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*(9*b*c - 8*a*d)*x^16*
(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c),
-((b*x^8)/a)])/(280*a^3*c*(b*c - a*d)*x*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^8+c}}{b^2dx^{26}+\left(b^2c+2\,abd\right)x^{18}+\left(2\,abc+a^2d\right)x^{10}+a^2cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^8 + c)/(b^2*d*x^26 + (b^2*c + 2*a*b*d)*x^18 + (2*a*b*c +
a^2*d)*x^10 + a^2*c*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

3.928 $\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

```
[Out] -(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 2, 1/2, 5/8, -((b*x^8)/a), -((d*x^8)/c
)])/(3*a^2*x^3*Sqrt[c + d*x^8])
```

Rubi [A] time = 0.0561598, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 2, 1/2, 5/8, -((b*x^8)/a), -((d*x^8)/c
)])/(3*a^2*x^3*Sqrt[c + d*x^8])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \frac{\sqrt{1+\frac{dx^8}{c}} \int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= -\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

Mathematica [B] time = 0.270698, size = 226, normalized size = 3.53

$$\frac{-13x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(8a^2d^2-56abcd+33b^2c^2\right)F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+65a\left(c+dx^8\right)\left(8a^2d-8ab\left(c-dx^8\right)-1\right.}{1560a^3cx^3\left(a+bx^8\right)\sqrt{c+dx^8}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (65*a*(c + d*x^8)*(8*a^2*d - 11*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 13*(33*b^2
*c^2 - 56*a*b*c*d + 8*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*(11*b*c - 8*a*d)*x^
16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c
), -((b*x^8)/a)])/(1560*a^3*c*(b*c - a*d)*x^3*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(bx^8+a\right)^2}\frac{1}{\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

3.929 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^5 \, dx$

Optimal. Leaf size=123

$$-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c^2) + ((2*b*c - a*d)*Sqrt[c + d/
x^2]*x^4)/(8*c) + (a*(c + d/x^2)^(3/2)*x^6)/(6*c) - (d^2*(2*b*c - a*d)*ArcT
anh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rubi [A] time = 0.0942834, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c^2) + ((2*b*c - a*d)*Sqrt[c + d/
x^2]*x^4)/(8*c) + (a*(c + d/x^2)^(3/2)*x^6)/(6*c) - (d^2*(2*b*c - a*d)*ArcT
anh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^4}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{\left(3bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3}\,dx,x,\frac{1}{x^2}\right)}{6c}$$
$$= \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{(d(2bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{16c}$$
$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{\left(d^2(2bc-ad)\right)\text{Subst}\left(\right.}{32c^2}$$
$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{(d(2bc-ad))\,\text{Subst}\left(\right.}{16}$$
$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{d^2(2bc-ad)\tanh^{-1}\left(\right.}{16c^{5/2}}$$

Mathematica [A] time = 0.198508, size = 121, normalized size = 0.98

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}x\sqrt{\frac{cx^2}{d}+1}\left(a\left(8c^2x^4+2cdx^2-3d^2\right)+6bc\left(2cx^2+d\right)\right)+3d^{3/2}(ad-2bc)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right)}{48c^{5/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(d + 2*c*x^2) + a*
(-3*d^2 + 2*c*d*x^2 + 8*c^2*x^4)) + 3*d^(3/2)*(-2*b*c + a*d)*ArcSinh[(Sqrt[
c]*x)/Sqrt[d]]))/(48*c^(5/2)*Sqrt[1 + (c*x^2)/d])
```

Maple [A] time = 0.017, size = 162, normalized size = 1.3

$$\frac{x}{48}\sqrt{\frac{cx^2+d}{x^2}}\left(8\,c^{3/2}\left(cx^2+d\right)^{3/2}x^3a-6\,\sqrt{c}\left(cx^2+d\right)^{3/2}xad+12\,c^{3/2}\left(cx^2+d\right)^{3/2}xb+3\,\sqrt{c}\sqrt{cx^2+d}xad^2-6\,c^{3/2}\sqrt{cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)*x*(8*c^(3/2)*(c*x^2+d)^(3/2)*x^3*a-6*c^(1/2)*(c*
x^2+d)^(3/2)*x*a*d+12*c^(3/2)*(c*x^2+d)^(3/2)*x*b+3*c^(1/2)*(c*x^2+d)^(1/2)
*x*a*d^2-6*c^(3/2)*(c*x^2+d)^(1/2)*x*b*d+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*
d^3-6*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d^2)/(c*x^2+d)^(1/2)/c^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66413, size = 533, normalized size = 4.33

$$\left[-\frac{3\left(2\,bcd^2-ad^3\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(8\,ac^3x^6+2\left(6\,bc^3+ac^2d\right)x^4+3\left(2\,bc^2d-acd^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96\,c^3},\ -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^
2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d^3)
*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c^3
*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2
 + d)/x^2))/c^3]
```

Sympy [B] time = 37.8391, size = 226, normalized size = 1.84

$$\frac{acx^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{5a\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}}+\frac{bcx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3b\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd}{8c\sqrt{\frac{c}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**5*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2/
d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c**2
*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b*c*
x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d + 1
)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sqrt(d)
)/(8*c**(3/2))
```

Giac [A] time = 1.14942, size = 193, normalized size = 1.57

$$\frac{1}{48}\left(2\left(4\,ax^2\operatorname{sgn}(x)+\frac{6\,bc^4\operatorname{sgn}(x)+ac^3d\operatorname{sgn}(x)}{c^4}\right)x^2+\frac{3\left(2\,bc^3d\operatorname{sgn}(x)-ac^2d^2\operatorname{sgn}(x)\right)}{c^4}\right)\sqrt{cx^2+d}x+\frac{\left(2\,bcd^2\operatorname{sgn}(x)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(2
*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*d^
2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2) - 1
/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)
```

3.930 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx$

Optimal. Leaf size=90

$$\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

```
[Out] ((4*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(8*c) + (a*(c + d/x^2)^(3/2)*x^4)/(4*c)
 + (d*(4*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rubi [A] time = 0.0678375, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] ((4*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(8*c) + (a*(c + d/x^2)^(3/2)*x^4)/(4*c)
 + (d*(4*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^3}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{\left(2bc-\frac{ad}{2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x^2}\right)}{4c}$$

$$= \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{(d(4bc-ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{16c}$$

$$= \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{(4bc-ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{8c}$$

$$= \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} + \frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Mathematica [A] time = 0.104588, size = 100, normalized size = 1.11

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}x\sqrt{\frac{cx^2}{d}+1}\left(a\left(2cx^2+d\right)+4bc\right)+\sqrt{d}(4bc-ad)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right)}{8c^{3/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(4*b*c + a*(d + 2*c*x^2))
 + Sqrt[d]*(4*b*c - a*d)*ArcSinh[(Sqrt[c]*x)/Sqrt[d]]))/(8*c^(3/2)*Sqrt[1 +
 (c*x^2)/d])
```

Maple [A] time = 0.007, size = 122, normalized size = 1.4

$$\frac{x}{8}\sqrt{\frac{cx^2+d}{x^2}}\left(2\sqrt{c}\left(cx^2+d\right)^{3/2}xa-\sqrt{c}\sqrt{cx^2+d}xad+4c^{3/2}\sqrt{cx^2+d}xb-\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)ad^2+4\ln\left(\sqrt{c}x+\sqrt{cx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(1/2)*x*(2*c^(1/2)*(c*x^2+d)^(3/2)*x*a-c^(1/2)*(c*x^2+d
)^(1/2)*x*a*d+4*c^(3/2)*(c*x^2+d)^(1/2)*x*b-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a
*d^2+4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d)/(c*x^2+d)^(1/2)/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37069, size = 431, normalized size = 4.79

$$\left[-\frac{\left(4\,bcd-ad^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^2x^4+\left(4\,bc^2+acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^2},-\frac{\left(4\,bcd-ad^2\right)\sqrt{-c}\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2
+ d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2
+ d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2
+ d)/x^2))/c^2]
```

Sympy [A] time = 28.8253, size = 144, normalized size = 1.6

$$\frac{acx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3a\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}+\frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*sqrt(d)*x**3/(8*sqrt(c*x**2/d
 + 1)) + a*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - a*d**2*asinh(sqrt(c)*x/sqr
t(d))/(8*c**(3/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + b*d*asinh(sqrt(c)*x
/sqrt(d))/(2*sqrt(c))
```

Giac [A] time = 1.14636, size = 142, normalized size = 1.58

$$\frac{1}{8}\left(2\,ax^2\operatorname{sgn}(x)+\frac{4\,bc^2\operatorname{sgn}(x)+acd\operatorname{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x-\frac{\left(4\,bcd\operatorname{sgn}(x)-ad^2\operatorname{sgn}(x)\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{3}{2}}}+\frac{(4\,bcd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*
x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d
)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3/2)
```

3.931 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{c+\frac{d}{x^2}}(ad+2bc)}{2c}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

```
[Out] -((2*b*c + a*d)*Sqrt[c + d/x^2])/(2*c) + (a*(c + d/x^2)^(3/2)*x^2)/(2*c) +
((2*b*c + a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*Sqrt[c])
```

Rubi [A] time = 0.0579218, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\sqrt{c+\frac{d}{x^2}}(ad+2bc)}{2c}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```


```
[Out] -((2*b*c + a*d)*Sqrt[c + d/x^2])/(2*c) + (a*(c + d/x^2)^(3/2)*x^2)/(2*c) +
((2*b*c + a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*Sqrt[c])
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}-\frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx,x,\frac{1}{x^2}\right)}{4c}\\
&= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}-\frac{1}{4}(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\\
&= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}-\frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{2d}\\
&= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.160214, size = 71, normalized size = 0.85

$$
\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\left(\frac{x(ad+2bc)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+ax^2-2b\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-2*b + a*x^2 + ((2*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[
d]])/(Sqrt[c]*Sqrt[d]*Sqrt[1 + (c*x^2)/d])))/2
```

Maple [A] time = 0.011, size = 127, normalized size = 1.5

$$
\frac{1}{2\,d}\sqrt{\frac{cx^2+d}{x^2}}\left(2\,c^{3/2}\sqrt{cx^2+d}x^2b+\sqrt{c}\sqrt{cx^2+d}x^2ad-2\,\sqrt{c}\left(cx^2+d\right)^{3/2}b+\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)xad^2+2\,\ln\left(\sqrt{c}x+\sqrt{}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/2*((c*x^2+d)/x^2)^(1/2)*(2*c^(3/2)*(c*x^2+d)^(1/2)*x^2*b+c^(1/2)*(c*x^2+d
)^(1/2)*x^2*a*d-2*c^(1/2)*(c*x^2+d)^(3/2)*b+ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x
*a*d^2+2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*b*c*d)/(c*x^2+d)^(1/2)/d/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32003, size = 358, normalized size = 4.26

$$\left[\frac{(2\,bc+ad)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\left(acx^2-2\,bc\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\,c},-\frac{(2\,bc+ad)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-\left(acx^2\right.}{2\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x
^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a*d
)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c*x^
2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A] time = 32.2153, size = 107, normalized size = 1.27

$$\frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}}+b\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)-\frac{bcx}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + a*d*asinh(sqrt(c)*x/sqrt(d))/(2*sqrt(c))
 + b*sqrt(c)*asinh(sqrt(c)*x/sqrt(d)) - b*c*x/(sqrt(d)*sqrt(c*x**2/d + 1))
- b*sqrt(d)/(x*sqrt(c*x**2/d + 1))
```

Giac [A] time = 1.25489, size = 124, normalized size = 1.48

$$\frac{1}{2}\sqrt{cx^2+d}ax\text{sgn}(x)+\frac{2\,b\sqrt{c}d\text{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d}-\frac{\left(2\,bc^{\frac{3}{2}}\text{sgn}(x)+a\sqrt{c}d\text{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d) - 1/4*(2*b*c^(3/2)*sgn(x) + a*sqrt(c)*d*sgn(x))*log((sqrt(
c)*x - sqrt(c*x^2 + d))^2)/c
```

3.932 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

Optimal. Leaf size=59

$$-a\sqrt{c+\frac{d}{x^2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

```
[Out] -(a*Sqrt[c + d/x^2]) - (b*(c + d/x^2)^(3/2))/(3*d) + a*Sqrt[c]*ArcTanh[Sqrt
[c + d/x^2]/Sqrt[c]]
```

Rubi [A] time = 0.0431102, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-a\sqrt{c+\frac{d}{x^2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] -(a*Sqrt[c + d/x^2]) - (b*(c + d/x^2)^(3/2))/(3*d) + a*Sqrt[c]*ArcTanh[Sqrt
[c + d/x^2]/Sqrt[c]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)$$
$$= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}(ac)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$
$$= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{(ac)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}$$
$$= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

Mathematica [A] time = 0.116312, size = 82, normalized size = 1.39

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(\frac{3a\sqrt{c}\sqrt{d}x^3\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{\frac{cx^2}{d}+1}} - 3adx^2 - b\left(cx^2+d\right)\right)}{3dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-3*a*d*x^2 - b*(d + c*x^2) + (3*a*Sqrt[c]*Sqrt[d]*x^3*Arc
Sinh[(Sqrt[c]*x)/Sqrt[d]])/Sqrt[1 + (c*x^2)/d]))/(3*d*x^2)
```

Maple [B] time = 0.011, size = 110, normalized size = 1.9

$$\frac{1}{3x^2d}\sqrt{\frac{cx^2+d}{x^2}}\left(3c^{3/2}\sqrt{cx^2+d}x^4a - 3\sqrt{c}\left(cx^2+d\right)^{3/2}x^2a + 3\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)x^3acd - \sqrt{c}\left(cx^2+d\right)^{\frac{3}{2}}b\right)\frac{1}{\sqrt{cx^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x)
```

```
[Out] 1/3*((c*x^2+d)/x^2)^(1/2)*(3*c^(3/2)*(c*x^2+d)^(1/2)*x^4*a-3*c^(1/2)*(c*x^2
+d)^(3/2)*x^2*a+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x^3*a*c*d-c^(1/2)*(c*x^2+d)
^(3/2)*b)/x^2/(c*x^2+d)^(1/2)/d/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3588, size = 385, normalized size = 6.53

$$\left[\frac{3\,a\sqrt{c}dx^2\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left((bc+3\,ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,dx^2},-\frac{3\,a\sqrt{-c}dx^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left((bc\right.}{3\,dx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2)
- d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(3*
a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (
(b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]
```

Sympy [A] time = 17.0405, size = 75, normalized size = 1.27

$$\frac{a\left(-\frac{2c\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c+\frac{d}{x^2}}\right)}{2}+\frac{b\left(\begin{cases}-\frac{\sqrt{c}}{x^2} & \text{for } d=0\\ -\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)
```

```
[Out] a*(-2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*sqrt(c + d/x**2))/2 +
b*Piecewise((-sqrt(c)/x**2, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True)
)/2
```

Giac [B] time = 1.39762, size = 220, normalized size = 3.73

$$-\frac{1}{2}a\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x)+\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}\operatorname{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d\operatorname{sgn}(x)-6\left(\sqrt{\right.\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(3/2)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))
^4*a*sqrt(c)*d*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^2*sgn
(x) + b*c^(3/2)*d^2*sgn(x) + 3*a*sqrt(c)*d^3*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + d))^2 - d)^3
```

3.933 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

```
[Out] ((b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (b*(c + d/x^2)^(5/2))/(5*d^2)
```

Rubi [A] time = 0.0373817, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```


```
[Out] ((b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (b*(c + d/x^2)^(5/2))/(5*d^2)
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int (a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Mathematica [A] time = 0.0162534, size = 47, normalized size = 1.02

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(5adx^2-2bcx^2+3bd\right)}{15d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] -(Sqrt[c + d/x^2]*(d + c*x^2)*(3*b*d - 2*b*c*x^2 + 5*a*d*x^2))/(15*d^2*x^4)
```

Maple [A] time = 0.006, size = 48, normalized size = 1.

$$-\frac{\left(5\,adx^2-2\,bcx^2+3\,bd\right)\left(cx^2+d\right)}{15\,d^2x^4}\sqrt{\frac{cx^2+d}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4
```

Maxima [A] time = 0.940565, size = 66, normalized size = 1.43

$$-\frac{1}{15}b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c +
d/x^2)^(3/2)/d
```

Fricas [A] time = 1.32251, size = 132, normalized size = 2.87

$$\frac{\left(\left(2\,bc^2-5\,acd\right)x^4-3\,bd^2-\left(bcd+5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2)/(d^2*x^4)
```

Sympy [A] time = 3.30036, size = 58, normalized size = 1.26

$$-\frac{a\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}-\frac{b\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3,x)
```

```
[Out] -a*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True))
/2 - b*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2
```

Giac [B] time = 2.19649, size = 338, normalized size = 7.35

$$\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}\mathrm{sgn}\left(x\right)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{5}{2}}\mathrm{sgn}\left(x\right)-30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d\mathrm{sgn}\left(x\right)+10\left(\sqrt{c}x-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*sgn(x) + 30*(sqrt(c)*x -
 sqrt(c*x^2 + d))^6*b*c^(5/2)*sgn(x) - 30*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a
*c^(3/2)*d*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d*sgn(x) +
 20*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^2*sgn(x) + 10*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*b*c^(5/2)*d^2*sgn(x) - 10*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(3/2)*d^3*sgn(x) - 2*b*c^(5/2)*d^3*sgn(x) + 5*a*c^(3/2)*d^4*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

3.934 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$

Optimal. Leaf size=74

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(5/
2))/(5*d^3) - (b*(c + d/x^2)^(7/2))/(7*d^3)
```

Rubi [A] time = 0.0587087, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```


```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(5/
2))/(5*d^3) - (b*(c + d/x^2)^(7/2))/(7*d^3)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int x(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{c(bc-ad)\sqrt{c+dx}}{d^2}+\frac{(-2bc+ad)(c+dx)^{3/2}}{d^2}+\frac{b(c+dx)^{5/2}}{d^2}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Mathematica [A] time = 0.0221702, size = 69, normalized size = 0.93

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(7adx^2\left(2cx^2-3d\right)+b\left(-8c^2x^4+12cdx^2-15d^2\right)\right)}{105d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(7*a*d*x^2*(-3*d + 2*c*x^2) + b*(-15*d^2 + 12*
c*d*x^2 - 8*c^2*x^4)))/(105*d^3*x^6)
```

Maple [A] time = 0.005, size = 70, normalized size = 1.

$$\frac{\left(14\,acdx^4-8\,bc^2x^4-21\,ad^2x^2+12\,bcdx^2-15\,bd^2\right)\left(cx^2+d\right)}{105\,d^3x^6}\sqrt{\frac{cx^2+d}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*d
*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6
```

Maxima [A] time = 0.944198, size = 113, normalized size = 1.53

$$-\frac{1}{105}\,b\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)-\frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/105*b*(15*(c + d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d
/x^2)^(3/2)*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2
)*c/d^2)
```

Fricas [A] time = 1.34685, size = 188, normalized size = 2.54

$$-\frac{\left(2\left(4\,bc^3-7\,ac^2d\right)x^6-\left(4\,bc^2d-7\,acd^2\right)x^4+15\,bd^3+3\left(bcd^2+7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d^
3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^6)
```

Sympy [A] time = 3.83459, size = 78, normalized size = 1.05

$$-\frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5,x)
```

```
[Out] -a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d
/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3
```

Giac [B] time = 3.89088, size = 419, normalized size = 5.66

$$\frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}\mathrm{sgn}\left(x\right)+280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}\left(x\right)-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d\mathrm{sgn}\left(x\right)+140\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(5/2)*sgn(x) + 280*(sqrt(c)
*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*sgn(x) - 175*(sqrt(c)*x - sqrt(c*x^2 + d)
)^8*a*c^(5/2)*d*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d^2*sgn(x) + 84*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*x^2
+ d))^4*a*c^(5/2)*d^3*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)
*d^3*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^4*sgn(x) + 4*b
*c^(7/2)*d^4*sgn(x) - 7*a*c^(5/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d)
)^2 - d)^7
```

3.935 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$

Optimal. Leaf size=104

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(5/2))/(5*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (b*(c + d/x^
2)^(9/2))/(9*d^4)
```

Rubi [A] time = 0.0790056, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(5/2))/(5*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (b*(c + d/x^
2)^(9/2))/(9*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{c^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{3/2}}{d^3}+\frac{(-3bc+ad)(c+dx)^{5/2}}{d^3}+\frac{b(\right.\right.\right.$$
$$= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Mathematica [A] time = 0.0494806, size = 79, normalized size = 0.76

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{cx^2}{d}+1\right)\left(8c^2x^4-12cdx^2+15d^2\right)(6bc-9ad)-105bd^2\left(cx^2+d\right)\right)}{945d^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-105*b*d^2*(d + c*x^2) + (6*b*c - 9*a*d)*x^2*(1 + (c*x^2)
/d)*(15*d^2 - 12*c*d*x^2 + 8*c^2*x^4)))/(945*d^3*x^8)
```

Maple [A] time = 0.007, size = 94, normalized size = 0.9

$$-\frac{\left(24\,ac^2dx^6-16\,bc^3x^6-36\,acd^2x^4+24\,bc^2dx^4+45\,ad^3x^2-30\,bcd^2x^2+35\,bd^3\right)\left(cx^2+d\right)}{315\,d^4x^8}\sqrt{\frac{cx^2+d}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x)
```

```
[Out] -1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+24
*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8
```

Maxima [A] time = 0.931659, size = 159, normalized size = 1.53

$$-\frac{1}{315}\,b\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)-\frac{1}{105}\,a\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c +
 d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c + d
/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d/x^2)^(3/2)*c^2/d^3
)
```

Fricas [A] time = 1.55066, size = 238, normalized size = 2.29

$$\frac{\left(8\left(2\,bc^4-3\,ac^3d\right)x^8-4\left(2\,bc^3d-3\,ac^2d^2\right)x^6-35\,bd^4+3\left(2\,bc^2d^2-3\,acd^3\right)x^4-5\left(bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*b
*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*sqrt((c
*x^2 + d)/x^2)/(d^4*x^8)
```

Sympy [A] time = 4.62581, size = 112, normalized size = 1.08

$$-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7,x)
```

```
[Out] -a*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**
(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)
/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4
```

Giac [B] time = 3.46804, size = 500, normalized size = 4.81

$$\frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}\mathrm{sgn}\left(x\right)+630\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}\left(x\right)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{7}{2}}d\mathrm{sgn}\left(x\right)+378\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(7/2)*sgn(x) + 630*(sqrt(c
)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 +
d))^10*a*c^(7/2)*d*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d
*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^2*sgn(x) + 168*(sq
rt(c)*x - sqrt(c*x^2 + d))^6*b*c^(9/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*
x^2 + d))^6*a*c^(7/2)*d^3*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(
9/2)*d^3*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^4*sgn(x)
+ 18*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(9/2)*d^4*sgn(x) - 27*(sqrt(c)*x -
 sqrt(c*x^2 + d))^2*a*c^(7/2)*d^5*sgn(x) - 2*b*c^(9/2)*d^5*sgn(x) + 3*a*c^(
7/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

3.936 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$

Optimal. Leaf size=134

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (3*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5) + ((4*b
*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (b*(c + d/x^2)^(11/2))/(11*d^5)
```

Rubi [A] time = 0.101759, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (3*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5) + ((4*b
*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (b*(c + d/x^2)^(11/2))/(11*d^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{c^3(bc-ad)\sqrt{c+dx}}{d^4} - \frac{c^2(4bc-3ad)(c+dx)^{3/2}}{d^4} + \frac{3c(2bc-ad)(c+dx)^{5/2}}{d^4} + \frac{(-\right.\right.\right.$$
$$= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5} + \frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5} + \frac{(4bc-ad)\left(c\right.}{9d^5}$$

Mathematica [A] time = 0.0633148, size = 90, normalized size = 0.67

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{cx^2}{d}+1\right)\left(24c^2dx^4-16c^3x^6-30cd^2x^2+35d^3\right)(8bc-11ad)-315bd^3\left(cx^2+d\right)\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-315*b*d^3*(d + c*x^2) + (8*b*c - 11*a*d)*x^2*(1 + (c*x^2
)/d)*(35*d^3 - 30*c*d^2*x^2 + 24*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x^10)
```

Maple [A] time = 0.007, size = 118, normalized size = 0.9

$$\frac{\left(176\,ac^3dx^8-128\,bc^4x^8-264\,ac^2d^2x^6+192\,bc^3dx^6+330\,acd^3x^4-240\,bc^2d^2x^4-385\,ad^4x^2+280\,bcd^3x^2-315\,bd^4\right)\left(cx\right.}{3465\,d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*x
^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*c*
```

```
d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^10
```

Maxima [A] time = 0.957736, size = 205, normalized size = 1.53

$$-\frac{1}{3465}b\left(\frac{315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^5}-\frac{1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^5}+\frac{2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^5}-\frac{2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^5}+\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4}{d^5}\right)-\frac{1}{315}a\left(\frac{35\left(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/3465*b*(315*(c + d/x^2)^(11/2)/d^5 - 1540*(c + d/x^2)^(9/2)*c/d^5 + 2970
*(c + d/x^2)^(7/2)*c^2/d^5 - 2772*(c + d/x^2)^(5/2)*c^3/d^5 + 1155*(c + d/x
^2)^(3/2)*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7
/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4)
```

Fricas [A] time = 1.79989, size = 302, normalized size = 2.25

$$-\frac{\left(16\left(8\,bc^5-11\,ac^4d\right)x^{10}-8\left(8\,bc^4d-11\,ac^3d^2\right)x^8+6\left(8\,bc^3d^2-11\,ac^2d^3\right)x^6+315\,bd^5-5\left(8\,bc^2d^3-11\,acd^4\right)x^4+35\left(\right.\right.}{3465\,d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^10 - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8
+ 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*c*
d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^10)
```

Sympy [A] time = 5.68921, size = 146, normalized size = 1.09

$$-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{b\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9,x)
```

```
[Out] -a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d
/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)
/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c +
 d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5
```

Giac [B] time = 7.0966, size = 581, normalized size = 4.34

$$\frac{32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\operatorname{sgn}(x)+11088\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{9}{2}}d\operatorname{sgn}(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*sgn(x) + 11088*(sq
rt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*sgn(x) - 4851*(sqrt(c)*x - sqrt(c*
x^2 + d))^12*a*c^(9/2)*d*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^2*sgn(x
) + 2640*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^2*sgn(x) - 165*(sqrt(
c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^3*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x
^2 + d))^6*b*c^(11/2)*d^3*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c
^(9/2)*d^4*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^4*sgn(
x) - 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^5*sgn(x) - 88*(sqrt(c)
*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^5*sgn(x) + 121*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*a*c^(9/2)*d^6*sgn(x) + 8*b*c^(11/2)*d^6*sgn(x) - 11*a*c^(9/2)*d^7*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

3.937 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$

Optimal. Leaf size=150

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4}-\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{99c^2}-\frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-}{231c^3}$$

```
[Out] (-16*d^3*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^3)/(3465*c^5) + (8*d^2*(11*b*
c - 8*a*d)*(c + d/x^2)^(3/2)*x^5)/(1155*c^4) - (2*d*(11*b*c - 8*a*d)*(c + d
/x^2)^(3/2)*x^7)/(231*c^3) + ((11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^9)/(99*c
^2) + (a*(c + d/x^2)^(3/2)*x^11)/(11*c)
```

Rubi [A] time = 0.0725086, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4}-\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{99c^2}-\frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-}{231c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (-16*d^3*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^3)/(3465*c^5) + (8*d^2*(11*b*
c - 8*a*d)*(c + d/x^2)^(3/2)*x^5)/(1155*c^4) - (2*d*(11*b*c - 8*a*d)*(c + d
/x^2)^(3/2)*x^7)/(231*c^3) + ((11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^9)/(99*c
^2) + (a*(c + d/x^2)^(3/2)*x^11)/(11*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}+\frac{(11bc-8ad)\int \sqrt{c+\frac{d}{x^2}}x^8\,dx}{11c}$$

$$= \frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}-\frac{(2d(11bc-8ad))\int \sqrt{c+\frac{d}{x^2}}x^6\,dx}{33c^2}$$

$$= -\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}+\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}+\frac{(8d^2(11b \ldots}{\ldots}$$

$$= \frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4}-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}+\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^{9 \ldots}}{99c^2}$$

$$= -\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3465c^5}+\frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4}-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2} \ldots\right.}{231c^3}$$

Mathematica [A] time = 0.0708872, size = 108, normalized size = 0.72

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(240c^2d^2x^4-280c^3dx^6+315c^4x^8-192cd^3x^2+128d^4\right)+11bc\left(-30c^2dx^4+35c^3x^6+24cd^2x^2-16 \ldots\right.\right.}{3465c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x
^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3*d
```

```
*x^6 + 315*c^4*x^8)))/(3465*c^5)
```

Maple [A] time = 0.006, size = 113, normalized size = 0.8

$$\frac{x\left(315\,ax^8c^4-280\,ac^3dx^6+385\,bc^4x^6+240\,ac^2d^2x^4-330\,bc^3dx^4-192\,acd^3x^2+264\,bc^2d^2x^2+128\,ad^4-176\,bcd^3\right)\left(c\right.}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^6
+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*a*
d^4-176*b*c*d^3)*(c*x^2+d)/c^5
```

Maxima [A] time = 0.949773, size = 213, normalized size = 1.42

$$\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)b}{315\,c^4}+\frac{\left(315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d/
x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*b/c^4 + 1/3465*(315*(c
+ d/x^2)^(11/2)*x^11 - 1540*(c + d/x^2)^(9/2)*d*x^9 + 2970*(c + d/x^2)^(7/2
)*d^2*x^7 - 2772*(c + d/x^2)^(5/2)*d^3*x^5 + 1155*(c + d/x^2)^(3/2)*d^4*x^3
)*a/c^5
```

Fricas [A] time = 1.35243, size = 294, normalized size = 1.96

$$\frac{\left(315\,ac^5x^{11}+35\left(11\,bc^5+ac^4d\right)x^9+5\left(11\,bc^4d-8\,ac^3d^2\right)x^7-6\left(11\,bc^3d^2-8\,ac^2d^3\right)x^5+8\left(11\,bc^2d^3-8\,acd^4\right)x^3-16\left(1\right.\right.}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a*
c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a*c
*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] time = 9.08156, size = 1386, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**10*(c+d/x**2)**(1/2),x)
```

```
[Out] 315*a*c**9*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860
*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**
5*d**20) + 1295*a*c**8*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*
x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**
2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(3465
*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c*
*6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2)*x**10*s
qrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**
7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**4*d**(43/
2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 280*a*c*
*3*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**
17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20)
+ 560*a*c**2*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 448*a*c*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 128*a*d**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 35*b*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 11
0*b*c**6*d**(21/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*
d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 114*b*c**5*d**(23/2)*x
**10*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**
5*d**11*x**2 + 315*c**4*d**12) + 40*b*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
```

```
4*d**12) - 5*b*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 +
 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*b*c**2*d*
*(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
 945*c**5*d**11*x**2 + 315*c**4*d**12) - 40*b*c*d**(31/2)*x**2*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) - 16*b*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945
*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
```

Giac [A] time = 1.13441, size = 217, normalized size = 1.45

$$\frac{\frac{11\left(35\left(cx^2+d\right)^{\frac{9}{2}}-135\left(cx^2+d\right)^{\frac{7}{2}}d+189\left(cx^2+d\right)^{\frac{5}{2}}d^2-105\left(cx^2+d\right)^{\frac{3}{2}}d^3\right)b\mathrm{sgn}(x)}{c^3}+\frac{\left(315\left(cx^2+d\right)^{\frac{11}{2}}-1540\left(cx^2+d\right)^{\frac{9}{2}}d+2970\left(cx^2+d\right)^{\frac{7}{2}}d^2-2772\left(cx^2+d\right)^{\frac{5}{2}}d^3+115\cdots}{c^4}}{3465\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*(11*(35*(c*x^2 + d)^(9/2) - 135*(c*x^2 + d)^(7/2)*d + 189*(c*x^2 + d
)^(5/2)*d^2 - 105*(c*x^2 + d)^(3/2)*d^3)*b*sgn(x)/c^3 + (315*(c*x^2 + d)^(1
1/2) - 1540*(c*x^2 + d)^(9/2)*d + 2970*(c*x^2 + d)^(7/2)*d^2 - 2772*(c*x^2
+ d)^(5/2)*d^3 + 1155*(c*x^2 + d)^(3/2)*d^4)*a*sgn(x)/c^4)/c + 16/3465*(11*
b*c*d^(9/2) - 8*a*d^(11/2))*sgn(x)/c^5
```

3.938 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$

Optimal. Leaf size=117

$$\frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2}-\frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

```
[Out] (8*d^2*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(315*c^4) - (4*d*(3*b*c - 2*a
*d)*(c + d/x^2)^(3/2)*x^5)/(105*c^3) + ((3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x
^7)/(21*c^2) + (a*(c + d/x^2)^(3/2)*x^9)/(9*c)
```

Rubi [A] time = 0.0616844, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2}-\frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (8*d^2*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(315*c^4) - (4*d*(3*b*c - 2*a
*d)*(c + d/x^2)^(3/2)*x^5)/(105*c^3) + ((3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x
^7)/(21*c^2) + (a*(c + d/x^2)^(3/2)*x^9)/(9*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}+\frac{(9bc-6ad)\int \sqrt{c+\frac{d}{x^2}}x^6\,dx}{9c}\\ &= \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}-\frac{(4d(3bc-2ad))\int \sqrt{c+\frac{d}{x^2}}x^4\,dx}{21c^2}\\ &= -\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}+\frac{\left(8d^2(3bc-\right.}{}\\ &= \frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{315c^4}-\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\end{aligned}$$

Mathematica [A] time = 0.0535571, size = 86, normalized size = 0.74

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(-30c^2dx^4+35c^3x^6+24cd^2x^2-16d^3\right)+3bc\left(15c^2x^4-12cdx^2+8d^2\right)\right)}{315c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(3*b*c*(8*d^2 - 12*c*d*x^2 + 15*c^2*x^4) + a
*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x^4 + 35*c^3*x^6)))/(315*c^4)
```

Maple [A] time = 0.006, size = 89, normalized size = 0.8

$$\frac{x\left(35\,ax^6c^3-30\,ac^2dx^4+45\,bc^3x^4+24\,acd^2x^2-36\,bc^2dx^2-16\,ad^3+24\,bcd^2\right)\left(cx^2+d\right)}{315\,c^4}\sqrt{\frac{cx^2+d}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24*
a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4
```

Maxima [A] time = 0.969763, size = 167, normalized size = 1.43

$$\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)b}{105\,c^3}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-10\right.}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^
2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)
^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^
3)*a/c^4
```

Fricas [A] time = 1.33652, size = 232, normalized size = 1.98

$$\frac{\left(35\,ac^4x^9+5\left(9\,bc^4+ac^3d\right)x^7+3\left(3\,bc^3d-2\,ac^2d^2\right)x^5-4\left(3\,bc^2d^2-2\,acd^3\right)x^3+8\left(3\,bcd^3-2\,ad^4\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d^
2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^4
```

Sympy [B] time = 6.33436, size = 910, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**8*(c+d/x**2)**(1/2),x)
```

```
[Out] 35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5*a*c**3
*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x**4*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d*
*9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*
a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x**10*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*b*c
**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x
**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**
5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**2*d**(15/2)*x**4
*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**
6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) + 8*b*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

Giac [A] time = 1.13264, size = 180, normalized size = 1.54

$$\frac{\frac{3\left(15\left(cx^2+d\right)^{\frac{7}{2}}-42\left(cx^2+d\right)^{\frac{5}{2}}d+35\left(cx^2+d\right)^{\frac{3}{2}}d^2\right)b\,\mathrm{sgn}(x)}{c^2}+\frac{\left(35\left(cx^2+d\right)^{\frac{9}{2}}-135\left(cx^2+d\right)^{\frac{7}{2}}d+189\left(cx^2+d\right)^{\frac{5}{2}}d^2-105\left(cx^2+d\right)^{\frac{3}{2}}d^3\right)a\,\mathrm{sgn}(x)}{c^3}}{315\,c}-\frac{8\left(3\,bcd^{\frac{7}{2}}-2\,

3.939 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx$

Optimal. Leaf size=84

$$\frac{x^5 \left(c+\frac{d}{x^2}\right)^{3/2} (7bc-4ad)}{35c^2}-\frac{2dx^3 \left(c+\frac{d}{x^2}\right)^{3/2} (7bc-4ad)}{105c^3}+\frac{ax^7 \left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

```
[Out] (-2*d*(7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^3)/(105*c^3) + ((7*b*c - 4*a*d)*(
c + d/x^2)^(3/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(3/2)*x^7)/(7*c)
```

Rubi [A] time = 0.0398965, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{x^5 \left(c+\frac{d}{x^2}\right)^{3/2} (7bc-4ad)}{35c^2}-\frac{2dx^3 \left(c+\frac{d}{x^2}\right)^{3/2} (7bc-4ad)}{105c^3}+\frac{ax^7 \left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (-2*d*(7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^3)/(105*c^3) + ((7*b*c - 4*a*d)*(
c + d/x^2)^(3/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(3/2)*x^7)/(7*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}+\frac{(7bc-4ad)\int \sqrt{c+\frac{d}{x^2}}x^4\,dx}{7c}$$
$$=\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}-\frac{(2d(7bc-4ad))\int \sqrt{c+\frac{d}{x^2}}x^2\,dx}{35c^2}$$
$$=-\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{105c^3}+\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}$$

Mathematica [A] time = 0.0401127, size = 64, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(15c^2x^4-12cdx^2+8d^2\right)+7bc\left(3cx^2-2d\right)\right)}{105c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(7*b*c*(-2*d + 3*c*x^2) + a*(8*d^2 - 12*c*d*
x^2 + 15*c^2*x^4)))/(105*c^3)
```

Maple [A] time = 0.007, size = 65, normalized size = 0.8

$$\frac{x\left(15\,ax^4c^2-12\,acdx^2+21\,bc^2x^2+8\,ad^2-14\,bcd\right)\left(cx^2+d\right)}{105\,c^3}\sqrt{\frac{cx^2+d}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*d
^2-14*b*c*d)*(c*x^2+d)/c^3
```

Maxima [A] time = 0.94442, size = 122, normalized size = 1.45

$$\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15\,c^2}+\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(1
5*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/2)
*d^2*x^3)*a/c^3
```

Fricas [A] time = 1.26122, size = 181, normalized size = 2.15

$$\frac{\left(15\,ac^3x^7+3\left(7\,bc^3+ac^2d\right)x^5+\left(7\,bc^2d-4\,acd^2\right)x^3-2\left(7\,bcd^2-4\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*x
^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3
```

Sympy [B] time = 4.36141, size = 422, normalized size = 5.02

$$\frac{15ac^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{33ac^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{3a}{105c^5d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**6*(c+d/x**2)**(1/2),x)
```

```
[Out] 15*a*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*sqrt(c*x**2/d
 + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(1
9/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + b*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(3/2)*x**2*sqrt(c*x**2/
d + 1)/(15*c) - 2*b*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

<u>Giac</u> [A] time = 1.18924, size = 142, normalized size = 1.69

$$\frac{\frac{7\left(3\left(cx^2+d\right)^{\frac{5}{2}}-5\left(cx^2+d\right)^{\frac{3}{2}}d\right)b\mathrm{sgn}(x)}{c}+\frac{\left(15\left(cx^2+d\right)^{\frac{7}{2}}-42\left(cx^2+d\right)^{\frac{5}{2}}d+35\left(cx^2+d\right)^{\frac{3}{2}}d^2\right)a\mathrm{sgn}(x)}{c^2}}{105\,c}+\frac{2\left(7\,bcd^{\frac{5}{2}}-4\,ad^{\frac{7}{2}}\right)\mathrm{sgn}\left(x\right)}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(7*(3*(c*x^2 + d)^(5/2) - 5*(c*x^2 + d)^(3/2)*d)*b*sgn(x)/c + (15*(c*
x^2 + d)^(7/2) - 42*(c*x^2 + d)^(5/2)*d + 35*(c*x^2 + d)^(3/2)*d^2)*a*sgn(x
)/c^2)/c + 2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3
```

3.940 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx$

Optimal. Leaf size=53

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

```
[Out] ((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^5
)/(5*c)
```

Rubi [A] time = 0.024704, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] ((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^5
)/(5*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{5c}+\frac{(5bc-2ad)\int \sqrt{c+\frac{d}{x^2}}x^2\,dx}{5c}$$
$$= \frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{5c}$$

Mathematica [A] time = 0.0232075, size = 42, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(3acx^2-2ad+5bc\right)}{15c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

Maple [A] time = 0.004, size = 43, normalized size = 0.8

$$\frac{x\left(3\,ax^2c-2\,ad+5\,bc\right)\left(cx^2+d\right)}{15\,c^2}\sqrt{\frac{cx^2+d}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2
```

Maxima [A] time = 0.943847, size = 74, normalized size = 1.4

$$\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)a}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^
2)^(3/2)*d*x^3)*a/c^2
```

Fricas [A] time = 1.29921, size = 127, normalized size = 2.4

$$\frac{\left(3\,ac^2x^5 + \left(5\,bc^2 + acd\right)x^3 + \left(5\,bcd - 2\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c*
x^2 + d)/x^2)/c^2
```

Sympy [B] time = 3.08089, size = 119, normalized size = 2.25

$$\frac{a\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} - \frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2} + \frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/(1
5*c) - 2*a*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*sqrt(d)*x**2*sqrt(c*x*
*2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
```

Giac [A] time = 1.10309, size = 99, normalized size = 1.87

$$\frac{5\left(cx^2+d\right)^{\frac{3}{2}}b\,\mathrm{sgn}\left(x\right) + \frac{\left(3\left(cx^2+d\right)^{\frac{5}{2}} - 5\left(cx^2+d\right)^{\frac{3}{2}}d\right)a\,\mathrm{sgn}(x)}{c}}{15\,c} - \frac{\left(5\,bcd^{\frac{3}{2}} - 2\,ad^{\frac{5}{2}}\right)\mathrm{sgn}\left(x\right)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(5*(c*x^2 + d)^(3/2)*b*sgn(x) + (3*(c*x^2 + d)^(5/2) - 5*(c*x^2 + d)^(
3/2)*d)*a*sgn(x)/c)/c - 1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2
```

3.941 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx$

Optimal. Leaf size=66

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}+bx\sqrt{c+\frac{d}{x^2}}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)$$

```
[Out] b*Sqrt[c + d/x^2]*x + (a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*Sqrt[d]*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rubi [A] time = 0.0366468, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {451, 242, 277, 217, 206}

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}+bx\sqrt{c+\frac{d}{x^2}}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] b*Sqrt[c + d/x^2]*x + (a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*Sqrt[d]*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}+b\int \sqrt{c+\frac{d}{x^2}}\,dx\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)\\
&= b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)\\
&= b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0470653, size = 84, normalized size = 1.27

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{cx^2+d}\left(a\left(cx^2+d\right)+3bc\right)-3bc\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{3c\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[d + c*x^2]*(3*b*c + a*(d + c*x^2)) - 3*b*c*Sqrt[d]
*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(3*c*Sqrt[d + c*x^2])
```

Maple [A] time = 0.009, size = 83, normalized size = 1.3

$$-\frac{x}{3\,c}\sqrt{\frac{cx^2+d}{x^2}}\left(3\,\sqrt{d}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)bc-a\left(cx^2+d\right)^{\frac{3}{2}}-3\,\sqrt{cx^2+d}dbc\right)\frac{1}{\sqrt{cx^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/3*((c*x^2+d)/x^2)^(1/2)*x*(3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*b*c-a*(c*x^2+d)^(3/2)-3*(c*x^2+d)^(1/2)*b*c)/(c*x^2+d)^(1/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35655, size = 365, normalized size = 5.53

$$\left[\frac{3\,bc\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,\left(acx^3+(3\,bc+ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,c},\frac{3\,bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(acx^3+(3\,bc+ad\right.}{3\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/
x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c*s
qrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (a*c*x^3 + (
3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A] time = 3.04817, size = 107, normalized size = 1.62

$$\frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}+\frac{b\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}}-b\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{bd}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c) +
 b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) +
b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))
```

Giac [B] time = 1.16584, size = 157, normalized size = 2.38

$$\frac{bd\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}}-\frac{\left(3\,bcd\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+3\,bc\sqrt{-d}\sqrt{d}+a\sqrt{-d}d^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,c\sqrt{-d}}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}ac^2\operatorname{sgn}(x)+3\,\sqrt{cx^2}\cdots}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan(
sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(c*
sqrt(-d)) + 1/3*((c*x^2 + d)^(3/2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^3*s
gn(x))/c^3
```

3.942 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{c+\frac{d}{x^2}}(2ad+bc)}{2cx}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

```
[Out] -((b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + (a*(c + d/x^2)^(3/2)*x)/c - ((b*
c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])
```

Rubi [A] time = 0.0463275, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 453, 195, 217, 206}

$$-\frac{\sqrt{c+\frac{d}{x^2}}(2ad+bc)}{2cx}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] -((b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + (a*(c + d/x^2)^(3/2)*x)/c - ((b*
c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}+\frac{(-bc-2ad)\,\text{Subst}\left(\int \sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}+\frac{1}{2}(-bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}+\frac{1}{2}(-bc-2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)\\
&= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0784238, size = 75, normalized size = 0.88

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-\frac{x^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{cx^2+d}}+2ax^2-b\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]
/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(2*x)
```

Maple [A] time = 0.012, size = 135, normalized size = 1.6

$$-\frac{1}{2\,xd}\sqrt{\frac{cx^2+d}{x^2}}\left(2\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^2a+\sqrt{d}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^2bc-2\,\sqrt{cx^2+d}x^2ad-\sqrt{cx^2+d}x^2bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*((c*x^2+d)/x^2)^(1/2)/x*(2*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*x^2*a+d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^2*b*c-2*(c*x^2+d)^(1/2
)*x^2*a*d-(c*x^2+d)^(1/2)*x^2*b*c+(c*x^2+d)^(3/2)*b)/(c*x^2+d)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43861, size = 375, normalized size = 4.41

$$\left[\frac{(bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,adx^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\,dx}, \frac{(bc+2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(2\,adx^2-b\right.}{2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^
2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((b*
c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d))
+ (2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]
```

Sympy [A] time = 4.17904, size = 107, normalized size = 1.26

$$\frac{a\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}}-a\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{ad}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) + a
*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x)
- b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```

Giac [A] time = 1.18739, size = 103, normalized size = 1.21

$$\frac{2\,\sqrt{cx^2+d}ac\operatorname{sgn}(x)+\frac{\left(bc^2\operatorname{sgn}(x)+2\,acd\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{\sqrt{cx^2+d}bc\operatorname{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + d)*a*c*sgn(x) + (b*c^2*sgn(x) + 2*a*c*d*sgn(x))*arctan(
sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - sqrt(c*x^2 + d)*b*c*sgn(x)/x^2)/c
```

3.943 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

Optimal. Leaf size=91

$$\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

```
[Out] ((b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d*x) - (b*(c + d/x^2)^(3/2))/(4*d*x) + (
c*(b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(3/2))
```

Rubi [A] time = 0.0507795, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 195, 217, 206}

$$\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] ((b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d*x) - (b*(c + d/x^2)^(3/2))/(4*d*x) + (
c*(b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(3/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx &= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(-bc+4ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx}{4d}\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}-\frac{(-bc+4ad)\,\mathrm{Subst}\left(\int \sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{4d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.115232, size = 100, normalized size = 1.1

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(\left(cx^2+d\right)\left(4adx^2+bcx^2+2bd\right)+cx^4\sqrt{\frac{cx^2}{d}+1}(4ad-bc)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{8dx^3\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] -(Sqrt[c + d/x^2]*((d + c*x^2)*(2*b*d + b*c*x^2 + 4*a*d*x^2) + c*(-(b*c) +
4*a*d)*x^4*Sqrt[1 + (c*x^2)/d]*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(8*d*x^3*(d +
 c*x^2))
```

Maple [B] time = 0.012, size = 175, normalized size = 1.9

$$-\frac{1}{8\,x^3d^2}\sqrt{\frac{cx^2+d}{x^2}}\left(4\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^4ac-\sqrt{d}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^4bc^2-4\,\sqrt{cx^2+d}x^4acd+\sqrt{cx^2+d}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/8*((c*x^2+d)/x^2)^(1/2)/x^3*(4*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/
x)*x^4*a*c-d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^4*b*c^2-4*(c*x^2+d
)^(1/2)*x^4*a*c*d+(c*x^2+d)^(1/2)*x^4*b*c^2+4*(c*x^2+d)^(3/2)*x^2*a*d-(c*x^
2+d)^(3/2)*x^2*b*c+2*(c*x^2+d)^(3/2)*b*d)/(c*x^2+d)^(1/2)/d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40902, size = 450, normalized size = 4.95

$$\left[-\frac{\left(bc^2-4\,acd\right)\sqrt{d}x^3\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,bd^2+\left(bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^2x^3},-\frac{\left(bc^2-4\,acd\right)\sqrt{-d}x^3\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2
 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*
sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/(d^2*x^3)]
```

Sympy [A] time = 6.51182, size = 144, normalized size = 1.58

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}}-\frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}}-\frac{bd}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2,x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*s
qrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*sqr
t(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*d/(
4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

Giac [A] time = 1.23753, size = 176, normalized size = 1.93

$$-\frac{\frac{\left(bc^3\operatorname{sgn}(x)-4\,ac^2d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}bc^3\operatorname{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\operatorname{sgn}(x)+\sqrt{cx^2+d}bc^3d\operatorname{sgn}(x)-4\sqrt{cx^2+d}ac^2d^2\operatorname{sgn}(x)}{c^2dx^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/8*((b*c^3*sgn(x) - 4*a*c^2*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(s
qrt(-d)*d) + ((c*x^2 + d)^(3/2)*b*c^3*sgn(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*
sgn(x) + sqrt(c*x^2 + d)*b*c^3*d*sgn(x) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x
))/(c^2*d*x^4))/c
```

3.944 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

Optimal. Leaf size=123

$$-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

```
[Out] ((b*c - 2*a*d)*Sqrt[c + d/x^2])/(8*d*x^3) - (b*(c + d/x^2)^(3/2))/(6*d*x^3)
 + (c*(b*c - 2*a*d)*Sqrt[c + d/x^2])/(16*d^2*x) - (c^2*(b*c - 2*a*d)*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(5/2))
```

Rubi [A] time = 0.0805208, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 279, 321, 217, 206}

$$-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] ((b*c - 2*a*d)*Sqrt[c + d/x^2])/(8*d*x^3) - (b*(c + d/x^2)^(3/2))/(6*d*x^3)
 + (c*(b*c - 2*a*d)*Sqrt[c + d/x^2])/(16*d^2*x) - (c^2*(b*c - 2*a*d)*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(-3bc+6ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx}{6d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} - \frac{(-3bc+6ad)\,\mathrm{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{6d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(c(bc-2ad))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{8d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{\left(c^2(bc-2ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,d\right.}{16d^2}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{\left(c^2(bc-2ad)\right)\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,\right.}{16d^2}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{5/2}}$$

Mathematica [C] time = 0.025872, size = 68, normalized size = 0.55

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(c^2x^6(bc-2ad)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{d}+1\right)-bd^3\right)}{6d^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-(b*d^3) + c^2*(b*c - 2*a*d)*x^6*Hypergeometr
ic2F1[3/2, 3, 5/2, 1 + (c*x^2)/d]))/(6*d^4*x^5)
```

Maple [B] time = 0.014, size = 220, normalized size = 1.8

$$\frac{1}{48\,x^5d^3}\sqrt{\frac{cx^2+d}{x^2}}\left(6\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^6ac^2-3\,\sqrt{d}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^6bc^3-6\,\sqrt{cx^2+d}\,x^6ac^2d+3\,\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)/x^5*(6*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/
x)*x^6*a*c^2-3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^6*b*c^3-6*(c*x
^2+d)^(1/2)*x^6*a*c^2*d+3*(c*x^2+d)^(1/2)*x^6*b*c^3+6*(c*x^2+d)^(3/2)*x^4*a
*c*d-3*(c*x^2+d)^(3/2)*x^4*b*c^2-12*(c*x^2+d)^(3/2)*x^2*a*d^2+6*(c*x^2+d)^(
3/2)*x^2*b*c*d-8*(c*x^2+d)^(3/2)*b*d^2)/(c*x^2+d)^(1/2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38809, size = 552, normalized size = 4.49

$$\left[-\frac{3\left(bc^3-2\,ac^2d\right)\sqrt{d}x^5\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(bc^2d-2\,acd^2\right)x^4-8\,bd^3-2\left(bcd^2+6\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96\,d^3x^5},\ \frac{3\left(bc^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(
b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3 -
2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)
) + (3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqr
```

```
t((c*x^2 + d)/x^2))/(d^3*x^5)]
```

Sympy [B] time = 10.8581, size = 226, normalized size = 1.84

$$-\frac{ac^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}}-\frac{ad}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{5}{2}}}{16d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}}{48dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{5b\sqrt{c}}{24x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)
```

```
[Out] -a*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*a*sqrt(c)/(8*x**3*sqrt(1 + d/(
c*x**2))) + a*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - a*d/(4*sqrt(c)
*x**5*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))) +
 b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqrt(1
+ d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d/(6*s
qrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

Giac [A] time = 1.24678, size = 207, normalized size = 1.68

$$\frac{\frac{3\left(bc^4\operatorname{sgn}(x)-2\,ac^3d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\operatorname{sgn}(x)-6\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\operatorname{sgn}(x)-8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\operatorname{sgn}(x)-3\sqrt{cx^2+d}bc^4d^2\operatorname{sgn}(x)+6\sqrt{cx^2+d}ac^3d}{c^3d^2x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b*c^4*sgn(x) - 2*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/
(sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) - 6*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4
*d^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c^3*d^3*sgn(x))/(c^3*d^2*x^6))/c
```

3.945 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx$

Optimal. Leaf size=123

$$\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}(6bc-ad)}{16c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

```
[Out] (d*(6*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c) + ((6*b*c - a*d)*(c + d/x^2)^(
3/2)*x^4)/(24*c) + (a*(c + d/x^2)^(5/2)*x^6)/(6*c) + (d^2*(6*b*c - a*d)*Arc
Tanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rubi [A] time = 0.0901921, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}(6bc-ad)}{16c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (d*(6*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c) + ((6*b*c - a*d)*(c + d/x^2)^(
3/2)*x^4)/(24*c) + (a*(c + d/x^2)^(5/2)*x^6)/(6*c) + (d^2*(6*b*c - a*d)*Arc
Tanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{\left(3bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d(6bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x^2}\right)}{16c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{\left(d^2(6bc-ad)\right)\text{Sul}}{}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d(6bc-ad))\,\text{Subs}}{}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} + \frac{d^2(6bc-ad)\tanh}{16c^{3/2}}$$

Mathematica [A] time = 0.118012, size = 123, normalized size = 1.

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}x\sqrt{\frac{cx^2}{d}+1}\left(a\left(8c^2x^4+14cdx^2+3d^2\right)+6bc\left(2cx^2+5d\right)\right)-3d^{3/2}(ad-6bc)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\right)}{48c^{3/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(5*d + 2*c*x^2) +
a*(3*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) - 3*d^(3/2)*(-6*b*c + a*d)*ArcSinh[(Sqr
t[c]*x)/Sqrt[d]]))/(48*c^(3/2)*Sqrt[1 + (c*x^2)/d])
```

Maple [A] time = 0.007, size = 162, normalized size = 1.3

$$\frac{x^3}{48}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(8\sqrt{c}\left(cx^2+d\right)^{5/2}xa-2\sqrt{c}\left(cx^2+d\right)^{3/2}xad+12\,c^{3/2}\left(cx^2+d\right)^{3/2}xb-3\sqrt{c}\sqrt{cx^2+d}xad^2+18\,c^{3/2}\sqrt{cx^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(3/2)*x^3*(8*c^(1/2)*(c*x^2+d)^(5/2)*x*a-2*c^(1/2)*(c*
x^2+d)^(3/2)*x*a*d+12*c^(3/2)*(c*x^2+d)^(3/2)*x*b-3*c^(1/2)*(c*x^2+d)^(1/2)
*x*a*d^2+18*c^(3/2)*(c*x^2+d)^(1/2)*x*b*d-3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a
*d^3+18*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d^2)/(c*x^2+d)^(3/2)/c^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.47824, size = 543, normalized size = 4.41

$$\left[-\frac{3\left(6\,bcd^{2}-ad^{3}\right)\sqrt{c}\log\left(-2\,cx^{2}+2\,\sqrt{c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\left(8\,ac^{3}x^{6}+2\left(6\,bc^{3}+7\,ac^{2}d\right)x^{4}+3\left(10\,bc^{2}d+acd^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}\cdots}{x^{2}}}}{96\,c^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b
*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 - a*
d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (8*a
*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/c^2]
```

Sympy [B] time = 52.0316, size = 253, normalized size = 2.06

$$\frac{ac^2x^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{11ac\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}+\frac{17ad^{\frac{3}{2}}x^3}{48\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{5}{2}}x}{16c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{3}{2}}}+\frac{bc^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3bc\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)
```

```
[Out] a*c**2*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 11*a*c*sqrt(d)*x**5/(24*sqrt(c
*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*sqrt(c*x**2/d + 1)) + a*d**(5/2)*x/(
16*c*sqrt(c*x**2/d + 1)) - a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(3/2)) +
b*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + b*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c))
```

Giac [A] time = 1.16028, size = 194, normalized size = 1.58

$$\frac{1}{48}\left(2\left(4\,acx^2\operatorname{sgn}(x)+\frac{6\,bc^5\operatorname{sgn}(x)+7\,ac^4d\operatorname{sgn}(x)}{c^4}\right)x^2+\frac{3\left(10\,bc^4d\operatorname{sgn}(x)+ac^3d^2\operatorname{sgn}(x)\right)}{c^4}\right)\sqrt{cx^2+dx}-\frac{\left(6\,bcd^2\operatorname{sgn}(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*c*x^2*sgn(x) + (6*b*c^5*sgn(x) + 7*a*c^4*d*sgn(x))/c^4)*x^2 +
3*(10*b*c^4*d*sgn(x) + a*c^3*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x - 1/16*(6*b
*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(3/2
) + 1/32*(6*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(3/2)
```

3.946 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^3\,dx$

Optimal. Leaf size=115

$$\frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c}-\frac{3d\sqrt{c+\frac{d}{x^2}}(ad+4bc)}{8c}+\frac{3d(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

```
[Out] (-3*d*(4*b*c + a*d)*Sqrt[c + d/x^2])/(8*c) + ((4*b*c + a*d)*(c + d/x^2)^(3/
2)*x^2)/(8*c) + (a*(c + d/x^2)^(5/2)*x^4)/(4*c) + (3*d*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rubi [A] time = 0.0844772, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$\frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c}-\frac{3d\sqrt{c+\frac{d}{x^2}}(ad+4bc)}{8c}+\frac{3d(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (-3*d*(4*b*c + a*d)*Sqrt[c + d/x^2])/(8*c) + ((4*b*c + a*d)*(c + d/x^2)^(3/
2)*x^2)/(8*c) + (a*(c + d/x^2)^(5/2)*x^4)/(4*c) + (3*d*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{(4bc+ad)\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)}{8c}$$

$$= \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{(3d(4bc+ad))\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)}{16c}$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{1}{16}(3d(4bc+ad)$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{1}{8}(3(4bc+ad))\,\mathrm{S}$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} + \frac{3d(4bc+ad)\,\mathrm{ta}}{8\sqrt{}}$$

Mathematica [A] time = 0.176663, size = 89, normalized size = 0.77

$$\frac{1}{8}\sqrt{c+\frac{d}{x^2}}\left(\frac{3\sqrt{d}x(ad+4bc)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{d}+1}} + 2acx^4 + 5adx^2 + 4bcx^2 - 8bd\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 + (3*Sqrt[d]*(
4*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/d]))
)/8
```

Maple [A] time = 0.013, size = 174, normalized size = 1.5

$$\frac{x^2}{8d}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(8c^{3/2}\left(cx^2+d\right)^{3/2}x^2b + 12c^{3/2}\sqrt{cx^2+d}x^2bd + 2\sqrt{c}\left(cx^2+d\right)^{3/2}x^2ad - 8\sqrt{c}\left(cx^2+d\right)^{5/2}b + 3\sqrt{c}\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(3/2)*x^2*(8*c^(3/2)*(c*x^2+d)^(3/2)*x^2*b+12*c^(3/2)*(
c*x^2+d)^(1/2)*x^2*b*d+2*c^(1/2)*(c*x^2+d)^(3/2)*x^2*a*d-8*c^(1/2)*(c*x^2+d
)^(5/2)*b+3*c^(1/2)*(c*x^2+d)^(1/2)*x^2*a*d^2+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2
))*x*a*d^3+12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*b*c*d^2)/(c*x^2+d)^(3/2)/d/c^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36592, size = 462, normalized size = 4.02

$$\left[\frac{3\left(4\,bcd + ad^{2}\right)\sqrt{c}\log\left(-2\,cx^{2} - 2\,\sqrt{c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}} - d\right) + 2\left(2\,ac^{2}x^{4} - 8\,bcd + \left(4\,bc^{2} + 5\,acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{16\,c}, -\frac{3\left(4\,bcd + ad^{2}\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2
 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5
*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] time = 89.8581, size = 216, normalized size = 1.88

$$\frac{ac^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3ac\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}}+\frac{3b\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2}+\frac{bc\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)
```

```
[Out] a*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + a*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + a*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c)) + 3*b*sqrt(c)*d*
asinh(sqrt(c)*x/sqrt(d))/2 + b*c*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 - b*c*sqrt(
d)*x/sqrt(c*x**2/d + 1) - b*d**(3/2)/(x*sqrt(c*x**2/d + 1))
```

Giac [A] time = 1.29828, size = 170, normalized size = 1.48

$$\frac{2b\sqrt{c}d^2\operatorname{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d}+\frac{1}{8}\left(2acx^2\operatorname{sgn}(x)+\frac{4bc^3\operatorname{sgn}(x)+5ac^2d\operatorname{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x-\frac{3\left(4bc^{\frac{3}{2}}d\operatorname{sgn}(x)+a\sqrt{c}d^2\operatorname{sgn}(x)\right)\ldots}{16\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*x
^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x - 3/
16*(4*b*c^(3/2)*d*sgn(x) + a*sqrt(c)*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^
2 + d))^2)/c
```

3.947 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx$

Optimal. Leaf size=110

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c}-\frac{1}{2}\sqrt{c+\frac{d}{x^2}}(3ad+2bc)+\frac{1}{2}\sqrt{c}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

```
[Out] -((2*b*c + 3*a*d)*Sqrt[c + d/x^2])/2 - ((2*b*c + 3*a*d)*(c + d/x^2)^(3/2))/
(6*c) + (a*(c + d/x^2)^(5/2)*x^2)/(2*c) + (Sqrt[c]*(2*b*c + 3*a*d)*ArcTanh[
Sqrt[c + d/x^2]/Sqrt[c]])/2
```

Rubi [A] time = 0.0727332, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c}-\frac{1}{2}\sqrt{c+\frac{d}{x^2}}(3ad+2bc)+\frac{1}{2}\sqrt{c}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] -((2*b*c + 3*a*d)*Sqrt[c + d/x^2])/2 - ((2*b*c + 3*a*d)*(c + d/x^2)^(3/2))/
(6*c) + (a*(c + d/x^2)^(5/2)*x^2)/(2*c) + (Sqrt[c]*(2*b*c + 3*a*d)*ArcTanh[
Sqrt[c + d/x^2]/Sqrt[c]])/2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x\,dx=-\left(\frac{1}{2}\operatorname{Subst}\left(\int\frac{(a+bx)(c+dx)^{3/2}}{x^2}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c}-\frac{\left(bc+\frac{3ad}{2}\right)\operatorname{Subst}\left(\int\frac{(c+dx)^{3/2}}{x}\,dx,x,\frac{1}{x^2}\right)}{2c}$$

$$=-\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c}-\frac{1}{4}(2bc+3ad)\operatorname{Subst}\left(\int\frac{\sqrt{c+dx}}{x}\,dx,x,\frac{1}{x^2}\right)$$

$$=-\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}}-\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c}-\frac{1}{4}(c(2bc+3ad))\operatorname{Su}$$

$$=-\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}}-\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c}-\frac{(c(2bc+3ad))\operatorname{Sub}}{}$$

$$=-\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}}-\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c}+\frac{1}{2}\sqrt{c}(2bc+3ad)\operatorname{ta}$$

Mathematica [C] time = 0.0827385, size = 78, normalized size = 0.71

$$\frac{1}{3}\sqrt{c+\frac{d}{x^2}}\left(-\frac{(3ad+2bc)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{d}\right)}{\sqrt{\frac{cx^2}{d}+1}}-\frac{b\left(cx^2+d\right)^2}{dx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-((b*(d + c*x^2)^2)/(d*x^2)) - ((2*b*c + 3*a*d)*Hypergeom
etric2F1[-3/2, -1/2, 1/2, -((c*x^2)/d)])/Sqrt[1 + (c*x^2)/d]))/3
```

Maple [B] time = 0.013, size = 216, normalized size = 2.

$$\frac{1}{6\,d^2}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(6\,c^{3/2}\left(cx^2+d\right)^{3/2}x^4ad+4\,c^{5/2}\left(cx^2+d\right)^{3/2}x^4b-6\,\sqrt{c}\left(cx^2+d\right)^{5/2}x^2ad-4\,c^{3/2}\left(cx^2+d\right)^{5/2}x^2b+9\,c^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x)
```

```
[Out] 1/6*((c*x^2+d)/x^2)^(3/2)*(6*c^(3/2)*(c*x^2+d)^(3/2)*x^4*a*d+4*c^(5/2)*(c*x
^2+d)^(3/2)*x^4*b-6*c^(1/2)*(c*x^2+d)^(5/2)*x^2*a*d-4*c^(3/2)*(c*x^2+d)^(5/
2)*x^2*b+9*c^(3/2)*(c*x^2+d)^(1/2)*x^4*a*d^2+6*c^(5/2)*(c*x^2+d)^(1/2)*x^4*
b*d+9*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x^3*a*c*d^3+6*ln(c^(1/2)*x+(c*x^2+d)^(1
/2))*x^3*b*c^2*d^2-2*c^(1/2)*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2/c^(1/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36788, size = 456, normalized size = 4.15

$$\left[\frac{3\,(2\,bc+3\,ad)\sqrt{c}x^2\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\left(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x^2},-\frac{3\,(2\,bc+3\,ad)\sqrt{-\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x
^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c*x
^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 -
2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]
```

Sympy [A] time = 33.146, size = 187, normalized size = 1.7

$$\frac{3a\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2}+\frac{ac\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{ac\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}+bc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)-\frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}+bd\Bigg(\Bigg\{$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)
```

```
[Out] 3*a*sqrt(c)*d*asinh(sqrt(c)*x/sqrt(d))/2 + a*c*sqrt(d)*x*sqrt(c*x**2/d + 1)
/2 - a*c*sqrt(d)*x/sqrt(c*x**2/d + 1) - a*d**(3/2)/(x*sqrt(c*x**2/d + 1)) +
 b*c**(3/2)*asinh(sqrt(c)*x/sqrt(d)) - b*c**2*x/(sqrt(d)*sqrt(c*x**2/d + 1)
) - b*c*sqrt(d)/(x*sqrt(c*x**2/d + 1)) + b*d*Piecewise((-sqrt(c)/(2*x**2),
Eq(d, 0)), (-(c + d/x**2)**(3/2)/(3*d), True))
```

Giac [B] time = 1.71049, size = 304, normalized size = 2.76

$$\frac{1}{2}\sqrt{cx^2+d}acx\operatorname{sgn}(x)-\frac{1}{4}\left(2bc^{\frac{3}{2}}\operatorname{sgn}(x)+3a\sqrt{c}d\operatorname{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)+\frac{2\left(6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}d\operatorname{sgn}(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*c*x*sgn(x) - 1/4*(2*b*c^(3/2)*sgn(x) + 3*a*sqrt(c)*d*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2) + 2/3*(6*(sqrt(c)*x - sqrt(c*x
^2 + d))^4*b*c^(3/2)*d*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)
*d^2*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d^2*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^3*sgn(x) + 4*b*c^(3/2)*d^3*sgn(x)
 + 3*a*sqrt(c)*d^4*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3
```

3.948 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x} dx$

Optimal. Leaf size=76

$$ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-ac\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

```
[Out] -(a*c*Sqrt[c + d/x^2]) - (a*(c + d/x^2)^(3/2))/3 - (b*(c + d/x^2)^(5/2))/(5
*d) + a*c^(3/2)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
```

Rubi [A] time = 0.0541351, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-ac\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```


```
[Out] -(a*c*Sqrt[c + d/x^2]) - (a*(c + d/x^2)^(3/2))/3 - (b*(c + d/x^2)^(5/2))/(5
*d) + a*c^(3/2)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}(ac)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}\left(ac^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{\left(ac^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}$$

$$= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

Mathematica [C] time = 0.046791, size = 90, normalized size = 1.18

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(5ad^2x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{d}\right)+3b\left(cx^2+d\right)^2\sqrt{\frac{cx^2}{d}+1}\right)}{15dx^4\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] -(Sqrt[c + d/x^2]*(3*b*(d + c*x^2)^2*Sqrt[1 + (c*x^2)/d] + 5*a*d^2*x^2*Hype
rgeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)/d)]))/(15*d*x^4*Sqrt[1 + (c*x^2)/
d])
```

Maple [B] time = 0.014, size = 153, normalized size = 2.

$$\frac{1}{15\,x^2d^2}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(10\,c^{5/2}\left(cx^2+d\right)^{3/2}x^6a+15\,c^{5/2}\sqrt{cx^2+d}x^6ad-10\,c^{3/2}\left(cx^2+d\right)^{5/2}x^4a+15\,\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(3/2)*(10*c^(5/2)*(c*x^2+d)^(3/2)*x^6*a+15*c^(5/2)*(c*
x^2+d)^(1/2)*x^6*a*d-10*c^(3/2)*(c*x^2+d)^(5/2)*x^4*a+15*ln(c^(1/2)*x+(c*x^
2+d)^(1/2))*x^5*a*c^2*d^2-5*c^(1/2)*(c*x^2+d)^(5/2)*x^2*a*d-3*c^(1/2)*(c*x^
2+d)^(5/2)*b*d)/x^2/(c*x^2+d)^(3/2)/d^2/c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34711, size = 493, normalized size = 6.49

$$\left[\frac{15\,ac^{\frac{3}{2}}dx^4\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(\left(3\,bc^2+20\,acd\right)x^4+3\,bd^2+\left(6\,bcd+5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,dx^4},-\frac{15\,a\sqrt{-c}cdx^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2
) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*s
qrt((c*x^2 + d)/x^2))/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(-c)
*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d
^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^4)]
```

Sympy [A] time = 37.2231, size = 73, normalized size = 0.96

$$-\frac{ac^2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-ac\sqrt{c+\frac{d}{x^2}}-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*c**2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - a*c*sqrt(c + d/x**2) - a
*(c + d/x**2)**(3/2)/3 - b*(c + d/x**2)**(5/2)/(5*d)
```

Giac [B] time = 1.80624, size = 343, normalized size = 4.51

$$-\frac{1}{2}ac^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x)+\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d\operatorname{sgn}(x)-9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/15*(15*(sqrt
(c)*x - sqrt(c*x^2 + d))^8*b*c^(5/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 +
d))^8*a*c^(3/2)*d*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2)*d^2
*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d^2*sgn(x) + 110*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + d))^2*a*c^(3/2)*d^4*sgn(x) + 3*b*c^(5/2)*d^4*sgn(x) + 20*a*c^(3/2)*d^
5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

3.949 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

```
[Out] ((b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (b*(c + d/x^2)^(7/2))/(7*d^2)
```

Rubi [A] time = 0.0364029, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (b*(c + d/x^2)^(7/2))/(7*d^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int (a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

Mathematica [A] time = 0.0183914, size = 49, normalized size = 1.07

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(cx^2+d\right)^2\left(7adx^2-2bcx^2+5bd\right)}{35d^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] -(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(35*d^2*x^
6)
```

Maple [A] time = 0.006, size = 48, normalized size = 1.

$$-\frac{\left(7\,adx^2-2\,bcx^2+5\,bd\right)\left(cx^2+d\right)}{35\,d^2x^4}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x)
```

```
[Out] -1/35*((c*x^2+d)/x^2)^(3/2)*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4
```

Maxima [A] time = 0.947076, size = 66, normalized size = 1.43

$$-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d} - \frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2} - \frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/5*a*(c + d/x^2)^(5/2)/d - 1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^
(5/2)*c/d^2)*b
```

Fricas [B] time = 1.37909, size = 180, normalized size = 3.91

$$\frac{\left(\left(2\,bc^3-7\,ac^2d\right)x^6-\left(bc^2d+14\,acd^2\right)x^4-5\,bd^3-\left(8\,bcd^2+7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (8
*b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^2*x^6)
```

Sympy [A] time = 10.1932, size = 138, normalized size = 3.

$$-\frac{ac\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}-\frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d}-\frac{bc\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] -a*c*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True
))/2 - a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d - b*c*(-c*(c
+ d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d/x**2)**(3
/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
```

Giac [B] time = 5.43887, size = 500, normalized size = 10.87

$$\frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}\mathrm{sgn}\left(x\right)+70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}\mathrm{sgn}\left(x\right)-70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}d\mathrm{sgn}\left(x\right)+70\left(\sqrt{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)*sgn(x) + 70*(sqrt(c)*x
- sqrt(c*x^2 + d))^10*b*c^(7/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*x^2 + d))^1
0*a*c^(5/2)*d*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*d*sgn(x
) + 105*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^2*sgn(x) + 140*(sqrt(c)
*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^2*sgn(x) - 140*(sqrt(c)*x - sqrt(c*x^2
+ d))^6*a*c^(5/2)*d^3*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)
*d^3*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^4*sgn(x) + 14*
(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*d^4*sgn(x) - 14*(sqrt(c)*x - sqrt
(c*x^2 + d))^2*a*c^(5/2)*d^5*sgn(x) - 2*b*c^(7/2)*d^5*sgn(x) + 7*a*c^(5/2)*
d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7
```

3.950 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=74

$$\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(7/
2))/(7*d^3) - (b*(c + d/x^2)^(9/2))/(9*d^3)
```

Rubi [A] time = 0.0557358, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(7/
2))/(7*d^3) - (b*(c + d/x^2)^(9/2))/(9*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{(-2bc+ad)(c+dx)^{5/2}}{d^2}+\frac{b(c+dx)^{7/2}}{d^2}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Mathematica [A] time = 0.0250015, size = 71, normalized size = 0.96

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(9adx^2\left(2cx^2-5d\right)+b\left(-8c^2x^4+20cdx^2-35d^2\right)\right)}{315d^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 + 2
0*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)
```

Maple [A] time = 0.007, size = 70, normalized size = 1.

$$\frac{\left(18\,acdx^4-8\,bc^2x^4-45\,ad^2x^2+20\,bcdx^2-35\,bd^2\right)\left(cx^2+d\right)}{315\,d^3x^6}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*d
*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6
```

Maxima [A] time = 0.937936, size = 113, normalized size = 1.53

$$-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)a-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^(5/2)*c/d^2)*a - 1/315*(35*(
c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x^2)^(5/2)*c^
2/d^3)*b
```

Fricas [A] time = 1.59785, size = 239, normalized size = 3.23

$$-\frac{\left(2\left(4\,bc^4-9\,ac^3d\right)x^8-\left(4\,bc^3d-9\,ac^2d^2\right)x^6+35\,bd^4+3\left(bc^2d^2+24\,acd^3\right)x^4+5\left(10\,bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b*
d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2)/(d^3*x^8)
```

Sympy [B] time = 11.8264, size = 194, normalized size = 2.62

$$-\frac{ac\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2}-\frac{bc\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -a*c*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - a*(c**2*(c +
 d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
 - b*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**
2)**(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(
5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3
```

Giac [B] time = 4.18228, size = 581, normalized size = 7.85

$$\frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{7}{2}}\mathrm{sgn}\left(x\right)+840\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{9}{2}}\mathrm{sgn}\left(x\right)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}d\mathrm{sgn}\left(x\right)+12\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(7/2)*sgn(x) + 840*(sqrt(c)
*x - sqrt(c*x^2 + d))^12*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 + d
))^12*a*c^(7/2)*d*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*
d*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(7/2)*d^2*sgn(x) + 1764
*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d^2*sgn(x) - 819*(sqrt(c)*x - sq
rt(c*x^2 + d))^8*a*c^(7/2)*d^3*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^6
*b*c^(9/2)*d^3*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(7/2)*d^4*s
gn(x) + 144*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(9/2)*d^4*sgn(x) - 9*(sqrt(
c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^5*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*b*c^(9/2)*d^5*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(7/2
)*d^6*sgn(x) + 4*b*c^(9/2)*d^6*sgn(x) - 9*a*c^(7/2)*d^7*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + d))^2 - d)^9
```

3.951 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=104

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(7/2))/(7*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (b*(c + d/x^
2)^(11/2))/(11*d^4)
```

Rubi [A] time = 0.0720556, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```


```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(7/2))/(7*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (b*(c + d/x^
2)^(11/2))/(11*d^4)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{c^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{5/2}}{d^3}+\frac{(-3bc+ad)(c+dx)^{7/2}}{d^3}\right.\right.\right.\\ &= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}\end{aligned}$$

Mathematica [A] time = 0.0303498, size = 94, normalized size = 0.9

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(-11adx^2\left(8c^2x^4-20cdx^2+35d^2\right)-3b\left(40c^2dx^4-16c^3x^6-70cd^2x^2+105d^3\right)\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x^
4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x
^10)
```

Maple [A] time = 0.008, size = 94, normalized size = 0.9

$$-\frac{\left(88\,ac^2dx^6-48\,bc^3x^6-220\,acd^2x^4+120\,bc^2dx^4+385\,ad^3x^2-210\,bcd^2x^2+315\,bd^3\right)\left(cx^2+d\right)}{3465\,d^4x^8}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x)
```

```
[Out] -1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4+
120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^8
```

Maxima [A] time = 0.93332, size = 159, normalized size = 1.53

$$-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)a-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*(35*(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x
^2)^(5/2)*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)
^(9/2)*c/d^4 + 495*(c + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^
4)*b
```

Fricas [A] time = 1.73843, size = 305, normalized size = 2.93

$$\frac{\left(8\left(6\,bc^5-11\,ac^4d\right)x^{10}-4\left(6\,bc^4d-11\,ac^3d^2\right)x^8+3\left(6\,bc^3d^2-11\,ac^2d^3\right)x^6-315\,bd^5-5\left(3\,bc^2d^3+110\,acd^4\right)x^4-35\left(12\,\right.\right.}{3465\,d^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 +
3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c*d
^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^10)
```

Sympy [B] time = 13.995, size = 262, normalized size = 2.52

$$-\frac{ac\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3}-\frac{bc\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -a*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)
**(7/2)/7)/d**3 - a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/
2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3 - b*c*(-c**3
*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(
7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)/3 - 4*c*
*3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)*
*(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4
```

Giac [B] time = 7.53456, size = 662, normalized size = 6.37

$$\frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{9}{2}}\operatorname{sgn}(x)+6930\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{11}{2}}\operatorname{sgn}(x)-1155\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}d\operatorname{sgn}(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sqr
t(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c*x
^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*sgn(x
) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 4851*(sq
rt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x - sqrt
(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11/2)*d^4*
sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x) - 330*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c
^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9/2)*d^7*sgn(
x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + d))^2 - d)^11
```

3.952 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=134

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(7/2))/(7*d^5) - (c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5) + ((4*b*c
 - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (b*(c + d/x^2)^(13/2))/(13*d^5)
```

Rubi [A] time = 0.0960258, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(7/2))/(7*d^5) - (c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5) + ((4*b*c
 - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (b*(c + d/x^2)^(13/2))/(13*d^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int x^3(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{c^3(bc-ad)(c+dx)^{3/2}}{d^4} - \frac{c^2(4bc-3ad)(c+dx)^{5/2}}{d^4} + \frac{3c(2bc-ad)(c+dx)^{7/}}{d^4}\right.\right.\right.$$
$$= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} + \frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5} + \frac{(4bc-ad)}{11}$$

Mathematica [A] time = 0.0339789, size = 115, normalized size = 0.86

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(13adx^2\left(-40c^2dx^4+16c^3x^6+70cd^2x^2-105d^3\right)+b\left(-560c^2d^2x^4+320c^3dx^6-128c^4x^8+840cd^3x^2\right.\right.}{15015d^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c^
2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 + 32
0*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)
```

Maple [A] time = 0.009, size = 118, normalized size = 0.9

$$\frac{\left(208\,ac^3dx^8-128\,bc^4x^8-520\,ac^2d^2x^6+320\,bc^3dx^6+910\,acd^3x^4-560\,bc^2d^2x^4-1365\,ad^4x^2+840\,bcd^3x^2-1155\,bd^4\right)}{15015\,d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2*
x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*b*
```

```
c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^10
```

Maxima [A] time = 0.944169, size = 205, normalized size = 1.53

$$-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a-\frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-\frac{5460\left(c+\right.}{d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)^(9/2)*c/d^4 + 495*(c
+ d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^4)*a - 1/15015*(1155*(
c + d/x^2)^(13/2)/d^5 - 5460*(c + d/x^2)^(11/2)*c/d^5 + 10010*(c + d/x^2)^(
9/2)*c^2/d^5 - 8580*(c + d/x^2)^(7/2)*c^3/d^5 + 3003*(c + d/x^2)^(5/2)*c^4/
d^5)*b
```

Fricas [A] time = 2.15573, size = 360, normalized size = 2.69

$$-\frac{\left(16\left(8\,bc^6-13\,ac^5d\right)x^{12}-8\left(8\,bc^5d-13\,ac^4d^2\right)x^{10}+6\left(8\,bc^4d^2-13\,ac^3d^3\right)x^8+1155\,bd^6-5\left(8\,bc^3d^3-13\,ac^2d^4\right)x^6+35\right.}{15015\,d^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^12 - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^1
0 + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13*a
*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a*d^
6)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^12)
```

Sympy [B] time = 15.9664, size = 326, normalized size = 2.43

$$-\frac{ac\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{a\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^4}-\frac{bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -a*c*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c +
 d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - a*(c**4*(c + d/x**2)**(3/
2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c
 + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4 - b*c*(c**4*(c + d/x**2
)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 -
4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5 - b*(-c**5*(c + d
/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(5/2) - 10*c**3*(c + d/x**2)**(7/2)/7
+ 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c*(c + d/x**2)**(11/2)/11 + (c + d/x**2
)**(13/2)/13)/d**5
```

Giac [B] time = 12.433, size = 743, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + d))^18*a*c^(11/2)*sgn(x) + 48048*
(sqrt(c)*x - sqrt(c*x^2 + d))^16*b*c^(13/2)*sgn(x) - 3003*(sqrt(c)*x - sqrt
(c*x^2 + d))^16*a*c^(11/2)*d*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + d))^1
4*b*c^(13/2)*d*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(11/2)*d^
2*sgn(x) + 109824*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(13/2)*d^2*sgn(x) -
28314*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(11/2)*d^3*sgn(x) + 37752*(sqrt(
c)*x - sqrt(c*x^2 + d))^10*b*c^(13/2)*d^3*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + d))^10*a*c^(11/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*b*c^(13/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(11/2)*d^
5*sgn(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(13/2)*d^5*sgn(x) + 371
8*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(11/2)*d^6*sgn(x) + 624*(sqrt(c)*x -
sqrt(c*x^2 + d))^4*b*c^(13/2)*d^6*sgn(x) - 1014*(sqrt(c)*x - sqrt(c*x^2 + d
))^4*a*c^(11/2)*d^7*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(13/2)
*d^7*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(11/2)*d^8*sgn(x) + 8
*b*c^(13/2)*d^8*sgn(x) - 13*a*c^(11/2)*d^9*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + d))^2 - d)^13
```

3.953 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx$

Optimal. Leaf size=150

$$\frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4}-\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5}+\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{143c^2}-\frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-}{429c^3}$$

```
[Out] (-16*d^3*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^5)/(15015*c^5) + (8*d^2*(13*b
*c - 8*a*d)*(c + d/x^2)^(5/2)*x^7)/(3003*c^4) - (2*d*(13*b*c - 8*a*d)*(c +
d/x^2)^(5/2)*x^9)/(429*c^3) + ((13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^11)/(14
3*c^2) + (a*(c + d/x^2)^(5/2)*x^13)/(13*c)
```

Rubi [A] time = 0.0717679, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4}-\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5}+\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{143c^2}-\frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-}{429c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (-16*d^3*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^5)/(15015*c^5) + (8*d^2*(13*b
*c - 8*a*d)*(c + d/x^2)^(5/2)*x^7)/(3003*c^4) - (2*d*(13*b*c - 8*a*d)*(c +
d/x^2)^(5/2)*x^9)/(429*c^3) + ((13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^11)/(14
3*c^2) + (a*(c + d/x^2)^(5/2)*x^13)/(13*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} + \frac{(13bc-8ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx}{13c}$$

$$= \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} - \frac{(6d(13bc-8ad))\int \left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx}{143c^2}$$

$$= -\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} + \frac{(8a}{}$$

$$= \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)}{143c^2}$$

$$= -\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{15015c^5} + \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2d(13bc-8ad)\left(c\right.}{429c^3}$$

Mathematica [A] time = 0.0740671, size = 110, normalized size = 0.73

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(a\left(560c^2d^2x^4-840c^3dx^6+1155c^4x^8-320cd^3x^2+128d^4\right)+13bc\left(-70c^2dx^4+105c^3x^6+40cd^2x^2\right.\right.}{15015c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d
*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*c^
```

```
3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)
```

Maple [A] time = 0.009, size = 115, normalized size = 0.8

$$\frac{x^3\left(1155\,ax^8c^4-840\,ac^3dx^6+1365\,bc^4x^6+560\,ac^2d^2x^4-910\,bc^3dx^4-320\,acd^3x^2+520\,bc^2d^2x^2+128\,ad^4-208\,bcd^3\right)(}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*x^3*(1155*a*c^4*x^8-840*a*c^3*d*x^6+1365*b*c^
4*x^6+560*a*c^2*d^2*x^4-910*b*c^3*d*x^4-320*a*c*d^3*x^2+520*b*c^2*d^2*x^2+1
28*a*d^4-208*b*c*d^3)*(c*x^2+d)/c^5
```

Maxima [A] time = 0.959037, size = 213, normalized size = 1.42

$$\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)b}{1155\,c^4}+\frac{\left(1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}x^{13}-5460\left(c+\frac{d}{x^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")
```

```
[Out] 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c
+ d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(11
55*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c + d/
x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^(5/2
)*d^4*x^5)*a/c^5
```

Fricas [A] time = 1.34779, size = 352, normalized size = 2.35

$$\frac{\left(1155\,ac^6x^{13}+105\left(13\,bc^6+14\,ac^5d\right)x^{11}+35\left(52\,bc^5d+ac^4d^2\right)x^9+5\left(13\,bc^4d^2-8\,ac^3d^3\right)x^7-6\left(13\,bc^3d^3-8\,ac^2d^4\right)x^5+\right.}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*a*c^6*x^13 + 105*(13*b*c^6 + 14*a*c^5*d)*x^11 + 35*(52*b*c^5*
d + a*c^4*d^2)*x^9 + 5*(13*b*c^4*d^2 - 8*a*c^3*d^3)*x^7 - 6*(13*b*c^3*d^3 -
 8*a*c^2*d^4)*x^5 + 8*(13*b*c^2*d^4 - 8*a*c*d^5)*x^3 - 16*(13*b*c*d^5 - 8*a
*d^6)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] time = 19.192, size = 3351, normalized size = 22.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)
```

```
[Out] 693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45
045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 4504
5*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt(c*
x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d
**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30
) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10
 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 +
 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x**16*sqr
t(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c*
*9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d
**30) + 315*a*c**9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d + 1)/(9009*c**
11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**
8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1295*a*c**8*d**(3
7/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 756*a
*c**7*d**(61/2)*x**12*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c*
*10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7
*d**29*x**2 + 9009*c**6*d**30) + 1990*a*c**7*d**(39/2)*x**14*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 63*a*c**6*d**(63/2)*x**10*sqrt
(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**
9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d*
*30) + 1358*a*c**6*d**(41/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) - 630*a*c**5*d**(65/2)*x**8*sqrt(c*x**2/d + 1)/(9009*c**11
```

*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*
d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 343*a*c**5*d**(43/2
)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 1680*a*c
4*d(67/2)*x**6*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10
*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d*
*29*x**2 + 9009*c**6*d**30) + 35*a*c**4*d**(45/2)*x**8*sqrt(c*x**2/d + 1)/(
3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 1386
0*c**6*d**19*x**2 + 3465*c**5*d**20) - 2016*a*c**3*d**(69/2)*x**4*sqrt(c*x*
*2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**
27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30)
+ 280*a*c**3*d**(47/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) - 1152*a*c**2*d**(71/2)*x**2*sqrt(c*x**2/d + 1)/(9009*c**11*d**25
*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*
x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 560*a*c**2*d**(49/2)*x**4
*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c
7*d18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 256*a*c*d**(73/
2)*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90
090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*
c**6*d**30) + 448*a*c*d**(51/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 128*a*d**(53/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 315*b*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c
9*d16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6
*d**19*x**2 + 3465*c**5*d**20) + 1295*b*c**9*d**(35/2)*x**16*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1990*b*c**8*d**(37/2)*x**14*sq
rt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7
*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*b*c**7*d**(39
/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
+ 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*b*c
7*d(21/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10
*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 343*b*c**6*d**(41/2)*x**10*
sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c*
*7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*b*c**6*d**(2
3/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 9
45*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**5*d**(43/2)*x**8*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 114*b*c**5*d**(25/2)*x**10*
sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**
11*x**2 + 315*c**4*d**12) + 280*b*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 40*b*c**4*d**(27/2)*x**8*sqrt(c*x**2/

```
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) + 560*b*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**
16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*
x**2 + 3465*c**5*d**20) - 5*b*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 448*b*c**2*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 138
60*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c
**5*d**20) - 30*b*c**2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**
6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*b*c*d
**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*b*c*
d**(33/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4
 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*b*d**(35/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12)
```

Giac [B] time = 1.10463, size = 440, normalized size = 2.93

$$\frac{\frac{143\left(35\left(cx^2+d\right)^{\frac{9}{2}}-135\left(cx^2+d\right)^{\frac{7}{2}}d+189\left(cx^2+d\right)^{\frac{5}{2}}d^2-105\left(cx^2+d\right)^{\frac{3}{2}}d^3\right)bd\operatorname{sgn}(x)}{c^3}+\frac{13\left(315\left(cx^2+d\right)^{\frac{11}{2}}-1540\left(cx^2+d\right)^{\frac{9}{2}}d+2970\left(cx^2+d\right)^{\frac{7}{2}}d^2-2772\left(cx^2+d\right)^{\frac{5}{2}}d^3\cdots}{c^3}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")
```

```
[Out] 1/45045*(143*(35*(c*x^2 + d)^(9/2) - 135*(c*x^2 + d)^(7/2)*d + 189*(c*x^2 +
 d)^(5/2)*d^2 - 105*(c*x^2 + d)^(3/2)*d^3)*b*d*sgn(x)/c^3 + 13*(315*(c*x^2
+ d)^(11/2) - 1540*(c*x^2 + d)^(9/2)*d + 2970*(c*x^2 + d)^(7/2)*d^2 - 2772*
(c*x^2 + d)^(5/2)*d^3 + 1155*(c*x^2 + d)^(3/2)*d^4)*b*sgn(x)/c^3 + 13*(315*
(c*x^2 + d)^(11/2) - 1540*(c*x^2 + d)^(9/2)*d + 2970*(c*x^2 + d)^(7/2)*d^2
- 2772*(c*x^2 + d)^(5/2)*d^3 + 1155*(c*x^2 + d)^(3/2)*d^4)*a*d*sgn(x)/c^4 +
 5*(693*(c*x^2 + d)^(13/2) - 4095*(c*x^2 + d)^(11/2)*d + 10010*(c*x^2 + d)^
(9/2)*d^2 - 12870*(c*x^2 + d)^(7/2)*d^3 + 9009*(c*x^2 + d)^(5/2)*d^4 - 3003
*(c*x^2 + d)^(3/2)*d^5)*a*sgn(x)/c^4)/c + 16/15015*(13*b*c*d^(11/2) - 8*a*d
^(13/2))*sgn(x)/c^5
```

3.954 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx$

Optimal. Leaf size=117

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

```
[Out] (8*d^2*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^5)/(3465*c^4) - (4*d*(11*b*c -
6*a*d)*(c + d/x^2)^(5/2)*x^7)/(693*c^3) + ((11*b*c - 6*a*d)*(c + d/x^2)^(5/
2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(5/2)*x^11)/(11*c)
```

Rubi [A] time = 0.0569717, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (8*d^2*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^5)/(3465*c^4) - (4*d*(11*b*c -
6*a*d)*(c + d/x^2)^(5/2)*x^7)/(693*c^3) + ((11*b*c - 6*a*d)*(c + d/x^2)^(5/
2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(5/2)*x^11)/(11*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c} + \frac{(11bc-6ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx}{11c}$$

$$= \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c} - \frac{(4d(11bc-6ad))\int \left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx}{99c^2}$$

$$= -\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c} + \frac{(8d^2}{}$$

$$= \frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{3465c^4} - \frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)}{99c^2}$$

Mathematica [A] time = 0.0547788, size = 89, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(3a\left(-70c^2dx^4+105c^3x^6+40cd^2x^2-16d^3\right)+11bc\left(35c^2x^4-20cdx^2+8d^2\right)\right)}{3465c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)
+ 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)
```

Maple [A] time = 0.007, size = 91, normalized size = 0.8

$$\frac{x^3\left(315\,ax^6c^3-210\,ac^2dx^4+385\,bc^3x^4+120\,acd^2x^2-220\,bc^2dx^2-48\,ad^3+88\,bcd^2\right)\left(cx^2+d\right)}{3465\,c^4}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*x
^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4
```

Maxima [A] time = 0.940862, size = 167, normalized size = 1.43

$$\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315\,c^3}+\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-\right.}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^
2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/
x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^
3*x^5)*a/c^4
```

Fricas [A] time = 1.36704, size = 298, normalized size = 2.55

$$\frac{\left(315\,ac^5x^{11}+35\left(11\,bc^5+12\,ac^4d\right)x^9+5\left(110\,bc^4d+3\,ac^3d^2\right)x^7+3\left(11\,bc^3d^2-6\,ac^2d^3\right)x^5-4\left(11\,bc^2d^3-6\,acd^4\right)x^3+8\,(\right.}{3465\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d +
3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 - 6
*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4
```

Sympy [B] time = 14.4344, size = 2304, normalized size = 19.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)
```

```
[Out] 315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16
*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x*
*2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(346
5*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c
**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2)*x**14*s
qrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**1
1*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**12*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d*
*16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19
*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt(c*x**2/d + 1)/(3
15*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d*
*12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 +
 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 34
65*c**5*d**20) + 40*a*c**4*d**(27/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9
*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 560*a
*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*
d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**2
0) - 5*a*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 448*a*c**2*d**(49/
2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 30*a*c**
2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x*
*4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*a*c*d**(51/2)*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*a*c*d**(33/2)*x**2*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 16*a*d**(35/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**8*
d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*b*c**7*d**(21/2)*x**12*sqrt
(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x
**2 + 315*c**4*d**12) + 114*b*c**6*d**(23/2)*x**10*sqrt(c*x**2/d + 1)/(315*
c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12
```

```
) + 40*b*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(11/2
)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) - 5*b*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
+ 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 33*b*c**4*d
**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 +
 105*c**3*d**6) - 30*b*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**
9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 17*b
*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5
*x**2 + 105*c**3*d**6) - 40*b*c**2*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 3*b*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) - 16*b*c*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**
7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) +
 12*b*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d*
*5*x**2 + 105*c**3*d**6) + 8*b*d**(21/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

Giac [B] time = 1.12677, size = 363, normalized size = 3.1

$$\frac{\frac{33\left(15\left(cx^2+d\right)^{\frac{7}{2}}-42\left(cx^2+d\right)^{\frac{5}{2}}d+35\left(cx^2+d\right)^{\frac{3}{2}}d^2\right)bd\,\mathrm{sgn}(x)}{c^2}+\frac{11\left(35\left(cx^2+d\right)^{\frac{9}{2}}-135\left(cx^2+d\right)^{\frac{7}{2}}d+189\left(cx^2+d\right)^{\frac{5}{2}}d^2-105\left(cx^2+d\right)^{\frac{3}{2}}d^3\right)b\,\mathrm{sgn}(x)}{c^2}+\frac{11\left(35\left(cx^2+d\right)^{\frac{9}{2}}\right.}{}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")
```

```
[Out] 1/3465*(33*(15*(c*x^2 + d)^(7/2) - 42*(c*x^2 + d)^(5/2)*d + 35*(c*x^2 + d)^
(3/2)*d^2)*b*d*sgn(x)/c^2 + 11*(35*(c*x^2 + d)^(9/2) - 135*(c*x^2 + d)^(7/2
)*d + 189*(c*x^2 + d)^(5/2)*d^2 - 105*(c*x^2 + d)^(3/2)*d^3)*b*sgn(x)/c^2 +
 11*(35*(c*x^2 + d)^(9/2) - 135*(c*x^2 + d)^(7/2)*d + 189*(c*x^2 + d)^(5/2)
*d^2 - 105*(c*x^2 + d)^(3/2)*d^3)*a*d*sgn(x)/c^3 + (315*(c*x^2 + d)^(11/2)
- 1540*(c*x^2 + d)^(9/2)*d + 2970*(c*x^2 + d)^(7/2)*d^2 - 2772*(c*x^2 + d)^
(5/2)*d^3 + 1155*(c*x^2 + d)^(3/2)*d^4)*a*sgn(x)/c^3)/c - 8/3465*(11*b*c*d^
(9/2) - 6*a*d^(11/2))*sgn(x)/c^4
```

3.955 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx$

Optimal. Leaf size=84

$$\frac{x^7 \left(c+\frac{d}{x^2}\right)^{5/2} (9bc-4ad)}{63c^2}-\frac{2dx^5 \left(c+\frac{d}{x^2}\right)^{5/2} (9bc-4ad)}{315c^3}+\frac{ax^9 \left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

```
[Out] (-2*d*(9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^5)/(315*c^3) + ((9*b*c - 4*a*d)*(
c + d/x^2)^(5/2)*x^7)/(63*c^2) + (a*(c + d/x^2)^(5/2)*x^9)/(9*c)
```

Rubi [A] time = 0.0415588, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{x^7 \left(c+\frac{d}{x^2}\right)^{5/2} (9bc-4ad)}{63c^2}-\frac{2dx^5 \left(c+\frac{d}{x^2}\right)^{5/2} (9bc-4ad)}{315c^3}+\frac{ax^9 \left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (-2*d*(9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^5)/(315*c^3) + ((9*b*c - 4*a*d)*(
c + d/x^2)^(5/2)*x^7)/(63*c^2) + (a*(c + d/x^2)^(5/2)*x^9)/(9*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}+\frac{(9bc-4ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx}{9c}\\ &= \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{63c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}-\frac{(2d(9bc-4ad))\int\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx}{63c^2}\\ &= -\frac{2d(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{315c^3}+\frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{63c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}\end{aligned}$$

Mathematica [A] time = 0.0413909, size = 66, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(a\left(35c^2x^4-20cdx^2+8d^2\right)+9bc\left(5cx^2-2d\right)\right)}{315c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c*
d*x^2 + 35*c^2*x^4)))/(315*c^3)
```

Maple [A] time = 0.006, size = 67, normalized size = 0.8

$$\frac{x^3\left(35\,ax^4c^2-20\,acdx^2+45\,bc^2x^2+8\,ad^2-18\,bcd\right)\left(cx^2+d\right)}{315\,c^3}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*a
*d^2-18*b*c*d)*(c*x^2+d)/c^3
```

Maxima [A] time = 0.960015, size = 122, normalized size = 1.45

$$\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35\,c^2}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(3
5*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/2)
*d^2*x^5)*a/c^3
```

Fricas [A] time = 1.33022, size = 232, normalized size = 2.76

$$\frac{\left(35\,ac^4x^9+5\left(9\,bc^4+10\,ac^3d\right)x^7+3\left(24\,bc^3d+ac^2d^2\right)x^5+\left(9\,bc^2d^2-4\,acd^3\right)x^3-2\left(9\,bcd^3-4\,ad^4\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2*
d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^3
```

Sympy [B] time = 10.1723, size = 1340, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**8,x)
```

```
[Out] 35*a*c**8*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*a*c**
5*d**(11/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x*
*2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*
d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 3
3*a*c**4*d**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d
**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(31
5*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**
12) + 17*a*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210
*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*sqrt(c*x**2/d +
 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c
**4*d**12) + 3*a*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) + 12*a*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 2
10*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(21/2)*sqrt(c*x**2/d + 1)/(105*
c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 15*b*c**6*d**(9/2)*x
**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + 33*b*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**4*d**(13/2)*x**6*sqrt(c*x**2/
d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**3
*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2
 + 105*c**3*d**6) + 12*b*c**2*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*b*c*d**(19/2)*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + b*d**(
3/2)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(5/2)*x**2*sqrt(c*x**2/d + 1)/(15*c)
- 2*b*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

Giac [B] time = 1.10413, size = 288, normalized size = 3.43

$$\frac{\frac{21\left(3\left(cx^2+d\right)^{\frac{5}{2}}-5\left(cx^2+d\right)^{\frac{3}{2}}d\right)bd\operatorname{sgn}(x)}{c}+\frac{3\left(15\left(cx^2+d\right)^{\frac{7}{2}}-42\left(cx^2+d\right)^{\frac{5}{2}}d+35\left(cx^2+d\right)^{\frac{3}{2}}d^2\right)b\operatorname{sgn}(x)}{c}+\frac{3\left(15\left(cx^2+d\right)^{\frac{7}{2}}-42\left(cx^2+d\right)^{\frac{5}{2}}d+35\left(cx^2+d\right)^{\frac{3}{2}}d^2\right)ad\operatorname{sgn}(}{c^2}}{315\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] 1/315*(21*(3*(c*x^2 + d)^(5/2) - 5*(c*x^2 + d)^(3/2)*d)*b*d*sgn(x)/c + 3*(1
5*(c*x^2 + d)^(7/2) - 42*(c*x^2 + d)^(5/2)*d + 35*(c*x^2 + d)^(3/2)*d^2)*b*
sgn(x)/c + 3*(15*(c*x^2 + d)^(7/2) - 42*(c*x^2 + d)^(5/2)*d + 35*(c*x^2 + d
)^(3/2)*d^2)*a*d*sgn(x)/c^2 + (35*(c*x^2 + d)^(9/2) - 135*(c*x^2 + d)^(7/2)
*d + 189*(c*x^2 + d)^(5/2)*d^2 - 105*(c*x^2 + d)^(3/2)*d^3)*a*sgn(x)/c^2)/c
 + 2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3
```

3.956 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx$

Optimal. Leaf size=53

$$\frac{x^5 \left(c+\frac{d}{x^2}\right)^{5/2} (7bc-2ad)}{35c^2}+\frac{ax^7 \left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

```
[Out] ((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^7
)/(7*c)
```

Rubi [A] time = 0.0253675, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{x^5 \left(c+\frac{d}{x^2}\right)^{5/2} (7bc-2ad)}{35c^2}+\frac{ax^7 \left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] ((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^7
)/(7*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{7c}+\frac{(7bc-2ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx}{7c}$$
$$=\frac{(7bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{7c}$$

Mathematica [A] time = 0.0266944, size = 44, normalized size = 0.83

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(5acx^2-2ad+7bc\right)}{35c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

Maple [A] time = 0.004, size = 45, normalized size = 0.9

$$\frac{x^3\left(5\,ax^2c-2\,ad+7\,bc\right)\left(cx^2+d\right)}{35\,c^2}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x)
```

```
[Out] 1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2
```

Maxima [A] time = 0.936792, size = 74, normalized size = 1.4

$$\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)a}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^
2)^(5/2)*d*x^5)*a/c^2
```

Fricas [A] time = 1.31448, size = 174, normalized size = 3.28

$$\frac{\left(5\,ac^3x^7 + \left(7\,bc^3 + 8\,ac^2d\right)x^5 + \left(14\,bc^2d + acd^2\right)x^3 + \left(7\,bcd^2 - 2\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3
+ (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [B] time = 7.04296, size = 498, normalized size = 9.4

$$\frac{15ac^6d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{33ac^5d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{17ac^4d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{3a}{105c^5d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)
```

```
[Out] 15*a*c**6*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**3*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*c*
d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105
*c**3*d**6) + a*d**(3/2)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*x**2*sqrt(c
*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*c*sqrt(
```

```
d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/5 + b*d
**(5/2)*sqrt(c*x**2/d + 1)/(5*c)
```

Giac [B] time = 1.0995, size = 203, normalized size = 3.83

$$\frac{35\left(cx^2+d\right)^{\frac{3}{2}}bd\mathrm{sgn}\left(x\right)+7\left(3\left(cx^2+d\right)^{\frac{5}{2}}-5\left(cx^2+d\right)^{\frac{3}{2}}d\right)b\mathrm{sgn}\left(x\right)+\frac{7\left(3\left(cx^2+d\right)^{\frac{5}{2}}-5\left(cx^2+d\right)^{\frac{3}{2}}d\right)ad\mathrm{sgn}(x)}{c}+\frac{\left(15\left(cx^2+d\right)^{\frac{7}{2}}-42\left(cx^2+\right.\right.}{}}{105\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] 1/105*(35*(c*x^2 + d)^(3/2)*b*d*sgn(x) + 7*(3*(c*x^2 + d)^(5/2) - 5*(c*x^2
+ d)^(3/2)*d)*b*sgn(x) + 7*(3*(c*x^2 + d)^(5/2) - 5*(c*x^2 + d)^(3/2)*d)*a*
d*sgn(x)/c + (15*(c*x^2 + d)^(7/2) - 42*(c*x^2 + d)^(5/2)*d + 35*(c*x^2 + d
)^(3/2)*d^2)*a*sgn(x)/c)/c - 1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2
```

3.957 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx$

Optimal. Leaf size=86

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}+bdx\sqrt{c+\frac{d}{x^2}}$$

```
[Out] b*d*Sqrt[c + d/x^2]*x + (b*(c + d/x^2)^(3/2)*x^3)/3 + (a*(c + d/x^2)^(5/2)*
x^5)/(5*c) - b*d^(3/2)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rubi [A] time = 0.05509, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {451, 335, 277, 217, 206}

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}+bdx\sqrt{c+\frac{d}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] b*d*Sqrt[c + d/x^2]*x + (b*(c + d/x^2)^(3/2)*x^3)/3 + (a*(c + d/x^2)^(5/2)*
x^5)/(5*c) - b*d^(3/2)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} + b\int \left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx \\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - b\,\text{Subst}\left(\int \frac{\left(c+dx^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - (bd)\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= bd\sqrt{c+\frac{d}{x^2}}x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right) \\
&= bd\sqrt{c+\frac{d}{x^2}}x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\ldots}}\right) \\
&= bd\sqrt{c+\frac{d}{x^2}}x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0997874, size = 81, normalized size = 0.94

$$
\frac{1}{15}x\sqrt{c+\frac{d}{x^2}}\left(\frac{3a\left(cx^2+d\right)^2}{c} - \frac{15bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{cx^2+d}} + 5b\left(cx^2+4d\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3/
2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/15
```

Maple [A] time = 0.01, size = 99, normalized size = 1.2

$$
-\frac{x^3}{15\,c}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(-3\,a\left(cx^2+d\right)^{5/2}+15\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)bc-5\,\left(cx^2+d\right)^{3/2}bc-15\,\sqrt{cx^2+d}bcd\right)\left(cx^2+d\right)^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x)
```

```
[Out] -1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(-3*a*(c*x^2+d)^(5/2)+15*d^(3/2)*ln(2*(d^(1
/2)*(c*x^2+d)^(1/2)+d)/x)*b*c-5*(c*x^2+d)^(3/2)*b*c-15*(c*x^2+d)^(1/2)*b*c*
d)/(c*x^2+d)^(3/2)/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37161, size = 473, normalized size = 5.5

$$\left[\frac{15\,bcd^{\frac{3}{2}}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(3\,ac^2x^5+\left(5\,bc^2+6\,acd\right)x^3+\left(20\,bcd+3\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,c},\ \frac{15\,bc\sqrt{-d}d\arctan\left(\frac{\sqrt{-}\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d
)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*x)
*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqrt((
c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20
*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] time = 5.72276, size = 184, normalized size = 2.14

$$\frac{ac\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}+\frac{b\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{bc\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-bd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)
```

```
[Out] a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d + 1
)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x**
2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)
/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 + d/(
c*x**2)))
```

Giac [A] time = 1.09143, size = 189, normalized size = 2.2

$$\frac{bd^2\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}}-\frac{\left(15\,bcd^2\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+20\,bc\sqrt{-d}d^{\frac{3}{2}}+3\,a\sqrt{-d}d^{\frac{5}{2}}\right)\operatorname{sgn}(x)}{15\,c\sqrt{-d}}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}ac^4\operatorname{sgn}(x)+5\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*a
rctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))*s
gn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 + d)
^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5
```

3.958 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx$

Optimal. Leaf size=121

$$\frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}(2ad+3bc)}{2cx}-\frac{1}{2}\sqrt{d}(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

```
[Out] -(d*(3*b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + ((3*b*c + 2*a*d)*(c + d/x^2)
^(3/2)*x)/(3*c) + (a*(c + d/x^2)^(5/2)*x^3)/(3*c) - (Sqrt[d]*(3*b*c + 2*a*d
)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/2
```

Rubi [A] time = 0.0575027, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 242, 277, 195, 217, 206}

$$\frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}(2ad+3bc)}{2cx}-\frac{1}{2}\sqrt{d}(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```


```
[Out] -(d*(3*b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + ((3*b*c + 2*a*d)*(c + d/x^2)
^(3/2)*x)/(3*c) + (a*(c + d/x^2)^(5/2)*x^3)/(3*c) - (Sqrt[d]*(3*b*c + 2*a*d
)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/2
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c} + \frac{(3bc+2ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} dx}{3c}$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{(3bc+2ad)\,\text{Subst}\left(\int \frac{(c+dx^2)^{3/2}}{x^2} dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{(d(3bc+2ad))\,\text{Subst}\left(\int \sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{1}{2}(d(3bc+2ad))$$

$$= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{1}{2}(d(3bc+2ad))$$

$$= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{1}{2}\sqrt{d}(3bc+2ad$$

Mathematica [A] time = 0.0634105, size = 105, normalized size = 0.87

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{cx^2+d}\left(2acx^4+8adx^2+6bcx^2-3bd\right)-3\sqrt{d}x^2(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{6x\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*x
^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6*x
*Sqrt[d + c*x^2])
```

Maple [A] time = 0.01, size = 170, normalized size = 1.4

$$-\frac{x}{6\,d}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(6\,d^{5/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^2a+9\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^2bc-2\,\left(cx^2+d\right)^{3/2}x^2ad-3\,\left(cx^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x)
```

```
[Out] -1/6*((c*x^2+d)/x^2)^(3/2)*x*(6*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*x^2*a+9*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^2*b*c-2*(c*x^2+d)^(3
/2)*x^2*a*d-3*(c*x^2+d)^(3/2)*x^2*b*c+3*(c*x^2+d)^(5/2)*b-6*(c*x^2+d)^(1/2)
*x^2*a*d^2-9*(c*x^2+d)^(1/2)*x^2*b*c*d)/(c*x^2+d)^(3/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34881, size = 447, normalized size = 3.69

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x},\ \frac{3\,(3\,bc+2\,ad)\sqrt{-d}x\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 +
d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt((c
*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt
((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d
)*sqrt((c*x^2 + d)/x^2))/x]
```

Sympy [A] time = 7.11669, size = 202, normalized size = 1.67

$$\frac{a\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{ac\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{ad^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{b\cdots}{x\sqrt{1\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)
```

```
[Out] a*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + a*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3
+ a*d**(3/2)*sqrt(c*x**2/d + 1)/3 - a*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) +
 a*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) + b*c**(3/2)*x/sqrt(1 + d/(c*x**2)
) - b*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x) + b*sqrt(c)*d/(x*sqrt(1 + d/(c*x
**2))) - 3*b*c*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))/2
```

Giac [A] time = 1.17717, size = 155, normalized size = 1.28

$$\frac{2\left(cx^2+d\right)^{\frac{3}{2}}ac\operatorname{sgn}(x)+6\sqrt{cx^2+d}bc^2\operatorname{sgn}(x)+6\sqrt{cx^2+d}acd\operatorname{sgn}(x)-\frac{3\sqrt{cx^2+d}bcd\operatorname{sgn}(x)}{x^2}+\frac{3\left(3bc^2d\operatorname{sgn}(x)+2acd^2\operatorname{sgn}(x)\right)\arctan\cdots}{\sqrt{-d}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + d)^(3/2)*a*c*sgn(x) + 6*sqrt(c*x^2 + d)*b*c^2*sgn(x) + 6*sq
rt(c*x^2 + d)*a*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d*sgn(x)/x^2 + 3*(3*b*c^
2*d*sgn(x) + 2*a*c*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d))/c
```

3.959 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}(4ad+bc)}{8x}-\frac{3c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

```
[Out] (-3*(b*c + 4*a*d)*Sqrt[c + d/x^2])/(8*x) - ((b*c + 4*a*d)*(c + d/x^2)^(3/2)
)/(4*c*x) + (a*(c + d/x^2)^(5/2)*x)/c - (3*c*(b*c + 4*a*d)*ArcTanh[Sqrt[d]/
(Sqrt[c + d/x^2]*x)])/(8*Sqrt[d])
```

Rubi [A] time = 0.0623911, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 453, 195, 217, 206}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}(4ad+bc)}{8x}-\frac{3c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```


```
[Out] (-3*(b*c + 4*a*d)*Sqrt[c + d/x^2])/(8*x) - ((b*c + 4*a*d)*(c + d/x^2)^(3/2)
)/(4*c*x) + (a*(c + d/x^2)^(5/2)*x)/c - (3*c*(b*c + 4*a*d)*ArcTanh[Sqrt[d]/
(Sqrt[c + d/x^2]*x)])/(8*Sqrt[d])
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x}{c} + \frac{(-bc-4ad)\,\text{Subst}\left(\int \left(c+dx^2\right)^{3/2} dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x}{c} - \frac{1}{4}(3(bc+4ad))\,\text{Subst}\left(\int \sqrt{c+dx^2}\, dx, x, \frac{1}{x}\right)$$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x}{c} - \frac{1}{8}(3c(bc+4ad))\,\text{Subst}\Bigg($$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x}{c} - \frac{1}{8}(3c(bc+4ad))\,\text{Subst}\Bigg($$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x}{c} - \frac{3c(bc+4ad)\tanh^{-1}\Bigg(}{8\sqrt{d}}$$

Mathematica [C] time = 0.0321634, size = 68, normalized size = 0.61

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(cx^4(4ad+bc)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{d}+1\right)-5bd^2\right)}{20d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^2 + c*(b*c + 4*a*d)*x^4*Hypergeometr
ic2F1[2, 5/2, 7/2, 1 + (c*x^2)/d]))/(20*d^3*x^3)
```

Maple [B] time = 0.012, size = 213, normalized size = 1.9

$$-\frac{1}{8\,xd^2}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(12\,d^{5/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^4ac+3\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^4bc^2-4\,\left(cx^2+d\right)^{3/2}x^4acd-(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2),x)
```

```
[Out] -1/8*((c*x^2+d)/x^2)^(3/2)/x*(12*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x
)*x^4*a*c+3*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^4*b*c^2-4*(c*x^2+
d)^(3/2)*x^4*a*c*d-(c*x^2+d)^(3/2)*x^4*b*c^2+4*(c*x^2+d)^(5/2)*x^2*a*d+(c*x
^2+d)^(5/2)*x^2*b*c-12*(c*x^2+d)^(1/2)*x^4*a*c*d^2-3*(c*x^2+d)^(1/2)*x^4*b*
c^2*d+2*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.40256, size = 490, normalized size = 4.38

$$\left[\frac{3\left(bc^2+4\,acd\right)\sqrt{d}x^3\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(8\,acdx^4-2\,bd^2-\left(5\,bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,dx^3},\ \frac{3\left(bc^2+4\,acd\right)\sqrt{-d}x^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^
2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^
2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3*ar
ctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c*d*x^4 - 2*b*d^2
 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]
```

Sympy [B] time = 11.1277, size = 216, normalized size = 1.93

$$\frac{ac^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{a\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{a\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2}-\frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asin}}{8\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)
```

```
[Out] a*c**(3/2)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x)
+ a*sqrt(c)*d/(x*sqrt(1 + d/(c*x**2))) - 3*a*c*sqrt(d)*asinh(sqrt(d)/(sqrt(
c)*x))/2 - b*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - b*c**(3/2)/(8*x*sqrt(1 +
 d/(c*x**2))) - 3*b*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asin
h(sqrt(d)/(sqrt(c)*x))/(8*sqrt(d)) - b*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x
**2)))
```

Giac [A] time = 1.16603, size = 196, normalized size = 1.75

$$\frac{8\sqrt{cx^2+d}ac^2\operatorname{sgn}(x)+\frac{3\left(bc^3\operatorname{sgn}(x)+4ac^2d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{5\left(cx^2+d\right)^{\frac{3}{2}}bc^3\operatorname{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\operatorname{sgn}(x)-3\sqrt{cx^2+d}bc^3d\operatorname{sgn}(x)-4\sqrt{cx^2+d}}{c^2x^4}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + d)*a*c^2*sgn(x) + 3*(b*c^3*sgn(x) + 4*a*c^2*d*sgn(x))*a
rctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - (5*(c*x^2 + d)^(3/2)*b*c^3*sgn(x
) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^3*d*sgn(x) -
 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x))/(c^2*x^4))/c
```

3.960 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx}+\frac{c\sqrt{c+\frac{d}{x^2}}(bc-6ad)}{16dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

```
[Out] (c*(b*c - 6*a*d)*Sqrt[c + d/x^2])/(16*d*x) + ((b*c - 6*a*d)*(c + d/x^2)^(3/
2))/(24*d*x) - (b*(c + d/x^2)^(5/2))/(6*d*x) + (c^2*(b*c - 6*a*d)*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(3/2))
```

Rubi [A] time = 0.0638926, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 195, 217, 206}

$$\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx}+\frac{c\sqrt{c+\frac{d}{x^2}}(bc-6ad)}{16dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] (c*(b*c - 6*a*d)*Sqrt[c + d/x^2])/(16*d*x) + ((b*c - 6*a*d)*(c + d/x^2)^(3/
2))/(24*d*x) - (b*(c + d/x^2)^(5/2))/(6*d*x) + (c^2*(b*c - 6*a*d)*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(3/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(-bc+6ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx}{6d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} - \frac{(-bc+6ad)\,\mathrm{Subst}\left(\int \left(c+dx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right)}{6d}$$

$$= \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(c(bc-6ad))\,\mathrm{Subst}\left(\int \sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{8d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{\left(c^2(bc-6ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+\ldots}}\right.}{16d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{\left(c^2(bc-6ad)\right)\mathrm{Subst}\left(\int \frac{1}{1-d\ldots}\right.}{16d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\ldots}\right.}{16d^{3/2}}$$

Mathematica [A] time = 0.099036, size = 126, normalized size = 1.02

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(\left(cx^2+d\right)\left(6adx^2\left(5cx^2+2d\right)+b\left(3c^2x^4+14cdx^2+8d^2\right)\right)+3c^2x^6\sqrt{\frac{cx^2}{d}+1}(6ad-bc)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{48dx^5\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] -(Sqrt[c + d/x^2]*((d + c*x^2)*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 + 14*c
*d*x^2 + 3*c^2*x^4)) + 3*c^2*(-(b*c) + 6*a*d)*x^6*Sqrt[1 + (c*x^2)/d]*ArcTa
nh[Sqrt[1 + (c*x^2)/d]]))/(48*d*x^5*(d + c*x^2))
```

Maple [B] time = 0.013, size = 259, normalized size = 2.1

$$-\frac{1}{48\,x^3d^3}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(18\,d^{5/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^6ac^2-3\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^6bc^3-6\left(cx^2+d\right)^{3/2}x^6ac^2d\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x)
```

```
[Out] -1/48*((c*x^2+d)/x^2)^(3/2)/x^3*(18*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d
)/x)*x^6*a*c^2-3*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^6*b*c^3-6*(c
*x^2+d)^(3/2)*x^6*a*c^2*d+(c*x^2+d)^(3/2)*x^6*b*c^3+6*(c*x^2+d)^(5/2)*x^4*a
*c*d-(c*x^2+d)^(5/2)*x^4*b*c^2-18*(c*x^2+d)^(1/2)*x^6*a*c^2*d^2+3*(c*x^2+d)
^(1/2)*x^6*b*c^3*d+12*(c*x^2+d)^(5/2)*x^2*a*d^2-2*(c*x^2+d)^(5/2)*x^2*b*c*d
+8*(c*x^2+d)^(5/2)*b*d^2)/(c*x^2+d)^(3/2)/d^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.45102, size = 562, normalized size = 4.57

$$\left[-\frac{3\left(bc^3-6\,ac^2d\right)\sqrt{d}x^5\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(3\left(bc^2d+10\,acd^2\right)x^4+8\,bd^3+2\left(7\,bcd^2+6\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96\,d^2x^5},-\frac{3\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*
(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b*c^
3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2
+ d)) + (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x
```

```
^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5)]
```

Sympy [B] time = 17.6721, size = 253, normalized size = 2.06

$$-\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}}-\frac{ad^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}}-\frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - a*c**(3/2)/(8*x*sqrt(1 + d/(c*x**2
))) - 3*a*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*a*c**2*asinh(sqrt(d)/
(sqrt(c)*x))/(8*sqrt(d)) - a*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2))) - b
*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*b*c**(3/2)/(48*x**3*sqrt(1 + d
/(c*x**2))) - 11*b*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + b*c**3*asinh(
sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - b*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c*x
**2)))
```

Giac [A] time = 1.19783, size = 234, normalized size = 1.9

$$-\frac{\frac{3\left(bc^4\operatorname{sgn}(x)-6\,ac^3d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\operatorname{sgn}(x)+30\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\operatorname{sgn}(x)+8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\operatorname{sgn}(x)-48\left(cx^2+d\right)^{\frac{3}{2}}ac^3d^2\operatorname{sgn}(x)-3\sqrt{cx^2+}}{c^3dx^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/48*(3*(b*c^4*sgn(x) - 6*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))
/(sqrt(-d)*d) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) + 30*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) + 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 48*(c*x^2 + d)^(3/2)*a*
c^3*d^2*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4*d^2*sgn(x) + 18*sqrt(c*x^2 + d)*a*
c^3*d^3*sgn(x))/(c^3*d*x^6))/c
```

3.961 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=159

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}(3bc-8ad)}{128d^2x}-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

```
[Out] (c*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(64*d*x^3) + ((3*b*c - 8*a*d)*(c + d/x^
2)^(3/2))/(48*d*x^3) - (b*(c + d/x^2)^(5/2))/(8*d*x^3) + (c^2*(3*b*c - 8*a*
d)*Sqrt[c + d/x^2])/(128*d^2*x) - (c^3*(3*b*c - 8*a*d)*ArcTanh[Sqrt[d]/(Sqr
t[c + d/x^2]*x)])/(128*d^(5/2))
```

Rubi [A] time = 0.0887293, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 279, 321, 217, 206}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}(3bc-8ad)}{128d^2x}-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (c*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(64*d*x^3) + ((3*b*c - 8*a*d)*(c + d/x^
2)^(3/2))/(48*d*x^3) - (b*(c + d/x^2)^(5/2))/(8*d*x^3) + (c^2*(3*b*c - 8*a*
d)*Sqrt[c + d/x^2])/(128*d^2*x) - (c^3*(3*b*c - 8*a*d)*ArcTanh[Sqrt[d]/(Sqr
t[c + d/x^2]*x)])/(128*d^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(-3bc+8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx}{8d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} - \frac{(-3bc+8ad)\,\mathrm{Subst}\left(\int x^2\left(c+dx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right)}{8d}$$

$$= \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{(c(3bc-8ad))\,\mathrm{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{16d}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{\left(c^2(3bc-8ad)\right)\mathrm{Subst}\left(\int \right.}{64d}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x} - \frac{(c}{}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x} - \frac{(c}{}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3} + \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3} + \frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x} - \frac{c^3}{}$$

Mathematica [C] time = 0.0294272, size = 71, normalized size = 0.45

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(c^3x^8(8ad-3bc)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{d}+1\right)-5bd^4\right)}{40d^5x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^4 + c^3*(-3*b*c + 8*a*d)*x^8*Hyperge
ometric2F1[5/2, 4, 7/2, 1 + (c*x^2)/d]))/(40*d^5*x^7)
```

Maple [B] time = 0.02, size = 302, normalized size = 1.9

$$\frac{1}{384\,x^5d^4}\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(24\,d^{5/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^8ac^3-9\,d^{3/2}\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^8bc^4-8\,\left(cx^2+d\right)^{3/2}x^8ac^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x)
```

```
[Out] 1/384*((c*x^2+d)/x^2)^(3/2)/x^5*(24*d^(5/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d
)/x)*x^8*a*c^3-9*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^8*b*c^4-8*(c
*x^2+d)^(3/2)*x^8*a*c^3*d+3*(c*x^2+d)^(3/2)*x^8*b*c^4+8*(c*x^2+d)^(5/2)*x^6
*a*c^2*d-3*(c*x^2+d)^(5/2)*x^6*b*c^3-24*(c*x^2+d)^(1/2)*x^8*a*c^3*d^2+9*(c*
x^2+d)^(1/2)*x^8*b*c^4*d+16*(c*x^2+d)^(5/2)*x^4*a*c*d^2-6*(c*x^2+d)^(5/2)*x
^4*b*c^2*d-64*(c*x^2+d)^(5/2)*x^2*a*d^3+24*(c*x^2+d)^(5/2)*x^2*b*c*d^2-48*(
c*x^2+d)^(5/2)*b*d^3)/(c*x^2+d)^(3/2)/d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52357, size = 674, normalized size = 4.24

$$\left[-\frac{3\left(3\,bc^4-8\,ac^3d\right)\sqrt{d}x^7\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(3\,bc^3d-8\,ac^2d^2\right)x^6-48\,bd^4-2\left(3\,bc^2d^2+56\,acd^3\right)x^4-8\,(9\right.}{768\,d^3x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqrt
((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d
^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7*arc
tan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(3*b*c^3*d - 8*a*c^2
*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*
a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]
```

Sympy [B] time = 27.7369, size = 287, normalized size = 1.81

$$-\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11a\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{3}{2}}} - \frac{ad^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}} + \frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{5}{2}}}{128dx^3\sqrt{1\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] -a*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*a*c**(3/2)/(48*x**3*sqrt(1 +
 d/(c*x**2))) - 11*a*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + a*c**3*asin
h(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - a*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c
*x**2))) + 3*b*c**(7/2)/(128*d**2*x*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(128
*d*x**3*sqrt(1 + d/(c*x**2))) - 13*b*c**(3/2)/(64*x**5*sqrt(1 + d/(c*x**2))
) - 5*b*sqrt(c)*d/(16*x**7*sqrt(1 + d/(c*x**2))) - 3*b*c**4*asinh(sqrt(d)/(
sqrt(c)*x))/(128*d**(5/2)) - b*d**2/(8*sqrt(c)*x**9*sqrt(1 + d/(c*x**2)))
```

Giac [A] time = 1.2201, size = 289, normalized size = 1.82

$$\frac{\frac{3\left(3\,bc^5\operatorname{sgn}(x)-8\,ac^4d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2} + \frac{9\left(cx^2+d\right)^{\frac{7}{2}}bc^5\operatorname{sgn}(x)-24\left(cx^2+d\right)^{\frac{7}{2}}ac^4d\operatorname{sgn}(x)-33\left(cx^2+d\right)^{\frac{5}{2}}bc^5d\operatorname{sgn}(x)-40\left(cx^2+d\right)^{\frac{5}{2}}ac^4d^2\operatorname{sgn}(x)-33\left(cx^2+\cdots\right.}{c^4d^2x^8}}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d
))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7/2
)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^(5/
```

```
2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2 + d
)^(3/2)*a*c^4*d^3*sgn(x) + 9*sqrt(c*x^2 + d)*b*c^5*d^3*sgn(x) - 24*sqrt(c*x
^2 + d)*a*c^4*d^4*sgn(x))/(c^4*d^2*x^8))/c
```

3.962 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=90

$$\frac{x^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{8c^2}-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

```
[Out] ((4*b*c - 3*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^2) + (a*Sqrt[c + d/x^2]*x^4)/(4*
c) - (d*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rubi [A] time = 0.0673245, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{x^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{8c^2}-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] ((4*b*c - 3*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^2) + (a*Sqrt[c + d/x^2]*x^4)/(4*
c) - (d*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x^3\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} - \frac{\left(2bc-\frac{3ad}{2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} + \frac{(d(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c^2}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} + \frac{(4bc-3ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{8c^2}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} - \frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Mathematica [A] time = 0.0791951, size = 95, normalized size = 1.06

$$\frac{\sqrt{c}x\left(cx^2+d\right)\left(2acx^2-3ad+4bc\right)+d\sqrt{cx^2+d}(3ad-4bc)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d}}\right)}{8c^{5/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c]*x*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + d*(-4*b*c + 3*a*d)*Sqr
t[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[d + c*x^2]])/(8*c^(5/2)*Sqrt[c + d/x^
2]*x)
```

Maple [A] time = 0.01, size = 129, normalized size = 1.4

$$\frac{1}{8x}\sqrt{cx^2+d}\left(2c^{5/2}\sqrt{cx^2+d}x^3a-3c^{3/2}\sqrt{cx^2+d}xad+4c^{5/2}\sqrt{cx^2+d}xb+3\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)acd^2-4\ln\left(\sqrt{c}x+\sqrt{c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/8*(c*x^2+d)^(1/2)*(2*c^(5/2)*(c*x^2+d)^(1/2)*x^3*a-3*c^(3/2)*(c*x^2+d)^(1
/2)*x*a*d+4*c^(5/2)*(c*x^2+d)^(1/2)*x*b+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c
*d^2-4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c^2*d)/((c*x^2+d)/x^2)^(1/2)/x/c^(7/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.3239, size = 440, normalized size = 4.89

$$\left[-\frac{\left(4\,bcd-3\,ad^{2}\right)\sqrt{c}\log\left(-2\,cx^{2}-2\,\sqrt{c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\left(2\,ac^{2}x^{4}+\left(4\,bc^{2}-3\,acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{16\,c^{3}},\frac{\left(4\,bcd-3\,ad^{2}\right)\sqrt{-c}\,\mathrm{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^
2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 +
d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c
*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c^3]
```

Sympy [A] time = 32.2973, size = 150, normalized size = 1.67

$$\frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{a\sqrt{d}x^3}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}}+\frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) - a*sqrt(d)*x**3/(8*c*sqrt(c*x**2/d +
 1)) - 3*a*d**(3/2)*x/(8*c**2*sqrt(c*x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*
x/sqrt(d))/(8*c**(5/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - b*d*asinh(
sqrt(c)*x/sqrt(d))/(2*c**(3/2))
```

Giac [B] time = 1.17982, size = 220, normalized size = 2.44

$$\frac{1}{8}\,d^2\left(\frac{(4\,bc-3\,ad)\arctan\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2d}-\frac{4\,bc^2\sqrt{\frac{cx^2+d}{x^2}}-5\,acd\sqrt{\frac{cx^2+d}{x^2}}-\frac{4\left(cx^2+d\right)bc\sqrt{\frac{cx^2+d}{x^2}}}{x^2}+\frac{3\left(cx^2+d\right)ad\sqrt{\frac{cx^2+d}{x^2}}}{x^2}}{\left(c-\frac{cx^2+d}{x^2}\right)^2c^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^2*((4*b*c - 3*a*d)*arctan(sqrt((c*x^2 + d)/x^2)/sqrt(-c))/(sqrt(-c)*c
^2*d) - (4*b*c^2*sqrt((c*x^2 + d)/x^2) - 5*a*c*d*sqrt((c*x^2 + d)/x^2) - 4*
(c*x^2 + d)*b*c*sqrt((c*x^2 + d)/x^2)/x^2 + 3*(c*x^2 + d)*a*d*sqrt((c*x^2 +
 d)/x^2)/x^2)/((c - (c*x^2 + d)/x^2)^2*c^2*d))
```

3.963 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=59

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

```
[Out] (a*Sqrt[c + d/x^2]*x^2)/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/(2*c^(3/2))
```

Rubi [A] time = 0.0402656, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 78, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x^2)/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/(2*c^(3/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0367489, size = 79, normalized size = 1.34

$$
\frac{\sqrt{cx^2+d}(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d}}\right)+a\sqrt{c}x\left(cx^2+d\right)}{2c^{3/2}x\sqrt{c+\frac{d}{x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c]*x*(d + c*x^2) + (2*b*c - a*d)*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x
)/Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.007, size = 90, normalized size = 1.5

$$\frac{1}{2x}\sqrt{cx^2+d}\left(c^{\frac{3}{2}}\sqrt{cx^2+d}xa+2b\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)c^2-\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)acd\right)\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/2*(c*x^2+d)^(1/2)*(c^(3/2)*(c*x^2+d)^(1/2)*x*a+2*b*ln(c^(1/2)*x+(c*x^2+d)
^(1/2))*c^2-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d)/((c*x^2+d)/x^2)^(1/2)/x/c^
(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42454, size = 335, normalized size = 5.68

$$\left[\frac{2\,acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{4\,c^2},\ \frac{acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2 +
 d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)
/(c*x^2 + d)))/c^2]
```

Sympy [A] time = 38.1443, size = 66, normalized size = 1.12

$$\frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(
3/2)) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)
```

Giac [A] time = 1.17194, size = 107, normalized size = 1.81

$$-\frac{1}{2}d\left(\frac{a\sqrt{\frac{cx^2+d}{x^2}}}{\left(c-\frac{cx^2+d}{x^2}\right)c}+\frac{(2bc-ad)\arctan\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d*(a*sqrt((c*x^2 + d)/x^2)/((c - (c*x^2 + d)/x^2)*c) + (2*b*c - a*d)*a
rctan(sqrt((c*x^2 + d)/x^2)/sqrt(-c))/(sqrt(-c)*c*d))
```

3.964 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx$

Optimal. Leaf size=43

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

```
[Out] -((b*Sqrt[c + d/x^2])/d) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]
```

Rubi [A] time = 0.033241, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] -((b*Sqrt[c + d/x^2])/d) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.0285701, size = 73, normalized size = 1.7

$$
\frac{adx\sqrt{cx^2+d}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d}}\right)-b\sqrt{c}\left(cx^2+d\right)}{\sqrt{c}dx^2\sqrt{c+\frac{d}{x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] (-(b*Sqrt[c]*(d + c*x^2)) + a*d*x*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[
d + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)
```

Maple [A] time = 0.009, size = 69, normalized size = 1.6

$$\frac{1}{x^2 d}\sqrt{cx^2+d}\left(a\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)xd-b\sqrt{cx^2+d}\sqrt{c}\right)\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x/(c+d/x^2)^(1/2),x)
```

```
[Out] (c*x^2+d)^(1/2)*(a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*d-b*(c*x^2+d)^(1/2)*c^(1
/2))/((c*x^2+d)/x^2)^(1/2)/x^2/c^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37018, size = 298, normalized size = 6.93

$$\left[\frac{a\sqrt{c}d\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,bc\sqrt{\frac{cx^2+d}{x^2}}}{2\,cd},-\frac{a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bc\sqrt{\frac{cx^2+d}{x^2}}}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) -
 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*sqr
t((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]
```

Sympy [A] time = 14.1264, size = 63, normalized size = 1.47

$$-\frac{a\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\sqrt{c+\frac{d}{x^2}}}\right)}{c\sqrt{-\frac{1}{c}}}+\frac{b\left(\begin{cases}-\frac{1}{\sqrt{c}x^2} & \text{for } d=0\\ -\frac{2\sqrt{c+\frac{d}{x^2}}}{d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*atan(1/(sqrt(-1/c)*sqrt(c + d/x**2)))/(c*sqrt(-1/c)) + b*Piecewise((-1/(
sqrt(c)*x**2), Eq(d, 0)), (-2*sqrt(c + d/x**2)/d, True))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/(sqrt(c + d/x^2)*x), x)
```

3.965 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

```
[Out] ((b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (b*(c + d/x^2)^(3/2))/(3*d^2)
```

Rubi [A] time = 0.0356831, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]

[Out] ((b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (b*(c + d/x^2)^(3/2))/(3*d^2)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Mathematica [A] time = 0.0251518, size = 39, normalized size = 0.91

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(3adx^2+b\left(d-2cx^2\right)\right)}{3d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

```
[Out] -(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(3*d^2*x^2)
```

Maple [A] time = 0.008, size = 47, normalized size = 1.1

$$-\frac{\left(3\,adx^2-2\,bcx^2+bd\right)\left(cx^2+d\right)}{3\,d^2x^4}\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/3*(3*a*d*x^2-2*b*c*x^2+b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^2/x^4
```

Maxima [A] time = 0.929946, size = 65, normalized size = 1.51

$$-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\,\sqrt{c+\frac{d}{x^2}}c}{d^2}\right)-\frac{a\sqrt{c+\frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^2
)/d
```

Fricas [A] time = 1.32556, size = 88, normalized size = 2.05

$$\frac{\left((2\,bc-3\,ad)x^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)
```

Sympy [A] time = 5.26124, size = 139, normalized size = 3.23

$$-\frac{\begin{cases}\frac{\frac{a}{x^2}+\frac{b}{2x^4}}{\sqrt{c}} & \text{for}\ d=0\\ -\frac{\frac{2ac}{\sqrt{c+\frac{d}{x^2}}}+2a\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)+\frac{2bc\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2b\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] -Piecewise(((a/x**2 + b/(2*x**4))/sqrt(c), Eq(d, 0)), (-(2*a*c/sqrt(c + d/x
**2) + 2*a*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2)) + 2*b*c*(-c/sqrt(c + d/
x**2) - sqrt(c + d/x**2))/d + 2*b*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x
**2) - (c + d/x**2)**(3/2)/3)/d)/d, True))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/(sqrt(c + d/x^2)*x^3), x)
```

3.966 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5} dx$

Optimal. Leaf size=72

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

```
[Out] -((c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3) + ((2*b*c - a*d)*(c + d/x^2)^(3/2))/
(3*d^3) - (b*(c + d/x^2)^(5/2))/(5*d^3)
```

Rubi [A] time = 0.0516449, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] -((c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3) + ((2*b*c - a*d)*(c + d/x^2)^(3/2))/
(3*d^3) - (b*(c + d/x^2)^(5/2))/(5*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2\sqrt{c+dx}} + \frac{(-2bc+ad)\sqrt{c+dx}}{d^2} + \frac{b(c+dx)^{3/2}}{d^2}\right)dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3} + \frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Mathematica [A] time = 0.0299483, size = 60, normalized size = 0.83

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(b\left(-8c^2x^4+4cdx^2-3d^2\right)-5adx^2\left(d-2cx^2\right)\right)}{15d^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] (Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2*
x^4)))/(15*d^3*x^4)
```

Maple [A] time = 0.005, size = 70, normalized size = 1.

$$\frac{\left(10\,acdx^4-8\,bc^2x^4-5\,ad^2x^2+4\,bcdx^2-3\,bd^2\right)\left(cx^2+d\right)}{15\,d^3x^6}\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)*(c*x^2+d)/(
(c*x^2+d)/x^2)^(1/2)/d^3/x^6
```

Maxima [A] time = 0.935661, size = 112, normalized size = 1.56

$$-\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^3}\right)-\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c +
 d/x^2)*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2)
```

Fricas [A] time = 1.28958, size = 139, normalized size = 1.93

$$-\frac{\left(2\left(4\,bc^2-5\,acd\right)x^4+3\,bd^2-\left(4\,bcd-5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2)/(d^3*x^4)
```

Sympy [A] time = 9.56974, size = 206, normalized size = 2.86

$$-\frac{\begin{cases}\frac{\frac{a}{2x^4}+\frac{b}{3x^6}}{\sqrt{c}} & \text{for } d = \\ -\frac{\frac{2ac\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2a\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}+\frac{2bc\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2b\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{otherw}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**5/(c+d/x**2)**(1/2),x)
```

```
[Out] -Piecewise(((a/(2*x**4) + b/(3*x**6))/sqrt(c), Eq(d, 0)), (-(2*a*c*(-c/sqrt
(c + d/x**2) - sqrt(c + d/x**2))/d + 2*a*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(
c + d/x**2) - (c + d/x**2)**(3/2)/3)/d + 2*b*c*(c**2/sqrt(c + d/x**2) + 2*c
*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*b*(-c**3/sqrt(c + d/x**
2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/
5)/d**2)/d, True))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/(sqrt(c + d/x^2)*x^5), x)
```

3.967 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$

Optimal. Leaf size=101

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

```
[Out] (c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2
))/(3*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (b*(c + d/x^2)^(7/
2))/(7*d^4)
```

Rubi [A] time = 0.0693012, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```


```
[Out] (c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2
))/(3*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (b*(c + d/x^2)^(7/
2))/(7*d^4)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(a+bx)}{\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{c(3bc-2ad)\sqrt{c+dx}}{d^3}+\frac{(-3bc+ad)(c+dx)^{3/2}}{d^3}+\frac{b(c+dx)^{5/2}}{d^3}\right)dx,x
\\
&= \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}
\end{aligned}$$

Mathematica [A] time = 0.0267121, size = 91, normalized size = 0.9

$$\frac{\left(\frac{cx^2}{d}+1\right)\left(8c^2x^4-4cdx^2+3d^2\right)(6bc-7ad)}{105d^3x^6\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(cx^2+d\right)}{7dx^8\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] -(b*(d + c*x^2))/(7*d*Sqrt[c + d/x^2]*x^8) + ((6*b*c - 7*a*d)*(1 + (c*x^2)/
d)*(3*d^2 - 4*c*d*x^2 + 8*c^2*x^4))/(105*d^3*Sqrt[c + d/x^2]*x^6)
```

Maple [A] time = 0.007, size = 94, normalized size = 0.9

$$-\frac{\left(56\,ac^2dx^6-48\,bc^3x^6-28\,acd^2x^4+24\,bc^2dx^4+21\,ad^3x^2-18\,bcd^2x^2+15\,bd^3\right)\left(cx^2+d\right)}{105\,d^4x^8}\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/105*(56*a*c^2*d*x^6-48*b*c^3*x^6-28*a*c*d^2*x^4+24*b*c^2*d*x^4+21*a*d^3*
x^2-18*b*c*d^2*x^2+15*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^4/x^8
```

Maxima [A] time = 0.944889, size = 159, normalized size = 1.57

$$-\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^4}-\frac{21\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^4}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^4}-\frac{35\sqrt{c+\frac{d}{x^2}}\,c^3}{d^4}\right)-\frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^4 - 21*(c + d/x^2)^(5/2)*c/d^4 + 35*(c + d/x
^2)^(3/2)*c^2/d^4 - 35*sqrt(c + d/x^2)*c^3/d^4) - 1/15*a*(3*(c + d/x^2)^(5/
2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3)
```

Fricas [A] time = 1.34226, size = 192, normalized size = 1.9

$$\frac{\left(8\left(6\,bc^3-7\,ac^2d\right)x^6-4\left(6\,bc^2d-7\,acd^2\right)x^4-15\,bd^3+3\left(6\,bcd^2-7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*b*c^3 - 7*a*c^2*d)*x^6 - 4*(6*b*c^2*d - 7*a*c*d^2)*x^4 - 15*b*d
^3 + 3*(6*b*c*d^2 - 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^6)
```

Sympy [A] time = 13.6424, size = 270, normalized size = 2.67

$$-\frac{\begin{cases} \frac{\frac{a}{3x^6}+\frac{b}{4x^8}}{\sqrt{c}} \\ -\frac{\frac{2ac\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2a\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{2bc\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^3}+\frac{2b\left(\frac{c^4}{\sqrt{c+\frac{d}{x^2}}}+4c^3\sqrt{c+\frac{d}{x^2}}-2c\cdots\right.}{\cdots}}{d} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**7/(c+d/x**2)**(1/2),x)
```

```
[Out] -Piecewise(((a/(3*x**6) + b/(4*x**8))/sqrt(c), Eq(d, 0)), (-(2*a*c*(c**2/sq
rt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*a*(
-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) -
(c + d/x**2)**(5/2)/5)/d**2 + 2*b*c*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c
 + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**3 + 2*b*(c**
4/sqrt(c + d/x**2) + 4*c**3*sqrt(c + d/x**2) - 2*c**2*(c + d/x**2)**(3/2) +
 4*c*(c + d/x**2)**(5/2)/5 - (c + d/x**2)**(7/2)/7)/d**3)/d, True))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/(sqrt(c + d/x^2)*x^7), x)
```

3.968 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=82

$$\frac{x^3\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{15c^2}-\frac{2dx\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{15c^3}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

```
[Out] (-2*d*(5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(15*c^3) + ((5*b*c - 4*a*d)*Sqrt[c
 + d/x^2]*x^3)/(15*c^2) + (a*Sqrt[c + d/x^2]*x^5)/(5*c)
```

Rubi [A] time = 0.0333523, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$\frac{x^3\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{15c^2}-\frac{2dx\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{15c^3}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```


```
[Out] (-2*d*(5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(15*c^3) + ((5*b*c - 4*a*d)*Sqrt[c
 + d/x^2]*x^3)/(15*c^2) + (a*Sqrt[c + d/x^2]*x^5)/(5*c)
```


Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c} + \frac{(5bc-4ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx}{5c}$$

$$= \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c} - \frac{(2d(5bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^2}$$

$$= -\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^3} + \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}$$

Mathematica [A] time = 0.0440407, size = 56, normalized size = 0.68

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(a\left(3c^2x^4-4cdx^2+8d^2\right)+5bc\left(cx^2-2d\right)\right)}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^4
)))/(15*c^3)
```

Maple [A] time = 0.007, size = 67, normalized size = 0.8

$$\frac{\left(3\,ax^4c^2-4\,acdx^2+5\,bc^2x^2+8\,ad^2-10\,bcd\right)\left(cx^2+d\right)}{15\,xc^3}\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15/x*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*(c*x^2+d)/((c
*x^2+d)/x^2)^(1/2)/c^3
```

Maxima [A] time = 0.949943, size = 115, normalized size = 1.4

$$\frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\sqrt{c+\frac{d}{x^2}}dx\right)b}{3\,c^2}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\sqrt{c+\frac{d}{x^2}}d^2x\right)a}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)*b/c^2 + 1/15*(3*(c + d/
x^2)^(5/2)*x^5 - 10*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)*a/c
^3
```

Fricas [A] time = 1.35639, size = 132, normalized size = 1.61

$$\frac{\left(3\,ac^2x^5+\left(5\,bc^2-4\,acd\right)x^3-2\left(5\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqrt
((c*x^2 + d)/x^2)/c^3
```

Sympy [B] time = 3.24329, size = 338, normalized size = 4.12

$$\frac{3ac^4d^{\frac{9}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{2ac^3d^{\frac{11}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{3ac^2d^{\frac{13}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{12acd^{\frac{15}{2}}x^2\sqrt{c}}{15c^5d^4x^4+30c^4d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] 3*a*c**4*d**(9/2)*x**8*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5
*x**2 + 15*c**3*d**6) + 2*a*c**3*d**(11/2)*x**6*sqrt(c*x**2/d + 1)/(15*c**5
*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 3*a*c**2*d**(13/2)*x**4*sq
rt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 1
2*a*c*d**(15/2)*x**2*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x
**2 + 15*c**3*d**6) + 8*a*d**(17/2)*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 +
 30*c**4*d**5*x**2 + 15*c**3*d**6) + b*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c
) - 2*b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)*x^4/sqrt(c + d/x^2), x)
```

3.969 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=51

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

```
[Out] ((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c)
```

Rubi [A] time = 0.0202861, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 191}

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] ((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c)
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c} + \frac{(3bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c}$$
$$= \frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}$$

Mathematica [A] time = 0.0278078, size = 34, normalized size = 0.67

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(acx^2-2ad+3bc\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

Maple [A] time = 0.003, size = 44, normalized size = 0.9

$$\frac{\left(ax^2c-2\,ad+3\,bc\right)\left(cx^2+d\right)}{3\,xc^2}\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/3/x*(a*c*x^2-2*a*d+3*b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/c^2
```

Maxima [A] time = 0.9383, size = 66, normalized size = 1.29

$$\frac{b\sqrt{c+\frac{d}{x^2}}\,x}{c} + \frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\,\sqrt{c+\frac{d}{x^2}}dx\right)a}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)
*a/c^2
```

Fricas [A] time = 1.28396, size = 82, normalized size = 1.61

$$\frac{\left(acx^3 + (3\,bc - 2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [A] time = 2.37479, size = 70, normalized size = 1.37

$$\frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2} + \frac{b\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c) - 2*a*d**(3/2)*sqrt(c*x**2/d + 1)/(
3*c**2) + b*sqrt(d)*sqrt(c*x**2/d + 1)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x^2}\right)x^2}{\sqrt{c + \frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)*x^2/sqrt(c + d/x^2), x)
```

3.970 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=47

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

```
[Out] (a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

Rubi [A] time = 0.0292151, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {375, 451, 217, 206}

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
```

```
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx &= -\text{Subst}\left(\int \frac{a+bx^2}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)\\ &= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.0310319, size = 71, normalized size = 1.51

$$\frac{a\sqrt{d}\left(cx^2+d\right)-bc\sqrt{cx^2+d}\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{c\sqrt{d}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d
]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.01, size = 73, normalized size = 1.6

$$\frac{1}{cx}\sqrt{cx^{2}+d}\left(a\sqrt{cx^{2}+d}\sqrt{d}-b\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^{2}+d}+d}{x}\right)c\right)\frac{1}{\sqrt{\frac{cx^{2}+d}{x^{2}}}}\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(1/2),x)
```

```
[Out] (c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+
d)/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37344, size = 305, normalized size = 6.49

$$\left[\frac{2\,adx\sqrt{\frac{cx^{2}+d}{x^{2}}}+bc\sqrt{d}\log\left(-\frac{cx^{2}-2\,\sqrt{d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}+2\,d}{x^{2}}\right)}{2\,cd},\frac{adx\sqrt{\frac{cx^{2}+d}{x^{2}}}+bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x
*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) + b
*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c*d)]
```

Sympy [A] time = 2.611, size = 39, normalized size = 0.83

$$\frac{a\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/sqrt(c + d/x^2), x)
```

3.971 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2} dx$

Optimal. Leaf size=61

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

```
[Out] -(b*Sqrt[c + d/x^2])/(2*d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x
^2]*x)])/(2*d^(3/2))
```

Rubi [A] time = 0.0375054, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 335, 217, 206}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] -(b*Sqrt[c + d/x^2])/(2*d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x
^2]*x)])/(2*d^(3/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(-bc+2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{2d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{(-bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{2d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{(-bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0525333, size = 80, normalized size = 1.31

$$\frac{x^2\sqrt{cx^2+d}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)-b\sqrt{d}\left(cx^2+d\right)}{2d^{3/2}x^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] (-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt[
d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)
```

Maple [B] time = 0.012, size = 105, normalized size = 1.7

$$-\frac{1}{2x^{3}}\sqrt{cx^{2}+d}\left(2\,a\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^{2}+d}+d}{x}\right)x^{2}d^{2}-\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^{2}+d}+d}{x}\right)x^{2}bcd+d^{\frac{3}{2}}\sqrt{cx^{2}+d}b\right)\frac{1}{\sqrt{\frac{cx^{2}+d}{x^{2}}}}d^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*(c*x^2+d)^(1/2)*(2*a*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^2*d^2-ln(2*
(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^2*b*c*d+d^(3/2)*(c*x^2+d)^(1/2)*b)/((c*x^2
+d)/x^2)^(1/2)/x^3/d^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36996, size = 346, normalized size = 5.67

$$\left[-\frac{(bc-2\,ad)\sqrt{d}x\log\left(-\frac{cx^{2}-2\,\sqrt{d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}+2\,d}{x^{2}}\right)+2\,bd\sqrt{\frac{cx^{2}+d}{x^{2}}}}{4\,d^{2}x},-\frac{(bc-2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)+bd\sqrt{\frac{cx^{2}+d}{x^{2}}}}{2\,d^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x
^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*d)
*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*d*sqrt
((c*x^2 + d)/x^2))/(d^2*x)]
```

Sympy [A] time = 4.6894, size = 66, normalized size = 1.08

$$-\frac{a\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}}-\frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d
*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/(sqrt(c + d/x^2)*x^2), x)
```

3.972 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{c+\frac{d}{x^2}}(3bc-4ad)}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

```
[Out] -(b*Sqrt[c + d/x^2])/(4*d*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^2*x
) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(5/2))
```

Rubi [A] time = 0.0502702, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 321, 217, 206}

$$\frac{\sqrt{c+\frac{d}{x^2}}(3bc-4ad)}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -(b*Sqrt[c + d/x^2])/(4*d*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^2*x
) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(5/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(-3bc+4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx}{4d}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}-\frac{(-3bc+4ad)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{4d}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{8d^2}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^2}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^{5/2}}$$

Mathematica [A] time = 0.121172, size = 107, normalized size = 1.15

$$-\frac{\left(cx^2+d\right)\left(d\sqrt{\frac{cx^2}{d}+1}\left(4adx^2-3bcx^2+2bd\right)+cx^4(3bc-4ad)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{8d^3x^5\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -((d + c*x^2)*(d*Sqrt[1 + (c*x^2)/d]*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2) + c*(3
*b*c - 4*a*d)*x^4*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(8*d^3*Sqrt[c + d/x^2]*x^5
*Sqrt[1 + (c*x^2)/d])
```

Maple [A] time = 0.01, size = 146, normalized size = 1.6

$$-\frac{1}{8x^5}\sqrt{cx^2+d}\left(-4\ln\left(2\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^4acd^2+3\ln\left(2\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)x^4bc^2d+4d^{5/2}\sqrt{cx^2+d}x^2a-3d^{3/2}\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+d)^(1/2)*(-4*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^4*a*c*d^2+3*
ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*x^4*b*c^2*d+4*d^(5/2)*(c*x^2+d)^(1/2)*x
^2*a-3*d^(3/2)*(c*x^2+d)^(1/2)*x^2*b*c+2*d^(5/2)*(c*x^2+d)^(1/2)*b)/((c*x^2
+d)/x^2)^(1/2)/x^5/d^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72066, size = 459, normalized size = 4.94

$$\left[-\frac{\left(3\,bc^2-4\,acd\right)\sqrt{d}x^3\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,bd^2-\left(3\,bcd-4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^3x^3},\frac{\left(3\,bc^2-4\,acd\right)\sqrt{-d}x^3\arctan\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x
^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-
d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^
2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3)]
```

Sympy [A] time = 8.33416, size = 150, normalized size = 1.61

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}+\frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{5}{2}}}-\frac{b}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2
*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*d*
x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(5/2
)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/(sqrt(c + d/x^2)*x^4), x)
```

3.973 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^2(4bc-5ad)}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] (3*d*(4*b*c - 5*a*d))/(8*c^3*Sqrt[c + d/x^2]) + ((4*b*c - 5*a*d)*x^2)/(8*c^
2*Sqrt[c + d/x^2]) + (a*x^4)/(4*c*Sqrt[c + d/x^2]) - (3*d*(4*b*c - 5*a*d)*A
rcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rubi [A] time = 0.0848099, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{3x^2\sqrt{c+\frac{d}{x^2}}(4bc-5ad)}{8c^3}-\frac{x^2(4bc-5ad)}{4c^2\sqrt{c+\frac{d}{x^2}}}-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((4*b*c - 5*a*d)*x^2)/(4*c^2*Sqrt[c + d/x^2]) + (3*(4*b*c - 5*a*d)*Sqrt[c
+ d/x^2]*x^2)/(8*c^3) + (a*x^4)/(4*c*Sqrt[c + d/x^2]) - (3*d*(4*b*c - 5*a*d
)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{a+bx}{x^3(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(2bc-\frac{5ad}{2}\right)\text{Subst}\left(\int \frac{1}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{(3(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{8c^2}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3d(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c^3}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{8c^3}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

Mathematica [A] time = 0.14451, size = 111, normalized size = 0.94

$$\frac{\sqrt{c}x\left(a\left(2c^2x^4-5cdx^2-15d^2\right)+4bc\left(cx^2+3d\right)\right)+3d^{3/2}\sqrt{\frac{cx^2}{d}+1}(5ad-4bc)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{7/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 3*
d^(3/2)*(-4*b*c + 5*a*d)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/
(8*c^(7/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.012, size = 140, normalized size = 1.2

$$\frac{cx^2+d}{8\,x^3}\left(2\,c^{7/2}x^5a-5\,c^{5/2}x^3ad+4\,c^{7/2}x^3b-15\,c^{3/2}xad^2+12\,c^{5/2}xbd+15\,\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)\sqrt{cx^2+d}acd^2-12\,\ln\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/8*(c*x^2+d)*(2*c^(7/2)*x^5*a-5*c^(5/2)*x^3*a*d+4*c^(7/2)*x^3*b-15*c^(3/2)
*x*a*d^2+12*c^(5/2)*x*b*d+15*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*
a*c*d^2-12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*b*c^2*d)/((c*x^2+d
)/x^2)^(3/2)/x^3/c^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88141, size = 662, normalized size = 5.61

$$\left[-\frac{3\left(4\,bcd^2-5\,ad^3+\left(4\,bc^2d-5\,acd^2\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^3x^6+\left(4\,bc^3-5\,ac^2d\right)x^4+3\left(4\,b\right.\right.}{16\left(c^5x^2+c^4d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*b*
```

```
c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)
)/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*
x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a
*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d)]
```

Sympy [A] time = 52.4764, size = 177, normalized size = 1.5

$$a\left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{5\sqrt{d}x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}}+\frac{15d^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}}\right)+b\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{2c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**5/(4*c*sqrt(d)*sqrt(c*x**2/d + 1)) - 5*sqrt(d)*x**3/(8*c**2*sqrt(c*x*
*2/d + 1)) - 15*d**(3/2)*x/(8*c**3*sqrt(c*x**2/d + 1)) + 15*d**2*asinh(sqrt
(c)*x/sqrt(d))/(8*c**(7/2))) + b*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3
*sqrt(d)*x/(2*c**2*sqrt(c*x**2/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c*
*(5/2)))
```

Giac [A] time = 1.1696, size = 261, normalized size = 2.21

$$\frac{1}{8}d^2\left(\frac{3\,(4\,bc-5\,ad)\arctan\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3d}+\frac{8\,(bc-ad)}{c^3d\sqrt{\frac{cx^2+d}{x^2}}}-\frac{4\,bc^2\sqrt{\frac{cx^2+d}{x^2}}-9\,acd\sqrt{\frac{cx^2+d}{x^2}}-\frac{4\left(cx^2+d\right)bc\sqrt{\frac{cx^2+d}{x^2}}}{x^2}+\frac{7\left(cx^2+d\right)ad\sqrt{\frac{cx^2+d}{x^2}}}{x^2}}{\left(c-\frac{cx^2+d}{x^2}\right)^2c^3d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*d^2*(3*(4*b*c - 5*a*d)*arctan(sqrt((c*x^2 + d)/x^2)/sqrt(-c))/(sqrt(-c)
*c^3*d) + 8*(b*c - a*d)/(c^3*d*sqrt((c*x^2 + d)/x^2)) - (4*b*c^2*sqrt((c*x^
2 + d)/x^2) - 9*a*c*d*sqrt((c*x^2 + d)/x^2) - 4*(c*x^2 + d)*b*c*sqrt((c*x^2
 + d)/x^2)/x^2 + 7*(c*x^2 + d)*a*d*sqrt((c*x^2 + d)/x^2)/x^2)/((c - (c*x^2
```

```
+ d)/x^2)^2*c^3*d))
```

3.974 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -(2*b*c - 3*a*d)/(2*c^2*Sqrt[c + d/x^2]) + (a*x^2)/(2*c*Sqrt[c + d/x^2]) +
((2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rubi [A] time = 0.0583173, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {446, 78, 51, 63, 208}

$$-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] -(2*b*c - 3*a*d)/(2*c^2*Sqrt[c + d/x^2]) + (a*x^2)/(2*c*Sqrt[c + d/x^2]) +
((2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{a+bx}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c^2}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{2c^2d}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} + \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Mathematica [A] time = 0.115579, size = 89, normalized size = 1.03

$$\frac{\sqrt{c}x\left(acx^2+3ad-2bc\right)-\sqrt{d}\sqrt{\frac{cx^2}{d}+1}(3ad-2bc)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{5/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(-2*b*c + 3*a*d + a*c*x^2) - Sqrt[d]*(-2*b*c + 3*a*d)*Sqrt[1 + (
c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(2*c^(5/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.009, size = 114, normalized size = 1.3

$$\frac{cx^2+d}{2\,x^3}\left(c^{\frac{5}{2}}x^3a+3\,c^{3/2}xad-2\,c^{5/2}xb-3\,\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)\sqrt{cx^2+d}acd+2\,\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)\sqrt{cx^2+d}bc^2\right)\left(\frac{cx^2}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/2*(c*x^2+d)*(c^(5/2)*x^3*a+3*c^(3/2)*x*a*d-2*c^(5/2)*x*b-3*ln(c^(1/2)*x+(
c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*a*c*d+2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^
2+d)^(1/2)*b*c^2)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62802, size = 543, normalized size = 6.31

$$\left[-\frac{\left(2\,bcd-3\,ad^2+\left(2\,bc^2-3\,acd\right)x^2\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(ac^2x^4-\left(2\,bc^2-3\,acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\left(c^4x^2+c^3d\right)},-\frac{\left(2\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a*c
*d)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*d^2
 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x
^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/(c^4*x^2 + c^3*d)]
```

Sympy [B] time = 27.2976, size = 264, normalized size = 3.07

$$a\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)+b\left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^3x^2\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^3x^2\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2/
d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(5/2))) + b*(-2*c**3*x**2*sqrt
(1 + d/(c*x**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**3*x**2*log(d/(c*x**2
))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) + 2*c**3*x**2*log(sqrt(1 + d/(c*x**2))
+ 1)/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**2*d*log(d/(c*x**2))/(2*c**(9/2)*
x**2 + 2*c**(7/2)*d) + 2*c**2*d*log(sqrt(1 + d/(c*x**2)) + 1)/(2*c**(9/2)*x
**2 + 2*c**(7/2)*d))
```

Giac [A] time = 1.17672, size = 182, normalized size = 2.12

$$-\frac{1}{2}\,d\left(\frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2d}+\frac{2\,bc^2-2\,acd-\frac{2\left(cx^2+d\right)bc}{x^2}+\frac{3\left(cx^2+d\right)ad}{x^2}}{\left(c\sqrt{\frac{cx^2+d}{x^2}}-\frac{\left(cx^2+d\right)\sqrt{\frac{cx^2+d}{x^2}}}{x^2}\right)c^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*d*((2*b*c - 3*a*d)*arctan(sqrt((c*x^2 + d)/x^2)/sqrt(-c))/(sqrt(-c)*c^
2*d) + (2*b*c^2 - 2*a*c*d - 2*(c*x^2 + d)*b*c/x^2 + 3*(c*x^2 + d)*a*d/x^2)/
((c*sqrt((c*x^2 + d)/x^2) - (c*x^2 + d)*sqrt((c*x^2 + d)/x^2)/x^2)*c^2*d))
```

3.975 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx$

Optimal. Leaf size=52

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^
(3/2)
```

Rubi [A] time = 0.0400453, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^
(3/2)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0623979, size = 73, normalized size = 1.4

$$\frac{\sqrt{c}x(bc-ad)+ad^{3/2}\sqrt{\frac{cx^2}{d}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}dx\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (Sqrt[c]*(b*c - a*d)*x + a*d^(3/2)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/
Sqrt[d]])/(c^(3/2)*d*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.008, size = 75, normalized size = 1.4

$$\frac{cx^2+d}{dx^3}\left(c^{\frac{5}{2}}xb-c^{\frac{3}{2}}xad+\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)\sqrt{cx^2+d}\,acd\right)\left(\frac{cx^2+d}{x^2}\right)^{-\frac{3}{2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x)
```

```
[Out] (c*x^2+d)*(c^(5/2)*x*b-c^(3/2)*x*a*d+ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d
)^(1/2)*a*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(5/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60452, size = 427, normalized size = 8.21

$$\left[\frac{2\left(bc^2-acd\right)x^2\sqrt{\frac{cx^2+d}{x^2}}+\left(acdx^2+ad^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{2\left(c^3dx^2+c^2d^2\right)}, \frac{\left(bc^2-acd\right)x^2\sqrt{\frac{cx^2+d}{x^2}}-\left(acdx^2+ad^2\right)\sqrt{-c}\ a}{c^3dx^2+c^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) + (a*c*d*x^2 + a*d^2)*sqr
t(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/(c^3*d*x^2 +
c^2*d^2), ((b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) - (a*c*d*x^2 + a*d^2)*
sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^3*d*x^2
 + c^2*d^2)]
```

Sympy [A] time = 10.2146, size = 49, normalized size = 0.94

$$-\frac{a\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{c\sqrt{-c}}-\frac{ad-bc}{cd\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*atan(sqrt(c + d/x**2)/sqrt(-c))/(c*sqrt(-c)) - (a*d - b*c)/(c*d*sqrt(c +
 d/x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x), x)
```

3.976 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx$

Optimal. Leaf size=42

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

```
[Out] -((b*c - a*d)/(d^2*Sqrt[c + d/x^2])) - (b*Sqrt[c + d/x^2])/d^2
```

Rubi [A] time = 0.0354627, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```


```
[Out] -((b*c - a*d)/(d^2*Sqrt[c + d/x^2])) - (b*Sqrt[c + d/x^2])/d^2
```


Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx = -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a+bx}{(c+dx)^{3/2}} \, dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{-bc+ad}{d(c+dx)^{3/2}} + \frac{b}{d\sqrt{c+dx}}\right) dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

Mathematica [A] time = 0.0184008, size = 36, normalized size = 0.86

$$\frac{adx^2 - b\left(2cx^2 + d\right)}{d^2x^2\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

```
[Out] (a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)
```

Maple [A] time = 0.006, size = 46, normalized size = 1.1

$$\frac{\left(adx^2 - 2\,bcx^2 - bd\right)\left(cx^2 + d\right)}{d^2x^4}\left(\frac{cx^2+d}{x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x)
```

```
[Out] (a*d*x^2-2*b*c*x^2-b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^2/x^4
```

Maxima [A] time = 0.927368, size = 62, normalized size = 1.48

$$-b\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)+\frac{a}{\sqrt{c+\frac{d}{x^2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -b*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2)) + a/(sqrt(c + d/x^2)*d)
```

Fricas [A] time = 1.47852, size = 92, normalized size = 2.19

$$-\frac{\left((2\,bc-ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2+d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -((2*b*c - a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2)/(c*d^2*x^2 + d^3)
```

Sympy [A] time = 3.39004, size = 68, normalized size = 1.62

$$\begin{cases} \frac{a}{d\sqrt{c+\frac{d}{x^2}}}-\frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for } d\neq 0 \\ \frac{-\frac{a}{2x^2}-\frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x*
*2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), True
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^3), x)
```

3.977 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

```
[Out] (c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + ((2*b*c - a*d)*Sqrt[c + d/x^2])/d^3
 - (b*(c + d/x^2)^(3/2))/(3*d^3)
```

Rubi [A] time = 0.0521264, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + ((2*b*c - a*d)*Sqrt[c + d/x^2])/d^3
 - (b*(c + d/x^2)^(3/2))/(3*d^3)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{-2bc+ad}{d^2\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d^2}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}\end{aligned}$$

Mathematica [A] time = 0.0226575, size = 60, normalized size = 0.88

$$\frac{b\left(8c^2x^4+4cdx^2-d^2\right)-3adx^2\left(2cx^2+d\right)}{3d^3x^4\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[c
 + d/x^2]*x^4)
```

Maple [A] time = 0.006, size = 69, normalized size = 1.

$$-\frac{\left(6\,acdx^4-8\,bc^2x^4+3\,ad^2x^2-4\,bcdx^2+bd^2\right)\left(cx^2+d\right)}{3\,d^3x^6}\left(\frac{cx^2+d}{x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x)
```

```
[Out] -1/3*(6*a*c*d*x^4-8*b*c^2*x^4+3*a*d^2*x^2-4*b*c*d*x^2+b*d^2)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^3/x^6
```

Maxima [A] time = 0.956154, size = 109, normalized size = 1.6

$$-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\,\sqrt{c+\frac{d}{x^2}}\,c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right)-a\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^3 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d
/x^2)*d^3)) - a*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2))
```

Fricas [A] time = 1.58677, size = 150, normalized size = 2.21

$$\frac{\left(2\left(4\,bc^2-3\,acd\right)x^4-bd^2+\left(4\,bcd-3\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(cd^3x^4+d^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(4*b*c^2 - 3*a*c*d)*x^4 - b*d^2 + (4*b*c*d - 3*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2)/(c*d^3*x^4 + d^4*x^2)
```

Sympy [A] time = 7.43081, size = 61, normalized size = 0.9

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d^3}-\frac{c\left(ad-bc\right)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c+\frac{d}{x^2}}\left(ad-2bc\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -b*(c + d/x**2)**(3/2)/(3*d**3) - c*(a*d - b*c)/(d**3*sqrt(c + d/x**2)) - s
qrt(c + d/x**2)*(a*d - 2*b*c)/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^5), x)
```

3.978 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx$

Optimal. Leaf size=100

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

```
[Out] -((c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2])) - (c*(3*b*c - 2*a*d)*Sqrt[c + d/
x^2])/d^4 + ((3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (b*(c + d/x^2)^(5/2
))/(5*d^4)
```

Rubi [A] time = 0.0710595, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] -((c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2])) - (c*(3*b*c - 2*a*d)*Sqrt[c + d/
x^2])/d^4 + ((3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (b*(c + d/x^2)^(5/2
))/(5*d^4)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
c, d, e, f]))))
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3(c+dx)^{3/2}} + \frac{c(3bc-2ad)}{d^3\sqrt{c+dx}} + \frac{(-3bc+ad)\sqrt{c+dx}}{d^3} + \frac{b(c+dx)^{3/2}}{d^3}\right) dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}} - \frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Mathematica [A] time = 0.0256347, size = 81, normalized size = 0.81

$$\frac{-5adx^2\left(-8c^2x^4-4cdx^2+d^2\right)-3b\left(8c^2dx^4+16c^3x^6-2cd^2x^2+d^3\right)}{15d^4x^6\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] (-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2*
d*x^4 + 16*c^3*x^6))/(15*d^4*Sqrt[c + d/x^2]*x^6)
```

Maple [A] time = 0.007, size = 94, normalized size = 0.9

$$\frac{\left(40\,ac^2dx^6-48\,bc^3x^6+20\,acd^2x^4-24\,bc^2dx^4-5\,ad^3x^2+6\,bcd^2x^2-3\,bd^3\right)\left(cx^2+d\right)}{15\,d^4x^8}\left(\frac{cx^2+d}{x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x)
```

```
[Out] 1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^2
+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^4/x^8
```

Maxima [A] time = 0.949499, size = 157, normalized size = 1.57

$$-\frac{1}{5}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}d^4}\right)-\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\sqrt{c+\frac{d}{x^2}}c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/5*b*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c + d/x
^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4)) - 1/3*a*((c + d/x^2)^(3/2)/d^3 -
 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d/x^2)*d^3))
```

Fricas [A] time = 1.61303, size = 204, normalized size = 2.04

$$-\frac{\left(8\left(6\,bc^3-5\,ac^2d\right)x^6+4\left(6\,bc^2d-5\,acd^2\right)x^4+3\,bd^3-\left(6\,bcd^2-5\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(cd^4x^6+d^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d^
3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^4*x^6 + d^5*x^4)
```

Sympy [A] time = 10.0171, size = 90, normalized size = 0.9

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d^4}+\frac{c^2\,(ad-bc)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}(ad-3bc)}{3d^4}-\frac{\sqrt{c+\frac{d}{x^2}}\left(-2acd+3bc^2\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -b*(c + d/x**2)**(5/2)/(5*d**4) + c**2*(a*d - b*c)/(d**4*sqrt(c + d/x**2))
- (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(3*d**4) - sqrt(c + d/x**2)*(-2*a*c*d +
 3*b*c**2)/d**4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^7), x)
```

3.979 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx$

Optimal. Leaf size=126

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{d^5}+\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

```
[Out] (c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (c^2*(4*b*c - 3*a*d)*Sqrt[c + d/x
^2])/d^5 - (c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + ((4*b*c - a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (b*(c + d/x^2)^(7/2))/(7*d^5)
```

Rubi [A] time = 0.0889519, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{d^5}+\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (c^2*(4*b*c - 3*a*d)*Sqrt[c + d/x
^2])/d^5 - (c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + ((4*b*c - a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (b*(c + d/x^2)^(7/2))/(7*d^5)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^3(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int\left(\frac{c^3(bc-ad)}{d^4(c+dx)^{3/2}} - \frac{c^2(4bc-3ad)}{d^4\sqrt{c+dx}} + \frac{3c(2bc-ad)\sqrt{c+dx}}{d^4} + \frac{(-4bc+ad)(c+dx)^{3/2}}{d^4}\right.\right.\right.$$

$$= \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}} + \frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5} + \frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{b\left(c+\frac{d}{x^2}\right.}{7d^5}$$

Mathematica [A] time = 0.0296777, size = 104, normalized size = 0.83

$$\frac{b\left(-16c^2d^2x^4+64c^3dx^6+128c^4x^8+8cd^3x^2-5d^4\right)-7adx^2\left(8c^2dx^4+16c^3x^6-2cd^2x^2+d^3\right)}{35d^5x^8\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8*
c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*Sqrt[c +
d/x^2]*x^8)
```

Maple [A] time = 0.007, size = 118, normalized size = 0.9

$$-\frac{\left(112\,ac^3dx^8-128\,bc^4x^8+56\,ac^2d^2x^6-64\,bc^3dx^6-14\,acd^3x^4+16\,bc^2d^2x^4+7\,ad^4x^2-8\,bcd^3x^2+5\,bd^4\right)\left(cx^2+d\right)}{35\,d^5x^{10}}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x)
```

```
[Out] -1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*c
*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^5/x^10
```

Maxima [A] time = 0.955883, size = 204, normalized size = 1.62

$$-\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\sqrt{c+\frac{d}{x^2}}\,c^3}{d^5}-\frac{35\,c^4}{\sqrt{c+\frac{d}{x^2}}\,d^5}\right)-\frac{1}{5}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/x
^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)*d
^5)) - 1/5*a*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c
 + d/x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4))
```

Fricas [A] time = 1.48715, size = 252, normalized size = 2.

$$\frac{\left(16\left(8\,bc^4-7\,ac^3d\right)x^8+8\left(8\,bc^3d-7\,ac^2d^2\right)x^6-5\,bd^4-2\left(8\,bc^2d^2-7\,acd^3\right)x^4+\left(8\,bcd^3-7\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\left(cd^5x^8+d^6x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b*
d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((c*
x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)
```

Sympy [A] time = 12.6597, size = 122, normalized size = 0.97

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7d^5}-\frac{c^3\,(ad-bc)}{d^5\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{5d^5}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\left(-3acd+6bc^2\right)}{3d^5}-\frac{\sqrt{c+\frac{d}{x^2}}\left(3ac^2d-4bc^3\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -b*(c + d/x**2)**(7/2)/(7*d**5) - c**3*(a*d - b*c)/(d**5*sqrt(c + d/x**2))
- (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(5*d**5) - (c + d/x**2)**(3/2)*(-3*a*c*
d + 6*b*c**2)/(3*d**5) - sqrt(c + d/x**2)*(3*a*c**2*d - 4*b*c**3)/d**5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^9), x)
```

3.980 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}-\frac{8dx\sqrt{c+\frac{d}{x^2}}(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] (4*d*(5*b*c - 6*a*d)*x)/(15*c^3*Sqrt[c + d/x^2]) - (8*d*(5*b*c - 6*a*d)*Sqr
t[c + d/x^2]*x)/(15*c^4) + ((5*b*c - 6*a*d)*x^3)/(15*c^2*Sqrt[c + d/x^2]) +
 (a*x^5)/(5*c*Sqrt[c + d/x^2])
```

Rubi [A] time = 0.0488855, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 271, 192, 191}

$$\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}-\frac{8dx\sqrt{c+\frac{d}{x^2}}(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (4*d*(5*b*c - 6*a*d)*x)/(15*c^3*Sqrt[c + d/x^2]) - (8*d*(5*b*c - 6*a*d)*Sqr
t[c + d/x^2]*x)/(15*c^4) + ((5*b*c - 6*a*d)*x^3)/(15*c^2*Sqrt[c + d/x^2]) +
 (a*x^5)/(5*c*Sqrt[c + d/x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx &= \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{5c}\\
&= \frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(4d(5bc-6ad))\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{15c^2}\\
&= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(8d(5bc-6ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^3}\\
&= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}-\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.0522132, size = 80, normalized size = 0.72

$$\frac{3a\left(-2c^2dx^4+c^3x^6+8cd^2x^2+16d^3\right)+5bc\left(c^2x^4-4cdx^2-8d^2\right)}{15c^4x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*d
*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.006, size = 91, normalized size = 0.8

$$\frac{\left(3\,ax^{6}c^{3}-6\,ac^{2}dx^{4}+5\,bc^{3}x^{4}+24\,acd^{2}x^{2}-20\,bc^{2}dx^{2}+48\,ad^{3}-40\,bcd^{2}\right)\left(cx^{2}+d\right)}{15\,x^{3}c^{4}}\left(\frac{cx^{2}+d}{x^{2}}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/15*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+4
8*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4
```

Maxima [A] time = 0.946507, size = 173, normalized size = 1.56

$$\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}x^{3}-6\,\sqrt{c+\frac{d}{x^{2}}}dx}{c^{3}}-\frac{3\,d^{2}}{\sqrt{c+\frac{d}{x^{2}}}c^{3}x}\right)+\frac{1}{5}\,a\left(\frac{5\,d^{3}}{\sqrt{c+\frac{d}{x^{2}}}c^{4}x}+\frac{\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}x^{5}-5\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}dx^{3}+15\,\sqrt{c+\frac{d}{x^{2}}}d^{2}x}{c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c
+ d/x^2)*c^3*x)) + 1/5*a*(5*d^3/(sqrt(c + d/x^2)*c^4*x) + ((c + d/x^2)^(5/2
)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)/c^4)
```

Fricas [A] time = 1.32175, size = 200, normalized size = 1.8

$$\frac{\left(3\,ac^3x^7+\left(5\,bc^3-6\,ac^2d\right)x^5-4\left(5\,bc^2d-6\,acd^2\right)x^3-8\left(5\,bcd^2-6\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(c^5x^2+c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*x
^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)
```

Sympy [B] time = 11.3805, size = 561, normalized size = 5.05

$$a\left(\frac{c^5d^{\frac{19}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{5c^3d^{\frac{23}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{30c^2d^{\frac{25}{2}}x^4\sqrt{\frac{cx}{d}}}{5c^7d^9x^6+15c^6d^{10}x^4+15}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**5*d**(19/2)*x**10*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**1
0*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*sqrt(c*
x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5
*c**4*d**12) + 30*c**2*d**(25/2)*x**4*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6
+ 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(27/2)*
x**2*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d*
*11*x**2 + 5*c**4*d**12) + 16*d**(29/2)*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**
6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12)) + b*(c**3*d**(
9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*
d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4
*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d + 1)/(3*c**5*
d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(c*x**2/d + 1
)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)*x^4/(c + d/x^2)^(3/2), x)
```

3.981 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2x\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -((3*b*c - 4*a*d)*x)/(3*c^2*Sqrt[c + d/x^2]) + (2*(3*b*c - 4*a*d)*Sqrt[c +
d/x^2]*x)/(3*c^3) + (a*x^3)/(3*c*Sqrt[c + d/x^2])
```

Rubi [A] time = 0.0301135, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 192, 191}

$$\frac{2x\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((3*b*c - 4*a*d)*x)/(3*c^2*Sqrt[c + d/x^2]) + (2*(3*b*c - 4*a*d)*Sqrt[c +
d/x^2]*x)/(3*c^3) + (a*x^3)/(3*c*Sqrt[c + d/x^2])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}+\frac{(3bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}+\frac{(2(3bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c^2}$$

$$= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [A] time = 0.0376584, size = 57, normalized size = 0.72

$$\frac{a\left(c^2x^4-4cdx^2-8d^2\right)+3bc\left(cx^2+2d\right)}{3c^3x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] (3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d/
x^2]*x)
```

Maple [A] time = 0.008, size = 66, normalized size = 0.8

$$\frac{\left(ax^4c^2-4\,acdx^2+3\,bc^2x^2-8\,ad^2+6\,bcd\right)\left(cx^2+d\right)}{3\,x^3c^3}\left(\frac{cx^2+d}{x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+d
)/x^2)^(3/2)/x^3/c^3
```

Maxima [A] time = 0.942641, size = 122, normalized size = 1.54

$$b\left(\frac{\sqrt{c+\frac{d}{x^2}}x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}c^2x}\right)+\frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\sqrt{c+\frac{d}{x^2}}dx}{c^3}-\frac{3d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2)
^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))
```

Fricas [A] time = 1.31819, size = 147, normalized size = 1.86

$$\frac{\left(ac^2x^5+\left(3\,bc^2-4\,acd\right)x^3+2\left(3\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(c^4x^2+c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((c
*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)
```

Sympy [B] time = 7.56112, size = 267, normalized size = 3.38

$$a\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**
*2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x
**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d +
 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(
c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b*(x**
2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)*x^2/(c + d/x^2)^(3/2), x)
```

3.982 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx$

Optimal. Leaf size=45

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

Rubi [A] time = 0.0299469, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 453, 191}

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{a+bx^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{ax}{c\sqrt{c+\frac{d}{x^2}}} + \frac{(-bc+2ad)\operatorname{Subst}\left(\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{ax}{c\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [A] time = 0.0227865, size = 33, normalized size = 0.73

$$\frac{acx^2+2ad-bc}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] (-(b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.005, size = 43, normalized size = 1.

$$\frac{\left(ax^2c+2\,ad-bc\right)\left(cx^2+d\right)}{x^3c^2}\left(\frac{cx^2+d}{x^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2),x)
```

```
[Out] (a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^2
```

Maxima [A] time = 0.943443, size = 72, normalized size = 1.6

$$a\left(\frac{\sqrt{c+\frac{d}{x^2}}x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}c^2x}\right)-\frac{b}{\sqrt{c+\frac{d}{x^2}}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)*
c*x)
```

Fricas [A] time = 1.28034, size = 93, normalized size = 2.07

$$\frac{\left(acx^3-(bc-2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)
```

Sympy [A] time = 6.42385, size = 65, normalized size = 1.44

$$a\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)-\frac{b}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)
)) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/(c + d/x^2)^(3/2), x)
```

3.983 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx$

Optimal. Leaf size=59

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x
)])/d^(3/2)
```

Rubi [A] time = 0.0340979, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {452, 335, 217, 206}

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x
)])/d^(3/2)
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2}\,dx &= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}+\frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0318553, size = 71, normalized size = 1.2

$$\frac{\sqrt{d}(bc-ad)-bc\sqrt{cx^2+d}\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{cd^{3/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]
)/(c*d^(3/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] time = 0.01, size = 79, normalized size = 1.3

$$-\frac{cx^2+d}{x^3c}\left(ad^{\frac{5}{2}}-d^{\frac{3}{2}}bc+\ln\left(2\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)\sqrt{cx^2+d}bcd\right)\left(\frac{cx^2+d}{x^2}\right)^{-\frac{3}{2}}d^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x)
```

```
[Out] -(c*x^2+d)*(a*d^(5/2)-d^(3/2)*b*c+ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*(c*x^
2+d)^(1/2)*b*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.08589, size = 424, normalized size = 7.19

$$\left[\frac{2\left(bcd-ad^2\right)x\sqrt{\frac{cx^2+d}{x^2}}+\left(bc^2x^2+bcd\right)\sqrt{d}\log\left(-\frac{cx^2-2\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)}{2\left(c^2d^2x^2+cd^3\right)},\frac{\left(bcd-ad^2\right)x\sqrt{\frac{cx^2+d}{x^2}}+\left(bc^2x^2+bcd\right)\sqrt{-d}\arctan\right.}{c^2d^2x^2+cd^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt(
d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*x^
2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*
sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^2*d^2*x^2
 + c*d^3)]
```

Sympy [B] time = 10.8989, size = 206, normalized size = 3.49

$$-\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + b\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7
/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**
(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2
*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*l
og(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^2), x)
```

3.984 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx$

Optimal. Leaf size=92

$$-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}+\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -b/(2*d*Sqrt[c + d/x^2]*x^3) - (3*b*c - 2*a*d)/(2*d^2*Sqrt[c + d/x^2]*x) +
((3*b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(5/2))
```

Rubi [A] time = 0.050077, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 288, 217, 206}

$$-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}+\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```


```
[Out] -b/(2*d*Sqrt[c + d/x^2]*x^3) - (3*b*c - 2*a*d)/(2*d^2*Sqrt[c + d/x^2]*x) +
((3*b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(5/2))
```


Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}+\frac{(-3bc+2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx}{2d}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}-\frac{(-3bc+2ad)\,\mathrm{Subst}\left(\int \frac{x^2}{(c+dx^2)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2d}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}x}+\frac{(3bc-2ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{2d^2}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}x}+\frac{(3bc-2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^2}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}x}+\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^{5/2}}$$

Mathematica [C] time = 0.0244457, size = 57, normalized size = 0.62

$$\frac{x^2(2ad-3bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{d}+1\right)-bd}{2d^2x^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] (-(b*d) + (-3*b*c + 2*a*d)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/
d])/(2*d^2*Sqrt[c + d/x^2]*x^3)
```

Maple [A] time = 0.01, size = 132, normalized size = 1.4

$$\frac{cx^2+d}{2\,x^5}\left(2\,d^{5/2}x^2a-3\,d^{3/2}x^2bc-2\,\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)\sqrt{cx^2+d}x^2ad^2+3\,\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)\sqrt{cx^2+d}x^2bcd-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x)
```

```
[Out] 1/2*(c*x^2+d)*(2*d^(5/2)*x^2*a-3*d^(3/2)*x^2*b*c-2*ln(2*(d^(1/2)*(c*x^2+d)^
(1/2)+d)/x)*(c*x^2+d)^(1/2)*x^2*a*d^2+3*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*(c*x^2+d)^(1/2)*x^2*b*c*d-d^(5/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^5/d^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.08669, size = 551, normalized size = 5.99

$$\left[-\frac{\left(\left(3\,bc^2-2\,acd\right)x^3+\left(3\,bcd-2\,ad^2\right)x\right)\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(bd^2+\left(3\,bcd-2\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\left(cd^3x^3+d^4x\right)},-\frac{\left(\left(3\,bc^2-2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x^
2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d - 2
*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2 -
2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^
2 + d)/x^2)/(c*x^2 + d)) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(c*d^3*x^3 + d^4*x)]
```

Sympy [B] time = 19.5526, size = 262, normalized size = 2.85

$$a\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)+b\left(-\frac{}{2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x*
*2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sq
rt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c
*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(
7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) +
3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 +
d/(c*x**2))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^4), x)
```

3.985 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} \, dx$

Optimal. Leaf size=123

$$\frac{3\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -b/(4*d*Sqrt[c + d/x^2]*x^5) - (5*b*c - 4*a*d)/(4*d^2*Sqrt[c + d/x^2]*x^3)
+ (3*(5*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^3*x) - (3*c*(5*b*c - 4*a*d)*ArcT
anh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(7/2))
```

Rubi [A] time = 0.0689806, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 288, 321, 217, 206}

$$\frac{3\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] -b/(4*d*Sqrt[c + d/x^2]*x^5) - (5*b*c - 4*a*d)/(4*d^2*Sqrt[c + d/x^2]*x^3)
+ (3*(5*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^3*x) - (3*c*(5*b*c - 4*a*d)*ArcT
anh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(7/2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6}\,dx = -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} + \frac{(-5bc+4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6}\,dx}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{(-5bc+4ad)\,\mathrm{Subst}\left(\int \frac{x^4}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{(3(5bc-4ad))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{4d^2}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, -\ldots\right)}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{7/2}}$$

Mathematica [C] time = 0.0272797, size = 60, normalized size = 0.49

$$\frac{cx^4(5bc-4ad)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{d}+1\right)-bd^2}{4d^3x^5\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] (-(b*d^2) + c*(5*b*c - 4*a*d)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x^
2)/d])/(4*d^3*Sqrt[c + d/x^2]*x^5)
```

Maple [A] time = 0.011, size = 157, normalized size = 1.3

$$-\frac{cx^2+d}{8\,x^7}\left(12\,d^{5/2}x^4ac-15\,d^{3/2}x^4bc^2-12\,\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)\sqrt{cx^2+d}x^4acd^2+15\,\ln\left(2\,\frac{\sqrt{d}\sqrt{cx^2+d}+d}{x}\right)\sqrt{cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x)
```

```
[Out] -1/8*(c*x^2+d)*(12*d^(5/2)*x^4*a*c-15*d^(3/2)*x^4*b*c^2-12*ln(2*(d^(1/2)*(c
*x^2+d)^(1/2)+d)/x)*(c*x^2+d)^(1/2)*x^4*a*c*d^2+15*ln(2*(d^(1/2)*(c*x^2+d)^
(1/2)+d)/x)*(c*x^2+d)^(1/2)*x^4*b*c^2*d+4*d^(7/2)*x^2*a-5*d^(5/2)*x^2*b*c+2
*d^(7/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^7/d^(9/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.13007, size = 680, normalized size = 5.53

$$\left[-\frac{3\left(\left(5\,bc^3-4\,ac^2d\right)x^5+\left(5\,bc^2d-4\,acd^2\right)x^3\right)\sqrt{d}\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(5\,bc^2d-4\,acd^2\right)x^4-2\,bd^3+\left(5\,bcd^2\right.\right.}{16\left(cd^4x^5+d^5x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d)
*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*c^
```

```
2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d
)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2
*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^
2 + d)) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*
x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]
```

Sympy [A] time = 34.5534, size = 180, normalized size = 1.46

$$a\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right)+b\left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}}+\frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{15c^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{7}{2}}}-\frac{}{4\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**6,x)
```

```
[Out] a*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)*
x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2)))) + b*(15*c**(3
/2)/(8*d**3*x*sqrt(1 + d/(c*x**2))) + 5*sqrt(c)/(8*d**2*x**3*sqrt(1 + d/(c*
x**2))) - 15*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(7/2)) - 1/(4*sqrt(c)*d*
x**5*sqrt(1 + d/(c*x**2))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^6), x)
```

3.986 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx$

Optimal. Leaf size=105

$$\frac{(ex)^{m+1}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2}(-m-1);-p,-q;\frac{1-m}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e(m+1)}$$

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(1 + m)*AppellF1[(-1 - m)/2, -p, -q, (1
- m)/2, -(b/(a*x^2)), -(d/(c*x^2))])/(e*(1 + m)*(1 + b/(a*x^2))^p*(1 + d/(c
*x^2))^q)
```

Rubi [A] time = 0.11591, antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {497, 511, 510}

$$\frac{x(ex)^m\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2}(-m-1);-p,-q;\frac{1-m}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(-1 - m)/2, -p, -q, (1 - m)
/2, -(b/(a*x^2)), -(d/(c*x^2))])/((1 + m)*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))
^q)
```

Rule 497

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[((a + b/x^n)^p*(c
+ d/x^n)^q)/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}, x
] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q} x(ex)^m F_1\left(\frac{1}{2}(-1-m);-p,-q;\frac{1-m}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{1+m}$$

Mathematica [A] time = 0.128659, size = 115, normalized size = 1.1

$$\frac{x(ex)^m\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2}(m-2p-2q+1);-p,-q;\frac{1}{2}(m-2p-2q+3);-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{m-2p-2q+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(1 + m - 2*p - 2*q)/2, -p,
-q, (3 + m - 2*p - 2*q)/2, -((a*x^2)/b), -((c*x^2)/d)])/((1 + m - 2*p - 2*q
)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \left(\frac{ax^2 + b}{x^2}\right)^p \left(\frac{cx^2 + d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.987 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$

Optimal. Leaf size=84

$$\frac{1}{5}x^5\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{2};-p,-q;-\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[-5/2, -p, -q, -3/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(5*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.0885412, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 511, 510}

$$\frac{1}{5}x^5\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{2};-p,-q;-\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[-5/2, -p, -q, -3/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(5*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^q}{x^6}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^6}\,dx, x,\right.\right.$$

$$= \frac{1}{5}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^5 F_1\left(-\frac{5}{2};-p,-q;-\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Mathematica [A] time = 0.0963391, size = 106, normalized size = 1.26

$$-\frac{x^5 \left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{5}{2};-p,-q;-p-q+\frac{7}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2p+2q-5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[5/2 - p - q, -p, -q, 7/2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^4 \left(\frac{ax^2 + b}{x^2}\right)^p \left(\frac{cx^2 + d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="fricas")
```

```
[Out] integral(x^4*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

3.988 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$

Optimal. Leaf size=100

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,3;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

```
[Out] (b^2*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*(
a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^3*(1 + p)*((b*(c + d/x^2))/(
b*c - a*d))^q)
```

Rubi [A] time = 0.0830878, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {446, 137, 136}

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,3;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```


```
[Out] (b^2*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*(
a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^3*(1 + p)*((b*(c + d/x^2))/(
b*c - a*d))^q)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,3;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0879435, size = 100, normalized size = 1.

$$-\frac{x^4\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+2;-p,-q;-p-q+3;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]

[Out] -((a + b/x^2)^p*(c + d/x^2)^q*x^4*AppellF1[2 - p - q, -p, -q, 3 - p - q, -(
(a*x^2)/b), -((c*x^2)/d)])/(2*(-2 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d
)^q)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^3 \left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

3.989 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[-3/2, -p, -q, -1/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.0906104, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 511, 510}

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[-3/2, -p, -q, -1/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^q}{x^4}\,dx, x, \frac{1}{x}\right)\\
&= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q}{x^4}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^4}\,dx, x,\right.\right.\\
&= \frac{1}{3}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^3 F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0916593, size = 106, normalized size = 1.26

$$-\frac{x^3 \left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{3}{2};-p,-q;-p-q+\frac{5}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2p+2q-3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[3/2 - p - q, -p, -q, 5/2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2 \left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

3.990 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$

Optimal. Leaf size=98

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

```
[Out] -(b*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
 + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^2*(1 + p)*((b*(c + d/x^2))/(b
*c - a*d))^q)
```

Rubi [A] time = 0.068031, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {446, 137, 136}

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```


```
[Out] -(b*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
 + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^2*(1 + p)*((b*(c + d/x^2))/(b
*c - a*d))^q)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.08463, size = 100, normalized size = 1.02

$$-\frac{x^2\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+1;-p,-q;-p-q+2;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

```
[Out] -((a + b/x^2)^p*(c + d/x^2)^q*x^2*AppellF1[1 - p - q, -p, -q, 2 - p - q, -(
(a*x^2)/b), -((c*x^2)/d)])/(2*(-1 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d
)^q)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="fricas")
```

```
[Out] integral(x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

3.991 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

Optimal. Leaf size=79

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.0806013, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 511, 510}

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^q}{x^2} dx, x, \frac{1}{x}\right) \\
&= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q}{x^2} dx, x, \frac{1}{x}\right)\right) \\
&= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^2} dx, x, \frac{1}{x}\right)\right) \\
&= \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x F_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0833821, size = 104, normalized size = 1.32

$$-\frac{x\left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{1}{2}; -p, -q; -p-q+\frac{3}{2}; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{2p+2q-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.992 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$

Optimal. Leaf size=97

$$\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,1;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

```
[Out] ((a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a*(1 + p)*((b*(c + d/x^2))/(b*c -
a*d))^q)
```

Rubi [A] time = 0.0704423, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {446, 137, 136}

$$\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,1;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

```
[Out] ((a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a*(1 + p)*((b*(c + d/x^2))/(b*c -
a*d))^q)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\operatorname{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,1;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0851773, size = 95, normalized size = 0.98

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q;-p,-q;-p-q+1;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

```
[Out] -((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -((a*x^2)
/b), -((c*x^2)/d)])/(2*(p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

3.993 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/2, -p, -q, 3/2, -(b/(a*x^2)), -(d
/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x))
```

Rubi [A] time = 0.0624714, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 430, 429}

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/2, -p, -q, 3/2, -(b/(a*x^2)), -(d
/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x))
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx &= -\text{Subst}\left(\int \left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)\\ &= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)\right)\\ &= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.108723, size = 106, normalized size = 1.29

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q-\frac{1}{2};-p,-q;-p-q+\frac{1}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{x(2p+2q+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-1/2 - p - q, -p, -q, 1/2 - p - q,
-((a*x^2)/b), -((c*x^2)/d)])/((1 + 2*p + 2*q)*x*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

3.994 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$

Optimal. Leaf size=85

$$-\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

```
[Out] -((a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((
d*(a + b/x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d/x^2))/(b*c - a*d))^q)
```

Rubi [A] time = 0.0618262, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 70, 69}

$$-\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

```
[Out] -((a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((
d*(a + b/x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d/x^2))/(b*c - a*d))^q)
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^p(c+dx)^q\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\operatorname{Subst}\left(\int (a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0433858, size = 110, normalized size = 1.29

$$-\frac{\left(cx^2+d\right)\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(\frac{cx^2}{d}+1\right)^p\left(c+\frac{d}{x^2}\right)^q{}_2F_1\left(-p,-p-q-1;-p-q;\frac{(bc-ad)x^2}{b\left(cx^2+d\right)}\right)}{2dx^2(p+q+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

```
[Out] -((a + b/x^2)^p*(c + d/x^2)^q*(d + c*x^2)*(1 + (c*x^2)/d)^p*Hypergeometric2
F1[-p, -1 - p - q, -p - q, ((b*c - a*d)*x^2)/(b*(d + c*x^2))])/(2*d*(1 + p
+ q)*x^2*(1 + (a*x^2)/b)^p)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}} \left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{a x^{2}+b}{x^{2}}\right)^{p} \left(\frac{c x^{2}+d}{x^{2}}\right)^{q}}{x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

3.995 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

```
[Out] -((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/2, -p, -q, 5/2, -(b/(a*x^2)), -(d/
(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x^3)
```

Rubi [A] time = 0.0922017, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 511, 510}

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

```
[Out] -((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/2, -p, -q, 5/2, -(b/(a*x^2)), -(d/
(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x^3)
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx &= -\text{Subst}\left(\int x^2\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)\\ &= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)\right)\\ &= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,dx, x,\right.\right.\\ &= -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.188082, size = 106, normalized size = 1.26

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q-\frac{3}{2};-p,-q;-p-q-\frac{1}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{x^3(2p+2q+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-3/2 - p - q, -p, -q, -1/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((3 + 2*p + 2*q)*x^3*(1 + (a*x^2)/b)^p*(1 + (
c*x^2)/d)^q))
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{x^4}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

3.996 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{7/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(7/2)*AppellF1[-7/4, -p, -q, -3/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(7*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.12453, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2(ex)^{7/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(7/2)*AppellF1[-7/4, -p, -q, -3/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(7*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{7/2}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

Mathematica [A] time = 0.111831, size = 111, normalized size = 1.22

$$-\frac{2x(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{7}{4};-p,-q;-p-q+\frac{11}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{4p+4q-7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(5/2)*AppellF1[7/4 - p - q, -p, -q,
 11/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-7 + 4*p + 4*q)*(1 + (a*x^2)/
b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q} (ex)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}} \left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ex}e^{2}x^{2}\left(\frac{ax^{2}+b}{x^{2}}\right)^{p}\left(\frac{cx^{2}+d}{x^{2}}\right)^{q}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*e^2*x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{5}{2}}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.997 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2)*AppellF1[-5/4, -p, -q, -1/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(5*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.121727, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2)*AppellF1[-5/4, -p, -q, -1/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(5*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q}{x^6}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q}{x^6}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^6}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{5/2}F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

Mathematica [A] time = 0.112769, size = 111, normalized size = 1.22

$$-\frac{2x(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{5}{4};-p,-q;-p-q+\frac{9}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{4p+4q-5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(3/2)*AppellF1[5/4 - p - q, -p, -q,
 9/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 4*p + 4*q)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex}ex\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*e*x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.998 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2)*AppellF1[-3/4, -p, -q, 1/4, -(b/
(a*x^2)), -(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.120351, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2)*AppellF1[-3/4, -p, -q, 1/4, -(b/
(a*x^2)), -(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\sqrt{ex}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q}{x^4}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}\\
&= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q}{x^4}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}\\
&= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^4}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}\\
&= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{3/2}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}
\end{aligned}
$$

Mathematica [A] time = 0.103268, size = 111, normalized size = 1.22

$$
-\frac{2x\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{3}{4};-p,-q;-p-q+\frac{7}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{4p+4q-3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*Sqrt[e*x]*AppellF1[3/4 - p - q, -p, -q, 7
/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 4*p + 4*q)*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.999 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x]*AppellF1[-1/4, -p, -q, 3/4, -(b/(a
*x^2)), -(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.12595, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x]*AppellF1[-1/4, -p, -q, 3/4, -(b/(a
*x^2)), -(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q}{x^2}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}\\ &= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q}{x^2}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}\\ &= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^2}\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}\\ &= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\sqrt{ex}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}\end{aligned}$$

Mathematica [A] time = 0.109108, size = 111, normalized size = 1.25

$$-\frac{2x\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{1}{4};-p,-q;-p-q+\frac{5}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{\sqrt{ex}(4p+4q-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/4 - p - q, -p, -q, 5/4 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 4*p + 4*q)*Sqrt[e*x]*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \frac{1}{\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q}{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{\sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

3.1000 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}} dx$

Optimal. Leaf size=89

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/4, -p, -q, 5/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*Sqrt[e*x])
```

Rubi [A] time = 0.089408, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 430, 429}

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```



```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/4, -p, -q, 5/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*Sqrt[e*x])
```


Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx &= -\frac{2\,\text{Subst}\left(\int \left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}\\ &= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}\\ &= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q\,dx, x, \frac{1}{\sqrt{ex}}\right.}{e}\\ &= -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}\end{aligned}$$

Mathematica [A] time = 0.116951, size = 111, normalized size = 1.25

$$-\frac{2x\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q-\frac{1}{4};-p,-q;-p-q+\frac{3}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(ex)^{3/2}(4p+4q+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/4 - p - q, -p, -q, 3/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((1 + 4*p + 4*q)*(e*x)^(3/2)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q(ex)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

3.1001 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}} dx$

Optimal. Leaf size=91

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/4, -p, -q, 7/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*(e*x)^(3/2))
```

Rubi [A] time = 0.120471, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/4, -p, -q, 7/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*(e*x)^(3/2))
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\,\text{Subst}\left(\int x^2\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

Mathematica [A] time = 0.120905, size = 111, normalized size = 1.22

$$-\frac{2x\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q-\frac{3}{4};-p,-q;-p-q+\frac{1}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(ex)^{5/2}(4p+4q+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-3/4 - p - q, -p, -q, 1/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 4*p + 4*q)*(e*x)^(5/2)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q}{e^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

3.1002 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx$

Optimal. Leaf size=135

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{5}{64}\cosh^{-1}$$

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/64 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/96 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/24 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 - (5*ArcCosh[Sq
rt[x]])/64
```

Rubi [A] time = 0.0607087, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {280, 323, 330, 52}

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{5}{64}\cosh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/64 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/96 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/24 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 - (5*ArcCosh[Sq
rt[x]])/64
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
```

```
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2} - \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2} - \frac{5}{48}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2} - $$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + $$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + $$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + $$

Mathematica [A] time = 0.057695, size = 111, normalized size = 0.82

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(48x^{7/2}-48x^3-8x^{5/2}+8x^2-10x^{3/2}+10x-15\sqrt{x}+15\right)+30\sqrt{1-\sqrt{x}}\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{192\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 - 15*Sqrt[x] + 10*x - 10*x^(3/2) + 8*x^2 - 8
*x^(5/2) - 48*x^3 + 48*x^(7/2)) + 30*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt
[x]]/Sqrt[2]])/(192*Sqrt[-1 + Sqrt[x]])
```

Maple [A] time = 0.014, size = 75, normalized size = 0.6

$$-\frac{1}{192}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-48\,x^{7/2}\sqrt{-1+x}+8\,x^{5/2}\sqrt{-1+x}+10\,x^{3/2}\sqrt{-1+x}+15\,\sqrt{x}\sqrt{-1+x}+15\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x)
```

```
[Out] -1/192*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-48*x^(7/2)*(-1+x)^(1/2)+8*x^(
5/2)*(-1+x)^(1/2)+10*x^(3/2)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(
1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Maxima [A] time = 0.960385, size = 77, normalized size = 0.57

$$\frac{1}{4}\,(x-1)^{\frac{3}{2}}x^{\frac{5}{2}}+\frac{5}{24}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{5}{32}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{5}{64}\,\sqrt{x-1}\sqrt{x}-\frac{5}{64}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2)
*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 0.925295, size = 201, normalized size = 1.49

$$\frac{1}{192}\left(48\,x^3-8\,x^2-10\,x-15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{5}{128}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1003 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{1}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{1}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 - (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))
/3 - ArcCosh[Sqrt[x]]/8
```

Rubi [A] time = 0.0485153, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {280, 323, 330, 52}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{1}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{1}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 - (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))
/3 - ArcCosh[Sqrt[x]]/8
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
```

```
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{8}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{16}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{8}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{8}$$

Mathematica [A] time = 0.0371856, size = 99, normalized size = 0.95

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(8x^{5/2}-8x^2-2x^{3/2}+2x-3\sqrt{x}+3\right)+6\sqrt{1-\sqrt{x}}\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{24\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 - 3*Sqrt[x] + 2*x - 2*x^(3/2) - 8*x^2 + 8*x^(
5/2)) + 6*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(24*Sqrt[-1
+ Sqrt[x]])
```

Maple [A] time = 0.007, size = 65, normalized size = 0.6

$$-\frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-8\,x^{5/2}\sqrt{-1+x}+2\,x^{3/2}\sqrt{-1+x}+3\,\sqrt{x}\sqrt{-1+x}+3\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)\frac{1}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x)
```

```
[Out] -1/24*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-8*x^(5/2)*(-1+x)^(1/2)+2*x^(3/
2)*(-1+x)^(1/2)+3*x^(1/2)*(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(
1/2)
```

Maxima [A] time = 0.932527, size = 63, normalized size = 0.61

$$\frac{1}{3}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{1}{4}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{8}\,\sqrt{x-1}\sqrt{x}-\frac{1}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(x - 1)^(3/2)*x^(3/2) + 1/4*(x - 1)^(3/2)*sqrt(x) + 1/8*sqrt(x - 1)*sqr
t(x) - 1/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 1.05156, size = 184, normalized size = 1.77

$$\frac{1}{24}\left(8\,x^2-2\,x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{16}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*l
og(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1004 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/2 - ArcCosh[Sqrt[x]]/4
```

Rubi [A] time = 0.0324258, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {280, 323, 330, 52}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/2 - ArcCosh[Sqrt[x]]/4
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
```

```
0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx &= \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx \\
&= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{8}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx \\
&= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right. \\
&= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0298912, size = 87, normalized size = 1.19

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(2x^{3/2}-2x-\sqrt{x}+1\right)+2\sqrt{1-\sqrt{x}}\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{4\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 - Sqrt[x] - 2*x + 2*x^(3/2)) + 2*Sqrt[1 - Sqr
t[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(4*Sqrt[-1 + Sqrt[x]])
```

Maple [A] time = 0.007, size = 52, normalized size = 0.7

$$-\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-2\,x^{3/2}\sqrt{-1+x}+\sqrt{x}\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)\frac{1}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x)
```

```
[Out] -1/4*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-2*x^(3/2)*(-1+x)^(1/2)+x^(1/2)*
(-1+x)^(1/2)+ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Maxima [A] time = 0.920499, size = 50, normalized size = 0.68

$$\frac{1}{2}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{4}\,\sqrt{x-1}\sqrt{x}-\frac{1}{4}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(x - 1)^(3/2)*sqrt(x) + 1/4*sqrt(x - 1)*sqrt(x) - 1/4*log(2*sqrt(x - 1)
 + 2*sqrt(x))
```

Fricas [A] time = 0.982566, size = 170, normalized size = 2.33

$$\frac{1}{4}\,(2\,x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{8}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1005 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

Rubi [A] time = 0.0208203, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {280, 330, 52}

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\ &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0210303, size = 72, normalized size = 1.95

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(\sqrt{x}-1\right)+2\sqrt{1-\sqrt{x}}\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] ((-1 + Sqrt[x])*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt
[1 - Sqrt[x]]/Sqrt[2]])/Sqrt[-1 + Sqrt[x]]
```

Maple [B] time = 0.007, size = 72, normalized size = 2.

$$\sqrt{-1+\sqrt{x}}\left(1+\sqrt{x}\right)^{\frac{3}{2}}-\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}-\sqrt{\left(1+\sqrt{x}\right)\left(-1+\sqrt{x}\right)}\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\frac{1}{\sqrt{-1+\sqrt{x}}}\frac{1}{\sqrt{1+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x)
```

```
[Out] (-1+x^(1/2))^(1/2)*(1+x^(1/2))^(3/2)-(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-(
(1+x^(1/2))*(-1+x^(1/2)))^(1/2)/(1+x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)*ln(x^(
1/2)+(-1+x)^(1/2))
```

Maxima [A] time = 0.937178, size = 35, normalized size = 0.95

$$\sqrt{x-1}\sqrt{x} - \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 0.971627, size = 151, normalized size = 4.08

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/sqrt(x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1006 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\sqrt{\sqrt{x}-1}\sqrt{x}\sqrt{\sqrt{x}+1}+2\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcCosh[Sqrt[x]]
```

Rubi [A] time = 0.0307674, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {327, 280, 330, 52}

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\sqrt{\sqrt{x}-1}\sqrt{x}\sqrt{\sqrt{x}+1}+2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcCosh[Sqrt[x]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2
*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(
a1*a2*c^(2*n)*(m + 1)), Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p
, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IG
tQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
```

```
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \right. \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0385511, size = 74, normalized size = 1.1

$$
\frac{2\left(-\frac{\sqrt{\sqrt{x}+1}\left(\sqrt{x}-1\right)}{\sqrt{x}} - 2\sqrt{1-\sqrt{x}}\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)\right)}{\sqrt{\sqrt{x}-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-(((-1 + Sqrt[x])*Sqrt[1 + Sqrt[x]])/Sqrt[x]) - 2*Sqrt[1 - Sqrt[x]]*Arc
Sin[Sqrt[1 - Sqrt[x]]/Sqrt[2]]))/Sqrt[-1 + Sqrt[x]]
```

Maple [A] time = 0.007, size = 47, normalized size = 0.7

$$2\,\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\sqrt{x}-\sqrt{-1+x}\right)}{\sqrt{x}\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x)
```

```
[Out] 2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(ln(x^(1/2)+(-1+x)^(1/2))*x^(1/2)-(-
1+x)^(1/2))/x^(1/2)/(-1+x)^(1/2)
```

Maxima [A] time = 1.39772, size = 36, normalized size = 0.54

$$-\frac{2\,\sqrt{x-1}}{\sqrt{x}}+2\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxima
")
```

```
[Out] -2*sqrt(x - 1)/sqrt(x) + 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 1.01254, size = 166, normalized size = 2.48

$$-\frac{x\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)+2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\,x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="fricas
")
```

```
[Out] -(x*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1) + 2*sqrt(x
)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

Giac [A] time = 3.15214, size = 65, normalized size = 0.97

$$-\frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4}-\log\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] -16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4) - log((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4)
```

3.1007 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

Optimal. Leaf size=31

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Rubi [A] time = 0.0093252, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {265}

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

Mathematica [A] time = 0.0114971, size = 31, normalized size = 1.

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Maple [A] time = 0.006, size = 23, normalized size = 0.7

$$\frac{-2+2x}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x)
```

```
[Out] 2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-1+x)/x^(3/2)
```

Maxima [A] time = 1.42084, size = 14, normalized size = 0.45

$$\frac{2(x-1)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxima
")
```

```
[Out] 2/3*(x - 1)^(3/2)/x^(3/2)
```

Fricas [A] time = 0.994732, size = 93, normalized size = 3.

$$\frac{2\left((x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x^2\right)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="fricas
")
```

```
[Out] 2/3*((x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x^2)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

Giac [B] time = 1.07377, size = 65, normalized size = 2.1

$$\frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

3.1008 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

Rubi [A] time = 0.0203429, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
```

]

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}} + \frac{2}{5}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$$
$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}$$

Mathematica [A] time = 0.0139119, size = 36, normalized size = 0.57

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}(2x+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(3 + 2*x))/(15*x^(5/2))
```

Maple [A] time = 0.006, size = 28, normalized size = 0.4

$$\frac{(-2+2x)(2x+3)}{15}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x)
```

```
[Out] 2/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-1+x)*(2*x+3)/x^(5/2)
```

Maxima [A] time = 1.43155, size = 28, normalized size = 0.44

$$\frac{4(x-1)^{\frac{3}{2}}}{15x^{\frac{3}{2}}} + \frac{2(x-1)^{\frac{3}{2}}}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxima
")
```

```
[Out] 4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)
```

Fricas [A] time = 0.994124, size = 108, normalized size = 1.71

$$\frac{2\left(2\,x^{3}+\left(2\,x^{2}+x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/
x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11736, size = 122, normalized size = 1.94

$$\frac{128\left(15\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}-20\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+80\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+64\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) +
1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5
```

3.1009 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx$

Optimal. Leaf size=94

$$\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2)) + (8*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(35*x^(5/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(3/2))
```

Rubi [A] time = 0.0312135, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2)) + (8*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(35*x^(5/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(3/2))
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
```

```
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{8}{35}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0162812, size = 41, normalized size = 0.44

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}\left(8x^2+12x+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7/
2))
```

Maple [A] time = 0.007, size = 33, normalized size = 0.4

$$\frac{(-2+2x)\left(8x^2+12x+15\right)}{105}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x)
```

```
[Out] 2/105*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-1+x)*(8*x^2+12*x+15)/x^(7/2)
```

Maxima [A] time = 1.40464, size = 42, normalized size = 0.45

$$\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{3}{2}}}+\frac{8\,(x-1)^{\frac{3}{2}}}{35\,x^{\frac{5}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="maxima
")
```

```
[Out] 16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3/
2)/x^(7/2)
```

Fricas [A] time = 0.971025, size = 124, normalized size = 1.32

$$\frac{2\left(8\,x^4+\left(8\,x^3+4\,x^2+3\,x-15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{105\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sq
rt(x) - 1))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12485, size = 150, normalized size = 1.6

$$\frac{4096\left(35\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}-70\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+168\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+224\left(\sqrt{\sqrt{x}+1}-\sqrt{\cdots}\right.\right.}{105\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x)
+ 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^
8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^7
```

3.1010 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$

Optimal. Leaf size=125

$$\frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}}+\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}}+\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(9*x^(9/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(21*x^(7/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(5/2)) + (32*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)
)/(315*x^(3/2))
```

Rubi [A] time = 0.041845, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}}+\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}}+\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(9*x^(9/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(21*x^(7/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(5/2)) + (32*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)
)/(315*x^(3/2))
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
```

```
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{2}{3}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{8}{21}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}}+\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}}+\end{aligned}$$

Mathematica [A] time = 0.018402, size = 46, normalized size = 0.37

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}\left(16x^3+24x^2+30x+35\right)}{315x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))/
(315*x^(9/2))
```

Maple [A] time = 0.013, size = 38, normalized size = 0.3

$$\frac{(-2+2x)\left(16x^3+24x^2+30x+35\right)}{315}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x)
```

```
[Out] 2/315*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-1+x)*(16*x^3+24*x^2+30*x+35)/x
^(9/2)
```

Maxima [A] time = 1.4097, size = 55, normalized size = 0.44

$$\frac{32\,(x-1)^{\frac{3}{2}}}{315\,x^{\frac{3}{2}}}+\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{5}{2}}}+\frac{4\,(x-1)^{\frac{3}{2}}}{21\,x^{\frac{7}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] 32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)^
(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)
```

Fricas [A] time = 0.918929, size = 138, normalized size = 1.1

$$\frac{2\left(16\,x^{5}+\left(16\,x^{4}+8\,x^{3}+6\,x^{2}+5\,x-35\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{315\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="frica
s")
```

```
[Out] 2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1))/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15701, size = 178, normalized size = 1.42

$$\frac{16384\left(315\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{20}-756\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}+1344\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+2304\left(\sqrt{\sqrt{x}+1}\right.\right.}{315\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac"
)
```

```
[Out] 16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt(
x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqrt(x
) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^4 + 4)^9
```

3.1011 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}+\frac{5}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{5}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 + (5*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/
2))/3 + (5*ArcCosh[Sqrt[x]])/8
```

Rubi [A] time = 0.0452857, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}+\frac{5}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{5}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 + (5*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/
2))/3 + (5*ArcCosh[Sqrt[x]])/8
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
```

```
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5}{16}\int$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5}{8}\,\mathrm{Su}$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5}{8}\cos$$

Mathematica [A] time = 0.0349079, size = 67, normalized size = 0.64

$$\frac{1}{24}\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\left(8x^2+10x+15\right)+30\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2) + 30*ArcT
anh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/24
```

Maple [A] time = 0.013, size = 65, normalized size = 0.6

$$\frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(8\,x^{5/2}\sqrt{-1+x}+10\,x^{3/2}\sqrt{-1+x}+15\,\sqrt{x}\sqrt{-1+x}+15\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)\frac{1}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

```
[Out] 1/24*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*x^(5/2)*(-1+x)^(1/2)+10*x^(3/2
)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^
(1/2)
```

Maxima [A] time = 0.936321, size = 63, normalized size = 0.61

$$\frac{1}{3}\,\sqrt{x-1}x^{\frac{5}{2}}+\frac{5}{12}\,\sqrt{x-1}x^{\frac{3}{2}}+\frac{5}{8}\,\sqrt{x-1}\sqrt{x}+\frac{5}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(x
) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 0.947352, size = 186, normalized size = 1.79

$$\frac{1}{24}\left(8\,x^{2}+10\,x+15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{5}{16}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/16
*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1012 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{3}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*x^(3/2))/2 + (3*ArcCosh[Sqrt[x]])/4
```

Rubi [A] time = 0.0324438, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}+\frac{3}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*x^(3/2))/2 + (3*ArcCosh[Sqrt[x]])/4
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
```

```
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx &= \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\ &= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{3}{8}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\ &= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{3}{4}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x\right.\\ &= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0254039, size = 62, normalized size = 0.85

$$\frac{1}{4}\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}(2x+3)+6\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x) + 6*ArcTanh[Sqrt[(-
1 + Sqrt[x])/(1 + Sqrt[x])]])/4
```

Maple [A] time = 0.01, size = 55, normalized size = 0.8

$$\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(2\,x^{3/2}\sqrt{-1+x}+3\,\sqrt{x}\sqrt{-1+x}+3\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)\frac{1}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

```
[Out] 1/4*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x^(3/2)*(-1+x)^(1/2)+3*x^(1/2)*
(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Maxima [A] time = 0.924397, size = 50, normalized size = 0.68

$$\frac{1}{2}\sqrt{x-1}x^{\frac{3}{2}}+\frac{3}{4}\sqrt{x-1}\sqrt{x}+\frac{3}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1) +
 2*sqrt(x))
```

Fricas [A] time = 0.950867, size = 170, normalized size = 2.33

$$\frac{1}{4}(2x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{3}{8}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1013 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=35

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

Rubi [A] time = 0.0206963, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx$$

$$= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{x}\right)$$

$$= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.010379, size = 55, normalized size = 1.57

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} + 2\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcTanh[Sqrt[-1 + Sqrt[x]]
/Sqrt[1 + Sqrt[x]]]
```

Maple [A] time = 0.01, size = 41, normalized size = 1.2

$$\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(\sqrt{x}\sqrt{-1+x} + \ln\left(\sqrt{x} + \sqrt{-1+x}\right)\right)\frac{1}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

```
[Out] (-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x^(1/2)*(-1+x)^(1/2)+ln(x^(1/2)+(-1+x
)^(1/2)))/(-1+x)^(1/2)
```

Maxima [A] time = 0.923579, size = 32, normalized size = 0.91

$$\sqrt{x-1}\sqrt{x}+\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A] time = 0.966081, size = 151, normalized size = 4.31

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [C] time = 6.58692, size = 83, normalized size = 2.37

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{x}\right)}{2\pi^{\frac{3}{2}}}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{2i\pi}}{x}\right)}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0),
()), 1/x)/(2*pi**(3/2)) - I*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()),
((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*pi)/x)/(2*pi**(3/2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.1014 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcCosh[Sqrt[x]]
```

Rubi [A] time = 0.0106923, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {330, 52}

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 2*ArcCosh[Sqrt[x]]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{x}\right)$$
$$= 2\cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [B] time = 0.0061587, size = 26, normalized size = 3.25

$$4\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 4*ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]]
```

Maple [B] time = 0.002, size = 40, normalized size = 5.

$$2\,\frac{\sqrt{\left(1+\sqrt{x}\right)\left(-1+\sqrt{x}\right)}\ln\left(\sqrt{x}+\sqrt{-1+x}\right)}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

```
[Out] 2*((1+x^(1/2))*(-1+x^(1/2)))^(1/2)/(1+x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)*ln(
x^(1/2)+(-1+x)^(1/2))
```

Maxima [B] time = 0.917757, size = 22, normalized size = 2.75

$$2\,\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [B] time = 0.969394, size = 85, normalized size = 10.62

$$-\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] -log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError
```

3.1015 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Rubi [A] time = 0.0098801, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {265}

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

Mathematica [A] time = 0.0091006, size = 29, normalized size = 1.

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Maple [A] time = 0.01, size = 20, normalized size = 0.7

$$2\,\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

```
[Out] 2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Maxima [A] time = 1.42919, size = 14, normalized size = 0.48

$$\frac{2\,\sqrt{x-1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*sqrt(x - 1)/sqrt(x)
```

Fricas [A] time = 1.01802, size = 74, normalized size = 2.55

$$\frac{2\left(\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [A] time = 1.094, size = 34, normalized size = 1.17

$$\frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)
```

3.1016 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

Rubi [A] time = 0.0201345, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
```

]

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{2}{3}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx$$
$$= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}$$

Mathematica [A] time = 0.011744, size = 36, normalized size = 0.57

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}(2x+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(1 + 2*x))/(3*x^(3/2))
```

Maple [A] time = 0.01, size = 25, normalized size = 0.4

$$\frac{4x+2}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

```
[Out] 2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x+1)/x^(3/2)
```

Maxima [A] time = 1.40682, size = 28, normalized size = 0.44

$$\frac{4\sqrt{x-1}}{3\sqrt{x}}+\frac{2\sqrt{x-1}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 4/3*sqrt(x - 1)/sqrt(x) + 2/3*sqrt(x - 1)/x^(3/2)
```

Fricas [A] time = 1.00233, size = 99, normalized size = 1.57

$$\frac{2\left((2\,x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\,x^2\right)}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Giac [A] time = 1.06921, size = 65, normalized size = 1.03

$$\frac{128\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

3.1017 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{8\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(5*x^(5/2)) + (8*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(15*x^(3/2)) + (16*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
])/(15*Sqrt[x])
```

Rubi [A] time = 0.0333635, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{8\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(5*x^(5/2)) + (8*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(15*x^(3/2)) + (16*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
])/(15*Sqrt[x])
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
```

```
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx &= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{4}{5}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx\\
&= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{8}{15}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx\\
&= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{16\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.014021, size = 41, normalized size = 0.44

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(8x^2+4x+3\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(3 + 4*x + 8*x^2))/(15*x^(5/2))
```

Maple [A] time = 0.01, size = 30, normalized size = 0.3

$$\frac{16\,x^2+8\,x+6}{15}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

```
[Out] 2/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*x^2+4*x+3)/x^(5/2)
```

Maxima [A] time = 1.43081, size = 42, normalized size = 0.45

$$\frac{16\sqrt{x-1}}{15\sqrt{x}}+\frac{8\sqrt{x-1}}{15\,x^{\frac{3}{2}}}+\frac{2\sqrt{x-1}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 16/15*sqrt(x - 1)/sqrt(x) + 8/15*sqrt(x - 1)/x^(3/2) + 2/5*sqrt(x - 1)/x^(5
/2)
```

Fricas [A] time = 0.987849, size = 111, normalized size = 1.18

$$\frac{2\left(8\,x^3+\left(8\,x^2+4\,x+3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1361, size = 93, normalized size = 0.99

$$\frac{4096\left(5\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+10\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+8\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
4)^5
```

3.1018 $\int x^2(-a+bx^n)^p(a+bx^n)^p\,dx$

Optimal. Leaf size=78

$$\frac{1}{3}x^3(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[3/(2*n), -p, 1 + 3/(2*n
), (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rubi [A] time = 0.0414691, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {366, 365, 364}

$$\frac{1}{3}x^3(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[3/(2*n), -p, 1 + 3/(2*n
), (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 366

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^Fra
cPart[p])/(a1*a2 + b1*b2*x^(2*n))^FracPart[p], Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 (-a+bx^n)^p (a+bx^n)^p \, dx &= \left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \int x^2 \left(-a^2+b^2x^{2n}\right)^p \, dx \\ &= \left((-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int x^2 \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \, dx \\ &= \frac{1}{3}x^3 (-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0295024, size = 80, normalized size = 1.03

$$\frac{1}{3}x^3 (bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{HypergeometricPFQ}\left(\left\{\frac{3}{2n},-p\right\},\left\{\frac{3}{2n}+1\right\},\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{3/(2*n), -p}, {1 + 3/(
2*n)}, (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int x^2 (-a+bx^n)^p (a+bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

```
[Out] int(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (bx^n - a)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + a)^p (bx^n - a)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(-a + bx^n\right)^p \left(a + bx^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral(x**2*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (bx^n - a)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")

[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

3.1019 $\int x(-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=70

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
 (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rubi [A] time = 0.0307147, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {366, 365, 364}

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
 (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 366

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^Fra
cPart[p])/(a1*a2 + b1*b2*x^(2*n))^FracPart[p], Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x(-a+bx^n)^p(a+bx^n)^p\,dx &= \left((-a+bx^n)^p(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int x\left(-a^2+b^2x^{2n}\right)^p\,dx\\ &= \left((-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int x\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\ &= \frac{1}{2}x^2(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0144613, size = 72, normalized size = 1.03

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{HypergeometricPFQ}\left(\left\{\frac{1}{n},-p\right\},\left\{\frac{1}{n}+1\right\},\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{n^(-1), -p}, {1 + n^(-
1)}, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int x(-a+bx^n)^p(a+bx^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

```
[Out] int(x*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (bx^n - a)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + a)^p (bx^n - a)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(-a + bx^n\right)^p \left(a + bx^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral(x*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (bx^n - a)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")

[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

3.1020 $\int (-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=73

$$x(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rubi [A] time = 0.0273191, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {253, 246, 245}

$$x(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int(-a+bx^n)^p(a+bx^n)^p\,dx &= \left((-a+bx^n)^p(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int\left(-a^2+b^2x^{2n}\right)^p\,dx\\ &= \left((-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\ &= x(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0126175, size = 73, normalized size = 1.

$$x(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
 (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int(-a+bx^n)^p(a+bx^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*x^n)^p*(a+b*x^n)^p,x)
```

```
[Out] int((-a+b*x^n)^p*(a+b*x^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(bx^n+a)^p(bx^n-a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + a)^p (bx^n - a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (-a + bx^n)^p (a + bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (bx^n - a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

3.1021 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{\left(a^2-b^2x^{2n}\right)(bx^n-a)^p\,(a+bx^n)^p\;{}_2F_1\left(1,p+1;p+2;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

```
[Out] -((-a + b*x^n)^p*(a + b*x^n)^p*(a^2 - b^2*x^(2*n))*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Rubi [A] time = 0.0671256, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {267, 126, 266, 65}

$$-\frac{\left(a^2-b^2x^{2n}\right)(bx^n-a)^p\,(a+bx^n)^p\;{}_2F_1\left(1,p+1;p+2;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

```
[Out] -((-a + b*x^n)^p*(a + b*x^n)^p*(a^2 - b^2*x^(2*n))*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Rule 267

```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1*
x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
&& EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+bx)^p(a+bx)^p}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(-a^2+b^2x^2\right)^p}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(-a^2+b^2x\right)^p}{x}\,dx, x, x^{2n}\right)}{2n}\\
&= -\frac{(-a+bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right) {}_2F_1\left(1,1+p;2+p;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0241013, size = 73, normalized size = 1.01

$$\frac{\left(b^2x^{2n}-a^2\right)(bx^n-a)^p (a+bx^n)^p \,{}_2F_1\left(1,p+1;p+2;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

```
[Out] ((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Maple [F] time = 0.238, size = 0, normalized size = 0.

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*x^n)^p*(a+b*x^n)^p/x,x)
```

```
[Out] int((-a+b*x^n)^p*(a+b*x^n)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^p(bx^n-a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n+a)^p(bx^n-a)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x,x)
```

```
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p (bx^n - a)^p}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

3.1022 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx$

Optimal. Leaf size=76

$$-\frac{(bx^n-a)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

```
[Out] -((((-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[-1/(2*n), -p, 1 - 1/(2*n)
, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Rubi [A] time = 0.0520478, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {366, 365, 364}

$$-\frac{(bx^n-a)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```


```
[Out] -((((-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[-1/(2*n), -p, 1 - 1/(2*n)
, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```


Rule 366

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^Fra
cPart[p])/(a1*a2 + b1*b2*x^(2*n))^FracPart[p], Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx &= \left((-a+bx^n)^p\,(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int \frac{\left(-a^2+b^2x^{2n}\right)^p}{x^2}\,dx\\ &= \left((-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int \frac{\left(1-\frac{b^2x^{2n}}{a^2}\right)^p}{x^2}\,dx\\ &= -\frac{(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.015318, size = 78, normalized size = 1.03

$$-\frac{(bx^n-a)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2n},-p\right\},\left\{1-\frac{1}{2n}\right\},\frac{b^2x^{2n}}{a^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

```
[Out] -(((-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{-1/(2*n), -p}, {1 - 1/(2
*n)}, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x)
```

```
[Out] int((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p (bx^n - a)^p}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p (bx^n - a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a + bx^n)^p (a + bx^n)^p}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x**2,x)
```

```
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p (bx^n - a)^p}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

3.1023 $\int \frac{1+x^6}{x(1-x^6)}\,dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rubi [A] time = 0.0137918, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 72}

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx, x, x^6\right)$$
$$= \frac{1}{6}\,\mathrm{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)dx, x, x^6\right)$$
$$= \log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

Mathematica [A] time = 0.0052172, size = 15, normalized size = 1.

$$\log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Maple [B] time = 0.01, size = 36, normalized size = 2.4

$$-\frac{\ln(1+x)}{3} - \frac{\ln\left(x^2+x+1\right)}{3} - \frac{\ln(-1+x)}{3} - \frac{\ln\left(x^2-x+1\right)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x/(-x^6+1),x)
```

```
[Out] -1/3*ln(1+x)-1/3*ln(x^2+x+1)-1/3*ln(-1+x)-1/3*ln(x^2-x+1)+ln(x)
```

Maxima [A] time = 0.916257, size = 20, normalized size = 1.33

$$-\frac{1}{3}\log\left(x^6-1\right) + \frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^6 - 1) + 1/6*log(x^6)
```

Fricas [A] time = 0.944319, size = 38, normalized size = 2.53

$$-\frac{1}{3}\log\left(x^6-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^6 - 1) + log(x)
```

Sympy [A] time = 0.1166, size = 10, normalized size = 0.67

$$\log\left(x\right)-\frac{\log\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x/(-x**6+1),x)
```

```
[Out] log(x) - log(x**6 - 1)/3
```

Giac [A] time = 1.06437, size = 22, normalized size = 1.47

$$\frac{1}{6}\log\left(x^6\right)-\frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

3.1024 $\int (ex)^m (a+bx^n)^p \left(a(1+m)+b(1+m+n+np)x^n\right) dx$

Optimal. Leaf size=22

$$\frac{(ex)^{m+1} (a+bx^n)^{p+1}}{e}$$

```
[Out] ((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

Rubi [A] time = 0.0148312, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {449}

$$\frac{(ex)^{m+1} (a+bx^n)^{p+1}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m (a+bx^n)^p \left(a(1+m)+b(1+m+n+np)x^n\right) dx = \frac{(ex)^{1+m} (a+bx^n)^{1+p}}{e}$$

Mathematica [C] time = 0.12954, size = 110, normalized size = 5.

$$x(ex)^m (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(\frac{bx^n(m+np+n+1)\,{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1} + a\,{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] (x*(e*x)^m*(a + b*x^n)^p*(a*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n,
 -((b*x^n)/a)] + (b*(1 + m + n + n*p)*x^n*Hypergeometric2F1[(1 + m + n)/n,
-p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n)))/(1 + (b*x^n)/a)^p
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int (ex)^m (a + bx^n)^p \left(a(1+m) + b\left(pn + m + n + 1\right)x^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x)
```

```
[Out] int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x)
```

Maxima [A] time = 1.19691, size = 49, normalized size = 2.23

$$\left(ae^m xx^m + be^m xe^{(m\log(x)+n\log(x))}\right)(bx^n + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="max
ima")
```

```
[Out] (a*e^m*x*x^m + b*e^m*x*e^(m*log(x) + n*log(x)))*(b*x^n + a)^p
```

Fricas [A] time = 1.03107, size = 107, normalized size = 4.86

$$\left(bxx^n e^{(m\log(e)+m\log(x))} + axe^{(m\log(e)+m\log(x))}\right)(bx^n + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="fri
cas")
```

```
[Out] (b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)))*(b*x^n + a)
^p
```

Sympy [B] time = 9.47734, size = 39, normalized size = 1.77

$$ae^m xx^m (a + bx^n)^p + be^m xx^m x^n (a + bx^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(n*p+m+n+1)*x**n),x)
```

```
[Out] a*e**m*x*x**m*(a + b*x**n)**p + b*e**m*x*x**m*x**n*(a + b*x**n)**p
```

Giac [A] time = 1.09913, size = 51, normalized size = 2.32

$$(bx^n + a)^p bxx^m x^n e^m + (bx^n + a)^p axx^m e^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="gia
c")
```

```
[Out] (b*x^n + a)^p*b*x*x^m*x^n*e^m + (b*x^n + a)^p*a*x*x^m*e^m
```

3.1025 $\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=114

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)])/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rubi [A] time = 0.0624926, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {508, 364}

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)])/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \frac{b\int \frac{(ex)^m}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{(ex)^m}{c+dx^n}\,dx}{bc-ad}$$
$$= \frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)} - \frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}$$

Mathematica [A] time = 0.0689063, size = 88, normalized size = 0.77

$$\frac{x(ex)^m\left(ad\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)-bc\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)]) + a*d*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))/(
a*c*(-(b*c) + a*d)*(1 + m))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(a + bx^n)(c + dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

3.1026 $\int \frac{x^2}{(a+bx^n)(c+dx^n)} \, dx$

Optimal. Leaf size=89

$$\frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

```
[Out] (b*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*(b*c - a*d)
) - (d*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c -
a*d))
```

Rubi [A] time = 0.0418385, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {508, 364}

$$\frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*(b*c - a*d)
) - (d*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c -
a*d))
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \frac{b\int \frac{x^2}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{x^2}{c+dx^n}\,dx}{bc-ad}$$
$$= \frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)} - \frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

Mathematica [A] time = 0.0538311, size = 78, normalized size = 0.88

$$\frac{bcx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right) - adx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3abc^2-3a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)] - a*d*x^3*Hyper
geometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*a*b*c^2 - 3*a^2*c*d)
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + bx^n)(c + dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(x**2/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

3.1027 $\int \frac{x}{(a+bx^n)(c+dx^n)} \, dx$

Optimal. Leaf size=89

$$\frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

```
[Out] (b*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*(b*c - a*d)
) - (d*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c -
a*d))
```

Rubi [A] time = 0.033347, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {508, 364}

$$\frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*(b*c - a*d)
) - (d*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c -
a*d))
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{b\int \frac{x}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{x}{c+dx^n}\,dx}{bc-ad}$$
$$= \frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)} - \frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

Mathematica [A] time = 0.051509, size = 78, normalized size = 0.88

$$\frac{bcx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right) - adx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2abc^2-2a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)] - a*d*x^2*Hyper
geometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*a*b*c^2 - 2*a^2*c*d)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(x/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + bx^n)(c + dx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(x/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^n + a)(dx^n + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

3.1028 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=72

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rubi [A] time = 0.0231652, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {391, 245}

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \frac{b\int \frac{1}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^n}\,dx}{bc-ad}$$
$$= \frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Mathematica [A] time = 0.0323504, size = 64, normalized size = 0.89

$$\frac{x\left(ad\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-bc\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{ac(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx^n)(a+bx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)(c + dx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

3.1029 $\int \frac{1}{x(a+bx^n)(c+dx^n)} dx$

Optimal. Leaf size=63

$$-\frac{b\log(a+bx^n)}{an(bc-ad)}+\frac{d\log(c+dx^n)}{cn(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)*n) + (d*Log[c + d*x^n])/(c
*(b*c - a*d)*n)
```

Rubi [A] time = 0.0678896, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log(a+bx^n)}{an(bc-ad)}+\frac{d\log(c+dx^n)}{cn(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)*n) + (d*Log[c + d*x^n])/(c
*(b*c - a*d)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{acx} + \frac{b^2}{a(-bc+ad)(a+bx)} + \frac{d^2}{c(bc-ad)(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= \frac{\log(x)}{ac} - \frac{b\log(a+bx^n)}{a(bc-ad)n} + \frac{d\log(c+dx^n)}{c(bc-ad)n}$$

Mathematica [A] time = 0.0476577, size = 56, normalized size = 0.89

$$\frac{-bc\log(a+bx^n) + ad\log(c+dx^n) - adn\log(x) + bcn\log(x)}{abc^2n - a^2cdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*n*Log[x] - a*d*n*Log[x] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a*
b*c^2*n - a^2*c*d*n)
```

Maple [A] time = 0.013, size = 69, normalized size = 1.1

$$\frac{b\ln(a+bx^n)}{n(ad-bc)a} - \frac{d\ln(c+dx^n)}{nc(ad-bc)} + \frac{\ln(x^n)}{nca}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] 1/n*b/(a*d-b*c)/a*ln(a+b*x^n)-1/n*d/c/(a*d-b*c)*ln(c+d*x^n)+1/n/c/a*ln(x^n)
```

Maxima [A] time = 0.947872, size = 93, normalized size = 1.48

$$-\frac{b\log\left(\frac{bx^n+a}{b}\right)}{abcn - a^2dn} + \frac{d\log\left(\frac{dx^n+c}{d}\right)}{bc^2n - acdn} + \frac{\log(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n -
 a*c*d*n) + log(x)/(a*c)
```

Fricas [A] time = 1.08512, size = 123, normalized size = 1.95

$$-\frac{bc\log\left(bx^n+a\right)-ad\log\left(dx^n+c\right)-(bc-ad)n\log\left(x\right)}{\left(abc^2-a^2cd\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^2
 - a^2*c*d)*n)
```

Sympy [A] time = 86.2728, size = 335, normalized size = 5.32

$$\begin{cases} \frac{\frac{\log\left(x\right)}{c}-\frac{\log\left(\frac{c}{d}+x^n\right)}{cn}}{a} & \text{for}\ b=0 \\ \frac{\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^n\right)}{an}}{c} & \text{for}\ d=0 \\ \frac{-\frac{x^{-n}}{cn}+\frac{d\log\left(x^{-n}+\frac{d}{c}\right)}{c^2n}}{b} & \text{for}\ a=0 \\ \frac{cn\log\left(x\right)}{ac^2n+acdnx^n}-\frac{c\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n}+\frac{dnx^n\log\left(x\right)}{ac^2n+acdnx^n}-\frac{dx^n\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n}-\frac{dx^n}{ac^2n+acdnx^n} & \text{for}\ b=\frac{ad}{c} \\ \frac{-\frac{x^{-n}}{an}+\frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2n}}{d} & \text{for}\ c=0 \\ \frac{\log\left(x\right)}{(a+b)(c+d)} & \text{for}\ n=0 \\ \frac{adn\log\left(x\right)}{a^2cdn-abc^2n}-\frac{ad\log\left(\frac{c}{d}+x^n\right)}{a^2cdn-abc^2n}-\frac{bcn\log\left(x\right)}{a^2cdn-abc^2n}+\frac{bc\log\left(\frac{a}{b}+x^n\right)}{a^2cdn-abc^2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), ((log(x)/a - lo
g(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-x**(-n)/(c*n) + d*log(x**(-n) + d/c)/
(c**2*n))/b, Eq(a, 0)), (c*n*log(x)/(a*c**2*n + a*c*d*n*x**n) - c*log(c/d +
 x**n)/(a*c**2*n + a*c*d*n*x**n) + d*n*x**n*log(x)/(a*c**2*n + a*c*d*n*x**n
) - d*x**n*log(c/d + x**n)/(a*c**2*n + a*c*d*n*x**n) - d*x**n/(a*c**2*n + a
*c*d*n*x**n), Eq(b, a*d/c)), ((-x**(-n)/(a*n) + b*log(x**(-n) + b/a)/(a**2*
n))/d, Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(n, 0)), (a*d*n*log(x)/(a**2
*c*d*n - a*b*c**2*n) - a*d*log(c/d + x**n)/(a**2*c*d*n - a*b*c**2*n) - b*c*
n*log(x)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a/b + x**n)/(a**2*c*d*n - a*b*
c**2*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)
```

3.1030 $\int \frac{1}{x^2(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=90

$$\frac{d\,_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)}-\frac{b\,_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

```
[Out] -((b*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c - a
*d)*x)) + (d*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*
(b*c - a*d)*x)
```

Rubi [A] time = 0.0430339, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {508, 364}

$$\frac{d\,_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)}-\frac{b\,_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((b*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c - a
*d)*x)) + (d*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*
(b*c - a*d)*x)
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^n)(c+dx^n)} dx = \frac{b \int \frac{1}{x^2(a+bx^n)} dx}{bc-ad} - \frac{d \int \frac{1}{x^2(c+dx^n)} dx}{bc-ad}$$
$$= -\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)x} + \frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)x}$$

Mathematica [A] time = 0.0530143, size = 74, normalized size = 0.82

$$\frac{bc\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{bx^n}{a}\right) - ad\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{dx^n}{c}\right)}{acx(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] - a*d*Hypergeo
metric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)])/(a*c*(-(b*c) + a*d)*x)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a+bx^n)(c+dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bdx^2x^{2n} + (bc + ad)x^2x^n + acx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^2*x^(2*n) + (b*c + a*d)*x^2*x^n + a*c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + bx^n) (c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n)*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

3.1031 $\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=95

$$\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

```
[Out] -(b*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a*(b*c - a*d
)*x^2) + (d*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)*x^2)
```

Rubi [A] time = 0.0428551, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {508, 364}

$$\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -(b*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a*(b*c - a*d
)*x^2) + (d*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)*x^2)
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx = \frac{b\int \frac{1}{x^3(a+bx^n)}\,dx}{bc-ad} - \frac{d\int \frac{1}{x^3(c+dx^n)}\,dx}{bc-ad}$$
$$= -\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2} + \frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}$$

Mathematica [A] time = 0.0519527, size = 77, normalized size = 0.81

$$\frac{bc\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{bx^n}{a}\right) - ad\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{dx^n}{c}\right)}{2acx^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)] - a*d*Hypergeomet
ric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)])/(2*a*c*(-(b*c) + a*d)*x^2)
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bdx^3x^{2n} + (bc + ad)x^3x^n + acx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^3*x^(2*n) + (b*c + a*d)*x^3*x^n + a*c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3(a + bx^n)(c + dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**n)*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

3.1032 $\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=175

$$\frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2e(m+1)n(bc-ad)^2}+\frac{d^2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)^2}+\frac{b(ex}{aen(bc-ad}$$

```
[Out] (b*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) + (b*(a*d*(1 + m - 2*n) -
 b*c*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*e*(1 + m)*n) + (d^2*(e*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)^2
*e*(1 + m))
```

Rubi [A] time = 0.288443, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {504, 597, 364}

$$\frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2e(m+1)n(bc-ad)^2}+\frac{d^2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)^2}+\frac{b(ex}{aen(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) + (b*(a*d*(1 + m - 2*n) -
 b*c*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*e*(1 + m)*n) + (d^2*(e*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)^2
*e*(1 + m))
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} - \frac{\int \frac{(ex)^m(bc(1+m-n)+adn+bd(1+m-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} - \frac{\int \left(\frac{b(-ad(1+m-2n)+bc(1+m-n))(ex)^m}{(bc-ad)(a+bx^n)} + \frac{ad^2n(ex)^m}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{d^2\int \frac{(ex)^m}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1+m-2n)-bc(1+m-n)))\int \frac{(ex)^m}{a+bx^n}\,dx}{a(bc-ad)^2n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx}{a}\right.}{a^2(bc-ad)^2e(1+m)n}$$

Mathematica [A] time = 0.185273, size = 141, normalized size = 0.81

$$\frac{x(ex)^m\left(\frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(m-2n+1)-bc(m-n+1))\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2(m+1)n} + \frac{d^2\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{cm+c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*(e*x)^m*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 + m - 2*n) - b
*c*(1 + m - n))*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)
])/(a^2*(1 + m)*n) + (d^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -(
```

```
(d*x^n)/c)])/(c + c*m)))/(b*c - a*d)^2
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)

[Out] int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^2e^m \int \frac{x^m}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\,dx + \frac{be^m x x^m}{a^2bcn - a^3dn + \left(ab^2cn - a^2bdn\right)x^n} - \left(b^2ce^m(m-n+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")

[Out] d^2*e^m*integrate(x^m/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*x^n), x) + b*e^m*x*x^m/(a^2*b*c*n - a^3*d*n + (a*b^2*c*
n - a^2*b*d*n)*x^n) - (b^2*c*e^m*(m - n + 1) - a*b*d*e^m*(m - 2*n + 1))*int
egrate(x^m/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^
2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m}{b^2dx^{3\,n} + a^2c + \left(b^2c + 2\,abd\right)x^{2\,n} + \left(2\,abc + a^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*
b*c + a^2*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(a + bx^n)^2 (c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**n)**2*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{(bx^n + a)^2 (dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1033 $\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=142

$$\frac{bx^3(ad(3-2n)-bc(3-n))\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^2n(bc-ad)^2}+\frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}+\frac{bx^3}{an(bc-ad)\,(a+bx^n)}$$

```
[Out] (b*x^3)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(3 - 2*n) - b*c*(3 - n))*x^
3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^2*(b*c - a*d)^2*
n) + (d^2*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c
 - a*d)^2)
```

Rubi [A] time = 0.224274, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {504, 597, 364}

$$\frac{bx^3(ad(3-2n)-bc(3-n))\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^2n(bc-ad)^2}+\frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}+\frac{bx^3}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^3)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(3 - 2*n) - b*c*(3 - n))*x^
3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^2*(b*c - a*d)^2*
n) + (d^2*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c
 - a*d)^2)
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}dx &= \frac{bx^3}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{x^2(bc(3-n)+adn+bd(3-n)x^n)}{(a+bx^n)(c+dx^n)}dx}{a(bc-ad)n} \\ &= \frac{bx^3}{a(bc-ad)n(a+bx^n)} - \frac{\int \left(\frac{b(-ad(3-2n)+bc(3-n))x^2}{(bc-ad)(a+bx^n)} + \frac{ad^2nx^2}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n} \\ &= \frac{bx^3}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{x^2}{c+dx^n}dx}{(bc-ad)^2} + \frac{(b(ad(3-2n)-bc(3-n)))\int \frac{x^2}{a+bx^n}dx}{a(bc-ad)^2n} \\ &= \frac{bx^3}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(3-2n)-bc(3-n))x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n} + \frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\right.}{3c(bc-}\end{aligned}$$

Mathematica [A] time = 0.159323, size = 135, normalized size = 0.95

$$\frac{x^3\left(a\left(ad^2n(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)+3bc(bc-ad)\right)+bc(a+bx^n)(ad(3-2n)+bc(n-3))\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)\right)}{3a^2cn(bc-ad)^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x^3*(b*c*(a*d*(3 - 2*n) + b*c*(-3 + n))*(a + b*x^n)*Hypergeometric2F1[1, 3
/n, (3 + n)/n, -((b*x^n)/a)] + a*(3*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*H
ypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])))/(3*a^2*c*(b*c - a*d)^2
*n*(a + b*x^n))
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^3}{a^2bcn - a^3dn + (ab^2cn - a^2bdn)x^n} + d^2 \int \frac{x^2}{b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^2d - 2\,abcd^2 + a^2d^3)x^n}\,dx - \left(abd(2\,n-3) - b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] b*x^3/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + d^2*integrate(x
^2/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*x^n), x) - (a*b*d*(2*n - 3) - b^2*c*(n - 3))*integrate(x^2/(a^2*b^2*c^2*n
- 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)
*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{b^2dx^{3\,n} + a^2c + (b^2c + 2\,abd)x^{2\,n} + (2\,abc + a^2d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c
+ a^2*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Integral(x**2/((a + b*x**n)**2*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1034 $\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=143

$$\frac{bx^2(2ad(1-n)-bc(2-n))\,_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2n(bc-ad)^2}+\frac{d^2x^2\,_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}+\frac{bx^2}{an(bc-ad)(a+bx^n)}$$

```
[Out] (b*x^2)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(2*a*d*(1 - n) - b*c*(2 - n))*x^
2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*
n) + (d^2*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c
 - a*d)^2)
```

Rubi [A] time = 0.181516, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {504, 597, 364}

$$\frac{bx^2(2ad(1-n)-bc(2-n))\,_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2n(bc-ad)^2}+\frac{d^2x^2\,_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}+\frac{bx^2}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^2)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(2*a*d*(1 - n) - b*c*(2 - n))*x^
2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*
n) + (d^2*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c
 - a*d)^2)
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{x(bc(2-n)+adn+bd(2-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}\\ &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} - \frac{\int \left(\frac{b(-2ad(1-n)+bc(2-n))x}{(bc-ad)(a+bx^n)} + \frac{ad^2nx}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n}\\ &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{x}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(2ad(1-n)-bc(2-n)))\int \frac{x}{a+bx^n}\,dx}{a(bc-ad)^2n}\\ &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} + \frac{b(2ad(1-n)-bc(2-n))x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n} + \frac{d^2x^2\,{}_2F_1\left(1,\frac{2}{n};\right.}{2c(bc-}\end{aligned}$$

Mathematica [A] time = 0.155101, size = 134, normalized size = 0.94

$$\frac{x^2\left(a\left(ad^2n(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)+2bc(bc-ad)\right)+bc(a+bx^n)(bc(n-2)-2ad(n-1))\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)\right)}{2a^2cn(bc-ad)^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x^2*(b*c*(b*c*(-2 + n) - 2*a*d*(-1 + n))*(a + b*x^n)*Hypergeometric2F1[1,
2/n, (2 + n)/n, -((b*x^n)/a)] + a*(2*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*
Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])))/(2*a^2*c*(b*c - a*d)^
2*n*(a + b*x^n))
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^2 \int \frac{x}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\,dx + \frac{bx^2}{a^2bcn - a^3dn + \left(ab^2cn - a^2bdn\right)x^n} - \left(2\,abd(n-1) - b^2c($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(x/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) + b*x^2/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*
n)*x^n) - (2*a*b*d*(n - 1) - b^2*c*(n - 2))*integrate(x/(a^2*b^2*c^2*n - 2*
a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{b^2dx^{3\,n} + a^2c + \left(b^2c + 2\,abd\right)x^{2\,n} + \left(2\,abc + a^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Integral(x/((a + b*x**n)**2*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1035 $\int \frac{1}{(a+bx^n)^2(c+dx^n)} dx$

Optimal. Leaf size=122

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\,(a+bx^n)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rubi [A] time = 0.137692, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 245}

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}\\ &= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^2n}\\ &= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1\right.}{c(bc-}\end{aligned}$$

Mathematica [A] time = 0.139951, size = 108, normalized size = 0.89

$$\frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^2 \int \frac{1}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\,dx - \left(abd(2\,n-1) - b^2c(n-1)\right)\int \frac{}{a^2b^2c^2n - 2\,a^3bcdn + a^4d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2
*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a
^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x
^n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2dx^{3\,n} + a^2c + \left(b^2c + 2\,abd\right)x^{2\,n} + \left(2\,abc + a^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)**2*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1036 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=101

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{cn(bc-ad)^2}+\frac{b}{an(bc-ad)(a+bx^n)}$$

```
[Out] b/(a*(b*c - a*d)*n*(a + b*x^n)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^n])/(a^2*(b*c - a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)
```

Rubi [A] time = 0.105012, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{cn(bc-ad)^2}+\frac{b}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*(a + b*x^n)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^n])/(a^2*(b*c - a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2cx}+\frac{b^2}{a(-bc+ad)(a+bx)^2}+\frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)}-\frac{d^3}{c(bc-ad)^2(c+dx)}\right)dx,x,x^n\right)}{n}$$
$$= \frac{b}{a(bc-ad)n\left(a+bx^n\right)}+\frac{\log(x)}{a^2c}-\frac{b(bc-2ad)\log\left(a+bx^n\right)}{a^2(bc-ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}$$

Mathematica [A] time = 0.174486, size = 97, normalized size = 0.96

$$\frac{-\frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2}+\frac{n\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{c(bc-ad)^2}+\frac{b}{a(bc-ad)(a+bx^n)}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b/(a*(b*c - a*d)*(a + b*x^n)) + (n*Log[x])/(a^2*c) - (b*(b*c - 2*a*d)*Log[
a + b*x^n])/(a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2))/n
```

Maple [A] time = 0.012, size = 131, normalized size = 1.3

$$-\frac{b}{n\left(ad-bc\right)a\left(a+bx^n\right)}+2\,\frac{b\ln\left(a+bx^n\right)d}{n\left(ad-bc\right)^2a}-\frac{b^2\ln\left(a+bx^n\right)c}{n\left(ad-bc\right)^2a^2}-\frac{d^2\ln\left(c+dx^n\right)}{nc\left(ad-bc\right)^2}+\frac{\ln\left(x^n\right)}{na^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] -1/n*b/(a*d-b*c)/a/(a+b*x^n)+2/n*b/(a*d-b*c)^2/a*ln(a+b*x^n)*d-1/n*b^2/(a*d
-b*c)^2/a^2*ln(a+b*x^n)*c-1/n*d^2/c/(a*d-b*c)^2*ln(c+d*x^n)+1/n/a^2/c*ln(x^
n)
```

Maxima [A] time = 0.94676, size = 204, normalized size = 2.02

$$-\frac{d^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n-2\,abc^2dn+a^2cd^2n}-\frac{\left(b^2c-2\,abd\right)\log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n}+\frac{b}{a^2bcn-a^3dn+\left(ab^2cn-a^2bdn\right)x^n}+\frac{\log\left(x\right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c
- 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n) +
 b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)
```

Fricas [B] time = 1.14898, size = 458, normalized size = 4.53

$$\frac{ab^2c^2 - a^2bcd + \left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)nx^n \log\left(x\right) + \left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)n \log\left(x\right) - \left(ab^2c^2 - 2\,a^2bcd + \left(b^3c^2 - 2\,ab\right.\right.}{\left(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2\right)nx^n + \left(a^3b^2c^3 - 2\,a^4bc^2d + a^5cd^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x) +
 (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d +
(b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*log
(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^3*b^
2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)
```

Sympy [A] time = 12.6121, size = 2227, normalized size = 22.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Piecewise(((log(x)/c - log(c/d + x**n)/(c*n))/a**2, Eq(b, 0)), ((-x**(-2*n)
/(2*c*n) + d*x**(-n)/(c**2*n) + d**2*log(x)/c**3 - d**2*log(c/d + x**n)/(c*
*3*n))/b**2, Eq(a, 0)), (2*c**2*d**2*n*log(x)/(2*b**2*c**5*n + 4*b**2*c**4*
d*n*x**n + 2*b**2*c**3*d**2*n*x**(2*n)) - 2*c**2*d**2*log(c/d + x**n)/(2*b*
*2*c**5*n + 4*b**2*c**4*d*n*x**n + 2*b**2*c**3*d**2*n*x**(2*n)) + 4*c*d**3*
n*x**n*log(x)/(2*b**2*c**5*n + 4*b**2*c**4*d*n*x**n + 2*b**2*c**3*d**2*n*x*
*(2*n)) - 4*c*d**3*x**n*log(c/d + x**n)/(2*b**2*c**5*n + 4*b**2*c**4*d*n*x*
*n + 2*b**2*c**3*d**2*n*x**(2*n)) - 4*c*d**3*x**n/(2*b**2*c**5*n + 4*b**2*c
```

```
**4*d*n*x**n + 2*b**2*c**3*d**2*n*x**(2*n)) + 2*d**4*n*x**(2*n)*log(x)/(2*b
**2*c**5*n + 4*b**2*c**4*d*n*x**n + 2*b**2*c**3*d**2*n*x**(2*n)) - 2*d**4*x
**(2*n)*log(c/d + x**n)/(2*b**2*c**5*n + 4*b**2*c**4*d*n*x**n + 2*b**2*c**3
*d**2*n*x**(2*n)) - 3*d**4*x**(2*n)/(2*b**2*c**5*n + 4*b**2*c**4*d*n*x**n +
 2*b**2*c**3*d**2*n*x**(2*n)), Eq(a, b*c/d)), ((-a**2/(a**4*n*x**n + a**3*b
*n*x**(2*n)) + 2*a*b*x**n*log(x**(-n) + b/a)/(a**4*n*x**n + a**3*b*n*x**(2*
n)) + 2*b**2*x**(2*n)*log(x**(-n) + b/a)/(a**4*n*x**n + a**3*b*n*x**(2*n))
+ 2*b**2*x**(2*n)/(a**4*n*x**n + a**3*b*n*x**(2*n)))/d, Eq(c, 0)), (log(x)/
((a + b)**2*(c + d)), Eq(n, 0)), ((a*n*log(x)/(a**3*n + a**2*b*n*x**n) - a*
log(a/b + x**n)/(a**3*n + a**2*b*n*x**n) + b*n*x**n*log(x)/(a**3*n + a**2*b
*n*x**n) - b*x**n*log(a/b + x**n)/(a**3*n + a**2*b*n*x**n) - b*x**n/(a**3*n
 + a**2*b*n*x**n))/c, Eq(d, 0)), (a**3*d**2*n*log(x)/(a**5*c*d**2*n - 2*a**
4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**3*b**2*c**2*d
*n*x**n + a**2*b**3*c**3*n*x**n) - a**3*d**2*log(c/d + x**n)/(a**5*c*d**2*n
 - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**3*b**
2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) - 2*a**2*b*c*d*n*log(x)/(a**5*c*d*
*2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**3
*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) + 2*a**2*b*c*d*log(a/b + x**n)
/(a**5*c*d**2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3
*n - 2*a**3*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) + a**2*b*d**2*n*x**
n*log(x)/(a**5*c*d**2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b
**2*c**3*n - 2*a**3*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) - a**2*b*d*
*2*x**n*log(c/d + x**n)/(a**5*c*d**2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*
n*x**n + a**3*b**2*c**3*n - 2*a**3*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n*x*
*n) + a*b**2*c**2*n*log(x)/(a**5*c*d**2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d*
*2*n*x**n + a**3*b**2*c**3*n - 2*a**3*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n
*x**n) - a*b**2*c**2*log(a/b + x**n)/(a**5*c*d**2*n - 2*a**4*b*c**2*d*n + a
**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**3*b**2*c**2*d*n*x**n + a**2*b
**3*c**3*n*x**n) - 2*a*b**2*c*d*n*x**n*log(x)/(a**5*c*d**2*n - 2*a**4*b*c**
2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**3*b**2*c**2*d*n*x**n
 + a**2*b**3*c**3*n*x**n) + 2*a*b**2*c*d*x**n*log(a/b + x**n)/(a**5*c*d**2*
n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**3*b*
*2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) + a*b**2*c*d*x**n/(a**5*c*d**2*n
- 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**3*b**2
*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) + b**3*c**2*n*x**n*log(x)/(a**5*c*d
**2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3*n - 2*a**
3*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) - b**3*c**2*x**n*log(a/b + x*
*n)/(a**5*c*d**2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c
**3*n - 2*a**3*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n) - b**3*c**2*x**n
/(a**5*c*d**2*n - 2*a**4*b*c**2*d*n + a**4*b*c*d**2*n*x**n + a**3*b**2*c**3
*n - 2*a**3*b**2*c**2*d*n*x**n + a**2*b**3*c**3*n*x**n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)
```

3.1037 $\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)} dx$

Optimal. Leaf size=142

$$-\frac{b(bc(n+1)-ad(2n+1))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2nx(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)^2}+\frac{b}{anx(bc-ad)(a+bx^n)}$$

```
[Out] b/(a*(b*c - a*d)*n*x*(a + b*x^n)) - (b*(b*c*(1 + n) - a*d*(1 + 2*n))*Hyperg
eometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n*x
) - (d^2*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*(b*c
 - a*d)^2*x)
```

Rubi [A] time = 0.214042, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {504, 597, 364}

$$-\frac{b(bc(n+1)-ad(2n+1))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2nx(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)^2}+\frac{b}{anx(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x*(a + b*x^n)) - (b*(b*c*(1 + n) - a*d*(1 + 2*n))*Hyperg
eometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n*x
) - (d^2*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*(b*c
 - a*d)^2*x)
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)} dx = \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{\int \frac{adn-bc(1+n)-bd(1+n)x^n}{x^2(a+bx^n)(c+dx^n)} dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{\int \left(\frac{b(-bc(1+n)+ad(1+2n))}{(bc-ad)x^2(a+bx^n)} + \frac{ad^2n}{(-bc+ad)x^2(c+dx^n)}\right) dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx(a+bx^n)} + \frac{d^2 \int \frac{1}{x^2(c+dx^n)} dx}{(bc-ad)^2} + \frac{(b(bc(1+n)-ad(1+2n))) \int \frac{1}{x^2(a+bx^n)} dx}{a(bc-ad)^2 n}$$

$$= \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{b(bc(1+n)-ad(1+2n))\, {}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx} - \frac{d^2\, {}_2F_1\left(1,-\frac{1}{n}\right.}{c(bc-}$$

Mathematica [A] time = 0.154346, size = 133, normalized size = 0.94

$$\frac{bc(a+bx^n)(ad(2n+1)-bc(n+1))\, {}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{bx^n}{a}\right) - a\left(ad^2n(a+bx^n)\, {}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{dx^n}{c}\right) + bc(ad-bc)\right)}{a^2cnx(bc-ad)^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*c*(-(b*c*(1 + n)) + a*d*(1 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1, -n^(
-1), (-1 + n)/n, -((b*x^n)/a)] - a*(b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n
)*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)]))/(a^2*c*(b*c - a
*d)^2*n*x*(a + b*x^n))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^2 \int \frac{1}{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^2x^n + \left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)x^2}\,dx - \left(abd(2\,n+1) - b^2c(n+1)\right)\int \frac{}{\left(ab^3c^2n - 2\,a^2b^2cd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2*x^n + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*x^2), x) - (a*b*d*(2*n + 1) - b^2*c*(n + 1))*integr
ate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^2*x^n + (a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^2), x) + b/((a*b^2*c*n - a^2*b*d*n)*x*x^n
 + (a^2*b*c*n - a^3*d*n)*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2dx^2x^{3\,n} + a^2cx^2 + \left(b^2c + 2\,abd\right)x^2x^{2\,n} + \left(2\,abc + a^2d\right)x^2x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^2*x^(3*n) + a^2*c*x^2 + (b^2*c + 2*a*b*d)*x^2*x^(2*n) +
 (2*a*b*c + a^2*d)*x^2*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + bx^n)^2 (c + dx^n)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n)**2*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^2), x)
```

3.1038 $\int \frac{1}{x^3(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=145

$$\frac{b(2ad(n+1)-bc(n+2))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2nx^2(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)^2}+\frac{b}{anx^2(bc-ad)\left(a+bx^n\right)}$$

```
[Out] b/(a*(b*c - a*d)*n*x^2*(a + b*x^n)) + (b*(2*a*d*(1 + n) - b*c*(2 + n))*Hype
rgeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*n*
x^2) - (d^2*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)^2*x^2)
```

Rubi [A] time = 0.21373, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {504, 597, 364}

$$\frac{b(2ad(n+1)-bc(n+2))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2nx^2(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)^2}+\frac{b}{anx^2(bc-ad)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x^2*(a + b*x^n)) + (b*(2*a*d*(1 + n) - b*c*(2 + n))*Hype
rgeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*n*
x^2) - (d^2*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)^2*x^2)
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{b}{a(bc-ad)nx^2(a+bx^n)} - \frac{\int \frac{adn-bc(2+n)-bd(2+n)x^n}{x^3(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx^2(a+bx^n)} - \frac{\int \left(\frac{b(2ad(1+n)-bc(2+n))}{(bc-ad)x^3(a+bx^n)} + \frac{ad^2n}{(-bc+ad)x^3(c+dx^n)}\right)dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx^2(a+bx^n)} + \frac{d^2\int \frac{1}{x^3(c+dx^n)}\,dx}{(bc-ad)^2} - \frac{(b(2ad(1+n)-bc(2+n)))\int \frac{1}{x^3(a+bx^n)}\,dx}{a(bc-ad)^2n}$$

$$= \frac{b}{a(bc-ad)nx^2(a+bx^n)} + \frac{b(2ad(1+n)-bc(2+n))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2nx^2} - \frac{d^2\,{}_2F_1\left(1,-\right.}{2c(bc}$$

Mathematica [A] time = 0.152146, size = 136, normalized size = 0.94

$$\frac{bc(a+bx^n)(2ad(n+1)-bc(n+2))\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{bx^n}{a}\right) - a\left(ad^2n(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{dx^n}{c}\right)+2bc(ad-bc)\right)}{2a^2cnx^2(bc-ad)^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*c*(2*a*d*(1 + n) - b*c*(2 + n))*(a + b*x^n)*Hypergeometric2F1[1, -2/n, (
-2 + n)/n, -((b*x^n)/a)] - a*(2*b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n)*Hy
pergeometric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)]))/(2*a^2*c*(b*c - a*d)^2
*n*x^2*(a + b*x^n))
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{1}{x^{3} \left(a + b x^{n}\right)^{2} \left(c + d x^{n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^{2} \int \frac{1}{\left(b^{2} c^{2} d - 2\, a b c d^{2} + a^{2} d^{3}\right) x^{3} x^{n} + \left(b^{2} c^{3} - 2\, a b c^{2} d + a^{2} c d^{2}\right) x^{3}}\, dx + \left(b^{2} c (n + 2) - 2\, a b d (n + 1)\right) \int \frac{}{\left(a b^{3} c^{2} n - 2\, a^{2} b^{2} c d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3*x^n + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*x^3), x) + (b^2*c*(n + 2) - 2*a*b*d*(n + 1))*integr
ate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^3*x^n + (a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^3), x) + b/((a*b^2*c*n - a^2*b*d*n)*x^2*x
^n + (a^2*b*c*n - a^3*d*n)*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b^{2} d x^{3} x^{3 \, n} + a^{2} c x^{3} + \left(b^{2} c + 2 \, a b d\right) x^{3} x^{2 \, n} + \left(2 \, a b c + a^{2} d\right) x^{3} x^{n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^3*x^(3*n) + a^2*c*x^3 + (b^2*c + 2*a*b*d)*x^3*x^(2*n) +
 (2*a*b*c + a^2*d)*x^3*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + bx^n)^2 (c + dx^n)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**n)**2*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^3), x)
```

3.1039 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal. Leaf size=130

$$\frac{bx^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}+\frac{b^3x^{4n}}{4dn}$$

```
[Out] -(((b*c - a*d)^3*x^n)/(d^4*n)) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*
n))/(2*d^3*n) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2*n) + (b^3*x^(4*n))/(4*d*
n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)
```

Rubi [A] time = 0.138426, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{bx^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}+\frac{b^3x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```


```
[Out] -(((b*c - a*d)^3*x^n)/(d^4*n)) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*
n))/(2*d^3*n) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2*n) + (b^3*x^(4*n))/(4*d*
n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^3}{d^4} + \frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x}{d^3} - \frac{b^2(bc-3ad)x^2}{d^2} + \frac{b^3x^3}{d} + \frac{c(bc-ad)^3}{d^4(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{(bc-ad)^3x^n}{d^4n} + \frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^{2n}}{2d^3n} - \frac{b^2(bc-3ad)x^{3n}}{3d^2n} + \frac{b^3x^{4n}}{4dn} + \frac{c(bc-ad)^3\log(c+}{d^5n}$$

Mathematica [A] time = 0.195873, size = 115, normalized size = 0.88

$$\frac{6bd^2x^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)-4b^2d^3x^{3n}(bc-3ad)+12dx^n(ad-bc)^3+12c(bc-ad)^3\log(c+dx^n)+3b^3d^4x^{4n}}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (12*d*(-(b*c) + a*d)^3*x^n + 6*b*d^2*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2
*n) - 4*b^2*d^3*(b*c - 3*a*d)*x^(3*n) + 3*b^3*d^4*x^(4*n) + 12*c*(b*c - a*d
)^3*Log[c + d*x^n])/(12*d^5*n)
```

Maple [B] time = 0.028, size = 284, normalized size = 2.2

$$\frac{e^{n\ln(x)}a^3}{dn} - 3\frac{e^{n\ln(x)}a^2bc}{d^2n} + 3\frac{e^{n\ln(x)}ab^2c^2}{d^3n} - \frac{e^{n\ln(x)}b^3c^3}{d^4n} + \frac{b^3\left(e^{n\ln(x)}\right)^4}{4dn} + \frac{3b\left(e^{n\ln(x)}\right)^2a^2}{2dn} - \frac{3b^2\left(e^{n\ln(x)}\right)^2ac}{2d^2n} + \frac{b^3\left(e^{n\ln(x)}\right.}{2d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a^3-3/d^2/n*exp(n*ln(x))*a^2*b*c+3/d^3/n*exp(n*ln(x))*a*
b^2*c^2-1/d^4/n*exp(n*ln(x))*b^3*c^3+1/4*b^3/d/n*exp(n*ln(x))^4+3/2*b/d/n*e
xp(n*ln(x))^2*a^2-3/2*b^2/d^2/n*exp(n*ln(x))^2*a*c+1/2*b^3/d^3/n*exp(n*ln(x
```

```
))^2*c^2+b^2/d/n*exp(n*ln(x))^3*a-1/3*b^3/d^2/n*exp(n*ln(x))^3*c-c/d^2/n*ln
(c+d*exp(n*ln(x)))*a^3+3*c^2/d^3/n*ln(c+d*exp(n*ln(x)))*a^2*b-3*c^3/d^4/n*l
n(c+d*exp(n*ln(x)))*a*b^2+c^4/d^5/n*ln(c+d*exp(n*ln(x)))*b^3
```

Maxima [A] time = 0.967582, size = 312, normalized size = 2.4

$$a^3\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) + \frac{1}{12}\,b^3\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3\,d^3x^{4\,n} - 4\,cd^2x^{3\,n} + 6\,c^2dx^{2\,n} - 12\,c^3x^n}{d^4n}\right) - \frac{1}{2}\,ab^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^3*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*log((d*x^
n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) - 12
*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x
^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*log((d*x^n
+ c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A] time = 1.04555, size = 363, normalized size = 2.79

$$\frac{3\,b^3d^4x^{4\,n} - 4\left(b^3cd^3 - 3\,ab^2d^4\right)x^{3\,n} + 6\left(b^3c^2d^2 - 3\,ab^2cd^3 + 3\,a^2bd^4\right)x^{2\,n} - 12\left(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4\right)x^n + \cdots}{12\,d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^3*d^4*x^(4*n) - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^(3*n) + 6*(b^3*c^2*
d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^(2*n) - 12*(b^3*c^3*d - 3*a*b^2*c^2*d^
2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^
2*d^2 - a^3*c*d^3)*log(d*x^n + c))/(d^5*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^3 x^{2\,n-1}}{dx^n + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)
```

3.1040 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal. Leaf size=90

$$\frac{x^n(bc-ad)^2}{d^3n}-\frac{bx^{2n}(bc-2ad)}{2d^2n}-\frac{c(bc-ad)^2\log\left(c+dx^n\right)}{d^4n}+\frac{b^2x^{3n}}{3dn}$$

```
[Out] ((b*c - a*d)^2*x^n)/(d^3*n) - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2*n) + (b^2*x^
(3*n))/(3*d*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)
```

Rubi [A] time = 0.0893512, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{x^n(bc-ad)^2}{d^3n}-\frac{bx^{2n}(bc-2ad)}{2d^2n}-\frac{c(bc-ad)^2\log\left(c+dx^n\right)}{d^4n}+\frac{b^2x^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] ((b*c - a*d)^2*x^n)/(d^3*n) - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2*n) + (b^2*x^
(3*n))/(3*d*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3} - \frac{b(bc-2ad)x}{d^2} + \frac{b^2x^2}{d} - \frac{c(bc-ad)^2}{d^3(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= \frac{(bc-ad)^2x^n}{d^3n} - \frac{b(bc-2ad)x^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} - \frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}$$

Mathematica [A] time = 0.11322, size = 82, normalized size = 0.91

$$\frac{\frac{x^n(bc-ad)^2}{d^3} - \frac{bx^{2n}(bc-2ad)}{2d^2} - \frac{c(bc-ad)^2\log(c+dx^n)}{d^4} + \frac{b^2x^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (((b*c - a*d)^2*x^n)/d^3 - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2) + (b^2*x^(3*n)
)/(3*d) - (c*(b*c - a*d)^2*Log[c + d*x^n])/d^4)/n
```

Maple [A] time = 0.023, size = 173, normalized size = 1.9

$$\frac{e^{n\ln(x)}a^2}{dn} - 2\frac{e^{n\ln(x)}abc}{d^2n} + \frac{e^{n\ln(x)}b^2c^2}{d^3n} + \frac{b^2\left(e^{n\ln(x)}\right)^3}{3dn} + \frac{b\left(e^{n\ln(x)}\right)^2a}{dn} - \frac{b^2\left(e^{n\ln(x)}\right)^2c}{2d^2n} - \frac{c\ln\left(c+de^{n\ln(x)}\right)a^2}{d^2n} + 2\frac{c^2\ln\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a^2-2/d^2/n*exp(n*ln(x))*a*b*c+1/d^3/n*exp(n*ln(x))*b^2*
c^2+1/3*b^2/d/n*exp(n*ln(x))^3+b/d/n*exp(n*ln(x))^2*a-1/2*b^2/d^2/n*exp(n*l
n(x))^2*c-c/d^2/n*ln(c+d*exp(n*ln(x)))*a^2+2*c^2/d^3/n*ln(c+d*exp(n*ln(x)))
*a*b-c^3/d^4/n*ln(c+d*exp(n*ln(x)))*b^2
```

Maxima [A] time = 0.950214, size = 203, normalized size = 2.26

$$a^2\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) - \frac{1}{6}b^2\left(\frac{6c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2d^2x^{3n} - 3cdx^{2n} + 6c^2x^n}{d^3n}\right) + ab\left(\frac{2c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2n} - 2cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) - 1/6*b^2*(6*c^3*log((d*x^n
+ c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + a*
b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A] time = 1.11146, size = 227, normalized size = 2.52

$$\frac{2b^2d^3x^{3n} - 3\left(b^2cd^2 - 2abd^3\right)x^{2n} + 6\left(b^2c^2d - 2abcd^2 + a^2d^3\right)x^n - 6\left(b^2c^3 - 2abc^2d + a^2cd^2\right)\log\left(dx^n + c\right)}{6d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x
^n + c))/(d^4*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^2 x^{2n-1}}{dx^n + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(2*n - 1)/(d*x^n + c), x)
```

3.1041 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx$

Optimal. Leaf size=60

$$-\frac{x^n(bc-ad)}{d^2n}+\frac{c(bc-ad)\log\left(c+dx^n\right)}{d^3n}+\frac{bx^{2n}}{2dn}$$

```
[Out] -(((b*c - a*d)*x^n)/(d^2*n)) + (b*x^(2*n))/(2*d*n) + (c*(b*c - a*d)*Log[c +
 d*x^n])/(d^3*n)
```

Rubi [A] time = 0.0540822, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{x^n(bc-ad)}{d^2n}+\frac{c(bc-ad)\log\left(c+dx^n\right)}{d^3n}+\frac{bx^{2n}}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)*x^n)/(d^2*n)) + (b*x^(2*n))/(2*d*n) + (c*(b*c - a*d)*Log[c +
 d*x^n])/(d^3*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{(bc-ad)x^n}{d^2n}+\frac{bx^{2n}}{2dn}+\frac{c(bc-ad)\log(c+dx^n)}{d^3n}$$

Mathematica [A] time = 0.0469774, size = 50, normalized size = 0.83

$$\frac{dx^n(2ad-2bc+bdx^n)+2c(bc-ad)\log(c+dx^n)}{2d^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3*
n)
```

Maple [A] time = 0.02, size = 87, normalized size = 1.5

$$\frac{e^{n\ln(x)}a}{dn}-\frac{be^{n\ln(x)}c}{d^2n}+\frac{b\left(e^{n\ln(x)}\right)^2}{2\,dn}-\frac{c\ln\left(c+de^{n\ln(x)}\right)a}{d^2n}+\frac{c^2\ln\left(c+de^{n\ln(x)}\right)b}{d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a-1/d^2/n*exp(n*ln(x))*b*c+1/2*b/d/n*exp(n*ln(x))^2-c/d^
2/n*ln(c+d*exp(n*ln(x)))*a+c^2/d^3/n*ln(c+d*exp(n*ln(x)))*b
```

Maxima [A] time = 0.96418, size = 112, normalized size = 1.87

$$a\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)+\frac{1}{2}\,b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/2*b*(2*c^2*log((d*x^n + c)
/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A] time = 1.04103, size = 119, normalized size = 1.98

$$\frac{bd^2x^{2n} - 2\left(bcd - ad^2\right)x^n + 2\left(bc^2 - acd\right)\log\left(dx^n + c\right)}{2\,d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n +
c))/(d^3*n)
```

Sympy [A] time = 36.535, size = 105, normalized size = 1.75

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for}\ d = 0 \wedge n = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for}\ n = 0 \\ \frac{\frac{ax^{2n}}{2n} + \frac{bx^{3n}}{3n}}{c} & \text{for}\ d = 0 \\ -\frac{ac\log\left(\frac{c}{d}+x^n\right)}{d^2n} + \frac{ax^n}{dn} + \frac{bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{bcx^n}{d^2n} + \frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a + b)*log(x)/(c + d),
 Eq(n, 0)), ((a*x**(2*n)/(2*n) + b*x**(3*n)/(3*n))/c, Eq(d, 0)), (-a*c*log(
c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d + x**n)/(d**3*n) - b*c
*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)x^{2n-1}}{dx^n + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(2*n - 1)/(d*x^n + c), x)
```

3.1042 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} \, dx$

Optimal. Leaf size=54

$$\frac{c \log (c + dx^n)}{dn(bc - ad)} - \frac{a \log (a + bx^n)}{bn(bc - ad)}$$

```
[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
*n)
```

Rubi [A] time = 0.0543653, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 72}

$$\frac{c \log (c + dx^n)}{dn(bc - ad)} - \frac{a \log (a + bx^n)}{bn(bc - ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{a\log(a+bx^n)}{b(bc-ad)n} + \frac{c\log(c+dx^n)}{d(bc-ad)n}$$

Mathematica [A] time = 0.0350836, size = 44, normalized size = 0.81

$$-\frac{ad\log(a+bx^n) - bc\log(c+dx^n)}{b^2cdn - abd^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*d*Log[a + b*x^n] - b*c*Log[c + d*x^n])/(b^2*c*d*n - a*b*d^2*n))
```

Maple [A] time = 0.02, size = 59, normalized size = 1.1

$$\frac{a\ln\left(a+be^{n\ln(x)}\right)}{(ad-bc)\,bn} - \frac{c\ln\left(c+de^{n\ln(x)}\right)}{dn\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] a/(a*d-b*c)/b/n*ln(a+b*exp(n*ln(x)))-c/d/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))
```

Maxima [A] time = 0.945042, size = 81, normalized size = 1.5

$$-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2cn - abdn} + \frac{c\log\left(\frac{dx^n+c}{d}\right)}{bcdn - ad^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n -
 a*d^2*n)
```

Fricas [A] time = 1.07062, size = 92, normalized size = 1.7

$$-\frac{ad\log\left(bx^n+a\right)-bc\log\left(dx^n+c\right)}{\left(b^2cd-abd^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a*d*log(b*x^n + a) - b*c*log(d*x^n + c))/((b^2*c*d - a*b*d^2)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2\,n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

3.1043 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=75

$$\frac{a}{bn(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{n(bc-ad)^2}-\frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Rubi [A] time = 0.0624375, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{a}{bn(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{n(bc-ad)^2}-\frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2} + \frac{bc}{(bc-ad)^2(a+bx)} - \frac{cd}{(bc-ad)^2(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= \frac{a}{b(bc-ad)n(a+bx^n)} + \frac{c\log(a+bx^n)}{(bc-ad)^2n} - \frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

Mathematica [A] time = 0.0974601, size = 58, normalized size = 0.77

$$\frac{\frac{a(bc-ad)}{b(a+bx^n)} + c\log(a+bx^n) - c\log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] ((a*(b*c - a*d))/(b*(a + b*x^n)) + c*Log[a + b*x^n] - c*Log[c + d*x^n])/((b
*c - a*d)^2*n)
```

Maple [A] time = 0.03, size = 109, normalized size = 1.5

$$\frac{e^{n\ln(x)}}{(ad-bc)\,n\left(a+be^{n\ln(x)}\right)} + \frac{c\ln\left(a+be^{n\ln(x)}\right)}{n\left(a^2d^2-2\,abcd+b^2c^2\right)} - \frac{c\ln\left(c+de^{n\ln(x)}\right)}{n\left(a^2d^2-2\,abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] 1/(a*d-b*c)/n*exp(n*ln(x))/(a+b*exp(n*ln(x)))+c/n/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(a+b*exp(n*ln(x)))-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(c+d*exp(n*ln(x))
)
```

Maxima [A] time = 0.95208, size = 163, normalized size = 2.17

$$\frac{c\log\left(\frac{bx^n+a}{b}\right)}{b^2c^2n - 2\,abcdn + a^2d^2n} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2n - 2\,abcdn + a^2d^2n} + \frac{a}{ab^2cn - a^2bdn + \left(b^3cn - ab^2dn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c*log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*log((d*x^n +
 c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n + (
b^3*c*n - a*b^2*d*n)*x^n)
```

Fricas [A] time = 1.07292, size = 244, normalized size = 3.25

$$\frac{abc - a^2d + \left(b^2cx^n + abc\right)\log\left(bx^n + a\right) - \left(b^2cx^n + abc\right)\log\left(dx^n + c\right)}{\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)nx^n + \left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*log(b*x^n + a) - (b^2*c*x^n + a*b*c)*l
og(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2 -
2*a^2*b^2*c*d + a^3*b*d^2)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{(bx^n + a)^2 (dx^n + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1044 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)} dx$

Optimal. Leaf size=105

$$\frac{a}{2bn(bc-ad)(a+bx^n)^2}-\frac{c}{n(bc-ad)^2(a+bx^n)}-\frac{cd\log(a+bx^n)}{n(bc-ad)^3}+\frac{cd\log(c+dx^n)}{n(bc-ad)^3}$$

```
[Out] a/(2*b*(b*c - a*d)*n*(a + b*x^n)^2) - c/((b*c - a*d)^2*n*(a + b*x^n)) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n
)
```

Rubi [A] time = 0.0899089, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{a}{2bn(bc-ad)(a+bx^n)^2}-\frac{c}{n(bc-ad)^2(a+bx^n)}-\frac{cd\log(a+bx^n)}{n(bc-ad)^3}+\frac{cd\log(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*n*(a + b*x^n)^2) - c/((b*c - a*d)^2*n*(a + b*x^n)) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n
)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^3(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^3} + \frac{bc}{(bc-ad)^2(a+bx)^2} - \frac{bcd}{(bc-ad)^3(a+bx)} + \frac{cd^2}{(bc-ad)^3(c+dx)}\right) dx, x, x^n\right)}{n}$$
$$= \frac{a}{2b(bc-ad)n(a+bx^n)^2} - \frac{c}{(bc-ad)^2n(a+bx^n)} - \frac{cd\log(a+bx^n)}{(bc-ad)^3n} + \frac{cd\log(c+dx^n)}{(bc-ad)^3n}$$

Mathematica [A] time = 0.11534, size = 97, normalized size = 0.92

$$\frac{\frac{a}{2b(bc-ad)(a+bx^n)^2} - \frac{c}{(bc-ad)^2(a+bx^n)} - \frac{cd\log(a+bx^n)}{(bc-ad)^3} + \frac{cd\log(c+dx^n)}{(bc-ad)^3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (a/(2*b*(b*c - a*d)*(a + b*x^n)^2) - c/((b*c - a*d)^2*(a + b*x^n)) - (c*d*L
og[a + b*x^n])/(b*c - a*d)^3 + (c*d*Log[c + d*x^n])/(b*c - a*d)^3)/n
```

Maple [A] time = 0.056, size = 203, normalized size = 1.9

$$\frac{1}{\left(a+be^{n\ln(x)}\right)^2}\left(-\frac{bce^{n\ln(x)}}{\left(a^2d^2-2\,abcd+b^2c^2\right)n} + \frac{a\left(-adb-b^2c\right)}{\left(2\,a^2d^2-4\,abcd+2\,b^2c^2\right)b^2n}\right) + \frac{cd\ln\left(a+be^{n\ln(x)}\right)}{n\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x)
```

```
[Out] (-b*c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/2*a*(-a*b*d-b^2*c)/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(a+b*exp(n*ln(x)))^2+c*d/n/(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(a+b*exp(n*ln(x)))-c*d/n/(a^3*d^3-3*a^2*b*c*d
```

```
^2+3*a*b^2*c^2*d-b^3*c^3)*ln(c+d*exp(n*ln(x)))
```

Maxima [B] time = 0.96995, size = 328, normalized size = 3.12

$$-\frac{cd\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n}+\frac{cd\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n}-\frac{}{2\left(a^2b^3c^2n-2\,a^3b^2cdn+a^4bd^2n+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^
3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*
d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n - 2*a
^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n)*x^
(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)
```

Fricas [B] time = 1.10648, size = 540, normalized size = 5.14

$$-\frac{ab^2c^2-a^3d^2+2\left(b^3c^2-ab^2cd\right)x^n+2\left(b^3cdx^{2n}+2\,ab^2cdx^n+a^2bcd\right)\log\left(bx^n+a\right)-2\left(b^3cdx^{2n}+2\,ab^2cdx^n+a^2b\right.}{2\left(\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)nx^{2n}+2\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)nx^n+\left(a^2b^4c^3-3\,a^3b^3c^2d+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*n
) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*a*
b^2*c*d*x^n + a^2*b*c*d)*log(d*x^n + c))/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*
b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^
2*c*d^2 - a^5*b*d^3)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{(bx^n + a)^3 (dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

3.1045 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n} dx$

Optimal. Leaf size=158

$$\frac{bx^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)}{3d^3n}-\frac{b^2x^{4n}(bc-3ad)}{4d^2n}-\frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}+\frac{cx^n(bc-ad)^3}{d^5n}-\frac{x^{2n}(bc-ad)^3}{2d^4n}+\frac{b^3x^{5n}}{5dn}$$

```
[Out] (c*(b*c - a*d)^3*x^n)/(d^5*n) - ((b*c - a*d)^3*x^(2*n))/(2*d^4*n) + (b*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3*n) - (b^2*(b*c - 3*a*d)*x^(4*
n))/(4*d^2*n) + (b^3*x^(5*n))/(5*d*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/
(d^6*n)
```

Rubi [A] time = 0.150764, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$\frac{bx^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)}{3d^3n}-\frac{b^2x^{4n}(bc-3ad)}{4d^2n}-\frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}+\frac{cx^n(bc-ad)^3}{d^5n}-\frac{x^{2n}(bc-ad)^3}{2d^4n}+\frac{b^3x^{5n}}{5dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```


```
[Out] (c*(b*c - a*d)^3*x^n)/(d^5*n) - ((b*c - a*d)^3*x^(2*n))/(2*d^4*n) + (b*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3*n) - (b^2*(b*c - 3*a*d)*x^(4*
n))/(4*d^2*n) + (b^3*x^(5*n))/(5*d*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/
(d^6*n)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^3}{c+dx}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)^3}{d^5} + \frac{(-bc+ad)^3x}{d^4} + \frac{b(b^2c^2-3abcd+3a^2d^2)x^2}{d^3} - \frac{b^2(bc-3ad)x^3}{d^2} + \frac{b^3x^4}{d} - \frac{c^2(bc-ad)^3}{d^5(c+dx)}\right) dx, x, x\right.}{n}$$

$$= \frac{c(bc-ad)^3x^n}{d^5n} - \frac{(bc-ad)^3x^{2n}}{2d^4n} + \frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^{3n}}{3d^3n} - \frac{b^2(bc-3ad)x^{4n}}{4d^2n} + \frac{b^3x^{5n}}{5dn} - \cdots$$

Mathematica [A] time = 0.215564, size = 138, normalized size = 0.87

$$\frac{20bd^3x^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)-15b^2d^4x^{4n}(bc-3ad)-60c^2(bc-ad)^3\log\left(c+dx^n\right)+30d^2x^{2n}(ad-bc)^3+60cdx^n(bc\cdots}{60d^6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (60*c*d*(b*c - a*d)^3*x^n + 30*d^2*(-(b*c) + a*d)^3*x^(2*n) + 20*b*d^3*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n) - 15*b^2*d^4*(b*c - 3*a*d)*x^(4*n) +
12*b^3*d^5*x^(5*n) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)
```

Maple [B] time = 0.036, size = 342, normalized size = 2.2

$$\frac{b^3(x^n)^5}{5dn} + \frac{3b^2(x^n)^4a}{4dn} - \frac{b^3(x^n)^4c}{4d^2n} + \frac{b(x^n)^3a^2}{dn} - \frac{b^2(x^n)^3ac}{d^2n} + \frac{b^3(x^n)^3c^2}{3d^3n} + \frac{(x^n)^2a^3}{2dn} - \frac{3(x^n)^2a^2bc}{2d^2n} + \frac{3(x^n)^2ab^2c^2}{2d^3n} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x)
```

```
[Out] 1/5*b^3/d/n*(x^n)^5+3/4*b^2/d/n*(x^n)^4*a-1/4*b^3/d^2/n*(x^n)^4*c+b/d/n*(x^
n)^3*a^2-b^2/d^2/n*(x^n)^3*a*c+1/3*b^3/d^3/n*(x^n)^3*c^2+1/2/d/n*(x^n)^2*a^
3-3/2/d^2/n*(x^n)^2*a^2*b*c+3/2/d^3/n*(x^n)^2*a*b^2*c^2-1/2/d^4/n*(x^n)^2*b
^3*c^3-c/d^2/n*x^n*a^3+3*c^2/d^3/n*x^n*a^2*b-3*c^3/d^4/n*x^n*a*b^2+c^4/d^5/
```

```
n*x^n*b^3+c^2/d^3/n*ln(x^n+1/d*c)*a^3-3*c^3/d^4/n*ln(x^n+1/d*c)*a^2*b+3*c^4
/d^5/n*ln(x^n+1/d*c)*a*b^2-c^5/d^6/n*ln(x^n+1/d*c)*b^3
```

Maxima [A] time = 0.970937, size = 386, normalized size = 2.44

$$-\frac{1}{60}b^3\left(\frac{60c^5\log\left(\frac{dx^n+c}{d}\right)}{d^6n}-\frac{12d^4x^{5n}-15cd^3x^{4n}+20c^2d^2x^{3n}-30c^3dx^{2n}+60c^4x^n}{d^5n}\right)+\frac{1}{4}ab^2\left(\frac{12c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3d^3x^{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/60*b^3*(60*c^5*log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^(5*n) - 15*c*d^3*x
^(4*n) + 20*c^2*d^2*x^(3*n) - 30*c^3*d*x^(2*n) + 60*c^4*x^n)/(d^5*n)) + 1/4
*a*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n
) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^3*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A] time = 1.0844, size = 471, normalized size = 2.98

$$\frac{12b^3d^5x^{5n}-15\left(b^3cd^4-3ab^2d^5\right)x^{4n}+20\left(b^3c^2d^3-3ab^2cd^4+3a^2bd^5\right)x^{3n}-30\left(b^3c^3d^2-3ab^2c^2d^3+3a^2bcd^4-a^3d^5\right)x^{\cdots}}{60d^6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^3*d^5*x^(5*n) - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^(4*n) + 20*(b^3*c
^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^(3*n) - 30*(b^3*c^3*d^2 - 3*a*b^2*c
^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^(2*n) + 60*(b^3*c^4*d - 3*a*b^2*c^3*d^2
 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b
*c^3*d^2 - a^3*c^2*d^3)*log(d*x^n + c))/(d^6*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^3 x^{3n-1}}{dx^n + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)
```

3.1046 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal. Leaf size=118

$$\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

```
[Out] -((c*(b*c - a*d)^2*x^n)/(d^4*n)) + ((b*c - a*d)^2*x^(2*n))/(2*d^3*n) - (b*(
b*c - 2*a*d)*x^(3*n))/(3*d^2*n) + (b^2*x^(4*n))/(4*d*n) + (c^2*(b*c - a*d)^
2*Log[c + d*x^n])/(d^5*n)
```

Rubi [A] time = 0.115469, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```


```
[Out] -((c*(b*c - a*d)^2*x^n)/(d^4*n)) + ((b*c - a*d)^2*x^(2*n))/(2*d^3*n) - (b*(
b*c - 2*a*d)*x^(3*n))/(3*d^2*n) + (b^2*x^(4*n))/(4*d*n) + (c^2*(b*c - a*d)^
2*Log[c + d*x^n])/(d^5*n)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{c+dx}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^4}+\frac{(-bc+ad)^2x}{d^3}-\frac{b(bc-2ad)x^2}{d^2}+\frac{b^2x^3}{d}+\frac{c^2(bc-ad)^2}{d^4(c+dx)}\right)dx,x,x^n\right)}{n}$$
$$= -\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}$$

Mathematica [A] time = 0.146358, size = 103, normalized size = 0.87

$$\frac{12c^2(bc-ad)^2\log(c+dx^n)+6d^2x^{2n}(bc-ad)^2-4bd^3x^{3n}(bc-2ad)-12cdx^n(bc-ad)^2+3b^2d^4x^{4n}}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (-12*c*d*(b*c - a*d)^2*x^n + 6*d^2*(b*c - a*d)^2*x^(2*n) - 4*b*d^3*(b*c - 2
*a*d)*x^(3*n) + 3*b^2*d^4*x^(4*n) + 12*c^2*(b*c - a*d)^2*Log[c + d*x^n])/(1
2*d^5*n)
```

Maple [B] time = 0.026, size = 236, normalized size = 2.

$$\frac{b^2\left(e^{n\ln(x)}\right)^4}{4\,dn}+\frac{\left(e^{n\ln(x)}\right)^2a^2}{2\,dn}-\frac{\left(e^{n\ln(x)}\right)^2abc}{d^2n}+\frac{\left(e^{n\ln(x)}\right)^2b^2c^2}{2\,d^3n}+\frac{2\,b\left(e^{n\ln(x)}\right)^3a}{3\,dn}-\frac{b^2\left(e^{n\ln(x)}\right)^3c}{3\,d^2n}-\frac{ce^{n\ln(x)}a^2}{d^2n}+2\,\frac{c^2e^{n\ln}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] 1/4*b^2/d/n*exp(n*ln(x))^4+1/2/d/n*exp(n*ln(x))^2*a^2-1/d^2/n*exp(n*ln(x))^
2*a*b*c+1/2/d^3/n*exp(n*ln(x))^2*b^2*c^2+2/3*b/d/n*exp(n*ln(x))^3*a-1/3*b^2
/d^2/n*exp(n*ln(x))^3*c-c/d^2/n*exp(n*ln(x))*a^2+2*c^2/d^3/n*exp(n*ln(x))*a
*b-c^3/d^4/n*exp(n*ln(x))*b^2+c^2/d^3/n*ln(c+d*exp(n*ln(x)))*a^2-2*c^3/d^4/
n*ln(c+d*exp(n*ln(x)))*a*b+c^4/d^5/n*ln(c+d*exp(n*ln(x)))*b^2
```

Maxima [A] time = 0.961493, size = 259, normalized size = 2.19

$$\frac{1}{12}\,b^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)-\frac{1}{3}\,ab\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] 1/12*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3
*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/3*a*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^2*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Fricas [A] time = 1.06209, size = 306, normalized size = 2.59

$$\frac{3\,b^2d^4x^{4\,n}-4\left(b^2cd^3-2\,abd^4\right)x^{3\,n}+6\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^{2\,n}-12\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^n+12\left(b^2c^4-2\,abc^3\right.}{12\,d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^2 x^{3n-1}}{dx^n + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)
```

3.1047 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx$

Optimal. Leaf size=86

$$-\frac{c^2(bc-ad)\log\left(c+dx^n\right)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

```
[Out] (c*(b*c - a*d)*x^n)/(d^3*n) - ((b*c - a*d)*x^(2*n))/(2*d^2*n) + (b*x^(3*n))
/(3*d*n) - (c^2*(b*c - a*d)*Log[c + d*x^n])/(d^4*n)
```

Rubi [A] time = 0.0755024, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{c^2(bc-ad)\log\left(c+dx^n\right)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)*x^n)/(d^3*n) - ((b*c - a*d)*x^(2*n))/(2*d^2*n) + (b*x^(3*n))
/(3*d*n) - (c^2*(b*c - a*d)*Log[c + d*x^n])/(d^4*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{c+dx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^3} + \frac{(-bc+ad)x}{d^2} + \frac{bx^2}{d} - \frac{c^2(bc-ad)}{d^3(c+dx)}\right) dx, x, x^n\right)}{n}$$
$$= \frac{c(bc-ad)x^n}{d^3n} - \frac{(bc-ad)x^{2n}}{2d^2n} + \frac{bx^{3n}}{3dn} - \frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}$$

Mathematica [A] time = 0.0774008, size = 78, normalized size = 0.91

$$\frac{-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4} + \frac{cx^n(bc-ad)}{d^3} - \frac{x^{2n}(bc-ad)}{2d^2} + \frac{bx^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] ((c*(b*c - a*d)*x^n)/d^3 - ((b*c - a*d)*x^(2*n))/(2*d^2) + (b*x^(3*n))/(3*d
) - (c^2*(b*c - a*d)*Log[c + d*x^n])/d^4)/n
```

Maple [A] time = 0.023, size = 125, normalized size = 1.5

$$\frac{b\left(e^{n\ln(x)}\right)^3}{3\,dn} + \frac{\left(e^{n\ln(x)}\right)^2 a}{2\,dn} - \frac{\left(e^{n\ln(x)}\right)^2 bc}{2\,d^2n} - \frac{ce^{n\ln(x)}a}{d^2n} + \frac{c^2e^{n\ln(x)}b}{d^3n} + \frac{c^2\ln\left(c+de^{n\ln(x)}\right)a}{d^3n} - \frac{c^3\ln\left(c+de^{n\ln(x)}\right)b}{d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] 1/3*b/d/n*exp(n*ln(x))^3+1/2/d/n*exp(n*ln(x))^2*a-1/2/d^2/n*exp(n*ln(x))^2*
b*c-c/d^2/n*exp(n*ln(x))*a+c^2/d^3/n*exp(n*ln(x))*b+c^2/d^3/n*ln(c+d*exp(n*
ln(x)))*a-c^3/d^4/n*ln(c+d*exp(n*ln(x)))*b
```

Maxima [A] time = 0.960865, size = 151, normalized size = 1.76

$$-\frac{1}{6}\,b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/6*b*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) +
 6*c^2*x^n)/(d^3*n)) + 1/2*a*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n)
 - 2*c*x^n)/(d^2*n))
```

Fricas [A] time = 1.08035, size = 170, normalized size = 1.98

$$\frac{2\,bd^3x^{3\,n}-3\left(bcd^2-ad^3\right)x^{2\,n}+6\left(bc^2d-acd^2\right)x^n-6\left(bc^3-ac^2d\right)\log\left(dx^n+c\right)}{6\,d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)*
x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)x^{3\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")

[Out] integrate((b*x^n + a)*x^(3*n - 1)/(d*x^n + c), x)
```

3.1048 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=71

$$\frac{a^2\log(a+bx^n)}{b^2n(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

```
[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) - (c^2*Log[c + d*x^n
])/(d^2*(b*c - a*d)*n)
```

Rubi [A] time = 0.0691811, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 72}

$$\frac{a^2\log(a+bx^n)}{b^2n(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) - (c^2*Log[c + d*x^n
])/(d^2*(b*c - a*d)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= \frac{x^n}{bdn} + \frac{a^2\log(a+bx^n)}{b^2(bc-ad)n} - \frac{c^2\log(c+dx^n)}{d^2(bc-ad)n}$$

Mathematica [A] time = 0.0665049, size = 66, normalized size = 0.93

$$\frac{\frac{a^2\log(a+bx^n)}{b^2(bc-ad)} - \frac{c^2\log(c+dx^n)}{d^2(bc-ad)} + \frac{x^n}{bd}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x^n/(b*d) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^n])/
(d^2*(b*c - a*d)))/n
```

Maple [A] time = 0.026, size = 78, normalized size = 1.1

$$\frac{e^{n\ln(x)}}{bdn} + \frac{c^2\ln\left(c + de^{n\ln(x)}\right)}{d^2n(ad-bc)} - \frac{a^2\ln\left(a + be^{n\ln(x)}\right)}{(ad-bc)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] 1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))-a^2/(a*d-b*c)
/b^2/n*ln(a+b*exp(n*ln(x)))
```

Maxima [A] time = 0.937863, size = 109, normalized size = 1.54

$$\frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3cn - ab^2dn} - \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{bcd^2n - ad^3n} + \frac{x^n}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c*
d^2*n - a*d^3*n) + x^n/(b*d*n)
```

Fricas [A] time = 1.09306, size = 142, normalized size = 2.

$$\frac{a^2d^2\log\left(bx^n+a\right)-b^2c^2\log\left(dx^n+c\right)+\left(b^2cd-abd^2\right)x^n}{\left(b^3cd^2-ab^2d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n)
/((b^3*c*d^2 - a*b^2*d^3)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3\,n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

3.1049 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2*n)
```

Rubi [A] time = 0.0903976, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)} + \frac{c^2}{(bc-ad)^2(c+dx)}\right) dx, x, x^n\right)}{n}$$
$$= -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}$$

Mathematica [A] time = 0.096695, size = 90, normalized size = 0.95

$$\frac{-\frac{a^2}{b^2(bc-ad)(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (-(a^2/(b^2*(b*c - a*d)*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(b
^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2))/n
```

Maple [A] time = 0.03, size = 163, normalized size = 1.7

$$\frac{a^2}{(ad-bc)\,b^2n\left(a+b\mathrm{e}^{n\ln(x)}\right)} + \frac{c^2\ln\left(c+d\mathrm{e}^{n\ln(x)}\right)}{dn\left(a^2d^2-2\,abcd+b^2c^2\right)} + \frac{a^2\ln\left(a+b\mathrm{e}^{n\ln(x)}\right)d}{b^2\left(a^2d^2-2\,abcd+b^2c^2\right)n} - 2\,\frac{a\ln\left(a+b\mathrm{e}^{n\ln(x)}\right)c}{\left(a^2d^2-2\,abcd+b^2c^2\right)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] a^2/(a*d-b*c)/b^2/n/(a+b*exp(n*ln(x)))+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*
ln(c+d*exp(n*ln(x)))+a^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*ln(a+b*exp(n*ln(
x)))*d-2*a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/n*ln(a+b*exp(n*ln(x)))*c
```

Maxima [A] time = 0.949438, size = 198, normalized size = 2.08

$$\frac{c^2 \log\left(\frac{dx^n+c}{d}\right)}{b^2c^2dn - 2\,abcd^2n + a^2d^3n} - \frac{a^2}{ab^3cn - a^2b^2dn + \left(b^4cn - ab^3dn\right)x^n} - \frac{\left(2\,abc - a^2d\right)\log\left(\frac{bx^n+a}{b}\right)}{b^4c^2n - 2\,ab^3cdn + a^2b^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*b
^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*log((
b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)
```

Fricas [A] time = 1.10814, size = 324, normalized size = 3.41

$$-\frac{a^2bcd - a^3d^2 + \left(2\,a^2bcd - a^3d^2 + \left(2\,ab^2cd - a^2bd^2\right)x^n\right)\log\left(bx^n + a\right) - \left(b^3c^2x^n + ab^2c^2\right)\log\left(dx^n + c\right)}{\left(b^5c^2d - 2\,ab^4cd^2 + a^2b^3d^3\right)nx^n + \left(ab^4c^2d - 2\,a^2b^3cd^2 + a^3b^2d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)*
x^n)*log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*log(d*x^n + c))/((b^5*c^2*d
 - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^
3*b^2*d^3)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.1050 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal. Leaf size=120

$$-\frac{a^2}{2b^2n(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{n(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{n(bc-ad)^3}$$

```
[Out] -a^2/(2*b^2*(b*c - a*d)*n*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a*
d)^2*n*(a + b*x^n)) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c +
 d*x^n])/((b*c - a*d)^3*n)
```

Rubi [A] time = 0.105352, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$-\frac{a^2}{2b^2n(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{n(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] -a^2/(2*b^2*(b*c - a*d)*n*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a*
d)^2*n*(a + b*x^n)) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c +
 d*x^n])/((b*c - a*d)^3*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2} + \frac{bc^2}{(bc-ad)^3(a+bx)} - \frac{c^2d}{(bc-ad)^3(c+dx)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{a^2}{2b^2(bc-ad)n(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2(bc-ad)^2n(a+bx^n)} + \frac{c^2\log(a+bx^n)}{(bc-ad)^3n} - \frac{c^2\log(c+dx^n)}{(bc-ad)^3n}$$

Mathematica [A] time = 0.139842, size = 112, normalized size = 0.93

$$\frac{-\frac{a^2}{2b^2(bc-ad)(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2(bc-ad)^2(a+bx^n)} + \frac{c^2\log(a+bx^n)}{(bc-ad)^3} - \frac{c^2\log(c+dx^n)}{(bc-ad)^3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (-a^2/(2*b^2*(b*c - a*d)*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a*d
)^2*(a + b*x^n)) + (c^2*Log[a + b*x^n])/(b*c - a*d)^3 - (c^2*Log[c + d*x^n]
)/(b*c - a*d)^3)/n
```

Maple [A] time = 0.054, size = 214, normalized size = 1.8

$$\frac{1}{\left(a+b\mathrm{e}^{n\ln(x)}\right)^2}\left(\frac{(-ad+2\,bc)\,a\mathrm{e}^{n\ln(x)}}{nb\left(a^2d^2-2\,abcd+b^2c^2\right)} + \frac{a^2\,(-ad+3\,bc)}{\left(2\,a^2d^2-4\,abcd+2\,b^2c^2\right)b^2n}\right) + \frac{c^2\ln\left(c+d\mathrm{e}^{n\ln(x)}\right)}{n\left(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x)
```

```
[Out] ((-a*d+2*b*c)/n*a/b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*exp(n*ln(x))+1/2*a^2*(-a*d+
3*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(a+b*exp(n*ln(x)))^2+c^2/n/(a^3*d
^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(c+d*exp(n*ln(x)))-c^2/n/(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(a+b*exp(n*ln(x)))
```

Maxima [B] time = 0.972715, size = 354, normalized size = 2.95

$$\frac{c^2 \log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} - \frac{c^2 \log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} + \frac{}{2\left(a^2b^4c^2n - 2\,a^3b^3cdn + a^4b^2d^2n + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3
*d^3*n) - c^2*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d
^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n)/(
a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*c*d*
n + a^2*b^4*d^2*n)*x^(2*n) + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^3*d^2
*n)*x^n)
```

Fricas [B] time = 1.10577, size = 598, normalized size = 4.98

$$\frac{3\,a^2b^2c^2 - 4\,a^3bcd + a^4d^2 + 2\left(2\,ab^3c^2 - 3\,a^2b^2cd + a^3bd^2\right)x^n + 2\left(b^4c^2x^{2\,n} + 2\,ab^3c^2x^n + a^2b^2c^2\right)\log\left(bx^n + a\right) - 2\left(b^4c^2x^2\right.}{2\left(\left(b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3\right)nx^{2\,n} + 2\left(ab^6c^3 - 3\,a^2b^5c^2d + 3\,a^3b^4cd^2 - a^4b^3d^3\right)nx^n + \left(a^2b^5c^3 - 3\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log
(b*x^n + a) - 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log(d*x^n
 + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^(2*n)
 + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x^n +
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{(bx^n+a)^3(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

3.1051 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

```
[Out] (x^14*(b + c*x)^14)/14
```

Rubi [A] time = 0.0018061, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (x^14*(b + c*x)^14)/14
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}x^{14}(b+cx)^{14}$$

Mathematica [B] time = 0.0050095, size = 172, normalized size = 12.29

$$\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}+143b^9c^5x^{19}+\frac{143}{2}b^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [B] time = 0.003, size = 155, normalized size = 11.1

$$\frac{c^{14}x^{28}}{14}+bc^{13}x^{27}+\frac{13\,b^{2}c^{12}x^{26}}{2}+26\,b^{3}c^{11}x^{25}+\frac{143\,b^{4}c^{10}x^{24}}{2}+143\,b^{5}c^{9}x^{23}+\frac{429\,b^{6}c^{8}x^{22}}{2}+\frac{1716\,b^{7}c^{7}x^{21}}{7}+\frac{429\,b^{8}c^{6}x^{20}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(c*x+b)^13*(2*c*x+b),x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

Maxima [B] time = 0.921959, size = 208, normalized size = 14.86

$$\frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^{2}c^{12}x^{26}+26\,b^{3}c^{11}x^{25}+\frac{143}{2}\,b^{4}c^{10}x^{24}+143\,b^{5}c^{9}x^{23}+\frac{429}{2}\,b^{6}c^{8}x^{22}+\frac{1716}{7}\,b^{7}c^{7}x^{21}+\frac{429}{2}\,b^{8}c^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Fricas [B] time = 0.844423, size = 387, normalized size = 27.64

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^{2}+26x^{25}c^{11}b^{3}+\frac{143}{2}x^{24}c^{10}b^{4}+143x^{23}c^{9}b^{5}+\frac{429}{2}x^{22}c^{8}b^{6}+\frac{1716}{7}x^{21}c^{7}b^{7}+\frac{429}{2}x^{20}c^{6}b^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

Sympy [B] time = 0.105289, size = 175, normalized size = 12.5

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(c*x+b)**13*(2*c*x+b),x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Giac [B] time = 1.08615, size = 208, normalized size = 14.86

$$\frac{1}{14}c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

3.1052 $\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rubi [A] time = 0.0413561, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx=\frac{1}{2}\,\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)$$
$$=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Mathematica [B] time = 0.005508, size = 182, normalized size = 11.38

$$\frac{13}{4}b^2c^{12}x^{52}+13b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}+\frac{143}{2}b^9c^5x^{38}+\frac{143}{4}b^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [B] time = 0.002, size = 157, normalized size = 9.8

$$\frac{c^{14}x^{56}}{28}+\frac{bc^{13}x^{54}}{2}+\frac{13\,b^2c^{12}x^{52}}{4}+13\,b^3c^{11}x^{50}+\frac{143\,b^4c^{10}x^{48}}{4}+\frac{143\,b^5c^9x^{46}}{2}+\frac{429\,b^6c^8x^{44}}{4}+\frac{858\,b^7c^7x^{42}}{7}+\frac{429\,b^8c^6x^{40}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

Maxima [B] time = 0.957692, size = 211, normalized size = 13.19

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [B] time = 0.783857, size = 402, normalized size = 25.12

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

Sympy [B] time = 0.108721, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

Giac [B] time = 1.07567, size = 211, normalized size = 13.19

$$\frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.1053 $\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rubi [A] time = 0.0363611, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{1}{3}\,\mathrm{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)$$
$$=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Mathematica [B] time = 0.0060331, size = 186, normalized size = 11.62

$$\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^1$$

Antiderivative was successfully verified.

```
[In] Integrate[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [B] time = 0.003, size = 157, normalized size = 9.8

$$\frac{c^{14}x^{84}}{42}+\frac{bc^{13}x^{81}}{3}+\frac{13\,b^2c^{12}x^{78}}{6}+\frac{26\,b^3c^{11}x^{75}}{3}+\frac{143\,b^4c^{10}x^{72}}{6}+\frac{143\,b^5c^9x^{69}}{3}+\frac{143\,b^6c^8x^{66}}{2}+\frac{572\,b^7c^7x^{63}}{7}+\frac{143\,b^8c^6x^{60}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

Maxima [B] time = 0.929204, size = 211, normalized size = 13.19

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [B] time = 0.839113, size = 408, normalized size = 25.5

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^2 + \frac{26}{3}x^{75}c^{11}b^3 + \frac{143}{6}x^{72}c^{10}b^4 + \frac{143}{3}x^{69}c^9b^5 + \frac{143}{2}x^{66}c^8b^6 + \frac{572}{7}x^{63}c^7b^7 + \frac{143}{2}x^{60}c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

Sympy [B] time = 0.109181, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**41*(c*x**3+b)**13*(2*c*x**3+b),x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

Giac [B] time = 1.08485, size = 211, normalized size = 13.19

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^{2}c^{12}x^{78} + \frac{26}{3}b^{3}c^{11}x^{75} + \frac{143}{6}b^{4}c^{10}x^{72} + \frac{143}{3}b^{5}c^{9}x^{69} + \frac{143}{2}b^{6}c^{8}x^{66} + \frac{572}{7}b^{7}c^{7}x^{63} + \frac{143}{2}b^{8}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

3.1054 $\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi [A] time = 0.0160322, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {446, 74}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{-1+14n}\left(b+cx^n\right)^{13}\left(b+2cx^n\right)\,dx=\frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}$$
$$=\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

Mathematica [A] time = 0.124704, size = 21, normalized size = 1.

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] time = 0.033, size = 230, normalized size = 11.

$$\frac{c^{14}\left(x^n\right)^{28}}{14\,n}+\frac{bc^{13}\left(x^n\right)^{27}}{n}+\frac{13\,b^2c^{12}\left(x^n\right)^{26}}{2\,n}+26\,\frac{b^3c^{11}\left(x^n\right)^{25}}{n}+\frac{143\,b^4c^{10}\left(x^n\right)^{24}}{2\,n}+143\,\frac{b^5c^9\left(x^n\right)^{23}}{n}+\frac{429\,b^6c^8\left(x^n\right)^{22}}{2\,n}+\frac{1716}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.04405, size = 467, normalized size = 22.24

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^{2}c^{12}x^{26\,n} + 364\,b^{3}c^{11}x^{25\,n} + 1001\,b^{4}c^{10}x^{24\,n} + 2002\,b^{5}c^{9}x^{23\,n} + 3003\,b^{6}c^{8}x^{22\,n} + 3432\,b^{7}c^{7}x^{2}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+14*n)*(b+c*x**n)**13*(b+2*c*x**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36063, size = 255, normalized size = 12.14

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^{2}c^{12}x^{26\,n} + 364\,b^{3}c^{11}x^{25\,n} + 1001\,b^{4}c^{10}x^{24\,n} + 2002\,b^{5}c^{9}x^{23\,n} + 3003\,b^{6}c^{8}x^{22\,n} + 3432\,b^{7}c^{7}x^{2}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.1055 $\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)dx$

Optimal. Leaf size=13

$$x^m(a+bx^n)^p$$

```
[Out] x^m*(a + b*x^n)^p
```

Rubi [A] time = 0.0138087, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {449}

$$x^m(a+bx^n)^p$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] x^m*(a + b*x^n)^p
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)dx=x^m(a+bx^n)^p$$

Mathematica [C] time = 0.155663, size = 107, normalized size = 8.23

$$\frac{x^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(bx^n(m+np)\,{}_2F_1\left(\frac{m+n}{n},1-p;\frac{m}{n}+2;-\frac{bx^n}{a}\right)+a(m+n)\,{}_2F_1\left(\frac{m}{n},1-p;\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{a(m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] (x^m*(a + b*x^n)^p*(a*(m + n)*Hypergeometric2F1[m/n, 1 - p, (m + n)/n, -((b
*x^n)/a)] + b*(m + n*p)*x^n*Hypergeometric2F1[(m + n)/n, 1 - p, 2 + m/n, -(
(b*x^n)/a)]))/(a*(m + n)*(1 + (b*x^n)/a)^p)
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int x^{-1+m} (a+bx^n)^{-1+p} \left(am + b\left(pn+m\right) x^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x)
```

```
[Out] int(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x)
```

Maxima [A] time = 1.15387, size = 22, normalized size = 1.69

$$e^{\left(p \log(bx^n+a)+m \log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="maxim
a")
```

```
[Out] e^(p*log(b*x^n + a) + m*log(x))
```

Fricas [B] time = 1.01659, size = 77, normalized size = 5.92

$$\left(bxx^{m-1}x^n + axx^{m-1}\right)(bx^n + a)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="frica
s")
```

```
[Out] (b*x*x^(m - 1)*x^n + a*x*x^(m - 1))*(b*x^n + a)^(p - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a+b*x**n)**(-1+p)*(a*m+b*(n*p+m)*x**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60004, size = 95, normalized size = 7.31

$$bxx^n e^{\left(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x)\right)} + axe^{\left(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="giac"
)
```

```
[Out] b*x*x^n*e^(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x)) + a*x*e^(
p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x))
```

3.1056 $\int \frac{b+2cx}{x(b+cx)} \, dx$

Optimal. Leaf size=8

$$\log(x(b+cx))$$

```
[Out] Log[x*(b + c*x)]
```

Rubi [A] time = 0.0056778, antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {72}

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(x*(b + c*x)),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{b+2cx}{x(b+cx)} \, dx &= \int \left(\frac{1}{x}+\frac{c}{b+cx}\right) dx \\ &= \log(x)+\log(b+cx) \end{aligned}$$

Mathematica [A] time = 0.0047065, size = 9, normalized size = 1.12

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(x*(b + c*x)),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

Maple [A] time = 0.003, size = 9, normalized size = 1.1

$$\ln\left(x\left(cx+b\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x/(c*x+b),x)
```

```
[Out] ln(x*(c*x+b))
```

Maxima [A] time = 0.924054, size = 12, normalized size = 1.5

$$\log\left(cx+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")
```

```
[Out] log(c*x + b) + log(x)
```

Fricas [A] time = 0.940105, size = 24, normalized size = 3.

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

Sympy [A] time = 0.27032, size = 8, normalized size = 1.

$$\log\left(bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x)
```

```
[Out] log(b*x + c*x**2)
```

Giac [A] time = 1.06982, size = 15, normalized size = 1.88

$$\log\left(|cx + b|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")
```

```
[Out] log(abs(c*x + b)) + log(abs(x))
```

3.1057 $\int \frac{b+2cx^2}{x(b+cx^2)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rubi [A] time = 0.0151964, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^2\right)\\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0068275, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$\ln(x)+\frac{\ln\left(cx^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x/(c*x^2+b),x)
```

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Maxima [A] time = 0.934556, size = 23, normalized size = 1.53

$$\frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Fricas [A] time = 0.909115, size = 39, normalized size = 2.6

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] time = 0.313993, size = 12, normalized size = 0.8

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x/(c*x**2+b),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

Giac [A] time = 1.06698, size = 24, normalized size = 1.6

$$\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

3.1058 $\int \frac{b+2cx^3}{x(b+cx^3)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rubi [A] time = 0.0144698, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^3\right)$$
$$= \log(x) + \frac{1}{3}\log\left(b+cx^3\right)$$

Mathematica [A] time = 0.0068583, size = 15, normalized size = 1.

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$\ln(x)+\frac{\ln\left(cx^3+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x/(c*x^3+b),x)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Maxima [A] time = 0.926119, size = 23, normalized size = 1.53

$$\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Fricas [A] time = 0.939187, size = 39, normalized size = 2.6

$$\frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] time = 0.324456, size = 12, normalized size = 0.8

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x/(c*x**3+b),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

Giac [A] time = 1.05797, size = 20, normalized size = 1.33

$$\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

3.1059 $\int \frac{b+2cx^n}{x(b+cx^n)} dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

[Out] Log[x] + Log[b + c*x^n]/n

Rubi [A] time = 0.0153942, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(x*(b + c*x^n)),x]

[Out] Log[x] + Log[b + c*x^n]/n

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{b+2cx^n}{x\,(b+cx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^n\right)}{n}$$
$$= \log(x) + \frac{\log\left(b+cx^n\right)}{n}$$

Mathematica [A] time = 0.0137174, size = 15, normalized size = 1.

$$\frac{\log\left(b+cx^n\right)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Maple [A] time = 0.001, size = 17, normalized size = 1.1

$$\frac{\ln\left(x^n\left(b+cx^n\right)\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/x/(b+c*x^n),x)
```

```
[Out] 1/n*ln(x^n*(b+c*x^n))
```

Maxima [B] time = 0.931054, size = 63, normalized size = 4.2

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A] time = 1.09325, size = 42, normalized size = 2.8

$$\frac{n\log\left(x\right) + \log\left(cx^{n} + b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(c*x^n + b))/n
```

Sympy [A] time = 0.576391, size = 29, normalized size = 1.93

$$\begin{cases} \log\left(x\right) & \text{for}\: c = 0 \wedge \left(c = 0 \vee n = 0\right) \\ \frac{\left(b + 2c\right)\log\left(x\right)}{b + c} & \text{for}\: n = 0 \\ \log\left(x\right) + \frac{\log\left(\frac{b}{c} + x^{n}\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/x/(b+c*x**n),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
+ c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^{n} + b}{\left(cx^{n} + b\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/((c*x^n + b)*x), x)
```

3.1060 $\int \frac{b+2cx}{x^8(b+cx)^8} dx$

Optimal. Leaf size=14

$$-\frac{1}{7x^7(b+cx)^7}$$

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Rubi [A] time = 0.002028, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{b+2cx}{x^8(b+cx)^8} dx = -\frac{1}{7x^7(b+cx)^7}$$

Mathematica [A] time = 0.0204685, size = 14, normalized size = 1.

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Maple [B] time = 0.02, size = 177, normalized size = 12.6

$$132\,\frac{c^7}{b^{13}\,(cx+b)}+66\,\frac{c^7}{b^{12}\,(cx+b)^2}+30\,\frac{c^7}{b^{11}\,(cx+b)^3}+12\,\frac{c^7}{b^{10}\,(cx+b)^4}+4\,\frac{c^7}{b^9\,(cx+b)^5}+\frac{c^7}{b^8\,(cx+b)^6}+\frac{c^7}{7\,b^7\,(cx+b)^7}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x^8/(c*x+b)^8,x)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+1/b^8*c^7/(c*x+b)^6+1/7/b^7*c^7/(c*x+b)^7-1
/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4
/b^9*c^2/x^5+1/b^8*c/x^6
```

Maxima [B] time = 0.971204, size = 109, normalized size = 7.79

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Fricas [B] time = 0.951381, size = 171, normalized size = 12.21

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] time = 3.77256, size = 87, normalized size = 6.21

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x**8/(c*x+b)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Giac [A] time = 1.06779, size = 18, normalized size = 1.29

$$-\frac{1}{7\left(cx^2 + bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

3.1061 $\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rubi [A] time = 0.0135677, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)$$
$$= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Mathematica [A] time = 0.0284871, size = 16, normalized size = 1.

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Maple [B] time = 0.019, size = 197, normalized size = 12.3

$$-\frac{c^8}{2\,b^{13}}\left(-66\,\frac{b}{c\left(cx^2+b\right)^2}-\frac{b^5}{c\left(cx^2+b\right)^6}-132\,\frac{1}{c\left(cx^2+b\right)}-30\,\frac{b^2}{c\left(cx^2+b\right)^3}-\frac{b^6}{7\,c\left(cx^2+b\right)^7}-12\,\frac{b^3}{c\left(cx^2+b\right)^4}-4\,\frac{b}{c\left(cx^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x)
```

```
[Out] -1/2*c^8/b^13*(-66*b/c/(c*x^2+b)^2-b^5/c/(c*x^2+b)^6-132/c/(c*x^2+b)-30*b^2
/c/(c*x^2+b)^3-1/7*b^6/c/(c*x^2+b)^7-12*b^3/c/(c*x^2+b)^4-4*b^4/c/(c*x^2+b)
^5)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

Maxima [B] time = 0.983807, size = 109, normalized size = 6.81

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [B] time = 0.953453, size = 177, normalized size = 11.06

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07392, size = 20, normalized size = 1.25

$$-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

3.1062 $\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rubi [A] time = 0.0129464, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)$$
$$= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Mathematica [A] time = 0.0372091, size = 16, normalized size = 1.

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Maple [B] time = 0.013, size = 197, normalized size = 12.3

$$-\frac{c^8}{3\,b^{13}}\left(-66\,\frac{b}{c\left(cx^3+b\right)^2}-\frac{b^5}{c\left(cx^3+b\right)^6}-132\,\frac{1}{c\left(cx^3+b\right)}-30\,\frac{b^2}{c\left(cx^3+b\right)^3}-\frac{b^6}{7\,c\left(cx^3+b\right)^7}-12\,\frac{b^3}{c\left(cx^3+b\right)^4}-4\,\frac{b}{c\left(cx^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-66*b/c/(c*x^3+b)^2-b^5/c/(c*x^3+b)^6-132/c/(c*x^3+b)-30*b^2
/c/(c*x^3+b)^3-1/7*b^6/c/(c*x^3+b)^7-12*b^3/c/(c*x^3+b)^4-4*b^4/c/(c*x^3+b)
^5)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

Maxima [B] time = 0.97367, size = 109, normalized size = 6.81

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [B] time = 0.995954, size = 177, normalized size = 11.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09889, size = 20, normalized size = 1.25

$$-\frac{1}{21\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

3.1063 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rubi [A] time = 0.0132773, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {446, 74}

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Mathematica [A] time = 0.181622, size = 21, normalized size = 1.

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] time = 0.05, size = 203, normalized size = 9.7

$$-132\,\frac{c^6}{b^{13}nx^n}+66\,\frac{c^5}{b^{12}n\,(x^n)^2}-30\,\frac{c^4}{b^{11}n\,(x^n)^3}+12\,\frac{c^3}{b^{10}n\,(x^n)^4}-4\,\frac{c^2}{b^9n\,(x^n)^5}+\frac{c}{b^8n\,(x^n)^6}-\frac{1}{7\,b^7n\,(x^n)^7}+\frac{c^7\left(924\,(x^n)^6c^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [B] time = 1.15427, size = 826, normalized size = 39.33

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}+4018014\,b}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5nx^{12\,n}+35\,b^{17}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Fricas [B] time = 1.19689, size = 236, normalized size = 11.24

$$-\frac{1}{7\left(c^7nx^{14n} + 7bc^6nx^{13n} + 21b^2c^5nx^{12n} + 35b^3c^4nx^{11n} + 35b^4c^3nx^{10n} + 21b^5c^2nx^{9n} + 7b^6cnx^{8n} + b^7nx^{7n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cx^n + b)x^{-7\,n-1}}{(cx^n + b)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)
```

3.1064 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{24}\left(x^{16}+1\right)^{3/2}-\frac{\sqrt{x^{16}+1}}{8}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -Sqrt[1 + x^16]/8 - (1 + x^16)^(3/2)/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(
4*Sqrt[2])
```

Rubi [A] time = 0.027784, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 206}

$$-\frac{1}{24}\left(x^{16}+1\right)^{3/2}-\frac{\sqrt{x^{16}+1}}{8}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] -Sqrt[1 + x^16]/8 - (1 + x^16)^(3/2)/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(
4*Sqrt[2])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx &= \frac{1}{16}\operatorname{Subst}\left(\int \frac{x\sqrt{1+x}}{1-x}\,dx,x,x^{16}\right)\\ &= -\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{1-x}\,dx,x,x^{16}\right)\\ &= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx,x,x^{16}\right)\\ &= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+x^{16}}\right)\\ &= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0182098, size = 44, normalized size = 0.85

$$\frac{1}{24}\left(3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)-\sqrt{x^{16}+1}\left(x^{16}+4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] (-(Sqrt[1 + x^16]*(4 + x^16)) + 3*Sqrt[2]*ArcTanh[Sqrt[1 + x^16]/Sqrt[2]])/
24
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \frac{x^{31}}{-x^{16}+1}\sqrt{x^{16}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^31*(x^16+1)^(1/2)/(-x^16+1),x)
```

```
[Out] int(x^31*(x^16+1)^(1/2)/(-x^16+1),x)
```

Maxima [A] time = 1.42199, size = 72, normalized size = 1.38

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(x^16 + 1))/(sqrt
(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

Fricas [A] time = 0.997523, size = 138, normalized size = 2.65

$$-\frac{1}{24}\left(x^{16}+4\right)\sqrt{x^{16}+1}+\frac{1}{16}\sqrt{2}\log\left(\frac{x^{16}+2\sqrt{2}\sqrt{x^{16}+1}+3}{x^{16}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="fricas")
```

```
[Out] -1/24*(x^16 + 4)*sqrt(x^16 + 1) + 1/16*sqrt(2)*log((x^16 + 2*sqrt(2)*sqrt(x
^16 + 1) + 3)/(x^16 - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07295, size = 76, normalized size = 1.46

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^{16}+1}\right|}{2\left(\sqrt{2}+\sqrt{x^{16}+1}\right)}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="giac")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^16
+ 1))/(sqrt(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

3.1065 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x} dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

```
[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a
] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/Sq
rt[b]
```

Rubi [A] time = 0.0863921, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 105, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a
] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/Sq
rt[b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
```

```
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\right) - d\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)$$
$$= -\left((2c)\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\right) - \frac{(2d)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}$$
$$= \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{(2d)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{b}$$
$$= \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.460271, size = 142, normalized size = 1.53

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}x\sqrt{c+\frac{d}{x}}\sqrt{bc-ad}\sqrt{\frac{b(cx+d)}{x(bc-ad)}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{bcx+bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d/x]*x*Sqrt[(b*(d + c*x))/((b*c - a*d)
*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*d + b*c*x) + (2*S
qrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a]
```

Maple [B] time = 0.043, size = 143, normalized size = 1.5

$$-x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}\left(\ln\left(\frac{1}{x}\left(adx+bcx+2\sqrt{bd}\sqrt{(cx+d)(ax+b)}+2bd\right)\right)d\sqrt{ac}-\ln\left(\frac{1}{2}\left(2xac+2\sqrt{(cx+d)(ax+b)}\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x)
```

```
[Out] -((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((c*x
+d)*(a*x+b))^(1/2)+2*b*d)/x)*d*(a*c)^(1/2)-ln(1/2*(2*x*a*c+2*((c*x+d)*(a*x+
b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*c*(b*d)^(1/2))/(a*c)^(1/2)/((c*
x+d)*(a*x+b))^(1/2)/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

Fricas [B] time = 2.10637, size = 1628, normalized size = 17.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^
2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)
- 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt(
(a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), -sqrt(-c/a)
*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
 + a*d)) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt(
(c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), sqrt(-d/b)*arctan(1/2*(2*b*d*
x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*
```

```
x^2 + b*d^2 + (b*c*d + a*d^2)*x)) + 1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*
c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x), -sqrt(-c/a)*
arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
+ a*d)) + sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt
((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + \frac{d}{x}}}{x\sqrt{a + \frac{b}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)/x/(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + \frac{d}{x}}}{\sqrt{a + \frac{b}{x}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

3.1066 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=252

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}-\frac{(ad+7bc)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{24bd^2n}+\frac{5(bc-ad)(ad+7bc)\left(a+bx^n\right)^{3/2}\sqrt{c+d}}{96bd^3n}$$

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b*d^4*
n) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b*
d^3*n) - ((7*b*c + a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b*d^2*n) + (
(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d*n) + (5*(b*c - a*d)^3*(7*b*c + a*
d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(3/2
)*d^(9/2)*n)
```

Rubi [A] time = 0.233938, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}-\frac{(ad+7bc)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{24bd^2n}+\frac{5(bc-ad)(ad+7bc)\left(a+bx^n\right)^{3/2}\sqrt{c+d}}{96bd^3n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b*d^4*
n) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b*
d^3*n) - ((7*b*c + a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b*d^2*n) + (
(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d*n) + (5*(b*c - a*d)^3*(7*b*c + a*
d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(3/2
)*d^(9/2)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} - \frac{(7bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{8bdn}$$

$$= -\frac{(7bc+ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{(5(bc-ad)(7bc+ad))\,\text{Subst}\,(}{48bd^2n}$$

$$= \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a+bx^n)^{7/}}{4b}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(}{}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(}{}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(}{}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(}{}$$

Mathematica [A] time = 0.808172, size = 223, normalized size = 0.88

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(a^2bd^2(118dx^n-191c)+15a^3d^3+ab^2d\left(265c^2-172cdx^n+136d^2x^{2n}\right)+b^3\left(70c^2dx^n-105c^3-\right.\right.}{192b^2d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*c^
3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^(7/
2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int x^{-1+2n} (a+bx^n)^{\frac{5}{2}} \frac{1}{\sqrt{c+dx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^{\frac{5}{2}} x^{2n-1}}{\sqrt{dx^n+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A] time = 1.37345, size = 1342, normalized size = 5.33

$$\left[-\frac{15\left(7b^4c^4 - 20ab^3c^3d + 18a^2b^2c^2d^2 - 4a^3bcd^3 - a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^{2n} + b^2c^2 + 6abcd + a^2d^2 - 4\left(2\sqrt{bd}bdx^n + (b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^
2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*
x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3
```

```
*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3
- 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d
^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7*b^4*c^
4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*
d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*
d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2
*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c
))/(b^2*d^5*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1067 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=199

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}-\frac{(ad+5bc)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12bd^2n}+\frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n}+$$

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b*d^3*n) - (
(5*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b*d^2*n) + ((a + b*x^n
)^(5/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(3/2)*d^(7/2)*n)
```

Rubi [A] time = 0.164262, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 80, 50, 63, 217, 206}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}-\frac{(ad+5bc)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12bd^2n}+\frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b*d^3*n) - (
(5*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b*d^2*n) + ((a + b*x^n
)^(5/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(3/2)*d^(7/2)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{(5bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{6bdn}$$

$$= -\frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} + \frac{((bc-ad)(5bc+ad))\,\text{Subst}\left(\int\right.}{8bd^2n}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+}}{3bdn}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+}}{3bdn}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+}}{3bdn}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+}}{3bdn}$$

Mathematica [A] time = 0.557486, size = 178, normalized size = 0.89

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(3a^2d^2+2abd(7dx^n-11c)+b^2\left(15c^2-10cdx^n+8d^2x^{2n}\right)\right)-3(bc-ad)^{5/2}(ad+5bc)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\,\text{sin}}{24b^2d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^
n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*b*
c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n]
)/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int x^{-1+2n}(a+bx^n)^{\frac{3}{2}}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^{\frac{3}{2}}x^{2n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A] time = 1.22346, size = 1031, normalized size = 5.18

$$\left[\frac{3\left(5b^3c^3-9ab^2c^2d+3a^2bcd^2+a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2-4\left(2\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^
n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a
*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2
*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)
```

```
)/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3
)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqr
t(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^
2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d
^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^2*d^4*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1068 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=146

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b*d^2*n) + ((a + b*x^n)
^(3/2)*Sqrt[c + d*x^n])/(2*b*d*n) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqr
t[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(3/2)*d^(5/2)*n)
```

Rubi [A] time = 0.120963, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b*d^2*n) + ((a + b*x^n)
^(3/2)*Sqrt[c + d*x^n])/(2*b*d*n) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqr
t[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(3/2)*d^(5/2)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} - \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{8bd^2n}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \frac{}{\sqrt{c-}}\right.}{4b^2d^2n}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx}{b}}\right.}{4b^2d^2n}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b}}{\sqrt{b}\sqrt{c+d}}\right.}{4b^{3/2}d^{5/2}n}$$

Mathematica [A] time = 0.432025, size = 141, normalized size = 0.97

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\,(ad-3bc+2bdx^n) + (ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c -
a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int x^{-1+2\,n}\sqrt{a+bx^n}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n + a}x^{2n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A] time = 1.16631, size = 801, normalized size = 5.49

$$\left[-\frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n} + b^2c^2 + 6\,abcd + a^2d^2 - 4\left(2\,\sqrt{bd}bdx^n + (bc + ad)\sqrt{bd}\right)\sqrt{bx^n + a}\sqrt{dx^n + c}\right.}{16\,b^2d^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(
b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(2*b
^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3
*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*sqrt(
-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^
2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n - 3*
b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n + a}x^{2n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

3.1069 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

```
[Out] (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d*n) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)*n)
```

Rubi [A] time = 0.0875472, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d*n) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}\\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2dn}\\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2dn}\\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}
\end{aligned}$$

Mathematica [A] time = 0.199474, size = 123, normalized size = 1.38

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)-\sqrt{bc-ad}(ad+bc)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*
d]])/(b^2*d^(3/2)*n*Sqrt[c + d*x^n])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int x^{-1+2n}\frac{1}{\sqrt{a+bx^n}}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Fricas [A] time = 1.08688, size = 645, normalized size = 7.25

$$\left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}bd+(bc+ad)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2-4\left(2\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\right.}{4b^2d^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (
b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^
2)*x^n))/(b^2*d^2*n), 1/2*(2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a
*d)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*s
qrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*
d^2)*x^n)))/(b^2*d^2*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{\sqrt{bx^n + a}\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

3.1070 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n}+\frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

```
[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]*n)
```

Rubi [A] time = 0.0888679, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n}+\frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```


```
[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]*n)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{bn}\\
&= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2n}\\
&= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2n}\\
&= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n}
\end{aligned}$$

Mathematica [A] time = 0.716295, size = 122, normalized size = 1.34

$$\frac{2\left(\frac{ab(c+dx^n)}{(bc-ad)\sqrt{a+bx^n}}+\frac{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}\right)}{b^2n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*((a*b*(c + d*x^n))/((b*c - a*d)*Sqrt[a + b*x^n]) + (Sqrt[b*c - a*d]*Sqrt
[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c -
a*d]])/Sqrt[d]))/(b^2*n*Sqrt[c + d*x^n])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int x^{-1+2n}(a+bx^n)^{-\frac{3}{2}}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Fricas [B] time = 1.75529, size = 879, normalized size = 9.66

$$\left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}abd+\left(\left(b^2c-abd\right)\sqrt{bd}x^n+\left(abc-a^2d\right)\sqrt{bd}\right)\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4\left(2\sqrt{bd}bdx\right.\right.}{2\left(\left(b^4cd-ab^3d^2\right)nx^n+\left(ab^3cd-a^2b^2d^2\right)n\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)*
x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)*n*x
^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d
 - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arctan(1/2
*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n
 + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^4*c*d - a
*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

3.1071 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=95

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

```
[Out] (2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*n*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*n*Sqrt[a + b*x^n])
```

Rubi [A] time = 0.0654474, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {446, 78, 37}

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*n*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*n*Sqrt[a + b*x^n])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{(3bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{3b(bc-ad)n}$$

$$= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}$$

Mathematica [A] time = 0.0496638, size = 57, normalized size = 0.6

$$\frac{2\sqrt{c+dx^n}\,(-2ac+adx^n-3bcx^n)}{3n(bc-ad)^2\,(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a +
b*x^n)^(3/2))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int x^{-1+2n}\,(a+bx^n)^{-\frac{5}{2}}\,\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Fricas [A] time = 2.1961, size = 281, normalized size = 2.96

$$-\frac{2\,(2\,ac+(3\,bc-ad)x^n)\sqrt{bx^n+a}\sqrt{dx^n+c}}{3\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)nx^{2\,n}+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)nx^n+\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*a*c + (3*b*c - a*d)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)/((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

3.1072 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=358

$$\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n}-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n}+\frac{(bc-\ldots}{}$$

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
 + d*x^n])/(128*b^2*d^5*n) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(192*b^2*d^4*n) + ((63*b^2*c^2 + 14
*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(240*b^2*d^3*n) -
(3*(3*b*c + a*d)*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(40*b^2*d^2*n) + (x^n*(
a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d*n) - ((b*c - a*d)^3*(63*b^2*c^2 +
14*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c +
 d*x^n])])/(128*b^(5/2)*d^(11/2)*n)
```

Rubi [A] time = 0.404693, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n}-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n}+\frac{(bc-\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
 + d*x^n])/(128*b^2*d^5*n) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(192*b^2*d^4*n) + ((63*b^2*c^2 + 14
*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(240*b^2*d^3*n) -
(3*(3*b*c + a*d)*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(40*b^2*d^2*n) + (x^n*(
a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d*n) - ((b*c - a*d)^3*(63*b^2*c^2 +
14*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c +
 d*x^n])])/(128*b^(5/2)*d^(11/2)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+3n}\left(a+bx^n\right)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{5bdn}$$

$$= -\frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\left(63b^2c^2+14abcd+3a^2d^2 \right. \cdots}{80 \cdots}$$

$$= \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n \cdots}{\cdots}$$

$$= -\frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(\cdots \right.}{240b^2d^3 \cdots}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a \cdots \right.}{192b^2d \cdots}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a \cdots \right.}{192b^2d \cdots}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a \cdots \right.}{192b^2d \cdots}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a \cdots \right.}{192b^2d \cdots}$$

Mathematica [A] time = 1.59239, size = 274, normalized size = 0.77

$$\frac{\sqrt{c+dx^n}\left(\frac{5(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\left(-\frac{16d^3\left(a+bx^n\right)^3}{15(ad-bc)^3}-\frac{4d^2\left(a+bx^n\right)^2}{3(bc-ad)^2}-\frac{2d\left(a+bx^n\right)}{ad-bc}-\frac{2\sqrt{d}\sqrt{a+bx^n}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{4bd^5}-\frac{24(ad+3bc)(a+bx^n)^4}{bd}+64x^n\left(a\right.\right.}{320bdn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*x
^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a +
b*x^n))/(-(b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d^3*
(a + b*x^n)^3)/(15*(-(b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(
Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)])))/(4*b*d^5)))/(320*b*d*n*Sqrt[a + b*x^n])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int x^{-1+3n}\left(a+bx^n\right)^{\frac{5}{2}}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^n+a\right)^{\frac{5}{2}}x^{3n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A] time = 1.64667, size = 1719, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b
^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(384*b
^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3
- 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n)
 + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b
^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*s
qrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^5 - 175*a
*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a
^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d
))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d +
a*b*d^2)*x^n)) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^
2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d
^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^
3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4
 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^{\frac{5}{2}}x^{3n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1073 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=291

$$\frac{(3a^2d^2+10abcd+35b^2c^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n}-\frac{(bc-ad)(3a^2d^2+10abcd+35b^2c^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n}+\frac{(bc-ad)}{}$$

```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
+ d*x^n])/(64*b^2*d^4*n) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^
n)^(3/2)*Sqrt[c + d*x^n])/(96*b^2*d^3*n) - ((7*b*c + 3*a*d)*(a + b*x^n)^(5/
2)*Sqrt[c + d*x^n])/(24*b^2*d^2*n) + (x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n]
)/(4*b*d*n) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(5/2)*d^(9/2)*n
)
```

Rubi [A] time = 0.315857, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{(3a^2d^2+10abcd+35b^2c^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n}-\frac{(bc-ad)(3a^2d^2+10abcd+35b^2c^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n}+\frac{(bc-ad)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```


```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
+ d*x^n])/(64*b^2*d^4*n) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^
n)^(3/2)*Sqrt[c + d*x^n])/(96*b^2*d^3*n) - ((7*b*c + 3*a*d)*(a + b*x^n)^(5/
2)*Sqrt[c + d*x^n])/(24*b^2*d^2*n) + (x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n]
)/(4*b*d*n) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(5/2)*d^(9/2)*n
)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn}$$

$$= -\frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right.}{48}$$

$$= \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n}{}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+}{96b^2d^3n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+}{96b^2d^3n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+}{96b^2d^3n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+}{96b^2d^3n}$$

Mathematica [A] time = 0.740804, size = 241, normalized size = 0.83

$$\frac{3(bc-ad)^{5/2}\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(3a^2bd^2(5c-2dx^n)+9a\right.}{192b^3d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*
d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 - 7
```

```
0*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)*(3
5*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSi
nh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^3*d^(9/2)*n*Sqrt[c +
d*x^n])
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int x^{-1+3n} (a+bx^n)^{\frac{3}{2}} \frac{1}{\sqrt{c+dx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^{\frac{3}{2}} x^{3n-1}}{\sqrt{dx^n+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A] time = 1.38324, size = 1335, normalized size = 4.59

$$\left[\frac{3\left(35\,b^4c^4 - 60\,ab^3c^3d + 18\,a^2b^2c^2d^2 + 4\,a^3bcd^3 + 3\,a^4d^4\right)\sqrt{bd}\log\left(8\,b^2d^2x^{2n} + b^2c^2 + 6\,abcd + a^2d^2 + 4\left(2\,\sqrt{bd}bdx^n + (b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d
^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d
*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^
3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 -
 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)
*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^4*c^4 -
 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d
)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a
)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^
3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2
 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^5*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{3n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1074 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=221

$$\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}-\frac{(3ad+5bc)\left(a+bx^n\right.}{12b^2d^2n}$$

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b^2*
d^3*n) - ((5*b*c + 3*a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b^2*d^2*n)
 + (x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(8*b^(5/2)*d^(7/2)*n)
```

Rubi [A] time = 0.245118, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}-\frac{(3ad+5bc)\left(a+bx^n\right.}{12b^2d^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```


```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b^2*
d^3*n) - ((5*b*c + 3*a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b^2*d^2*n)
 + (x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(8*b^(5/2)*d^(7/2)*n)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{3bdn}$$

$$= -\frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} + \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\text{Subst}}{8b^2d^2n}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/}}{3bd}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/}}{3bd}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/}}{3bd}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/}}{3bd}$$

Mathematica [A] time = 0.496274, size = 191, normalized size = 0.86

$$\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3a^2d^2+2abd(dx^n-2c)+b^2\left(15c^2-10cdx^n+8d^2x^{2n}\right)\right)-3(bc-ad)^{3/2}\left(a^2d^2+2abcd+5b^2c^2\right)}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*x^n)
 + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(3/2)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c + d*x^n])
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int x^{-1+3n}\sqrt{a+bx^n}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n+a}\,x^{3n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Fricas [A] time = 1.20916, size = 1019, normalized size = 4.61

$$\left[-\frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4\left(2\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx\right.}}{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n
 + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*
b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b
*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt
```

```
(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^
n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n
)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*
(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^4*n)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n + a}x^{3n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

3.1075 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=150

$$-\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b^2*d^2*n) + (x^n*Sqrt[
a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d*n) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(5/2)*d^(5/2
)*n)
```

Rubi [A] time = 0.151673, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 90, 80, 63, 217, 206}

$$-\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b^2*d^2*n) + (x^n*Sqrt[
a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d*n) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(5/2)*d^(5/2
)*n)
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
```

```
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right)\text{Subst}\left(\int \dots\right.}{8b^2d^2n}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right)\text{Subst}\left(\int \dots\right.}{4b^3d^2n}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right)\text{Subst}\left(\int \dots\right.}{4b^3d^2n}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right)\tanh^{-1}\left(\dots\right.}{4b^{5/2}d^{5/2}n}$$

Mathematica [A] time = 0.304349, size = 157, normalized size = 1.05

$$\frac{\sqrt{bc-ad}\left(3a^2d^2+2abcd+3b^2c^2\right)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\,(-3ad-3bc+2bdx^n)}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt[
b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c -
a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)*n*
Sqrt[c + d*x^n])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int x^{-1+3\,n}\frac{1}{\sqrt{a+bx^n}}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Fricas [A] time = 1.16984, size = 810, normalized size = 5.4

$$\left[\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}\right.}{16\,b^3d^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(2*
b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*
d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*
sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c
)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n
 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{\sqrt{bx^n + a}\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

3.1076 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=133

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (Sqrt[a + b*
x^n]*Sqrt[c + d*x^n])/(b^2*d*n) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*d^(3/2)*n)
```

Rubi [A] time = 0.160575, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 89, 80, 63, 217, 206}

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (Sqrt[a + b*
x^n]*Sqrt[c + d*x^n])/(b^2*d*n) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*d^(3/2)*n)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
```

```
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{b^2(bc-ad)n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2b^2dn}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a-\right.}{b^3dn}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^3dn}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}$$

Mathematica [A] time = 0.434388, size = 185, normalized size = 1.39

$$\frac{\sqrt{bc-ad}\left(-3a^2d^2+2abcd+b^2c^2\right)\sqrt{a+bx^n}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\,(c+dx^n)\left(-3a^2d+ab\,(c-dx^n)+b^2cx^n\right)}{b^3d^{3/2}n(ad-bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-(b*Sqrt[d]*(c + d*x^n)*(-3*a^2*d + b^2*c*x^n + a*b*(c - d*x^n))) + Sqrt[b
*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(b^3
*d^(3/2)*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int x^{-1+3\,n}\,(a+bx^n)^{-\frac{3}{2}}\frac{1}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Fricas [B] time = 1.87666, size = 1143, normalized size = 8.59

$$\Bigg[\frac{4\,\big(ab^2cd-3\,a^2bd^2+\big(b^3cd-ab^2d^2\big)x^n\big)\sqrt{bx^n+a}\sqrt{dx^n+c}+\big(\big(b^3c^2+2\,ab^2cd-3\,a^2bd^2\big)\sqrt{bd}x^n+\big(ab^2c^2+2\,a^2bcd-3\,\cdots}{4\,\big(\big(b^5cd^2-ab^4d^3\big)nx\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n +
 (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^5*c*d
^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2*c*d -
 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) +
 ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2 + 2*a^2
*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a
*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d
+ (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2
```

```
- a^2*b^3*d^3)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{(bx^n + a)^{\frac{3}{2}}\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

3.1077 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=147

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\left(a+bx^n\right)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*n*(a + b*x^n)^(3/2)) + (4*a*(3*
b*c - 2*a*d)*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)^2*n*Sqrt[a + b*x^n]) + (2*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*Sqrt
[d]*n)
```

Rubi [A] time = 0.142043, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {446, 89, 78, 63, 217, 206}

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\left(a+bx^n\right)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```


```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*n*(a + b*x^n)^(3/2)) + (4*a*(3*
b*c - 2*a*d)*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)^2*n*Sqrt[a + b*x^n]) + (2*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*Sqrt
[d]*n)
```


Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{3b^2(bc-ad)n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{b^2n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a\cdots}\right.}{b^3n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^3n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

Mathematica [A] time = 0.68142, size = 217, normalized size = 1.48

$$\frac{2\sqrt{c+dx^n}\left(\frac{(3b^2c^2-a^2d^2)(a+bx^n)}{d(bc-ad)^2} + \frac{a^2}{ad-bc} - \frac{3(a+bx^n)\left(\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}} - \sqrt{d}\sqrt{a+bx^n}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)\right)}{d\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{3b^2n\,(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^n)
)/(d*(b*c - a*d)^2) - (3*(a + b*x^n)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/
(b*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sq
rt[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)])))/(3
*b^2*n*(a + b*x^n)^(3/2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int x^{-1+3n} \left(a+b x^{n}\right)^{-\frac{5}{2}} \frac{1}{\sqrt{c+d x^{n}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3\,n-1}}{\left(b x^{n}+a\right)^{\frac{5}{2}} \sqrt{d x^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Fricas [B] time = 3.04903, size = 1620, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)
*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)*x^
n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^7*
c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5
*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^
3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2
```

```
)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(
-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*arctan(1/2*(2
*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n +
c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^7*c^2*d - 2*
a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a
^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

3.1078 $\int x^p (b+cx)^p (b+2cx)\,dx$

Optimal. Leaf size=20

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

```
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Rubi [A] time = 0.003549, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

```
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

Mathematica [A] time = 0.0109254, size = 20, normalized size = 1.

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

```
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Maple [A] time = 0.003, size = 21, normalized size = 1.1

$$\frac{x^{1+p}\left(cx+b\right)^{1+p}}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(c*x+b)^p*(2*c*x+b),x)
```

```
[Out] x^(1+p)*(c*x+b)^(1+p)/(1+p)
```

Maxima [A] time = 1.08433, size = 39, normalized size = 1.95

$$\frac{\left(cx^2+bx\right)e^{\left(p\log(cx+b)+p\log(x)\right)}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x)*e^(p*log(c*x + b) + p*log(x))/(p + 1)
```

Fricas [A] time = 1.01109, size = 53, normalized size = 2.65

$$\frac{\left(cx^2+bx\right)(cx+b)^p x^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)
```

Sympy [A] time = 2.89365, size = 46, normalized size = 2.3

$$\begin{cases} \frac{bxx^p(b+cx)^p}{p+1} + \frac{cx^2x^p(b+cx)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(c*x+b)**p*(2*c*x+b),x)
```

```
[Out] Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Giac [A] time = 1.57675, size = 47, normalized size = 2.35

$$\frac{(cx+b)^p cx^2 x^p + (cx+b)^p bxx^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="giac")
```

```
[Out] ((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)
```

3.1079 $\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx$

Optimal. Leaf size=27

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

```
[Out] (x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

Rubi [A] time = 0.0089422, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{x^{2(1+p)}\left(b+cx^2\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.078724, size = 97, normalized size = 3.59

$$\frac{x^{2p+2}\left(b+cx^2\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2 + 2*p)*(b + c*x^2)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2
)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [A] time = 0.004, size = 26, normalized size = 1.

$$\frac{x^{2+2p}\left(cx^2+b\right)^{1+p}}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x)
```

```
[Out] 1/2*x^(2+2*p)*(c*x^2+b)^(1+p)/(1+p)
```

Maxima [A] time = 1.09265, size = 47, normalized size = 1.74

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2p\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A] time = 1.06487, size = 72, normalized size = 2.67

$$\frac{\left(cx^3+bx\right)\left(cx^2+b\right)^p x^{2p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^2 + b)^p*x^(2*p + 1)/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10196, size = 70, normalized size = 2.59

$$\frac{\left(cx^2+b\right)^p cx^3 e^{\left(2\,p\log(x)+\log(x)\right)}+\left(cx^2+b\right)^p bxe^{\left(2\,p\log(x)+\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2 + b)^p*c*x^3*e^(2*p*log(x) + log(x)) + (c*x^2 + b)^p*b*x*e^(2*p
*log(x) + log(x)))/(p + 1)
```

3.1080 $\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx$

Optimal. Leaf size=27

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

```
[Out] (x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

Rubi [A] time = 0.0088568, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] (x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.077174, size = 97, normalized size = 3.59

$$\frac{x^{3p+3}\left(b+cx^3\right)^p\left(\frac{cx^3}{b}+1\right)^{-p}\left(2c(p+1)x^3\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] (x^(3 + 3*p)*(b + c*x^3)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3
)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Maple [A] time = 0.004, size = 26, normalized size = 1.

$$\frac{x^{3+3p}\left(cx^{3}+b\right)^{1+p}}{3+3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x)
```

```
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(1+p)/(1+p)
```

Maxima [A] time = 1.09681, size = 47, normalized size = 1.74

$$\frac{\left(cx^{6}+bx^{3}\right)e^{\left(p\log\left(cx^{3}+b\right)+3p\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A] time = 1.07523, size = 72, normalized size = 2.67

$$\frac{\left(cx^{4}+bx\right)\left(cx^{3}+b\right)^{p}x^{3p+2}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + b*x)*(c*x^3 + b)^p*x^(3*p + 2)/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10001, size = 76, normalized size = 2.81

$$\frac{\left(cx^3+b\right)^p cx^4 e^{\left(3\,p\log(x)+2\,\log(x)\right)} + \left(cx^3+b\right)^p bxe^{\left(3\,p\log(x)+2\,\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/3*((c*x^3 + b)^p*c*x^4*e^(3*p*log(x) + 2*log(x)) + (c*x^3 + b)^p*b*x*e^(3
*p*log(x) + 2*log(x)))/(p + 1)
```

3.1081 $\int x^{-1+n(1+p)}\left(b+cx^n\right)^p\left(b+2cx^n\right)\,dx$

Optimal. Leaf size=27

$$\frac{x^{n(p+1)}\left(b+cx^n\right)^{p+1}}{n(p+1)}$$

```
[Out] (x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

Rubi [A] time = 0.0124291, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{n(p+1)}\left(b+cx^n\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] (x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+n(1+p)}\left(b+cx^n\right)^p\left(b+2cx^n\right)\,dx=\frac{x^{n(1+p)}\left(b+cx^n\right)^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.12544, size = 101, normalized size = 3.74

$$\frac{\left(b+cx^n\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] ((b + c*x^n)^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3 +
 p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*(1 + (c*x^n)/b)^p)
```

Maple [F] time = 0.08, size = 0, normalized size = 0.

$$\int x^{-1+n(1+p)} \left(b+c x^{n}\right)^{p} \left(b+2 c x^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x)
```

```
[Out] int(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x)
```

Maxima [A] time = 1.16396, size = 53, normalized size = 1.96

$$\frac{\left(c x^{2 n}+b x^{n}\right) e^{\left(n p \log(x)+p \log(c x^{n}+b)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Fricas [A] time = 1.07237, size = 77, normalized size = 2.85

$$\frac{(c x x^{n}+b x)(c x^{n}+b)^{p} x^{n p+n-1}}{n p+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="fricas")
```

```
[Out] (c*x*x^n + b*x)*(c*x^n + b)^p*x^(n*p + n - 1)/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n*(1+p))*(b+c*x**n)**p*(b+2*c*x**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.09821, size = 89, normalized size = 3.3

$$\frac{(cx^n + b)^p cxx^n e^{\left(np\log(x) + n\log(x) - \log(x)\right)} + (cx^n + b)^p bxe^{\left(np\log(x) + n\log(x) - \log(x)\right)}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="giac")
```

```
[Out] ((c*x^n + b)^p*c*x*x^n*e^(n*p*log(x) + n*log(x) - log(x)) + (c*x^n + b)^p*b
*x*e^(n*p*log(x) + n*log(x) - log(x)))/(n*p + n)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```